Rapid mixing of hypergraph independent sets

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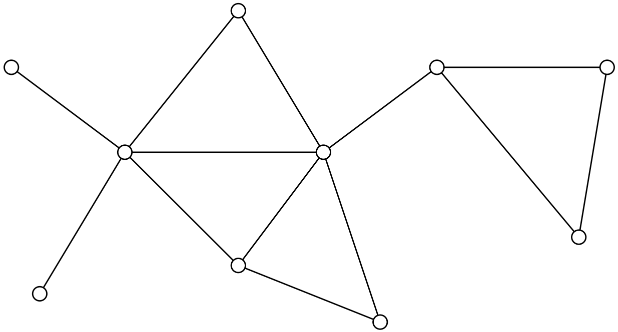
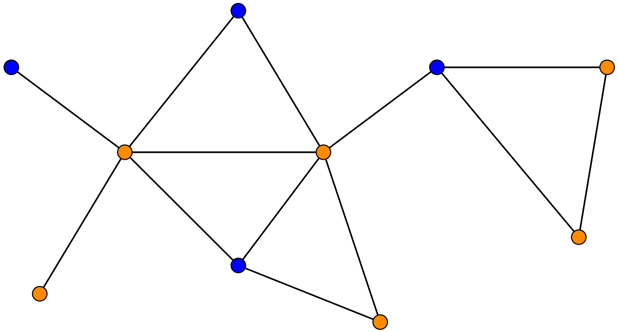
Joint work with Jonathan Hermon and Allan Sly

Independent sets on graph



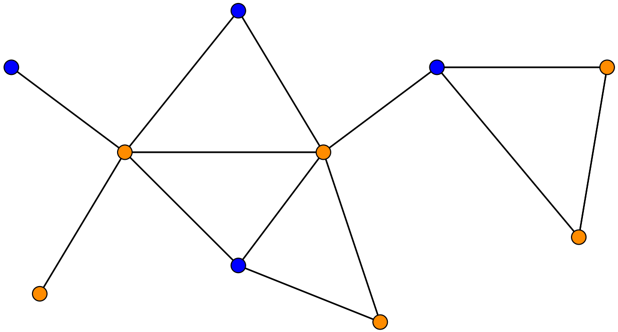
Independent sets on graph



An independent set ⇔ No edge has more than one 1.

⇔ Every edge has at least one 0.

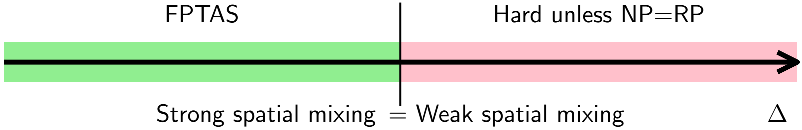
Independent sets on graph



An independent set $\Leftrightarrow$ No edge has more than one 1.
$\Leftrightarrow$ Every edge has at least one 0.

Want to approximate the partition function $Z(G)$

Phase Transition of Counting Complexity



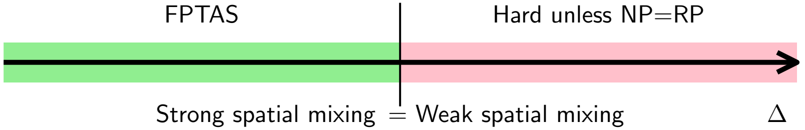
[Weitz 06], [Sly 10]

FPTAS for all graphs with maximum degree d

if and only if

Correlation decay on d-regular tree.

Phase Transition of Counting Complexity



[Weitz 06], [Sly 10], [Li-Lu-Yin 12], [Sly-Sun 12]

Similar result holds for other anti-ferromagnetic 2-spin systems.

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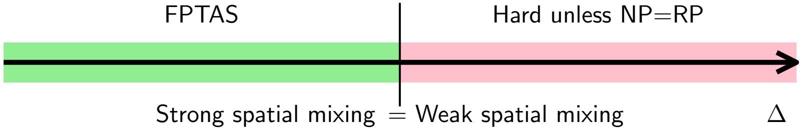

[Weitz 06], [Sly 10], [Li-Lu-Yin 12], [Sly-Sun 12]

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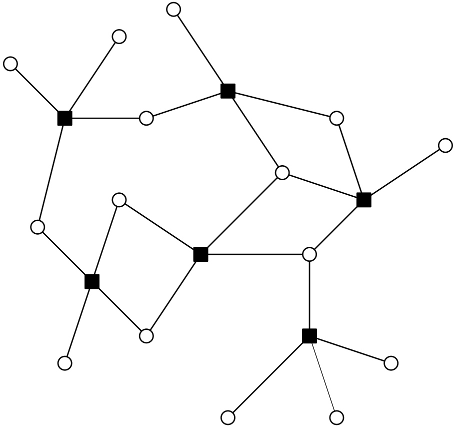
What about hypergraphs?

Independent Set on Hypergraphs



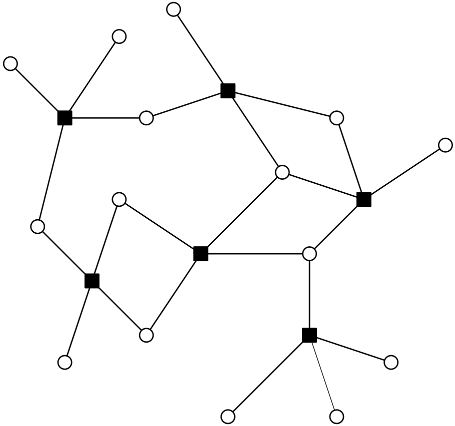
$G = (V, F, E)$

V: vertices(circles).

F: hyperedges(squares).

