Rapid mixing of hypergraph independent sets

Yumeng Zhang
University of California, Berkeley

Simons Institute
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Joint work with Jonathan Hermon and Allan Sly
Independent sets on graph

- An independent set $\mathcal{I}$: No edge has more than one $1$.
- An independent set $\mathcal{I}$: Every edge has at least one $0$.

Want to approximate the partition function $Z_{pG}$.
Independent sets on graph

An independent set $\iff$ No edge has more than one 1.
$\iff$ Every edge has at least one 0.
Independent sets on graph

An independent set \iff No edge has more than one 1.
\iff Every edge has at least one 0.

Want to approximate the partition function \( Z(G) \)
Phase Transition of Counting Complexity

FPTAS for all graphs with maximum degree $d$

if and only if

Correlation decay on $d$-regular tree.

[Weitz 06], [Sly 10]
Phase Transition of Counting Complexity

FPTAS | Hard unless NP=RP

Strong spatial mixing = Weak spatial mixing

[Weitz 06], [Sly 10], [Li-Lu-Yin 12], [Sly-Sun 12]

Similar result holds for other anti-ferromagnetic 2-spin systems.
Phase Transition of Counting Complexity

FPTAS

Hard unless \( \text{NP} = \text{RP} \)

Strong spatial mixing = Weak spatial mixing

\[ \Delta \]

[Weitz 06], [Sly 10], [Li-Lu-Yin 12], [Sly-Sun 12]

Similar result holds for other anti-ferromagnetic 2-spin systems.

What about hypergraphs?
Independent Set on Hypergraphs

\[ G = (V, F, E) \]

- \( V \): vertices (circles).
- \( F \): hyperedges (squares).

Degree \( \Delta \), Size \( k \)
Independent Set on Hypergraphs

$G = (V, F, E)$

$V$: vertices (circles).

$F$: hyperedges (squares).

Degree $\Delta$, Size $k$

No edge has more than one 1 $\Leftrightarrow$ Every edge has at least one 0.
Independent Set on Hypergraphs

\( G = (V, F, E) \)

\( V \): vertices (circles).
\( F \): hyperedges (squares).

Degree \( \Delta \), Size \( k \)

Hypergraph independent set: Every edge has at least one 0.
Independent Set on Hypergraphs

Hypergraph independent set: Every edge has at least one 0.
—Also known as monotone CNF.
Phase Transition of Counting Complexity

Graph $\Leftrightarrow$ Tree  
Hypergraph $\Leftrightarrow$ Hyper-tree?
Phase Transition of Counting Complexity

Graph $\Leftrightarrow$ Tree  

Hypergraph $\Leftrightarrow$ Hyper-tree?

SSM  

WSM

$O(\frac{2^k}{k})$

• SSM: correlation decay under boundary of arbitrary shape.
• WSM: correlation decay under boundary of complete tree.
Phase Transition of Counting Complexity

SSM  
FPRAS

6  $O(k)$  

?  $O(2^{k/2})$

WSM  
Hard unless NP=RP

$O(2^k/k)$  

$\Delta$

[BDK06] MCMC algo. for $\Delta \leq (k - 1)/2$.

[BGGGS16] FPTAS for $\Delta \leq k$.

[BGGGS16] No approx. algo. unless NP=RP for $\Delta \geq 5 \cdot 2^{k/2}$.
Phase Transition of Counting Complexity

SSM       FPRAS   WSM  
6          O(k)    ?     O(2^k/k)  Hard unless NP=RP

O(2^k/2)  O(2^k/k)  

[Moitra17] FPTAS for \( \Delta \leq e^{k/20} \).

[GJL17] Exact sampling for \( \Delta \leq c2^{k/2} \) with min overlap.

