RAMSEY THEOREMS FOR STRUCTURES CONTAINING BOTH RELATIONS & OPERATIONS

JAROSLAV NEŠETŘIL
CHARLES UNIVERSITY
PRAGUE

SIMONS INSTITUTE
BERKELEY DEC 12, 2017

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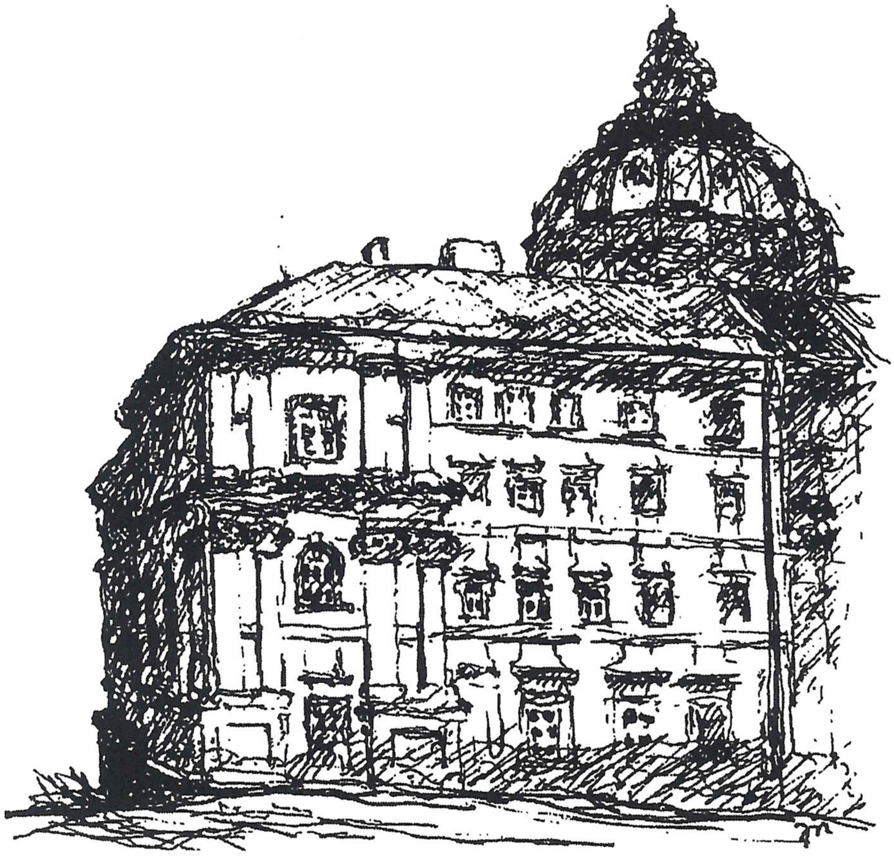

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GRAPH RAMSEY THEOREM $\boxed{\text{GRT}}$

EVERY LARGE GRAPH G ($|G| \geqslant N$)

CONTAINS:

EITHER LARGE CLIQUE OR LARGE INDEPENDENT SET

$\omega(G) \geqslant n$ $\alpha(G) \geqslant n$.

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CONTAINS:

EITHER LARGE CLIQUE OR LARGE INDEPENDENT SET

$$\omega(G) \geqslant n \qquad \alpha(G) \geqslant n.$$

$\boxed{\text{GRT}}$

FOR EVERY PARTITION $\binom{[N]}{2} = a_1 \cup \ldots \cup a_k$

THERE EXISTS $Y \subseteq [N] = \{1, 2, \ldots, N\}$, i_0

SUCH THAT $|Y| = n$

$$\binom{Y}{2} \subseteq a_{i_0}.$$

$$N \longrightarrow (n)^2_k$$

ERDÖS-RADO PARTITION ARROW

FINITE RAMSEY THEOREM $\boxed{\text{FRT}}$

$\forall p, k, n \quad \exists N:$

FOR EVERY PARTITION $\binom{[N]}{p} = a_1 \cup \ldots \cup a_k$

THERE EXISTS $Y \subseteq [N]$, i_0 SUCH THAT

$$\binom{Y}{p} \subseteq a_{i_0}.$$

$$N \longrightarrow (n)^p_k.$$

OTHER EARLY EXAMPLES

(PIDGEON HOLE $p=1$)

VAN DER WAERDEN

SCHUR

HILBERT

OTHER EARLY EXAMPLES

(PIDGEON HOLE $p=1$)

VAN DER WAERDEN

SCHUR

HILBERT

---

60 – 70ies

COMBINATORIAL CUBES — HALES-JEWETT, GRAHAM-ROTHSCHILD

(m,p,c) SETS — (RADO CONJECTURE) DEUBER, LEEB

FINITE VECTOR SPACES — (ROTA CONJECTURE) GRAHAM, LEEB, ROTHSCHILD

K_k-FREE GRAPHS — (ERDÖS, GALVIN, HAJNAL) N. RÖDL

DEFINITION OF $\boxed{\text{RAMSEY CLASS}}$

(LEEB, N., RÖDL)

$\mathcal{K}$ CLASS OF STRUCTURES + SUBOBJECTS (EMBEDDINGS)

$A, B \in \mathcal{K}$ $\quad \binom{B}{A} =$ ALL SUBOBJECTS OF B ISOMORPHIC TO A

INTERESTING HISTORY

$\binom{n}{k}$ A. ETTINGHAUSEN

MENDEL

AABAB

$\binom{X}{k}$

$\binom{B}{A}$

ANDREAS VON ETTINGHAUSEN:

DIE KOMBINATORISCHE ANALYSIS

ALS

VORBEREITUNGSLEHRE ZUM STUDIUM

DER

THEORETISCHEN HÖHEREN MATHEMATIK

1826

J. N. & HELENA NEŠETŘILOVÁ

REMARKS ON MATHEMATICS OF MENDEL

FOLIA MENDELIANA 2017(?)

