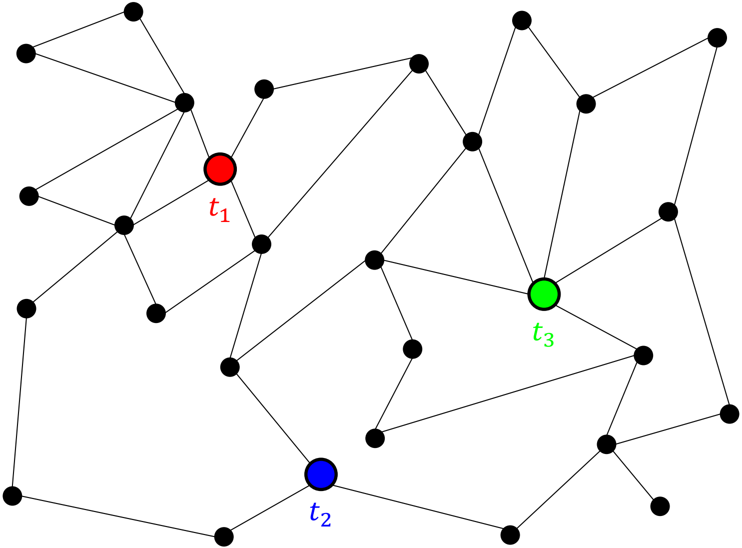
Simplex Partitioning and the Multiway Cut Problem

Roy Schwartz

Problem Definition



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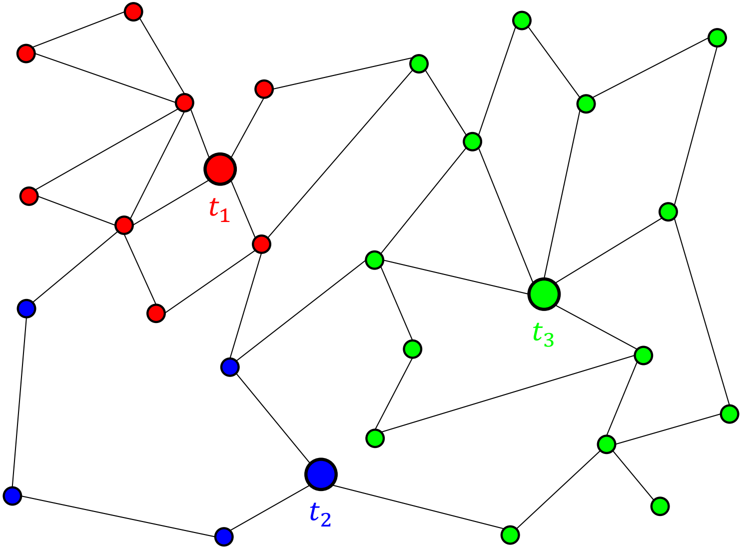

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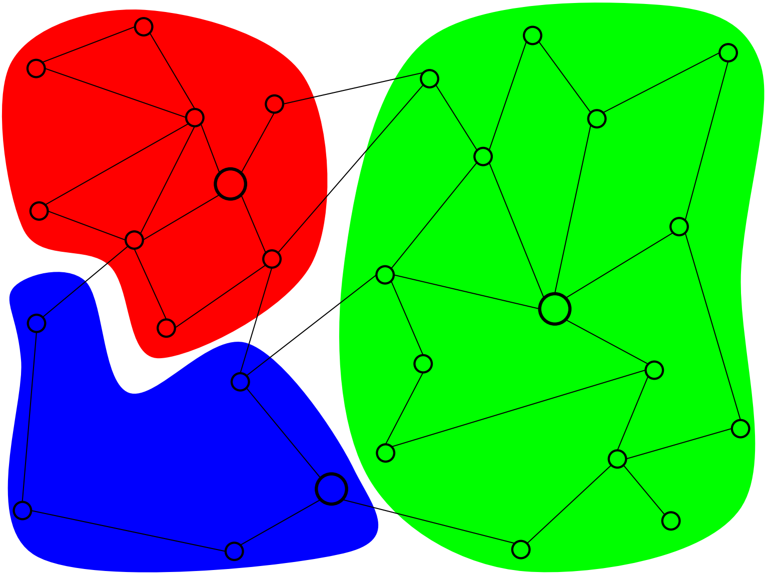

