Dynamic Network Models for Epidemic Estimation

Reading group on Epidemics
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Time Scale of the Dynamic <u>of</u> the Network vs Dynamic <u>on</u> the Network

- **Static or quenched networks**

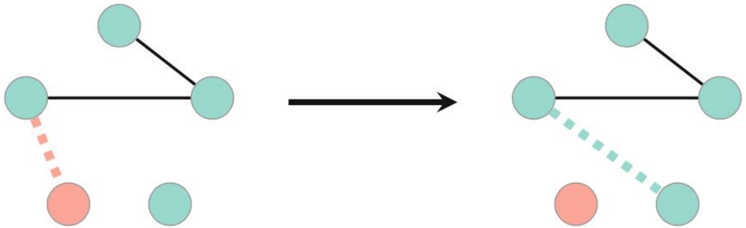
- **Dynamic/ adaptive Networks:**
  The individual connections of an infected node may change considerably during the node's infectious period.
  Often, these changes may be in response to observed infection status

- **Annealed / fast networks:**
  We can ignore the possibility of a single edge transmitting twice.
  Well approximated with mean-field models at node level

Model 1: Rewiring

S nodes break their link to I nodes and reconnect to a uniform random S node with a fixed rate.

[Epidemic Dynamics on an Adaptive Network, Thilo Gross, Carlos J. Dommar D'Lima, and Bernd Blasius - Physical Review Letters '06]



Final infection size may become larger under rewiring networks (compared to static networks)!

[Leung, K., Ball, F., Sirl, D. and Britton, T. (2018). *Individual preventive social distancing during an epidemic may have negative population-level outcomes*. J. R. Soc. Interface]

Rewiring: SIS pairwise approximation

For the SIS epidemic with per-contact transmission rate τ, recovery rate γ and a rewiring rate ω, the pairwise model (4.3) can be extended to the system below.

SIS pairwise model with contact-conserving rewiring

$$[\dot{S}] = \gamma[I] - \tau[SI], \tag{8.1a}$$

$$[\dot{I}] = \tau[SI] - \gamma[I], \tag{8.1b}$$

$$[\dot{SI}] = -(\tau+\gamma)[SI] + \tau([SSI] - [ISI]) + \gamma[II] \underbrace{-\omega[SI]}_{\text{loss due to rewiring}}, \tag{8.1c}$$

$$[\dot{II}] = -2\gamma[II] + 2\tau([ISI] + [SI]), \tag{8.1d}$$

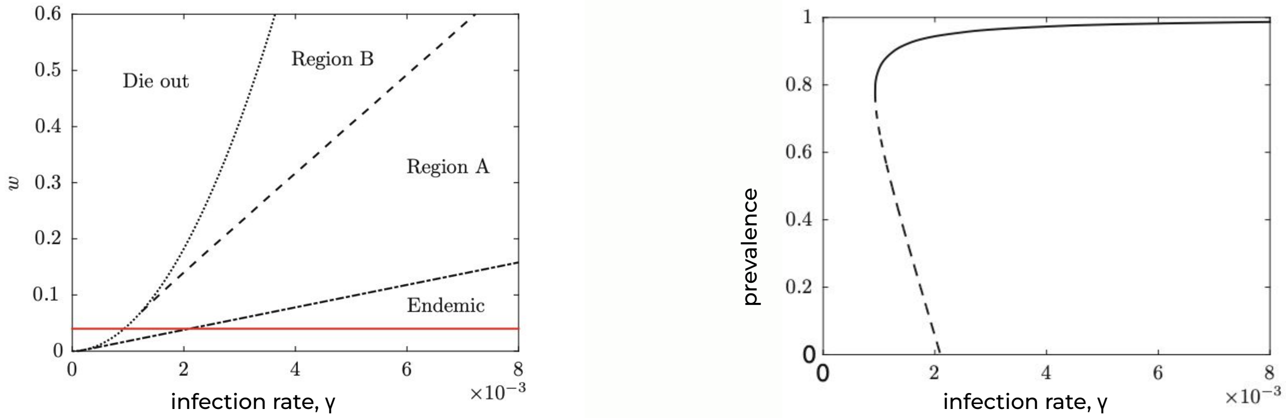
$$[\dot{SS}] = 2\gamma[SI] - 2\tau[SSI] \underbrace{+2\omega[SI]}_{\text{gain due to rewiring}}. \tag{8.1e}$$

Following [128], we take the closures $[SSI] = \frac{[SS][SI]}{[S]}$ and $[ISI] = \frac{[SI][SI]}{[S]}$.

