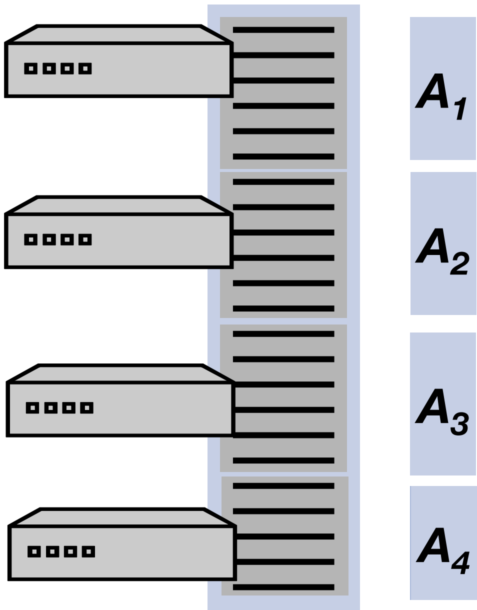


# Tall-and-skinny QRs and SVDs in MapReduce

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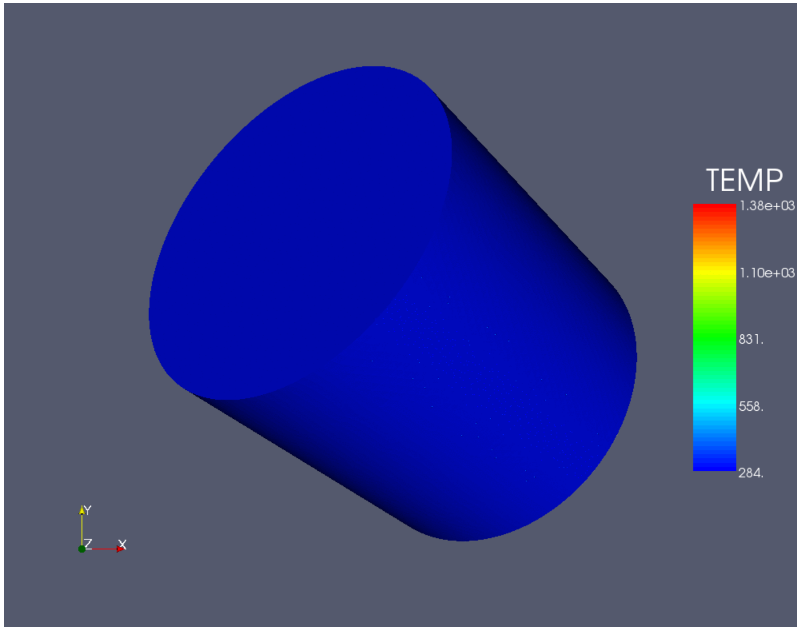
Mainly funded by Sandia National Labs CSAR project, recently by NSF CAREER, and Purdue Research Foundation.

David F. Gleich

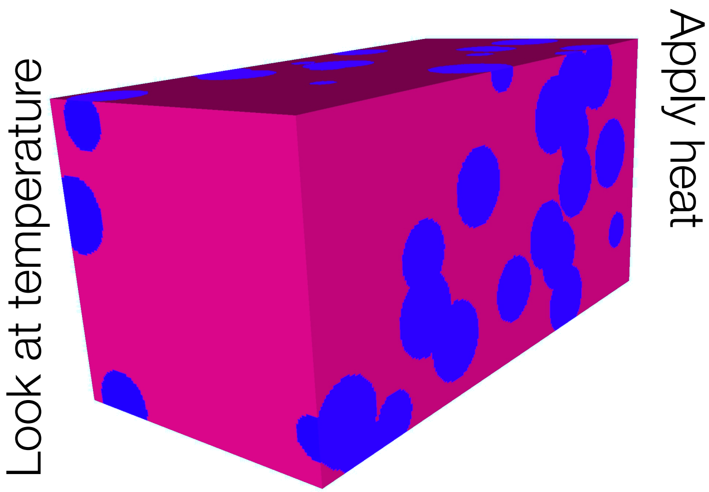
Computer Science
Purdue University

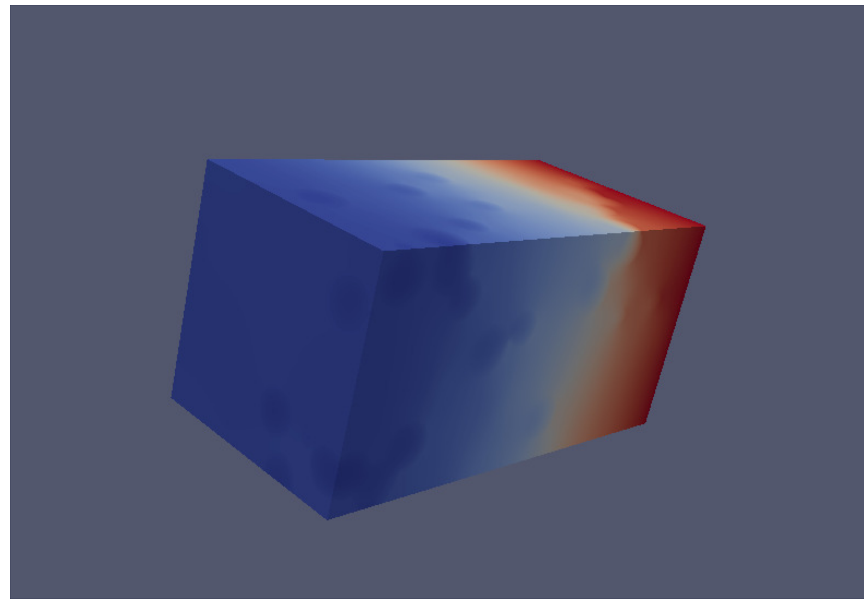
# Big simulation data



# Nonlinear heat transfer model in random media

Each run takes 5 hours on 8 processors, outputs 4M (node) by 9 (time-step) simulation