Die combinatorische Analysis

als

Vorbereitungslehre zum Studium

der

theoretischen höhern Mathematik,

dargestellt

von

Andreas v. Ettingshausen,

Professor der höhern Mathematik an der k. k. Universität zu Wien.

Wien, 1826.

Druck und Verlag von J. B. Wallishausser.

DEFINITION OF $\boxed{\text{RAMSEY CLASS}}$

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$\mathcal{K}$ HAS $\boxed{\text{A-RAMSEY PROPERTY}}$ IF

FOR EVERY $B \in \mathcal{K}$, $k \in \mathbb{N}$, THERE EXISTS $C \in \mathcal{K}$ SUCH THAT

$$C \longrightarrow (B)^A_k \quad :$$

DEFINITION OF $\boxed{\text{RAMSEY CLASS}}$

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FOR EVERY PARTITION $\binom{C}{A} = a_1 \cup \ldots \cup a_k$

THERE EXISTS $B' \in \binom{C}{B}$, i_0

SUCH THAT $\binom{B'}{A} \subseteq a_{i_0}$

DEFINITION OF $\boxed{\text{RAMSEY CLASS}}$

(LEEB, N., RÖDL)

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$$C \longrightarrow (B)^A_k \quad :$$

FOR EVERY PARTITION $\binom{C}{A} = \mathcal{a}_1 \cup \dots \cup \mathcal{a}_k$
THERE EXISTS $B' \in \binom{C}{B}$, i_0
SUCH THAT $\binom{B'}{A} \subseteq \mathcal{a}_{i_0}$

$\mathcal{K}$ IS $\boxed{\text{RAMSEY}}$ IF $\mathcal{K}$ HAS A-RAMSEY FOR EVERY $A \in \mathcal{K}$.

RAMSEY CLASSES

||

TOP OF THE LINE OF RAMSEY PROPERTIES

"ONE CAN COLOUR ANYTHING (POINTS, EDGES, ...)
IN ANY NUMBER OF COLOURS AND
YET OBTAIN ANY GIVEN OBJECT
MONOCHROMATIC"

EXAMPLES

- FRT (FINITE SUBSETS, $\subseteq$)
- FRT (FINITE LINEAR ORDERINGS
  +
  MONOTONNE MAPPINGS
  INJECTIONS)
- FINITE VECTOR SPACES
- PARAMETER SETS
- FINITE TREES (VIEWED AS SEMILATTICE)

EXAMPLES

- FRT (FINITE SUBSETS, $\subseteq$)
- FRT (FINITE LINEAR ORDERINGS)
  +
  MONOTONNE MAPPINGS
  INJECTIONS
- FINITE VECTOR SPACES
- PARAMETER SETS
- FINITE TREES (VIEWED AS SEMILATTICE)

NOT GRAPHS

NOT POSETS

⋮

CHARACTERISATION OF RAMSEY CLASSES ?

IMPORTANT BEYOND COMBINATORICS

EXTREMELY AMENABLE TOPOLOGICAL GROUPS

$\wr$

RAMSEY CLASSES

(B. WEISS; E. GLASNER; KECHRIS, PESTOV, TODORCEVIC; ...)

RAMSEY - GROMOV - MILMAN PHENOMENA

CHARACTERISATION OF RAMSEY CLASSES ?

TWO OBSTACLES FOR RAMSEY CLASSES

LACK OF SYMMETRY

LACK OF RIGIDITY

$\boxed{\text{SYMMETRY}}$ RESTS ON FOLLOWING:

$\boxed{\text{THM}}$ (N. 89) (KECHRIS, PESTOV, TODORCEVIC 2005

EVERY HEREDITARY RAMSEY CLASS WITH JOINT EMBEDDING PROPERTY IS AMALGAMATION CLASS.

$\boxed{\text{SYMMETRY}}$ RESTS ON FOLLOWING:

$\boxed{\text{THM}}$ (N. 89) (KECHRIS, PESTOV, TODORCEVIC 2005)

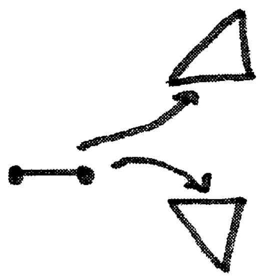
EVERY HEREDITARY RAMSEY CLASS WITH JOINT EMBEDDING PROPERTY IS AMALGAMATION CLASS.

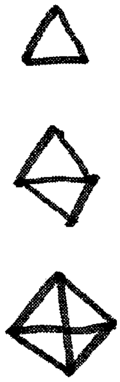
AMALGAMATION:

FOR EVERY A, B_1, B_2, $A \xrightarrow{\alpha_i} B_i$ THERE EXISTS C, $B_i \xrightarrow{\beta_i} C$, SUCH THAT $\beta_1 \circ \alpha_1 = \beta_2 \circ \alpha_2$.