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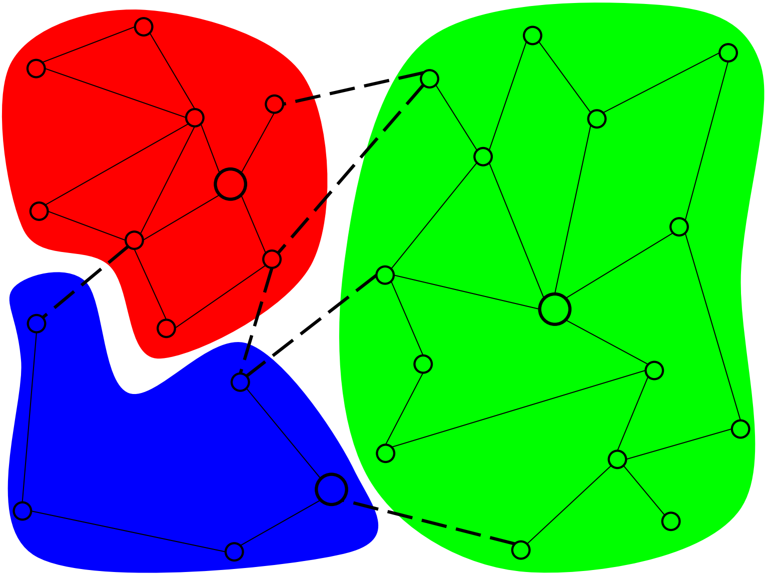

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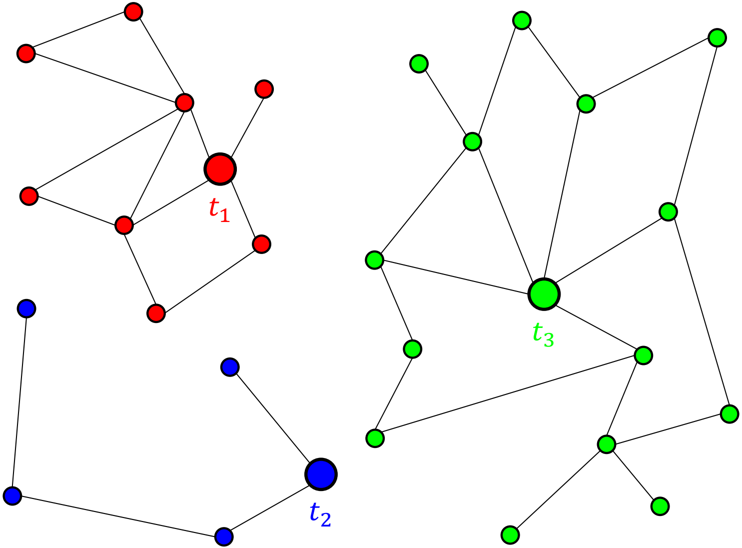

Problem Definition (cont.)

Definition

- Input: $G = (V, E)$ and $T = \{t_1, t_2, \ldots, t_k\} \subseteq V$.
- Output: $\{S_1, S_2, \ldots, S_k\}$ a partition of V minimizing:

$$\frac{1}{2} \sum_{i=1}^{k} \delta(S_i),$$

s.t. $t_i \in S_i \quad \forall i$.

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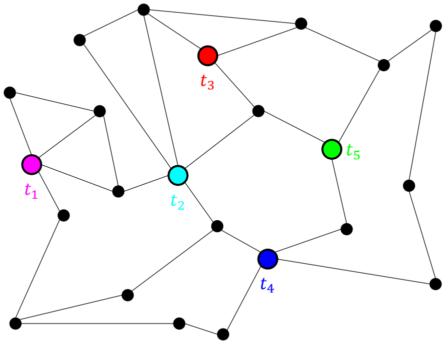
Note: For $k = 2$ problem is easy (min $s - t$ cut).

Combinatorial Approach

[Dahlhaus-Johnson-Papadimitriou-Seymour-Yannakakis-92]



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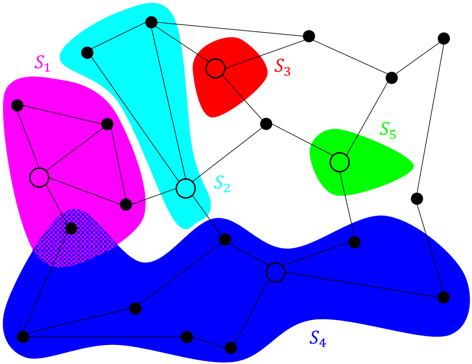