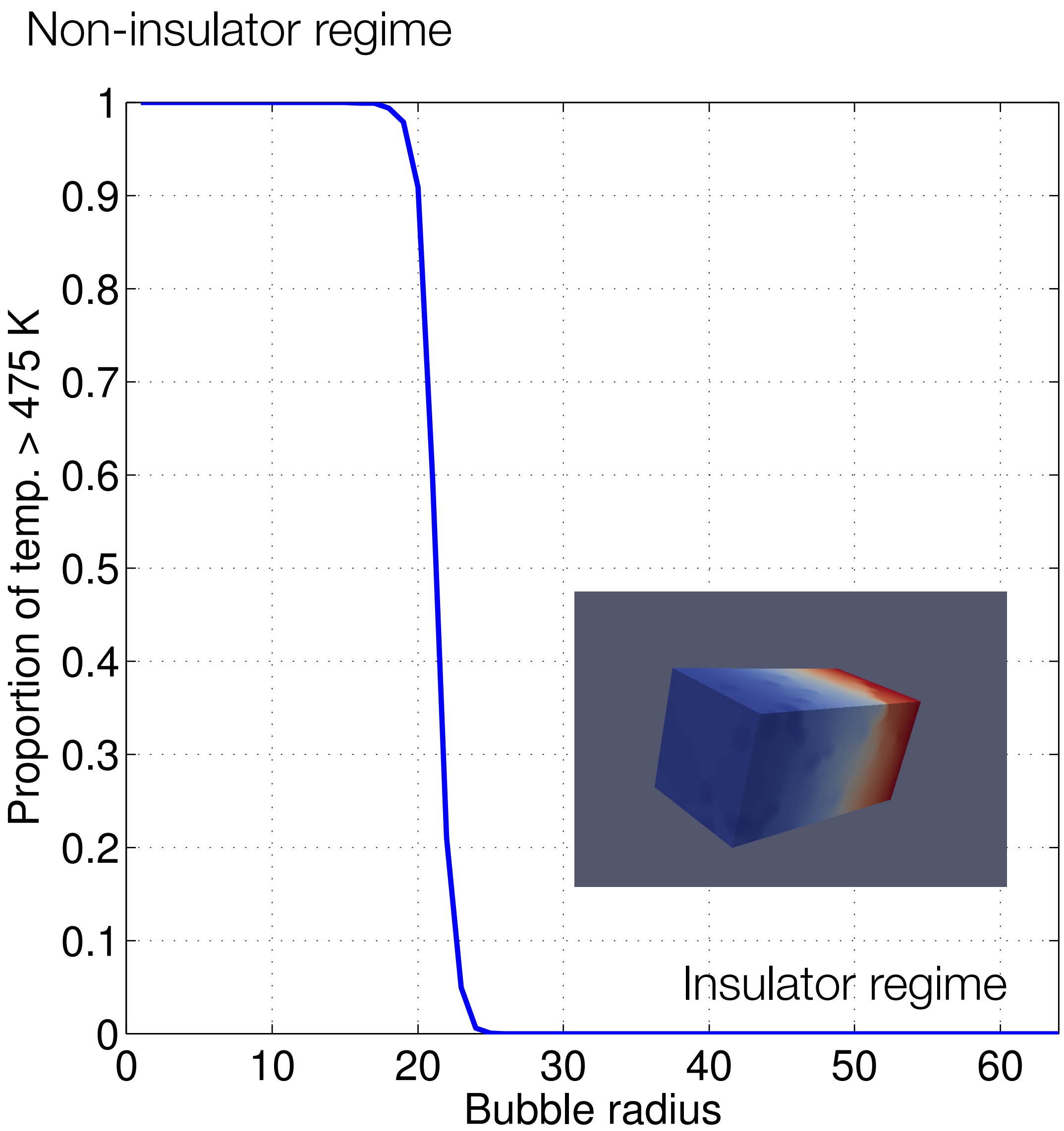


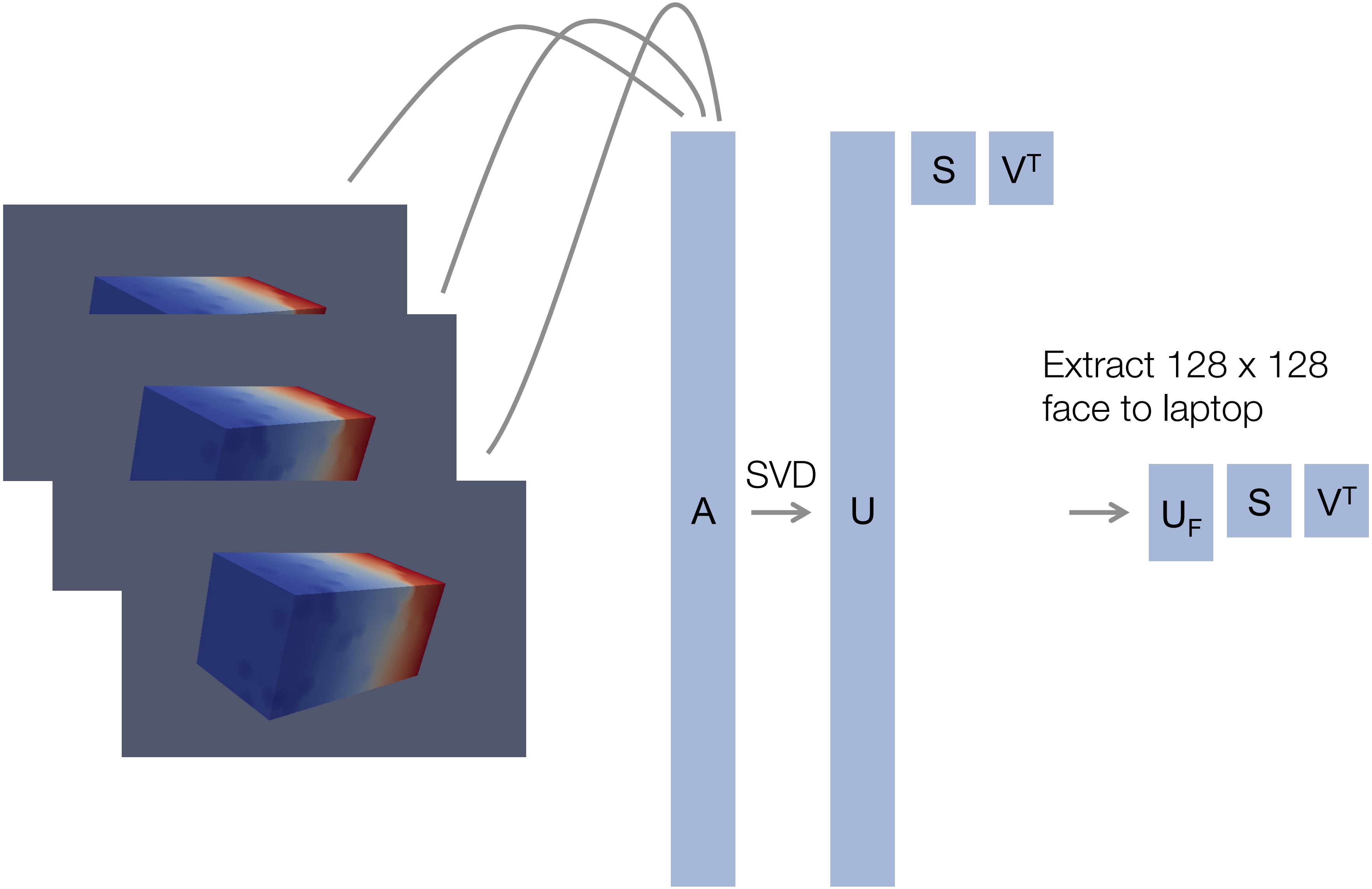



We did 8192 runs (128 samples of bubble locations, 64 bubble radii)

4.5 TB of data in Exodus II (NetCDF)

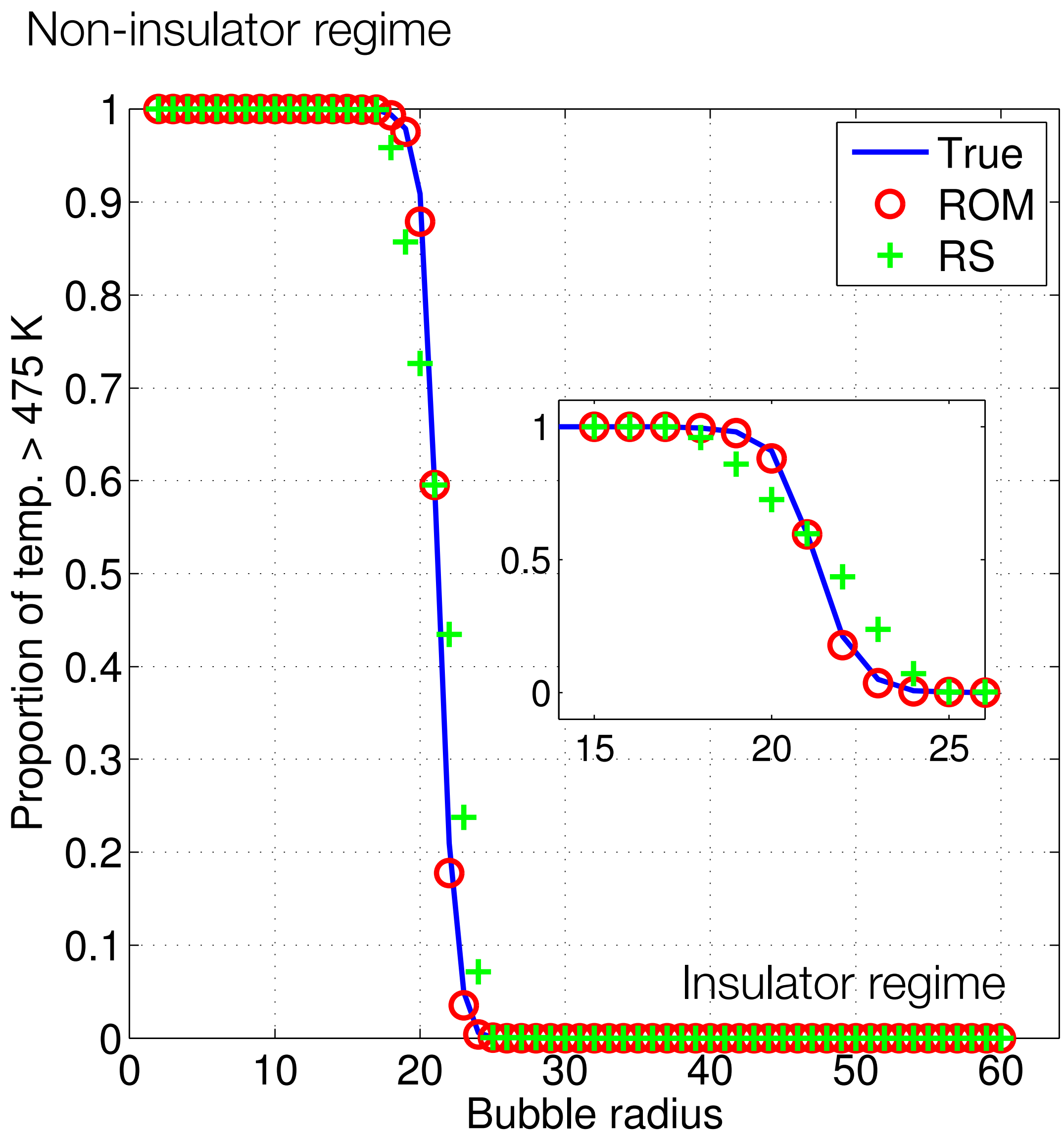
https://www.opensciencedatacloud.org/publicdata/heat-transfer/