Today  MCMC algo. for \( \Delta \leq c2^{k/2} \) (with monotonicity).
Main result

**Glauber dynamics**  Pick a vertex, flip a coin, set to new value whenever possible.

**Mixing time**  \( t_{\text{mix}} = \min\{t : d_{\text{TV}}(\mathbb{P}^t(\sigma, \cdot), \pi) < 1/4, \forall \sigma \in \Omega\} \).
Main result

**Glauber dynamics**  Pick a vertex, flip a coin, set to new value whenever possible.

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**Main Theorem**

For any \( k \)-hypergraph with maximum degree \( \Delta \leq c2^{k/2} \), the Glauber dynamics mixes in \( O(n \log n) \) time.
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**Corollary**

Rapid mixing + self-reducibility \( \Rightarrow \) FPRAS
Path Coupling

Theorem [Bubley-Dyer 97]

Let $X_t, X'_t$ be two runs of the Glauber dynamics. If there exists a coupling of $X, X'$ such that for any $d(X_0, X'_0) = 1$,

$$\mathbb{E}[d(X_1, X'_1) \mid X_0, X'_0] \leq 1 - \alpha/n$$

then the mixing time is $O(n \log n)$. 
New discrepancy is created if the other vertices in the same hyperedge are all 1, which is increasingly unlikely (2\^k).
Path Coupling

\[ \mathbb{E}[d(X_1, X_1') \mid X_0, X_0'] = 1 - \frac{1}{n} + \Delta \cdot \frac{1}{n} \cdot \frac{1}{2}. \]

New discrepancy is created if the other vertices in the same hyperedge are all 1,
Path Coupling

\[ \mathbb{E}[d(X_1, X'_1) \mid X_0, X'_0] = 1 - \frac{1}{n} + \Delta \cdot \frac{1}{n} \cdot \frac{1}{2}. \]

New discrepancy is created if the other vertices in the same hyperedge are all 1, which is increasingly unlikely \((2^{-k})\).
Path Coupling

There are lots of interactions between bad events, but each individual interaction is weak.
Path Coupling

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Union bounds such as the “vanilla” path coupling will blow up.
Path Coupling

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Union bounds such as the “vanilla” path coupling will blow up.

In this talk, we will take a more direct approach:

- Characterize the “interactions” – via analyzing the propagation of the discrepancies in the space-time graph.
- Bound their appearance – via an auxiliary percolation.
Grand Coupling
and
the Propagation of Discrepancies
Grand Coupling

Continuous-time Glauber dynamics

- Place an independent Pois(1) clock at each vertex.
- At each alarm, flip a coin, set the value whenever possible.
Grand Coupling

Continuous-time Glauber dynamics

• Place an independent Pois(1) clock at each vertex.
• At each alarm, flip a coin, set the value whenever possible.

Let \((X^\sigma_t)_{t \geq 0}\) be the process on \(G\) starting from \(\sigma \in \Omega_G\); 

\((Y^s_t)_{t \geq s}\) be the process on empty graph starting at time \(s\) from the all 1 configuration.
Continuous-time Glauber dynamics

- Place an independent Pois(1) clock at each vertex.
- At each alarm, flip a coin, set the value whenever possible.

Let \(( X_t^\sigma \))_{t \geq 0} be the process on \( G \) starting from \( \sigma \in \Omega_G \);
\(( Y_t^s )_{t \geq s} \) be the process on empty graph starting at time \( s \) from the all 1 configuration.

\(( Y_t^s )_{t \geq s} \) is identical to the lazy simple random walk on \( \{0, 1\}^n \).
Grand Coupling

If we use the same clocks and coins for updating \( X_t \)'s and \( Y_t \)'s, then

\[
X_t^\sigma(v) \leq Y_t^0(v) \leq Y_t^s(v), \quad \forall \sigma \in \Omega_G, 0 \leq s \leq t.
\]
If we use the same clocks and coins for updating $X_t$'s and $Y_t$'s, then

$$X_t^\sigma(v) \leq Y_t^0(v) \leq Y_t^s(v), \quad \forall \sigma \in \Omega_G, 0 \leq s \leq t.$$ 

