Uncertainty in Algorithmic Mechanism Design

Anna R. Karlin
University of Washington
An Example

Classical Optimization Problem:

**Maximum Weighted Matching**

**Input:** Weighted Bipartite Graph

**Output:** Matching that maximizes the sum of matched edge weights.
An Example

Classical Optimization Problem:

**Maximum Weighted Matching**

**Input:** Weighted Bipartite Graph

**Output:** Matching that maximizes the sum of matched edge weights.

![Graph]

- Edges with weights: 2, 3, 1, 5, 1
Example Application

Selling advertisement slots

- A search engine has advertising slots for sale
- Advertisers are willing to pay different amount to have their ad shown in a particular slot.

Suppose search engine wants to make as much money as possible.
The values are private!

Algorithm must solicit values.

Advertisers may lie to get a better deal.
The values are private!

Algorithm must solicit values.

Advertisers may lie to get a better deal.

This is a game!
Google designs the game.
The advertisers play the game.
Big Picture

• Many problems where input is the private data of agents who will act selfishly to promote their own best interests
  • Resource allocation
  • Routing and congestion control
  • Electronic commerce

Mechanism design:
How do we optimize in a selfish world?
Real world mechanism design settings

• Sponsored search auctions; display advertising

“What most people don’t know is that all that money comes in pennies at a time.”
Hal Varian, Google Chief Economist

Google revenue in 2015 was approximately $74,500,000,000.
Real world mechanism design settings

- Sponsored search auctions; display advertising
- FCC spectrum auctions
- Kidney exchange
- Healthcare systems
- Recommendation systems
- Routing on the Internet
- Resource allocation in the cloud
- Platform design for a sharing economy
- Energy and electricity markets
- Bitcoin
- Participatory democracy
- Crowdsourcing
What characterizes these problems?

• Many participants with
  • diverse incentives
  • private information of each agent unknown to designer and other agents (maybe even to themselves!)
  • varying attitudes towards risk.
  • varying degrees of myopia

• Complex optimization problems

• Dynamic and repeated interaction
Plan for talk

• Meander...
  • Posted prices via prophet inequalities
  • Prior-independent and prior-free auctions, sample complexity
  • AGT and learning

Apologies: incomplete references.
Applications of prophet inequalities
Prophet inequalities, reminder

- Sequence of prizes: $V_1 \sim F_1$, $V_2 \sim F_2$, $\ldots$, $V_n \sim F_n$
- You know all the priors.
- See them one at a time and make an irrevocable decision at that moment whether or not to keep it. Once done, game over.
- Compete with prophet who gets expected reward $\mathbb{E}(\max_i V_i)$

**Version 1:** Take the first prize that is above $\frac{1}{2} \mathbb{E}(\max_i V_i)$

**Version 2:** Choose threshold $t$ such that $\Pr(\text{there is any value is higher than } t) = \frac{1}{2}$
Take the first one above $t$.

** Guarantee:** expected value of prize selected $\frac{1}{2} \mathbb{E}(\max_i V_i)$
Bidder’s goal: maximize utility = value - payment

Maximize social surplus: allocate to bidder with maximum value.

Vickrey Second-price Auction: Allocate to highest bidder at second highest bid

Incentivizes truth-telling, i.e. \( b_1 := v_1 \) no matter what others bid

\[
\begin{align*}
\nu_1 &= 100 \\
\nu_2 &= 80 \\
\text{Truthful bidding:} \\
\nu_1 &= 100 \text{ at } b_1 \\
\nu_2 &= 80 \text{ at } b_2 \\
\text{With collusion, say} \\
\nu_2 &= 10 \\
\text{Bidder 1 pays him}$ 50 \\
u_1 &= 100 - 10 - 50 = 40 \\
u_2 &= 50
\end{align*}
\]
Issues with Vickrey Auction

• Collusion.
• Can be slow and inconvenient.
• May require more communication than we would like.
• All bidders needs to be present over the time the auction is run.

