Tensors in CS and geometry workshop, Nov. 11 (2014)

Tensors and complexity in quantum information science

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Outline

Subject: math of tensors <-> physics of entanglement

Aim: talk by a quantum information theorist about elementary concepts/results and many open questions

1. Classification of multipartite entanglement
   - geometry of complex projective spaces
   - polynomial invariants as entanglement measures

2. Entanglement and its complexity
   - complexity of quantum computation in terms of tensors
   - "coarser" classification of multipartite entanglement

Classification of multipartite entanglement

Miyake, Phys. Rev. A 67, 012108 (2003):dual variety, hyperdeterminants
Miyake, Verstraete, Phys. Rev. A 69, 012101 (2004): 2x2x4 case

Stochastic LOCC

Entanglement: quantum correlation, never increasing on average under Local Operations and Classical Communication(LOCC)

single copy of a pure state (ray) in $H = \mathbb{C}^{k_1} \otimes \cdots \otimes \mathbb{C}^{k_n}$

$$\left|\Psi\right\rangle = \sum_{i_1,\ldots,i_n=0}^{k_1-1,\ldots,k_n-1} \psi_{i_1\cdots i_n} \left|i_1\right\rangle \otimes \cdots \otimes \left|i_n\right\rangle$$

$$\left|\Psi'\right\rangle = M_1 \otimes \cdots \otimes M_n \left|\Psi\right\rangle$$

$\left|\Psi\right\rangle$

$\left|\Psi'\right\rangle$

SLOCC: probabilistic $M_j = SL(k_j)$ (invertible)

cf. LOCC: deterministic $M_j = U(k_j)$ etc.

CP map, followed by the postselection of a successful outcome

SLOCC classification: classification of orbits of

$$SL(k_1) \times \cdots \times SL(k_n)$$

(in bipartite cases: classification by the Schmidt rank)

Bipartite entanglement under SLOCC

two k-level qudits system (pure states) on $H = \mathbb{C}^k \otimes \mathbb{C}^k$

$$|\Psi\rangle = \sum_{i_1,i_2=0}^{k-1} \psi_{i_1 i_2} |i_1\rangle \otimes |i_2\rangle$$

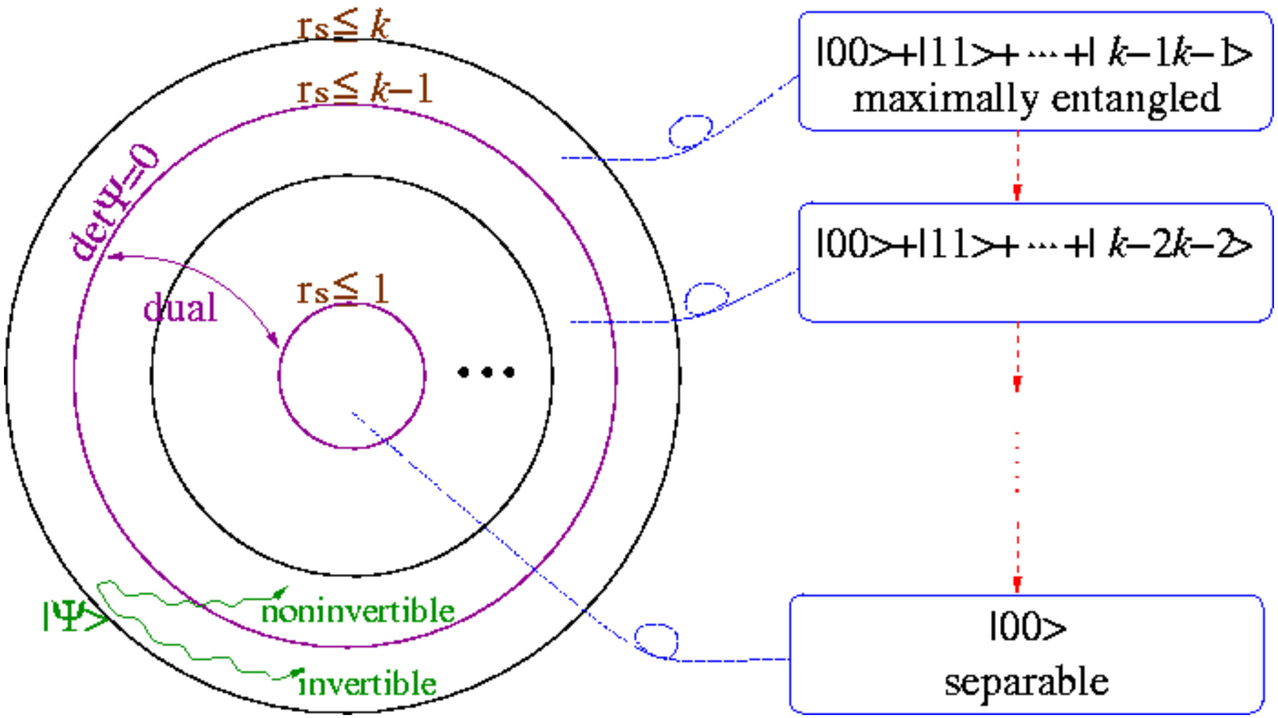
$$\left(\psi_{i_1 i_2}\right) \xrightarrow{G=M_1\otimes M_2} M_1\left(\psi_{i_1 i_2}\right)M_2^T$$

basic "matrix" transformation

the rank (Schmidt local rank) r_s is

invariant under invertible G: k equivalence classes

decreasing under noninvertible G: totally ordered monotonicity



$\mathbb{C}P^{k^2-1}$

Secant variety

Segre variety

Encounter with tensors

What is the SLOCC classification of 2x2x2 (3-qubit) case?

[Gelfand, Kapranov, Zelevinsky *Discriminants, Resultants, and Multidimensional Determinants* (1994), Chapter 14, Example 4.5]

[rediscovery in QIS: Dür, Vidal, Cirac, PRA 62, 062314 (2000); over 1300 citations]

