Lower Bound Issues in Computational Social Choice

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What is Computational Social Choice?

Computational Social Choice (COMSOC) aims at improving our understanding of

- **social choice mechanisms** and
- **algorithmic decision making**,

including topics such as voting, rank aggregation, fair division, matching (under preferences), and resource allocation.

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"Closest friend": Algorithmic Game Theory.

Motivation

COMSOC problems frequently lead to NP-hardness.

Some typical voting-related examples:

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- Control.
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$\rightsquigarrow$ **FPT-related studies highly welcome in the COMSOC world**, also justified by many natural parameterizations (most natural to go multivariate).

http://vignette3.wikia.nocookie.net/villains/images/2/29/Dalek_Parliament.JPG

Motivating Example: Sequential Voting I

1956 Education Act in the USA:

Alternatives	**(b)ill**:	Funding to primary and secondary schools.
	(a)mended bill:	Funding, but not to segregated schools.
	(s)tatus quo:	No bill.

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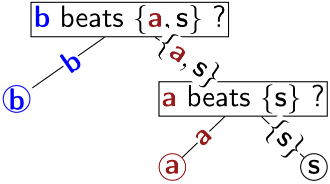
Two popular procedures

Euro-Latin procedure:



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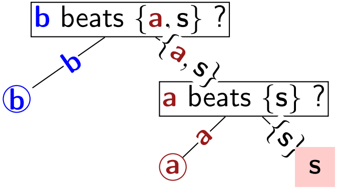

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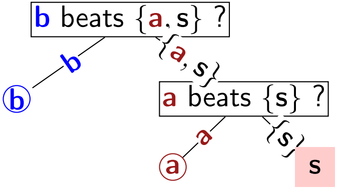
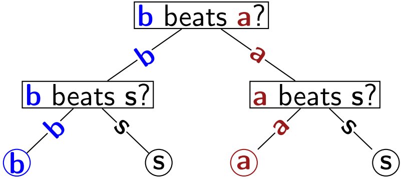


Anglo-American procedure:



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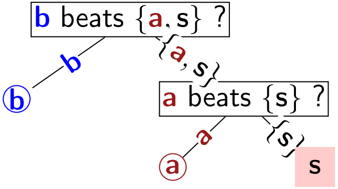

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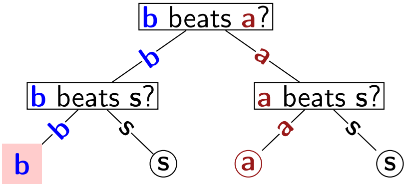

Motivating Example: Sequential Voting II

The agenda (order of alternatives) is essential:

Agenda Control

In: Election $E = (C, V)$ and a specific alternative **a** $\in C$.

Task: Find an agenda for C such that **a** wins.

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Motivating Example: Sequential Voting III

Two further relevant computational problems for sequential voting:

Manipulation

In: Election $E = (C, V)$, **a** $\in C$, $k \in \mathbb{N}$, and agenda $\mathcal{L}$ for C.

Task: Add k voters such that **a** wins under $\mathcal{L}$.

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Possible (or Necessary) Winner

In: Election $E = (C, V)$ with incomplete preferences, **a** $\in C$, and a partial agenda $\mathcal{B}$ for C.

?: Can **a** win in an (or in every) election completing E under an (or under every) agenda completing $\mathcal{B}$?

Motivating Example: Sequential Voting IV

Summary of computational complexity results ([Bredereck/Chen/N./Walsh, IJCAI 2015]):

- n: number of voters.
- m: number of alternatives.
- k: number of added voters.

Problem	Anglo-American	Euro-Latin
Agenda Control	$O(n \cdot m^2 + m^3)$ ♠	$O(n \cdot m^2)$
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Necessary Winner	coNP-hard	$O(n \cdot m^3)$
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W. Necessary Winner	coNP-hard when $m = 4$ ♣ $O(n)$ when $m \leq 3$	$O(n \cdot m^3)$

Main Venues of COMSOC

Central biannual workshop ("COMSOC Workshop Series"):
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Journals with a significant fraction of COMSOC stuff:
ACM Transactions on Economics and Computation, Artificial Intelligence, Autonomous Agents and Multi-Agent Systems, Journal of Artificial Intelligence Research, Journal of Mathematical Economics, Mathematical Social Sciences, Social Choice and Welfare, and scattered around in theory journals (e.g., Information and Computation).

