Counting Complexity and Phase Transitions (Spring 2016)

Final Program Report

Jin-Yi Cai, Martin Dyer (Organizing Co-Chairs)

Overview

Alan Turing founded the discipline of computability theory. For historical as well as technical reasons the theory is formulated mainly in terms of decision problems. This tradition has been retained for complexity theory, where the most classical complexity classes such as P and NP were defined in terms of decision problems. However for important classes of computations, such as computing probabilities and counting discrete structures, and evaluating a host of well-studied quantities from statistical physics, a more direct perspective is to study computations that produce a numerical quantity, such as those that can be expressed a sum-of-product computation.

These are counting problems in a general sense. The focus of this Simons Institute program was the study of such counting problems, as well as related questions involving the random sampling of combinatorial objects and their typical structure. This included generalizations to weighted counterparts, such as estimating partition functions and sampling from Gibbs distributions. These have been active research topics in Theoretical Computer Science, Statistics and Statistical Physics for some time, but recent years have seen many exciting developments. However, many important problems remain open. The major goal of the program was to coordinate activities between researchers working on these problems, in all areas of exact and approximate counting, and in their applications. While this was the overarching objective of the program, the program activities could generally be grouped into several overlapping sub-topics. A (non-exhaustive) list of these sub-topics follows.

Spin systems and Statistical Physics

A considerable amount of program time was devoted to studying applications to Statistical Physics, as well as applying new ideas that were first suggested as heuristics in physics and attempting to make them mathematically rigorous in the process. Sometimes non-rigorous approaches can suggest interesting phenomena that bring new perspectives to bear on old problems. Of particular interest were problems of mixing in time and space, approximating partition functions, random sampling from Gibbs measures, phase transitions (including those found in random constraint satisfaction problems), and their relationship to computational complexity. The program continued remarkable progress in this area over the past decade.

Markov chain methods

Markov chain Monte Carlo (MCMC) methods have historically provided the most successful approach to approximate counting and estimation of partition functions, although much recent work has adopted an alternative approach based on spatial mixing. There is no reason to believe that either of these two approaches is uniformly better than the other, and indeed there may even be hope of uniting them. The important applications of the MCMC method, in Statistics and elsewhere, also provided a powerful stimulus to this line of research. The principal interest was in estimation of mixing times for Markov chains of interest, and techniques for performing estimations from them. This line of work also includes sampling algorithms that are closely related to approximate counting.

Homomorphisms and CSPs

There has been spectacular progress in recent years in establishing the computational complexity of the general task of counting the number of solutions to Constraint Satisfaction Problems (CSPs). The leading figures in these developments were all long-term participants in the program. In the exact counting case, this work has resulted in very general dichotomy theorems, which completely classify broad classes of problems into those that are tractable and those that are intractable. For approximate counting, such dichotomy theorems appear unlikely, but substantial progress on classification has been made. Significant new results in both exact and approximate counting were obtained during the program. A further interest was in the study of random CSPs and thresholds for existence of solutions. Impressive progress has been made recently in this area, and was continued during the program.

Holant problems

The study of the holant, and holographic algorithms for computing it, was initiated by Valiant. It is a far-reaching generalization of graph matchings, and can also be viewed as an edge-based variant of the (vertex-based) CSP. The gadgets used are matchgates and their variants, and these have proved useful both algorithmically and in proofs of hardness. While establishing a complete classification for the complexity of computing the general holant appears more difficult than for counting CSPs, significant results have been obtained in restricted cases. Most results have been obtained for the Boolean case with symmetric signatures, but attention is now being directed towards higher domain problems and asymmetric signatures. Notable progress in these areas was made during the program.

Review of Program Activities

By any reasonable measure, the program was a resounding success. This is reflected not only in the significant list of publications it has already produced in high-impact conferences and journals, such as the *IEEE Symposium on Foundations of Computer Science (FOCS)*, the *ACM Symposium on the Theory of Computing (STOC)*, the *ACM-SIAM Symposium on Discrete Algorithms (SODA),* the *International Colloquium on Automata, Languages and Programming (ICALP),* the *Journal of the ACM* and

*SIAM Journal on Comput*ing, but also in establishing a more collaborative research agenda that spans the whole of the counting program, and in fostering long term research connections among the program participants.

The program started with a Boot Camp, as is the usual practice for Simons Institute programs, the purpose of which was to bring various participants of different backgrounds together and to present an overview of the scope and thrust of the program. Judging by the feedback from the participants, this was quite a success. Of particular note was the uniformly high standard of the presentations by the speakers, who obviously took a good deal of effort in their preparations. The Boot Camp was designed to involve many of the younger long-term participants, and they communicated their enthusiasm as well as their knowledge. This Boot Camp set the tone for the rest of the program in a most collaborative spirit.

The semester-long program also hosted three workshops: "Approximate Counting, Markov Chains and Phase Transitions", Feb. 22 – Feb. 26, 2016; "The Classification Program of Counting Complexity", Mar. 28 – Apr. 1, 2016; and "Random Instances and Phase Transitions", May 2 – May 6, 2016. These workshops were well attended, not only by long-term program participants, but also by invited speakers and others from outside the program who self-registered. The commonality with the parallel program on "Algorithmic Challenges in Genomics" was also reflected in the attendance at workshops and seminars, though research interaction with that program took place mostly on an individual basis. The program also organized a reunion workshop at the one-year anniversary, in May 2017, to report on the progress made and to reflect on the challenges ahead. From the talks and discussions at the reunion workshop it was clear that a great deal of progress was either made directly during, or initiated at the Simons Institute by people who were brought together by the program.

