Logic and Databases

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Lecture 5
Alternative Semantics of Queries

- **Bag Semantics**
  
  We focused on the containment problem for conjunctive queries under bag semantics.

- **Probabilistic Databases**
  
  We focused on the data complexity of conjunctive query on tuple independence databases.

Today, we will discuss

- **Inconsistent Databases**
  
  The focus will be on the data complexity of conjunctive queries in this framework.
Logic and Databases

- Two main uses of logic in databases:
  - Logic is used as a database query language to express questions asked against databases.
  - Logic is used as a specification language to express integrity constraints in databases.

- So far, we have discussed the use of logic as a database query language.

- In what follows, we will discuss some aspects of the use of logic as a specification language to express integrity constraints.
Integrity Constraints in Databases

- **Integrity Constraints** are semantic restrictions that the data at hand ought to obey.

- Extensive study of various types of integrity constraints in relational databases during the 1970s and early 1980s:
  - Key constraints and functional dependencies
  - Inclusion dependencies, join dependencies, multi-valued dependencies, ...

- Eventually, it was realized that all these different types of dependencies can be specified in fragments of first-order logic.
Two Unifying Classes of Integrity Constraints

Definition

- **Equality-generating dependency (egd):**
  \[ \forall x (\phi(x) \rightarrow x_i = x_j), \]
  where \( \phi(x) \) is a conjunction of atoms.

  **Special Cases:**
  Key constraints, functional dependencies.

- **Tuple-generating dependency (tgd):**
  \[ \forall x (\phi(x) \rightarrow \exists y \psi(x, y)), \]
  where \( \phi(x) \) is a conjunction of atoms with vars. in \( x \), and \( \psi(x, y) \) is a conjunction of atoms with vars. in \( x \) and \( y \).

  **Special Cases:**
  LAV (local-as-view) constraints, GAV (global-as-view) constraints.
Study of Integrity Constraints in Databases

- Initial focus on the decidability and complexity of the implication problem for integrity constraints: Given $\Sigma$ and $\Sigma'$, does $\Sigma \models \Sigma'$?

- More recent extensive study of egds and tgds in data integration and data exchange. They have been used to design schema-mapping languages for formalizing data inter-operability tasks.

- More recent extensive study of the decidability and complexity of query answering over inconsistent databases, i.e., databases that violate integrity constraints specified by egds and tgds.
Equality-Generating Dependencies

Definition

▶ Functional Dependency \( R : X \rightarrow Y \)
If two tuples in \( R \) agree on \( X \), then they agree on \( Y \).

▶ Key Constraint \( R : X \rightarrow Y \), where \( Y \) is the set of attributes of \( R \) that are not in \( X \).

Example \( R(A, B, C, D) \)

▶ Functional Dependency \( R : A, B \rightarrow D \) as an egd:
\[
\forall a, b, c, c', d, d'(R(a, b, c, d) \land R(a, b, c', d') \rightarrow d = d')
\]

▶ Key Constraint \( R : A, B \rightarrow C, D \) as two egds:
\[
\forall a, b, c, c', d, d'(R(a, b, c, d) \land R(a, b, c', d') \rightarrow c = c')
\]
\[
\forall a, b, c, c', d, d'(R(a, b, c, d) \land R(a, b, c', d') \rightarrow d = d')
\]
Inconsistent Databases

- In designing databases, one specifies a schema $S$ and a set $\Sigma$ of integrity constraints on $S$.
- An inconsistent database is a database $I$ that does not satisfy $\Sigma$.
- Inconsistent databases arise in a variety of contexts and for different reasons:
  - For lack of support of particular integrity constraints.
  - In data integration of heterogeneous data obeying different integrity constraints.
  - In data warehousing and in Extract-Transform-Load (ETL) applications, where data has to be “cleaned” before it can be processed.
Coping with Inconsistent Databases

Two different approaches:

- **Data Cleaning**: Based on heuristics or specific domain knowledge, the inconsistent database is transformed to a consistent one by modifying (adding, deleting, updating) tuples in relations.
  - This is the main approach in industry (e.g., IBM InfoSphere Quality Stage, Microsoft DQS).
  - More engineering than science as quite often arbitrary choices have to be made.
Coping with Inconsistent Databases

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▶ **Data Cleaning**: Based on heuristics or specific domain knowledge, the inconsistent database is transformed to a consistent one by modifying (adding, deleting, updating) tuples in relations.
  - This is the main approach in industry (e.g., IBM InfoSphere Quality Stage, Microsoft DQS).
  - More engineering than science as quite often arbitrary choices have to be made.

