Networks & Economics 1

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Behavior on Networks:

• Contagion and diffusion

• Learning – processing beliefs

• Peer influence in choices and behaviors
  • Care about how peers act
  • A “complex” form of interaction – behaviors are fully interdependent
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• Contagion and diffusion

• Learning – processing beliefs

• Peer influence in choices and behaviors
  • Care about how peers act
  • A “complex” form of interaction – behaviors are fully interdependent
Peer Effects

- Information – influencing and correlating beliefs and opinions
- Opportunities – we rely on others for access to jobs, education, group memberships,…
- Traditions, culture, norms, pressures – social influences for us to adopt specific behaviors, generally correlated with others
- Complementarities – benefits from coordinating (e.g., using same technology or language, studying, stealing, being corrupt, etc.)
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Start with a Canonical Special Case:

• Each player chooses action $x_i$ in \{0,1\}

• payoff depends on
  – how many neighbors choose each action
  – how many neighbors a player has
• Each player chooses action \( x_i \) in \{0,1\}

• Consider cases where \( i \)'s payoff is

\[
u_{d_i}(x_i, m_{N_i})
\]

depends on \( d_i(g) \) and \( m_{N_i(g)} \) - number of neighbors of \( i \) choosing 1
Games on Networks - Outline

• Basic Definitions

• Examples

• Strategic Complements/Substitutes

• Equilibrium existence and structure
Example:

Player prefers to adopt new technology if at least 40 percent of neighbors do.
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Example: Complements

• agent $i$ is willing to choose 1 if and only if at least $t$ neighbors do:

• Payoff action 0: $u_{d_i}(0, m_{N_i}) = 0$

• Payoff action 1: $u_{d_i}(1, m_{N_i}) = m_{N_i} - \frac{0.4}{d_i}$
A Nash Equilibrium

• A pure strategy Nash equilibrium on a network

• Specify a choice for each person $x_i$ in $X_i$

• Nobody should want to change their action given what their friends are doing: $u_i(x_i, x_{-i}) \geq u_i(x_i', x_{-i})$ for all $i, x_i'$
Another Example: Best-Shot Public Goods

Player prefers to buy if no neighbors do
Another Example: Best-Shot Public Goods

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Another Example:
Best-Shot Public Goods

Player prefers to buy if no neighbors do
Example: Best-Shot

- agent $i$ is willing to choose 1 if and only if no neighbors do:

- Payoff action 0: $u_{d_i}(0,m_{N_i}) = 1$ if $m_{N_i} > 0$
  
  $= 0$ if $m_{N_i} = 0$

- Payoff action 1: $u_{d_i}(1,m_{N_i}) = 1 - c$
Independent Set: a set $S$ of nodes such that no two nodes in $S$ are linked,

Maximal: every node in $N$ is either in $S$ or linked to a node in $S$

Equilibria: Adopters = a maximal independent set
Another Example: Best-Shot Public Goods

Player prefers to buy if no neighbors do
Games on Networks - Outline

- Basic Definitions
- Examples
- Strategic Complements/Substitutes
- Equilibrium existence and structure
Externalities:

- Others’ behaviors affect one’s utility/welfare

- Others’ behaviors affect one’s *decisions, actions, consumptions, opinions*...
  - others’ actions affect the *relative* payoffs to one’s behaviors
• $b =$ benefit of a book/etc
• $c =$ cost  $b > c$

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<th>Friend buys</th>
<th>Friend does not</th>
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Strategic Substitutes

- $b =$ benefit of a book/etc.
- $c =$ cost  $b > c$

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externality
Externalities:

• Others’ behaviors affect one’s utility/welfare

• Game theory: others’ behaviors affect one’s decisions, actions, consumptions, opinions...
  – others’ actions affect the relative payoffs to one’s behaviors
• $b =$ benefit of a book/cd/etc.
• $c =$ cost $b > c$

Friend has

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b = benefit of playing game with friend

• c = cost of learning to play

Friend plays  
Play              b - c

Friend do not  
Not               0

externality

- c

0
Strategic Complements

- $b =$ benefit of playing game with friend
- $c =$ cost of learning to play

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• with negative externality, e.g., doping:
Strategic Substitutes

- with negative externality, e.g., congestion game:

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Strategic Complements/Substitutes

• strategic complements -- for all $d, m \geq m'$
  – Increasing differences:
    $$u_d(1,m) - u_d(0,m) \geq u_d(1,m') - u_d(0,m')$$

• strategic substitutes -- for all $d, m \geq m'$
  – Decreasing differences:
    $$u_d(1,m) - u_d(0,m) \leq u_d(1,m') - u_d(0,m')$$
Strategic Complements/Substitutes

- **Complements**: Choice to take an action by my friends increases my relative payoff to taking that action (e.g., friend learns to play a video game)

- **Substitutes**: Choice to take an action by my friends decreases my relative payoff to taking that action (e.g., roommate buys a stereo/fridge)
Examples