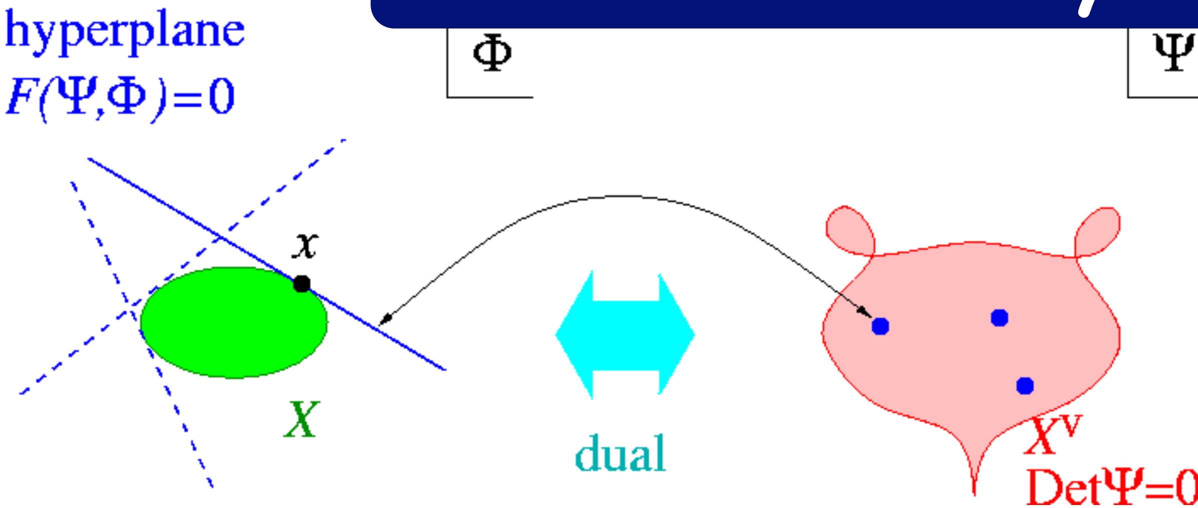
PHYSICAL REVIEW A, VOLUME 62, 062314

Three qubits can be entangled in two inequivalent ways

W. Dür, G. Vidal, and J. I. Cirac
Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria
(Received 26 May 2000; published 14 November 2000)

$$|\mathrm{GHZ}\rangle = (1/\sqrt{2})(|000\rangle + |111\rangle) \qquad |W\rangle = (1/\sqrt{3})(|001\rangle + |010\rangle + |100\rangle),$$

Dual variety



point: a state

hyperplane:
a set of states $|\Phi\rangle$, orthogonal to $|\Psi\rangle$

$$F(\Psi,\Phi)=\sum_{i_1,\ldots,i_n}\psi_{i_1\ldots i_n}\varphi_{i_1\ldots i_n}=0$$

$|\Psi\rangle \in$ dual set $X^\vee \Leftrightarrow \exists x\in X$ s.t. hyperplane $F(\Psi,\Phi)=0$ is tangent at x

X: set of completely separable states

$$\begin{cases} F(\Psi,x)=\displaystyle\sum_{i_1,\ldots,i_n=0}^{k_1-1,\ldots,k_n-1}\psi_{i_1\ldots i_n}x^{(1)}_{i_1}\cdots x^{(n)}_{i_n}=0 \\ \dfrac{\partial}{\partial x^{(j)}_{i_j}}F(\Psi,x)=0,\quad \forall j=1,\ldots,n,\ i_j=0,\ldots,k_j-1 \end{cases}$$

have at least a nonzero solution $x=(x^{(1)},\ldots,x^{(n)})\in X$

$\Leftrightarrow X^\vee:\ \mathrm{Det}\,\Psi=0$ set of degenerate entangled states

(if codim $X^\vee=1$, i.e., $k_j-1\le\sum_{j'\neq j}(k_{j'}-1)\ \forall j$)

Hyperdeterminat

$$\mathrm{Det}_{22}\Psi = \det\Psi = \psi_{00}\psi_{11} - \psi_{01}\psi_{10} \quad \text{deg. 2}$$

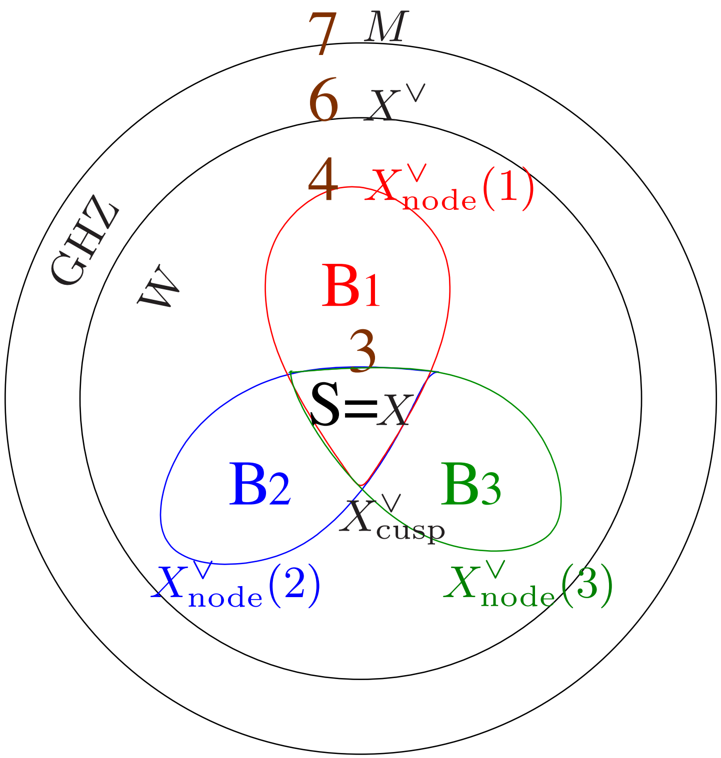
$$\begin{aligned}\mathrm{Det}_{222}\Psi = {} & \psi_{000}{}^2\psi_{111}{}^2 + \psi_{001}{}^2\psi_{110}{}^2 + \psi_{010}{}^2\psi_{101}{}^2 + \psi_{100}{}^2\psi_{011}{}^2 - 2(\psi_{000}\psi_{001}\psi_{110}\psi_{111} \\ & + \psi_{000}\psi_{010}\psi_{101}\psi_{111} + \psi_{000}\psi_{100}\psi_{011}\psi_{111} + \psi_{001}\psi_{010}\psi_{101}\psi_{110} + \psi_{001}\psi_{100}\psi_{011}\psi_{110} \\ & + \psi_{010}\psi_{100}\psi_{011}\psi_{101}) + 4(\psi_{000}\psi_{011}\psi_{101}\psi_{110} + \psi_{001}\psi_{010}\psi_{100}\psi_{111}) \quad \text{deg. 4}\end{aligned}$$

