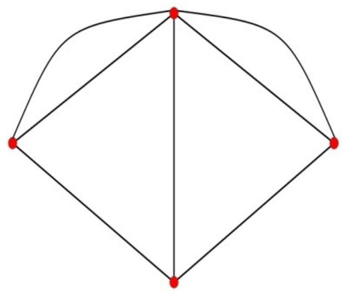
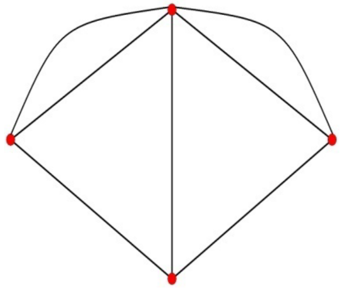
Networks & Economics 1



Matthew O. Jackson

Behavior on Networks:



- Contagion and diffusion

- Learning – processing beliefs

- Peer influence in choices and behaviors
  - Care about how peers act
  - A ``complex'' form of interaction – behaviors are fully interdependent

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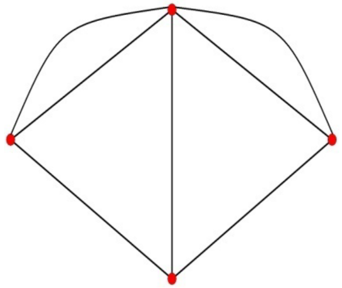

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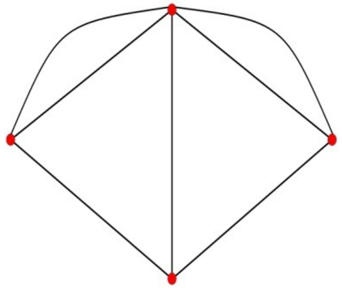
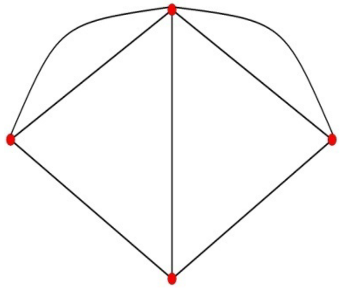
Peer Effects



- Information – influencing and correlating beliefs and opinions
- Opportunities – we rely on others for access to jobs, education, group memberships,…
- Traditions, culture, norms, pressures – social influences for us to adopt specific behaviors, generally correlated with others
- Complementarities – benefits from coordinating (e.g., using same technology or language, studying, stealing, being corrupt, etc.)

Peer Effects



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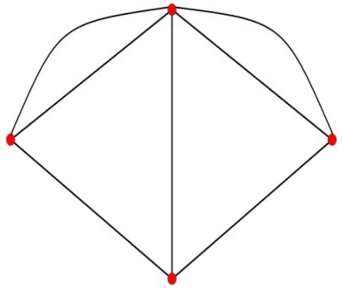
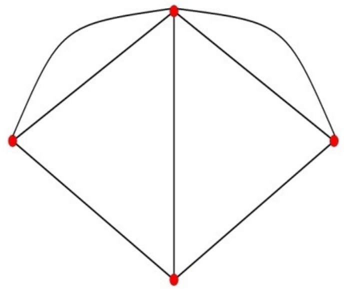
Start with a Canonical Special Case:



- Each player chooses action x_i in {0,1}
- payoff depends on
  – how many neighbors choose each action
  – how many neighbors a player has

Definitions



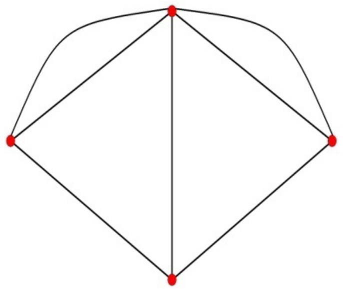
- Each player chooses action x_i in {0,1}

- Consider cases where i's payoff is

$$u_{d_i}(x_i , m_{N_i})$$

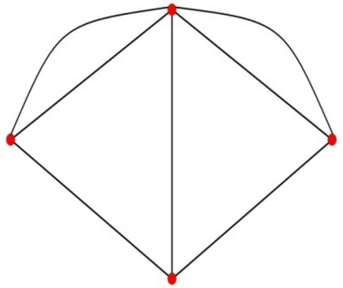
depends on $d_i(g)$ and $m_{N_i(g)}$ - number of neighbors of i choosing 1