The program had one weekly seminar, on Friday afternoons, organized by Andreas Galanis and Heng Guo. This had a steady stream of interesting talks, which were always well attended. In addition, three informal working groups were established within the program. The working group on phase transitions in random instances was organised by Lenka Zdeborova; a group on proving stronger results in counting complexity under stronger complexity assumptions (mainly the Exponential Time Hypotheses) was organized by Radu Curticapean and Holger Dell; and a group on relationships between counting and quantum computing was organized by John Lapinskas and Steve Homer. These groups were lively and well attended, and continued meeting throughout the program.

There were two Open Lectures, directed at the larger UC Berkeley campus community. The first lecture (by Alan Frieze of Carnegie-Mellon University) gave an overview of phase transitions in random structures, which was an important theme of the program. The second (by Leslie Valiant of Harvard University) covered holographic algorithms and holographic reductions for proving complexity results. Again, this reflected an important theme of the program. Both attracted sizable audiences, and appeared to be well received.

There was also an active social program, organized by the staff of the Institute and long-term participants Holger Dell and John Lapinskas. This involved, inter alia, afternoon teas, receptions at the Institute, group walks, outings to concerts and films, and a weekly board games evening. This helped to give an air of collegiality to the program, particularly amongst the younger participants.

Research interactions were mostly conducted at an informal level in the many collaboration spaces at the Institute, in addition to the formal seminars and workshops described earlier. Judging from the excellent research results obtained by the program participants, as documented in numerous journal and conference publications as well as in the individual research reports submitted by participants at the time of the reunion workshop, it is clear that these informal interactions were extremely productive, and resulted in some of the most striking advances of the program.

Research Outcomes

The program made significant progress in all areas of its proposed research activity. We now briefly describe several major results that were achieved by program participants. A common theme was that these advances were made possible by bringing the participants together for an extended period at the Institute. This is by no means an exhaustive list of achievements, but is intended to give some examples and highlights. Several of these papers have been accepted for presentation at flagship conferences such as the IEEE FOCS and ACM STOC.

- There was great interest in developing rigorous versions of the one-step replica symmetry breaking and belief propagation methods from Statistical Physics for random combinatorial problems. Recent work has made substantial progress in understanding the phase transitions in random constraint satisfaction problems. In particular, for several of these models, the exact satisfiability threshold has been rigorously determined, confirming predictions of statistical physics. A particular breakthrough in this area coming out of the program was paper [116] (presented at FOCS 2016) by Allan Sly, Nike Sun and Yaming Zhang on the number of solutions for random regular NAE-SAT. In this work the authors gave a complete solution throughout the condensation phase where earlier methods had failed. A substantial portion of the work was carried out during the program and benefited from insights from the statistical physicists who were also present as long-term visitors. The authors consider random regular k-NAE-SAT: knowing the satisfiability threshold, it is natural to study, in the satisfiable regime, the number of solutions of a typical instance. They prove that these solutions have a well-defined free energy (limiting exponential growth rate), with an explicit value matching the one-step replica symmetry breaking prediction from physics. The proof develops new techniques for analyzing a certain "survey propagation model" associated to this problem. It is believed that these methods may be applicable to a wide class of related problems.

- In a similar vein, as reported at the reunion workshop, a chain of collaborative research resulted in a proof of a conjectured formula for the mutual information in statistical inference problems induced by random graphs (paper [51], which appeared recently in STOC 2017) by Amin Coja-Oghlan, Florent Krzakala, Will Perkins and Lenka Zdeborova. Early in the program, Zdeborova and Krzakala felt that the Guerra interpolation method can lead to a rigorous derivation of the replica results (that they had developed previously) for a class of Bayes-optimal inference problems. Then they talked to Jiaming Xu, another program participant whom they met at the Institute, and through discussions they realized that a proof for one side of the bound can indeed be obtained. This resulted in the joint paper [100]. Then as a follow up they realized, with Nicolas Macris who was visiting for the third workshop of the program, that they could also work out the converse bounds, resulting in paper [7]. Finally, Zdeborova and Krzakala also met Perkins during his visit to the Institute and realized that their results in [100] and [7] on dense systems could can be extended to sparse systems, which finally led to paper [51]. This paper is also concerned with the cavity method, and the quest to make various predictions of it rigorous. The result established in the paper implies a well-known conjecture by Decelle et al., and allows a precise pinpointing of the condensation phase transition in random constraint satisfaction problems such as random graph coloring. The proof provides a conceptual underpinning of the replica symmetric variant of the cavity method.

- Jin-Yi Cai and his collaborators produced some comprehensive results in the classification program of the complexity of exact counting. With Zhiguo Fu, he showed that holographic algorithms with matchgates are universal for planar #CSP over the Boolean domain. This work was presented in the 2017 STOC [41], and the full paper runs to about 100 pages. They proved the following sweeping classification theorem that classifies all counting constraint satisfaction problems (#CSPs) over Boolean variables into exactly three categories: (1) Polynomial-time tractable; (2) #P-hard for general instances, but solvable in polynomial-time over planar graphs; and (3) #P-hard over planar graphs. The classification applies to all sets of local, not necessarily symmetric, constraint functions on Boolean variables that take complex values. It is shown that Valiant's holographic algorithm with matchgates is a universal strategy for all problems in category (2). This completely resolves the question, in the #CSP framework over the Boolean domain, about the power of Valiant's holographic algorithms. The remaining challenge is for the holant problem, where known classification theorems only apply to symmetric constraint functions. As a first step in achieving a holant dichotomy, Cai in collaboration with Zhiguo Fu and Mingji Xia gave a complexity classification for the six-vertex model, which is of independent interest in the physics community and has been well studied for many decades. As reported in the reunion workshop, they have extended the classification to the planar case of the six-vertex model, as well as to the eight-vertex model. These are base cases in a potential inductive proof for a general holant dichotomy. Also in preparation for the classification of holant problems, Cai, Pinyan Lu and Mingji Xia achieved a complexity dichotomy for holant^c over the Boolean domain, that had eluded them for at least five years. The breakthrough was the discovery of a new class of tractable holant^c problems, specified by local affine functions. These are reported in papers [40,42,43,45]. Also, a book by Cai and Xi Chen, "Complexity Dichotomies for Counting Problems" was finally finished during the Simons Institute program, and will be published by Cambridge University Presss in September 2017.