$S_i =$ min cut separating t_i

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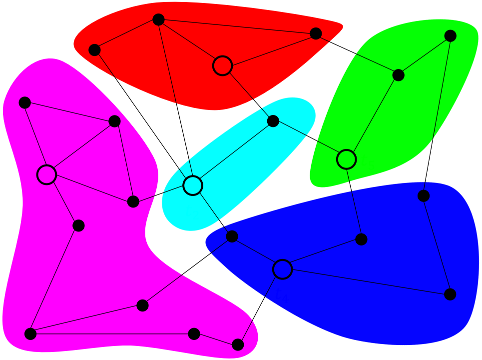

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$$\begin{cases} \delta(S_i) \leq \delta(OPT_i) \\ \sum_{i=1}^{k} \delta(OPT_i) = 2 \cdot OPT \end{cases}$$

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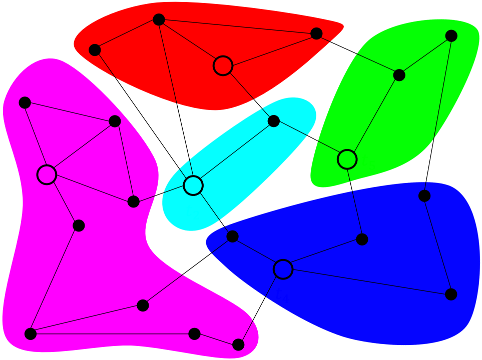

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$\Downarrow$

2-approximation

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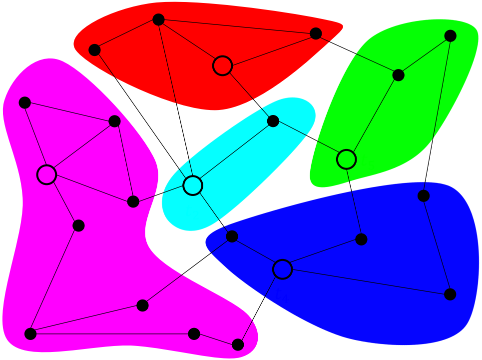

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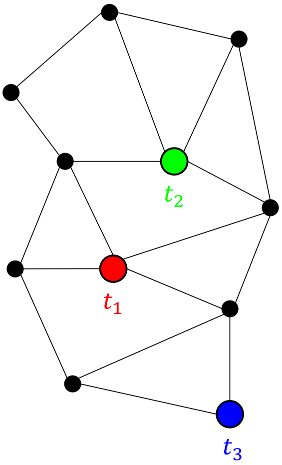
2-approximation

NP-hard and APX-complete $(k \geq 3)$

Geometric Approach

[Călinescu-Karloff-Rabani-98]



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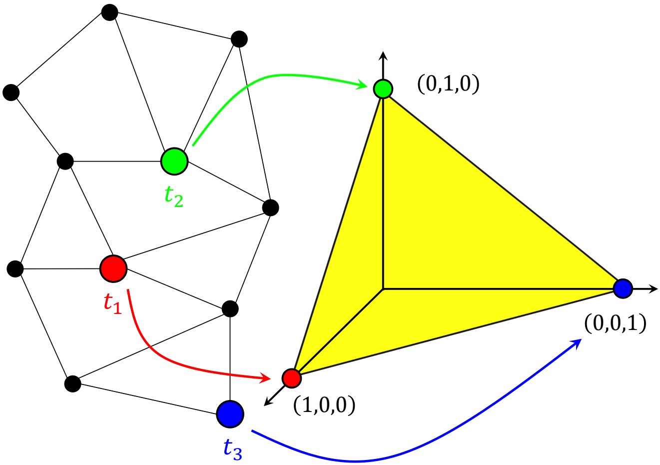

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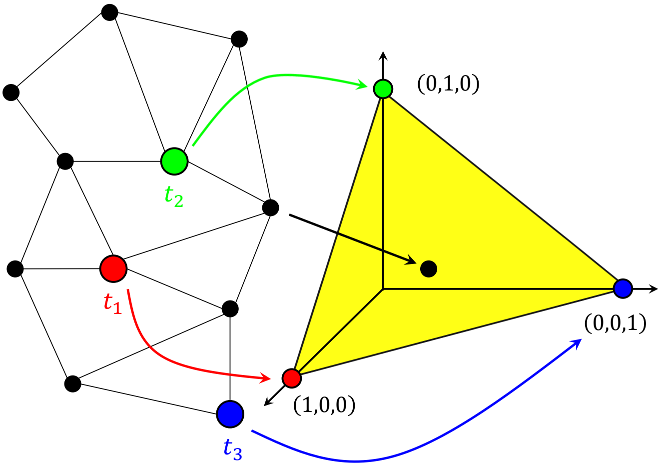

Geometric Approach (cont.)