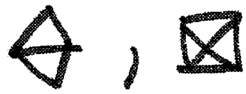
$$
\begin{tikzcd}
 & B_1 \arrow[dr, "\beta_1"] & \\
A \arrow[ur, "\alpha_1"] \arrow[dr, "\alpha_2"'] & & C \\
 & B_2 \arrow[ur, "\beta_2"'] &
\end{tikzcd}
$$

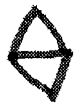
AMALGAMATION NOT UNIQUE





ALL POSSIBLE

STRONG AMALGAMATION 

FREE AMALGAMATION 

$\mathcal{K}$ IS $\boxed{\text{AMALGAMATION CLASS}}$

IF HEREDITARY, JEP AND

FOR EVERY A, B_1, B_2, $\alpha_i : A \to B_i$ IN $\mathcal{K}$

THERE EXIST C, $\beta_i : B_i \to C$ IN $\mathcal{K}$

FORMING AMALGAM.

¿ WHY SYMMETRY ?

FRAÏSSÉ THM

FOR EVERY CLASS $\mathcal{K}$ OF FINITE STRUCTURES

① $\mathcal{K}$ IS AMALGAMATION CLASS

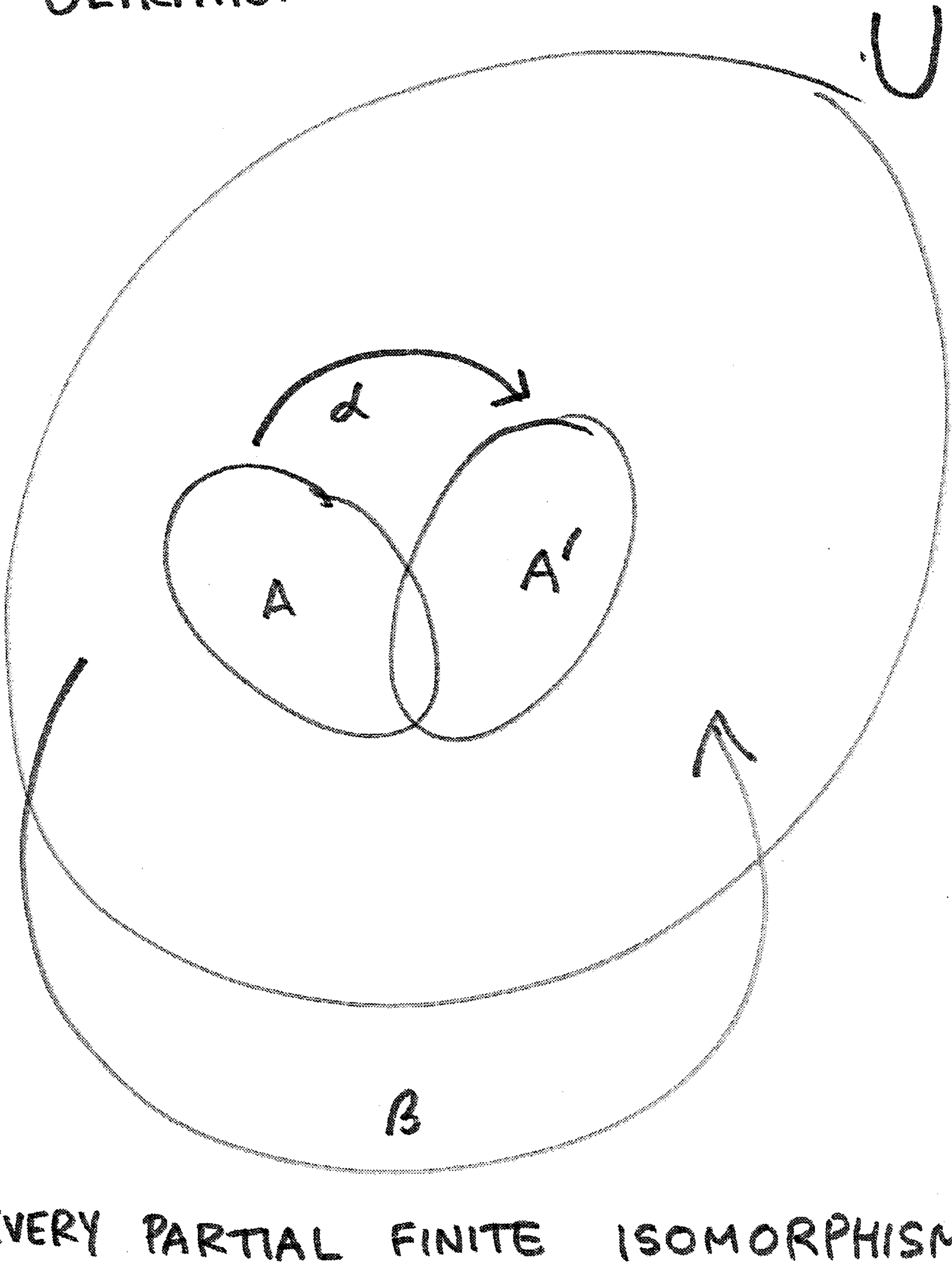
② $\mathcal{K}$ IS THE CLASS OF ALL

$\text{Age}(U)$ $\left\{ \begin{array}{l} \text{FINITE SUBSTRUCTURES OF} \\ \text{AN ULTRAHOMOGENEOUS} \\ \text{STRUCTURE } U \end{array} \right.$

U UNIQUELY DETERMINED....