- Alexander Barvinok pioneered a new approach to deriving deterministic algorithms for approximating partition function polynomials via Taylor series expansions. This approach is discussed in detail in his recent book [9], which was completed during his stay at the program. Indeed, Barvinok reports on extensive discussions with Guus Regts at one of the workshops that were very helpful to him in completing his book. Soon after the semester ended, Patel and Regts made substantial progress in transforming quasi-polynomial interpolation algorithms based on Barvinok's approach into genuinely polynomial ones on bounded degree graphs, and then Peters and Regts proved Sokal's conjecture on the roots of the independence polynomial of a graph. For Barvinok, this last result was a vindication of the interpolation approach, as it allows the method to match Weitz's complexity bound for the approximation of the independence polynomial. The original method gives only pseudo-polynomial time algorithms, but further developments by Patel and Regts, and by Liu, Sinclair and Srivastava [104] (all participants in the program) have refined this to give polynomial time algorithms in certain important cases, including the ferromagnetic Ising model. In particular, the work of Liu, Sinclair and Srivastava [104] gives the first deterministic approximation algorithm (FPTAS) for the ferromagnetic Ising partition function for bounded degree graphs that works throughout the range of parameter values except at zero field. Previous algorithms have either been randomized (MCMC), or have worked only in the uniqueness regime. Their approach combines the work of Barvinok and Patel and Regts metioned above, together with the classical Lee-Yang theorem, which localizes the zeros of the Ising model partition function. They extend their results to hypergraphs, proving along the way an extension of the Lee-Yang theorem to hypergraphs, and to general ferromagnetic two-spin systems.

- Solving CNF formulas (Boolean formulas in Conjunctive Normal Form) is one of the central problems in computer science, and the Simons program led to new progress on the computational complexity transition in counting/sampling solutions to a CNF formula. Suppose each clause in the formula has k literals, and each variable appears in at most d clauses. The classical Lovasz Local Lemma, and its later algorithmic version due to Moser and Tardos, have established an easy-to-hard transition at around $d = 2^k$ for deciding/searching for a solution. However, the previously best known algorithm for counting/sampling has only a linear dependency between k and d, and hardness results were known around $d = 2^{k/2}$. During the program, Hermon, Sly and Zhang [93] (all long-term participants of the program) found a new argument to analyze the mixing time of Glauber dynamics in monotone formulas, establishing a rapid mixing bound on $d <= c\ 2^{k/2}$ which is tight up to constants. Independently, Guo, Jerrum and Liu (also long-term program participants: Guo was a Research Fellow and Liu a graduate student) found a new sampling algorithm called "partial rejection sampling", which can be applied to any constraint sampling problem, and whose running time is stochastically dominated by that of rejection sampling. Their paper [84], presented at STOC 2017, proposed a new algorithmic framework that connects the algorithmic Lovasz Local Lemma and uniform sampling. Applied to CNF formulas, this new algorithm is efficient under an almost tight dependency between d and k, though under some additional min-intersection conditions. (Interestingly, yet another concurrent and independent contribution was made by Moitra, outside the program, who gave an algorithm that works with a (not tight) exponential dependency between d and k, with no additional conditions.) Although the sharp transition threshold for sampling CNF solutions remains elusive, these exciting new developments have greatly enhanced our understanding and provided new tools to attack this important problem.

- Research Fellow Radu Curticapean produced an impressive stream of work on counting graph matchings. This produced papers on matchings with k unmatched vertices in planar graphs [58], on a new "parity separation" method for weighted matchings [59], on counting edge-injective homomorphisms and k-matchings in restricted graph classes (with Holger Dell and Marc Roth) [62], and on tight conditional lower bounds for counting perfect matchings on graphs of bounded treewidth, cliquewidth and genus (with Dániel Marx) [63]. In a breakthrough in late 2016, in joint work with Research Fellow Holger Dell and Dániel Marx, they managed to unify several disparate branches of parameterized counting complexity by introducing so-called "graph motif parameters": these are graph parameters that count small structures in graphs. Formally, fix a graph H and let #Ind(H->*) denote the function that maps input graphs to the number of induced H-copies appearing in them. A graph motif parameter is any function that can be written as a finite linear combination of such basis functions, involving a finite set of graphs H. This encompasses essentially all problems that ask to count some kind of small pattern in graphs, including those that were defined previously in [62]. They developed new algorithms for computing graph motif parameters on input graphs (in many cases these are now the fastest algorithms known, e.g., for counting k-matchings or k-paths). They also gave a full dichotomy for the complexity of evaluating graph motif parameters: if one fixes a recursively enumerable class of graph motif parameters, evaluating some f from this class is either FPT in the description length of f, or #W[1]-hard. The resulting paper [61] appeared at STOC 2017.

