ADVERSE SELECTION AND AUCTION DESIGN FOR INTERNET DISPLAY ADVERTISING

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“Half the money I spend on advertising is wasted; the trouble is, I don’t know which half.”

- John Wanamaker, Advertising pioneer
Old-Fashioned “Brand” Ads
New-Fashioned “Performance” Ads
Display Advertisement Types

**Brand Ads**

- **Goal:** awareness and image
  - Reach and repetition.
- **Common Characteristics**
  - Targeted to a large group
  - Large number of Impressions
  - Guaranteed delivery
- **Sample Advertisers**
  - Ford (weekend auto sale)
  - Disney (movie openings)
  - Shopping Center (location)

**Performance Ads**

- **Goal:** measurable action now
  - Click, fill form, or buy.
- **Common Characteristics**
  - Targeted to an individual (based on cookies)
  - Smaller number of impressions
  - Bought one by one
- **Sample Advertisers**
  - Hertz (car rental)
  - Amazon (re-targeting)
  - Quicken mortgage (refinance)
Danger of Adverse Selection

Brand Advertisers
- May select impressions en masse ("road block" ads)
- Receive deferred, aggregated data about performance of the whole ad campaign
- Cannot easily distinguish low-performing ads and publishers

Performance Advertisers
- Mostly use private cookies to select impressions
- Receive immediate, detailed data about the performance of individual ads
- Can quickly identify low-performing ads and publishers

If the value of ad impressions is positively correlated for both types of advertisers, then brand advertisers may suffer adverse selection.
…and “Not-Quite-Optimal” Market Design
There are $N + 1$ advertisers, with $N \geq 2$

The value of an impression to advertiser $i$ is $X_i = CM_i$

$C$ is the (random) common value factor and
$M_i$ is the (random) match value factor for bidder $i$

Key Assumptions

1. Advertiser 0 (the “brand advertiser’) does not observe $X_0$
2. Performance advertisers $n = 1, \ldots, N$ observe their values $X_n$
   Define $X = (X_1, \ldots, X_n)$.
3. The common value factor $C$ is statistically independent of $M \overset{\text{def}}{=} (M_0, \ldots, M_N)$
A Market Design Challenge

- Compare the restricted-worst-case performance of different mechanisms on efficiency grounds

- The mechanisms considered are:
  1. A benchmark: “Omniscient” mechanism with C observed
  2. “Optimal” (expected-efficiency maximizing) mechanisms
  3. Second-price auction
  4. “Modified second-bid auction” in which the highest performance bidder wins if the ratio of the highest to second-highest performance bid exceeds a threshold.
The Omniscient Benchmark

OMN, in which the auctioneer observes both the bids and C
OMN Benchmark

- If the auctioneer could separately gather *perfect information* about the common factor $C$ and decide the allocation accordingly (no incentive constraints), it could achieve this value:

$$V(OMN) = E[\max(C \cdot E[M_0], X_1, \ldots, X_n)]$$

- Performance of other mechanisms will be compared to $V(OMN)$. 
Bayesian Optimal Mechanism

OPT ...and its drawbacks
Optimal Mechanism Formulation

- \( z_i(X) \) is probability that \( i \) wins, given \( X \)
- \( p_i(X) \) is \( i \)'s expected payment, given \( X \)

Efficiency Objective
- Goal is to maximize \( E \left[ \sum_{i=0}^{n} X_i z_i(X) \right] \)
  - subject to dominant-strategy incentive constraints and participation constraints
- Let OPT be the mechanism that does that
 Assume that $M_1, \ldots, M_n$ are i.i.d and that...

\[
P\{C = 1\} = P\{C = 2\} = \frac{1}{2}
\]

\[
P\{M_n = 1\} = P\{M_n = 2\} = P\{M_n = 4\} = \frac{1}{3}
\]

\[3 < E[M_0] < 4\]

So, optimally, only a performance advertiser $n$ with $M_n = 4$ ought to be assigned this impression.
Example Solved

- The expected-efficiency-maximizing assignment with $N = 2$ is:
  - If $X(1) \in \{1,2\}$, then $M(1) \leq 2 < E[M_0] \Rightarrow$ brand advertiser
  - If $X(1) = 8$, then $M(1) = 4 > E[M_0] \Rightarrow$ top performance advertiser
  - If $X(1) = 4$, assignment hinges on whether $E[M(1)|X(1),X(2)] \geq E[M_0]$.
    - If $X(2) = 1$, then $M(1) = 4 \Rightarrow$ top performance advertiser
    - If $X(2) = 2$, then $E[M(1)|X(1),X(2)] = 3 < E[M_0] \Rightarrow$ brand advertiser
      - In this case, $\Pr\{C = 1, M(1) = 4, M(2) = 2|X(1),X(2)\} = \Pr\{C = 2, M(1) = 2, M(2) = 1|X(1),X(2)\} = \frac{1}{2}$.
    - If $X(2) = 4$, then $E[M(1)|X(1),X(2)] = 3 < E[M_0]$, $\Rightarrow$ brand advertiser
      - In this case, $\Pr\{C = 1, M(1) = M(2) = 4|X(1),X(2)\} = \Pr\{C = 2, M(1) = M(2) = 2|X(1),X(2)\} = \frac{1}{2}$. 
Main Concerns about OPT