Degree Δ, Size k

Independent Set on Hypergraphs



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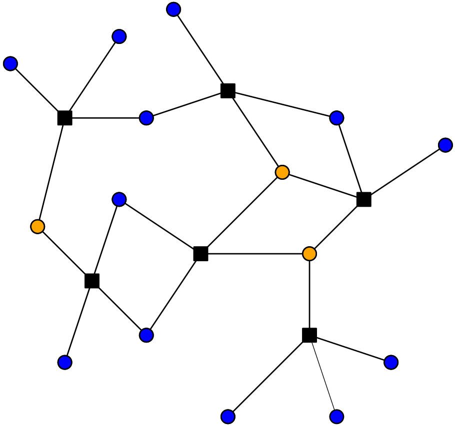
Independent Set on Hypergraphs



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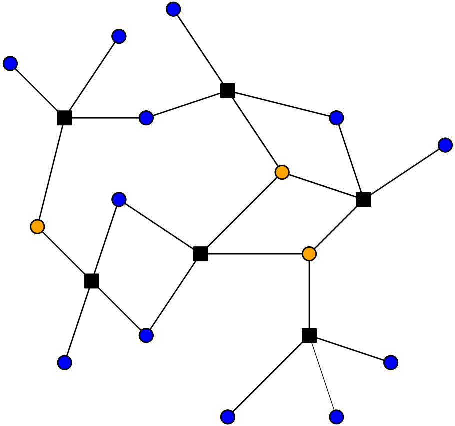

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V: vertices(circles).

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Hypergraph independent set: Every edge has at least one 0.

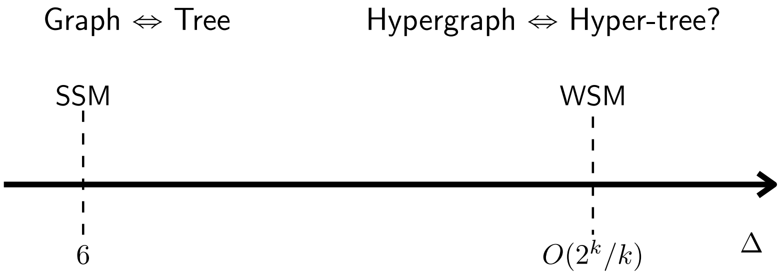
—Also known as monotone CNF.

Phase Transition of Counting Complexity

Graph $\Leftrightarrow$ Tree

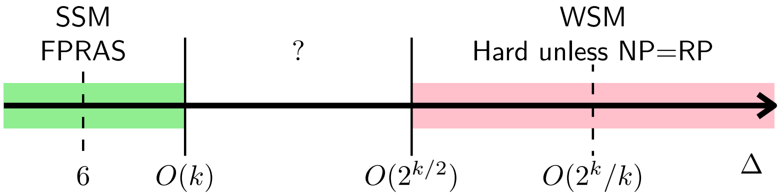
Hypergraph $\Leftrightarrow$ Hyper-tree?

Phase Transition of Counting Complexity



- SSM: correlation decay under boundary of arbitrary shape.
- WSM: correlation decay under boundary of complete tree.

Phase Transition of Counting Complexity

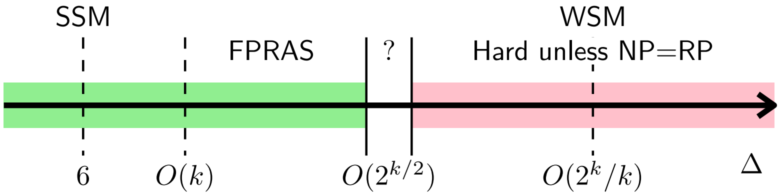


[BDK06] MCMC algo. for $\Delta \leqslant (k-1)/2$.

[BGGGS16] FPTAS for $\Delta \leqslant k$.

[BGGGS16] No approx. algo. unless NP=RP for $\Delta \geqslant 5 \cdot 2^{k/2}$.

Phase Transition of Counting Complexity



[Moitra17] FPTAS for $\Delta \leqslant e^{k/20}$.

[GJL17] Exact sampling for $\Delta \leqslant c2^{k/2}$ with min overlap.

Today MCMC algo. for $\Delta \leqslant c2^{k/2}$ (with monotonicity).

Main result

Glauber dynamics Pick a vertex, flip a coin,
set to new value whenver possible.

Mixing time $t_{\text{mix}} = \min\{t : d_{\text{TV}}(\mathbb{P}^t(\sigma, \cdot), \pi) < 1/4, \forall \sigma \in \Omega\}$.

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Corollary

Rapid mixing + self-reducibility $\Rightarrow$ FPRAS

Path Coupling

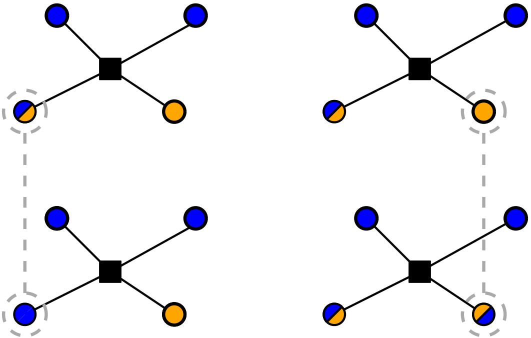
Theorem [Bubley-Dyer 97]

Let X_t, X'_t be two runs of the Glauber dynamics. If there exists a coupling of X, X' such that for **any** $d(X_0, X'_0) = 1$,

$$\mathbb{E}[d(X_1, X'_1) \mid X_0, X'_0] \leqslant 1 - \alpha/n$$

then the mixing time is $O(n \log n)$.