COMSOC History

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- Seminal papers from complexity perspective:
  - Determining winners for many voting rules is NP-hard.
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- Active research area with regular contributions since 2002.
- Name "COMSOC" and biannual workshop since 2006.

Outline of the Rest of the Talk

Four COMSOC problems, six issues:

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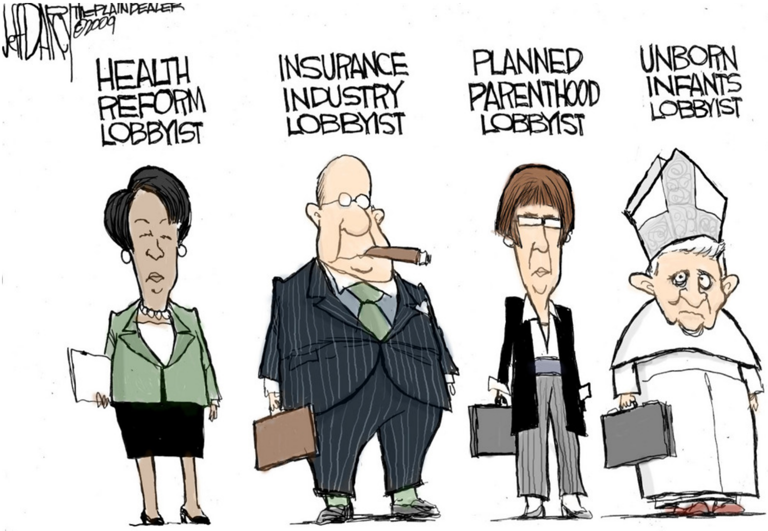
Majority-Wise Accepted Ballot: W[2] vs W[2](Maj).



http://media.cleveland.com/darcy/photo/12ggjeffdarcyjpg-21ffa9c8fee1f80c.jpg

Multi-Issue Elections and Lobbying

Multi-issue election

Votes from an electorate who can accept or reject each of several issues.

Election with 3 issues and 5 voters

Issues:	Emissions trading	Nuclear power	Tax raise
Voter 1	×	×	✓
Voter 2	✓	×	✓
Voter 3	×	✓	×
Voter 4	✓	×	×
Voter 5	×	✓	×
Result	×	×	×

Lobbying

In: $A \in \{0,1\}^{n \times m}$ and $k \leq n$.

?: Can we change at most k rows of A such that each column has more 1s than 0s?

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Can we do better?

Assuming ETH, Lobbying cannot be solved in $2^{o(n)}(n+m)^{O(1)}$ time.
Proof due to reduction from Vertex Cover (parameterized by the number of vertices).

Lobbying: ETH-Based Lower Bounds II

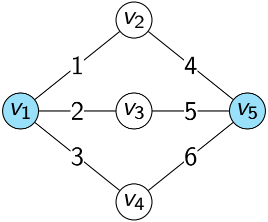
Lobbying not solvable in $2^{o(n)}(n+m)^{O(1)}$ time:

Vertex Cover

In: A graph $G = (V, E)$ and a number h

?: Is there a size-h vertex cover?

Use incidence matrix of input graph $G = (V, E)$ of VC and add $|V| - 2h + 1$ dummy rows and columns.



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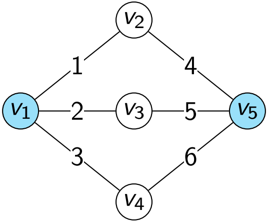

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Vertex Cover

	1	2	3	4	5	6
v_1:	1	1	1	0	0	0
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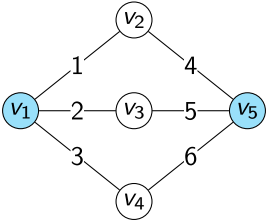

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d_1:	0	0	0	0	0	0
d_2:	0	0	0	0	0	0

Lobbying: ETH-Based Lower Bounds II

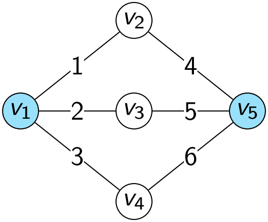
Lobbying not solvable in $2^{o(n)}(n+m)^{O(1)}$ time:

Vertex Cover

In: A graph $G = (V, E)$ and a number h

?: Is there a size-h vertex cover?