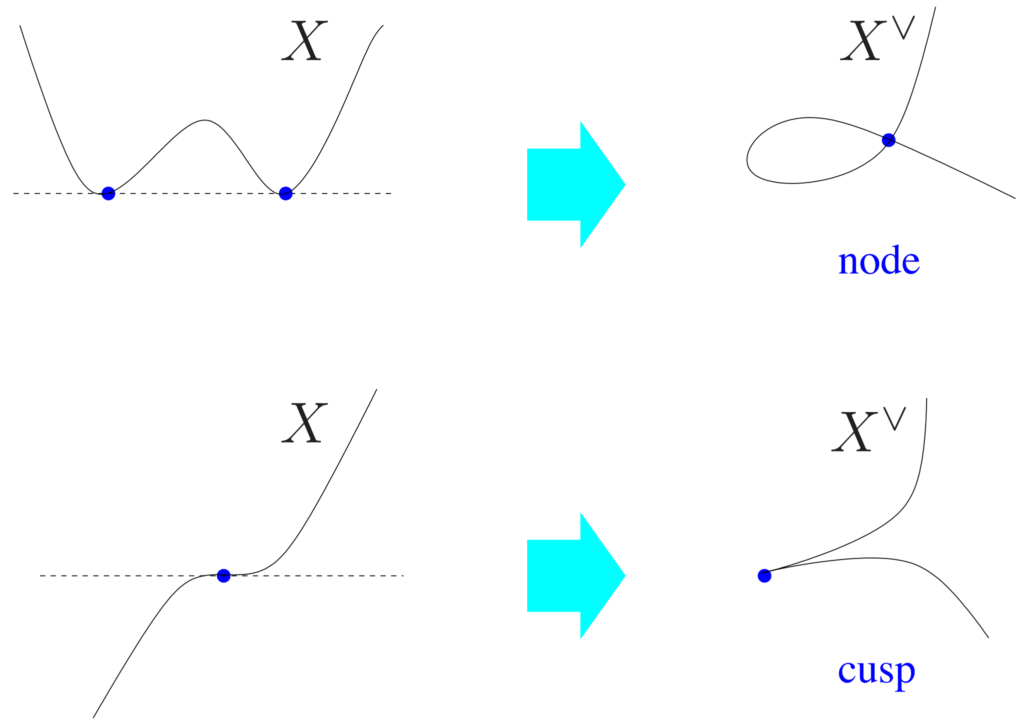
- SLOCC invariant
  entanglement monotones for genuine multipartite component
  $C = 2\left|\mathrm{Det}_{22}\Psi\right|$ concurrence for 2-qubit pure states
  $\tau = 4\left|\mathrm{Det}_{222}\Psi\right|$ 3-tangle for 3-qubit pure states
- the degree of n-qubit hyperdeterminants grows very rapidly

n	2	3	4	5	6	7	8	9	10	…
deg.	2	4	24	128	880	6,816	60,032	589,312	6,384,384	...

- generating function for the entanglement structure
  singularities of the hypersurface → degenerate entangled classes

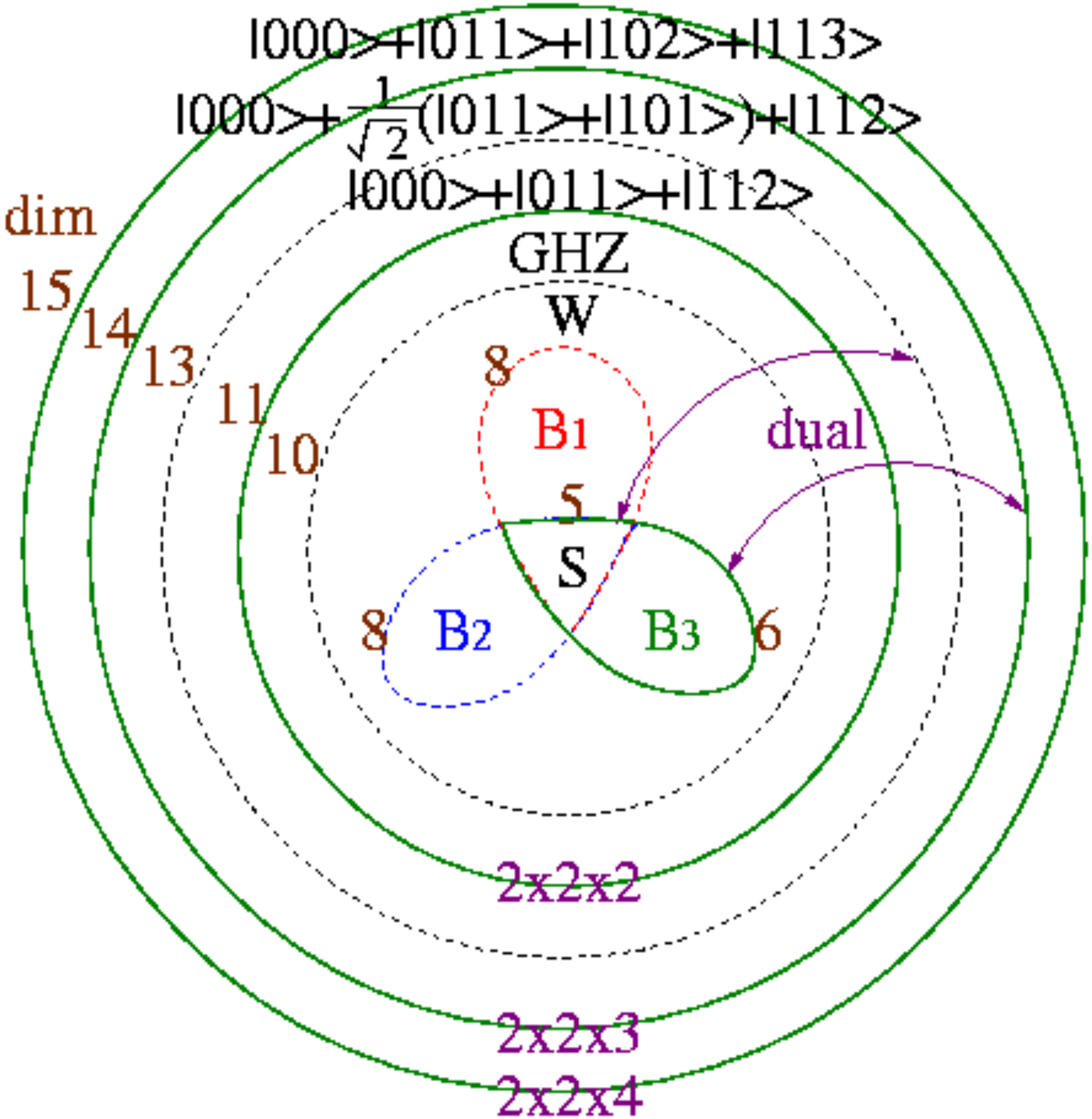
3-qubit (2x2x2) SLOCC classification





dim 7: $|000\rangle + |111\rangle$, GHZ $\in M(=\mathbb{C}P^7)-X^\vee$.
dim 6: $|001\rangle+|010\rangle+|100\rangle$, W $\in X^\vee - X^\vee_{\rm sing} = X^\vee - X^\vee_{\rm cusp}$.
dim 4: $|001\rangle + |010\rangle$, $|001\rangle + |100\rangle$, $|010\rangle + |100\rangle$,
biseparable $B_j \in X^\vee_{\rm node}(j) - X$ for $j = 1, 2, 3$.
$X^\vee_{\rm node}(j) = \mathbb{C}P^1_{j\text{-th}} \times \mathbb{C}P^3$ are three closed irreducible components of $X^\vee_{\rm sing} = X^\vee_{\rm cusp}$.
dim 3: $|000\rangle$, completely separable S
$\in X = \bigcap_{j=1,2,3} X^\vee_{\rm node}(j) = \mathbb{C}P^1 \times \mathbb{C}P^1 \times \mathbb{C}P^1$.