• But, if we are willing to settle for approximate optimality, and we have a prior...
• That is, we know that \( V_i \sim F_i \) independently.
Use posted price based on prophet inequality

- Choose threshold $t$ such that $\Pr(\text{there is any bidder whose value is higher than } t) = \frac{1}{2}$
- Post a price of $t$; whoever grabs item first, gets it.
- This guarantees that the expected surplus of the outcome (value of winning bidder) is $\frac{1}{2} \mathbb{E}(\max_i v_i)$

**Features:**
- Very simple to implement: auction described by one number
- Very simple for agents to “play”.
- Agent doesn’t even need to know exact value.
- Robust: it doesn’t matter if bidders aren’t all there at the same time, or come in some arbitrary or even worst-case order.
- Resilient to collusion.
- It doesn’t matter if distributions change above or below the threshold!
Beautiful generalization
[Feldman, Gravin, Lucier]
Combinatorial auction:
m items, n bidders each with private submodular \( v_i(\cdot) \)
Goal: allocate items to maximize \( \sum_i v_i(S_i) \)
Combinatorial auctions, surplus maximization, submodular bidders

- Algorithmic version: 1-1/e approx.
  - [Dobzinski, Shapira][Vondrak][Feige]

- The best known truthful auction achieves $O(\sqrt{\log m})$ approx.
  - [Dobzinski]

- Simultaneous first-price auctions achieve 1-1/e approx. at every BNE
  - [Christodoulou, Kovacs, Schapira][Hassidim, Kaplan, Mansour, Nisan] [Syrgkanis, Tardos]

- [Feldman, Gravin, Lucier] constant approximation via posted prices, given priors.
Given priors, compute posted prices

Utility of bidder = value of bundle - price

utility of 

= \( v(\text{kindle} + \text{tap}) - (99 + 114) \)

Prices guarantee constant fraction of the optimal expected surplus!
Note

• Here we don’t even know how to design a truthful mechanism that gets close to the same approximation ratio as we can get in the algorithmic setting.

• One of the big open questions in mechanism design:
  • to what extent is computability with incentive compatibility harder than without. [Nisan, Ronen]
Profit Maximization: prophet inequalities very useful here too!
What if we want to design an auction to maximize auctioneer revenue?

assuming known priors
completely solved by [Myerson]
Simplest case: 1 bidder

\[ V \sim F \]

Optimal auction: posted price

Best price to offer: \[ p^* = \arg\max_p \quad p(1 - F(p)) \]

Called \textit{monopoly price} for the distribution.
Maximizing revenue

Bayesian setting: \( V_i \sim F \)

Under *i.i.d. assumption* 
[Myerson]

- The optimal auction is just Vickrey with a monopoly reserve price.
- Reserve price \( r \): like an opening bid on eBay
- Truthful auction

**Vickrey auction:**

- Winner = highest bidder above \( r \), if any
- Price = maximum of \( r \) and 2\(^{nd} \) highest bid
Beyond i.i.d.

Bayesian setting: $V_i \sim F_i$ independently

[Myerson]

• Ask bidders to report values
• For each bidder, compute virtual value

$$\psi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$$

$\mathbb{E}$(revenue of an auction) = $\mathbb{E}$ (virtual value of winner)

Optimal auction*: Allocate to bidder with highest $\psi(v_i)$ (if positive) breaking ties by value.

* distributions regular
Example

- Bidder 1 has value drawn $\exp(1)$
- Bidder 2 has value drawn $U[0,1]$
- $(v_1, v_2) = (1.5, 0.8) \Rightarrow$ bidder 2 wins (and pays 0.75)

Indep, but not identical $\rightarrow$

*optimal auction complicated, unintuitive and depends on detailed distributional information!*

Is there a simpler, more practical, more robust way, if we are willing to sacrifice optimality?
Prophet inequalities to the rescue!
[Chawla, Hartline, Kleinberg] [Chawla, Hartline, Malec, Sivan]

• Myerson tells us that our revenue will be the expected virtual value of winning bidder.

• To apply prophet inequality, think of \( \varphi_i(V_i)^+ \) as i\(^{th}\) prize.

• Choose \( t \) so that \( \mathbb{P}(\max_i \varphi_i(V_i)^+ \geq t) = \frac{1}{2} \)

• For each bidder this induces a threshold price. \( \varphi_i(p_i) = t \)

• Sell to first bidder willing to pay his personal reserve price.