- Work on applying the correlation decay methods from Statistical Physics to problems of approximate counting was done by Ivona Bezáková, Andreas Galanis, Leslie Goldberg, Heng Guo, and Daniel Štefankovič, who showed that these methods can be applicable even when strong spatial mixing fails [16].

- Other notable work on Markov chain Monte Carlo was done by Antonio Blanca and Alistair Sinclair [19,20], and by Heng Guo and Mark Jerrum [83] on the random-cluster model; by Nayantara Bhatnagar and Dana Randall on simulated tempering and swapping in mean-field models [18]; and by Colin Cooper, Martin Dyer, Catherine Greenhill and Andrew Handley on the flip Markov chain for connected regular graphs [54]. This last paper introduces a two-stage canonical path method for Markov chain analysis, which appears superior to the classical Diaconis and Saloff-Coste comparison method whenever that is applicable.

- Charilaos Efthymiou, Tom Hayes, Daniel Štefankovič, Eric Vigoda and Yitong Yin produced some very interesting work on the convergence of MCMC and loopy belief propagation in the tree uniqueness region for the hard-core model. This work, which appeared in the 2016 FOCS conference [69], seeks to combine the belief propagation and MCMC methods to achieve the best of both worlds. This may well provide an important direction for future research.

- Andreas Galanis, Leslie Goldberg and Mark Jerrum proved a trichotomy theorem for the complexity of approximately counting graph homomorphisms [79].

Concluding Remarks

Overall, the program on "Counting Complexity and Phase Transitions" was a great success. It covered a variety of topics, building on the core topic areas in the proposal as well as some newer developments, such as the structure of random CSPs and the algorithmic Lovasz Local Lemma. It fostered a great collaborative research environment. Substantive collaborations from people across subareas in the field resulted in a number of notable successes as outlined above. As evidenced by publications that have already appeared within one year of the program, as well as others in the pipeline, many of which were presented during the reunion workshop, new collaborations have been fruitfully established. Many of these research collaborations are direct results of the program, without which they are unlikely to have happened. As the main organizers, we feel that the accomplishments of the program have made our experience organizing it a fond memory and a worthwhile service to the community. And of course, the leadership and the staff members at the Simons Institute made it all possible; we extend our thanks to them.

Counting Complexity and Phase Transitions, Spring 2016

[1] D. J. ALDOUS. The Incipient Giant Component in Bond Percolation on General Finite Weighted Graphs. *arXiv preprint arXiv:1604.06741v1,* 2016.

[2] D. J. ALDOUS. Weak Concentration for First Passage Percolation Times on Graphs and General Increasing Set-valued Processes. *arXiv preprint arXiv:1604.06418v1,* 2016.

[3] P. AYRE and C. GREENHILL. Frozen hypergraphs and contiguity. Preprint, 2017.

[4] J. BANKS, C. MOORE, J. NEEMAN, and P. NETRAPALLI. Information-theoretic thresholds for community detection in sparse networks. In *Proceedings of the 29th Annual Conference on Learning Theory (COLT)*, 2016.

[5] J. BANKS, C. MOORE, R. VERSHYNIN, N. VERZELEN, and J. XU. Information-theoretic bounds and phase transitions in clustering, sparse PCA, and submatrix localization. In *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, 2017.

[6] J. BARBIER, M. DIA, N. MACRIS, and F. KRZAKALA. The mutual information in random linear estimation. In *Proceedings of the 54th Allerton Conference on Communication, Control, and Computing (Allerton)*, pp. 625–632, 2016.

[7] J. BARBIER, M. DIA, N. MACRIS, F. KRZAKALA, T. LESIEUR, and L. ZDEBOROVA. Mutual information for symmetric rank-one matrix estimation: A proof of the replica formula. In *Advances in Neural Information Processing Systems (NIPS)*, pp. 424–432, 2016.

[8] A. BARVINOK. Approximating permanents and hafnians. *Discrete Analysis,* 2017.

[9] A. BARVINOK. *Combinatorics and Complexity of Partition Functions*. Springer International Publishing, 2017..

[10] A. BARVINOK. Computing the permanent of (some) complex matrices. *Foundations of Computational Mathematics,* 16, 2, pp. 329–342, 2016.

[11] A. BARVINOK and P. SOBERÓN. Computing the partition function for graph homomorphisms with multiplicities. *Journal of Combinatorial Theory, Series A,* 137, pp. 1–26, 2016.

[12] R. BASU and N. BHATNAGAR. Limit Theorems for Longest Monotone Subsequences in Random Mallows Permutations. *arXiv preprint arXiv:1601.02003,* 2016. Accepted to Annales de Institut Henri Poincare.

[13] L. BECCHETTI, A. CLEMENTI, E. NATALE, F. PASQUALE, and L. TREVISAN. Find Your Place: Simple Distributed Algorithms for Community Detection. In *Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 2016.

[14] P. BERENBRINK, A. CLEMENTI, R. ELSÄSSER, P. KLING, F. MALLMANN-TRENN, and E. NATALE. Ignore or Comply? On Breaking Symmetry in Consensus. *arXiv preprint arXiv:1702.04921,* 2017. To appear in PODC 2017.

[15] I. BEZÁKOVÁ, R. CURTICAPEAN, H. DELL, and F. V. FOMIN. Finding detours is fixed-parameter tractable. In *Proceedings of the 44th International Colloquium on Automata, Languages, and Programming (ICALP)*, 2017.