2x2x4 SLOCC classification



- SLOCC orbits are added outside.
- Duality suggests one subsystem is too large, compared to the rest.

$2\times 2\times n\ (n\geq 4);\ \ 2\times 2\times 4$

- LOCC conversions of distributed 4 qubits.

parties	2	3	4
classes	4	9	∞

coarser finer

Entanglement measures in 2x2x4 case

$$|\Psi\rangle = \sum_{i,j,k=0}^{1,1,n-1} \psi_{ijk} |i\rangle_A \otimes |j\rangle_B \otimes |k\rangle_C$$

By calculating local ranks (r_1, r_2, r_3) of $\rho_i = \mathrm{tr}_{\forall j \neq i} |\Psi\rangle\langle\Psi|$

- (2,2,4) generic class (dim 15)
- (2,2,3) Clare's local operation: $\forall \psi_{ijk} = 0 \quad (k \geq 3)$

$$\mathrm{Det}_{223}\Psi = \begin{vmatrix} \psi_{000} & \psi_{010} & \psi_{100} \\ \psi_{001} & \psi_{011} & \psi_{101} \\ \psi_{002} & \psi_{012} & \psi_{102} \end{vmatrix} \begin{vmatrix} \psi_{010} & \psi_{100} & \psi_{110} \\ \psi_{011} & \psi_{101} & \psi_{111} \\ \psi_{012} & \psi_{102} & \psi_{112} \end{vmatrix} - \begin{vmatrix} \psi_{000} & \psi_{010} & \psi_{110} \\ \psi_{001} & \psi_{011} & \psi_{111} \\ \psi_{002} & \psi_{012} & \psi_{112} \end{vmatrix} \begin{vmatrix} \psi_{000} & \psi_{100} & \psi_{110} \\ \psi_{001} & \psi_{101} & \psi_{111} \\ \psi_{002} & \psi_{102} & \psi_{112} \end{vmatrix} \begin{cases} \neq 0 & \text{dim14} \\ = 0 & \text{dim13} \end{cases}$$

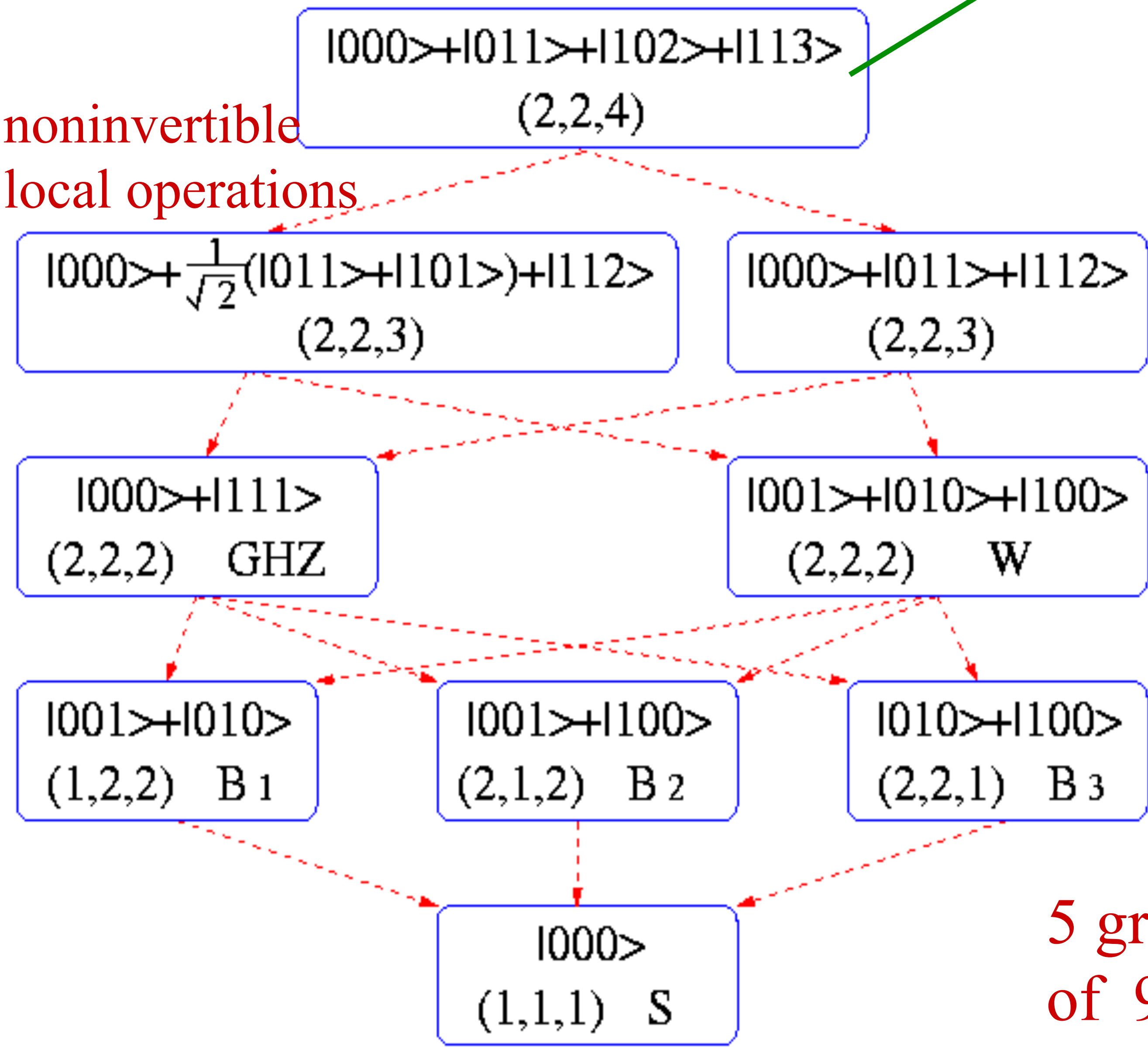
- (2,2,2) Clare's local operation: $\forall \psi_{ijk} = 0 \quad (k \geq 2)$

$$\begin{aligned} \mathrm{Det}_{222}\Psi = {} & \psi_{000}{}^2\psi_{111}{}^2 + \psi_{001}{}^2\psi_{110}{}^2 + \psi_{010}{}^2\psi_{101}{}^2 + \psi_{100}{}^2\psi_{011}{}^2 - 2(\psi_{000}\psi_{001}\psi_{110}\psi_{111} \\ & + \psi_{000}\psi_{010}\psi_{101}\psi_{111} + \psi_{000}\psi_{100}\psi_{011}\psi_{111} + \psi_{001}\psi_{010}\psi_{101}\psi_{110} + \psi_{001}\psi_{100}\psi_{011}\psi_{110} \\ & + \psi_{010}\psi_{100}\psi_{011}\psi_{101}) + 4(\psi_{000}\psi_{011}\psi_{101}\psi_{110} + \psi_{001}\psi_{010}\psi_{100}\psi_{111}) \begin{cases} \neq 0 & \text{GHZ (dim11)} \\ = 0 & \text{W} \quad \text{(dim10)} \end{cases} \end{aligned}$$

