Quantum computing and the entanglement frontier

\[ |j_0\rangle = |0\rangle \]

\[ |j_1\rangle = |0\rangle \]

\[ |j_2\rangle = |0\rangle \]

\[ |s_0\rangle \]

\[ |s_1\rangle \]

John Preskill
Visions of TOC
29 May 2013
### Frontiers of Physics

<table>
<thead>
<tr>
<th>short distance</th>
<th>long distance</th>
<th>complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higgs boson</td>
<td>Large scale structure</td>
<td>“More is different”</td>
</tr>
<tr>
<td>Neutrino masses</td>
<td>Cosmic microwave background</td>
<td>Many-body entanglement</td>
</tr>
<tr>
<td>Supersymmetry</td>
<td>Dark matter</td>
<td>Phases of quantum matter</td>
</tr>
<tr>
<td>Quantum gravity</td>
<td>Dark energy</td>
<td>Quantum computing</td>
</tr>
<tr>
<td>String theory</td>
<td></td>
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</tbody>
</table>
Quantum Information Science:

*Can we control complex quantum systems and if so what are the scientific and technological implications?*

Not the frontier of short (subnuclear) distances or long (cosmological) distances, but rather the frontier of highly complex quantum states: *The entanglement frontier*
Truism:
the macroscopic world is classical.
the microscopic world is quantum.

Goal of Quantum Information Science:
controllable quantum behavior in scalable systems

Why?
Classical systems cannot simulate quantum systems efficiently (a widely believed but unproven conjecture).

But to control quantum systems we must slay the dragon of decoherence …

Is this merely really, really hard?
Or is it ridiculously hard?
Toward quantum supremacy

Sufficiently complex quantum systems will behave in ways that cannot be predicted using digital computers --- these systems will “surpass understanding” and surprise us.

What quantum tasks are feasible?
What quantum tasks are hard to simulate classically?

Or … might it be that the extravagant “exponential” classical resources required for classical description and simulation of generic quantum states are illusory, because quantum states in Nature have succinct descriptions?
2013
Quantum Theory
Though quantum theory is over 100 years old, quantum and classical systems differ in profound ways we are just beginning to understand …
Information is encoded in the state of a physical system.
Information is encoded in the state of a quantum system.
Put Weirdness to work!
Theoretical Quantum Information Science is driven by ...

Three *Great* Ideas:

1) Quantum Entanglement
2) Quantum Computation
3) Quantum Error Correction
Classical Bit
Classical Bit
Classical Bit

What went in, comes out.
Quantum Bit (“Qubit”)

The two doors are two complementary observables, such as two ways to measure the polarization state of a photon.
Quantum Bit (“Qubit”)

If you open the *same* door that you closed, you can recover the bit from the box.
Quantum Bit ("Qubit")
Quantum Bit ("Qubit")

If you open a different door than you closed, the color is random (red 50% of the time and green 50% of the time).
No cloning!
Photon polarization as a qubit

\[ |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \]

\[ |1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \]
Quantum Correlations

Open either door in Pasadena, and the color of the ball is *random*.

Same thing in Andromeda.
Quantum Correlations

But if we both open the same door, we always find the same color.
Quantum Correlations

Quantum information can be *nonlocal*, shared equally by a box in Pasadena and a box in Andromeda.

This phenomenon, called *quantum entanglement*, is a crucial feature that distinguishes quantum information from classical information.
Classical Correlations
Classical Correlations

Quantum Correlations

Aren’t boxes like soxes?
Einstein’s 1935 paper, with Podolsky and Rosen (EPR), launched the theory of quantum entanglement. To Einstein, quantum entanglement was so unsettling as to indicate that something is missing from our current understanding of the quantum description of Nature.
“Another way of expressing the peculiar situation is: the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of its *parts* … I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought…

By the interaction the two representatives [quantum states] have become *entangled.*”

“It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter’s mercy in spite of his having no access to it.”

Quantum Entanglement

Bell ‘64
Quantum information can be nonlocal; quantum correlations are a stronger resource than classical correlations.
Alice and Bob play a cooperative two-player game.

If they share correlated classical bits and play their best strategy, they win with probability 75% (averaged over the inputs they receive).
Quantum entanglement

Alice and Bob play a cooperative two-player game.

If they share entangled qubits and play their best strategy, they win with probability 85.4% (averaged over the inputs they receive).

If they share entangled qubits and play their best strategy, they win with probability 85.4% (averaged over the inputs they receive).
Quantum entanglement

In experimental tests, physicists have played the game and have won with probability above 75%.