$\Delta_k = \left\{ \mathbf{x} \in \mathbb{R}_+^k \;:\; \sum_{i=1}^k x_i = 1 \right\}$

$$\begin{aligned} \min \quad & \sum_{(u,v)\in E} \frac{1}{2} \|\mathbf{u} - \mathbf{v}\|_1 & \\ s.t. \quad & \mathbf{u} \in \Delta_k & \forall u \in V \\ & \mathbf{t}_i = \mathbf{e}_i & \forall i = 1,\ldots,k \end{aligned}$$

Geometric Approach (cont.)

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$$
\begin{array}{lll}
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\end{array}
$$

Gap Implies Hardness:

gap $\left\{ \begin{array}{l} \frac{8}{7+(k-1)^{-1}} \\ \frac{6}{5+(k-1)^{-1}} \end{array} \right.$

$\frac{8}{7+(k-1)^{-1}}$ [Freund-Karloff-00]

$\frac{6}{5+(k-1)^{-1}}$ [Angelidakis-Makarychev-Manurangsi-16]

(assuming UGC) [Manokaran-Naor-Raghavendra-S-08]

Geometric Approach (cont.)

Question: How to extract a partitioning from Δ_k?

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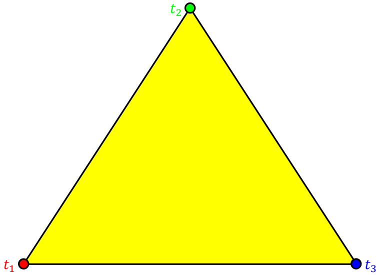

Random partition $\{S_1, \ldots, S_k\}$ of Δ_k s.t.:

1. $\mathbf{e}_i \in S_i$
2. $\Pr[u \text{ and } v \text{ separated}] \le \alpha \cdot \frac{1}{2}||\mathbf{u} - \mathbf{v}||_1$

Geometric Approach (cont.)

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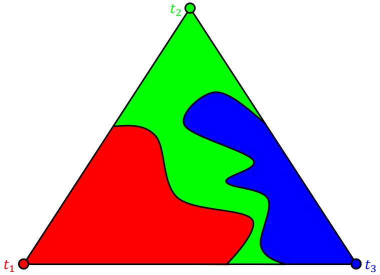

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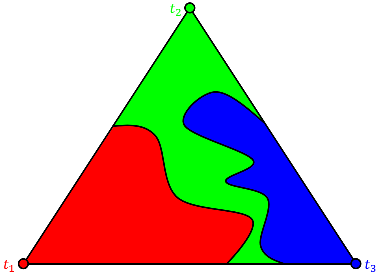

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$\Downarrow$

α-approximation for MULTIWAY-CUT

Geometric Approach (cont.)

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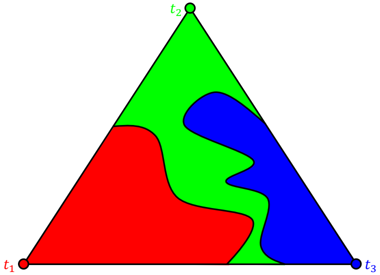

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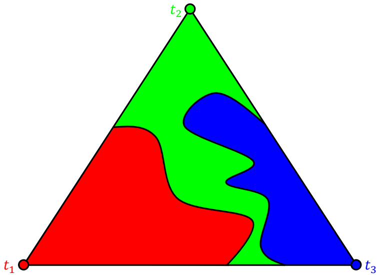
α-approximation for MULTIWAY-CUT

Note: all algorithms besides the greedy 2-approximation bound α.

Edge Structure



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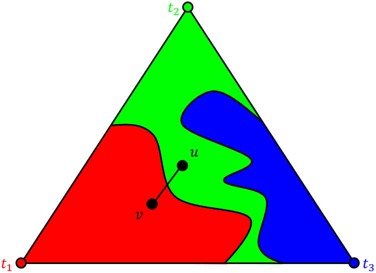

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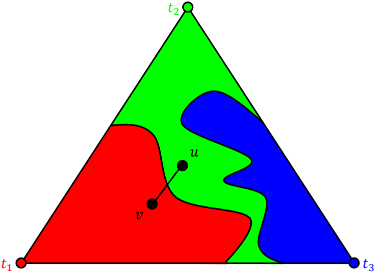

