Ground state connectivity of local Hamiltonians

Sevag Gharibian

Jamie Sikora

Department of Computer Science
Virginia Commonwealth University
U.S.A.

Center for Quantum Technologies
National University of Singapore
Singapore

May 6, 2015

Outline

Outline

Is there life after the Local Hamiltonian Problem?

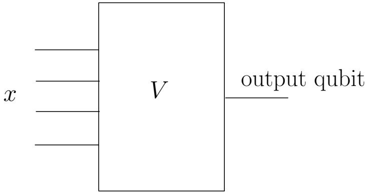
Signs of life

Examples.

1. Computing ground state degeneracy and density of states of a local Hamiltonian is $\#\mathrm{BQP}$-complete (Brown, Flammia, Schuch, 2011).
2. Determining the smallest subset S of interaction terms of a given local Hamiltonian such that S has a frustrated ground space is cq-Σ_2-hard to approximate (Gharibian, Kempe, 2012).
3. Evaluating local observables against a local Hamiltonian is $\mathrm{P}^{\mathrm{QMA}[\log n]}$-complete (Ambainis, 2014).

QCMA

Quantum-Classical Merlin Arthur (QCMA)



Classical proof x, quantum verifier V

QCMA

Known QCMA-complete problems:

- Does local Hamiltonian H have an efficiently preparable quantum ground state? [Wocjan, Janzing, Beth, 2003]
- Does a quantum circuit C act almost as the identity on standard basis states? [Wocjan, Janzing, Beth, 2003]
- Given a braid, can it be conjugated by another braid from a given class such that the Jones polynomial of its plat closure is nearly maximal? [Wocjan, Yard, 2006]
- Given a continuous-time classical random walk on a restricted class of graphs, and time T, do there exist vertices i and j such that the difference of the probabilities of being at i and j is at least $c \cdot \exp(-\mu T)$? [Janzing, Wocjan, 2006]
- Given a quantum circuit C accepting a non-empty monotone set, what is the smallest Hamming weight string accepted by C? [G, Kempe, 2012]

Reconfiguration problems

Problem 1.
Given Boolean formula ϕ and satisfying assignments $x, y \in \{0,1\}^n$, $\exists$ sequence of bit flips mapping x to y s.t. all intermediate assignments are satisfying?

[Gopalan, Kolaitis, Maneva, Papadimitriou, 2009]
Problem 1 is in P or PSPACE-complete, depending on constraint types in ϕ.

Reconfiguration problems

Problem 1.
Given Boolean formula ϕ and satisfying assignments $x, y \in \{0, 1\}^n$, $\exists$ sequence of bit flips mapping x to y s.t. all intermediate assignments satisfy ϕ?

[Gopalan, Kolaitis, Maneva, Papadimitriou, 2009]
Problem 1 is in P or PSPACE-complete, depending on constraint types in ϕ.

Problem 2.
Same as Problem 1, except now we find the shortest such sequence.

[Mouawad, Nishimura, Pathak, Raman, 2014]
Problem 2 is in P, NP-complete, or PSPACE-complete, depending on constraint types in ϕ.

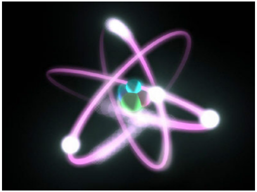
Quantum: Intuitive definition

Input:

- A local Hamiltonian H,
- Two ground states of H, $|\psi_{\text{init}}\rangle$ and $|\psi_{\text{final}}\rangle$.

Output:

Does there exist a "sequence of operations" mapping $|\psi_{\text{init}}\rangle$ to $|\psi_{\text{final}}\rangle$ *through* the ground space of H?



More formal definition

Ground State Connectivity [GSCON(k, t, q)]

Input:

- A k-local Hamiltonian H acting on n qubits,
- Ground states $|\psi_{\text{init}}\rangle$ and $|\psi_{\text{final}}\rangle$ of H, specified by quantum circuits.

Output:

Does there exist a sequence of q-qubit unitaries $(U_i)_{i=1}^{t}$ such that:

- $U_t \cdots U_1 |\psi_{\text{init}}\rangle \approx |\psi_{\text{final}}\rangle$,
- $\forall\ 1 \leq i \leq t$, $|\psi_i\rangle := U_i \cdots U_1 |\psi_{\text{init}}\rangle$ lies "in the ground space" of H, i.e.

$$\langle\psi_i|H|\psi_i\rangle \text{ is "small" ?}$$

More formal definition

Ground State Connectivity [GSCON(k, t, q)]

Input:

- A k-local Hamiltonian H acting on n qubits,
- Ground states $|\psi_{\text{init}}\rangle$ and $|\psi_{\text{final}}\rangle$ of H, specified by quantum circuits.