- (1,2,2), (2,1,2), (2,2,1) Biseparable 1,2,3 (dim 8,8,6)
- (1,1,1) Separable (dim 5)

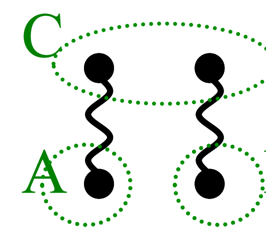
SLOCC conversions among different classes

noninvertible
local operations



<u>unique</u> "maximally entangled" class, lying on the top. Its representative is 2 Bell pairs:

$$\left(|00\rangle+|11\rangle\right)_{AC_1}\otimes\left(|00\rangle+|11\rangle\right)_{BC_2}$$



5 graded partial order of 9 entangled classes

4-qubit hyperdeterminant

$$\mathrm{Det}_{2222}\Psi = c_1^2c_2^2c_3^2 - 4c_0c_2^3c_3^2 - 4c_1^2c_2^3c_4 - 4c_1^3c_3^3 - 6c_0c_1^2c_3^2c_4 + 16c_0c_2^4c_4$$
$$+18c_0c_1c_2c_3^3 + 18c_1^3c_2c_3c_4 - 27c_0^2c_3^4 - 27c_1^4c_4^2 - 80c_0c_1c_2^2c_3c_4 - 128c_0^2c_2^2c_4^2$$
$$+144c_0c_1^2c_2c_4^2 + 144c_0^2c_2c_3^2c_4 - 192c_0^2c_1c_3c_4^2 + 256c_0^3c_4^3$$

deg. 24

$$c_j = \frac{1}{(4-j)!\,j!}\frac{\partial^4}{\partial x_0^{4-j}\partial x_1^j}\mathrm{Det}_{222}\Psi\left(\psi_{0i_1i_2i_3}x_0 + \psi_{1i_1i_2i_3}x_1\right)$$

4-qubit generic entangled states:

$$\alpha\left(|0000\rangle + |1111\rangle + |0011\rangle + |1100\rangle\right) + \beta\left(|0000\rangle + |1111\rangle - |0011\rangle - |1100\rangle\right)$$
$$\gamma\left(|0101\rangle + |1010\rangle + |0110\rangle + |1001\rangle\right) + \delta\left(|0101\rangle + |1010\rangle - |0110\rangle - |1001\rangle\right)$$

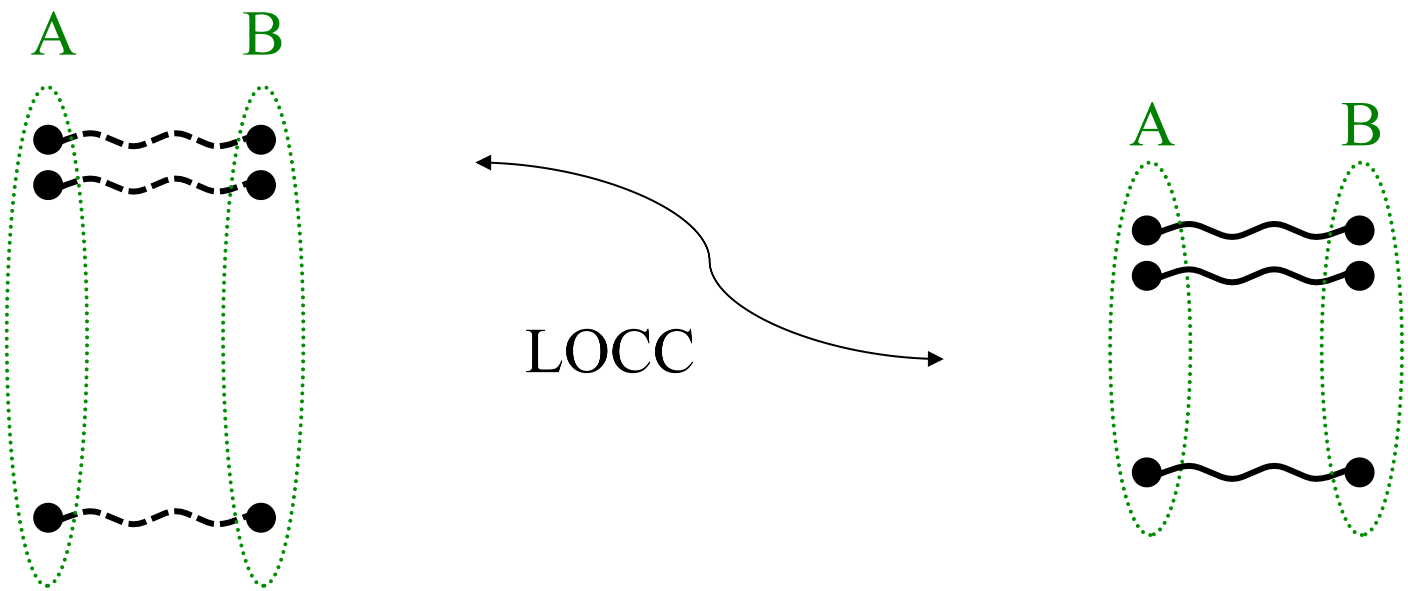
$$\mathrm{Det}_{2222}\Psi = \left(\left(\alpha^2-\beta^2\right)\left(\alpha^2-\gamma^2\right)\left(\alpha^2-\delta^2\right)\left(\beta^2-\gamma^2\right)\left(\beta^2-\delta^2\right)\left(\gamma^2-\delta^2\right)\right)^2 \neq 0$$

(Vandermonde determinant)^2

- infinitely many SLOCC orbits in generic entanglement
  → segmentation on the top of the partial order
- GHZ and W are not generic (i.e., $\mathrm{Det}_2{}^n\,\Psi = 0$) for n >= 4 qubits

Asymptotic bipartite case

Classification of entanglement when there are infinitely many identical copies



$$\left|\Psi^{+}\right\rangle^{\otimes M}=\left(\sqrt{\lambda}\,|00\rangle+\sqrt{1-\lambda}\,|11\rangle\right)^{\otimes M} \quad \left|\Phi^{+}\right\rangle^{\otimes N}=\left(|00\rangle+|11\rangle\right)^{\otimes N}$$

LOCC is reversible when $N=ME(\Psi)=M(-\lambda\log\lambda-(1-\lambda)\log(1-\lambda))$

Entanglement entropy
(uniqueness of entanglement measures)

embedding into a symmetric subspace (cf. Veronese mapping)

Asymptotic case and MREGS

Classification of multipartite entanglement with infinitely many identical copies is still open.