Use incidence matrix of input graph $G = (V, E)$ of VC and add $|V| - 2h + 1$ dummy rows and columns.



$h = 2$

Vertex Cover

	1	2	3	4	5	6	dummies	
v_1:	1	1	1	0	0	0	0	0
v_2:	1	0	0	1	0	0	0	0
v_3:	0	1	0	0	1	0	0	0
v_4:	0	0	1	0	0	1	0	0
v_5:	0	0	0	1	1	1	0	0
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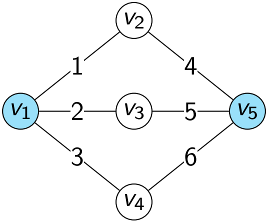

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$k = |V| - h = 3$

Lobbying

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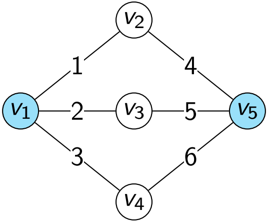

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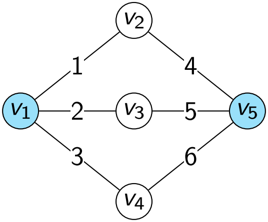

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$k = |V| - h = 3$

Lobbying

$n = 2|V| - 2h + 1 \leq 2|V|$ and Vertex Cover is not solvable in $2^{o(|V|)}$ time.

$\rightsquigarrow$ Lobbying is not solvable in $2^{o(n)} \cdot (n+m)^{O(1)}$ time.

Lobbying: ILP-Based FPT (Lenstra) I

Lobbying

In: $A \in \{0,1\}^{n \times m}$ and $k \leq n$.

?: Can we change at most k rows of A such that each column has more 1s than 0s?

Columns vs. rows: FPT for the $n :=$ #rows was easy to see, what about FPT for $m :=$ #columns?

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Result

- ILP-FPT for parameter number m of columns (next slide),
- but no poly kernel wrt. (m, number of modified rows k).

[Bredereck/Chen/Hartung/Kratsch/N./Suchý/Woeginger, JAIR 2014]

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Challenges:

- Combinatorial algorithm?
- Non-trivial running time lower bounds?

Lobbying: ILP-Based FPT (Lenstra) II

Exact solution via ILP-formulation with 2^m variables

$$\sum_{i=1}^{2^m} x_i \leq k$$

$$\text{constraints: } \forall 1 \leq i \leq 2^m : 0 \leq x_i \leq r_i$$

$$\forall 1 \leq j \leq m : \sum_{i=1}^{2^m} x_i \cdot \bar{A}[i,j] \geq g_j$$

- variable x_i: #rows of type i in the solution
- integer coefficient g_j: #additional ones needed for column j
- integer coefficient r_i: #rows of type i in the matrix
- binary coefficient $\bar{A}[i,j]$: 1 iff row i has a zero in column j

Lobbying: ILP-Based FPT (Lenstra) III

Exact solution via ILP-formulation with 2^m variables

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$$\forall 1 \leq j \leq m : \sum_{i=1}^{2^m} x_i \cdot \bar{A}[i,j] \geq g_j$$

- Linear-time algorithm for $m \leq 4$, factor-log m approximation.
- FPT follows by Lenstra's famous results

  [Lenstra, Mathematics of Operations Research, 1983]
- and implies running time $O^*((2^m)^{2.5 \cdot 2^m + o(2^m)})$

  [Frank and Tardos, Combinatorica, 1987]

  [Kannan, Mathematics of Operations Research, 1987]

Lobbying: ILP-Based FPT (Lenstra) IV

Note: Dozens of other voting problems have ILP-FPT classification wrt. number of alternatives (columns), all lacking lower bounds / combinatorial algorithms.