Path Coupling



$$\mathbb{E}[d(X_1, X_1') \mid X_0, X_0'] = 1 - \frac{1}{n} + \Delta \cdot \frac{1}{n} \cdot \frac{1}{2}.$$

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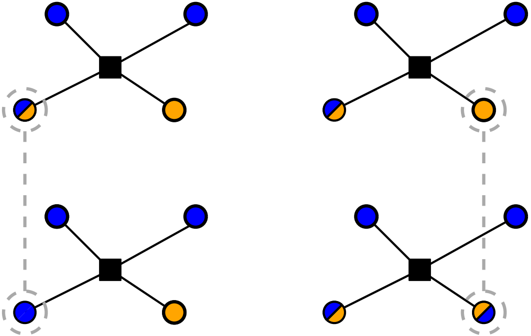

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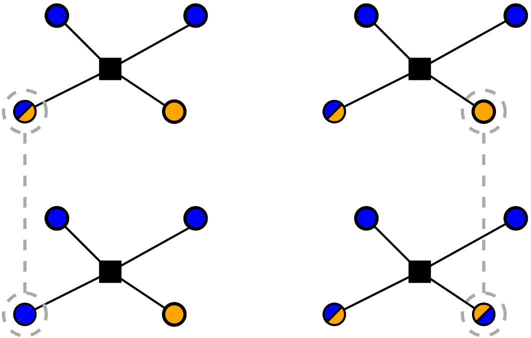

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New discrepancy is created if the other vertices in the same hyperedge are all 1, which is **increasingly unlikely** (2^{-k}).

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Union bounds such as the "vanilla" path coupling will blow up.

In this talk, we will take a more direct approach:

- Characterize the "interactions" – via analyzing the propagation of the discrepancies in the space-time graph.
- Bound their appearance – via an auxillary percolation.

Grand Coupling

and

the Propagation of Discrepancies

Grand Coupling

Continuous-time Glauber dynamics

- Place an independent Pois(1) clock at each vertex.
- At each alarm, flip a coin, set the value whenever possible.

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Continuous-time Glauber dynamics

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Let $(X_t^\sigma)_{t \geqslant 0}$ be the process on G starting from $\sigma \in \Omega_G$;
$(Y_t^s)_{t \geqslant s}$ be the process on empty graph starting at time s from the all 1 configuration.

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Let $(X_t^\sigma)_{t\geqslant 0}$ be the process on G starting from $\sigma \in \Omega_G$;
$(Y_t^s)_{t\geqslant s}$ be the process on empty graph starting at time s from the all 1 configuration.

$(Y_t^s)_{t\geqslant s}$ is identical to the lazy simple random walk on $\{0, 1\}^n$.

Grand Coupling

If we use the same clocks and coins for updating X_t's and Y_t's, then

$$X_t^{\sigma}(v) \leqslant Y_t^{0}(v) \leqslant Y_t^{s}(v), \quad \forall \sigma \in \Omega_G, 0 \leqslant s \leqslant t.$$

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Let $t_{\text{coup}} \equiv \min\{t : X_t^\sigma = X_t^\tau,\ \forall \sigma, \tau \in \Omega_G\}$. It follows that

$$t_{\text{mix}} \leqslant \min\{T : \mathbb{P}(t_{\text{coup}} > T) \leqslant 1/4\}.$$

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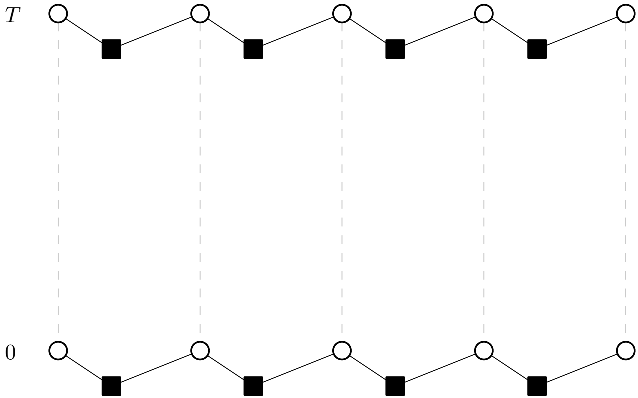
It is enough to bound $\mathbb{P}(t_{\text{coup}} > T)$ for $T = O(n \log n)$.

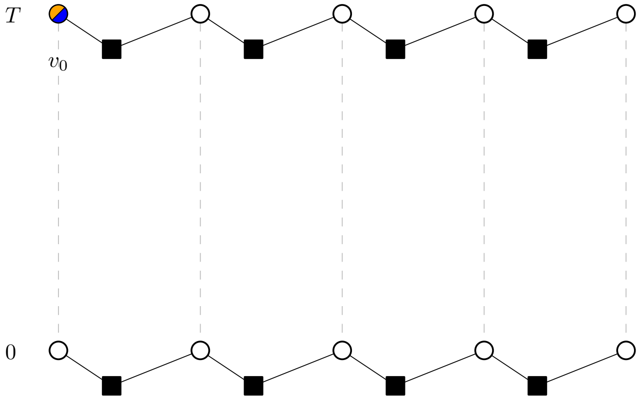
Tracing back the discrepancies

$t_{\text{coup}} \geqslant T$



Tracing back the discrepancies

$$t_{\mathsf{coup}} \geqslant T \Rightarrow \exists \sigma, \tau \in \Omega_G, v_0 \in V \text{ such that } X_T^{\sigma}(v_0) \neq X_T^{\tau}(v_0).$$