▶ **Database Repairs**: A framework for coping with inconsistent databases in a principled way and without “cleaning” dirty data first.
Database Repairs

Definition (Arenas, Bertossi, Chomicki – 1999)

Σ a set of integrity constraints and I an inconsistent database. A database J is a repair of I w.r.t. Σ if

- J is a consistent database (i.e., J $\models$ Σ);
- J differs from I in a minimal way.

Fact

Several different types of repairs have been considered:

- Set-based repairs (subset, superset, ⊕-repairs).
- Cardinality-based repairs
- Attribute-based repairs
- Preferred repairs
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- Cardinality-based repairs
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- Preferred repairs
Subset Repairs

**Definition**

\( \Sigma \) a set of integrity constraints and \( I \) an inconsistent database. \( J \) is a *subset-repair* of \( I \) w.r.t. \( \Sigma \) if

- \( J \subset I \)
- \( J \models \Sigma \) (i.e., \( J \) is consistent)
- there is no \( J' \) such that \( J' \models \Sigma \) and \( J \subset J' \subset I \).

**Note**

From now on, we will use the term *repair*, instead of the term *subset repair*. 
Subset Repairs

Example

Key constraint

\[ \Sigma = \{ \forall x \forall y \forall ((R(x, y) \land R(x, z)) \rightarrow y = z) \} \]

Database

\[ I = \{ R(a_1, b_1), R(a_1, b_2), R(a_2, b_1), R(a_2, b_2) \} \]

I has four (subset) repairs w.r.t. \( \Sigma \):

- \( J_1 = \{ R(a_1, b_1), R(a_2, b_1) \} \)
- \( J_2 = \{ R(a_1, b_1), R(a_2, b_2) \} \)
- \( J_3 = \{ R(a_1, b_2), R(a_2, b_1) \} \)
- \( J_4 = \{ R(a_1, b_2), R(a_2, b_2) \} \).

Exponentially many repairs, in general.
Consistent Query Answering (CQA)

Definition (Arenas, Bertossi, Chomicki)
\[ \Sigma \text{ a set of integrity constraints, } q \text{ a query, and } I \text{ a database.} \]

The consistent answers of \( q \) on \( I \) w.r.t. \( \Sigma \) is the set

\[ \text{CON}(q, I, \Sigma) = \bigcap \{ q(J) : J \text{ is a repair of } I \text{ w.r.t. } \Sigma \}. \]

Note:

- The motivation comes from the semantics of queries in the context of incomplete information and possible worlds.
- The consistent answers of \( q \) in \( I \) are the certain answers of \( q \) on \( I \), when the set of all possible worlds is the set of all repairs of \( I \) w.r.t. \( \Sigma \).
Consistent Query Answering (CQA)

Example (Revisited)

$$\Sigma = \{ \forall x \forall y \forall z((R(x, y) \land R(x, z) \rightarrow y = z) \}$$

$$I = \{ R(a_1, b_1), R(a_1, b_2), R(a_2, b_1), R(a_2, b_2) \}$$

Recall that $I$ has four repairs w.r.t. $\Sigma$:

- $J_1 = \{ R(a_1, b_1), R(a_2, b_1) \}$, $J_2 = \{ R(a_1, b_1), R(a_2, b_2) \}$
- $J_3 = \{ R(a_1, b_2), R(a_2, b_1) \}$, $J_4 = \{ R(a_1, b_2), R(a_2, b_2) \}$.

- If $q(x)$ is the query $\exists y R(x, y)$, then
  \[ \text{CON}(q, I, \Sigma) = \{ a_1, a_2 \}. \]

- If $q(x)$ is the query $\exists z R(z, x)$, then
  \[ \text{CON}(q, I, \Sigma) = \emptyset. \]
Overview of Research on Database Repairs

Main themes explored so far:

- **Complexity of CQA for conjunctive queries:**
  From polynomial-time computability to **undecidability**.

