Reinforcement Learning with Rich Observations

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What is Reinforcement Learning?
How to Learn?

Practice

Powerful modeling, simple exploration

e.g.: Atari Deep Reinforcement Learning

Theory

Sophisticated exploration in small-state MDPs

e.g. $E^3$, R-MAX algorithms

Limited theory for rich observations

Goal

Develop Reinforcement Learning approaches guaranteed to learn an optimal policy with a small number of samples despite rich observations.
Our Results

<table>
<thead>
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</tr>
</thead>
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</tr>
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</tr>
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Key ideas

- New measure of the hardness of exploration
- Algorithm with sample complexity scaling with this measure
- Applications in several RL settings
Model
Markov Decision Processes (MDPs)

$x_1 \sim \Gamma_1$
Markov Decision Processes (MDPs)

\[ x_1 \sim \Gamma_1 \]

Take action \( a_1 \), Observe \( r_1(a_1) \)
Markov Decision Processes (MDPs)

- Episodic: $H$ actions in a trajectory
- Layered: Distinct states at each level
- Markovian: $x_n$ only depends on $(x_{n-1}, a_{n-1}), r_n$ on $(x_n, a_n)$

$x_1 \sim \Gamma_1$

Take action $a_1$, Observe $r_1(a_1)$

New state $x_2 \sim \Gamma(x_1, a_1)$
Goal of Learning

- Maximize long-term reward

\[
\sum_{h=1}^{H} r_h(a_h)
\]
Goal of Learning

- Maximize long-term reward...using policies

\[ \sum_{h=1}^{H} r_h(\pi(x_h)) \]

- Policies are mappings from states to actions
Example: Navigation in a toy setting

Robotic agent navigating in real-world (left)
States: Position in a grid
Actions: Forward/Back/Left/Right
Reward: 1 on reaching target, -100 for dying
Example: Navigation in a real setting

Robotic agent navigating in real-world (right)
States: Camera view in front of the robot
Actions: Forward/Back/Left/Right
Reward: 1 on reaching target, -100 for dying
Example: Web Search

- User comes with an intent
- Issues a query to the search engine
- Receives ranked list of results
- Issues another query
- ...

States: Query, info on user
Actions: Search results
Reward: 1 when user finds a satisfactory result
Existing results

Learn $\epsilon$-optimal policy using $\text{poly}\left(|X|, A, H, \frac{1}{\epsilon}\right)$ samples

Small number of states necessary for learning
Lower bound

There is an MDP with $|A|^H$ states where finding an $\epsilon$-optimal policy requires $\Omega\left(\frac{|A|^H}{\epsilon^2}\right)$ trajectories.

Intuition: Embed a bandit problem with $|A|^H$ arms.

Compact $\mathcal{F}$ not sufficient for generalization in RL
Gathering the right data has large sample complexity
Large-state MDPs

- Too many “unique” states in real-world tasks
- Cannot reason separately for each state
- Need information sharing between similar states
  - aka generalization
- Typically done via value-function approximation
Function Approximation
Optimal value function

- **Optimal value function** $Q^*$
  - Maps $(x, a)$ pair to a long-term reward
  - Take action $a$ in state $x$ and follow the optimal policy thereafter
  - Removes the need to reason over multiple decisions

- **Optimal policy** $\pi^*(x) = \text{argmax}_a Q^*(x, a)$
Value of a policy

- Value: Long-term reward on following a policy

\[ V(\pi) = E_{x \sim \Gamma_1} [V(x, \pi)], \text{ where} \]

\[ V(x, \pi) = E_{r \sim D_x} \left[ r(\pi(x)) + E_{x' \sim \Gamma(x, \pi(x))} V(x', \pi) \right] \]

- Distribution of initial state
- Distribution of next state
- Instantaneous reward
Optimal value function

- Optimal value function: Best reward from each state

\[ V^* = \mathbb{E}_{x \sim \Gamma_1} [V^*(x)], \text{ where} \]

\[ V^*(x) = \max_a \mathbb{E}_{r \sim D_x} \left[ r(a) + \mathbb{E}_{x' \sim \Gamma(x,a)} V^*(x') \right] \]
Optimal value function