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General idea behind these ILP-FPT results:

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2. Only the number of objects from each type is important.
3. Plus additional linear constraints.

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⇝ Used to solve Weighted Set Multicover parameterized by "universe size" in FPT time

[Bredereck/Faliszewski/N./Skowron/Talmon, ADT 2015]

Lobbying: LOGSNP-Completeness I

Lobbying

In: $A \in \{0,1\}^{n \times m}$ and $k \leq n$.

?: Can we change at most k rows of A such that each column has more 1s than 0s?

Special situation: Each column needs $\leq g$ many additional ones.

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A simple heuristic: Always take a row filled at least half with 0s.

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Consequently, there is a certificate for yes-instances using only $O(g \cdot \log^2(n+m))$ many bits.

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$\rightsquigarrow$ Solvable in $O((n)^{g \log(m)+1} \cdot m)$ time—is this tight?

Lobbying: LOGSNP-Completeness II

[Papadimitriou and Yannakakis, Journal of Computer and System Sciences 1996]

The class LogSNP

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LogSNP-completeness:

Polynomial-time reduction from/to the LogSNP-complete Rich Hypergraph Cover to/from Lobbying with constant g.

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Rich Hypergraph Cover

In: A ground set $X = \{x_1, \ldots, x_n\}$ with an even number n of elements, m subsets $S_1, \ldots, S_m$ of X with $|S_i| \geq n/2$ for all i, and an integer k.

?: Does there exist a $Y \subseteq X$ with $Y \cap S_i \neq \emptyset$ for all i and $|Y| \leq k$?

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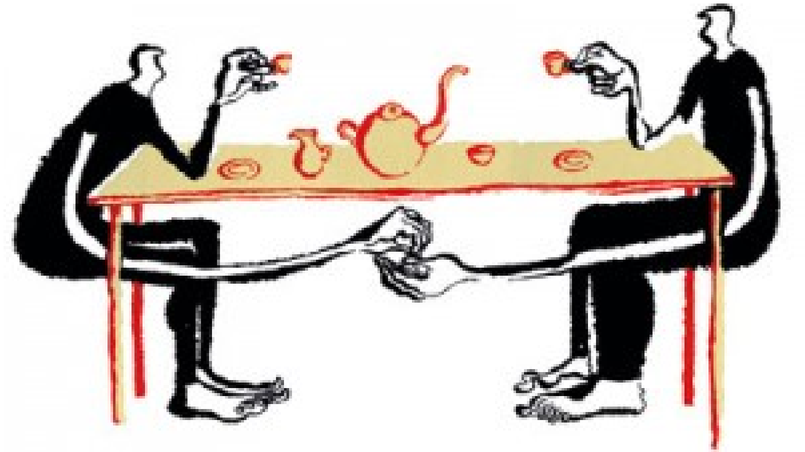
Challenge: Further examples for LOGSNP-hardness.



/blog/wp-content/uploads/2014/10

Shift Bribery FPT Approximability I

Four symphonies:

a	b	c	d
“Arensky II”	“Brahms I”	“Copland III”	“Dvořák IX”

Three national symphony charts

Austria: $a \succ b \succ c \succ d$

Belgium: $a \succ b \succ c \succ d$

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Winner: Compute the "Borda top-ranked" symphony.

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		Borda score
Austria:	$a \succ b \succ c \succ d$	$a: \; 3+3+0=6$
Belgium:	$a \succ b \succ c \succ d$	$b: \; 2+2+3=7$
Chile:	$b \succ d \succ c \succ a$	$c: \; 1+1+1=3$
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$b: \; 2+2+3=7$

$c: \; 1+1+1=3$

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Winner: Compute the “Borda top-ranked” symphony.

Goal: Make c win by shifting it higher, for as little money as possible.