Let $t_{\text{coup}} \equiv \min\{t : X_t^\sigma = X_t^\tau, \forall \sigma, \tau \in \Omega_G\}$. It follows that

$$t_{\text{mix}} \leq \min\{T : \mathbb{P}(t_{\text{coup}} > T) \leq 1/4\}.$$
If we use the same clocks and coins for updating $X_t$’s and $Y_t$’s, then

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$$t_{\text{mix}} \leq \min\{T : \mathbb{P}(t_{\text{coup}} > T) \leq 1/4\}.$$ 

It is enough to bound $\mathbb{P}(t_{\text{coup}} > T)$ for $T = O(n \log n)$. 
Tracing back the discrepancies

\[ t_{\text{coup}} \geq T \]
Tracing back the discrepancies

\[ t_{\text{coup}} \geq T \Rightarrow \exists \sigma, \tau \in \Omega_G, v_0 \in V \text{ such that } X_T^\sigma(v_0) \neq X_T^\tau(v_0). \]
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Tracing back the discrepancies

t_{\text{coup}} \geq T \Rightarrow \exists \sigma, \tau \in \Omega_\sigma, v_0 \in V \text{ such that } X^\sigma_\tau(v_0) \neq X^\tau_\sigma(v_0).
\[ t_0 = T_-(v_0, T) \]
\[ t_1 = T_-(v_1, t_0) \]
\[ t_2 = T_-(v_2, t_1) \]
\[ t_3 = T_-(v_3, t_2) \]
\[ 0 = T_-(v_4, t_3) \]
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\[ t_2 = T_-(v_2, t_1) \]
\[ t_3 = T_-(v_3, t_2) \]
\[ 0 = T_-(v_4, t_3) \]

\[ t_{\text{coup}} \geq T \implies \exists ((v_\ell, a_\ell, t_\ell))_{0 \leq \ell \leq L} \text{ such that for all } 0 \leq \ell \leq L, \]
\[ (v_\ell, t_\ell) \in \text{Updates}, \quad t_\ell = T_-(v_\ell, t_{\ell-1}), \quad X_{t_\ell}^\sigma \lor X_{t_\ell}^T (\partial a_\ell \setminus \{v_\ell\}) = 1 \]
\[ t_0 = T_-(v_0, T) \]
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\[ t_2 = T_-(v_2, t_1) \]
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\[(v_\ell, t_\ell) \in \text{Updates}, \quad t_\ell = T_-(v_\ell, t_{\ell-1}), \quad Y^0_{t_\ell}(\partial a_\ell) = 1 \]
Discrepancy Paths

We say that \(((v_\ell, a_\ell, t_\ell))_{0 \leq \ell \leq L}\) is a discrepancy path if

\[(v_\ell, t_\ell) \in \text{Updates}, \quad t_\ell = T_-(v_\ell, t_{\ell-1}), \quad Y^0_{t_\ell}(\partial a_\ell) = 1\]

It follows that

\[t_{\text{coup}} > T \Rightarrow \exists \text{ discrepancy path with } t_{-1} = T.\]
Discrepancy Paths

To control the existence of discrepancy paths with $t_{-1} = T$:

- Construct a space-time percolation such that every discrepancy path maps to an open path in the percolation.
- Bound the open cluster of the percolation containing $t = 0$. 
To control the existence of **discrepancy paths** with \( t_{-1} = T \):

- Construct a space-time percolation such that every discrepancy path maps to an open path in the percolation.
- Bound the open cluster of the percolation containing \( t = 0 \).