- Optimal value function: Best reward from each state

\[ V^* = E_{s \sim \Gamma_1}[V^*(x)], \text{ where} \]

\[ V^*(x) = \max_a E_{r \sim D_x}[r(a) + E_{x' \sim \Gamma(x,a)} V^*(x')] \]

Optimal policy: \( \pi^*(x) = \arg\max_a Q^*(x,a) \)
Function approximation

Given a class $\mathcal{F} : X \times A \to \mathbb{R}$, find a good approximation to $Q^*$, assuming $Q^* \in \mathcal{F}$

Associated greedy policy: $\pi_f = \text{argmax}_a f(x, a)$

- Key intuition: Use a class $\mathcal{F}$ that generalizes well in supervised learning
  - Consider $\mathcal{F}$ of small VC-dimension/Rademacher complexity/finite size...
A solution sketch

- Start with an initial guess $f_1 \in \mathcal{F}$ for $Q^*$
- Act according to $f_1$, collect trajectories
  $$(x_1, a_1, r_1, \ldots, x_H, a_H, r_H)$$ where $a_h = \pi_{f_1}(x_h)$
- Use the trajectories to obtain a better estimate $f_2 \in \mathcal{F}$
- Repeat
A solution sketch

- Start with an initial guess $f_1 \in \mathcal{F}$ for $Q^*$
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- Use the trajectories to obtain a better estimate $f_2 \in \mathcal{F}$
- Repeat
Our Setting
Bellman Equations

\[ Q^*(x, a) = \mathbb{E}_{r \sim D_x}[r(a) + \mathbb{E}_{x' \sim \Gamma(x,a)} V^*(x')] \]

- Take \( a \) at current step, act optimally thereafter
Bellman Equations

\[ Q^*(x, a) = \mathbb{E}_{r \sim D_x} \left[ r(a) + \mathbb{E}_{x' \sim \Gamma(x, a)} \max_{a'} Q^*(x', a') \right] \]

- Take \( a \) at current step, act optimally thereafter
Bellman Equations

\[ Q^*(x, a) = \mathbb{E}_{r \sim D_x} \left[ r(a) + \mathbb{E}_{x' \sim \Gamma(x, a)} Q^*(x', \pi^*(x')) \right] \]

- Take \( a \) at current step, act optimally thereafter
- Holds for each \( x \), hence any distribution over \( x \)
Bellman Equations

\[ \varepsilon(f, \pi, h) = \mathbb{E}[f(x_h, a_h) - r_h - f(x_{h+1}, a_{h+1})], \]

where \( a_1, \ldots, a_{h-1} \sim \pi \) and \( a_h, a_{h+1} \) according to \( \pi_f \)
Bellman Equations

\[ \varepsilon(f, \pi, h) = \mathbb{E}[f(x_h, a_h) - r_h - f(x_{h+1}, a_{h+1})], \]

where \( a_1, \ldots, a_{h-1} \sim \pi \) and \( a_h, a_{h+1} \) according to \( \pi_f \)

Standard result: \( \varepsilon(Q^*, \pi, h) = 0 \), for all \( \pi \)

Gives a test for checking if \( f \approx Q^* \)
Using Bellman Equations

- Given candidate \( f \in \mathcal{F} \), check \( \varepsilon(f, \pi, h) \) for all \( \pi, h \)
- Reject \( f \) if \( \varepsilon(f, \pi, h) \gg 0 \) for any \( \pi, h \)
- Restrict to \( \pi = \pi_g \) for \( g \in \mathcal{F} \)

Challenge: Computing \( \varepsilon(f, \pi, h) \) requires samples from \( \pi \)
For all \( \pi_g \) requires \( O(|\mathcal{F}|) \) samples!
Key challenges

- Too many functions in any interesting $F$
- Data based on one $f$ might not prove sub-optimality for others
- Need to collect the right data
- Like bandits, but with exponentially many arms!
Bellman factorization and rank