FRAÏSSE LIMIT

ULTRAHOMOGENEOUS



EVERY PARTIAL FINITE ISOMORPHISM EXTENDS TO GLOBAL ONE

POINT —
LINE —
⋮

SYMMETRIC

CHARACTERISATION OF RAMSEY CLASSES $\rightsquigarrow$ CHARACTERISATION OF ULTRAHOMOGENEOUS STRUCTURES

CHARACTERISATION OF RAMSEY CLASSES $\rightsquigarrow$ CHARACTERISATION OF ULTRAHOMOGENEOUS STRUCTURES

$$
\begin{tikzcd}
\text{RAMSEY CLASSES} \arrow[r, Rightarrow] & \text{AMALGAMATION CLASSES} \arrow[d, Rightarrow] \\
\text{LIFTS OF ULTRAHOMOGENEOUS} \arrow[u, Rightarrow] & \text{ULTRAHOMOGENEOUS STRUCTURES} \arrow[l, Rightarrow]
\end{tikzcd}
$$

$$\begin{tikzcd} \text{CHARACTERISATION OF RAMSEY CLASSES} \arrow[r, rightsquigarrow] & \text{CHARACTERISATION OF ULTRAHOMOGENEOUS STRUCTURES} \end{tikzcd}$$

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RAMSEY
CLASSIFICATION
PROGRAM

STRUCTURE + ADDITIONAL INFORMATION

$\underbrace{\hspace{6em}}$

LIFT
(EXPANSION)

MOST FREQUENT LIFT IS LINEAR ORDERING

⇝

ORDERING PROPERTY

STATISTICS OF ORDERINGS

(O. ANGEL, KECHRIS, LYONS, N., RÖDL, BOLLOBAS, JANSON, ...)

$$\begin{tikzcd}
\text{RAMSEY CLASSES} \arrow[r, Rightarrow] & \text{AMALGAMATION CLASSES} \arrow[d, Rightarrow] \\
\text{LIFTS OF AGE} \arrow[u, Rightarrow] & \text{ULTRAHOMOGENEOUS STRUCTURES} \arrow[l, Rightarrow]
\end{tikzcd}$$

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\begin{tikzcd}
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\text{LIFTS OF AGE} \arrow[u, Rightarrow] & \text{ULTRAHOMOGENEOUS STRUCTURES} \arrow[l, Rightarrow] \\
\text{LOCALLY FINITE (PRECOMPACT)} \arrow[u, rightsquigarrow] &
\end{tikzcd}
$$

LOCALLY FINITE (PRECOMPACT) LIFT (FINITELY MANY RELATIONS FOR EVERY ARITY)

$\cup$

FINITE LIFT

THM (EVANS, HUBIČKA, N. 17)

THERE EXISTS AN ULTRAHOMOGENEOUS STRUCTURE $\mathbf{A}$ WITH THE FOLLOWING PROPERTIES:

1. $\mathbf{A}$ IS ω-CATEGORICAL.
2. THERE IS NO ω-CATEGORICAL LIFT (EXPANSION) WHOSE AGE IS RAMSEY CLASS.

2'. EVERY EXTREMELY AMENABLE SUBGROUP H OF AUT($\mathbf{A}$) HAS INFINITELY MANY ORBITS OF PAIRS.

3. $\mathbf{A}$ HAS OPTIMAL RAMSEY EXPANSION

3'. AUT($\mathbf{A}$) KONTAINS MAXIMAL EXTREMELY AMENABLE SUBGROUP.

ω-CATEGORICAL LIFTS DO NOT SUFFICE FOR RAMSEY CLASSES

KPT CORRESPONDENCE MORE COMPLICATED

EXAMPLE : HRUSHOVSKI SPARSE GRAPHS
FOR A

(FAMILY OF EXAMPLES)

+

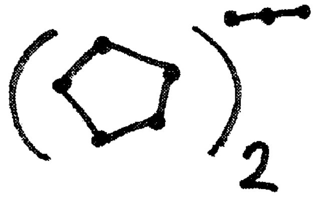
STRONG RAMSEY RESULTS FOR RELATIONS $\boxed{\text{AND}}$ OPERATIONS.