Goal: $a \oplus b = x \land y$

Quantum correlations are a stronger resource than classical correlations.
Quantum entanglement

In experimental tests, physicists have played the game and have won with probability above 75%.

Quantum correlations are a stronger resource than classical correlations.

* Spooky action at a distance!!

* Spukhafte Fernwirkungen!!*

\[
\text{Goal: } a \oplus b = x \land y
\]
Quantum entanglement

In experimental tests, physicists have played the game and have won with probability above 75%.

Goal: $a \oplus b = x \land y$

* Spooky action at a distance!!
Boxes are not like soxes!
Quantum information vs. Classical information

1) **Randomness.** Clicks in a Geiger counter are intrinsically random, not pseudorandom. Can't predict outcome even with the most complete possible knowledge of the state.

2) **Uncertainty.** Operators A and B do not commute means that measuring A influences the outcome of a subsequent measurement of B.

3) **Entanglement.** The whole is more definite than the parts. Even if we have the complete possible knowledge of the (pure) state of joint system AB, the (mixed) state of A may be highly uncertain.
Nearly all the information in a typical entangled “quantum book” is encoded in the correlations among the “pages”.

You can't access the information if you read the book one page at a time.
To describe 300 qubits, we would need more numbers than the number of atoms in the visible universe!
We can’t even hope to *describe* the state of a few hundred qubits in terms of classical bits.

Might a computer that operates on qubits rather than bits (a *quantum computer*) be able to perform tasks that are beyond the capability of any conceivable classical computer?
Peter Shor
Finding Prime Factors

\[ 1807082088687 \times 4048059516561 \times 6440590556627 \times 8102516769401 \times 3491701270214 \times 500566254024 \times 4048387341127 \times 5908123033717 \times 8188796656318 \times 2013214880557 = ? \times ? \]
## Finding Prime Factors

<table>
<thead>
<tr>
<th>Factors</th>
<th>Prime Factors</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1807082088687</td>
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<td></td>
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<tr>
<td>8188796656318</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013214880557</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The boundary between “hard” and “easy” seems to be different in a quantum world than in a classical world.

Shor
<table>
<thead>
<tr>
<th>Classical Computer</th>
<th>Quantum Computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor 193 digits in 30 CPU years (2.2 GHz).</td>
<td>Factor 193 digits in 0.1 second.</td>
</tr>
<tr>
<td>Factor 500 digits in $10^{12}$ CPU years.</td>
<td>Factor 500 digits in 2 seconds.</td>
</tr>
</tbody>
</table>

MISSION:

IMPOSSIBLE

Peter Shor
Problems

Classically Easy

Quantumly Easy

Quantumly Hard

Classically Easy
What’s in here?
Quantum algorithms

*Quantum computers have limitations:* Spectacular quantum speedups seem to be possible only for problems with special structure, *not* for NP-complete problems like 3-SAT. (Quantum physics speeds up unstructured search quadratically, not exponentially.)

*Beyond NP:* Speedups for problems outside NP are also common and important. Indeed the “natural” application for a quantum computer is simulating time evolution of quantum systems, e.g. collisions in molecular chemistry or quantum field theory.

Many more quantum algorithms at math.nist.gov/quantum/zoo/
Quantum algorithms for quantum field theories

Classical methods have limited precision, particularly at strong coupling.

A quantum computer can simulate particle collisions, even at high energy and strong coupling, using resources (number of qubits and gates) scaling polynomially with precision, energy, and number of particles.

Does the quantum circuit model capture the computational power of Nature?

What about quantum gravity?

Decoherence

\[ \frac{1}{\sqrt{2}} \left( \begin{array}{c}
+ \\
\end{array} \right) \]

Environment
Decoherence explains why quantum phenomena, though observable in the microscopic systems studied in the physics lab, are not manifest in the macroscopic physical systems that we encounter in our ordinary experience.
How can we protect a quantum computer from decoherence and other sources of error?
What about errors?
What about errors?
What about errors?

Error!
What about errors?
What about errors?
What about errors?
What about errors?

Redundancy protects against errors.
No cloning!
What about *quantum* errors?
What about *quantum* errors?
What about *quantum* errors?

Error!
What about *quantum* errors?
What about *quantum* errors?
What about quantum errors?

To fix the errors, must we know what door the dragon opened?
What about *quantum* errors?
What about *quantum* errors?
What about *quantum* errors?

A door-number-2 error ("phase error") occurs if the dragon remembers (i.e., copies) the color that he sees through door number 1. It is easier to remember a bit than to flip a bit; therefore, phase errors are particularly pervasive.
To resist decoherence, we must prevent the environment from “learning” about the state of the quantum computer during the computation.
If a quantum computation works, and you ask the quantum computer later what it just did, it should answer: “I forget...”
What about quantum errors?