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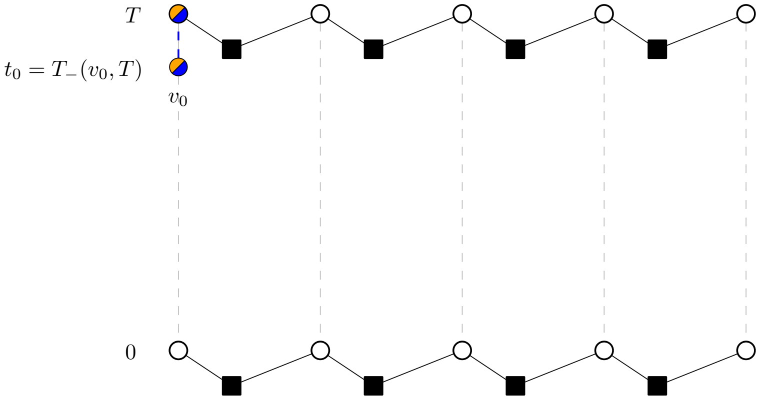

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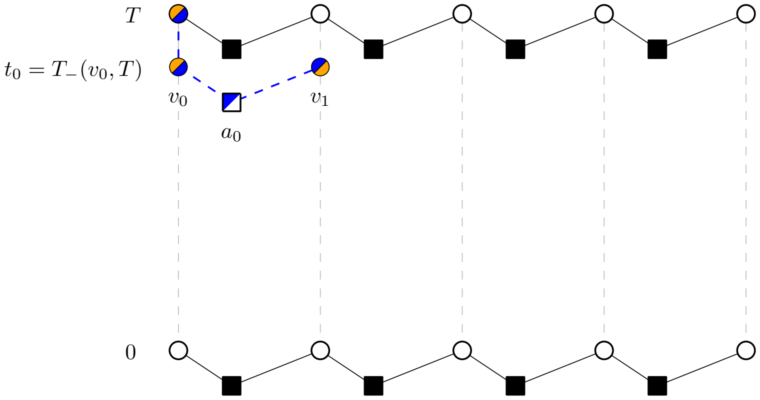

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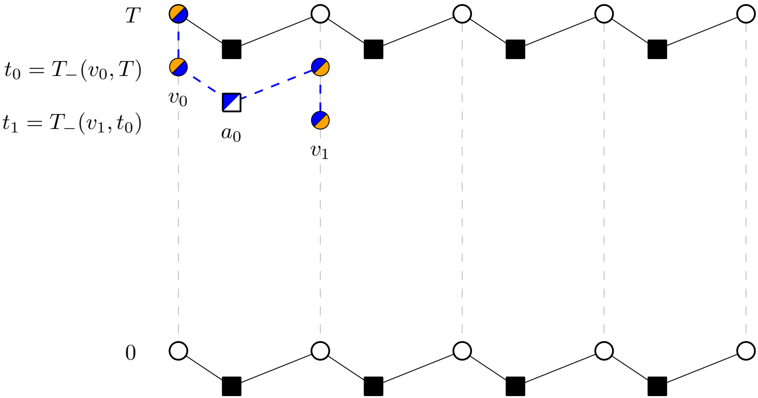

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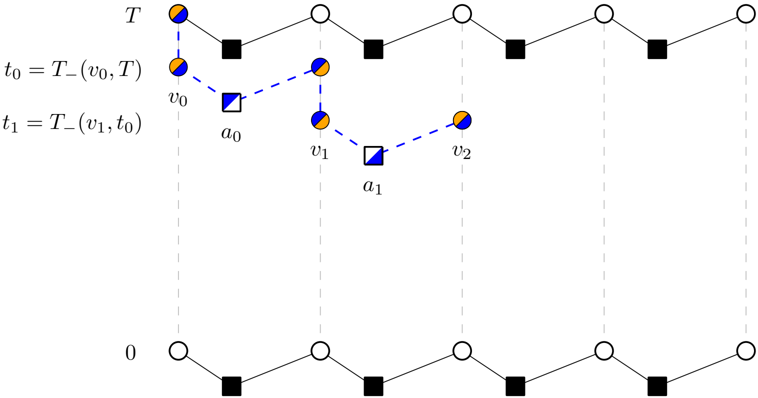

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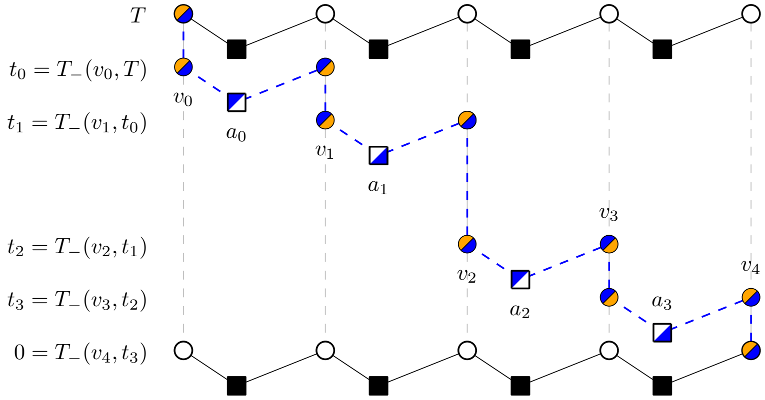
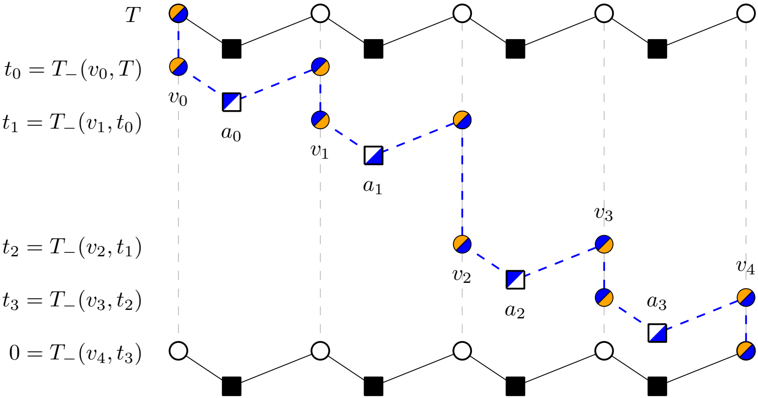
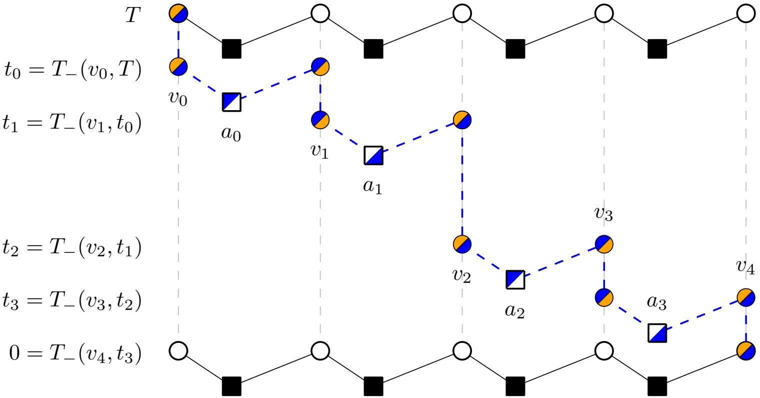