[16] I. BEZÁKOVÁ, A. GALANIS, L. A. GOLDBERG, H. GUO, and D. STEFANKOVIC. Approximation via correlation decay when strong spatial mixing fails. In *Proceedings of the 43rd International Colloquium on Automata, Languages, and Programming (ICALP)*, 2016.

[17] I. BEZÁKOVÁ and Z. LANGLEY. Contiguous Minimum Single-Source-Multi-Sink Cuts in Weighted Planar Graphs. Submitted.

[18] N. BHATNAGAR and D. RANDALL. Simulated Tempering and Swapping on Mean-Field Models. *Journal of Statistical Physics,* 164, 3, pp. 495–530, 2016.

[19] A. BLANCA and A. SINCLAIR. Dynamics for the mean-field random-cluster model. *arXiv preprint arXiv: 1412.6180,* 2014. Journal version in preparation, 2017.

[20] A. BLANCA and A. SINCLAIR. Random-cluster dynamics in Z^2. *Probability Theory and Related Fields,* pp. 1–27, 2016.

[21] M. BLÄSER, C. BRAND, and S. SINHA. A fully polynomial-time randomized approximation scheme for Kostka numbers of two-rowed diagrams. Submitted.

[22] L. BOCZKOWSKI, A. KORMAN, and E. NATALE. Self-Stabilizing Clock Synchronization with 3-bit messages. In *Proceedings of the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 2017.

[23] M. BORASSI. A Note on the Complexity of Computing the Number of Reachable Vertices in a Digraph. *Information Processing Letters,* 2016.

[24] M. BORASSI, P. CRESCENZI, and L. TREVISAN. An Axiomatic and an Average-Case Analysis of Algorithms and Heuristics for Metric Properties of Graphs. *arXiv preprint arXiv:1604.01445v1,* 2016.

[25] M. BORASSI and E. NATALE. KADABRA is an ADaptive Algorithm for Betweenness via Random Approximation. *arXiv preprint arXiv:1604.08553v3,* 2016.

[26] C. BORDENAVE, P. CAPUTO, D. CHAFAÏ, and D. PIRAS. Spectrum of large random Markov chains: heavy-tailed weights on the oriented complete graph. *arXiv preprint arXiv:1610.01836,* 2016. To appear in Random Matrices: Theory and Applications.

[27] C. BRAND, H. DELL, and M. ROTH. Fine-grained dichotomies for the Tutte plane and Boolean #CSP. In *Proceedings of the 11th International Symposium on Parameterized and Exact Computation (IPEC)*, 2016.

[28] R. C. BREWSTER, F. FOUCAUD, P. HELL, and R. NASERASR. The complexity of signed graph and 2-edge-coloured graph homomorphisms. *Discrete Mathematics,* 2016.

[29] R. BRICEÑO. Combinatorial aspects of spatial mixing and new conditions for pressure representation. PhD Thesis, UBC Mathematics Department, 2016.

[30] R. BRICEÑO, A. A. BULATOV, and V. DALMAU. Dismantlable relational structures and Gibbs measures [tentative title]. In preparation.

[31] R. BRICEÑO, B. MARCUS, and R. PAVLOV. Pointwise (topological) strong spatial mixing [tentative title]. In preparation.

[32] A. A. BULATOV. A dichotomy theorem for nonuniform CSPs. *arXiv preprint arXiv:1703.03021v2,* 2017.

[33] A. A. BULATOV. Constraint Satisfaction Problems over semilattice block Mal'tsev algebras. *arXiv preprint arXiv:1701.02623v2,* 2017. To appear in Proceedings of the 32nd Annual ACM/IEEE Symposium on Logic in Computer Science (LICS), 2017.

[34] A. A. BULATOV. Graphs of finite algebras, edges, and connectivity. *arXiv preprint arXiv:1601.07403,* 2016.

[35] A. A. BULATOV. Graphs of relational structures: restricted colours. In *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, 2016.

[36] A. A. BULATOV, R. BRICEÑO, and V. DALMAU. Finite duality and Gibbs measures supported on relational structures. In preparation.

[37] A. A. BULATOV, L. A. GOLDBERG, M. JERRUM, D. RICHERBY, and S. ŽIVNÝ. Functional clones and expressibility of partition functions. *Theoretical Computer Science,* 2017.

[38] P. BÜRGISSER, C. IKENMEYER, and G. PANOVA. No occurrence obstructions in geometric complexity theory. In *Proceedings of the 57th Annual Symposium on Foundations of Computer Science (FOCS)*, pp. 386–395, 2016.

[39] P. BÜRGISSER and A. LERARIO. Probabilistic Schubert Calculus. *arXiv preprint arXiv:1612.06893v2,* 2017.

[40] J.-Y. CAI and Z. FU. Complexity Classification of the Eight-Vertex Model. *arXiv preprint arXiv:1702.07938v2,* 2017.

[41] J.-Y. CAI and Z. FU. Holographic Algorithm with Matchgates Is Universal for Planar $CSP Over Boolean Domain. In *Proceedings of the 49th Annual ACM Symposium on the Theory of Computing (STOC)*, 2017.

[42] J.-Y. CAI, Z. FU, and S. SHAO. A Complexity Trichotomy for the Six-Vertex Model. *arXiv preprint arXiv:1704.01657v2,* 2017.

[43] J.-Y. CAI, Z. FU, and M. XIA. Complexity Classification Of The Six-Vertex Model. *arXiv preprint arXiv:1702.02863v1,* 2017.