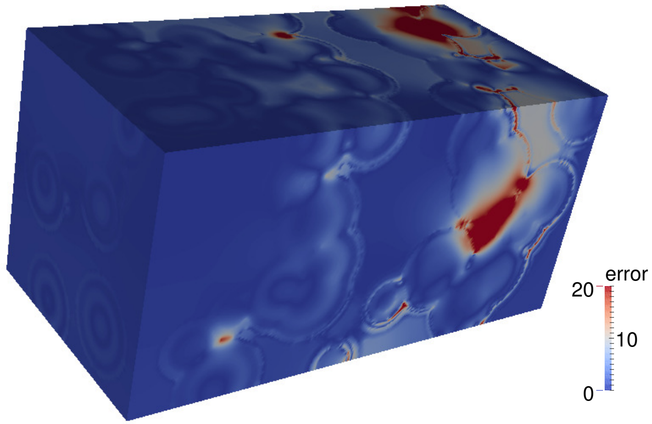

Non-insulator regime
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Proportion of temp. > 475 K
0
10
20
30
40
50
60
Bubble radius
Insulator regime



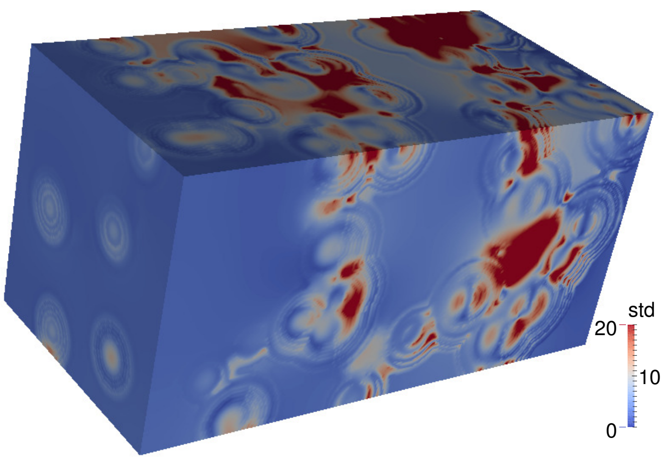

Each simulation is a column

5B-by-64 matrix
2.2TB


Non-insulator regime
Proportion of temp. > 475 K
Bubble radius
True
ROM
RS
Insulator regime
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
1
0.5
0
15
20
25




(c) Error, $s = 1.95$ cm




(d) Std, $s = 1.95$ cm

| $s$ | $R(s, \bar{\tau})$ | $\mathcal{E}(s, \bar{\tau})$ |
|---|---|---|
| 0.08 | 16 | 1.00e-04 |
| 0.23 | 15 | 2.00e-04 |
| 0.39 | 14 | 4.00e-04 |
| 0.55 | 13 | 6.00e-04 |
| 0.70 | 13 | 8.00e-04 |
| 0.86 | 12 | 1.10e-03 |
| 1.01 | 11 | 1.50e-03 |
| 1.17 | 10 | 2.10e-03 |
| 1.33 | 9 | 3.10e-03 |
| 1.48 | 8 | 4.50e-03 |
| 1.64 | 8 | 6.50e-03 |
| 1.79 | 7 | 8.20e-03 |
| 1.95 | 7 | 1.07e-02 |
| 2.11 | 6 | 1.23e-02 |
| 2.26 | 6 | 1.39e-02 |

*Constantine, Gleich,*
*Hou & Templeton arXiv 2013.*

# Dynamic Mode Decomposition

Dynamic mode decomposition of a rectangular supersonic screeching jet

Joseph W. Nichols

July 20, 2012

A

# Is this BIG Data?

BIG Data has two properties

- too big for one hard drive
- ‘skewed’ distribution

BIG Data = “Big Internet Giant” Data

BIG Data = “Big In’Gineering” Data

“Engineering”

A matrix $\boldsymbol{A} : m \times n, m \geq n$
is tall and skinny when $O(n^2)$
work and storage is “cheap”
compared to $m$.

-- Austin Benson

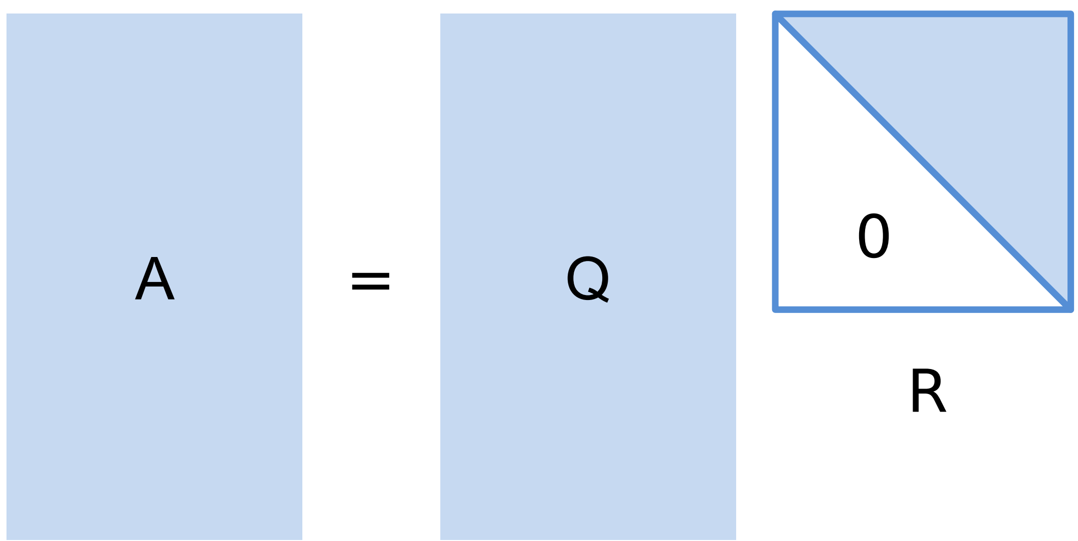
# Quick review of QR

Let $\boldsymbol{A} : m \times n$, $m \geq n$, real

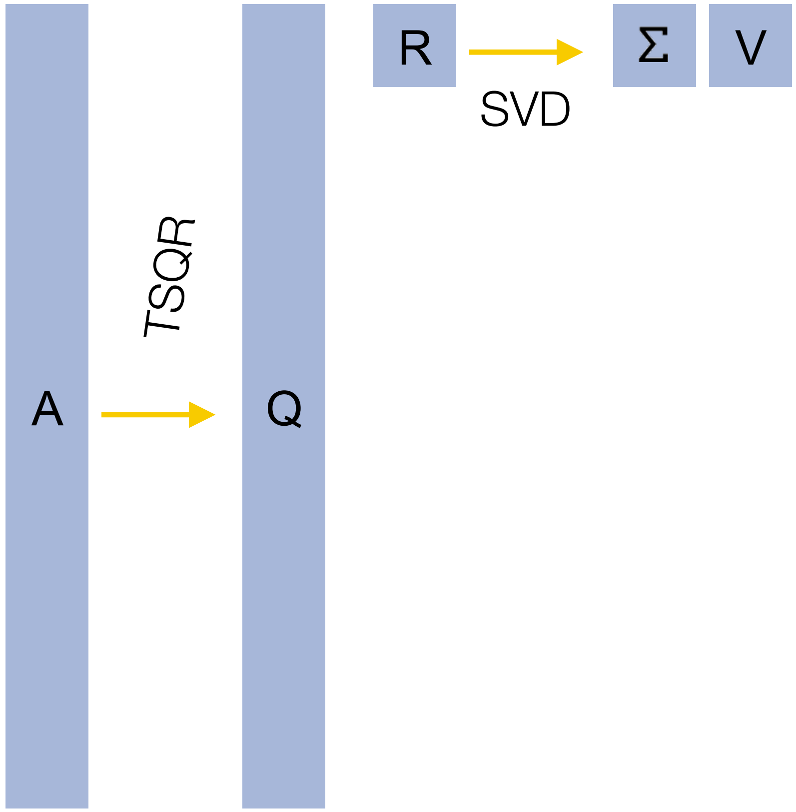
$\boldsymbol{A} = \boldsymbol{Q}\boldsymbol{R}$

$\boldsymbol{Q}$ is $m \times n$ orthogonal ($\boldsymbol{Q}^T \boldsymbol{Q} = \boldsymbol{I}$)

$\boldsymbol{R}$ is $n \times n$ upper triangular



# Tall-and-skinny SVD and RSVD




Let $\boldsymbol{A}$ : $m \times n$, $m \geq n$, real

$$\boldsymbol{A} = \boldsymbol{U\Sigma V^T}$$

$\boldsymbol{U}$ is $m \times n$ orthogonal

$\boldsymbol{\Sigma}$ is $m \times n$ nonneg, diag.