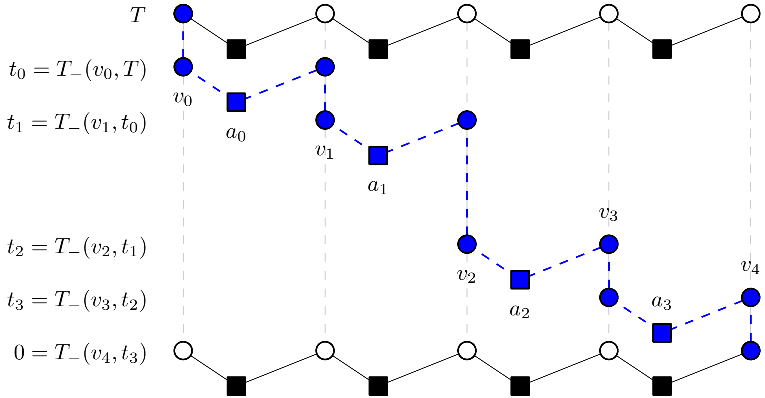


T
$t_0 = T_-(v_0, T)$
$t_1 = T_-(v_1, t_0)$
$t_2 = T_-(v_2, t_1)$
$t_3 = T_-(v_3, t_2)$
$0 = T_-(v_4, t_3)$
v_0
a_0
v_1
a_1
v_2
a_2
v_3
a_3
v_4



$$t_{\mathrm{coup}} \geqslant T \;\Rightarrow\; \exists\big((v_\ell, a_\ell, t_\ell)\big)_{0\leqslant \ell \leqslant L} \text{ such that for all } 0 \leqslant \ell \leqslant L,$$

$$(v_\ell, t_\ell) \in \text{Updates}, \quad t_\ell = T_-(v_\ell, t_{\ell-1}), \quad X^{\sigma}_{t_\ell} \vee X^{\tau}_{t_\ell}(\partial a_\ell \setminus \{v_\ell\}) = \underline{1}$$



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Discrepancy Paths

We say that $((v_\ell, a_\ell, t_\ell))_{0 \leqslant \ell \leqslant L}$ is a **discrepancy path** if

$$(v_\ell, t_\ell) \in \text{Updates}, \quad t_\ell = T_-(v_\ell, t_{\ell-1}), \quad Y^0_{t_\ell}(\partial a_\ell) = \underline{1}$$

It follows that

$$t_{\text{coup}} > T \Rightarrow \exists \textbf{ discrepancy path} \text{ with } t_{-1} = T.$$

Discrepancy Paths

To control the existence of **discrepancy paths** with $t_{-1} = T$:

- Construct a space-time percolation such that every discrepancy path maps to an open path in the percolation.
- Bound the open cluster of the percolation containing $t = 0$.

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Will show:
A weaker proof in 10 mins + modifications to the tight result.

Auxillary Percolation

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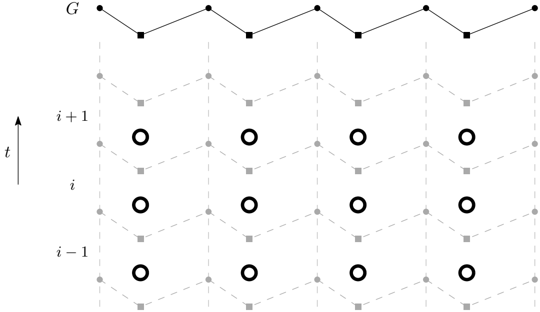
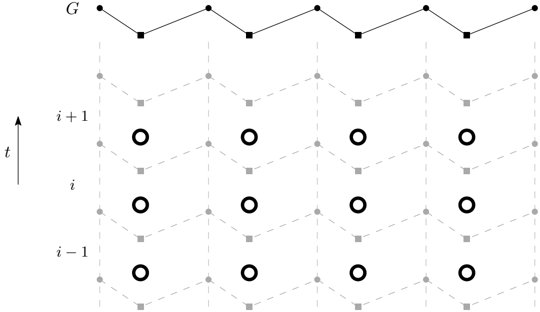


Break the time line into chunks of length $2k$ and define

$$H \equiv F \times \mathbb{Z}_+ \equiv F \times \{0, 1, \dots\}.$$

Each site $(a, i) \in H$ represents the block $\partial a \times [2ik, 2(i+1)k)$.