Will show:
A weaker proof in 10 mins \( + \) modifications to the tight result.
Auxillary Percolation
Break the time line into chunks of length $2k$ and define

$$H \equiv F \times \mathbb{Z}_+ \equiv F \times \{0, 1, \ldots \}.$$ 

Each site $(a, i) \in H$ represents the block $\partial a \times [2ik, 2(i + 1)k)$. 
Auxillary Percolation

We say that \((a, i) \in H\) is **bad** if \(Y^t_{2(i-1)k}(\nu) = 1\) for

**Horizontal:** some \(t \in [2ik, 2(i + 1)k]\) and all \(\nu \in \partial a\).

or **Vertical:** all \(t \in [2ik, 2(i + 1)k]\) and some \(\nu \in \partial a\).
We say that \((a, i) \in H\) is **bad** if \(Y^2_{2(i-1)k}(v) = 1\) for

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or **Vertical**: all $t \in [2ik, 2(i + 1)k)$ and some $v \in \partial a$.

This definition is local.
Control the Percolation

Draw an edge between \((a, i), (b, j)\) \(\in H\) if and only if

\[ \partial a \cap \partial b \neq \emptyset, \quad |i - j| \leq 1. \]
Control the Percolation

Draw an edge between \((a, i), (b, j) \in H\) if and only if

\[ \partial a \cap \partial b \neq \emptyset, \quad |i - j| \leq 1. \]

Claim: If \((a, i)\) is not adjacent to \((b, j)\), then

\[ \{ (a, i) \text{ is bad} \} \upparrow \{ (b, j) \text{ is bad} \}. \]
$G_i + 1$
Fix a discrepancy path, analyze the sites \((a, i)\)'s along it.
Fix a discrepancy path, analyze the sites $(a, i)$'s along it.
Fix a discrepancy path, analyze the sites \((a, i)\)'s along it.

\[\Rightarrow\] Each \((a, i)\) is either bad or next to some \((b, i)\) that is bad.
Fix a \textbf{discrepancy path}, analyze the sites \((a, i)\)'s along it.

\Rightarrow Each \((a, i)\) is either \textbf{bad} or next to some \((b, i)\) that is bad.

\Rightarrow \textbf{discrepancy path} \subseteq \textbf{open cluster} of the percolation if we allow the cluster to connect through diagonals.
Control the Percolation
We say that a path $\gamma = ((a_\ell, i_\ell))_{0 \leq \ell \leq L}$ in $H$ is a **minimal path** if $(a_\ell, i_\ell)$ is not adjacent to $(a_{\ell'}, i_{\ell'})$ for all $\ell \geq \ell' + 2$.

$$\Rightarrow \{ (a_\ell, i_\ell) \text{ is bad} \} \perp \{ (a_{\ell'}, i_{\ell'}) \text{ is bad} \}, \ \forall \ell \geq \ell' + 2.$$
We say that a path $\gamma = ((a_\ell, i_\ell))_{0 \leq \ell \leq L}$ in $H$ is a **minimal path** if $(a_\ell, i_\ell)$ is not adjacent to $(a_{\ell'}, i_{\ell'})$ for all $\ell \geq \ell' + 2$.

$$\Rightarrow \{ (a_\ell, i_\ell) \text{ is bad} \} \perp \{ (a_{\ell'}, i_{\ell'}) \text{ is bad} \}, \quad \forall \ell \geq \ell' + 2.$$ 

Any cluster contains a **minimal path** as subset.
Control the Percolation

$t_{\text{coup}} > T$

$\Rightarrow \exists \text{ discrepancy path} \text{ with } t_{-1} = T.$

$\Rightarrow \exists \text{ open cluster in } H \text{ connecting } F \times \{0\} \text{ and } F \times \{\lfloor T/k \rfloor\}.$

$\Rightarrow \exists \text{ open minimal path} \text{ in } H \text{ with length } L = \lceil T/k \rceil.$
Recall that $Y_t(\partial a)$ is just the LSRW on $\{0,1\}^k$. 

$$\Rightarrow \mathbb{P}((a, i) \text{ is bad}) \leq 2k^22^{-k} + (k + 1)e^{-k} \approx 2k^22^{-k}.$$
Recall that \( Y_t(\partial a) \) is just the LSRW on \( \{0, 1\}^k \).