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α-approximation for MULTIWAY-CUT

$$\mathbf{u} = (u_1, \ldots, u_{i-1}, \quad u_i \quad, u_{i+1}, \ldots, u_{j-1}, \quad u_j \quad, u_{j+1}, \ldots, u_k)$$
$$\mathbf{v} = (u_1, \ldots, u_{i-1}, \quad u_i + \varepsilon \quad, u_{i+1}, \ldots, u_{j-1}, \quad u_j - \varepsilon \quad, u_{j+1}, \ldots, u_k)$$

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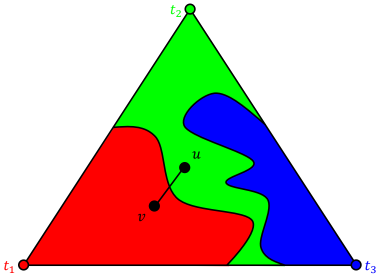

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Cut density of (i,j)-edge is $\alpha_{i,j} \triangleq \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \Pr[(u,v) \text{ is cut}]$.

Building Block I

[Buchbinder-Naor-S-13] [Ge-He-Ye-Zhang-11]

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$$Z_1, \ldots, Z_k \sim \exp(1) \text{ i.i.d} \qquad S_i = \left\{ u \in \Delta_k \ : \ \frac{Z_i}{u_i} \leq \frac{Z_1}{u_1}, \ldots, \frac{Z_k}{u_k} \right\}$$

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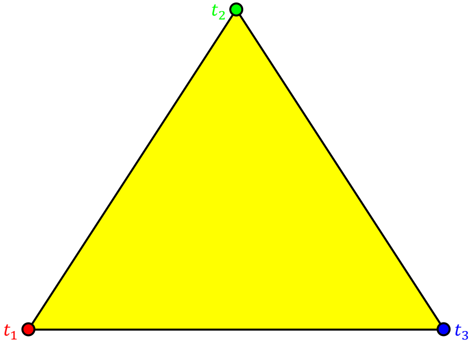

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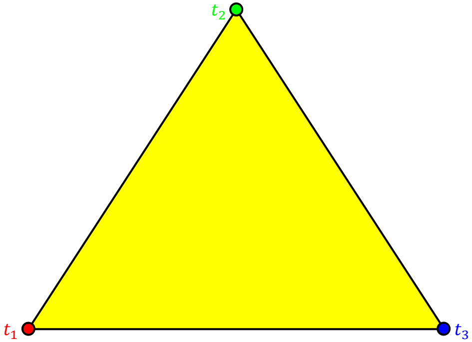

$$Z = (Z_1, Z_2, Z_3) \,/\, (Z_1 + Z_2 + Z_3) \in \Delta_3$$

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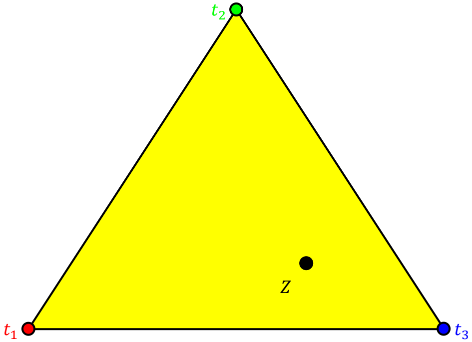

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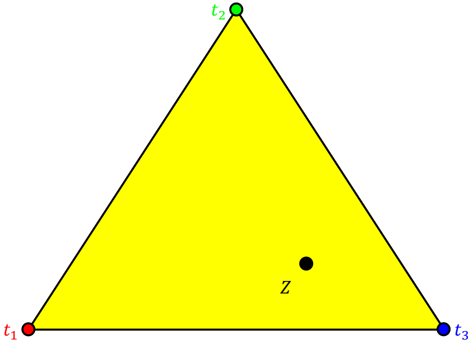

How is Z distributed ?

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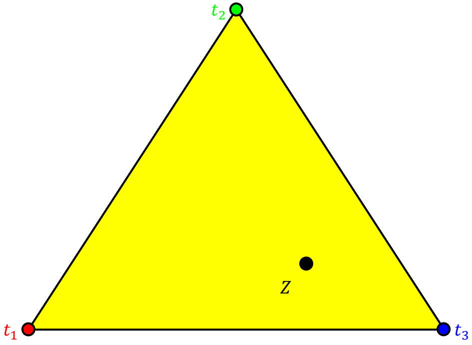