- Consider the $|\mathcal{F}| \times |\mathcal{F}|$ matrix:
  \[ \varepsilon(\mathcal{F}, h)_{f,g} = \varepsilon(f, \pi_g, h) \]

  Bellman rank of an MDP is the rank of $\varepsilon(\mathcal{F}, h)$

- Bounded by number of states
- Bounded by rank of transition matrix $\Gamma$
- Bounded by number of “hidden” states
Example: Navigation in a real setting

Robotic agent navigating in real-world (right)
States: Camera view in front of the robot
Transitions determined by grid-view (left)
Bellman rank bounded by size of grid!
Example: Web Search

- User comes with an intent
- Issues a query to the search engine
- Receives ranked list of results
- Issues another query
- ...

States: Query, info on user
Transitions often depend on user intent
Bellman rank bounded by number of possible intents (topics)
Summary so far

- Given function approximators $f \in \mathcal{F}$
- Want to find an $f$ such that
  - $f$ is valid
  - $f$ yields a good policy, that is $V(\pi_f)$ is large

- Algorithm intuition: Low Bellman rank gives concise basis for checking validity (exploration)

- Challenge: We do not know the basis, just its existence
Our Algorithm
Reminder...

- $Q^*$ is valid for state distribution under any policy $\pi$
- $Q^*$ captures the optimal value:

$$V(\pi^*) = \mathbb{E}_{x \sim \Gamma_1} \max_a [Q^*(x, a)]$$
$$= \mathbb{E}_{x \sim \Gamma_1} Q^*(x, \pi^*(x))$$
Optimism Led Iterative Value-function Elimination (OLIVE)

- $F_0 = F$
- For $t=1,2,...$
  - Choose $f_t$ to maximize $\hat{V} = E_{x \sim \Gamma_1}[f(x, \pi_f(x))]$
  - Collect trajectories using $\pi_t = \pi_{f_t}$
  - If $V(\pi_t) \geq \hat{V} - \epsilon$
    - Return $\pi_t$
  - Reject all $f$ with large $\epsilon(f, \pi_t, h)$ for any $h$
  - Set $F_t$ to be the set of surviving $f$

Optimism under uncertainty, guess for $V(\pi^*)$ if $f = Q^*$

Checking our optimistic belief

Prune the possible solutions
Suppose $Q^* \in \mathcal{F}$, and the Bellman rank is at most $M$. OLIVE returns a policy $\pi$ satisfying $V(\pi) \geq V(\pi^*) - \epsilon$ and with probability $1 - \delta$, the number of trajectories needed is at most

$$O\left(\frac{M^2H^3|A|^2 \log(|\mathcal{F}|/\delta)}{\epsilon^2}\right)$$
Implications

- **Retains sample-efficiency for small-state MDPs**
  - albeit better results exist here

- **New results for several settings**
  - Low-rank MDPs
  - Reactive POMDPs
  - Reactive PSRs
  - ...

- **Unifying treatment for sample-efficient RL**
Proof intuition (correctness)

Algorithm always retains $Q^*$, terminates when:

$$V(\pi_t) \geq \hat{V} - \epsilon \geq V^* - \epsilon$$

- Either $f$ found in first step is near-optimal
- Or, we will reject it
- Shows correctness
Proof intuition (sample efficiency)

- Bellman error matrix has low rank
- Each elimination step decreases rank by 1 if we check for $\varepsilon(f, \pi, h) = 0$
- Extension to noisy checking: Ellipsoidal argument
  - Reduce the volume of “Bellman error vectors” by constant fraction each time
Extensions

- Do not require $Q^* \in \mathcal{F}$
  - Find the valid $f$ with largest $V(\pi_f)$

- Adapt to the knowledge of $M$

- Allow errors in Bellman factorization and validity

- Allow infinite classes $\mathcal{F}$ with low VC-like dimensions
Wrapping up

- New structural condition for efficient exploration in RL
- First sample-complexity results in a broad setup called Contextual Decision Processes
  - Unifying treatment for several RL models
- Algorithm robust to modeling assumptions
- Key open problem: Computational efficiency
Thank You!
Details at: https://arxiv.org/abs/1610.09512