Shift Bribery: FPT Approximation II

Symphony charts example again

Austria: $a \succ b \succ c \succ d$

Belgium: $a \succ b \succ c \succ d$

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$\mathcal{R}$ Shift Bribery

In: Election $E = (C, V = (v_1, \ldots, v_n))$,

?:

Shift Bribery: FPT Approximation II

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Austria: $a \succ b \succ c \succ d$

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?:

Shift Bribery: FPT Approximation II

Symphony charts example again

Austria: $a \succ b \succ c \succ d$ (shift prices: 0, 3)

Belgium: $a \succ b \succ c \succ d$ (shift prices: 0, 2)

Chile: $b \succ d \succ c \succ a$ (shift prices: 0, 1)

$\mathcal{R}$ Shift Bribery

In: Election $E = (C, V = (v_1, \ldots, v_n))$, specific candidate $c \in C$, price function list $\Pi = (\pi_1, \ldots, \pi_n)$,

?:

Shift Bribery: FPT Approximation II

Symphony charts example again

0 3

Austria: $a \succ b \succ c \succ d$

0 2

Belgium: $a \succ b \succ c \succ d$

0 1

Chile: $b \succ d \succ c \succ a$

$\mathcal{R}$ Shift Bribery

In: Election $E = (C, V = (v_1, \ldots, v_n))$, specific candidate $c \in C$, price function list $\Pi = (\pi_1, \ldots, \pi_n)$, budget $B \in \mathbb{N}$.

?:

Shift Bribery: FPT Approximation II

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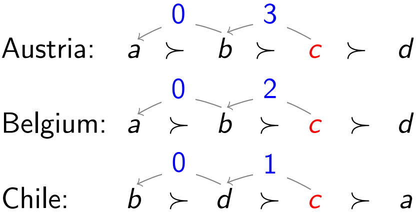

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?: $\exists$ shift action $\vec{s} = (s_1, \ldots, s_n)$ with $\Pi(\vec{s}) \leq B$ such that p is a winner according to rule $\mathcal{R}$?

Shift Bribery: FPT Approximation III

FPT-Approximation Scheme

A factor-$(1+\varepsilon)$ algorithm solving in FPT time wrt. ε and the parameter.

Theorem

For any voting rule $\mathcal{R}$ where Winner Determination is in FPT wrt. to the number n of voters, $\mathcal{R}$ Shift Bribery admits a factor-$(1+\varepsilon)$ approximation scheme; the running time is $O^*(\lceil n/\varepsilon \rceil^n)$.

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Basic idea of the algorithm

- Guess the maximum budget $\pi_{\max}$ to spend on a single voter.
- Rescale and round the price functions to not exceed a given bound (dependent on ε and n).
- Find a cheapest successful shift action $\vec{s}$ for the rescaled instance in $f(\varepsilon, n)$ time.

One can show that this action $\vec{s}$ costs at most $(1+\varepsilon)$OPT.

[Bredereck/Chen/Falizewski/Nichterlein/N., AAAI 2014]

Combinatorial Shift Bribery: Inapproximability

Combinatorial flavor clearly makes the problem more complex...

Can we at least find approximate solutions?

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No: we obtain strong inapproximability results like this:

Theorem

Combinatorial Shift Bribery is inapproximable even in FPT-time with respect to the parameter budget B, even if there are only two candidates.

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No: we obtain strong inapproximability results like this:

Theorem

Combinatorial Shift Bribery is inapproximable even in FPT-time with respect to the parameter budget B, even if there are only two candidates.

Idea:

- Reduction from the W[2]-complete Set Cover problem.
- No solution for the constructed Combinatorial Shift Bribery instance can cost more than B (negative effects are essential for this).

[Bredereck/Faliszewski/N./Talmon, AAMAS 2015]

http://archive.feedblitz.com/217976/ 4087397

Network-Based Vertex Dissolution: ParaNP-Hardness I

Political (Re-)Districting: Example:
Given: Districts with $s = 2$ voters per district.

Goal: Increase the number of voters per district by $\Delta_s = 3$.
Method: Dissolve some districts and redistribute their voters.