• Complements:
  – smoking & other behavior among teens, peers, ...
  – technology adoption – care about fraction others compatible...
  – educate/drop out work force
  – learn a language
  – corruption, crime
  – cheating, doping

• Substitutes
  – information gathering
  – local public goods (shareable products...)
  – competing firms (oligopoly with local markets)
  – vaccinations (near herd immunity)...
Useful Observation

• Complements: there is a threshold $t(d)$, such that $i$ prefers 1 if $m_{N_i} > t(d)$ and 0 if $m_{N_i} < t(d)$

• Substitutes: there is a threshold $t(d)$, such that $i$ prefers 1 if $m_{N_i} < t(d)$ and 0 if $m_{N_i} > t(d)$

• Can be indifferent at the threshold
Games on Networks - Outline

• Basic Definitions

• Examples

• Strategic Complements/Substitutes

• Equilibrium existence and structure
Equilibrium

• Nash equilibrium: Every player’s action is optimal for that player given the actions of others

• Often look for pure strategy equilibria

• May require some mixing in case of substitutes
Proposition

In a game on a network of strategic complements where the individual strategy sets are complete lattices: the set of pure strategy equilibria are a (nonempty) complete lattice.
• Complete Lattice: for every set of equilibria $X$
  – there exists an equilibrium $x'$ such that $x' \geq x$ for all $x$ in $X$, and
  – there exists an equilibrium $x''$ such that $x'' \leq x$ for all $x$ in $X$. 
Contrast: Complements and Substitutes

• In a game of complements: pure strategy equilibria are a (nonempty) complete lattice

• In a game of strategic substitutes:
  – Best shot game: pure strategy equilibria exist and are related to maximal independent sets
  – Others: pure strategy may not exist, but mixed will (with finite action spaces, or appropriate measure spaces)
  – Equilibria often do not form a lattice
When can multiple actions be sustained:

• Coordination game

• Care about fraction of neighbors taking action 1:
  prefer to take action 1 if fraction q or more take 1
• Let $S$ be the group that take action 1

• Each $i$ in $S$ must have fraction of at least $q$ neighbors in $S$

• Each $i$ not in $S$ must have a fraction of at least $1-q$ neighbors outside of $S$
Morris 2000: A group $S$ is $r$-cohesive relative to $g$ if
\[
\min_{i \in S} |N_i(g) \cap S| / d_i(g) \geq r
\]
At least a fraction $r$ of each member of $S$’s neighbors are in $S$

Cohesiveness of $S$ is $\min_{i \in S} |N_i(g) \cap S| / d_i(g)$
Both groups are 2/3 cohesive
There exists a pure strategy equilibrium where both actions are played if and only if there is a group $S$ that is at least $q$ cohesive and such that its complement is at least $1-q$ cohesive.
Example:

Player prefers to adopt new technology if at least 40 percent of neighbors do.
$1/3 < q < 1/2$

$1/2 < q < 2/3$
$1/4 < q < 1/3$

$2/3 < q < 3/4$
$q < \frac{1}{4}$

$\frac{3}{4} < q$
Growing block models:  blocks  $b$ in $B(n)$

Probability of linking nodes from blocks $b$, $b'$ is  $p_{bb'}(n)$

expected degree of node in $b$ to nodes in $b'$  $d_{bb'}(n)$

overall expected degree of node in $b$  $d_b(n)$

$\left( > (1 + \varepsilon) \log(n) \text{ for all } b, n \right)$
Convergent growing block models:

\[ |B(n)| = k \text{ for large } k \]

\[ \frac{d_{bb}(n)}{d_b(n)} \text{ converges for all } b, b' \]

Weakly homophilous:

\[ \frac{d_{bb}(n)}{d_b(n)} > \frac{d_{b'b}(n)}{d_{b'}(n)} + \varepsilon \]
Consider a growing sequence of stochastic block networks.

- Any sequence of sets of adopters that are equilibria for some open set of $q$, are a superset of the blocks with a probability going to 1.

- If the sequence of block models is convergent and weakly homophilous, then there exists some open set of $q$, for which any given block is an equilibrium for those $q$ with a probability going to 1.
Proof ideas

Thm by McDiarmid, Skerman 2018 – modularity of G(n,p) goes to 0

Relate modularity to equilibrium structure: if equilibrium splits some block, then modularity of that block has lower bound.
Community structures: equilibria define groups of people whose behaviors are always tied, communities differ based on behavior (q).

Seeding: communities help for seeding.

Complex contagion differs from simple: clustering needed for diffusion.

Equilibria can be ordered by degree distributions in random networks (Bayesian games, mean field games, graphon games).
Estimate $q$ from data...