Output:

Does there exist a sequence of q-qubit unitaries $(U_i)_{i=1}^{t}$ such that:

- $U_t \cdots U_1 |\psi_{\text{init}}\rangle \approx |\psi_{\text{final}}\rangle$,
- $\forall\ 1 \leq i \leq t$, $|\psi_i\rangle := U_i \cdots U_1 |\psi_{\text{init}}\rangle$ lies "in the ground space" of H, i.e.

$$\langle \psi_i | H | \psi_i \rangle \text{ is "small" ?}$$

Notes:

- Physical motivations. Quantum memories, stablizer codes.
- $|\psi_i\rangle$ in actual ground space. True in our QCMA-completeness setting since QCMA=QCMA_1 (Jordan, Kobayashi, Nagaj, Nishimura, 2012).

Our Results

Theorem 1
GSCON ($k = 5$, $t = \text{poly}(n)$, $q = 2$) is QCMA-complete.

Theorem 2
GSCON ($k = 3$, $t = \exp(n)$, $q = 1$) is PSPACE-complete.

Theorem 3
SUCCINCT GSCON ($k = 5$, $t = \exp(n)$, $q = 1$) is NEXP-complete.

k: locality of Hamiltonian
t: number of unitaries
q: locality of unitaries

Outline

Outline

1. Ground State Connectivity
2. QCMA-hardness
   - The construction: *Go* qubits
   - The proof: K-orthogonality and the Traversal Lemma
3. Tightness of the Traversal Lemma
4. Conclusions

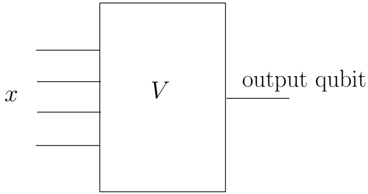
The construction

Goal

Given a QCMA verification circuit V, map V to

- local Hamiltonian H, and
- ground states $|\psi_{\text{init}}\rangle$ and $|\psi_{\text{final}}\rangle$ of H

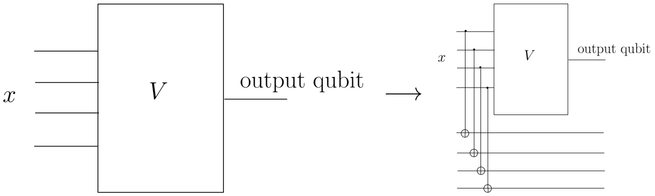
satisfying the conditions of GSCON ($k = 5$, $t = \text{poly}(n)$, $q = 2$).



The construction

Step 1

Force proofs to be in computational basis:



Henceforth denote the new circuit as V.

The construction

Step 2

Map V to a 3-local Hamiltonian H_V using Kempe and Regev's circuit-to-Hamiltonian construction [Kempe, Regev, 2003].

Properties of H_V:

- If there exists a proof $|\psi\rangle$ accepted by V with "high" probability, then H_V has a "small" eigenvalue.
- If V rejects all proofs $|\psi\rangle$ with "high probability", then all of H_V's eigenvalues are "large".

Step 3

Adding a 3-qubit “GO” register g, define:

$$H = H_V \otimes P_g,$$

where $P = I - |000\rangle\langle 000| - |111\rangle\langle 111|$ acts on the GO register.

The construction

Step 3

Adding a 3-qubit "GO" register g, define:

$$H = H_V \otimes P_g,$$

where $P = I - |000\rangle\langle 000| - |111\rangle\langle 111|$ acts on the GO register.

Step 4

Define

$$\begin{aligned} |\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\ |\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \end{aligned}$$

where we denote the first register as the V register. This concludes the construction.

The construction

Step 3

Adding a 3-qubit “GO” register g, define:

$$H = H_V \otimes P_g,$$

where $P = I - |000\rangle\langle 000| - |111\rangle\langle 111|$ acts on the GO register.

Step 4

Define

$$\begin{aligned} |\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\ |\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \end{aligned}$$

where we denote the first register as the V register. This concludes the construction.

Why does this work?

Outline

Analysis: YES case

Recall:

$$
\begin{aligned}
H &= H_V \otimes P_g \qquad \text{for } P = (I - |000\rangle\langle 000| - |111\rangle\langle 111|) \\
|\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\
|\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \\
|\psi_i\rangle &= U_i \cdots U_1 |\psi_{\text{init}}\rangle.
\end{aligned}
$$

Prover acts as follows:

- Prepare the history state of H_V in register V.
  Note: Can be done using poly-many 2-qubit gates.