[44] J.-Y. CAI, H. GUO, and T. WILLIAMS. Clifford Gates in the Holant Framework. *arXiv preprint arXiv:1705.00942v1,* 2017.

[45] J.-Y. CAI, P. LU, and M. XIA. Complexity dichotomy of Holant^c problems over the Boolean domain. *arXiv preprint arXiv:1702.02693,* 2017.

[46] P. CAPUTO, F. MARTINELLI, and F. L. TONINELLI. Entropic repulsion in |∇ ϕ|^p surfaces: a large deviation bound for all p≥1. *arXiv preprint arXiv:1701.03327v1,* 2017.

[47] P. CAPUTO and A. SINCLAIR. Entropy Production in Non-Linear Recombination Models. To appear in Bernoulli, 2017.

[48] X. CHEN, Y. CHENG, and B. TANG. On the Recursive Teaching Dimension of VC Classes. In *Proceedings of the 30th Annual Conference on Neural Information Processing Systems (NIPS)*, 2016.

[49] X. CHEN, R. SERVEDIO, L.-Y. TAN, and E. WAINGARTEN. Adaptivity is Exponentially Powerful for Testing Monotonicity of Halfspaces. Manuscript, 2016.

[50] X. CHEN, E. WAINGARTEN, and J. XIE. Beyond Talagrand Functions: New Lower Bounds for Testing Monotonicity and Unateness. In *Proceedings of the 49th Annual ACM Symposium on the Theory of Computing (STOC)*, 2017.

[51] A. COJA-OGHLAN, F. KRZAKALA, W. PERKINS, and L. ZDEBOROVA. Information-theoretic thresholds

from the cavity method. In *Proceedings of the 49th Annual ACM SIGACT Symposium on Theory of Computing (STOC)*, 2017.
[52] C. COOPER, M. DYER, and A. FRIEZE. Voting processes with rewiring. In preparation.
[53] C. COOPER, M. DYER, A. FRIEZE, and N. RIVERA. Discordant voting processes on finite graphs. In *Proceedings of the 43rd International Colloquium on Automata, Languages, and Programming (ICALP)*, 2016.
[54] C. COOPER, M. DYER, C. GREENHILL, and A. HANDLEY. The flip Markov chain for connected regular graphs. *arXiv preprint arXiv:1701.03856v1,* 2017.
[55] C. COOPER, A. FRIEZE, and T. RADZIK. Unvisited-edge and unvisited-vertex biassed random walks. In preparation.
[56] C. COOPER, T. RADZIK, N. RIVERA, and T. SHIRAGA. Fast plurality consensus in regular expanders. *arXiv preprint arXiv:1605.08403v1,* 2016. Submitted to ESA 2017.
[57] M. C. COOPER and S. ŽIVNÝ. The Power of Arc Consistency for CSPs Defined by Partially-Ordered Forbidden Patterns. In *Proceedings of the Thirty-First Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, 2016.
[58] R. CURTICAPEAN. Counting matchings with k unmatched vertices in planar graphs. In *Proceedings of the 24th Annual European Symposium on Algorithms (ESA)*, 2016.
[59] R. CURTICAPEAN. Parity separation: A scientifically proven method for permanent weight loss. In *Proceedings of the 43rd International Colloquium on Automata, Languages, and Programming (ICALP)*, 2016.
[60] R. CURTICAPEAN, H. DELL, F. FOMIN, L. A. GOLDBERG, and J. LAPINSKAS. A Fixed-Parameter Perspective on #BIS. *arXiv preprint arXiv:1702.05543v1,* 2017.
[61] R. CURTICAPEAN, H. DELL, and D. MARX. Homomorphisms Are a Good Basis for Counting Small Subgraphs. In *Proceedings of the 49th Annual ACM SIGACT Symposium on Theory of Computing (STOC)*, 2017.
[62] R. CURTICAPEAN, H. DELL, and M. ROTH. Counting edge-injective homomorphisms and matchings on restricted graph classes. In *Proceedings of the 34th International Symposium on Theoretical Aspects of Computer Science (STACS)*, 2017.
[63] R. CURTICAPEAN and D. MARX. Tight conditional lower bounds for counting perfect matchings on graphs of bounded treewidth, cliquewidth, and genus. In *Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pp. 1650–1669, 2016.
[64] V. DALMAU, M. KOZIK, A. KROKHIN, K. MAKARYCHEV, and Y. MAKARYCHEV. Robust approximation algorithms for near-unanimity CSPs. In *Proceedings of the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 2017.
[65] J. DONG, S. GIGAN, F. KRZAKALA, and G. WAINRIB. Scaling up Echo-State Networks with multiple light scattering. *arXiv preprint arXiv:1609.05204,* 2016. Submited to NIPS.
[66] M. DYER, L. A. GOLDBERG, and D. RICHERBY. Counting 4 X 4 matrix partitions of graphs. *Discrete Applied Mathematics,* 213, pp. 76–92, 2016.
[67] M. DYER, M. JERRUM, and H. MÜLLER. On the switch Markov chain for perfect matchings. *Journal of the ACM (JACM),* 64, 2, pp. 12, 2017.
[68] M. DYER and H. MÜLLER. Counting perfect matchings and the switch chain. *arXiv preprint arXiv: 1705.05790v1,* 2017.
[69] C. EFTHYMIOU, T. P. HAYES, D. STEFANKOVIC, E. VIGODA, and Y. YIN. Convergence of MCMC and Loopy BP in the Tree Uniqueness Region for the Hard-Core Model. In *Proceedings of the 57th Annual IEEE Symposium on Foundations of Computer Science (FOCS)*, 2016.
[70] A. EL ALAOUI, A. RAMDAS, F. KRZAKALA, L. ZDEBOROVA, and M. I. JORDAN. Decoding from Pooled Data: Phase Transitions of Message Passing. In *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, 2017.
[71] A. EL ALAOUI, A. RAMDAS, F. KRZAKALA, L. ZDEBOROVA, and M. I. JORDAN. Decoding from Pooled Data: Sharp Information-Theoretic Bounds. *arXiv preprint arXiv:1611.09981v1,* 2016.
[72] J. ELLIS-MONAGHAN, A. GOODALL, J. MAKOWSKY, and I. MOFFATT. Graph Polynomials: Towards a Comparative Theory (Dagstuhl Seminar 16241). In *Dagstuhl Reports*, vol. 6, 2016.
[73] W. FENG, Y. SUN, and Y. YIN. What can be Sampled Locally? In *Proceedings of the 36th ACM Symposium on Principles of Distributed Computing (PODC)*, 2017.
[74] L. FLORESCU. Belief propagation analysis for random hypergraph q-coloring. In preparation.
[75] P. FRAIGNIAUD and E. NATALE. Noisy Rumor Spreading and Plurality Consensus. In *Proceedings of the ACM Symposium on Principles of Distributed Computing (PODC)*, 2016.