- **Repair Checking:** Given \( I \) and \( J \), is \( J \) a repair of \( I \) w.r.t. \( \Sigma \)?
  From polynomial-time computability to **coNP-completeness**.

- **Prototype CQA Systems** for selected classes of constraints and selected classes of queries (mainly, conjunctive queries).
Definition Assume that

- $\Sigma$ is a set of key constraints with one key per relation.
- $q$ is a Boolean conjunctive query (no free variables).

$\text{CERTAINTY}(q, \Sigma)$ is the following decision problem:
Given a database $I$, is $\text{CON}(q, I, \Sigma)$ true?
(i.e., is $q$ true on every repair of $I$?)

Fact

- Repair checking is in P (in fact, it is in L).
- $\text{CERTAINTY}(q, \Sigma)$ is in coNP.
Complexity of CQA: An Illustration

Binary relations \( R \) and \( S \) having the first attribute as key, i.e.,
\[
\Sigma = \{ R(u, v) \land R(u, w) \rightarrow v = w, \quad S(u, v) \land S(u, w) \rightarrow v = w \}.
\]

- Let \( q_1 \) be the Boolean query \( \exists x, y, z (R(x, y) \land S(y, z)) \).
- Let \( q_2 \) be the Boolean query \( \exists x, y (R(x, y) \land S(y, x)) \).
- Let \( q_3 \) be the Boolean query \( \exists x, y, z (R(x, y) \land S(z, y)) \).

Question:
What can we say about \( \text{CERTAINTY}(q_i, \Sigma) \), where \( i = 1, 2, 3 \)?
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$$\Sigma = \{ R(u, v) \land R(u, w) \rightarrow v = w, \quad S(u, v) \land S(u, w) \rightarrow v = w \}.$$

- Let $q_1$ be the query $\exists x, y, z (R(x, y) \land S(y, z))$.
  $\text{CERTAINTY}(q_1, \Sigma)$ is in P; in fact, it is FO-rewritable as $\exists x, y, z (R(x, y) \land S(y, z) \land \forall y'(R(x, y') \rightarrow \exists z' S(y', z')))$. 

- Let $q_2$ be the query $\exists x, y (R(x, y) \land S(y, x))$.
  $\text{CERTAINTY}(q_2, \Sigma)$ is in P, but it is not FO-rewritable.

- Let $q_3$ be the query $\exists x, y, z (R(x, y) \land S(z, y))$.
  $\text{CERTAINTY}(q_3, \Sigma)$ is coNP-complete.
Question: Can we classify the complexity of CERTAINTY($q, \Sigma$)?
Classifying the Complexity of CQA

**Question:** Can we classify the complexity of \( \text{CERTAINTY}(q, \Sigma) \)?

**Conjecture (Dichotomy Conjecture for \( \text{CERTAINTY}(q, \Sigma) \))**

If \( \Sigma \) is a set of key constraints with one key per relation and \( q \) is a Boolean conjunctive query, then one of the following holds:

- \( \text{CERTAINTY}(q, \Sigma) \) is in \( P \).
- \( \text{CERTAINTY}(q, \Sigma) \) is \( \text{coNP} \)-complete.

Moreover, the dichotomy is **effective**: we can decide in PTIME whether \( \text{CERTAINTY}(q, \Sigma) \) is in \( P \) or it is \( \text{coNP} \)-complete.
Ladner’s Theorem and Dichotomies in Complexity

Theorem (Ladner - 1975)
If \( P \neq \text{NP} \), then there is a decision problem \( Q \) such that

- \( Q \) is in \( \text{NP} \), but not in \( P \).
- \( Q \) is not \( \text{NP} \)-complete.

The Fine Structure of \( \text{NP} \)

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Ladner’s Theorem and Dichotomies in Complexity

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The Fine Structure of $NP$

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Dichotomy Conjecture for $CERTAINTY(q, \Sigma)$

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Progress towards the Dichotomy for $\text{CERTAINTY}(q, \Sigma)$

**Theorem (Koutris and Wijsen - 2015)**

If $\Sigma$ is a set of key constraints with one key per relation and $q$ is a Boolean self-join free conjunctive query, then one of the following holds:

- $\text{CERTAINTY}(q, \Sigma)$ is in $\text{P}$.
- $\text{CERTAINTY}(q, \Sigma)$ is $\text{coNP}$-complete.

