Information Collection and Reporting through Strategic Agents

Marco OTTAVIANI Bocconi University

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Quantifying Uncertainty: Stochastic, Adversarial, and Beyond @ Simons Institute

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Economics of Statistics

Statistics, based on decision theory, does not explicitly account for:

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- Talk overviews game theory models of:
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- Implications for regulation

Plan

Data: HARD information

1. Strategic sample selection selective disclosure in "hard" data reporting

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 Persuasion bias optional stopping in data collection

Plan

Data: HARD information

- 1. Strategic sample selection selective disclosure in "hard" data reporting
- 2. Persuasion bias optional stopping in data collection

Predictions: SOFT information

- Forecasting contest competition for best record
- 4. Reputational forecasting reputation for accurate information

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 - Favorable outcomes become more likely
 - but are also less convincing that treatment is effective
- How does sample selection affect the value of information?

Impact of ANTICIPATED Selection

- Compare information value of two experiments:
 - Random : $X = \theta + \varepsilon$ with $\varepsilon \sim F$ [BLUE]
 - ▶ Selected : max of k iid draws: $Y = \theta + \varepsilon_{(k)}$ with $\varepsilon_{(k)} \sim F^k$ [RED]



x, y

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Cournot (1843) on P-Hacking

"§101 ... A person not knowing how the data were analysed and whom the experimenter told the result of that analysis concerning the system ... but not how many attempts he made to achieve that result, is unable to judge with a determined chance of error whether the chances ... are equal or not..."

"... However, **unsuccessful tests usually leave no traces**; **the public only knows the results** which the experimenter thought to be deserving notice. It follows that a person alien to the testing is **absolutely unable** to regulate bets on whether the result is, or is not attributable to anomalies of chance."

Here we illustrate idea for simple hypothesis testing:

	θ_L	θ_H
reject	R	R
accept	θ_L	θ_H

$$\theta_L < R < \theta_H$$
, prior $p = \Pr(\theta_H)$

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- ▶ Location experiment $x = \theta + \varepsilon$, with $\varepsilon \sim F$ independent from θ
 - Assume **logconcave** density $f \Leftrightarrow$ monotone likelihood ratio property

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- ▶ Location experiment $x = \theta + \varepsilon$, with $\varepsilon \sim F$ independent from θ
 - ► Assume logconcave density f ⇔ monotone likelihood ratio property
- With a single draw, cutoff rule optimal: accept iff

$$\frac{f(x-\theta_H)}{f(x-\theta_L)} \ge \frac{1-p}{p} \frac{R-\theta_L}{\theta_H-R} \Leftrightarrow x \ge \bar{x}$$

Random Experiment



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Selected Experiment



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$$\underbrace{1 - F^{k}(\bar{y} - \theta_{L}) = 1 - F(\bar{x} - \theta_{L})}_{\Rightarrow \bar{y}} \Rightarrow \bar{y} = (F^{k})^{-1}F(\bar{x} - \theta_{L}) + \theta_{L}$$

Using cutoff \bar{y} in Y that matches False Positives



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Theorem (general sample size n): Fixing sample size n, as pre-sample size k increases, experiment becomes more (less) informative in every monotone problem if reverse hazard rate RHR f(x|0)/F(x|0) is log-supermodular (log-submodular, w/ support unbounded above)

$$\frac{f(\boldsymbol{x}|\boldsymbol{\theta}')/F(\boldsymbol{x}|\boldsymbol{\theta}')}{f(\boldsymbol{x}|\boldsymbol{\theta})/F(\boldsymbol{x}|\boldsymbol{\theta})}$$

increasing (decreasing) in x, for all $\theta' > \theta$

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 - Neutral selection: loglinear RHR $\frac{f(x)}{F(x)}$ (Gumbel)
 - ► Harmful selection: logconvex RHR $\frac{f(x)}{F(x)}$ (e.g., Exponential)

NEUTRAL Selection: Loglinear f/F

Gumbel noise: $F(\varepsilon) = e^{-e^{-\varepsilon}}$



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HARMFUL Selection: f LESS Logconcave than F

Exponential noise: $F(\varepsilon) = 1 - e^{-\varepsilon}$



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BENEFICIAL Selection: f MORE Logconcave than F

Normal noise: $\varepsilon \sim \mathcal{N}$



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Data: HARD information

- 1. Hypothesis testing with selected data
 - selection benefits/harms if data has thinner/thicker tail than Gumbel

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- 2. Wald persuasion games with costly information collection
 - optional stopping in data collection

- Informer benefits from approval of drug with uncertain efficacy
 - Evaluator = FDA regulator

 $\theta_e^H > 0$ in state *H* and $\theta_e^L < 0$ in state *L*

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 - instantaneous trial result from state-dependent Brownian motion

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- Informer sequentially acquires & diffuses costly information
 - instantaneous trial result from state-dependent Brownian motion
- Evaluator has coarse instruments for regulation
 - ► approve/reject, ask for additional evidence [impose liability]

TWO players:

- 1. Informer i
- a. directly controls information acquisition & pays info cost
- b. but always wants approval and does not directly value info

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Wald's social planner w

- a. controls all decisions (rejection/approval) & info acquisition
- b. obtains total payoff $v_w = \theta_e + v_i$ (evaluator+informer) & pays info cost

Wald Welfare Benchmark: Value Function



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3. Evaluator *e* Commitment: $\max_{S} u_e|_{s=b_i(S)}$

Informer Best Reply [RED]



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• *i*'s best reply RED $s = b_i(S)$ given S [s on horizontal axis]

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- ► TOP: Informer stops as soon as evaluator persuaded: $S^i = \hat{q}_e = \frac{-\theta_e^i}{\theta_e^{ij} \theta_e^{ij}}$

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- ► TOP: Informer stops as soon as evaluator persuaded: $S^i = \hat{q}_e = \frac{-\theta_e^i}{\theta_a^\mu \theta_a^\mu}$
- ▶ BOTTOM: Informer withdraws when pessimistic enough: $s^i = b_i(\hat{q}_e)$

Sequential Foundation of Bayesian Persuasion

Comparison to Kamenica-Gentzkow's (2011) commitment solution?