One qubit of quantum information can be encoded in the nonlocal correlations among five qubits.
What about *quantum* errors?
What about *quantum* errors?

Though the dragon does damage one of the boxes, and he might learn something about the color of the ball in that box, this information does not tell him anything about the *encoded* qubit. Therefore the damage is *reversible*. Error!
What about quantum errors?
What about *quantum* errors?

By making carefully designed *collective* measurements on the five qubits (using a quantum computer), the beaver learns what damage the dragon inflicted, and how to reverse it. But he, too, learns nothing about the state of the encoded qubit.
What about *quantum* errors?

Redundancy protects against *quantum* errors!
Alexei Kitaev
A. Kitaev

Anqons and Fault Tolerance

9 April 1997

Classical Fault Tolerance
- Not needed! Why?

Magnetic disk:

\[ H = -J \sum_i \sigma_i^z \sigma_{i+1}^z \]  -- A "repetition code"

Rep. code has no quantum analog

Closest thing is "toric code"

Forms qubits on edges of lattice

Stabilizer generators:

- All mutually commuting

9 April 1997 … An exciting day!
Topology

- Quantum Computer

- Noise!
Aharonov-Bohm Phase \[ \exp(i \epsilon \Phi) \]
Aharonov-Bohm Phase \( \exp(ie\Phi) \)
Nonabelian anyons

Quantum information can be stored in the collective state of exotic particles in two spatial dimensions (“anyons”).

The information can be processed by exchanging the positions of the anyons (even though the anyons never come close to one another).
Quantum information can be stored in the collective state of exotic particles in two spatial dimensions (“anyons”).

The information can be processed by exchanging the positions of the anyons (even though the anyons never come close to one another).
Topological quantum computation (Kitaev ’97, FLW ’00)

create pairs

braid

braid

braid

annihilate pairs?

create pairs

Kitaev

Freedman
Topological quantum computation (Kitaev ’97, FLW ‘00)

annihilate pairs?

create pairs

braid

braid

braid

Kitaev

Freedman
Topological quantum computation

The computation is intrinsically resistant to decoherence. If the paths followed by the particles in spacetime execute the right braid, then the quantum computation is guaranteed to give the right answer!
Kitaev’s magic trick: sawing an electron in half!
Majorana fermion

conventional superconductor

Majorana fermion

add an electron

topological superconductor
Majorana fermion

conventional superconductor

topological superconductor

Majorana fermion

conventional superconductor
Majorana fermion\[\rightarrow\]
topological superconductor\[\downarrow\]
Majorana fermion\[\rightarrow\]

Majorana fermion

conventional superconductor

topological superconductor

Majorana fermion

conventional superconductor

conventional superconductor
topological superconductor

Majorana fermion

conventional superconductor

Majorana fermion

conventional superconductor

conventional superconductor
Majorana fermion

conventional superconductor

Majorana fermion

conventional superconductor

topological superconductor
Dave Wineland

2012 Nobel Prize in Physics
Ion Trap Quantum Computer
Two $^9\text{Be}^+$ ions in an ion trap at the National Institute of Standards and Technology (NIST) in Boulder, CO.
Ion Trap Quantum Computer
Ion Trap Quantum Computer
Ion Trap Quantum Computer
Ion Trap Quantum Computer
Ion Trap Quantum Computer
Ion Trap Quantum Computer

Dave Wineland
Wineland Lab, NIST

Ion trap quantum computer: The Reality
Persistent current in a superconducting circuit

Magnetic field of a single electron
Quantum Hardware

- Two-level ions in a Paul trap, coupled to “phonons.”
- Superconducting circuits with Josephson junctions.
- Electron spin (or charge) in quantum dots.
- Cold neutral atoms in optical lattices.
- Two-level atoms in a high-finesse microcavity, strongly coupled to cavity modes of the electromagnetic field.
- Linear optics with efficient single-photon sources and detectors.
- Nuclear spins in semiconductors, and in liquid state NMR.
- Nitrogen vacancy centers in diamond.
- Anyons in fractional quantum Hall systems, quantum wires, etc.
Classical vs. Quantum Factoring

Factoring 2048 bit number …

**Classical algorithm**: 10 year run time and requires a server farm covering 1/4 of North America, at cost of $10^6$ trillion. Consumes $10^6$ terawatt ($10^5$ times world output). Would consume world's supply of fossil fuels in one day.