Moreover, this dichotomy is decidable in quadratic time.

Key Notion: The attack graph associated with $\Sigma$ and $q$.
- The nodes of the attack graph are the atoms of $q$.
- The edges of the attack graph are determined by the functional dependencies on the variables of an atom that are implied by the keys of the other atoms.
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The Attack Graph

Σ a set of key constraints with one key per relation. 
q a Boolean self-join free conjunctive query

▶ \( K(q) = \{ \text{key}(F) \rightarrow \text{Var}(F) : F \text{ is an atom of } q \} \).

▶ \( F^{+,q} = \{ x \in \text{Var}(q) : K(q \setminus F) \models \text{key}(F) \rightarrow x \} \).

▶ F attacks G, denoted \( F \rightsquigarrow G \), if there is a sequence \( F_1, \ldots, F_n \) such that
  ▶ \( F_1 = F \) and \( F_n = G \).
  ▶ \( \text{Var}(F_i) \cap \text{Var}(F_{i+1}) \not\subseteq F^{+,q} \), for every \( i \leq n - 1 \),

▶ An attack \( F \rightsquigarrow G \) is weak if \( K(q) \models \text{key}(F) \rightarrow \text{key}(G) \); otherwise, the attack is strong.

▶ A cycle in the attack graph is strong if it contains at least one strong attack.
Progress towards the Dichotomy for $\textsc{Certainty}(q, \Sigma)$

Theorem (Koutris and Wijsen - 2015)

Let $\Sigma$ be a set of key constraints with one key per relation and let $q$ is a Boolean self-join free conjunctive query.

- If the attack graph is acyclic, then $\textsc{Certainty}(q, \Sigma)$ is in $P$ and, in fact, it FO-rewritable; otherwise, $\textsc{Certainty}(q, \Sigma)$ is $L$-hard, hence it is not FO-rewritable.

- If the attack graph contains no strong cycle, then $\textsc{Certainty}(q, \Sigma)$ is in $P$.

- If the attack graph contains a strong cycle, then $\textsc{Certainty}(q, \Sigma)$ is $\text{coNP}$-complete.

Moreover, these conditions can be checked in quadratic time.
Applying the Koutris-Wisjen Dichotomy Theorem

Theorem (K... and Pema - 2012)

Assume $\Sigma$ consists of a key for $R$ and a key for $S$, and let $q$ be a Boolean query with two atoms, one $R$-atom and one $S$-atom. If $\text{CERTAINTY}(q, \Sigma)$ is not FO-rewritable, then the following hold:

- If $\text{key}(R) \cup \text{key}(S) \subseteq \text{Var}(R) \cap \text{Var}(S)$, then $\text{CERTAINTY}(q, \Sigma)$ is in $P$.
- If $\text{key}(R) \cup \text{key}(S) \not\subseteq \text{Var}(R) \cap \text{Var}(S)$, then $\text{CERTAINTY}(q, \Sigma)$ is $\text{coNP}$-complete.

Examples:

- Let $q_2$ be the query $\exists x, y (R(x, y) \land S(y, x))$. $\text{CERTAINTY}(q_2, \Sigma)$ is in $P$, because $\text{key}(R) \cup \text{key}(S) = \{x, y\}$, $\text{Var}(R) \cap \text{Var}(S) = \{x, y\}$.
- Let $q_3$ be the query $\exists x, y, z (R(x, y) \land S(z, y))$. $\text{CERTAINTY}(q_3, \Sigma)$ is $\text{coNP}$-complete, because $\text{key}(R) \cup \text{key}(S) = \{x, z\}$, $\text{Var}(R) \cap \text{Var}(S) = \{y\}$. 
Applying the Koutris-Wisjen Dichotomy Theorem

Theorem (K... and Pema - 2012)

Assume \( \Sigma \) consists of a key for \( R \) and a key for \( S \), and let \( q \) be a Boolean query with two atoms, one \( R \)-atom and one \( S \)-atom. If \( \text{CERTAINTY}(q, \Sigma) \) is not FO- rewritable, then the following hold:

- If \( \text{key}(R) \cup \text{key}(S) \subseteq \text{Var}(R) \cap \text{Var}(S) \), then \( \text{CERTAINTY}(q, \Sigma) \) is in \( P \).
- If \( \text{key}(R) \cup \text{key}(S) \not\subseteq \text{Var}(R) \cap \text{Var}(S) \), then \( \text{CERTAINTY}(q, \Sigma) \) is \( \text{coNP} \)-complete.

