Dynamic Pricing in Ridesharing Platforms
A Queueing Approach

Sid Banerjee | Ramesh Johari | Carlos Riquelme
Cornell | Stanford | Stanford
rjohari@stanford.edu

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Ridesharing and Pricing
Ridesharing platforms

Examples of major platforms: Lyft, Uber, Sidecar
This talk: Pricing and ridesharing

Ridesharing is somewhat unique among online platforms:

*The platform sets the transaction price.*

Our goal: Understand optimal pricing strategy.
Our contributions

1. A model that combines:
   - Strategic behavior of passengers and drivers
   - Pricing behavior of the platform
   - Queueing behavior of the system

2. What are the advantages of *dynamic* pricing over *static* pricing?
   - *Static:* Constant over several hour periods
   - *Dynamic:* Pricing changes in response to system state; "surge", "prime time"
Related work

Our work sits at a nexus between several different lines of research:

1. *Matching queues* (cf. [Adan and Weiss 2012])
2. *Strategic queueing models* (cf. [Naor 1969])
3. *Two-sided platforms* (cf. [Rochet and Tirole 2003, 2006])
4. *Revenue management* (cf. [Talluri and van Ryzin 2006])
5. *Large-scale matching markets* (cf. [Azevedo and Budish 2013])
6. *Mean field equilibrium* (cf. [Weintraub et al. 2008])
Model
Two types: Strategic and queueing

We need a *strategic model* that captures:

1. Platform pricing
2. Passenger incentives
3. Driver incentives

We need a *queueing model* that captures:

1. Driver time spent idling vs. driving
2. Ride requests blocked vs. served
1. Focus on a *block* of time (e.g., several hours) over which arrival rates are roughly stable

2. Focus on a single region (e.g., a single city neighborhood)
   - For technical simplicity
   - Insights generalize to networks of regions

3. Focus on throughput: rate of completed rides
   - For technical simplicity
   - Same results for profit, when system is supply-limited
   - Similar numerical results for welfare; theory ongoing
Strategic modeling: Platform pricing

Platforms:

- Earn a (fixed) fraction $\gamma$ of every dollar spent (e.g., 20%)
- Need both drivers (supply) and passengers (demand)
- Use pricing to align the two sides

Load-dependent pricing:
If # of available drivers = $A$, then price offered to ride = $P(A)$
Strategic model: Platform pricing

In practice:

- Platforms charge a time- and distance-dependent base price
- Platforms manipulate price through a multiplier
- Base price typically is not varied

In our model: \( price \equiv multiplier \).
Strategic model: Passengers

How do passengers enter?

- Passenger \equiv \text{one ride request}
- Sees \textit{instantaneous} ride price
- Enters if price < reservation value \( V \)
- \( V \sim F_V \), i.i.d. across ride requests

\[ \mu_0 = \text{exogenous rate of "app opens".} \]
\[ \mu = \text{actual rate of rides requested}. \]

Then when \( A \) available drivers present:

\[ \mu = \mu_0 F_V(P(A)). \]
Strategic model: Drivers

How do drivers enter?

▷ Sensitive to *expected earnings over the block*

▷ Choose to enter if:
  
  reservation earnings rate $\mathcal{C} \times$
  
  expected total time in system
  
  $<$ expected earnings while in system

▷ $\mathcal{C} \sim F_{\mathcal{C}}$, i.i.d. across drivers

$\Lambda_0 =$ exogenous rate of driver arrival.

$\lambda =$ actual rate at which drivers enter.

Then:

$$
\lambda = \Lambda_0 F_{\mathcal{C}} \left( \frac{\text{expected earnings in system}}{\text{expected time in system}} \right)
$$
Queueing model

1. Drivers enter at rate $\lambda$.
2. When $A$ drivers available, ride requests arrive at rate $\mu(A)$.
3. If a driver is available, ride is served; else blocked.
4. Rides last exponential time, mean $\tau$.
5. After ride completion:
   - With probability $q_{\text{exit}}$: Driver signs out
   - With probability $1 - q_{\text{exit}}$: Driver becomes available
Queueing model: Steady state

Jackson network of two queues: $M/M(n)/1$ and $M/M/\infty$ $\implies$ product-form steady state distribution $\pi$. 

![Diagram](image-url)
Putting it together: Equilibrium

Given pricing policy $P(\cdot)$, system equilibrium is $(\lambda, \mu, \pi, \iota, \eta)$ such that:

1. $\pi$ is the steady state distribution, given $\lambda$ and $\mu$
2. $\eta$ is the expected earnings per ride, given $P(\cdot)$ and $\pi$
3. $\iota$ is the expected idle time per ride, given $\pi$ and $\lambda$
4. $\lambda$ is the entry rate of drivers, given $\iota$ and $\eta$:

   $$\lambda = \Lambda_0 F_C \left( \frac{\eta}{\iota + \tau} \right)$$

5. $\mu(A)$ is the arrival rate of ride requests when $A$ drivers are available, given $P(\cdot)$:

   $$\mu = \mu_0 \bar{F}_V(P(A)).$$
Putting it together: Equilibrium

If price increases when number of available drivers decreases:

- Equilibria always exist under appropriate continuity of $F_C$, $F_V$.
- Equilibria are unique under reasonable conditions.
Large Market Limit
The challenge

- To understand optimal pricing, we need to characterize system equilibria.
- In particular, need sensitivity of equilibria to changes in pricing policy.
- Our approach: asymptotics to simplify analysis.
Large market asymptotics

Consider a sequence of systems indexed by $n$.

- In $n$'th system, exogenous arrival rates are $n\Lambda_0$, $n\mu_0$.
- In $n$'th system, pricing policy is $P_n(\cdot)$.
- In each system, this gives rise to a system equilibrium.