- The example highlights three concerns about OPT
  1. Sensitivity: OPT is sensitive to detailed distributional assumptions.
  2. False-name bidding: Performance advertiser \( n \) with value \( X_n = 4 \) can only benefit by submitting a additional, false-name bid of \( X_\hat{n} = 1 \) (because that leads the auctioneer to infer that \( M_n = 4 \)).
  3. Adverse selection: The brand advertiser wins 4/9 of high-value impressions, but 7/9 of low-value ones.
    - Most problematic if the brand advertiser feels uninformed about the impressions and who else may be bidding.
Modified Second Bid auction characterized by its properties
A mechanism is

- **anonymous among performance advertisers** if...
- **strategy-proof** if...
- **fully strategy-proof** if, in addition, it is both
  - **bidder false-name proof**: no bidder can benefit by submitting multiple bids, and
  - **publisher false-name proof**: the seller cannot benefit by submitting “low” bids (below all performance bids)
- **adverse-selection free** if for every joint distribution on \((C, M)\) such that \(C\) and \(M\) are independent, \(z_0(X)\) is statistically independent of \(C\).
Definition. A direct mechanism is a modified second bid auction if for some $\alpha \geq 1$,
- If $\frac{X_1}{X_2} > \alpha$, then the highest performance advertiser wins & pays $\alpha X_2$.
- If $\frac{X_1}{X_2} \leq \alpha$, then the brand advertiser wins (and pays its contract price).

Theorem. A deterministic mechanism $(z, p)$ is anonymous, fully strategy-proof, and adverse selection free if and only if it MSB.
Comparing $MSB_\alpha$ and $SP_r$ to OMN

$MSB_\alpha$: modified second-bid auction

$SP_r$: second-price auction with reserve
Assumptions for Comparison

- Evaluate $\text{MSB}_\alpha$ and $\text{SP}_r$ mechanisms in worst case over a limited family of environments, in which...
  - $M_1, ..., M_N$ are IID from a distribution $F$.
  - $C$ is drawn from distribution $G$.
  - $N \geq 2$ and $E[M_0] \geq 0$ are arbitrary.
Theorem. (Comparing $SP_r$ and $MSB_\alpha$ to $OMN$)

1. Assuming Nash equilibrium bidding by the brand advertiser, both MSB and SP have similar worst case performance:

\[
\inf_{F,G,N\geq 2,E[M_0]\geq 0} \max_\alpha \frac{V(MSB_\alpha)}{V(OMN)} = \frac{1}{2}
\]

\[
\inf_{F,G,N\geq 2,E[M_0]\geq 0} \max_r \frac{V(SP_r)}{V(OMN)} = \frac{1}{2}
\]

2. Further restricting $F$ and/or $G$ to be drawn from power law distributions $\mathcal{P}$,

\[
\inf_{F\in\mathcal{P},G\in\mathcal{P},N\geq 2,E[M_0]\geq 0} \max_r \frac{V(SP_r)}{V(OMN)} = \frac{1}{2}
\]

\[
\inf_{F\in\mathcal{P},G,N\geq 2,E[M_0]\geq 0} \max_\alpha \frac{V(MSB_\alpha)}{V(OMN)} \approx 0.948
\]
Theorem. Fix a number of bidders \( N \) and assume that the publisher shares in the rents from brand advertising in any fixed proportions, say \( (\delta, 1 - \delta) \).

If \( F \) is a power law distribution, then there is some \( \alpha \) such that \( MSB_\alpha \) achieves at least 94.8\% of the expected revenue from the corresponding expected-revenue-maximizing strategy-proof auction \( REVMAX \).
Conclusion

- Adverse selection can be neutralized, even without encouraging false-name bidding, provided that $X_n = CM_n$ and $C$ and $M$ are independent.

- The cost of doing that, even without observing the common value factor $C$, is low provided that the tails of the distribution are fat (power law).

- For real applications, we need to evaluate...
  - Is adverse selection important?
  - Are values independent?
  - Are match-value distributions fat-tailed?
24

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