Results: They describe the existence of *endemic steady state* based of the ODEs above.

[Epidemic Dynamics on an Adaptive Network, Thilo Gross, Carlos J. Dommar D'Lima, and Bernd Blasius - Physical Review Letters '06]

Rewiring: SIS pairwise approximation



Global bifurcation diagram for *N* = 100,000 nodes, avg degree= 20 and γ = 0.002. The solid line corresponds to the cross section for *ω* = 0.04, which is plotted in detail in the right panel.

In both regions A and B, two non-zero steady states exist. In region A, one is stable and the other is not. In region B, both are unstable.

[Epidemic Dynamics on an Adaptive Network, Thilo Gross, Carlos J. Dommar D'Lima, and Bernd Blasius - Physical Review Letters '06]

Rewiring: SI and SIR

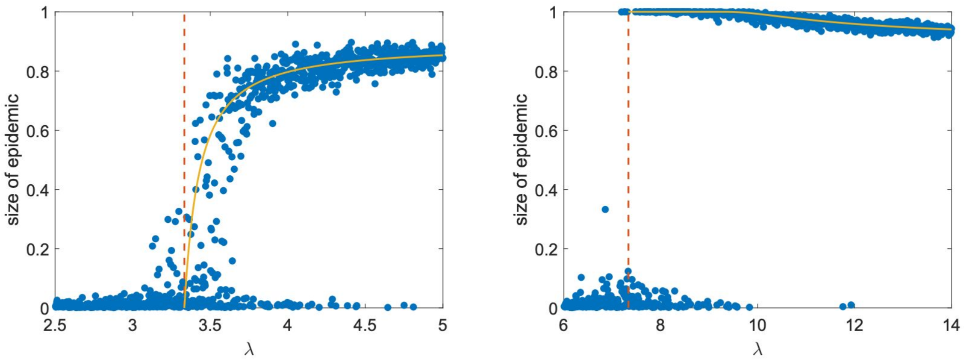


Figure 4: 1,000 simulations of final size of SIR epidemic with rewiring only to susceptibles when $n = 10,000, \mu = 2.5, \gamma = 1, \alpha = 1$ and varying λ; $\omega = 4$ in the left panel and $\omega = 10$ in the right panel. See text for details.

ω: edge deletion/rewiring rate

λ: infection rate

Ball et al. proposed some sufficient conditions on when the discontinuity happen (when the initial graph is a Erdos Renyi random graph).

Ball, Frank, and Tom Britton. "Epidemics on networks with preventive rewiring." *Random Structures & Algorithms*

Model 2: Link Activation/Deletion

Any existing edge is deleted at random according to a Poisson process with rate ω.
Any pair of nodes without an edge is joined following a Poisson process with rate α.
The rates depend on node status.

SIS pairwise model with random link activation and deletion

$$[\dot{S}] = \gamma[I] - \tau[SI], \tag{8.7a}$$

$$[\dot{I}] = \tau[SI] - \gamma[I], \tag{8.7b}$$

$$[\dot{SI}] = -(\tau+\gamma)[SI] + \tau([SSI] - [ISI]) + \gamma[II] + \underbrace{\alpha([S][I] - [SI])}_{\text{link activation}} \underbrace{-\omega[SI]}_{\text{link deletion}}, \tag{8.7c}$$

$$[\dot{II}] = -2\gamma[II] + 2\tau([ISI] + [SI]) \underbrace{+\alpha([I]([I]-1) - [II])}_{\text{link activation}} \underbrace{-\omega[II]}_{\text{link deletion}}, \tag{8.7d}$$

$$[\dot{SS}] = 2\gamma[SI] - 2\tau[SSI] \underbrace{+\alpha([S]([S]-1) - [SS])}_{\text{link activation}} \underbrace{-\omega[SS]}_{\text{link deletion}}, \tag{8.7e}$$

with the closures

$$[SSI] = \frac{(n_S - 1)}{n_S}\frac{[SS][SI]}{[S]} \quad \text{and} \quad [ISI] = \frac{(n_S - 1)}{n_S}\frac{[SI][SI]}{[S]}, \tag{8.8}$$

employed to generate a solvable self-consistent system, where $n_S(t) = ([SS] + [SI])/[S]$ is the average degree of susceptible nodes.