Network-Based Vertex Dissolution: ParaNP-Hardness I

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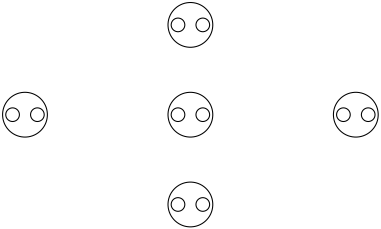

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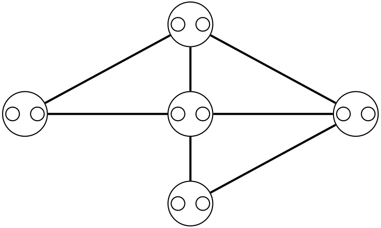
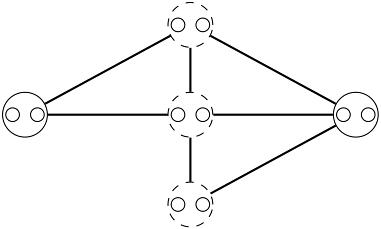


Problem: Cannot move voters arbitrarily!
Move voters of a dissolved district only to an adjacent non-dissolved district.

Network-Based Vertex Dissolution: ParaNP-Hardness I

Political (Re-)Districting: Example:

Given: Districts with $s = 2$ voters per district.

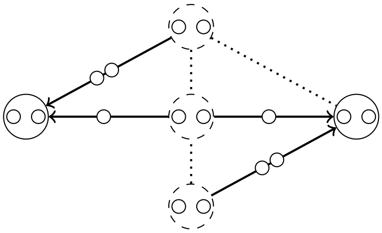


Solution: Dissolve the middle districts.

Network-Based Vertex Dissolution: ParaNP-Hardness I

Political (Re-)Districting: Example:

Given: Districts with $s = 2$ voters per district.



Network-Based Vertex Dissolution: ParaNP-Hardness I

Political (Re-)Districting: Example:

Result: Districts with 5 voters per district.

Network-Based Vertex Dissolution: ParaNP-Hardness II

What do we know from structural results?

$s \backslash \Delta$	1	2	3	4	5	6
1	?	?	?	?	?	?
2	?	?	?	?	?	?
3	?	?	?	?	?	?
4	?	?	?	?	?	?
5	?	?	?	?	?	?
6	?	?	?	?	?	?

Network-Based Vertex Dissolution: ParaNP-Hardness II

$s \backslash \Delta$	1	2	3	4	5	6
1	P	?	?	?	?	?
2	?	P	?	?	?	?
3	?	?	P	?	?	?
4	?	?	?	P	?	?
5	?	?	?	?	P	?
6	?	?	?	?	?	P

Lemma 1: There exists an (s,s)-dissolution for an undirected graph G if and only if G has a perfect matching.

Network-Based Vertex Dissolution: ParaNP-Hardness II

$s \backslash \Delta$	1	2	3	4	5	6
1	P	NP-h	NP-h	NP-h	NP-h	NP-h
2	?	P	?	NP-h	?	NP-h
3	?	?	P	?	?	NP-h
4	?	?	?	P	?	?
5	?	?	?	?	P	?
6	?	?	?	?	?	P

Idea: There exists a $(t \cdot \Delta_s, \Delta_s)$-dissolution for an undirected graph G if and only if G has a t-star partition.
Partitioning a graph into t-stars is NP-hard.

Network-Based Vertex Dissolution III: Para-NP Hardness

Symmetry with respect to s and Δ_s:

Lemma 2: There exists an (s, Δ_s)-dissolution for an undirected graph G if and only if there exists a (Δ_s, s)-dissolution for G.

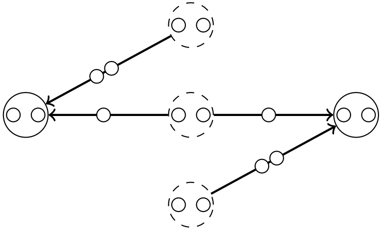
Network-Based Vertex Dissolution III: Para-NP Hardness

Symmetry with respect to s and Δ_s:

Lemma 2: There exists an (s, Δ_s)-dissolution for an undirected graph G if and only if there exists a (Δ_s, s)-dissolution for G.