Sequential Foundation of Bayesian Persuasion

- Comparison to Kamenica-Gentzkow's (2011) commitment solution?
- We recover KG without info frictions if (1) $c \rightarrow 0$ & (2) $r \rightarrow 0$, so:
 - KG solution becomes sequentially optimal without commitment



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• Evaluator e gains commitment power: precursor of FDA in 1905

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- EQUILIBRIUM: Stationary Markov Perfect Equilibrium solving
 - *i* controls withdrawal standard: $s^N = b_i(S^N)$ RED
 - e controls adoption standard: $S^N = B_e(s^N)$ BLUE

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• e sets more stringent approval standard: $S^N > S^i$

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- e sets more stringent approval standard: $S^N > S^i$
- *i* withdraws earlier: $s^N > s^i$

3. Evaluator Commitment Solution



Evaluator commits ex ante to approval when belief reaches S^e

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▶ approve iff q ≥ S^e

3. Evaluator Commitment Solution



- Evaluator commits ex ante to approval when belief reaches S^e
 - ▶ approve iff q ≥ S^e
- Stackelberg tangency with e's BLUE iso-payoffs
- Evaluator benefits to be more lenient S^e < S^N
 - commits to free-ride less to encourage more info collection
- Compared to Nash, FALSE POSITIVES ↑ & FALSE NEGATIVES ↓

Data: HARD information

- 1. Hypothesis testing with selected data
 - anticipated selection benefits/harms if data has thinner/thicker tail than Gumbel

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- 2. Wald persuasion games with costly information collection
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Predictions: SOFT information

- 1. Forecasting contest
 - excessive differentiation
- 2. Reputational forecasting
 - conformism with naive audience
 - information loss with rational audience

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Honest Forecasting: Benchmark Statistical Model

- Unknown state with prior $x \sim N(\mu, \frac{1}{\nu})$
- Single forecaster with private signal s about state x
- Honest forecaster (naive statistician) minimizes forecast error

$$\min_{m} E\left[\left(m-x\right)^{2}|s\right]$$

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Best statistical forecast is posterior expectation

$$m = E[x|s]$$

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- Forecasts are orthogonal to the forecast error
- Forecasts are dispersed, but less than the state

Forecasting Contest

Ottaviani-Sørensen (2006, J of Financial Economics)

Francis Galton's (1907) ox weight competition:


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Francis Galton's (1907) ox weight competition:

1. Large number of forecasters, each observes signal s_i

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2. Forecasters simultaneously submit forecasts *m_i*

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Francis Galton's (1907) ox weight competition:

- 1. Large number of forecasters, each observes signal s_i
- 2. Forecasters simultaneously submit forecasts m_i
- 3. True state is publicly observed x
- 4. Forecaster whose forecast is closest to the state wins



At the posterior expectation

$$m_i = E\left[x|s_i
ight]$$

a small deviation away from the prior mean μ results in

- a second-order loss due to lower chance of winning, but
- ► a first-order gain due to reduced competition

Excessive Differentiation in Forecasting Contest



Figure: Equilibrium forecasts are more variable than posterior expectation

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Ottaviani-Sørensen (2006, RAND J Economics)

1. Single forecaster observes signal s with accuracy t

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E[t|m,x]

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E[t|m,x]

- Objective of forecaster is to obtain favorable evaluation E[t|m,x]
 - Forecast m is a (cheap talk) signal
 - Is honest m = E[x|s] an equilibrium?

Misreporting Incentives



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▶ If evaluator (naïvely) expects honest forecasting m = E[x|s]

Will forecaster want to report honestly?

Misreporting Incentives



▶ If evaluator (naïvely) expects honest forecasting m = E[x|s]

- Will forecaster want to report honestly?
- With location signal, forecaster has incentive to lie reporting

$$m{E}[m{x}|\hat{m{s}}=m{E}[m{x}|m{s}]]$$
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Reputational Cheap Talk Equilibrium

Equilibrium communication is coarse (as in Crawford and Sobel, 1982)

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- Forecasters with signals in an interval send identical message
- Loss of forecast accuracy!

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- Forecasters with signals in an interval send identical message
- Loss of forecast accuracy!
- E.g., there exists a two-message equilibrium:
 - Report whether s is above or below prior mean E[x]

Forecasting Summary

- Concern for accuracy leads to:
 - excessive conformity if the market is naïve
 - Ioss of information if the market is rational

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Forecasting Summary

- Concern for accuracy leads to:
 - excessive conformity if the market is naïve
 - Ioss of information if the market is rational
- Competition for best accuracy record leads to excessive differentiation

Game Theory of Data Collection & Reporting

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