$\boldsymbol{V}$ is $n \times n$ orthogonal

There are good MPI implementations.

What's left to do?

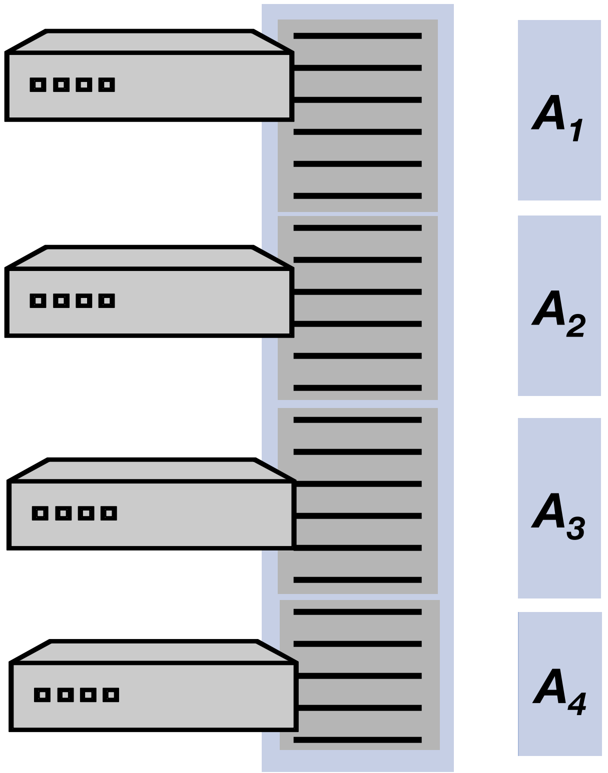
Moving data to an MPI cluster may be infeasible or costly

# How to store tall-and-skinny matrices in Hadoop

$A : m \times n$, $m \gg n$

Key is an arbitrary row-id
Value is the $1 \times n$ array
for a row (or $b \times n$ block)

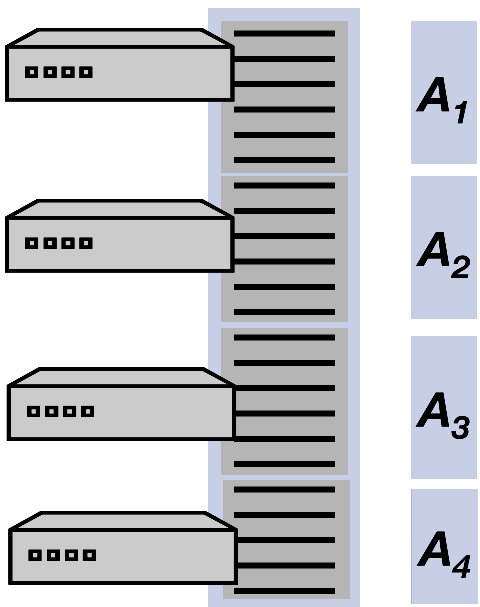
Each submatrix $A_i$ is an
the input to a map task.



# Still, isn't this easy to do?

**Current MapReduce algs** use the normal equations

$$\mathbf{A} = \mathbf{QR} \qquad \mathbf{A}^T\mathbf{A} \xrightarrow{\text{Cholesky}} \mathbf{R}^T\mathbf{R} \qquad \mathbf{Q} = \mathbf{A}\mathbf{R}^{-1}$$




**Map**
$A_{ii}$ to $A_i^T A_i$

**Reduce**
$R^T R = \text{Sum}(A_i^T A_i)$

**Map 2**
$A_{ii}$ to $A_i R^{-1}$

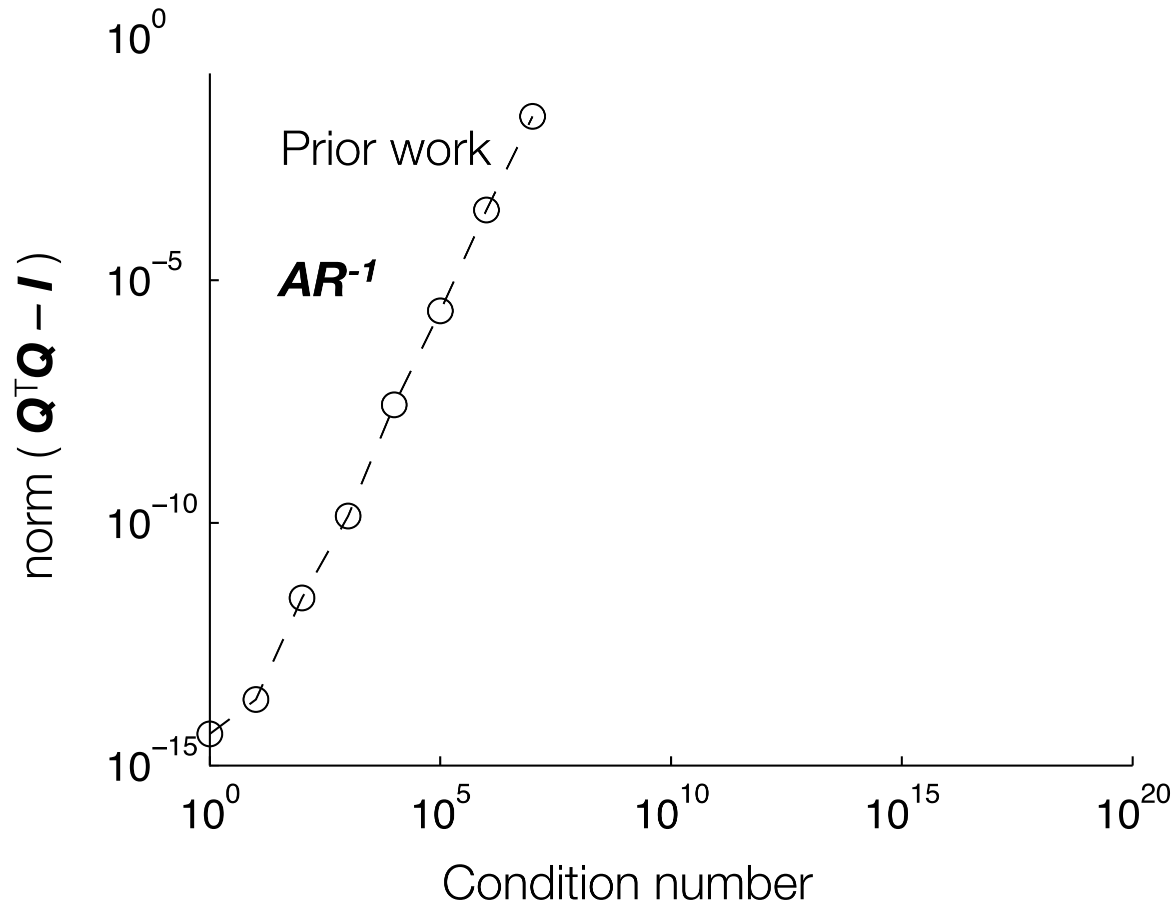
**Two problems**

R inaccurate if A ill-conditioned

Q not numerically orthogonal (Householder assures this)