$s = 2$

$\Delta_s = 3$



Network-Based Vertex Dissolution III: Para-NP Hardness

Symmetry with respect to s and Δ_s:

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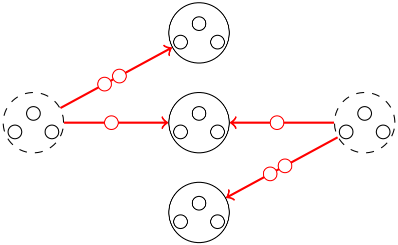
$s = 3$

$\Delta_s = 2$



Interpret the voter movement backwards.
We put the voters to receive into the districts.
Move to each dissolved districts two voters.

Network-Based Vertex Dissolution II: ParaNP-Hardness III

P vs NP dichotomy:

$s \backslash \Delta_s$	1	2	3	4	5	6
1	P	NP-h	NP-h	NP-h	NP-h	NP-h
2	?	P	?	NP-h	?	NP-h
3	?	?	P	?	?	NP-h
4	?	?	?	P	?	?
5	?	?	?	?	P	?
6	?	?	?	?	?	P

Network-Based Vertex Dissolution II: ParaNP-Hardness III

$s \backslash \Delta$	1	2	3	4	5	6
1	P	NP-h	NP-h	NP-h	NP-h	NP-h
2	NP-h	P	?	NP-h	?	NP-h
3	NP-h	?	P	?	?	NP-h
4	NP-h	NP-h	?	P	?	?
5	NP-h	?	?	?	P	?
6	NP-h	NP-h	NP-h	?	?	P

Idea: There exists an (s, Δ_s)-dissolution for an undirected graph G if and only if there exists a (Δ_s, s)-dissolution for G.

Network-Based Vertex Dissolution II: ParaNP-Hardness III

$s \backslash \Delta_s$	1	2	3	4	5	6
1	P	NP-h	NP-h	NP-h	NP-h	NP-h
2	NP-h	P	NP-h	NP-h	NP-h	NP-h
3	NP-h	NP-h	P	NP-h	NP-h	NP-h
4	NP-h	NP-h	NP-h	P	NP-h	NP-h
5	NP-h	NP-h	NP-h	NP-h	P	NP-h
6	NP-h	NP-h	NP-h	NP-h	NP-h	P

Finally: The rest of the cells are covered by an NP-hardness reduction from **Exact Cover by t-Sets**

Network-Based Vertex Dissolution II: ParaNP-Hardness III

$s \backslash \Delta_s$	1	2	3	4	5	6
1	P	NP-h	NP-h	NP-h	NP-h	NP-h
2	NP-h	P	NP-h	NP-h	NP-h	NP-h
3	NP-h	NP-h	P	NP-h	NP-h	NP-h
4	NP-h	NP-h	NP-h	P	NP-h	NP-h
5	NP-h	NP-h	NP-h	NP-h	P	NP-h
6	NP-h	NP-h	NP-h	NP-h	NP-h	P

Theorem

If $s = \Delta_s$, then DISSOLUTION solvable in $O(n^\omega)$ time (where ω is the matrix multiplication exponent); otherwise NP-complete.

[Bevern/Bredereck/Chen/Froese/N./Woeginger, SIAM J. Discrete Math. 2015]

Network-Based Vertex Dissolution: ParaNP-Hardness IV

P vs. NP dichotomy for planar case:

$s \backslash \Delta_s$	1	2	3	4	5	6
1	P	NP-h	NP-h	NP-h	NP-h	NP-h
2	NP-h	P	?	NP-h	?	NP-h
3	NP-h	?	P	?	?	NP-h
4	NP-h	NP-h	?	P	?	?
5	NP-h	?	?	?	P	?
6	NP-h	NP-h	NP-h	?	?	P

Planar: a lot of open cases!

Network-Based Vertex Dissolution: ParaNP-Hardness IV

P vs. NP dichotomy for planar case:

$s \backslash \Delta_s$	1	2	3	4	5	6
1	P	NP-h	NP-h	NP-h	NP-h	NP-h
2	NP-h	P	?	NP-h	?	NP-h
3	NP-h	?	P	?	?	NP-h
4	NP-h	NP-h	?	P	?	?
5	NP-h	?	?	?	P	?
6	NP-h	NP-h	NP-h	?	?	P

Planar: a lot of open cases!