RIGIDITY

(EXPLANATION OF LIFTS)

FINITE GRAPHS FAILS TO FORM RAMSEY CLASS:

FOR EVERY GRAPH C

$C \not\longrightarrow$ 

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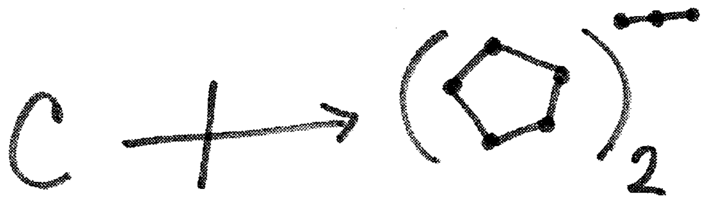
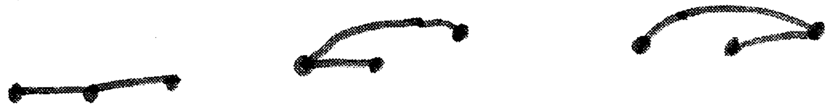


(THERE ARE 3 TYPES OF ORDERED P_3 IN EVERY COPY OF C_5)



BUT

THE CLASS OF ALL FINITE ORDERED GRAPHS + MONOTONNE EMBEDDINGS IS RAMSEY CLASS

(N., RÖDL)

$\boxed{\text{BUT}}$

THE CLASS OF ALL FINITE $\boxed{\text{ORDERED}}$ GRAPHS + $\boxed{\text{MONOTONNE}}$ EMBEDDINGS IS RAMSEY CLASS

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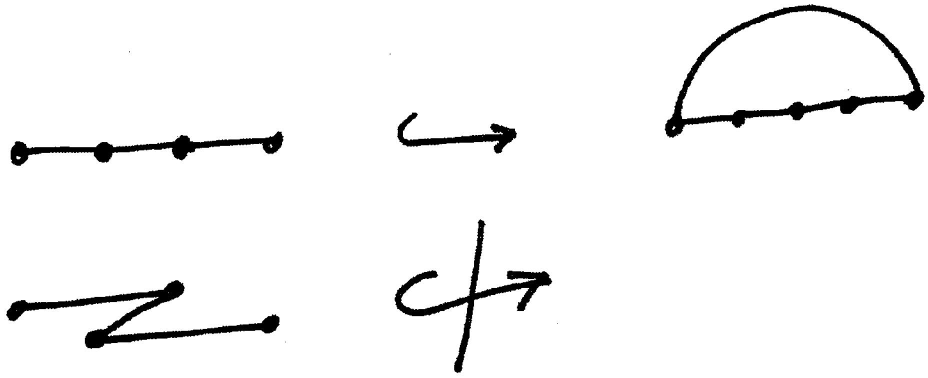
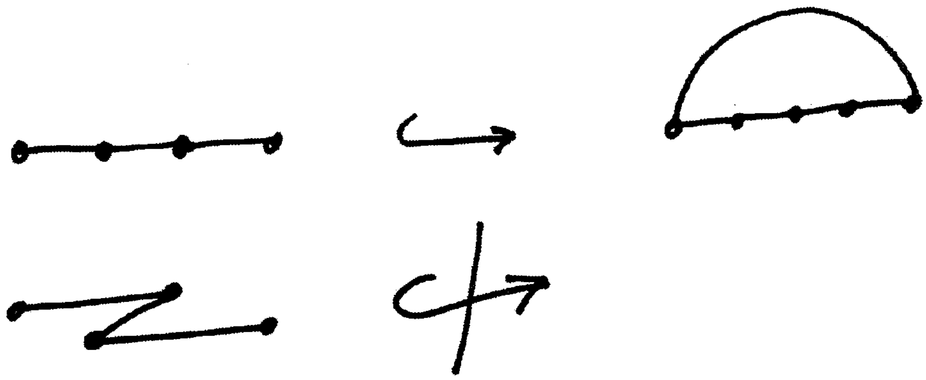


$\boxed{\text{BUT}}$

THE CLASS OF ALL FINITE $\boxed{\text{ORDERED}}$ GRAPHS + $\boxed{\text{MONOTONNE}}$ EMBEDDINGS IS RAMSEY CLASS (N., RÖDL)



$\boxed{\text{THM}}$ (ABRAMSON, HARRINGTON) N. RÖDL

THE CLASS REL(Δ) OF ALL LINEARLY ORDERED FINITE RELATIONAL STRUCTURES OF TYPE Δ (FINITE LANGUAGE) IS RAMSEY.

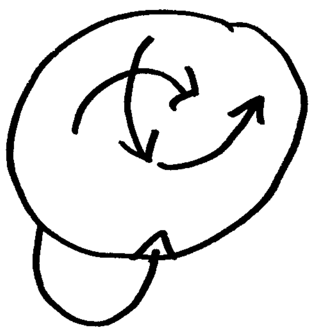
$\boxed{\text{THM}}$ (EVANS, HUBIČKA, N. 16)

LET $\mathcal{K}$ BE A CLASS OF FINITE L-STRUCTURES WHERE L CONTAINS BOTH RELATIONS AND PARTIAL FUNCTIONS. ASSUME THAT $\mathcal{K}$ IS A FREE AMALGAMATION CLASS. THEN