# Numerical stability was a problem for prior approaches

Previous methods couldn't ensure that the matrix Q was orthogonal



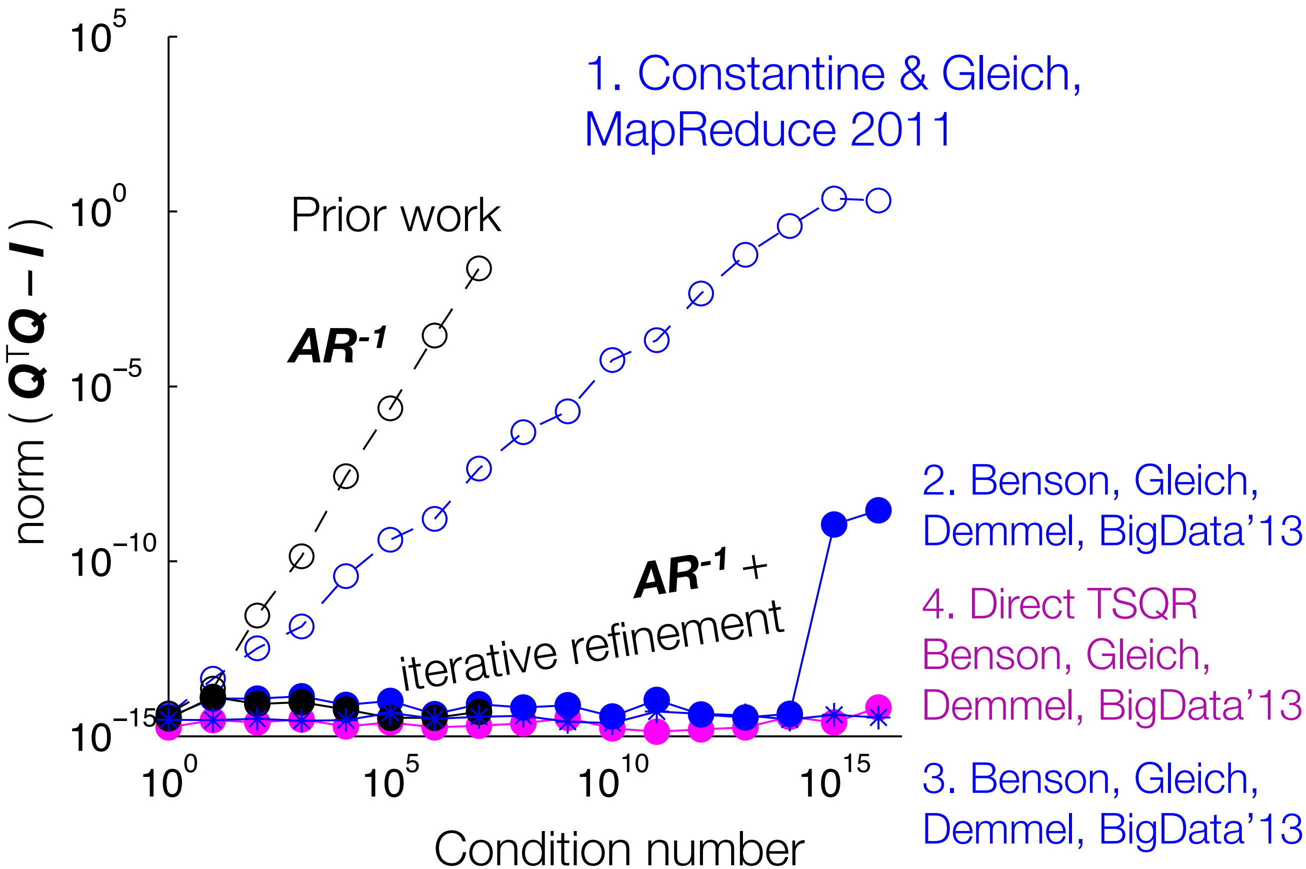
# Four things that are better

1. A simple algorithm to compute R accurately. (but doesn't help get Q orthogonal).
2. "Fast algorithm" to get Q numerically orthogonal in most cases.
3. Multi-pass algorithm to get Q numerically orthogonal in virtually all cases.
4. A direct algorithm for a numerically orthogonal Q in all cases.

Constantine & Gleich MapReduce 2011
Benson, Gleich & Demmel IEEE BigData 2013

# Numerical stability was a problem for prior approaches

Previous methods couldn't ensure that the matrix Q was orthogonal



# MapReduce is great for TSQR!
# You don't need $A^T A$

**Data** A tall and skinny (TS) matrix by rows

Input 500,000,000-by-50 matrix
Each record 1-by-50 row
HDFS Size 183.6 GB

Time to compute read A **253 sec.** write A **848 sec.**
Time to compute R in qr(A) **526 sec.** w/ **$Q=AR^{-1}$ 1618 sec**.
Time to compute Q in qr(A) **3090 sec.** (numerically stable)

```
git clone https://github.com/arbenson/mrtsqr
```

# Communication avoiding QR (Demmel et al. 2008)

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$$

$$A = \underbrace{\begin{bmatrix} Q_1 & & & \\ & Q_2 & & \\ & & Q_3 & \\ & & & Q_4 \end{bmatrix}}_{8n \times 4n} \underbrace{\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}}_{4n \times n}$$

First, do QR factorizations of each local matrix $\boldsymbol{A}_i$

Second, compute a QR factorization of the new "R"

$$A = \overbrace{\underbrace{\begin{bmatrix} Q_1 & & & \\ & Q_2 & & \\ & & Q_3 & \\ & & & Q_4 \end{bmatrix}}_{8n \times 4n} \underbrace{\tilde{Q}}_{4n \times n}}^{=Q} \underbrace{\tilde{R}}_{n \times n}$$

# Serial QR factorizations (Demmel et al. 2008)

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$$

Compute QR of $\boldsymbol{A}_1$,
read $\boldsymbol{A}_2$, update QR, …

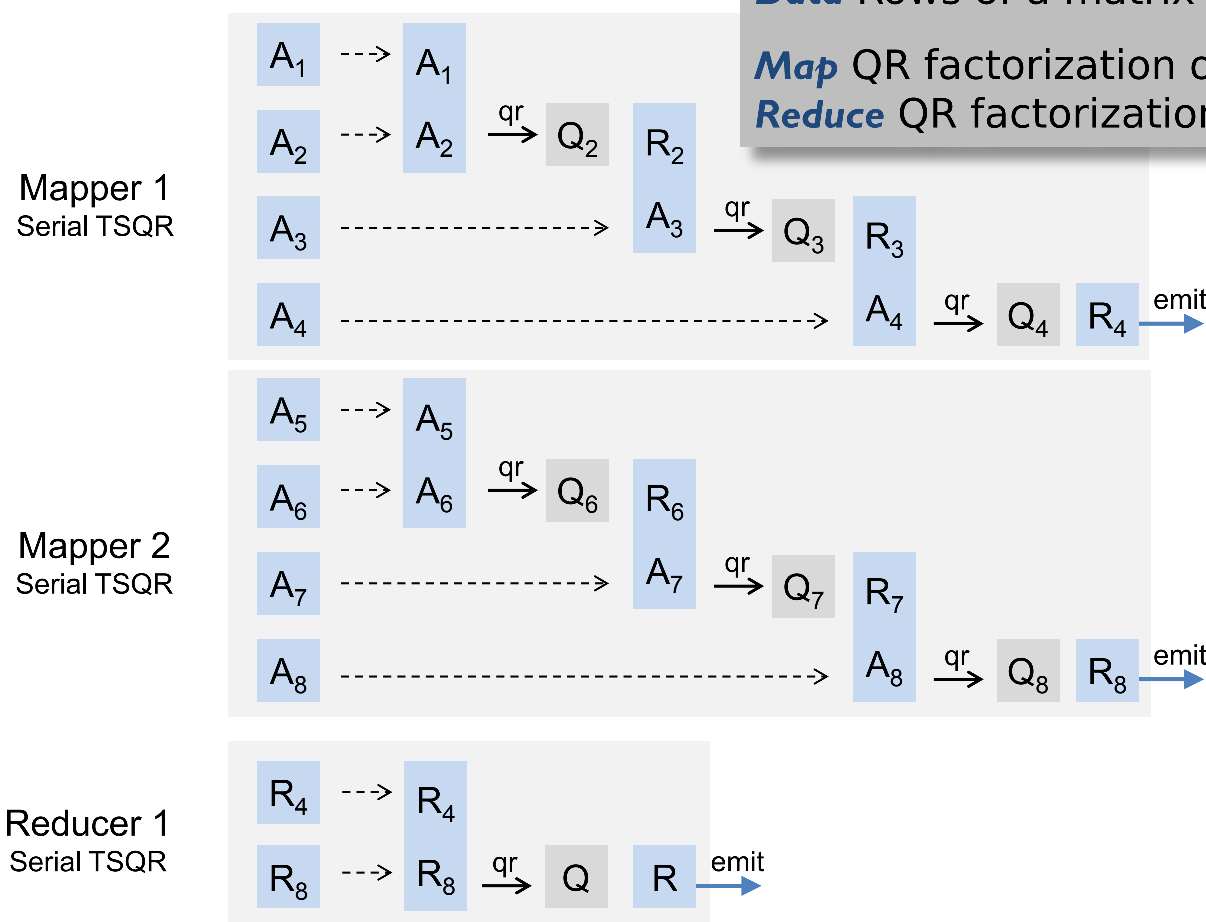
$$A_1 = Q_1 R_1; \begin{bmatrix} R_1 \\ A_2 \end{bmatrix} = Q_2 R_2; \begin{bmatrix} R_2 \\ A_3 \end{bmatrix} = Q_3 R_3; \begin{bmatrix} R_3 \\ A_4 \end{bmatrix} = Q_4 R_4$$

$$A = \overbrace{\underbrace{\begin{bmatrix} Q_1 & & & \\ & I_{2n} & & \\ & & I_{2n} & \\ & & & I_{2n} \end{bmatrix}}_{8n \times 7n} \underbrace{\begin{bmatrix} Q_2 & & \\ & I_{2n} & \\ & & I_{2n} \end{bmatrix}}_{7n \times 5n} \underbrace{\begin{bmatrix} Q_3 & \\ & I_{2n} \end{bmatrix}}_{5n \times 3n} \underbrace{Q_4}_{3n \times n}}^{=Q} \underbrace{R}_{n \times n}$$