Auxillary Percolation



We say that $(a, i) \in H$ is **bad** if $Y_t^{2(i-1)k}(v) = 1$ for

Horizontal: some $t \in [2ik, 2(i+1)k)$ and all $v \in \partial a$.

or **Vertical:** all $t \in [2ik, 2(i+1)k)$ and some $v \in \partial a$.

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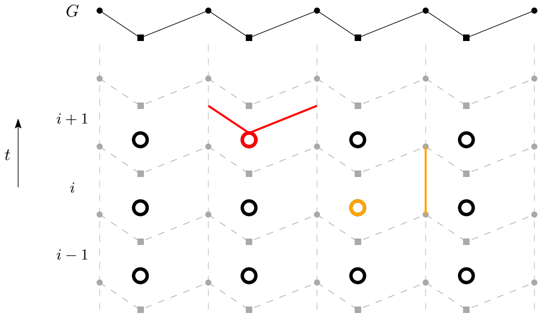

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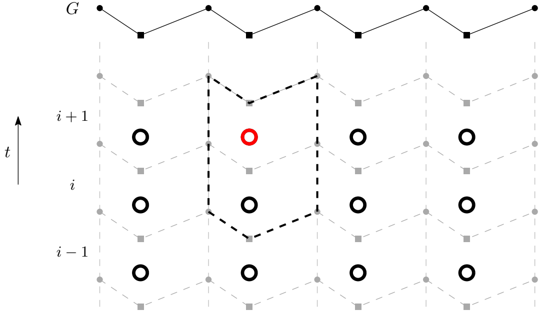

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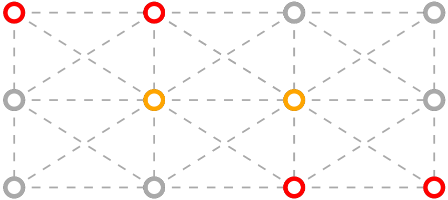
This definition is local.

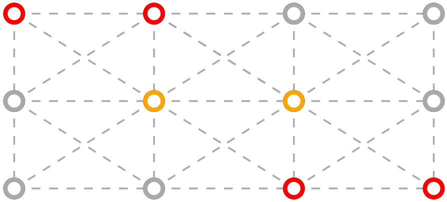
Control the Percolation



Draw an edge between $(a, i), (b, j) \in H$ if and only if

$$\partial a \cap \partial b \neq \varnothing, \quad |i - j| \leqslant 1.$$

Control the Percolation

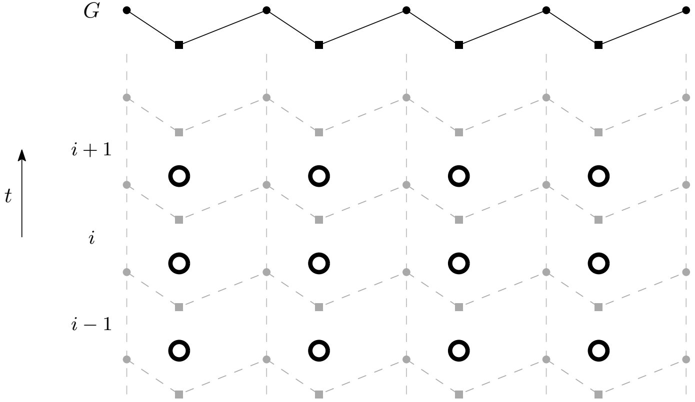


Draw an edge between $(a, i), (b, j) \in H$ if and only if

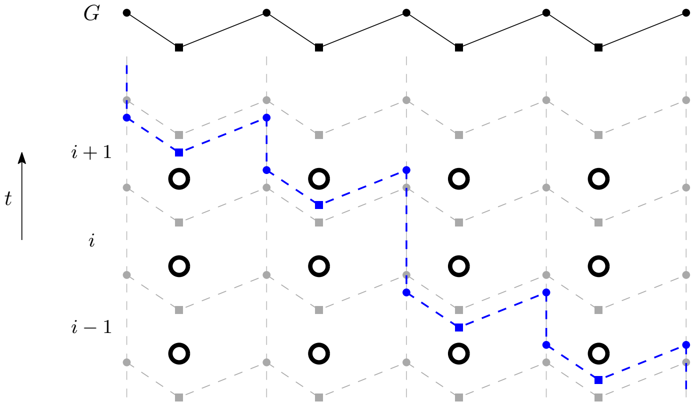
$$\partial a \cap \partial b \neq \varnothing, \quad |i - j| \leqslant 1.$$

Claim: If (a, i) is not adjacent to (b, j), then

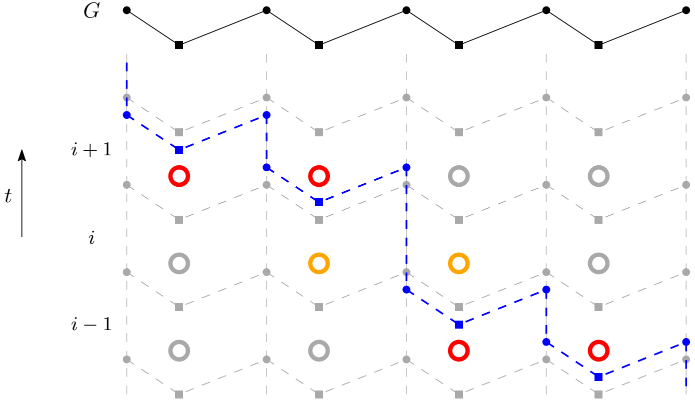
$$\{(a, i) \text{ is } \textbf{bad}\} \perp\!\!\!\perp \{(b, j) \text{ is } \textbf{bad}\}.$$



G
i + 1
t
i
i − 1



Fix a **discrepancy path**, analyze the sites (a, i)'s along it.