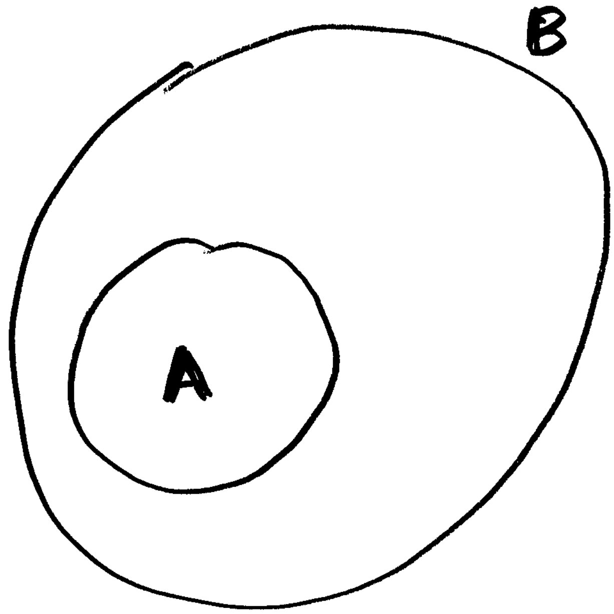
1. $\vec{\mathcal{K}}$ IS RAMSEY CLASS
2. THERE IS A CLASS OF (ADMISSIBLE) ORDERINGS $\widetilde{\mathcal{K}}$ WHICH IS RAMSEY AND HAS ORDERING PROPERTY (W.R.T. $\mathcal{K}$)
3. $\mathcal{K}$ HAS EXTENSION PROPERTY FOR PARTIAL AUTOMORPHISMS (EPPA) PROVIDING ALL FUNCTION SYMBOLS ARE UNARY

RE EPPA:

A



STRUCTURE
+
ALL PARTIAL
ISOMORPHISM



A SUBSTRUCTURE
OF B
+
EVERY PARTIAL ISO
OF A
EXTENDS TO AN
AUTOMORPHISM OF B

DOES B EXISTS ?

(HERWIG, LASKAR, HODKINSON, OTTO, ...)
HRUSHOVSKI, VERSHIK, SOLECKI

TEST FOR AMENABILITY

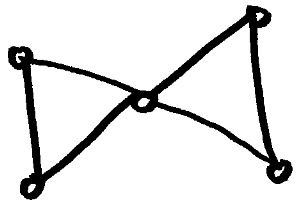
ALTERNATIVE VIEWS:

- STRUCTURES WITH ALGEBRAIC CLOSURE
  +
  CLOSURE PRESERVING EMBEDDINGS

- STRUCTURES WITH SPECIAL ("STRONG") MAPS

APPLICATIONS

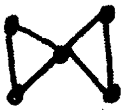
(1) BOW-TIE FREE GRAPHS

NO 

(AS NON-INDUCED SUBGRAPH)

RAMSEY + EPPA

EXAMPLE

$\mathrm{FORB}_{\mathrm{MONO}}($  $)$

"BOWTIE FREE" GRAPHS

UNIVERSAL ω-CATEGORICAL
(KOMJATH, CHERLIN, SHELAH, SHE)

THM HUBIČKA, N.

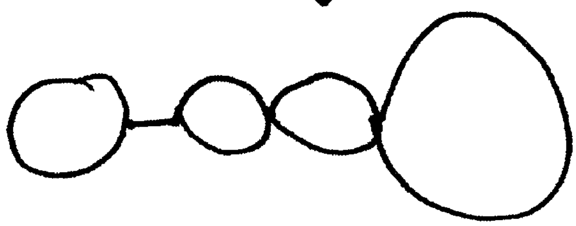
CLASS OF BOWTIE FREE GRAPHS HAS FINITE RAMSEY LIFT.

(STRUCTURE THM
+
COMPLEX LIFT
+
CONVEX ORDERING)

EXAMPLE

$\mathrm{FORB}_{\mathrm{MONO}}(\{F\})$ UNIVERSAL

$\Updownarrow$ CHERLIN CONJ.

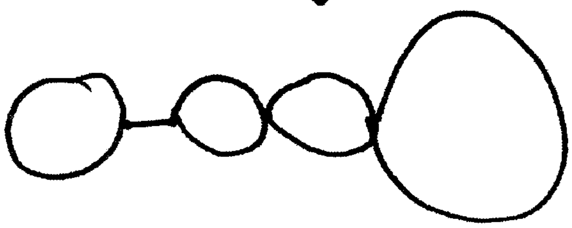


CHAIN OF COMPLETE GRAPHS

EXAMPLE

$\mathrm{FORB}_{\mathrm{MONO}}(\{F\})$ UNIVERSAL

$\Updownarrow$ CHERLIN CONJ.

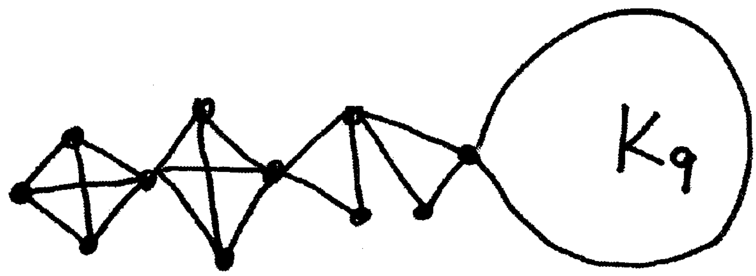


CHAIN OF COMPLETE GRAPHS

THM ALL CHERLIN CLASSES HAVE RAMSEY LIFT.