[76] A. FRIEZE. Online purchasing under uncertainty. In preparation.
[77] A. FRIEZE and T. JOHANSSON. On the insertion time of random walk cuckoo hashing. *arXiv preprint arXiv: 1602.04652,* 2016. Submitted to FOCS.
[78] P. FULLA and S. ŽIVNÝ. On Planar Valued CSPs. In *Proceedings of the 41st International Symposium on Mathematical Foundations of Computer Science (MFCS)*, 2016.
[79] A. GALANIS, L. A. GOLDBERG, and M. JERRUM. A Complexity Trichotomy for Approximately Counting List H-Colorings. *ACM Transactions on Computation Theory (TOCT),* 9, 2, 2017.
[80] A. GALANIS, L. A. GOLDBERG, and D. STEFANKOVIC. Inapproximability of the independent set polynomial below the Shearer threshold. In *Proceedings of the 44th International Colloquium on Automata, Languages, and Programming (ICALP)*, 2017.
[81] S. GANGULY and F. MARTINELLI. Upper triangular matrix walk: Cutoff for finitely many columns. *arXiv preprint arXiv:1612.08741v1,* 2016.
[82] L. GUALÀ, S. LEUCCI, E. NATALE, and R. TAURASO. Large Peg-Army Maneuvers. In *Proceedings of the Eighth International Conference on Fun with Algorithms (FUN)*, 2016.
[83] H. GUO and M. JERRUM. Random cluster dynamics for the Ising model is rapidly mixing. In *Proceedings of the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 2017.
[84] H. GUO, M. JERRUM, and J. LIU. Uniform Sampling through the Lovász Local Lemma. In *Proceedings of the 49th Annual ACM Symposium on the Theory of Computing (STOC)*, 2017.
[85] B. HAJEK, Y. WU, and J. XU. Achieving the exact cluster recovery threshold via semidefinite programming. In *Proceedings of the IEEE Transactions on Information Theory*, vol. 62, pp. 2788–2797, 2016.
[86] B. HAJEK, Y. WU, and J. XU. Information limits for recovering a hidden community. In *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, 2016.
[87] B. HAJEK, Y. WU, and J. XU. Semidefinite Programs for Exact Recovery of a Hidden Community. In *Proceedings of the 29th Annual Conference on Learning Theory (COLT)*, 2016.
[88] N. J. A. HARVEY, P. SRIVASTAVA, and J. VONDRÁK. Computing the independence polynomial in Shearer's region for the LLL. *arXiv preprint arXiv:1608.02282v1,* 2016.
[89] P. HELL and C. HERNÁNDEZ-CRUZ. Minimal digraph obstructions for small matrices. *arXiv preprint arXiv: 1605.09587v1,* 2016.
[90] P. HELL and M. M. NEVISI. Minimum Cost Homomorphisms with Constrained Costs. In *Proceedings of the 22nd International Computing and Combinatorics Conference (COCOON'16)*, pp. 194–206, 2016.
[91] P. HELL and A. RAFIEY. Bi-Arc Digraphs and Conservative Polymorphisms. *arXiv preprint arXiv: 1608.03368v3,* 2016.
[92] M. HERMANN. Complexity of Counting Problems. Book in preparation.
[93] J. HERMON, A. SLY, and Y. ZHANG. Rapid Mixing of Hypergraph Independent Set. *arXiv preprint arXiv: 1610.07999v1,* 2016.
[94] S. HOMER, J. APPAVOO, A. WATERLAND, S. ELDRIDGE, A. JOSHI, and M. SELTZER. Programmable Smart Machines. Preprint.
[95] R. KANNAN and S. VEMPALA. Beyond Spectral: Matching Bounds for the Planted Gaussian Problem. To appear in Proceedings of COLT, 2017.
[96] M. KARPINSKI. Approximate Complexity of Counting Matchings in Diagonal Hypergraphs. In preparation.
[97] M. KARPINSKI. Approximate Counting of Matchings in Restricted Classes of Hypergraphs. In preparation.
[98] T. KOTEK and J. MAKOWSKY. Efficient computation of generalized Ising polynomials on graphs with fixed clique-width. In *International Conference on Topics in Theoretical Computer Science*, pp. 135–146, 2015. Submitted to Graphs and Combinatorics 2016, in revision.
[99] T. KOTEK, J. MAKOWSKY, and E. RAVVE. On sequences of polynomials arising from graph invariants. *arXiv preprint arXiv:1701.08564,* 2017. Submitted to European Journal of Combinatorics.
[100] F. KRZAKALA, J. XU, and L. ZDEBOROVÁ. Mutual information in rank-one matrix estimation. In *Information Theory Workshop (ITW)*, pp. 71–75, 2016.
[101] N. LABAI and J. MAKOWSKY. Hankel Matrices for Weighted Visibly Pushdown Automata. In *International Conference on Language and Automata Theory and Applications (LATA)*, pp. 464–477, 2016.
[102] N. LABAI and J. MAKOWSKY. On the exact learnability of graph parameters: The case of partition functions. In *Proceedings of the 41st International Symposium on Mathematical Foundations of Computer Science (MFCS)*, 2016.
[103] T. LESIEUR, C. DE BACCO, J. BANKS, F. KRZAKALA, C. MOORE, and L. ZDEBOROVÁ. Phase