• Gives 2 approximation!
Observations

- Although constant virtual price $t$ gives bidder-specific posted prices, it’s still way simpler than optimal auction.
- All the nice properties we saw earlier.
- Called **oblivious posted pricing** because don’t care what order the agents show up.
- If the designer is allowed to consider the agents in the order of his choosing, better approximations possible. Called **sequential posted pricing**.
Sequential Posted Pricing

\[ V_i \sim F_i \]

[Chawla, Hartline, Kleinberg]
[Chawla, Hartline, Malec, Sivan] [Alaei] [Yan]

- Relax the problem by considering the **optimal ex-ante relaxation** i.e., agent i wins with probability \( q_i \)

\[
\text{maximize} \sum_i R_i(q_i) \\
\text{subject to} \sum_i q_i \leq 1 \\
q_i \geq 0
\]

- If distributions nice (regular), then optimal ex-ante pricing is a posted pricing with solution say

\[
\sum_i R_i(q_i) = \sum_i q_i p_i \\
q_i = 1 - F_i(p_i)
\]
Sequential Posted Pricing

[Alaei] [Yan][Agrawal, Ding, Saberi, Ye]

- Example: if distributions regular, then ex-ante pricing is a posted pricing
  \[
  \sum_i R_i(q_i) = \sum_i q_i p_i
  \]

- Suggests: order by decreasing \( p_i \).

- Then
  \[
  \text{Rev} = \sum_i q_i p_i \prod_{j<i} (1 - \frac{1}{i+1})
  \]

Example: \( p_i = 1, q_i = 1/n \), ex-ante opt: 1

\[
\text{Rev} = \sum_i \frac{1}{n} 1(1 - 1/n)^{i-1} \to 1 - 1/e
\]
Revenue maximization beyond single-item auctions

• All of this even more interesting beyond single item, e.g. many agents can be served, with combinatorial feasibility constraint, e.g. matroid.

• See [Chawla, Hartline, Malec, Sivan]...
Summary

- Many results on posted price auctions (often based on prophet inequalities). Many extensions beyond single item auctions.
- In mechanism design, we love posted prices! Simple, strategyproof, robust, collusion-resistant, less information revealed, don’t need everyone present at once, they are what we see in the real world, etc.

- Take-away: Auctions can be approximately optimal without being complex! “simple vs optimal” [Hartline, Roughgarden]
- These results depend on deep understanding of optimal auction for single item auctions.
- Take-away: The optimal (or approx optimal) auction serves as benchmark for evaluating more practical designs.
- Huge on-going quest to understand optimal and approximately optimal auctions in ever-more complex settings.

- All of the above depended on the fact that we had an accurate prior in our hands.
Where does the prior come from?
The prior...

• Where does it come from?
  • Previous experience in the market
  • On the fly market analysis

• What if the prior is not accurate? Results potentially sensitive to small errors in the prior.

• What if the prior is changing over time?

• Even if we can get our hands on it, we may not want to redesign the mechanism every time the prior changes
Prior independence
[Dhangwatnotai, Roughgarden, Yan]

• Prior-independent mech design; unknown $F$
  • Assume values come from some prior, design single auction (with no knowledge of $F$) so that, no matter what $F$ is

$$\mathbb{E}_{v \sim F}(A(v)) \geq c \cdot \mathbb{E}(\text{opt}_F(v))$$

Not possible without any assumptions on $F$. However, benign assumptions suffice!
Single item, i.i.d. setting (regular distributions)

• [Bulow, Klemperer]

$$\mathbb{E}(\text{Rev Vickrey}) \geq \mathbb{E}(\text{Rev opt}_{F})$$

with $n + 1$ bidders

with $n$ bidders

• Interpretation:

  • A little more competition is more important than precise knowledge of prior.
  • High value from one extra sample
  • Random price from the distribution almost as good as the optimal reserve price.
Prior-independent results inspired by [BK]

[Dhangwatnotai, Roughgarden, Yan]
• Pick one random bidder and use his bid as the reserve for others – “market research on the fly”
• “single sample mechanism” gets 2 approximation to optimal mechanism tailored to the distributions, no matter what the distribution*
• Many extensions.
• Beginning of flurry of activity on prior-independent auctions.