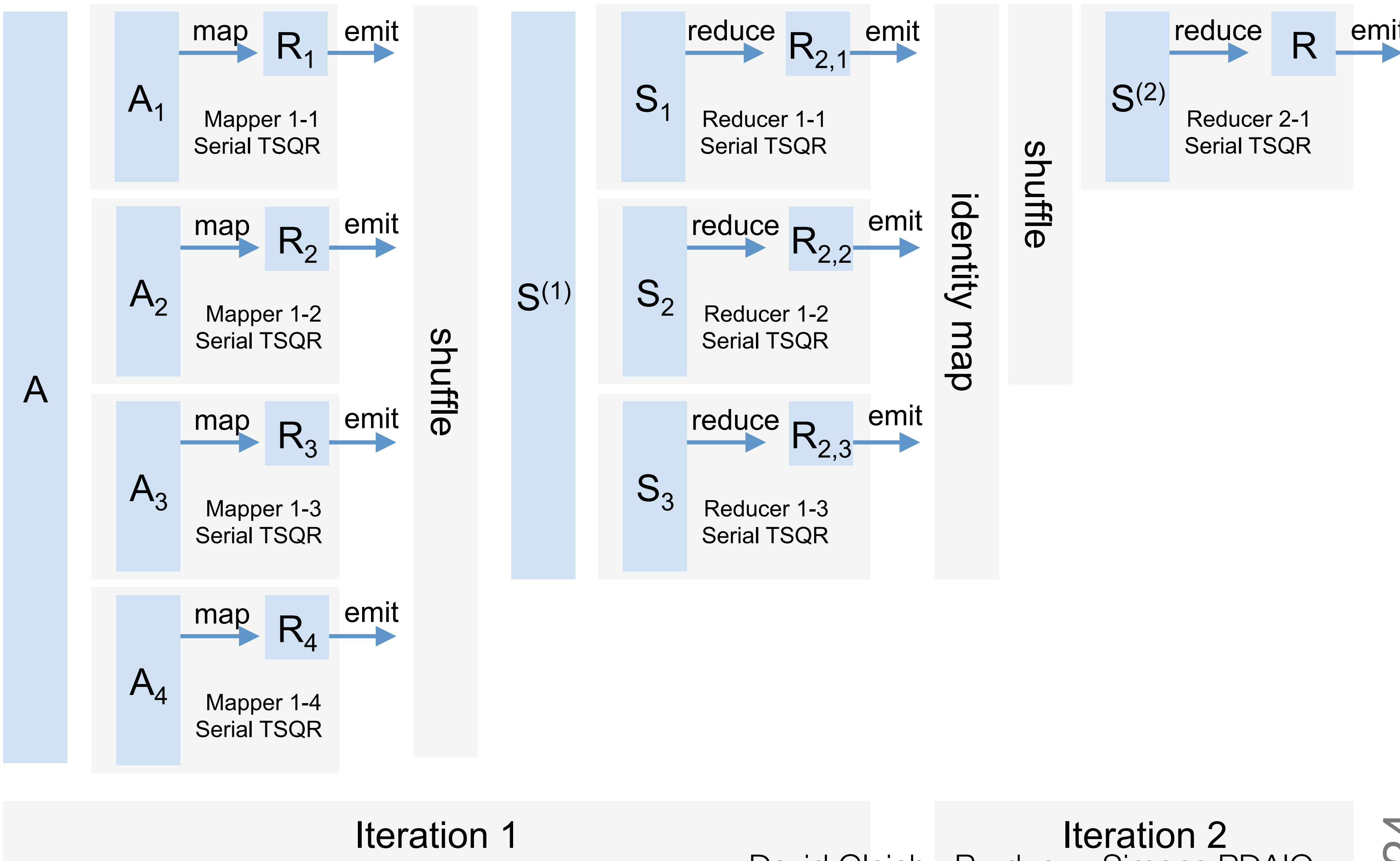
# Communication avoiding QR (Demmel et al. 2008) on MapReduce (Constantine and Gleich, 2011)

**Algorithm**
***Data*** Rows of a matrix

***Map*** QR factorization of rows
***Reduce*** QR factorization of rows



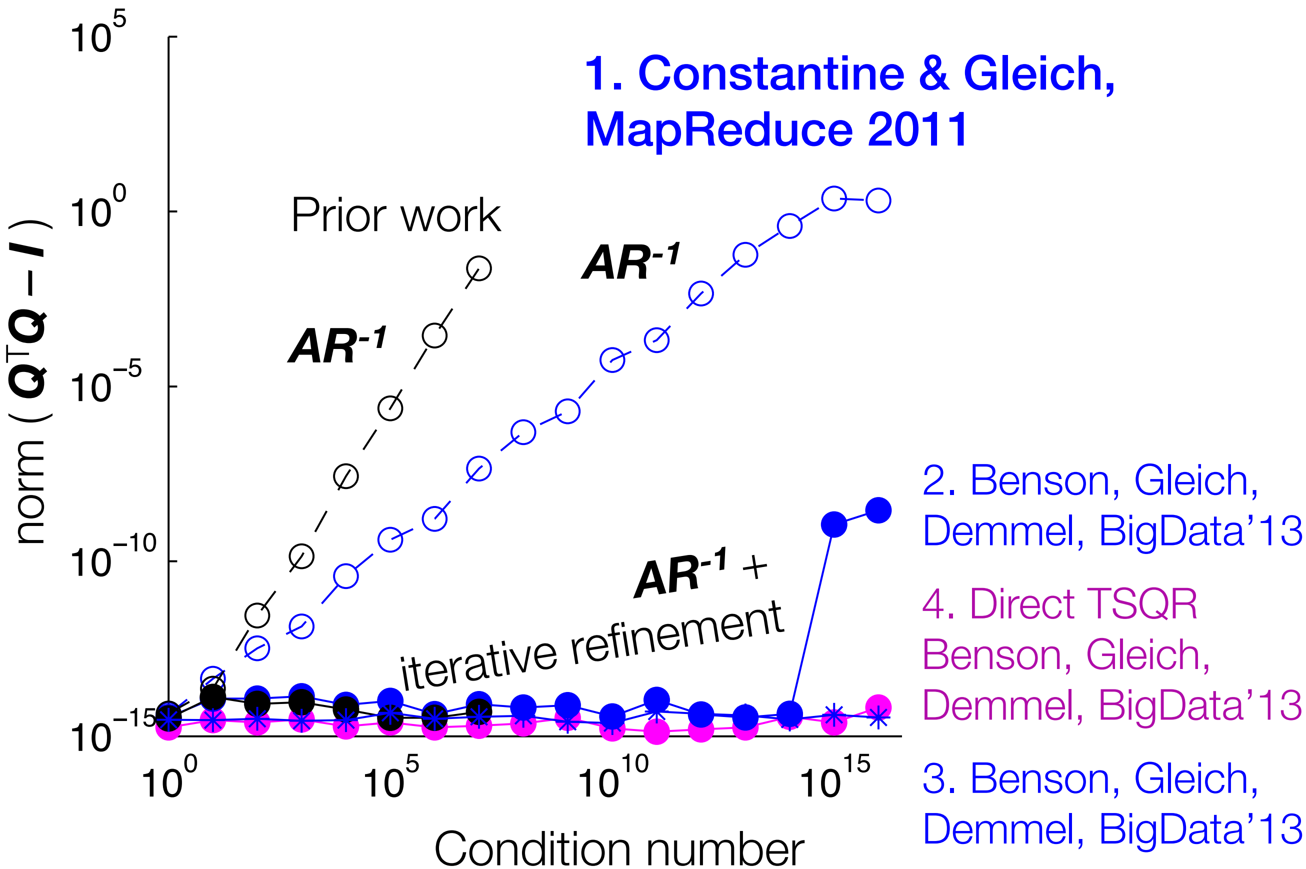
# Too many maps cause too much data to one reducer!