Minimally Reversibly Entanglement Generating Set (MREGS)?

$$\left|\Psi\right\rangle^{\otimes M} \leftrightarrow \begin{cases} (\left|000\right\rangle + \left|111\right\rangle)^{\otimes n_1} \\ \left(\left|001\right\rangle + \left|010\right\rangle + \left|100\right\rangle\right)^{\otimes n_2} \\ (\left|00\right\rangle + \left|11\right\rangle)^{\otimes n_3} \\ ? \end{cases}$$

- Does it exist?
- If so, is it made by finite generators?

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   - geometry of complex projective spaces
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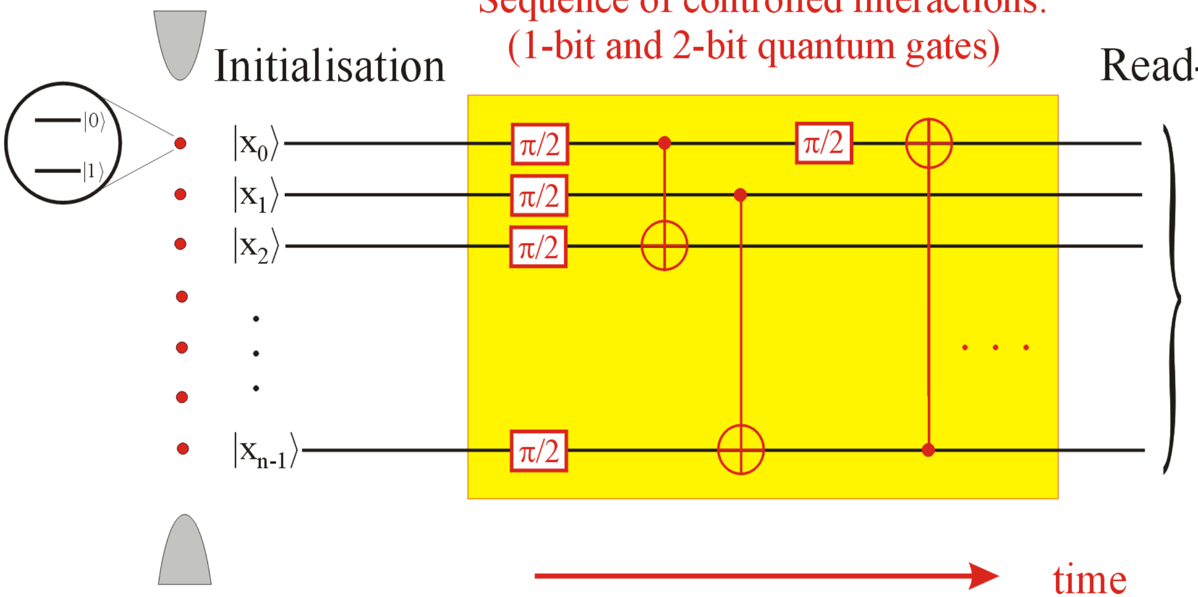
2. Entanglement and its complexity
   - complexity of quantum computation in terms of tensors
   - "coarser" classification of multipartite entanglement

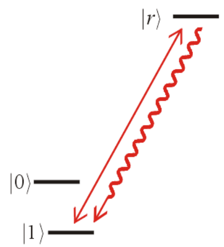
Entanglement and its complexity

Van den Nest, Miyake, Dür, Briegel, Phys. Rev. Lett. 97, 150504 (2006)
Van den Nest, Dür, Miyake, Briegel, New. J. Phys. 9, 204 (2007)

quantum circuit as tensor composition

entanglement can be generated by a quantum circuit





2^n coefficients

$$|\psi\rangle = \sum_{x\in\{0,1\}^n} \alpha_x |x\rangle$$

$$|x\rangle = |x_0, x_1, \ldots x_{n-1}\rangle$$

a 1-qubit gate: SU(2) unitary map $\quad \mathbb{C}^2 \mapsto \mathbb{C}^2$

a 2-qubit gate: SU(4) unitary map $\quad \mathbb{C}^2 \otimes \mathbb{C}^2 \mapsto \mathbb{C}^2 \otimes \mathbb{C}^2$

changing the quality of entanglement

2-qubit Controlled-Z gate

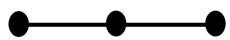
$$CZ = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11| = |0\rangle\langle 0| \otimes 1 + |1\rangle\langle 1| \otimes \sigma_z$$

$C = \{1, 2\}:$ 

$$|+\rangle|+\rangle = \left(|0\rangle + |1\rangle\right)|+\rangle$$

$$\rightarrow |0\rangle|+\rangle + |1\rangle|-\rangle \equiv |0\rangle_z |0\rangle_x + |1\rangle_z |1\rangle_x$$

Bell state

$C = \{1, 2, 3\}:$ 

GHZ state

$$|+\rangle|+\rangle|+\rangle = |+\rangle\left(|0\rangle + |1\rangle\right)|+\rangle$$

$$\rightarrow |+\rangle|0\rangle|+\rangle + |-\rangle|1\rangle|-\rangle \equiv |0\rangle_x |0\rangle_z |0\rangle_x + |1\rangle_x |1\rangle_z |1\rangle_x$$

$C = \{1, 2, 3, 4\}:$ 

1D cluster state

(not GHZ)

$$|+\rangle|+\rangle|+\rangle|+\rangle = \left(|0\rangle + |1\rangle\right)|+\rangle|+\rangle|+\rangle$$

$$\rightarrow \left(|0\rangle_z + |1\rangle_z \sigma_z\right)\left(|0\rangle_x |0\rangle_z |0\rangle_x + |1\rangle_x |1\rangle_z |1\rangle_x\right)$$

$$= |0\rangle_z |0\rangle_x |0\rangle_z |0\rangle_x + |0\rangle_z |1\rangle_x |1\rangle_z |1\rangle_x + |1\rangle_z |0\rangle_x |0\rangle_z |0\rangle_x - |1\rangle_z |1\rangle_x |1\rangle_z |1\rangle_x$$

Multipartite entanglement as graphs

[review: Hein et al., quant-ph/0602096]

1. For a graph G, vertices = qubits, edges = Ising-type interaction pattern. degree is the number of edges from a vertex.