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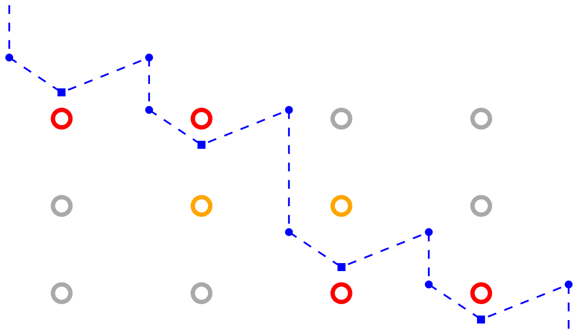

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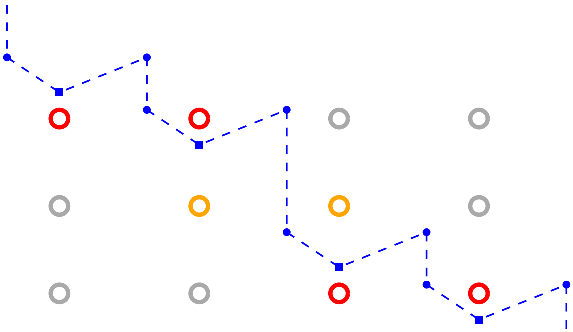
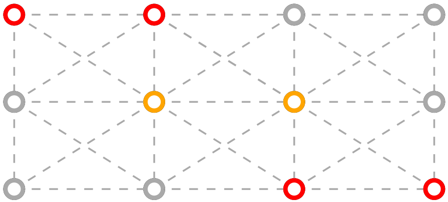
$\Rightarrow$ Each (a, i) is either **bad** or next to some (b, i) that is bad.



Fix a **discrepancy path**, analyze the sites (a, i)'s along it.
$\Rightarrow$ Each (a, i) is either **bad** or next to some (b, i) that is bad.
$\Rightarrow$ **discrepancy path** $\subseteq$ **open cluster** of the percolation
if we allow the cluster to connect through diagonals.

Control the Percolation



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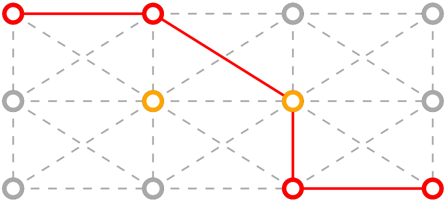

We say that a path $\gamma = ((a_\ell, i_\ell))_{0 \leqslant \ell \leqslant L}$ in H is a **minimal path** if (a_ℓ, i_ℓ) is not adjacent to $(a_{\ell'}, i_{\ell'})$ for all $\ell \geqslant \ell' + 2$.

$$\Rightarrow \ \{(a_\ell, i_\ell) \text{ is } \textbf{bad}\} \perp\!\!\!\perp \{(a_{\ell'}, i_{\ell'}) \text{ is } \textbf{bad}\}, \ \forall \ell \geqslant \ell' + 2.$$

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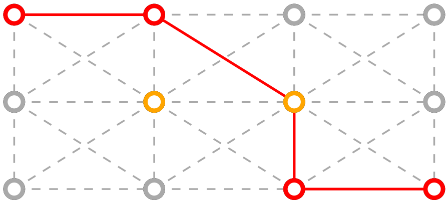

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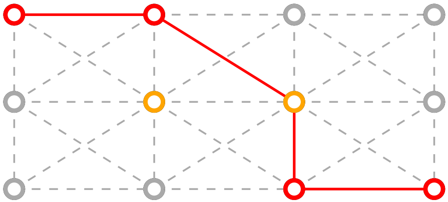
Any cluseter contains a **minimal path** as subset.

Control the Percolation



$t_{\text{coup}} > T$

$\Rightarrow \exists$ **discrepancy path** with $t_{-1} = T$.

$\Rightarrow \exists$ **open cluster** in H connecting $F \times \{0\}$ and $F \times \{\lfloor T/k \rfloor\}$.

$\Rightarrow \exists$ **open minimal path** in H with length $L = \lfloor T/k \rfloor$.

Control the Percolation

Recall that $Y_t(\partial a)$ is just the LSRW on $\{0, 1\}^k$.

$$\Rightarrow \mathbb{P}((a, i) \text{ is } \textbf{bad}) \leqslant 2k^2 2^{-k} + (k+1)e^{-k} \approx 2k^2 2^{-k}.$$

Control the Percolation

Recall that $Y_t(\partial a)$ is just the LSRW on $\{0, 1\}^k$.

$$\Rightarrow \mathbb{P}((a, i) \text{ is } \textbf{bad}) \leqslant 2k^2 2^{-k} + (k+1)e^{-k} \approx 2k^2 2^{-k}.$$

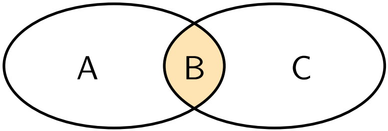
Note that the maximum degree of H is $(2\Delta k + 1)$

$$\begin{aligned}
\mathbb{P}(t_{\text{coup}} > T) &\leqslant \mathbb{P}(\exists \textbf{ open minimal path} \text{ with length } \lfloor T/k \rfloor) \\
&\leqslant \sum_{((a_\ell, i_\ell))_{0 \leqslant \ell \leqslant L}} \prod_{r=0}^{\lfloor L/2 \rfloor} \mathbb{P}((a_{2r}, i_{2r}) \text{ is } \textbf{bad}) \\
&\leqslant n(2\Delta k + 1)^L (2k^2 2^{-k})^{\lfloor L/2 \rfloor} \\
&\to 0, \quad \text{if } \Delta \leqslant O(k^{-2} 2^{k/2}),\ L = \Omega(\log n).
\end{aligned}$$

Removing the factor of k^2

Removing the factor of k^2

For a discrepancy passes to an adjacent hyperedge before being killed by an update, we need:



a vertex in C is updated:

1. when C is all 1;
2. when B is all 1 before each vertex in B is updated once.