\[
\Rightarrow \mathbb{P}(\text{(a, i) is bad}) \leq 2k^22^{-k} + (k + 1)e^{-k} \approx 2k^22^{-k}.
\]

Note that the maximum degree of \( H \) is \((2\Delta k + 1)\)

\[
\mathbb{P}(t_{\text{coup}} > T) \leq \mathbb{P}(\exists \text{ open minimal path with length } \lfloor T/k \rfloor) \\
\leq \sum_{\ell=0}^{\lfloor L/2 \rfloor} \prod_{r=0}^{\ell} \mathbb{P}(\text{(a}_{2r}, i_{2r}) \text{ is bad}) \\
\leq n(2\Delta k + 1)^L(2k^22^{-k})^{\lfloor L/2 \rfloor} \\
\rightarrow 0, \quad \text{if } \Delta \leq O(k^{-2}2^{k/2}), \ L = \Omega(\log n).
Removing the factor of $k^2$
Removing the factor of $k^2$

For a discrepancy passes to an adjacent hyperedge before being killed by an update, we need:

A vertex in $C$ is updated:

1. when $C$ is all 1;
2. when $B$ is all 1 before each vertex in $B$ is updated once.
Removing the factor of $k^2$

Recall the key step from previous slide:

$$\mathbb{P}(t_{\text{coup}} \leq T) \leq Cn(\Delta k)^L(k^22^{-k})^{[L/2]}$$

The “branching number” consists of two parts:
Removing the factor of $k^2$

Recall the key step from previous slide:

$$\mathbb{P}(t_{\text{coup}} \leq T) \leq Cn(\Delta k)^L(k^22^{-k})^{[L/2]}$$

The “branching number” consists of two parts:

1. The number of adjacent hyperedges

   $\leq \Delta k$
Removing the factor of $k^2$

Recall the key step from previous slide:

$$\mathbb{P}(t_{\text{coup}} \leq T) \leq Cn(\Delta k)^L (k^2 2^{-k})^{[L/2]}$$

The “branching number” consists of two parts:

1. The number of adjacent hyperedges $\leq \Delta k$
2. The probability that a discrepancy passes to an adjacent hyperedge before being killed by an update.
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The “branching number” consists of two parts:

1. The number of adjacent hyperedges $\leq \Delta k$
2. The probability that a discrepancy passes to an adjacent hyperedge before being killed by an update. $\leq k$
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2. The probability that a discrepancy passes to an adjacent hyperedge before being killed by an update.
   — i.e. an vertex in $C$ is updated $\leq k$
Removing the factor of $k^2$

Recall the key step from previous slide:

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The “branching number” consists of two parts:

1. The number of adjacent hyperedges \( \leq \Delta k \)
2. The probability that a discrepancy passes to an adjacent hyperedge before being killed by an update.
   - i.e. an vertex in $C$ is updated \( \leq k \)
   - a. when $C$ is all one; \( \text{avg. } 2^{-k/2} \)
Removing the factor of $k^2$

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$$\mathbb{P}(t_{\text{coup}} \leq T) \leq Cn(\Delta k)^L(k^22^{-k})^{[L/2]}$$

The “branching number” consists of two part:

1. The number of adjacent hyperedges — $\leq \Delta k$

2. The probability that a discrepancy passes to an adjacent hyperedge before being killed by an update.
   — i.e. an vertex in $C$ is updated
     a. when $C$ is all one; — avg. $2^{-k/2}$
     b. before each vertex in $B$ is updated once. — $\leq k$
Removing the factor of $k^2$

Assume that adj. hyperedges share $|B| = m \leq k/2$ vertices.
Removing the factor of $k^2$

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1. The number of adjacent hyperedges $\Delta k/m$
Removing the factor of $k^2$

Assume that adj. hyperedges share $|B| = m \leq k/2$ vertices.