Analysis: YES case

Recall:

$$\begin{aligned} H &= H_V \otimes P_g \qquad \text{for } P = (I - |000\rangle\langle 000| - |111\rangle\langle 111|) \\ |\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\ |\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \\ |\psi_i\rangle &= U_i \cdots U_1 |\psi_{\text{init}}\rangle. \end{aligned}$$

Prover acts as follows:

- Prepare the history state of H_V in register V.
  Note: Can be done using poly-many 2-qubit gates.
- Flip each GO qubit to 1.

Analysis: YES case

Recall:

$$\begin{aligned} H &= H_V \otimes P_g \qquad \text{for } P = (I - |000\rangle\langle 000| - |111\rangle\langle 111|) \\ |\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\ |\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \\ |\psi_i\rangle &= U_i \cdots U_1 |\psi_{\text{init}}\rangle. \end{aligned}$$

Prover acts as follows:

- Prepare the history state of H_V in register V.
  Note: Can be done using poly-many 2-qubit gates.
- Flip each GO qubit to 1.
- Uncompute the history state of H_V in register V.

Analysis: NO case

Recall:

$$
\begin{aligned}
H &= H_V \otimes P_g \qquad \text{for } P = (I - |000\rangle\langle 000| - |111\rangle\langle 111|) \\
|\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\
|\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \\
|\psi_i\rangle &= U_i \cdots U_1 |\psi_{\text{init}}\rangle.
\end{aligned}
$$

Intuition:

- To map $|\psi_{\text{init}}\rangle$ to $|\psi_{\text{final}}\rangle$ we need to flip each *GO* qubit to 1.

  $\Rightarrow$ But we are only allowed 2-qubit unitaries U_i to do the flips.

Analysis: NO case

Recall:

$$\begin{aligned} H &= H_V \otimes P_g \qquad \text{for } P = (I - |000\rangle\langle 000| - |111\rangle\langle 111|) \\ |\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\ |\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \\ |\psi_i\rangle &= U_i \cdots U_1 |\psi_{\text{init}}\rangle. \end{aligned}$$

Intuition:

- To map $|\psi_{\text{init}}\rangle$ to $|\psi_{\text{final}}\rangle$ we need to flip each *GO* qubit to 1.
  $\Rightarrow$ But we are only allowed 2-qubit unitaries U_i to do the flips.
- Thus, for some step i our state $|\psi_i\rangle$ "must" have non-trivial overlap with $I_V \otimes P_g$.
  $\Rightarrow$ This "activates" the Hamiltonian H_V, which checks the V register.

Analysis: NO case

Recall:

$$\begin{aligned} H &= H_V \otimes P_g \qquad \text{for } P = (I - |000\rangle\langle 000| - |111\rangle\langle 111|) \\ |\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\ |\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \\ |\psi_i\rangle &= U_i \cdots U_1 |\psi_{\text{init}}\rangle. \end{aligned}$$

Intuition:

- To map $|\psi_{\text{init}}\rangle$ to $|\psi_{\text{final}}\rangle$ we need to flip each *GO* qubit to 1.
  $\Rightarrow$ But we are only allowed 2-qubit unitaries U_i to do the flips.
- Thus, for some step i our state $|\psi_i\rangle$ "must" have non-trivial overlap with $I_V \otimes P_g$.
  $\Rightarrow$ This "activates" the Hamiltonian H_V, which checks the V register.
- But, since we started with a NO instance of our QCMA problem, H_v has no small eigenvalues.
  $\Rightarrow$ Thus, for a NO instance, $\langle\psi_i|H|\psi_i\rangle$ cannot be "small".

Analysis: NO case

Recall:

$$\begin{aligned} H &= H_V \otimes P_g \qquad \text{for } P = (I - |000\rangle\langle 000| - |111\rangle\langle 111|) \\ |\psi_{\text{init}}\rangle &= |0\cdots 0\rangle_V \otimes |000\rangle_g, \\ |\psi_{\text{final}}\rangle &= |0\cdots 0\rangle_V \otimes |111\rangle_g, \\ |\psi_i\rangle &= U_i \cdots U_1 |\psi_{\text{init}}\rangle. \end{aligned}$$

Goal

Show that if we map $|000\rangle_g$ to $|111\rangle_g$ using 2-qubit unitaries U_i, there exists a step i such that

$$\langle\psi_i|P|\psi_i\rangle \text{ is “large”.}$$

k-Orthogonal

Two states $|\psi\rangle, |\phi\rangle \in (\mathbb{C}^d)^{\otimes n}$ are k-orthogonal if for all k-qudit unitaries U,

$$\langle\psi|U|\phi\rangle = 0.$$

Observation 1. k-Orthogonal $\Rightarrow$ orthogonal: Set $U = I$.