(HUBIČKA, N. 16)

FOR EXAMPLE:



APPLICATIONS

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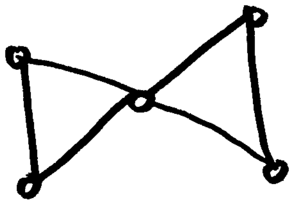
NO 

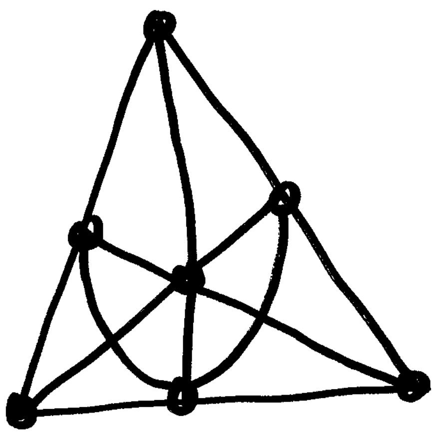
(AS NON-INDUCED SUBGRAPH)

RAMSEY + EPPA

② STEINER SYSTEMS

BLOCK DESIGNS

$f(x,y)$ = THE BLOCK CONTAINING x,y

RAMSEY 

(3) FORBIDDEN HOMOMORPHISMS FROM AN INFINITE REGULAR FAMILY $\mathcal{F}$ WHICH IS LOCALLY FINITE

RAMSEY LIFT = ORDERING + MAXIMAL

(J. HUBIČKA, N. 15)

RAMSEY + EPPA

(3) FORBIDDEN HOMOMORPHISMS FROM AN INFINITE REGULAR FAMILY $\mathcal{F}$ WHICH IS LOCALLY FINITE

RAMSEY LIFT = ORDERING
+
MAXIMAL PIECES

(J. HUBIČKA, N. 15)

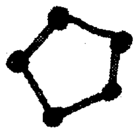
EXAMPLE

$\mathrm{FORB}_{\mathrm{HOM}}($  $)$

(3) FORBIDDEN HOMOMORPHISMS FROM AN INFINITE REGULAR FAMILY $\mathcal{F}$ WHICH IS LOCALLY FINITE

RAMSEY LIFT = ORDERING + MAXIMAL

(J. HUBIČKA, N. 15)

$\boxed{\text{EXAMPLE}}$ $\mathrm{FORB}_{\mathrm{HOM}}$ ()

PIECES  

FORB ( , )

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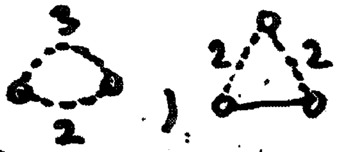
RAMSEY LIFT = ORDERING + MAXIMAL

(J. HUBIČKA, N. 15)

$\boxed{\text{EXAMPLE}}$ $\mathrm{FORB}_{\mathrm{HOM}}$ ()

PIECES 

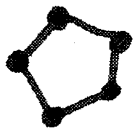
FORB ()

$\mathrm{FORB}_{\mathrm{HOM}}$ () NOT RAMSEY

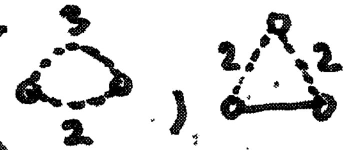
③ FORBIDDEN HOMOMORPHISMS FROM AN INFINITE REGULAR FAMILY $\mathcal{F}$ WHICH IS LOCALLY FINITE

RAMSEY LIFT = ORDERING + MAXIMAL

(J. HUBIČKA, N. 15)

$\boxed{\text{EXAMPLE}}$ $\mathrm{FORB}_{\mathrm{HOM}}$()

PIECES 

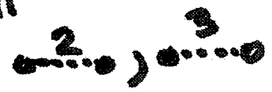
FORB ()

$\mathrm{FORB}_{\mathrm{HOM}}$() NOT RAMSEY

$\overrightarrow{\mathrm{FORB}_{\mathrm{HOM}}}$ ()

RAMSEY

$\cap$

REL(E, )

(4) METRIC SPACES

(a) FINITE METRIC SPACES

RAMSEY LIFT = LINEAR ORDER

N. 05

n

4 METRIC SPACES

a FINITE METRIC SPACES

RAMSEY LIFT = LINEAR ORDER

N. 05

b GRAPHS WITH ISOMETRIC EMBEDDINGS

RAMSEY LIFT = LINEAR ORDER

DELLAMONICA, RÖDL 12

4 METRIC SPACES

a FINITE METRIC SPACES

RAMSEY LIFT = LINEAR ORDER

N. 05

b GRAPHS WITH ISOMETRIC EMBEDDINGS

RAMSEY LIFT = LINEAR ORDER

DELLAMONICA, RÖDL 12

c S - METRIC SPACES

(SPACES WITH DISTANCES IN S)

THM FOR S THE FOLLOWING IS EQUIV.

1. FINITE CONVEXLY ORDERED S-METRIC SPACES ARE RAMSEY
2. THERE EXISTS S-URYSOHN SPACE

HUBIČKA, N. 15+ (USING SAUER 12)

MANY NEW RAMSEY CLASSES
OF METRIC SPACES DISCOVERED
BY DOCCOURSE 2016/7

(A. ARANDA, D. BRADLEY-WILLIAMS,
J. HUBIČKA, M. KARAMANLIS,
M. KONEČNÝ, M. PAWLIUK)

A MASTER THEOREM

(HUBIČKA, N. 16)

$\mathcal{R}$ RAMSEY CLASS, $\mathcal{K} \subseteq \mathcal{R}$

SATISFIES:

- $\mathcal{K}$ HEREDITARY
- $\mathcal{K}$ CLOSED ON STRONG AMALGAMATION
- ALL STRUCTURES IN $\mathcal{R}$ WHICH CANNOT BE COMPLETED IN $\mathcal{K}$ ARE LOCALLY FINITE.