How is Z distributed ? $Z \sim Unif(\Delta_3)$

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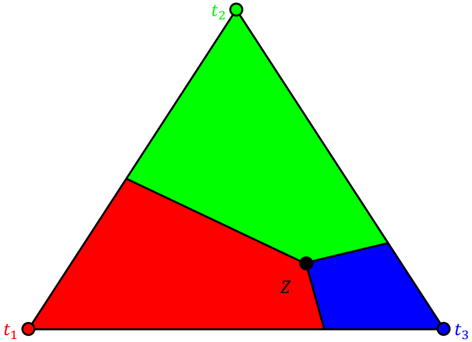

All boundaries of the Δ_3 partition Z induces

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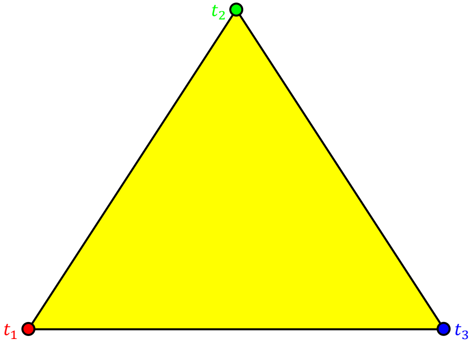

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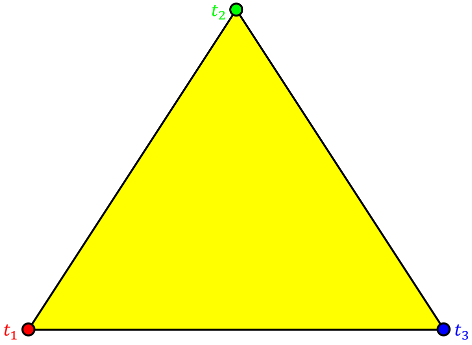

Fix a point $u \in \Delta_3$

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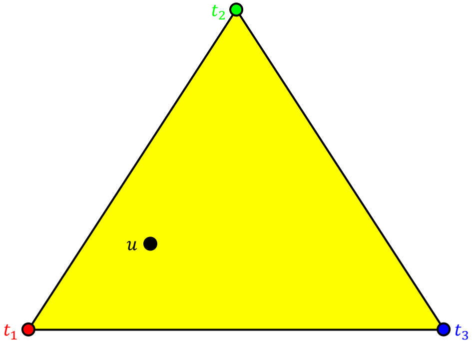

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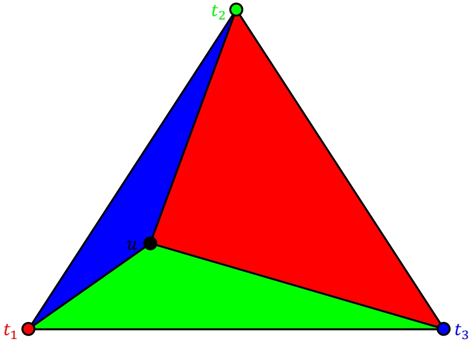

How is Δ_3 partitioned given u?

Building Block I (cont.)

Theorem [Buchbinder-Naor-S-13]

Building Block I has cut density:

$$\alpha_{i,j} \leq 2 - u_i - u_j .$$

Building Block I (cont.)

Theorem [Buchbinder-Naor-S-13]

Building Block I has cut density:

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Note: can use algorithm of [Kleinberg-Tardos-99] instead.

Building Block II

[Călinescu-Karloff-Rabani-98]

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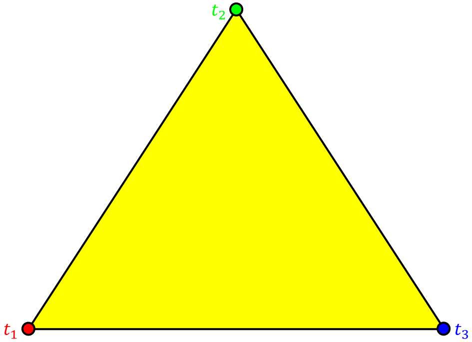
π random permutation on terminals $\qquad r \sim \text{Unif}[0,1]$



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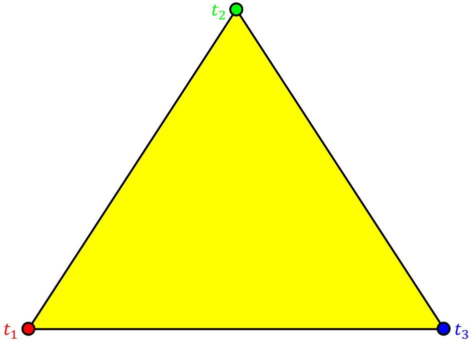

i^{th} step: assign all unassigned $\mathbf{u} \in \Delta_k$ with $u_{\sigma(i)} \geq r$ to $t_{\sigma(i)}$.