Interesting subproblem: What is the complexity of **Planar Exact Cover by t-Sets** for $t \geq 3$?

http://www.stephaniemcmillan.org/codegreen/comics/2011-03-21-push-their-agenda.jpg

Majority-wise Accepted Ballot: W[2] vs W[2](Maj) I

Unanimously Accepted Ballot

In: A set $\mathcal{P}$ of m proposals; a set $\mathcal{V}$ of n voters with favorite ballots $B_1, \ldots, B_n \subseteq \mathcal{P}$; an agenda $Q_+ \subseteq \mathcal{P}$.

?: Is there a ballot Q with $Q_+ \subseteq Q \subseteq \mathcal{P}$ which **every** single voter i accepts (that is, $|B_i \cap Q| > |Q|/2$)?

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?: Is there a ballot Q with $Q_+ \subseteq Q \subseteq \mathcal{P}$ which a **strict majority** of the voters accepts (that is, $|B_i \cap Q| > |Q|/2$)?

Majority-wise Accepted Ballot: Example

Example with five proposals and four voters.

- $\mathcal{P} = \{p_1, p_2, p_3, p_4, p_5\}$
- $B_1 = \{p_2, p_3, p_4\}$, $B_2 = \{p_1, p_3, p_5\}$, $B_3 = \{p_1, p_2, p_4\}$, $B_4 = \{p_1, p_2, p_3\}$

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Ballot matrix:

p_1	p_2	p_3	p_4	p_5
0	1	1	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	0	0

Note: The ballot $\{p_1, p_2, p_3\}$ would be accepted by all voters. However, although less than half of the voters like p_5, the commission can push through p_5 by proposing $\{p_1, p_2, p_3, p_4, p_5\}$.

Majority-wise Accepted Ballot: Example

Example with five proposals and four voters.

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Ballot matrix:

p_1	p_2	p_3	p_4	p_5
0	1	1	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	0	0

Interesting: No monotonicity w.r.t. solution sizes.

Majority-wise Accepted Ballot: W[2] vs W[2](Maj) II

[Alon, Bredereck, Chen, Kratsch, N., Woeginger, ACM TEAC 2015]

With respect to the parameter size $|Q|$ of the solution, both variants are easily shown to be W[2]-hard (via Hitting Set).

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W[2]-membership can be show by a quite technical reduction to the W[2]-complete Independent Dominating Set problem.

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Unanimously Accepted Ballot

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Majoritywise Accepted Ballot

Showing W[2]-membership seems quite challenging.

Interestingly: Considering the class W[2](Maj) instead of W[2], that is, allowing only majority gates instead of AND/OR gates in the boolean circuits, one can show membership relatively easy.

Concluding Remarks

COMSOC offers a rich variety of combinatorial problems (touching permutations, partial orders, sets, matrices, graphs, etc.), many of these are NP-hard.

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Lower bounds in COMSOC so far mostly of the form

- W-hardness (W-completeness) or
- ParaNP-hardness or
- no polynomial-size problem kernel unless...

$\rightsquigarrow$ ETH-based lower bounds largely unexplored...

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General challenges:

- Lower bounds for "Lenstra-ILP-FPT results"?
- Exploration of LOGSNP-completeness for lower bounds?
- W[2] = W[2](Maj)?

  [Fellows, Flum, Hermelin, Müller, Rosamond, TOCS 2010]
- Running time lower bounds for FPT approximation schemes?

Some General COMSOC Literature

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- Felix Brandt, Vincent Conitzer, and Ulle Endriss. Computational Social Choice. In Gerhard Weiss, editor, Multiagent Systems, Chapter 6, MIT Press, 2nd edition, 2013.
- Nadja Betzler, Robert Bredereck, Jiehua Chen, Rolf Niedermeier: Studies in Computational Aspects of Voting – A Parameterized Complexity Perspective. The Multivariate Algorithmic Revolution and Beyond, Springer 2012.
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