SIS pairwise model with link-status-dependent activation and deletion

$$[\dot{S}] = \gamma[I] - \tau[SI] \tag{8.15a}$$

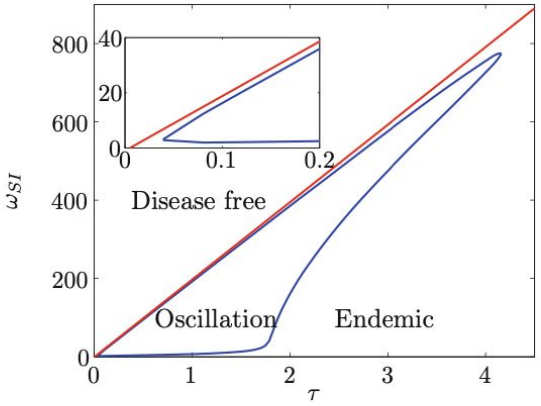
$$[\dot{I}] = \tau[SI] - \gamma[I], \tag{8.15b}$$

$$[\dot{SI}] = -(\tau+\gamma)[SI] + \tau([SSI] - [ISI]) + \gamma([II]) + \alpha_{SI}([S][I] - [SI]) - \omega_{SI}[SI], \tag{8.15c}$$

$$[\dot{II}] = -2\gamma[II] + 2\tau([ISI] + [SI]) + \alpha_{II}([I]([I]-1) - [II]) - \omega_{II}[II], \tag{8.15d}$$

$$[\dot{SS}] = 2\gamma[SI] - 2\tau[SSI] + \alpha_{SS}([S]([S]-1) - [SS]) - \omega_{SS}[SS]. \tag{8.15e}$$

Model 2: Link Activation/Deletion



Bifurcation diagram for the pairwise ODE model (8.15) in the (τ, ω_{SI}) parameter space for $N = 200$, $\gamma = 1$ and $\alpha_{SS} = 0.04$.

SIS pairwise model with link-status-dependent activation and deletion

$$[\dot{S}] = \gamma[I] - \tau[SI] \tag{8.15a}$$

$$[\dot{I}] = \tau[SI] - \gamma[I], \tag{8.15b}$$

$$\begin{aligned}[\dot{SI}] = &-(\tau+\gamma)[SI] + \tau([SSI] - [ISI]) + \gamma([II]) \\ &+ \alpha_{SI}([S][I] - [SI]) - \omega_{SI}[SI],\end{aligned} \tag{8.15c}$$

$$[\dot{II}] = -2\gamma[II] + 2\tau([ISI] + [SI]) + \alpha_{II}([I]([I]-1) - [II]) - \omega_{II}[II], \tag{8.15d}$$

$$[\dot{SS}] = 2\gamma[SI] - 2\tau[SSI] + \alpha_{SS}([S]([S]-1) - [SS]) - \omega_{SS}[SS]. \tag{8.15e}$$

Kiss, et al. "Modelling approaches for simple dynamic networks and applications to disease transmission models."
Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 468.2141 (2012):

Model 2: Oscillation Cycle

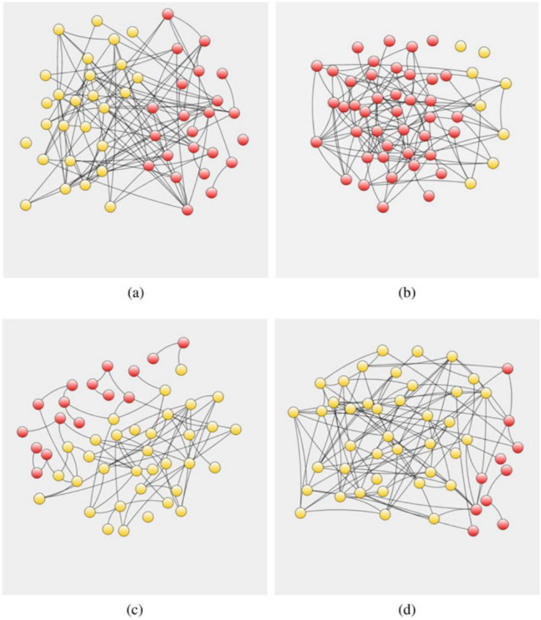
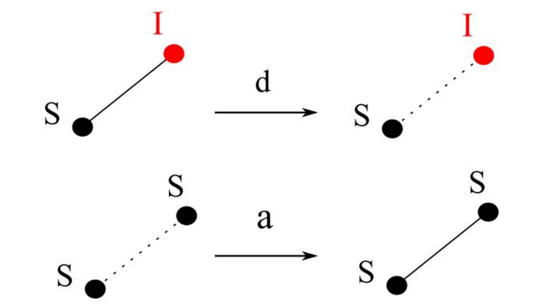