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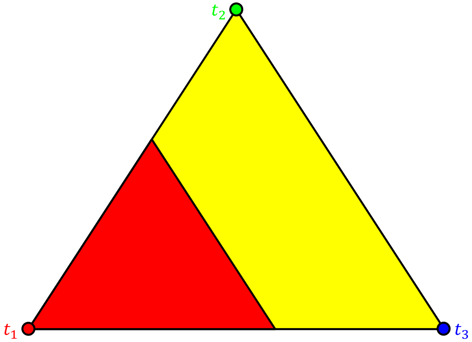

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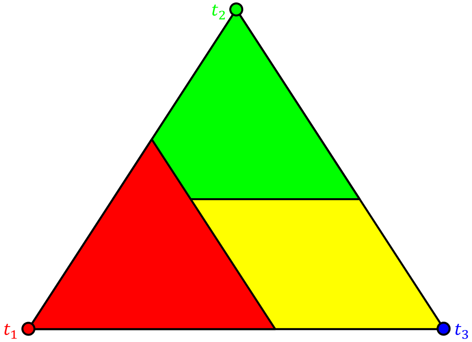

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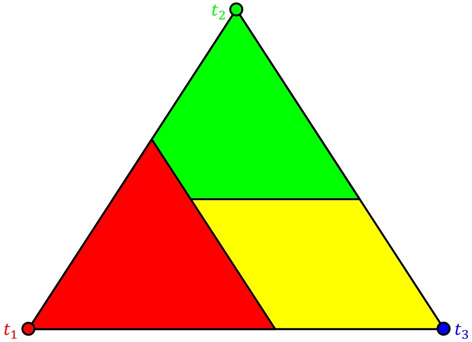

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Building Block II

[Călinescu-Karloff-Rabani-98]

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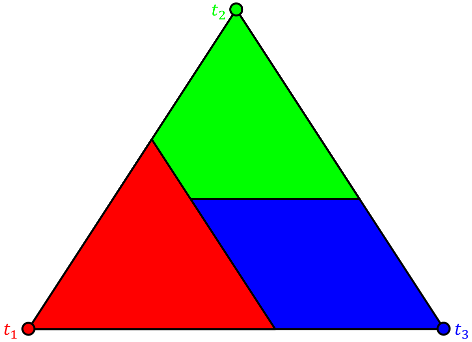

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Theorem [Călinescu-Karloff-Rabani-98]

Assuming $u_1 \geq \ldots \geq u_k$ Building Block II has cut density:

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Note: Building Block II provides a $(3/2)$-approximation for MULTIWAY-CUT.

Building Block II (cont.)

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- Threshold Dependency: independent, descending.

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$$\begin{cases} \text{Expo. Clocks} & \text{w.p } 2/3 \\ \text{CKR (quadratic distort)} & \text{w.p } 1/3 \end{cases}$$

[Buchbinder-Naor-S-13]

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[Buchbinder-Naor-S-13]

Cut density: $\alpha_{i,j} \leq \frac{2}{3}\left(2 - u_i - u_j\right) + \frac{1}{3}\left(\frac{2u_i}{i} + \frac{2u_j}{j}\right) \leq \frac{4}{3}$.

Algorithms Roadmap

1.5	[Călinescu-Karloff-Rabani-98]	CKR

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Question: simpler approach?

Distorting the Simplex

$$\begin{cases} \mathbf{u} = (u_1, \ldots, u_k) \\ g : [0,1] \to [0,1] \end{cases} \quad \rightsquigarrow \quad (g(u_1), \ldots, g(u_k))$$

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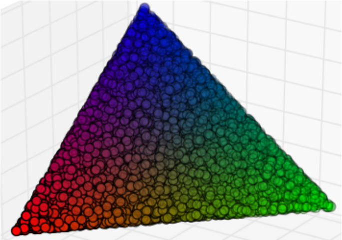

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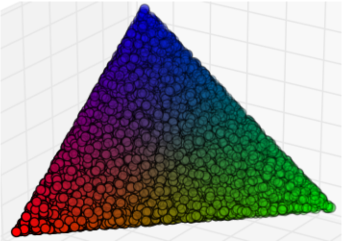

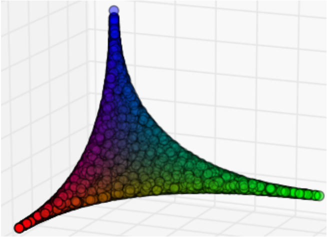

Distorting the Simplex (cont.)