transitions and optimal algorithms in high-dimensional Gaussian mixture clustering. In *Proceedings of the 54th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pp. 601–608, 2016.

[104] J. LIU, A. SINCLAIR, and P. SRIVASTAVA. The Ising Partition Function: Zeros and Deterministic Approximation. *arXiv preprint arXiv:1704.06493v1,* 2017. To appear in Proceedings of the 58th Annual IEEE Symposium on Foundations of Computer Science (FOCS), 2017.

[105] M. LIU, X. PAN, and Y. YIN. Randomized approximate nearest neighbor search with limited adaptivity. In *Proceedings of the 28th ACM Symposium on Parallelism in Algorithms and Architectures (SPAA)*, 2016.

[106] P. LU, K. YANG, C. ZHANG, and M. ZHU. An FPTAS for Counting Proper Four-Colorings on Cubic Graphs. In *Proceedings of the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pp. 1798–1817, 2017.

[107] J. MAKOWSKY, A. GOODALL, M. HERMANN, T. KOTEK, and S. NOBLE. On the complexity of generalized chromatic polynomials. Preprint. Submitted to Annals of Applied Probability (Special issue on Progress in Tutte polynomials).

[108] J. MAKOWSKY and T. KOTEK. The complexity of the Tutte polynomial. Preprint. To appear in the CRC Handbook on the Tutte polynomial.

[109] J. MAKOWSKY and E. RAVVE. Semantic Equivalence of Graph Polynomials Definable in Second Order Logic. In *Proceedings of the 23rd International Workshop on Logic, Language, Information, and Computation (WoLLIC)*, pp. 279–296, 2016.

[110] J. MAKOWSKY, E. RAVVE, and T. KOTEK. A logician's view of graph polynomials. *arXiv preprint arXiv: 1703.02297v1,* 2017. Submitted to the Annals of Pure and Applied Logic.

[111] F. MARTINELLI and C. TONINELLI. Towards a universality picture for the relaxation to equilibrium of kinetically constrained models. *arXiv preprint arXiv:1701.00107v1,* 2016.

[112] E. MOSSEL and J. XU. Density evolution in the degree-correlated stochastic block model. In *Proceedings of the 29th Annual Conference on Learning Theory (COLT)*, 2016.

[113] A. SAADE, F. KRZAKALA, M. LELARGE, and L. ZDEBOROVÁ. Fast Randomized Semi-Supervised Clustering. *arXiv preprint arXiv:1605.06422,* 2016. Submited to NIPS 2017.

[114] A. SAADE, F. KRZAKALA, and L. ZDEBOROVÁ. Spectral Bounds for the Ising Ferromagnet on an Arbitrary Given Graph. *Journal of Statistical Mechanics: Theory and Experiment,* 2017.

[115] A. SAADE, M. LELARGE, F. KRZAKALA, and L. ZDEBOROVÁ. Clustering from Sparse Pairwise Measurements. In *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, pp. 780–784, 2016.

[116] A. SLY, N. SUN, and Y. ZHANG. The number of solutions for random regular NAE-SAT. In *Proceedings of the 57th Annual Symposium on Foundations of Computer Science (FOCS)*, pp. 724–731, 2016.

[117] J. THAPPER and S. ŽIVNÝ. The power of Sherali-Adams relaxations for general-valued CSPs. *arXiv preprint arXiv:1606.02577,* 2016. To appear in SIAM Journal on Computation.

[118] Y. YIN. Simple average-case lower bounds for approximate near-neighbor from isoperimetric inequalities. In *Proceedings of the 43rd International Colloquium on Automata, Languages and Programming (ICALP)*, 2016.

[119] Y. YIN and C. ZHANG. Spatial mixing and approximate counting for Potts model on graphs with bounded average degree. In *Proceedings of the 20th International Workshop on Randomization and Computation (RANDOM)*, 2016.

[120] Y. YIN and J. ZHAO. Counting hypergraph matchings up to the uniqueness threshold. In *Proceedings of the 20th International Workshop on Randomization and Computation (RANDOM)*, 2016.

[121] C. ZHANG. FPTAS for Counting Proper Four Colorings on Cubic Graphs. In *Proceedings of the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 2017.

[122] C. ZHANG. Sampling in the Potts Model on Sparse Random Graphs. In *Proceedings of the 19th International Workshop on Approximation Algorithms for Combinatorial Optimization Problems (APPROX)*, 2016.