**Quantum algorithm** (brute force): 10K logical qubits and 10M physical (superconducting) qubits. 1 cm spacing to allow room for lost of wires. Costs $100B ($10K per physical qubit) and runs in 16 hours. Consumes 10 MWatt. (We need to get the cost down.)
Quantum error correction

Classical memory $\Leftrightarrow$ ferromagnet order

Robust bit

Quantum memory $\Leftrightarrow$ topological order

Robust qubit

Red path (door 1) or green path (door 2)

Realize physically, or simulate with generic hardware.

Some recently reported error rates

Ion trap – one-qubit gates:
~ $2 \times 10^{-5}$  [NIST]

Ion trap – two-qubit gates:
~ $5 \times 10^{-3}$  [Innsbruck]

Superconducting circuits – one-qubit gate
~ $2.5 \times 10^{-3}$  [Yale]

Quantum error correction becomes effective when gate error rates are low enough, and the overhead cost of error correction improves as hardware becomes more reliable.

Error rates are estimated by performing “circuits” of variable size, and observing how the error in the final readout grows with circuit size.
Three Questions About Quantum Computers

1. *Why* build one?

How will we use it, and what will we learn from it?

2. *Can* we build one?

Are there obstacles that will prevent us from building quantum computers as a matter of principle?

3. *How* will we build one?

What kind of quantum hardware is potentially scalable to large systems?
Classical correlations are polygamous
Quantum correlations are *monogamous*

- Betty
  - Adam: fully entangled
  - Charlie: unentangled
Quantum correlations are *monogamous*
Monogamy is **frustrating**!

Betty

- fully entangled

Adam

- cryptography
- quantum matter
- black holes

Charlie

- unentangled
Information Puzzle: Is a black hole a quantum cloner?

Suppose that the collapsing body’s quantum information is encoded in the emitted Hawking radiation; the information is thermalized, not destroyed.

The green time slice crosses both the collapsing body behind the horizon and nearly all of the radiation outside the horizon. *Thus the same (quantum) information is in two places at the same time.*

A quantum cloning machine has operated, which is not allowed by the linearity of quantum mechanics.

We’re stuck: either information is destroyed or cloning occurs. Either way, quantum physics needs revision.
Perhaps the lesson is that, for mysterious reasons that should be elucidated by a complete theory of quantum gravity, it is wrong to think of the “outside” and “inside” portions of the time slice as two separate subsystems of a composite system.

\[ \mathcal{H} \not\cong \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}} \]

Rather, the inside and outside are merely complementary descriptions of the same system. Which description is appropriate depends on whether the observer enters the black hole or stays outside.
Black hole complementarity challenged

Three reasonable beliefs, not all true! [AMPS 2012]:

(1) The black hole “scrambles” information, but does not destroy it.

(2) An observer who falls through the black hole horizon sees nothing unusual (at least for a while).

(3) An observer who stays outside the black hole sees nothing unusual.

Conservative resolution: A “firewall” at the horizon.
Complementarity Challenged

(1) For an old black hole, recently emitted radiation (B) is highly entangled with radiation emitted earlier (C) by the time it reaches Charlie.

(2) If freely falling observer sees vacuum at the horizon, then the recently emitted radiation (B) is highly entangled with modes behind the horizon (A).

(3) If B is entangled with C by the time it reaches Charlie, it was already entangled with C at the time of emission from the black hole.

Monogamy of entanglement violated!
Complementarity Challenged

(1) If A and B not entangled, a firewall at the horizon! Freely falling observer burns without warning.

(2) If B and C not entangled, evolution is nonunitary and information is lost.

(3) If B and C are entangled as B reaches C, but not before, entanglement is generated nonlocally.

It seems that a single observer ought to be able to verify both the BC entanglement and the AB entanglement, hence invoking complementarity does not seem to provide a pleasing resolution.
nature is subtle
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

“Nature is subtle” is a play on Einstein’s famous pronouncement: “Raffiniert ist der Herrgott aber boshaf ist er nicht” (Subtle is the Lord, but malicious He is not).

For all his genius, Einstein underestimated the subtlety of nature when he derisively dismissed quantum entanglement as “Spukhafte Fernwirkungen” (Spooky action at a distance). The aim of quantum information science is to relish, explore, and exploit the glorious subtlety of the quantum world in all its facets and ramifications.
Quantum computing and the entanglement frontier

\[ |j_0\rangle = |0\rangle \quad H \quad H \quad \rightarrow \quad |j_1\rangle = |0\rangle \quad H \quad S \quad H \quad \rightarrow \quad |j_2\rangle = |0\rangle \quad H \quad T \quad S \quad H \quad \rightarrow \quad |s_0\rangle \quad U^4 \quad U^2 \quad U \quad |s_1\rangle \]

depth (increasingly coarse grained)

John Preskill
Visions of TOC
29 May 2013