Observation 2. Orthogonal $\not\Rightarrow$ k-orthogonal:
$\langle 100|000\rangle = 0$ but $|100\rangle$ and $|000\rangle$ are not 1-orthogonal.

Observation 3. $|000\rangle$ and $|111\rangle$ are 2-orthogonal.

Traversal Lemma

k-Orthogonal

Two states $|\psi\rangle, |\phi\rangle \in (\mathbb{C}^d)^{\otimes n}$ are k-orthogonal if for all k-qudit unitaries U,

$$\langle\psi|U|\phi\rangle = 0.$$

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that

$$|\langle w|U_m \cdots U_1|v\rangle| = 1,$$

there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Quantum Zeno effect

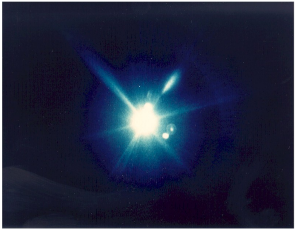
In a classical world:

Be a good parent $\Rightarrow$ Baby grows well

Quantum Zeno effect

In a quantum world:

Papa physicist



Baby photon

Be a good parent $\Rightarrow$ stunt your baby's growth

Proof

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Proof. By contradiction.

- Suppose for each i, $\langle v_i|P|v_i\rangle < \frac{1}{2m^2}$.

Proof

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Proof. By contradiction.

- Suppose for each i, $\langle v_i|P|v_i\rangle < \frac{1}{2m^2}$.
- $\Rightarrow (I - P)U_i \cdots U_1|v\rangle \approx U_i \cdots U_1|v\rangle$ for all i (Gentle Msmt. [Winter, 1999]).

Proof

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Proof. By contradiction.

- Suppose for each i, $\langle v_i|P|v_i\rangle < \frac{1}{2m^2}$.
- $\Rightarrow (I - P)U_i \cdots U_1|v\rangle \approx U_i \cdots U_1|v\rangle$ for all i (Gentle Msmt. [Winter, 1999]).
- $\Rightarrow (I - P)U_i(I - P)U_{m-1} \cdots (I - P)U_2(I - P)U_1|v\rangle \approx |v_i\rangle$.

Proof

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Proof. By contradiction.

- Suppose for each i, $\langle v_i|P|v_i\rangle < \frac{1}{2m^2}$.
- $\Rightarrow (I-P)U_i \cdots U_1|v\rangle \approx U_i \cdots U_1|v\rangle$ for all i (Gentle Msmt. [Winter, 1999]).
- $\Rightarrow (I-P)U_i(I-P)U_{m-1} \cdots (I-P)U_2(I-P)U_1|v\rangle \approx |v_i\rangle$.
- $\Rightarrow \langle w|(I-P)U_m(I-P)U_{m-1} \cdots (I-P)U_2(I-P)U_1|v\rangle > 0$.

Proof

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Proof. By contradiction.

- Suppose for each i, $\langle v_i|P|v_i\rangle < \frac{1}{2m^2}$.
- $\Rightarrow (I-P)U_i \cdots U_1|v\rangle \approx U_i \cdots U_1|v\rangle$ for all i (Gentle Msmt. [Winter, 1999]).
- $\Rightarrow (I-P)U_i(I-P)U_{m-1} \cdots (I-P)U_2(I-P)U_1|v\rangle \approx |v_i\rangle$.
- $\Rightarrow \langle w|(I-P)U_m(I-P)U_{m-1} \cdots (I-P)U_2(I-P)U_1|v\rangle > 0$.

But:

- Since $|v\rangle$ and $|w\rangle$ are 2-orthogonal, $(I-P)U_i \cdots (I-P)U_1|v\rangle \propto |v\rangle$.

Proof

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Proof. By contradiction.

- Suppose for each i, $\langle v_i|P|v_i\rangle < \frac{1}{2m^2}$.
- $\Rightarrow (I - P)U_i \cdots U_1|v\rangle \approx U_i \cdots U_1|v\rangle$ for all i (Gentle Msmt. [Winter, 1999]).
- $\Rightarrow (I - P)U_i(I - P)U_{m-1} \cdots (I - P)U_2(I - P)U_1|v\rangle \approx |v_i\rangle$.
- $\Rightarrow \langle w|(I - P)U_m(I - P)U_{m-1} \cdots (I - P)U_2(I - P)U_1|v\rangle > 0$.