Removing the factor of k^2

Recall the key step from previous slide:

$$\mathbb{P}(t_{\text{coup}} \leqslant T) \leqslant Cn(\Delta k)^L (k^2 2^{-k})^{\lfloor L/2 \rfloor}$$

The "branching number" consists of two part:

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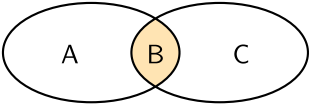
Removing the factor of k^2

Assume that adj. hyperedges share $|B| = m \leqslant k/2$ vertices.



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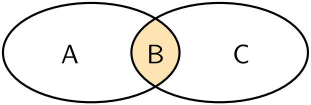

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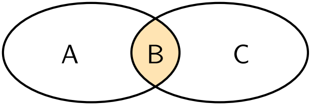

1. The number of adjacent hyperedges —$\Delta k/m$
2. An vertex in C is updated —$(k - m)$

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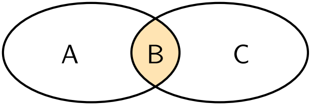

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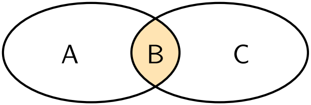

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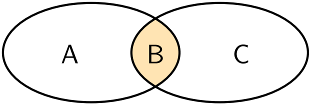

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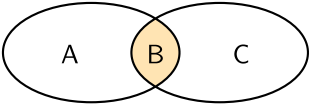

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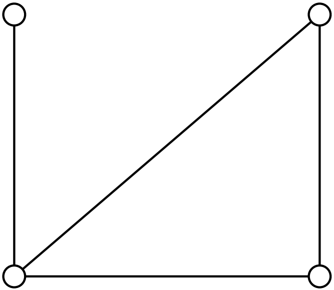
   (which corresponds to the time a SRW on hypercube $\{0, 1\}^m$ stays at $(1, \ldots, 1)$ before mixing.)

The branching factor is $\max_{m \leqslant k/2} \Delta \frac{k(k-m)}{m^2} 2^{-(k-m)} \lesssim \Delta 2^{-k/2}$.

Worst Case

The worst case $m = k/2$ corresponds to reduction construction in the proof of hardness.



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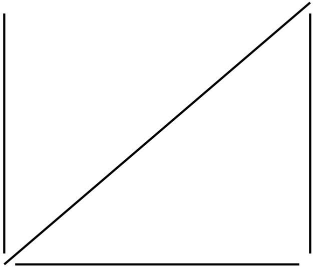

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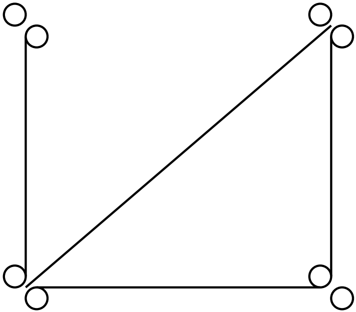

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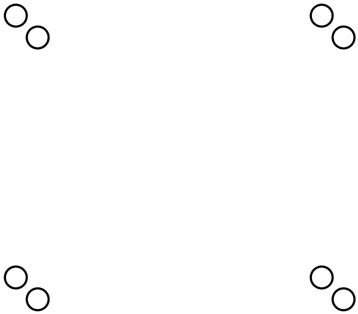

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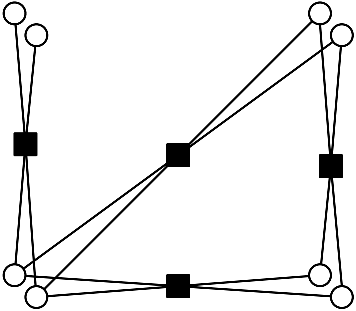

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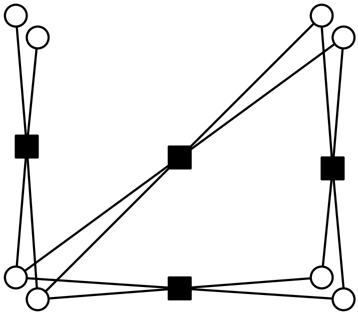

In fact, restricting to linear hypergraphs (i.e. max overlap 1) improves the bound to $\Delta \leqslant 2^k/\text{poly}(k)$

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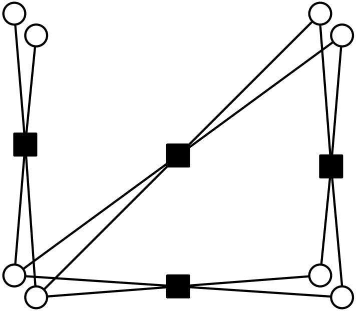

Meanwhile, in [GJL17] the best case is achieved when the minimum overlap is $\Omega(k)$.

Open problem

There are three results uploaded to ArXiv around the same time last year.

	Monotone	Min overlap	Δ	Type
[Moitra17]	No	No	$\leqslant e^{ck}$	FPTAS
[GJL17]	No	Yes	$\leqslant 2^{k/2}$	Exact
[HSZ17]	Yes	No	$\leqslant 2^{k/2}$	FPRAS

Any chance combining the results?

Open problem

1. Any "physical" interperatation to the threshold $O(2^{k/2})$?
2. How does the mixing time/sampling complexity changes when we impose extra restrictions to G?
   We also showed that rapid mixing holds for
   linear hypergraphs with $\Delta \leqslant O(2^k/k^3)$.
   random regular graph with $\Delta \leqslant O(2^k/k) = O(\mathrm{WSM})$.
3. General models?

Thank you!