1. The number of adjacent hyperedges $\Delta k/m$

2. An vertex in $C$ is updated $(k - m)$
Removing the factor of $k^2$

Assume that adj. hyperedges share $|B| = m \leq k/2$ vertices.

1. The number of adjacent hyperedges $\Delta k^2/m$
2. An vertex in $C$ is updated
   a. when $C$ is all one;
   b. when $B$ is all one and before every vertex in $B$ is updated at least once.

   $\approx 2^{-(k-m)}$
Removing the factor of $k^2$

Assume that adj. hyperedges share $|B| = m \leq k/2$ vertices.

1. The number of adjacent hyperedges $\Delta k/m$

2. An vertex in $C$ is updated $(k - m)$
   a. when $C$ is all one; $\approx 2^{-(k-m)}$
   b. when $B$ is all one and before every vertex in $B$ is updated at least once. $\approx m^{-1}$
Removing the factor of $k^2$

Assume that adj. hyperedges share $|B| = m \leq k/2$ vertices.

1. The number of adjacent hyperedges $\Delta k/m$
2. An vertex in C is updated $(k - m)$
   a. when C is all one; $\approx 2^{-(k-m)}$
   b. when B is all one and before every vertex in B is updated at least once. $\approx m^{-1}$

   (which corresponds to the time a SRW on hypercube $\{0,1\}^m$ stays at $(1, \ldots, 1)$ before mixing.)
Removing the factor of $k^2$

Assume that adj. hyperedges share $|B| = m \leq k/2$ vertices.

1. The number of adjacent hyperedges $\Delta$ $\frac{k(k-m)}{m^2} 2^{-(k-m)} \leq \Delta 2^{-k/2}$.

2. An vertex in $C$ is updated
   a. when $C$ is all one; $\Delta k/m$
   b. when $B$ is all one and before every vertex in $B$ is updated at least once.

   (which corresponds to the time a SRW on hypercube $\{0, 1\}^m$ stays at $(1, \ldots, 1)$ before mixing.)

The branching factor is $\max_{m \leq k/2} \Delta k(k-m)/m^2 2^{-(k-m)} \leq \Delta 2^{-k/2}$.
Worst Case

The worst case $m = k/2$ corresponds to reduction construction in the proof of hardness.
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In fact, restricting to linear hypergraphs (i.e. max overlap 1) improves the bound to $\Delta \leq 2^k / \text{poly}(k)$
Worst Case

The worst case $m = k/2$ corresponds to reduction construction in the proof of hardness.

Meanwhile, in [GJL17] the best case is achieved when the minimum overlap is $\Omega(k)$. 
There are three results uploaded to ArXiv around the same time last year.

<table>
<thead>
<tr>
<th></th>
<th>Monotone</th>
<th>Min overlap</th>
<th>$\Delta$</th>
<th>Type</th>
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<tbody>
<tr>
<td>[Moitra17]</td>
<td>No</td>
<td>No</td>
<td>$\leq e^{ck}$</td>
<td>FPTAS</td>
</tr>
<tr>
<td>[GJL17]</td>
<td>No</td>
<td>Yes</td>
<td>$\leq 2^{k/2}$</td>
<td>Exact</td>
</tr>
<tr>
<td>[HSZ17]</td>
<td>Yes</td>
<td>No</td>
<td>$\leq 2^{k/2}$</td>
<td>FPRAS</td>
</tr>
</tbody>
</table>

Any chance combining the results?
Open problem

1. Any “physical” interpretation to the threshold $O(2^{k/2})$?
2. How does the mixing time/sampling complexity change when we impose extra restrictions to $G$?
   We also showed that rapid mixing holds for linear hypergraphs with $\Delta \leq O(2^k/k^3)$.
   random regular graph with $\Delta \leq O(2^k/k) = O(\text{WSM})$.
3. General models?
Thank you!