But:

- Since $|v\rangle$ and $|w\rangle$ are 2-orthogonal, $(I - P)U_i \cdots (I - P)U_1|v\rangle \propto |v\rangle$.
- $\Rightarrow \langle w|(I - P)U_m(I - P)U_{m-1} \cdots (I - P)U_2(I - P)U_1|v\rangle = 0$.

Outline

Is the Traversal Lemma tight?

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Question

If we use more unitaries U_i, can we better “avoid” the subspace P?

Is the Traversal Lemma tight?

Traversal Lemma

Let $|v\rangle, |w\rangle \in (\mathbb{C}^d)^{\otimes n}$ be k-orthogonal states. For any sequence of k-qudit unitaries $(U_i)_{i=1}^m$ such that $|\langle w|U_m \cdots U_1|v\rangle| = 1$, there exists an $i \in [m]$ such that

$$\langle v_i|P|v_i\rangle \geq \frac{1}{2m^2}$$

for $P := I - |v\rangle\langle v| - |w\rangle\langle w|$ and $|v_i\rangle := U_i \cdots U_1|v\rangle$.

Question

If we use more unitaries U_i, can we better "avoid" the subspace P?

On tightness of the Traversal Lemma

Theorem

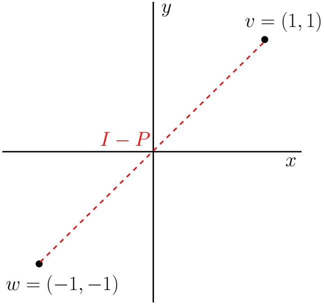
Let $|v\rangle = |000\rangle$, $|w\rangle = |111\rangle$, and $P := I - |v\rangle\langle v| - |w\rangle\langle w|$. Then, for any $0 < \Delta < 1/2$, there exists a sequence of $O(1/\Delta^2)$ 2-local unitaries mapping $|v\rangle$ to $|w\rangle$ such that each intermediate state $|v_i\rangle$ satisfies $\langle v_i|P|v_i\rangle \leq \Delta$.

On tightness of the Traversal Lemma

Theorem

Let $|v\rangle = |000\rangle$, $|w\rangle = |111\rangle$, and $P := I - |v\rangle\langle v| - |w\rangle\langle w|$. Then, for any $0 < \Delta < 1/2$, there exists a sequence of $O(1/\Delta^2)$ 2-local unitaries mapping $|v\rangle$ to $|w\rangle$ such that each intermediate state $|v_i\rangle$ satisfies $\langle v_i|P|v_i\rangle \leq \Delta$.

Proof Inspiration.

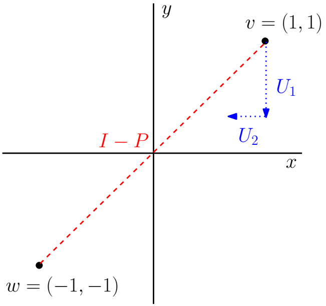


On tightness of the Traversal Lemma

Theorem

Let $|v\rangle = |000\rangle$, $|w\rangle = |111\rangle$, and $P := I - |v\rangle\langle v| - |w\rangle\langle w|$. Then, for any $0 < \Delta < 1/2$, there exists a sequence of $O(1/\Delta^2)$ 2-local unitaries mapping $|v\rangle$ to $|w\rangle$ such that each intermediate state $|v_i\rangle$ satisfies $\langle v_i|P|v_i\rangle \leq \Delta$.

Proof Inspiration.

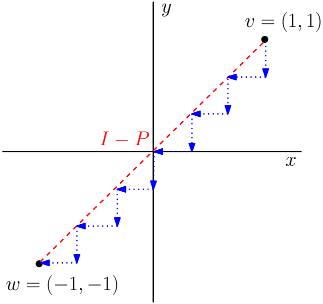


On tightness of the Traversal Lemma

Theorem

Let $|v\rangle = |000\rangle$, $|w\rangle = |111\rangle$, and $P := I - |v\rangle\langle v| - |w\rangle\langle w|$. Then, for any $0 < \Delta < 1/2$, there exists a sequence of $O(1/\Delta^2)$ 2-local unitaries mapping $|v\rangle$ to $|w\rangle$ such that each intermediate state $|v_i\rangle$ satisfies $\langle v_i|P|v_i\rangle \leq \Delta$.

Proof Inspiration.



Take baby steps!

Outline

Conclusions

Recap

- Defined notion of ground state connectivity for local Hamiltonians.
- Proved QCMA-completeness, PSPACE-completeness, and NEXP-completeness for its variants.
- Key technical tool: Traversal Lemma.

Open Questions

- Complexity of connectivity for exp. many 2-qubit unitaries?
- Efficient algorithms for special constraint types?
- Further physical applications/interpretations of ground state connectivity?