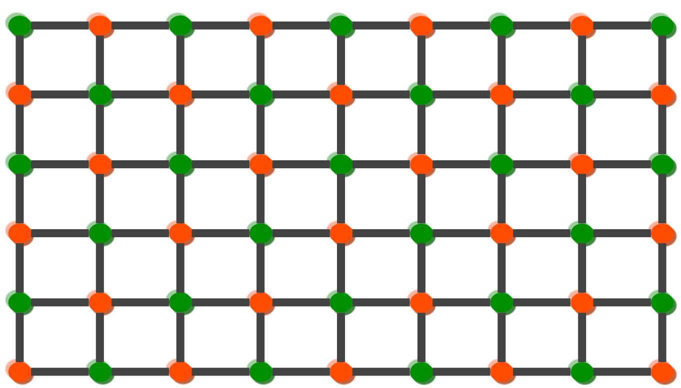
$$|G\rangle = \prod_{(a,b)\in edges} \mathrm{CZ}^{(a,b)} |+\rangle^{m}, \quad \text{where } \mathrm{CZ} = \mathrm{diag}(1,1,1,-1) \propto \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

$$|+\rangle = \tfrac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

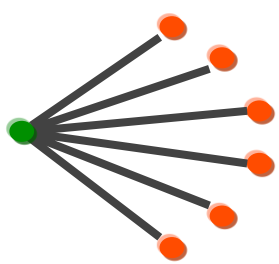
2. joint eigenstate of N commuting correlation operators for N qubits.

$$K_a |G\rangle = |G\rangle, \qquad K_a = \sigma_x^{(a)} \bigotimes_{b\in N_a} \sigma_z^{(b)}$$

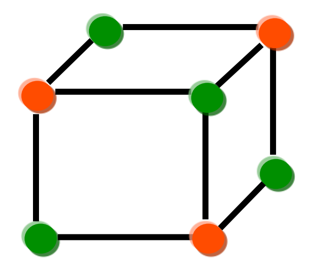
3. any stabilizer state can be written as a graph state, up to local unitaries.



2D cluster state



GHZ state



7-qubit codeword

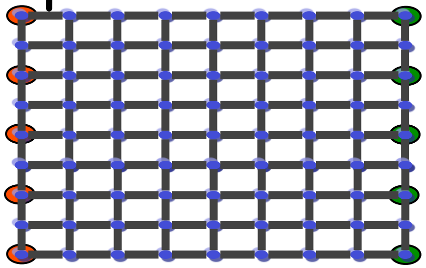
Universality of quantum computation

We call measurement-based computation universal, if

Using single-qubit projective measurements and classical feedforward of measurement outcomes (LOCC),

- it's capable to simulate <u>any</u> unitary gate operation on a (known) *n*-qubit input state deterministically
- it's capable to produce <u>any</u> corresponding output state

$$\underbrace{|\psi\rangle}_{N} \xrightarrow{\text{LOCC}} \underbrace{|\phi\rangle}_{n}\underbrace{|\text{measured}\rangle}_{N-n}$$



2D cluster states $\{|C_k\rangle, k=1,2,\ldots\}$ were shown to be universal

Definition of Universal resource (without encoding):

A set of states $\Psi = \{|\psi_1\rangle, |\psi_2\rangle, \ldots,\}$ is universal for QC, if

$$\exists \underbrace{|\psi_k\rangle}_{N} \xrightarrow{\text{LOCC}} \forall \underbrace{|\phi\rangle}_{n}\underbrace{|\text{measured}\rangle}_{N-n}$$

Criterion for universality

Idea: if any significant entanglement feature exhibited by a set of universal resource states (ex. the 2D cluster states) is not available from another set Ψ, then it cannot be universal

$$E(|\phi\rangle) \geq E(|\phi'\rangle) \text{ whenever } |\phi\rangle \geq_{LOCC} |\phi'\rangle$$

Criterion for universality

A set of states Ψ <u>cannot</u> be universal for MQC if

$$\sup_{\forall|\phi\rangle} E(|\phi\rangle) > \sup_{|\psi\rangle\in\Psi} E(|\psi\rangle)$$

$$\sup_{\forall|\phi\rangle} E(|\phi\rangle) = \sup_{k} E(|C_k\rangle)$$

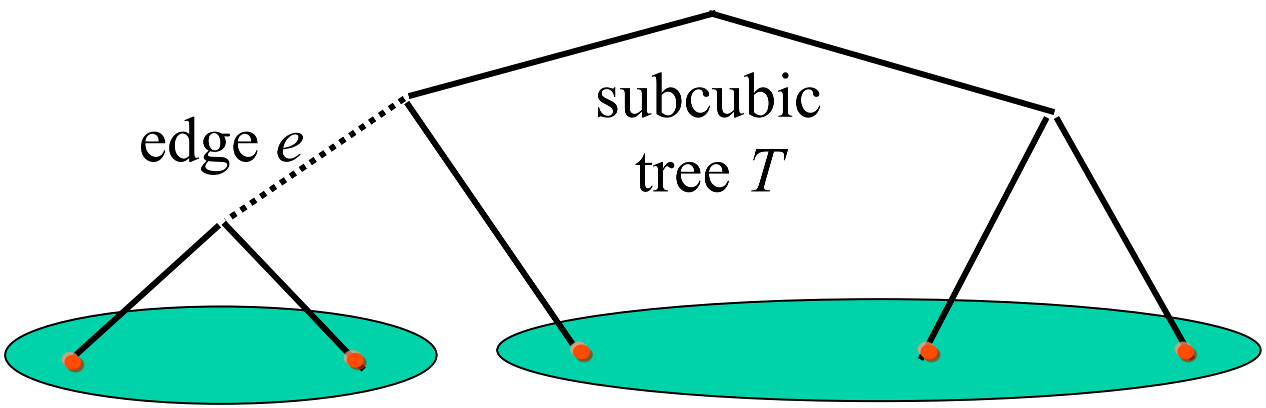
- entanglement width: distinguish 1D and 2D cluster states
- geometric measure
- Schmidt measure (logarithm of tensor rank)
- localizable entanglement

Entanglement width

Entanglement-width

maximal entropy of entanglement associated with certain bipartitions

$$\mathrm{Ewd}(|\psi\rangle) = \min_{\mathrm{T}} \max_{e \in T} E_{A_T^e}(|\psi\rangle)$$



- non-increasing under LOCC (i.e. fulfills (P1))
- equivalent to the rank-width in graph theory

2D cluster states show the divergence such as $\mathrm{Ewd}(|C_{k \times k}\rangle) \geq \log_2(k+2) - 1$

Any universal resource must have a diverging entanglement width.