Examples:

- Let \( q_2 \) be the query \( \exists x, y \left( R(x, y) \land S(y, x) \right) \). 
  \( \text{CERTAINTY}(q_2, \Sigma) \) is in \( P \), because 
  \( \text{key}(R) \cup \text{key}(S) = \{ x, y \}, \text{Var}(R) \cap \text{Var}(S) = \{ x, y \} \).
- Let \( q_3 \) be the query \( \exists x, y, z \left( R(x, y) \land S(z, y) \right) \). 
  \( \text{CERTAINTY}(q_3, \Sigma) \) is \( \text{coNP} \)-complete, because 
  \( \text{key}(R) \cup \text{key}(S) = \{ x, z \}, \text{Var}(R) \cap \text{Var}(S) = \{ y \} \).
Open Problems

- Prove the **Dichotomy Conjecture** for $\text{CERTAINTY}(q, \Sigma)$, where $\Sigma$ is a set of keys, one for each relation, and $q$ is an arbitrary Boolean conjunctive query.

- Prove a **Dichotomy Theorem** for $\text{CERTAINTY}(q, \Sigma)$, where $\Sigma$ is a set of functional dependencies and $q$ is a union of Boolean conjunctive queries.
Beyond Keys and Functional Dependencies

The Broader Classification Challenge:
Classify the complexity of $\text{CERTAINTY}(q, \Sigma)$, where $q$ is a FO-query and $\Sigma$ is a “well-behaved” set of egds and tgd.
Beyond Keys and Functional Dependencies

The Broader Classification Challenge:
Classify the complexity of $\text{CERTAINTY}(q, \Sigma)$, where $q$ is a FO-query and $\Sigma$ is a “well-behaved” set of egds and tgd$s.$

Fontaine - 2015:
Discovered an $a \text{ priori}$ unexpected connection between Consistent Query Answering and Constraint Satisfaction (equivalently, the query complexity of conjunctive queries).
Beyond Keys and Functional Dependencies

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Discovered an \textit{a priori} unexpected connection between Consistent Query Answering and Constraint Satisfaction (equivalently, the query complexity of conjunctive queries).

Theorem (Fontaine - 2015)
If the dichotomy theorem holds for \( \text{CERTAINTY}(q, \Sigma) \), where \( \Sigma \) is a finite set of GAV constraints and \( q \) is a union of Boolean conjunctive queries, then the Feder-Vardi Conjecture is true, i.e., a dichotomy theorem holds for the family \( P_D(CQ) \) of problems about the query complexity of CQ-evaluation.
Global-As-View (GAV) Constraints

Definition:

- Recall that a tgd is a constraint of the form
  \[ \forall x (\phi(x) \rightarrow \exists y \psi(x, y)) \],
  where \( \phi(x) \) and \( \psi(x, y) \) are conjunctions of atoms.

- A global-as-view (GAV) constraint is a tgd of the form
  \[ \forall x (\phi(x) \rightarrow T(x)) \],
  where \( T(x) \) is a single atom.
  In effect, a GAV constraint is a Horn clause.

Examples:

- \( \forall x, y (R(x, y) \rightarrow R(y, x)) \)
- \( \forall x, y, z (R(x, z) \land S(z, y) \rightarrow T(x, y)) \)
Fact
If $\Sigma$ is a finite set of GAV constraints and $q$ is a union of Boolean conjunctive queries, then $\oplus$-CERTAINTY$(q, \Sigma)$ is in $\text{coNP}$. 
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If $\Sigma$ is a finite set of GAV constraints and $q$ is a union of Boolean conjunctive queries, then $\oplus$-CERTAINTY($q, \Sigma$) is in coNP.