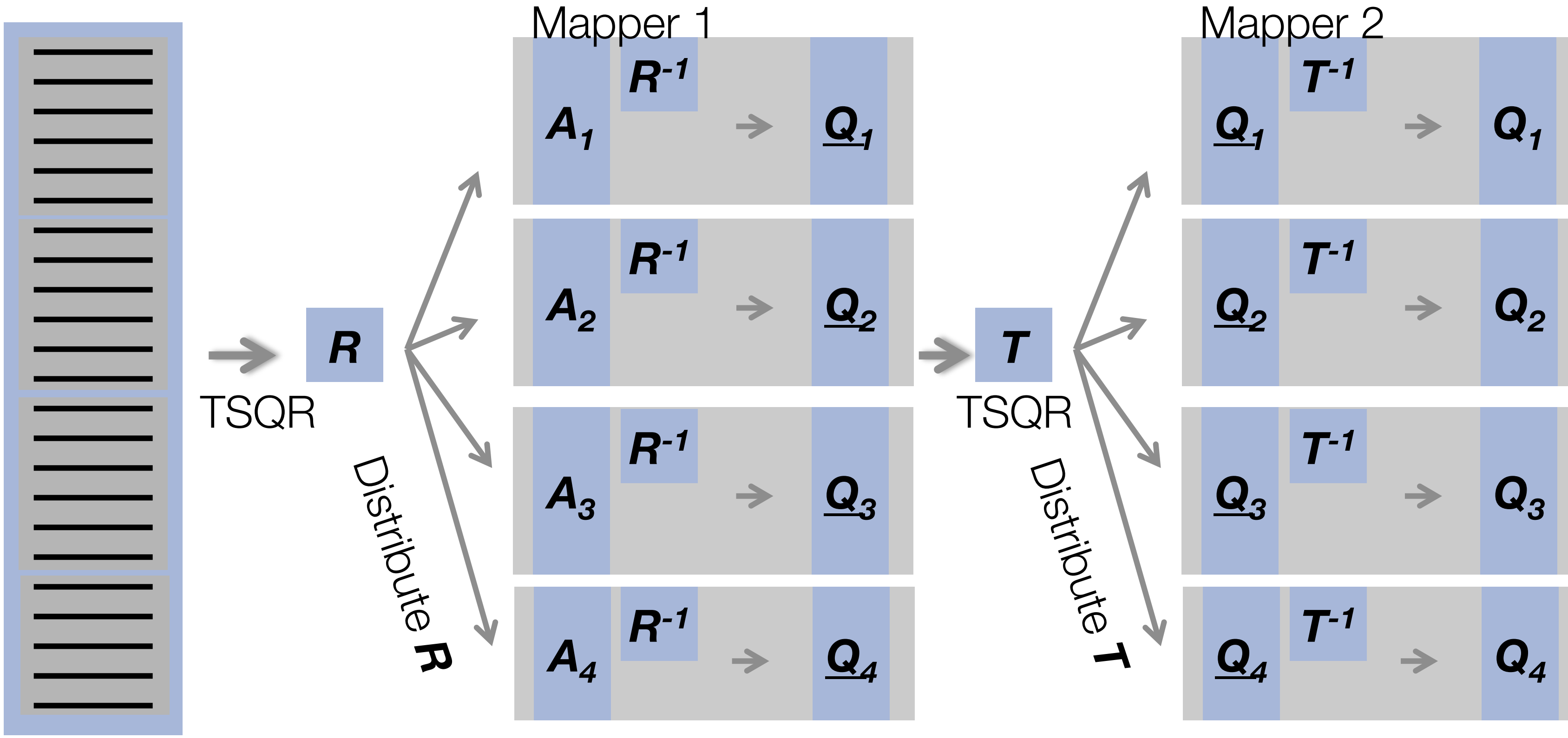
# Getting Q

# Numerical stability was a problem for prior approaches

Previous methods couldn't ensure that the matrix Q was orthogonal

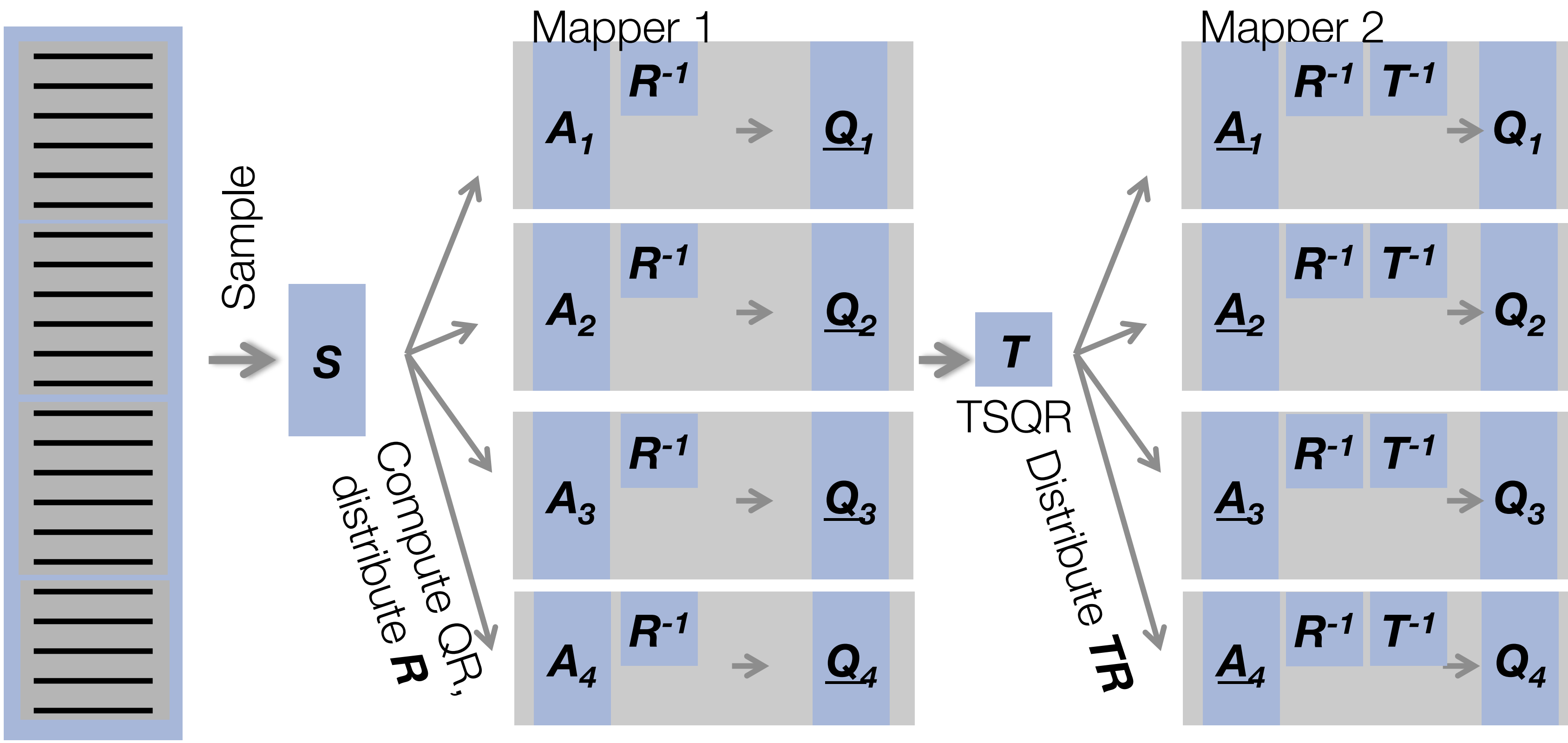


# Iterative refinement helps




Iterative refinement is like using Newton's method to solve Ax = b. It's folklore that "two iterations of iterative refinement are enough"