Criterion by localizable entanglement

Localizable entanglement $E_{\mathrm{L}}^{(\mathrm{a,b})}(|\psi\rangle)$

[Verstraete et al., PRL 92, 027901 (2004); Gour et al., PRA 72, 042329 (2005)]

entanglement (as measured by the concurrence) that can be at most created between two qubits (a,b) by LOCC

- non-increasing under LOCC (fulfills (P1))
- maximum number N_{LE} of qubits such that $E_{\mathrm{L}}^{(a,b)}(|\psi\rangle) = 1, \ \forall (a,b) \in N_{\mathrm{LE}}$

2D cluster states show the properties such as

$$E_{\mathrm{L}}^{(a,b)}(|C_{k\times k}\rangle) = 1, \ \forall (a,b)$$

$$N_{\mathrm{LE}}(|C_{k\times k}\rangle) = k^2 = m \qquad \text{: unbounded}$$

Non-universal resource states:

- all states such that $E_{\mathrm{L}} < 1$ (e.g. W states)
- all states such that E_{L} decays with distance (N_{LE} is bounded)

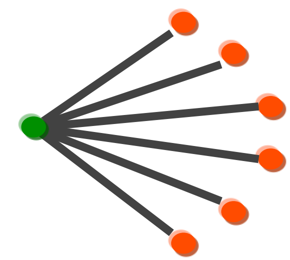
Non-universal graph states

Theorem 1:
Any set of the graph states whose entanglement-width is bounded for the number of qubits N is not a universal resource.

"interaction geometry determines the value as a resource"

- 1D cluster states (Ewd = 1, SM=N/2)

- GHZ (tree) states, fully connected graphs
- graphs with bounded tree-width or clique-width



complementary to classical simulatibility (note universal QC is simply believed not to be classically simulatable efficiently)

- Measurement-based computation on some resource states (e.g. 1D cluster states and GHZ states) [Nielsen 05; Markov & Shi 05; Jozsa 06; Van den Nest et al. 06; ...]

Tensor network as quantum computation

SIAM J. Comput.
Vol. 38, No. 3, pp. 963–981

SIMULATING QUANTUM COMPUTATION BY CONTRACTING TENSOR NETWORKS*

IGOR L. MARKOV† AND YAOYUN SHI†

Abstract. The treewidth of a graph is a useful combinatorial measure of how close the graph is to a tree. We prove that a quantum circuit with T gates whose underlying graph has a treewidth d can be simulated deterministically in $T^{O(1)} \exp[O(d)]$ time, which, in particular, is polynomial in T if $d = O(\log T)$.

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QUANTUM COMPUTATION AND THE EVALUATION OF TENSOR NETWORKS*

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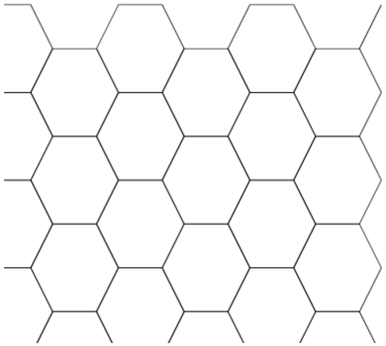
Abstract. We present a quantum algorithm that additively approximates the value of a tensor network to a certain scale. When combined with existing results, this provides a complete problem for quantum computation. The result is a simple new way of looking at quantum computation in which unitary gates are replaced by tensors and time is replaced by the order in which the tensor network is "swallowed." We use this result to derive new quantum algorithms that approximate the partition function of a variety of classical statistical mechanical models, including the Potts model.

Examples of universal resources

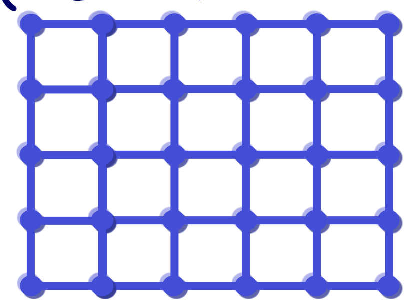
Theorem2:
graph states by 2D regular tilings are universal resources.

hexagonal lattice



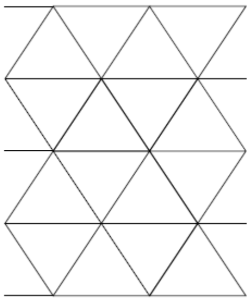
degree = 3

square lattice
(2D cluster state)



degree = 4

triangular lattice

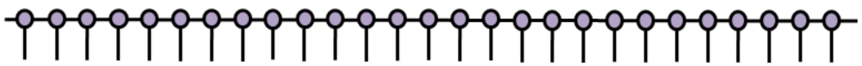


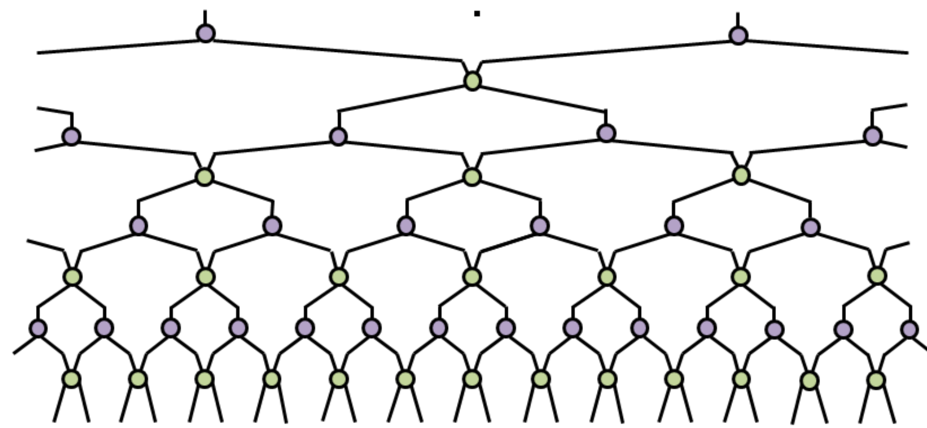
degree = 6

minimum possible degree for
uniform lattices to be universal

More by other tensor network states
[Gross, Eisert PRL 98, 220503 (2007); Gross et.al., PRA 76, 052315 (2008)]

Tensor network states





Workshops | Spring 2014

Tensor Networks and Simulations

Apr. 21 – Apr. 24, 2014

Program: Quantum Hamiltonian Complexity

Add to Calendar

View schedule & video »   View abstracts »

Organizers:

Ignacio Cirac (Max Planck Institute, Garching), Frank Verstraete (University of Vienna).

This workshop will focus on tensor network (MPS, PEPS, MERA) based simulations of strongly correlated quantum many body systems, relation to area laws, quantum spin glasses, as well as related work in mathematics, quantum chemistry and quantum information theory.

Complexity of entanglement

Universal resources are necessarily macroscopic entanglement!

entropic entanglement width

?

maximum size of unit localizable entanglement

Schmidt-rank width

universal resource families

$O(N) \rightarrow \infty$

classically efficiently simulatable $O(\log(N))$

geometric measure

Schmidt measure (logarithm of tensor rank)

Summary: open questions

Subject: math of tensors <-> physics of entanglement

Aim: talk by a quantum information theorist about elementary concepts/results and many open questions

1. Classification of multipartite entanglement
   - "complete" characterization and "monogamy" constraints (cf. two previous talks)
   - asymptotic case: minimally generating set

2. Entanglement and its complexity
   - necessary and sufficient condition for graph states to be universal
   - classes of tensor networks which are efficiently computable