We analyze pricing by looking at asymptotics of equilibria.
Static Pricing
What is static pricing?

Static pricing means: *price policy is constant.* Let \( P(A) = p \) for all \( A \).

**Theorem**

Let \( r_n(p) \) denote the equilibrium rate of completed rides in the \( n \)'th system. Then:

\[
r_n(p) \to \hat{r}(p) \triangleq \min\{ \Lambda_0 F_C(\gamma p/\tau)/q_{exit}, \mu_0 F_V(p) \}. 
\]

*Throughput* = \( \min \{ \text{available supply, available demand} \} \)
Static pricing: Illustration

Normalized Rate of Completed Rides \( \frac{r_{n_i}}{n} \) vs Price \( p \)
Static pricing: Interpretation

Note that at *any price*, queueing system is always stable:

- When supply < demand:
  Drivers become fully saturated
- When supply > demand:
  Drivers forecast high idle times and don't enter

*Balance price* $p_{bal}$: Price where supply = demand

**Corollary**

*The optimal static price is* $p_{bal}$. 

Dynamic pricing
What is dynamic pricing?

Meant to capture "surge" (Uber) and "prime time" (Lyft) pricing strategies.

We focus on *threshold pricing*:

- Threshold $\theta$
- High price $p_h$ charged when available drivers $< \theta$
- Low price $p_\ell < p_h$ charged when available drivers $> \theta$
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.

$$n = 10$$
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.

\[ n = 100 \]
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.
Optimal dynamic pricing

**Theorem**

Let $r_n^*$ be the rate of completed rides in the $n$'th system, using the optimal static price.

Let $r_n^{**}$ be the rate of completed rides in the $n$'th system, using the optimal threshold pricing strategy.

Then if $F_V$ has monotone hazard rate,

$$\frac{r_n^* - r_n^{**}}{n} \rightarrow 0 \text{ as } n \rightarrow \infty.$$
Optimal dynamic pricing

In other words:

_In the fluid limit, no dynamic pricing policy yields higher throughput than optimal static pricing._
Optimal dynamic pricing

In other words:

*In the fluid limit, no dynamic pricing policy yields higher throughput than optimal static pricing.*

This result is reminiscent of similar results in the classical revenue management literature (e.g., [Gallego and van Ryzin, 1994]).

The main differences arise due to the presence of a two sided market.
Proof sketch

Under threshold pricing:

- Drivers are sensitive to *two* quantities: idle time, and price.
- Show that optimal $\theta_n^* \to \infty$, but chosen so that idle time $\to 0$ as $n \to \infty$.
- In this limit, drivers are sensitive to the *average* price per ride:

$$p_{avg} = \pi_h p_h + \pi_\ell p_\ell,$$

where $\pi_h$, $\pi_\ell$ are $\approx$ probabilities of being below or above $\theta$, respectively.
- If $p_{avg}$ decreases, fewer drivers will enter.
Proof sketch (cont'd)

We note that:

1. If \( p_\ell < p_h \leq p_{\text{bal}} \), then \( p_{\text{avg}} = p_h \).
2. If \( p_{\text{bal}} \leq p_\ell < p_h \), then \( p_{\text{avg}} = p_\ell \).
3. If \( p_\ell < p_{\text{bal}} < p_h \), then \( \pi_\ell > 0, \pi_h > 0 \).

In first two cases, \textit{de facto} static pricing.
Proof sketch (cont'd)

We explore the third case. Suppose that we start with \( p_\ell < p_h = p_{\text{bal}} \) (so \( p_{\text{avg}} = p_h \)).

Now increase \( p_h \):

- Before \( \pi_\ell = 0 \), but now \( \pi_\ell > 0 \), so some customers pay \( p_\ell \); this lowers \( p_{\text{avg}} \).

- \( p_h \) higher, so customers arriving when \( A < \theta \) pay more; this increases \( p_{\text{avg}} \).

When \( F_V \) is MHR, we show that the first effect dominates the second, so throughput falls.
Robustness
The value of dynamic pricing

How does dynamic pricing help?

- When system parameters are known, performance does not exceed static pricing.
- When system parameters are unknown, dynamic pricing naturally "learns" them.
Robustness: Illustration

What happens to *static pricing* in a demand shock?
Robustness: Illustration

What happens to dynamic pricing in a demand shock?

Robustness of Pricing Policies to Demand Shocks

- Static pricing
- Dynamic pricing
- Optimal pricing
Robustness: Dynamic pricing

We can formally establish the observation in the previous illustration:

- Suppose $F_C$ is logconcave, and $\mu_0^{(1)} < \mu_0^{(2)}$ are fixed.
- Let $p_{bal,n}^{(1)}, p_{bal,n}^{(2)} =$ optimal static prices in the $n$'th system.
- Let $r_n^{(1)}, r_n^{(2)} =$ optimal throughput in the $n$'th system.
- Suppose now the true $\mu_0 \in [\mu_0^{(1)}, \mu_0^{(2)}]$.
- Using both prices $p_{bal,n}^{(1)}, p_{bal,n}^{(2)}$ is robust:
  - There exists a sequence of threshold pricing policies with throughput at any such $\mu_0$ (in the fluid scaling) $\geq$ the linear interpolation of $r_n^{(1)}$ and $r_n^{(2)}$.

(Same holds w.r.t. $\Lambda_0$.)
Conclusion
Platform optimization

This work is an example of *platform optimization*: Requires understanding *both* operations and economics.

Other topics under investigation:

1. Network modeling (multiple regions):
   Our main insights generalize
2. Effect of pricing on aggregate welfare
3. Modeling driver heat maps
4. Fee structure: changing the percentage
5. Effect of changing the matching algorithm