Theorem (Fontaine - 2015)
For every relational structure $\mathcal{B}$, there is a finite set $\Sigma$ of GAV constraints and a union $q$ of Boolean conjunctive queries, such that $\text{CSP}(\mathcal{B})$ is PTIME-equivalent to $\oplus$-CERTAINTY($q, \Sigma$).

Corollary
If the dichotomy theorem holds for $\oplus$-CERTAINTY($q, \Sigma$), where $\Sigma$ is a finite set of GAV constraints and $q$ is a union of Boolean conjunctive queries, then the dichotomy theorem holds for $\text{CSP}(\mathcal{B})$, where $\mathcal{B}$ is a relational structure.
Note

- CQA has been criticized as being too conservative: too many repairs may imply too few answers.
- CQA does not differentiate between repairs: all repairs are treated as equals.
Pragmatics of Consistent Query Answering

Note

- CQA has been criticized as being too conservative: too many repairs may imply too few answers.
- CQA does not differentiate between repairs: all repairs are treated as equals.

Staworko, Chomicki, and Marcinkowski - 2012
Introduced prioritized repairing that incorporates preferences between facts: if facts $f$ and $g$ conflict, we may prefer to resolve the conflict by deleting $g$ (and not $f$).

- $f$ may come from a more reliable source.
- $f$ may be more current.
Prioritizing Inconsistent Databases

Definition: Let $\Sigma$ be a set of functional dependencies (FDs). An inconsistent prioritizing database is a pair $(I, \succ)$, where

- $I$ is an inconsistent database w.r.t. $\Sigma$.
- $\succ$ is an acyclic binary relation on the facts of $I$ such that if $f \succ g$, then $f$ and $g$ violate one of the FDs in $\Sigma$.

Intuition:

- $f \succ g$ should be interpreted as “between the conflicting facts $f$ and $g$, we prefer to keep $f$ rather than $g$”.
- A preference relation between conflicting facts induces a preference relation between repairs.
- Thus, we can focus on “optimally preferred” repairs.
Globally Optimal Repairs

Definition (Staworko, Chomicki, Marcinkowski - 2012)
\( \Sigma \) set of FDs, \((I, \succ)\) an inconsistent prioritizing database.

- If \( J, K \) are two different consistent sub-databases of \( I \), then \( J \) is a global improvement of \( K \) if for every fact \( g \in K \setminus J \), there is a fact \( f \in J \setminus K \) such that \( f \succ g \).

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<th>J \setminus K</th>
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<tr>
<td>K \setminus J</td>
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- \( J \) is a globally optimal repair of \( I \) (in short, a \( g \)-repair of \( I \)) if \( J \) is consistent and has no global improvement.

Note: Every \( g \)-repair of \((I, \succ)\) is a (subset) repair of \( I \).
course, term $\rightarrow$ instructor and instructor, term $\rightarrow$ course

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Preferences:

- $f_2 \succ f_1$
- $f_4 \succ f_3$
- $f_5 \succ f_6$
- $f_5 \succ f_7$

$I$

$K$ is a repair of $I$

$J$ is a $g$-repair of $(I, \succ)$

<table>
<thead>
<tr>
<th>course</th>
<th>term</th>
<th>instructor</th>
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<tbody>
<tr>
<td>$f_3$</td>
<td>PL</td>
<td>Fall</td>
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<tr>
<td>$f_5$</td>
<td>PL</td>
<td>Spring</td>
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Repair Checking

\(\Sigma\) a fixed set of functional dependencies (FDs).

- **REPAIR CHECKING**: Given \(I\) and \(J\), is \(J\) a repair of \(I\)?
- Recall that REPAIR CHECKING in \(P\) (in fact, it is in \(L\)).

**Definition**

**\(g\)-REPAIR CHECKING**: Given \((I, \succ)\) and \(J\), is \(J\) a \(g\)-repair of \(I\)?

- It is easy to see that \(g\)-REPAIR CHECKING is in \(coNP\).
Repair Checking

\[ \Sigma \] a fixed set of functional dependencies (FDs).

- **REPAIR CHECKING**: Given \( I \) and \( J \), is \( J \) a repair of \( I \)?

- Recall that **REPAIR CHECKING** is in P (in fact, it is in L).