# What if iterative refinement is too slow?

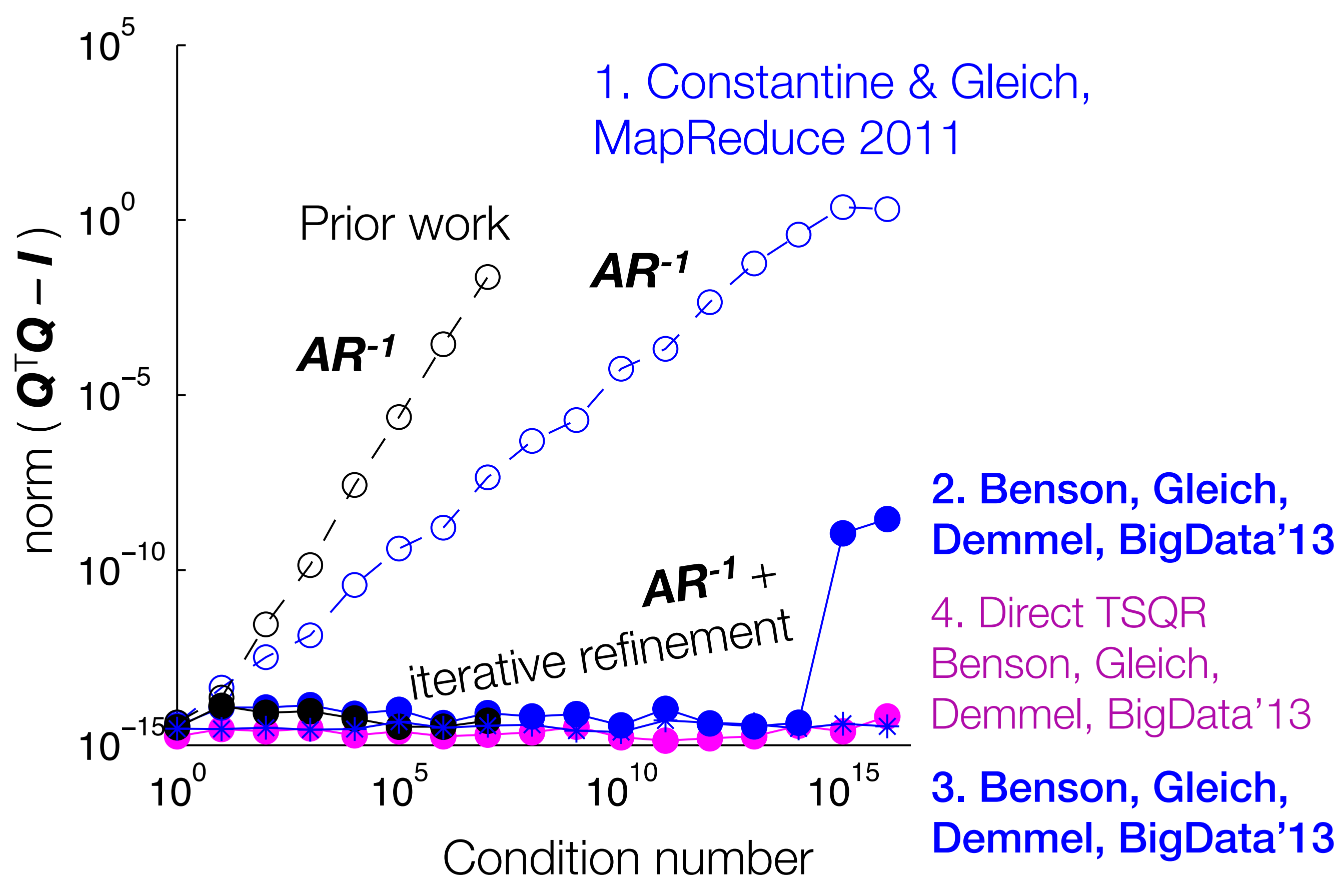



Estimate the "norm" by ***S***

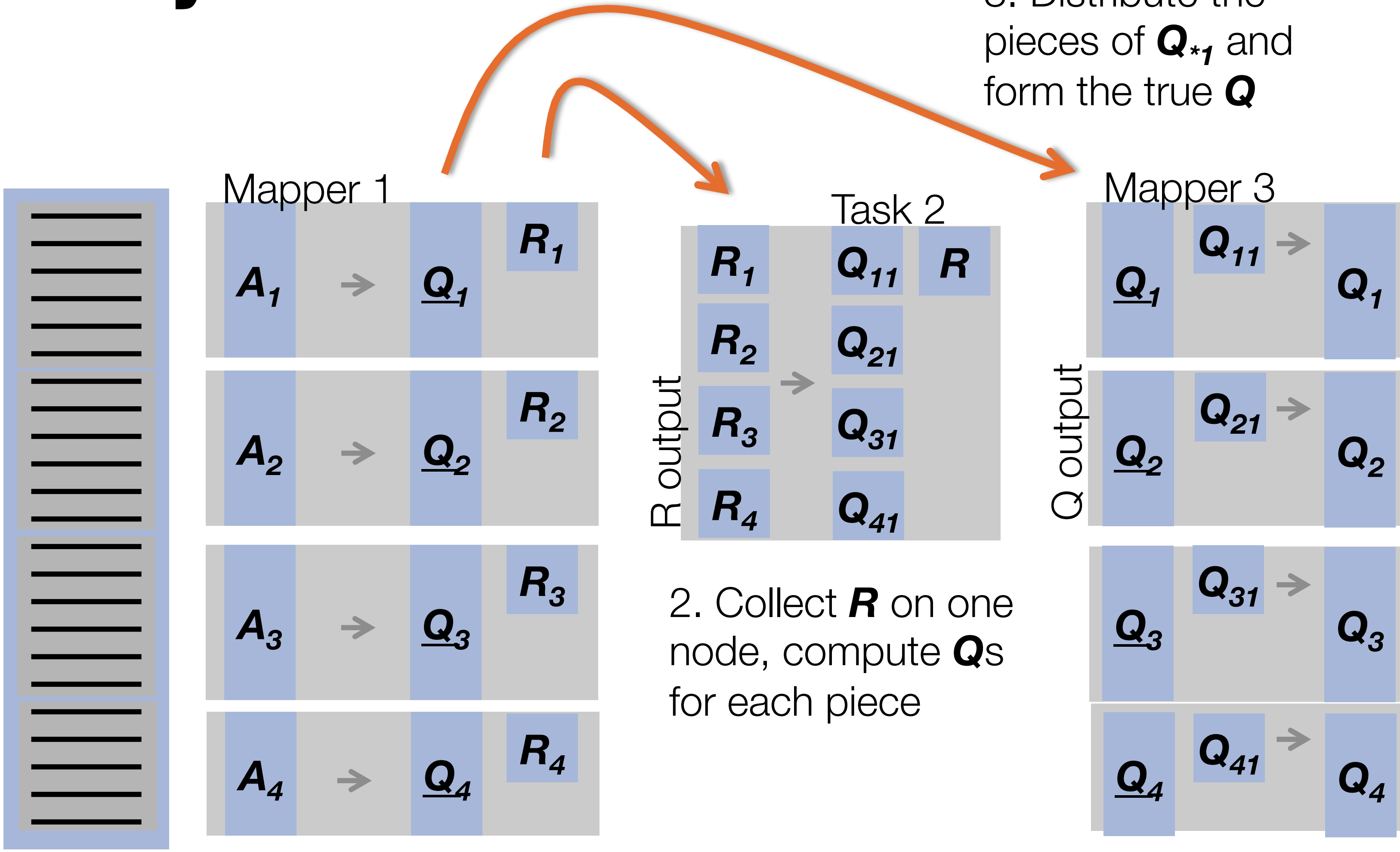
Based on recent work by "random matrix" community on approximating QR with a random subset of rows. Also assumes that you can get a subset of rows "cheaply" – possible, but nontrivial in Hadoop.

# Numerical stability was a problem for prior approaches

Previous methods couldn't ensure that the matrix Q was orthogonal



# Recreate Q by storing the history of the factorization




1. Output local $\underline{Q}$ and $R$ in separate files

2. Collect $R$ on one node, compute $Q$s for each piece

3. Distribute the pieces of $Q_{*1}$ and form the true $Q$

# Theoretical lower bound on runtime for a few cases on our small cluster

Model

| Rows | Cols | Old | R-only + no IR | R-only + PIR | R-only + IR | Direct TSQR |
|---|---|---|---|---|---|---|
| 4.0B | 4 | 1803 | 1821 | 1821 | 2343 | 2525 |
| 2.5B | 10 | 1645 | 1655 | 1655 | 2062 | 2464 |
| 0.6B | 25 | 804 | 812 | 812 | 1000 | 1237 |
| 0.5B | 50 | 1240 | 1250 | 1250 | 1517 | 2103 |

Actual

| Rows | Cols | Old | R-only + no IR | R-only + PIR | R-only + IR | Direct TSQR |
|---|---|---|---|---|---|---|
| 4.0B | 4 | 2931 | 3460 | 3620 | 4741 | 6128 |
| 2.5B | 10 | 2508 | 2509 | 3354 | 4034 | 4035 |
| 0.6B | 25 | 1098 | 1104 | 1476 | 2006 | 1910 |
| 0.5B | 50 | 921 | 1618 | 1960 | 2655 | 3090 |

All values in seconds

Only two params needed – read and write bandwidth for the cluster – in order to derive a performance model of the algorithm. This simple model is almost within a factor of two of the true runtime. (10-node cluster, 60 disks)

# Papers

Constantine & Gleich, MapReduce 2011

Benson, Gleich & Demmel, BigData'13

Constantine & Gleich, ICASSP 2012

Constantine, Gleich, Hou & Templeton, arXiv 2013

# Code

https://github.com/arbenson/mrtsqr

https://github.com/dgleich/simform

## Questions?

# BIG

Bloody Imposing Graphs

Building Impressions of Groundtruth

Blockwise Independent Guesses

Best Implemented at Google