[Roughgarden, Talgam-Cohen, Yan][Devanur, Hartline, K, Nguyen][Roughgarden, Talgam-Cohen] [Goldner, K]....

* Under regularity assumption, +..
\[ \mathbf{v}_1, \ldots, \mathbf{v}_s \sim \mathcal{F} \]

**Question:** How many samples are necessary and sufficient to get a \( 1 - \epsilon \) approximation to the optimal expected revenue?
Learning algorithm

\[ \mathbf{v}_1, \ldots, \mathbf{v}_s \sim \mathcal{F} \]

Auction \( A(\mathbf{v}_1, \ldots, \mathbf{v}_s) \)

Revenue of \( A \) on \( \mathbf{V} \)

\( \mathbf{v} \sim \mathcal{F} \)

Approach:
- Discretize the value space losing only \( O(\epsilon) \) fraction of revenue.
- Bounds number of mechanisms need to consider.
- Chernoff bounds => best mechanism in class is \( 1 - \epsilon \) approximation on new random sample.
Results:

• Without any assumptions on distribution, can’t do anything.

• In i.i.d. setting, $\text{poly}(\epsilon^{-1})$ samples suffices.

• In non-i.i.d. setting $n \text{ poly}(\epsilon^{-1})$ samples suffice.
\[ \mathbf{v}_1, \ldots, \mathbf{v}_s \sim \mathcal{F} \]

Learning algorithm
Selects auction in \( C \)

\[ \mathbf{v} \sim \mathcal{F} \]

Auction \( A(\mathbf{v}_1, \ldots, \mathbf{v}_s) \)

Revenue of \( A \) on \( \mathbf{v} \)

- Apply notion of "pseudodimension" from learning theory.
- Pseudodimension \( d(C) \) matches intuitive notion of simplicity/complexity of family of auctions \( C \).
- Implies good sample complexity bounds. Roughly \( H^2 \epsilon^{-2} d \) samples suffice.
- Results imply that when it’s possible to compute a near-optimal simple auction with a known prior, can do so with unknown prior (with polynomial number of samples).
• **Bayesian optimal** mechanism design
  - *Given priors, design mechanism that maximizes or approximates the optimal expected revenue*

• **Prior-independent** opt mechanism design – unknown $F$
  - *Knowing that values are drawn from some large class of distributions, design single auction so that*

$$E_{v \sim F}(A(v)) \geq c \cdot E(\text{opt}_F(v))$$

• **Prior-free** optimal mechanism design
  - *Design truthful auctions so that for every input*

$$A(v) = \Omega(B(v))$$

where $B(v)$ is some profit benchmark.
We’d like $B(v)$ to be $\text{Opt}(v)$, but in the setting of truthful auction design, there is no such thing as an optimal auction!

- **Prior-free** optimal mechanism design
  - *Design auction so that for every input $v$

  $A(v) = \Omega(B(v))$

  where $B(v)$ is some profit benchmark.
Example: digital good auctions
[Goldberg, Hartline, Wright] [GHKWS]

- Auctioneer has unlimited supply of items.
- n agents, each has private value $v_i$ for getting one item
- Auction as before takes as input set of bids, and chooses as output, a subset of winners.

- Design a truthful auction that obtains good revenue pointwise.
  \[
  A(v) = \Omega(B(v))
  \]

- What should benchmark $B(v)$ be?
- As I said, there is no optimal truthful auction.
• Good benchmark: optimal fixed price profit: order values
\[ v_1 \geq v_2 \geq \ldots \geq v_n \]
\[ B(v) = \max_i iv_i \]

• Nice thing about this benchmark:
  • If you did have a prior, then this quantity is at least as large as
    \( \text{opt}_F(v) : \text{sell at price } \max_p p(1 - F(p)) \)
  • If we can compete with this benchmark, then
    \textbf{simultaneously competitive} with all Bayesian optimal auctions.

• Question: Can we construct a truthful auction that gets revenue \( c B(\mathbf{v}) \) on every input?
Competing with best fixed price

• **Truthful mechanism:** price an agent charged can’t depend on their own value.