Fig. 8.10: Snapshots of the network during the main phases of the oscillatory cycle: (a) the growing phase of the epidemic with $\langle K \rangle$ close to its maximum, (b) close to the maximum prevalence and a decreasing average degree, (c) decreasing prevalence with $\langle K \rangle$ close to its minimum and, finally, (d) minimal prevalence but with growing average degree. Parameter values are $N = 50$, $\tau = \gamma = 1$, $\omega_{SI} = 1.3$ and $\alpha_{SS} = 0.04$, with all the other activation and deletion rates being equal to zero. Susceptible and infected nodes are denoted by red and yellow filled circles, respectively.

Kiss, et al. "Modelling approaches for simple dynamic networks and applications to disease transmission models." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 468.2141 (2012):

Model 3: Link deactivation on a fixed network



Results: for SIS they compute the threshold does not depend on activation rate:

d: deactivation rate.

⟨K⟩: avg degree

γ : infection rate

$$\tau_c = \frac{d+\gamma}{\langle K \rangle},$$

Shkarayev, Maxim S., Ilker Tunc, and Leah B. Shaw. "Epidemics with temporary link deactivation in scale-free networks." *Journal of Physics A: Mathematical and Theoretical*

Shkarayev, Maxim S., Ilker Tunc, and Leah B. Shaw. Epidemics in adaptive social networks with temporary link deactivation. *Journal of statistical physics*, 2013.

Model 3: Link deactivation on re-activation

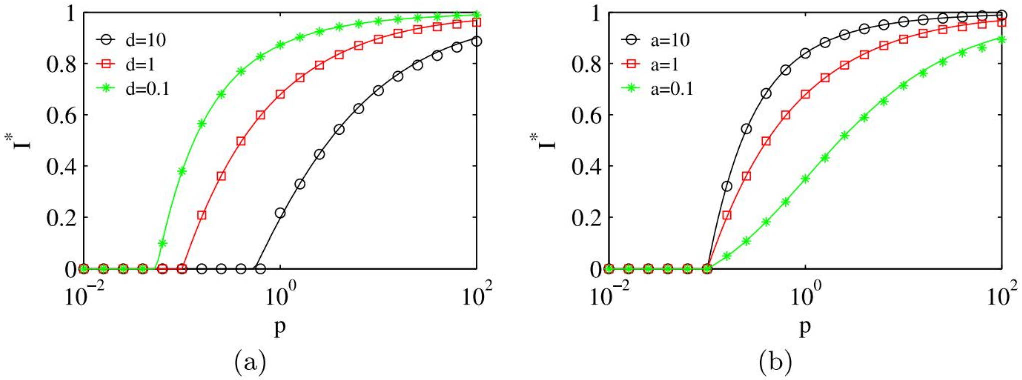


Fig. 3 (**a**) Fraction of infected nodes as a function of infection rate for $a = 1$ and fixed d values. (**b**) Fraction of infected nodes as a function of infection rate for $d = 1$ and fixed a values. *Curves* are mean-field solutions and *symbols* are simulation results. Bifurcation curves were obtained in simulations by sweeping p downward after discarding transients (Color figure online)

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Shkarayev, Maxim S., Ilker Tunc, and Leah B. Shaw. Epidemics in adaptive social networks with temporary link deactivation. *Journal of statistical physics*, 2013.

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Mathematics of Epidemics on Networks

From Exact to Approximate Models

Springer

A few Questions!

When should we choose dynamic models vs. static models?

Each dynamic model behaves differently.
How should we choose the right model for the population we are interested in?

Can we propose a unifying theory for epidemics on dynamic networks?