12th grade smoking, add health data

Estimate $q$ presuming equilibrium: $q = 0.39$

mis-predict 29% of behavior

Jackson-Storms 2019
Each person chooses a level of behavior $x_i$:

- level of criminal activity
- how fast drive
- how long stay in school
- how much study
- effort spent legislating
g_{ij} \quad \text{intensity of connection from i to j: how much i is influenced by what j does}

can be weighted and directed

u_i(x_i, x_{-i}) = a x_i - c x_i^2/2 + b \sum_j g_{ij} x_i x_j

Ballester, Calvo-Armengol and Zenou (2006)
A Linear-Quadratic Model

\[ u_i(x_i, x_{-i}) = a x_i - c x_i^2/2 + b \sum_j g_{ij} x_i x_j \]

\[ \downarrow \]

the direct
benefit of \( x_i \)
A Linear-Quadratic Model

\[ u_i(x_i, x_{-i}) = ax_i - c x_i^2/2 + b \sum_j g_{ij} x_i x_j \]

the cost of \( x_i \)
convex – higher
marginal costs as
increase \( x_i \)
A Linear-Quadratic Model

\[ u_i(x_i, x_{-i}) = a x_i - c x_i^2/2 + b \sum_j g_{ij} x_i x_j \]

interaction effect:
the higher \( x_j \) and the higher \( g_{ij} \), the more \( i \) benefits from increasing \( x_i \)
A Linear-Quadratic Model

\[ u_i(x_i, x_{-i}) = a x_i - c x_i^2/2 + b \sum_j g_{ij} x_i x_j \]

Maximize this function

the best response of \( x_i \) to \( x_{-i} \):
A Linear-Quadratic Model

\[ u_i(x_i, x_{-i}) = a x_i - c x_i^2/2 + b \sum_j g_{ij} x_i x_j \]

Maximize this function
the best response of \( x_i \) to \( x_{-i} \):

\[ a - c x_i + b \sum_j g_{ij} x_j = 0 \]

\[ x_i = \frac{(a + b \sum_j g_{ij} x_j)}{c} \]
A Linear-Quadratic Model

\[ x_i = \left( a + b \sum_j g_{ij} x_j \right)/c \]

in matrix form:  \[ x = A + G x \]

where \( A = (a/c, \ldots, a/c), \quad G_{ij} = b \ g_{ij}/c \)
A Linear-Quadratic Model

\[ x = A + Gx \]

or \[ x = A + G( A + G ( A + G ....))) = \sum_{k \geq 0} G^k A \]

or \[ x = (I - G)^{-1} A \] if invertible

(or if a=0, then \[ x = Gx \], so unit eigenvector)
• Actions are related to network structure:
  
  – higher neighbors’ actions, higher own action
  
  – higher own action, higher neighbors actions
  
  – feedback – for solution need b/c to be small and/or $g_{ij}$’s to be small (need $\Sigma_{k\geq0} G^k$ to converge)
• Relation to centrality measures:

\[ x = \sum_{k \geq 0} G^k A = \sum_{k \geq 0} G^k 1 \ (a/c) = (1 + \sum_{k \geq 1} G^k 1)(a/c) \]

Katz-Bonacich centrality:

\[ B(g) = \sum_{k \geq 1} g^k 1 \]

So, \[ x = (1 + B(G))(a/c) \]
A Linear-Quadratic Model

- Natural feedback, actions relate to the total feedback from various positions
- Capture network in tractable manner
- Centrality: relative number of weighted influences going from one node to another
Applications of Model:

- criminal behavior, delinquency (Patacchini, Zenou 12)
- study habits (Calvo, Patacchini, Zenou 09)
- political efforts, party divisions (Canen, Jackson, Trebbi 22)
- corporate control (Vitali et al 11, Larcker et al 13)
- drug trafficking (Dell 15)
- friendship paradox and teen behavior (Jackson 19)
Application to Student Performance

• Calvo-Armengol, Patacchini, Zenou (2009) applied this to Add Health data

• Let \( x_i \) be how hard a student studies

• Measure this by academic performance (a factor analysis of survey answers and grades)

• Estimate \( b/c \), see how much centrality matters in determining academic performance (w controls)
Estimates

• Estimate b/c to be .55

• Find a SD increase in Bonacich centrality increases performance by 7%
Games on Networks

• *Many* applications

• Externalities make the analysis important – individual incentives and societal welfare diverge

• Networks have systematic features that matter in ways that can be quantified
Network Formation Models

• Random models
  • *How* networks form

• Game theoretic/strategic models:
  • *Why* specific networks form
    • prefer to connect *because* someone is well connected
    • high clustering because lower cost for nearby connections
    • small worlds because value to bridges
  • Welfare analyses, inefficiencies, externalities, *policies*...
Questions:

• Which networks are formed by the people/nodes?

• Which networks are best for society?
An `Economic’ Analysis:

• Choose connections
  – benefits from connections
  – costs to maintaining relationships
  – time limits...

• Care about direct friendships, but also about indirect friendships
  – follow someone on media because they are connected...
References


Jackson, Matthew O. and Yves Zenou. 2014. “Games on Networks.” *Handbook of Game Theory, Elsevier*, edited by Young, H.P. and Zamir, S.