• Suggests: offer best fixed price from rest of values.

• Doesn’t work:
  • if bidder high, right price looks low (little revenue)
  • If bidder low, right price too high (rejects)

• [Goldberg, Hartline, Wright] No deterministic auction that treats bidders symmetrically can get any constant competitive ratio.
Constant competitive auction*

[GHW][GHKSW]

- Use random sampling:
  - Partition the bidders at random into two sets, $S$ and $S'$
  - Compute the best fixed price $p$ for $S$ and best fixed price $p'$ for $S'$
  - Offer price $p$ to bidders in $S'$ and price $p'$ to bidders in $S$

- Many other results.

- Tight bound of 2.42 on competitive ratio is now known.
  - [Goldberg, Hartline, K, Saks] [Chen, Gravin, Lu]

* assumes no dominant bidder
AGT and Learning
Issues

• Interactions in the marketplace and beyond are highly dynamic and/or repetitive.

• Agents know that information they reveal today may be used against them tomorrow.

• Lots and lots of data, but it may be strategic.
Repeated interactions very tricky

Example: Fishmonger sells a fish each day to a buyer. The buyer’s value $V$ is $U[0,1]$ but doesn’t change from day to day.

Single sale: price $\frac{1}{2}$
Expected revenue: $\frac{1}{4}$
Buyer’s value on day 1 = value on day 2
Seller knows it’s a U[0,1] draw

e.g. Buyer with value 0.6

Accepts both days
=> utility = 0.2
Rejects 1\textsuperscript{st}, accepts 2\textsuperscript{nd}
=> utility = 0.35
But if seller could commit...
If not, exp revenue at most 0.45

Without commitment, the seller’s revenue in n days is o(n)!

[Devanur, Peres, Sivan]

Remember: buyer utility is value - price
Learning – other questions

• How do you incentivize providers of data to put the effort in to give you high quality data?
• How do you learn from data when the data is both noisy and strategically presented? E.g., if data providers want to influence the outcome of the learning algorithm.

• Huge body of work emerging from various communities e.g. [Cai, Daskalakis, Papadimitriou]
Incentivizing exploration
[Mansour, Slivkins, Syrgkanis, Wu]
[Frazier, Kempe, Kleinberg, Kleinberg]...

• Waze recommends routes for drivers, but relies on the drivers to do the discovery.
• Retailers like amazon want products to be explored and reviewed, but rely on their users to do this.
• Principal’s goal is to collect information about many alternatives: necessitates exploring!

• Users’ goal is to select the best alternative for them right now: exploit!!

• How can the principal incentivize the users so that long term learning is as good as it can be? And how good can it be?
[Frazier, Kempe, Kleinberg, Kleinberg] model

- K alternatives ("arms"), each with type that governs reward distribution when selected

- Users observe all past rewards before making their selection.
(Bayesian) multi-arm bandits

• K alternatives ("arms") (Users observe all past rewards before making their selection.)

Principal’s goal:  
User’s goal:

\[
\text{maximize} \quad \sum_{t=0}^{\infty} \gamma^t r_t
\]

\[
\text{maximize} \quad r_t
\]
(Bayesian) multi-arm bandits

• K alternatives ("arms")

Optimal policy (Gittins index)

Myopic policy

maximize $\sum_{t=0}^{\infty} \gamma^t r_t$

maximize $r_t$
Incentive payments

At time $t$, announce bonus $c_{t,i} \geq 0$ for each arm $i$. User now chooses $i$ to maximize $E[r_{i,t}] + c_{i,t}$.

Paper precisely characterizes the tradeoff between the incentive payments and the opportunity cost (what you lose in rewards from not playing the optimal policy).
Conclusions

• Many missing topics including:
  • modeling of agents (e.g. value distributions that aren’t independent)
  • Mechanism design in more complex settings (multi-parameter)
  • complexity of equilibria – beyond worst case?
  • dynamic mechanism design.

• Many exciting applications that we don’t understand!
• For more see
  • Jason Hartline’s book
  • Tim Roughgarden’s lecture notes and videos
  • Simons Institute Economics and Computation semester workshop videos!

Thank you!!!!!!