$$\begin{cases} \mathbf{u} = (u_1, \ldots, u_k) \\ g_i : \Delta_k \to [0,1]^k \end{cases} \quad \rightsquigarrow \quad (g_1(\mathbf{u}), \ldots, g_k(\mathbf{u}))$$

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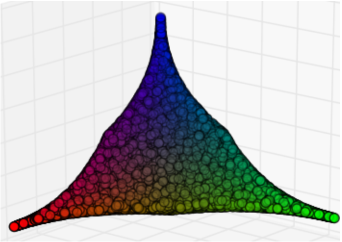
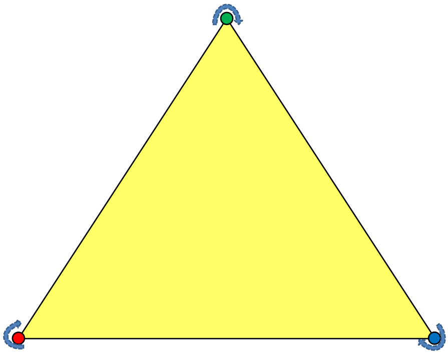


- expresses dependencies
- lower bound does not hold

Feasible Distortion?

- Terminals are fixed points



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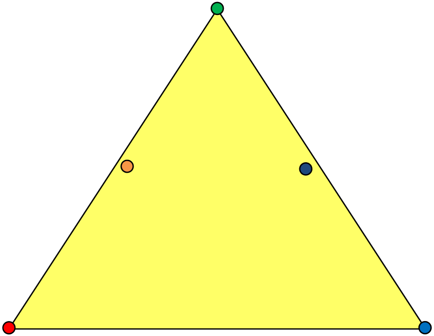

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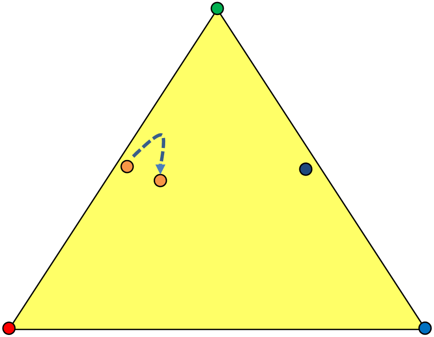

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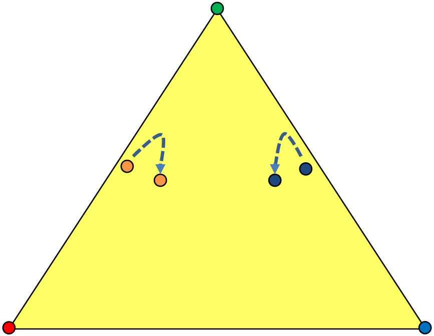

$\mathbf{v} = \sigma(\mathbf{u}) \qquad \mathbf{u} \to g(\mathbf{u}) \qquad \mathbf{v} \to \sigma(g(\mathbf{u}))$

Feasible Distortion?

- Terminals are fixed points
- Symmetry

1. sort coordinates
2. apply distortion according to position in order
3. place back

Feasible Distortion?

- Terminals are fixed points
- Symmetry
- Well defined

Tie breaking does not affect g:

$$g_i(u_1, \ldots, u_k) \triangleq g_1(\underbrace{u_i, \ldots, u_i}_{i \text{ times}}, u_{i+1}, \ldots, u_k)$$

Feasible Distortion?

- Terminals are fixed points
- Symmetry
- Well defined
- Monotonic : $u_i \geq u_j \Rightarrow g_i(\mathbf{u}) \geq g_j(\mathbf{u})$

Case Study – Breaking the $\frac{3+\sqrt{5}}{4} \approx 1.309$ Lower Bound

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Theorem [Buchbinder-S-Weizman-17]

Mixing Exponential Clocks with CKR with above g yields an approximation of:

$$\frac{17}{13} \approx 1.3077 < 1.309 \approx \frac{3+\sqrt{5}}{4}$$

for MULTIWAY-CUT.

Main Result

Theorem [Buchbinder-S-Weizman-17]

There exists a feasible $g : \Delta_k \to [0,1]^k$ such that mixing Exponential Clocks with CKR with g yields an approximation of 1.2969 for MULTIWAY-CUT.

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There exists a feasible $g : \Delta_k \to [0, 1]^k$ such that mixing Exponential Clocks with CKR with g yields an approximation of 1.2969 for MULTIWAY-CUT.

Note: (almost) matches the 1.2965 of [Sharma-Vondrák-14].

Open Questions

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2. What is the best possible cut density of Δ_k?