Definition

**g-REPAIR CHECKING**: Given \((I, \succ)\) and \( J \), is \( J \) a g-repair of \( I \)?

- It is easy to see that **g-REPAIR CHECKING** is in \text{coNP}.

**Theorem** (Staworko, Chomicki, Marcinkowski - 2012)

There is a set \( \Sigma \) of four FDs on a relation of arity 8 such that **g-REPAIR CHECKING** is \text{coNP}-complete.

**Question**:

Can we classify the complexity of **g-REPAIR CHECKING**?
Dichotomy Theorem for $g$-Repair Checking

Theorem (Fagin, Kimelfeld, K . . . - 2015)
Let $\Sigma$ be a set of FDs on a collection of relations.

- If $\Sigma$ induces a single FD or two key constraints on each relation, then $g$-REPAIR CHECKING is solvable in P.
- Otherwise, $g$-REPAIR CHECKING is coNP-complete.

Moreover, this dichotomy is effective.

Note
This is a data complexity result: the constraints are held fixed, the input consists of $(I, \succ)$ and $J$. 

Illustrating the Dichotomy for $g$-Repair Checking

<table>
<thead>
<tr>
<th>Courses</th>
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<tr>
<td>course</td>
<td>term</td>
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<tr>
<th>Functional Dependencies</th>
<th>Complexity</th>
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<tbody>
<tr>
<td>course, term $\rightarrow$ instructor</td>
<td>$P$ (two keys)</td>
</tr>
<tr>
<td>instructor, term $\rightarrow$ course</td>
<td>$P$ (one FD)</td>
</tr>
<tr>
<td>instructor $\rightarrow$ course</td>
<td>$coNP$-complete (two non-key FDs)</td>
</tr>
<tr>
<td>course $\rightarrow$ instructor</td>
<td></td>
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<tr>
<td>instructor $\rightarrow$ course</td>
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Proof Strategy for the Intractability Side

Two main steps:

1. Proof of intractability for six basic sets of FDs. All six basic sets of FDs are for a ternary relation $R(A, B, C)$:

<table>
<thead>
<tr>
<th>$A \rightarrow B$, $B \rightarrow A$</th>
<th>$A \rightarrow B$, $B \rightarrow C$</th>
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<tbody>
<tr>
<td>$A \rightarrow B$, $C \rightarrow B$</td>
<td>$AB \rightarrow C$, $C \rightarrow B$</td>
</tr>
<tr>
<td>$AB \rightarrow C$, $AC \rightarrow B$, $BC \rightarrow A$</td>
<td>$\rightarrow A$, $B \rightarrow C$</td>
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2. Proof of intractability for an arbitrary set of FDs, Use case analysis and distinct reductions from one of the six basic sets of FDs.
Open Problems for Preferred Repairs

- Classify the complexity of $g$-CERTAINTY($q, \Sigma$), where $q$ is a Boolean conjunctive query and $\Sigma$ is a set of FDs.
  - Is there a Trichotomy Theorem for $g$-CERTAINTY($q, \Sigma$)? (P, coNP-complete, $\Pi_2^P$-complete)

- What if the preference relation $\succ$ is specified syntactically?
  - Is there a “useful” language for expressing preferences such that $g$-repair checking and $g$-CERTAINTY($q, \Sigma$) are of lower complexity?
Topics Covered

▶ Logic and Database Query Languages
  ▶ Relational Algebra and Relational Calculus
  ▶ Conjunctive Queries and their variants
  ▶ Datalog
▶ Query Evaluation, Query Containment, Query Equivalence
▶ Other aspects of Conjunctive Query Evaluation
  ▶ Acyclic joins, treewidth, bounds on the size of natural joins
▶ Alternative Semantics
  ▶ Bag Databases, Probabilistic Databases, Inconsistent Databases
▶ Emphasis on the interplay between databases, logic, and computational complexity
Topics Not Covered

- Automata-theoretic techniques in databases
- Reasoning about database dependencies (the implication problem)
- Incomplete databases
- Information integration, data exchange, data warehousing
- Data privacy and security
- Data provenance

Guest Lecture by Val Tannen

- Beyond relational databases
  - Semi-structured data and XML
  - Graph databases, web data
Logic and Databases are inextricably intertwined