THEN $\mathcal{K}$ IS RAMSEY CLASS.

A MASTER THEOREM

(HUBIČKA, N. 16)

$\mathcal{R}$ RAMSEY CLASS, $\mathcal{K} \subseteq \mathcal{R}$

SATISFIES:

- $\mathcal{K}$ HEREDITARY
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- ALL STRUBTURES IN $\mathcal{R}$ WHICH CANNOT BE COMPLETED IN $\mathcal{K}$ ARE LOCALLY FINITE.

THEN $\mathcal{K}$ IS RAMSEY CLASS.

(LOCAL FINITE: FOR SOME $\mathcal{F}$, f

$F \not\to A \Rightarrow \exists F' \in \mathcal{F} \; (|F'| \leq f(|A|)$

$F' \not\to A))$

PROOF:

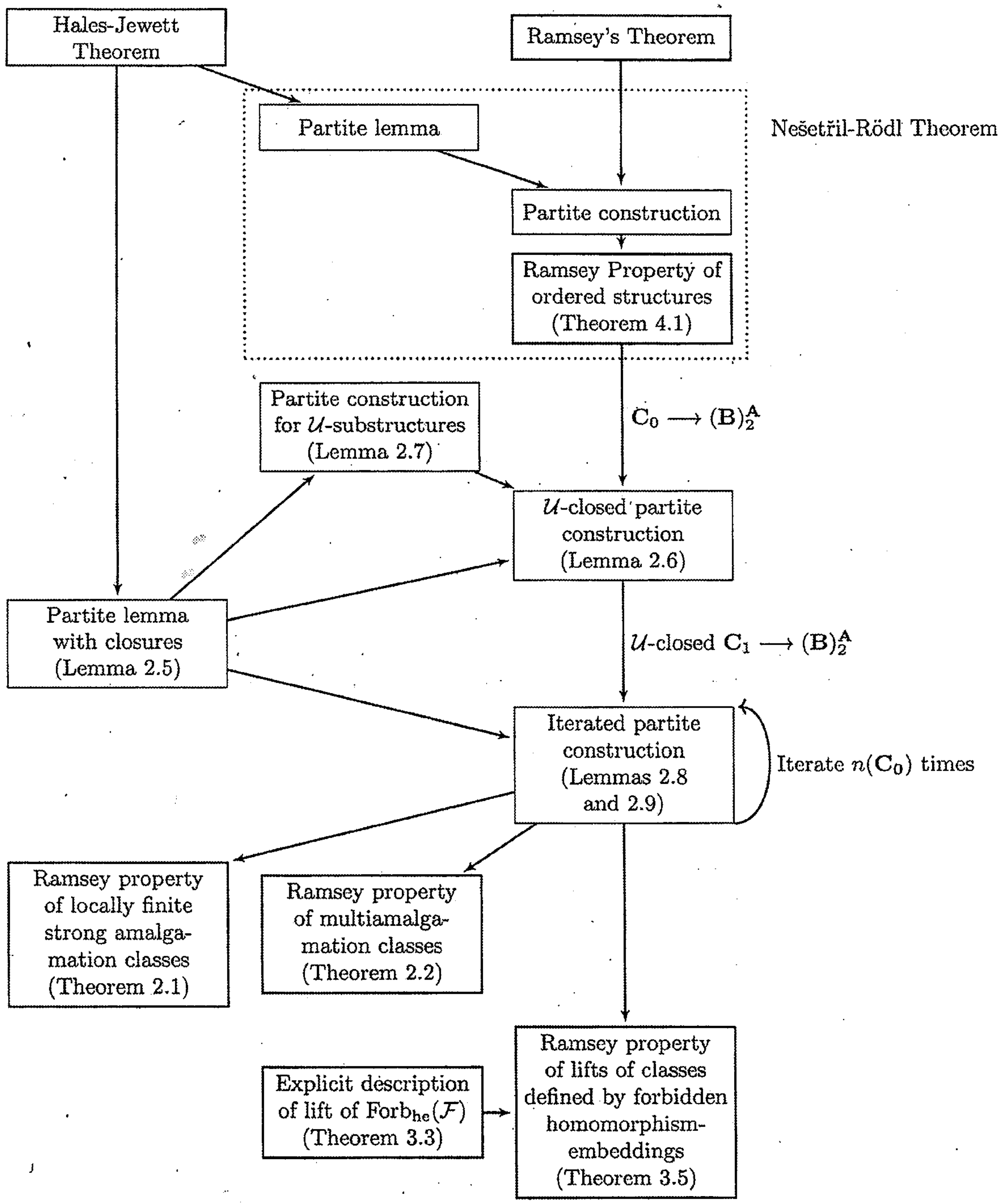

Figure 3: Structure of proofs of the main results.

MANY PROBLEMS REMAIN

? EPPA FOR STEINER (TRIPLE) SYSTEMS ?

? RAMSEY CLASSES OF MATROIDS ?

? RAMSEY CLASSES OF LATTICES ?

THANK YOU
FOR
YOUR ATTENTION