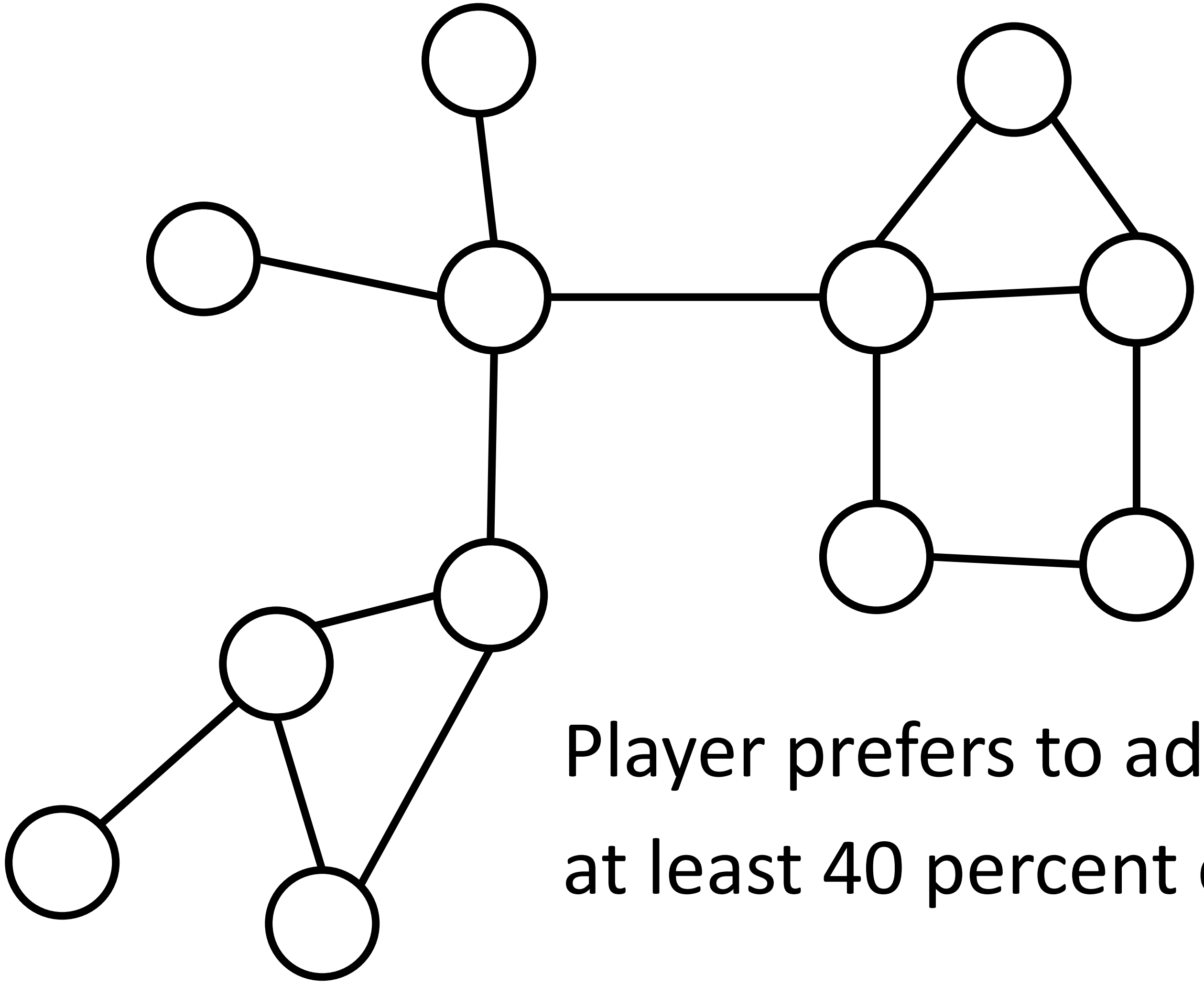
Games on Networks - Outline



- Basic Definitions
- Examples
- Strategic Complements/Substitutes
- Equilibrium existence and structure

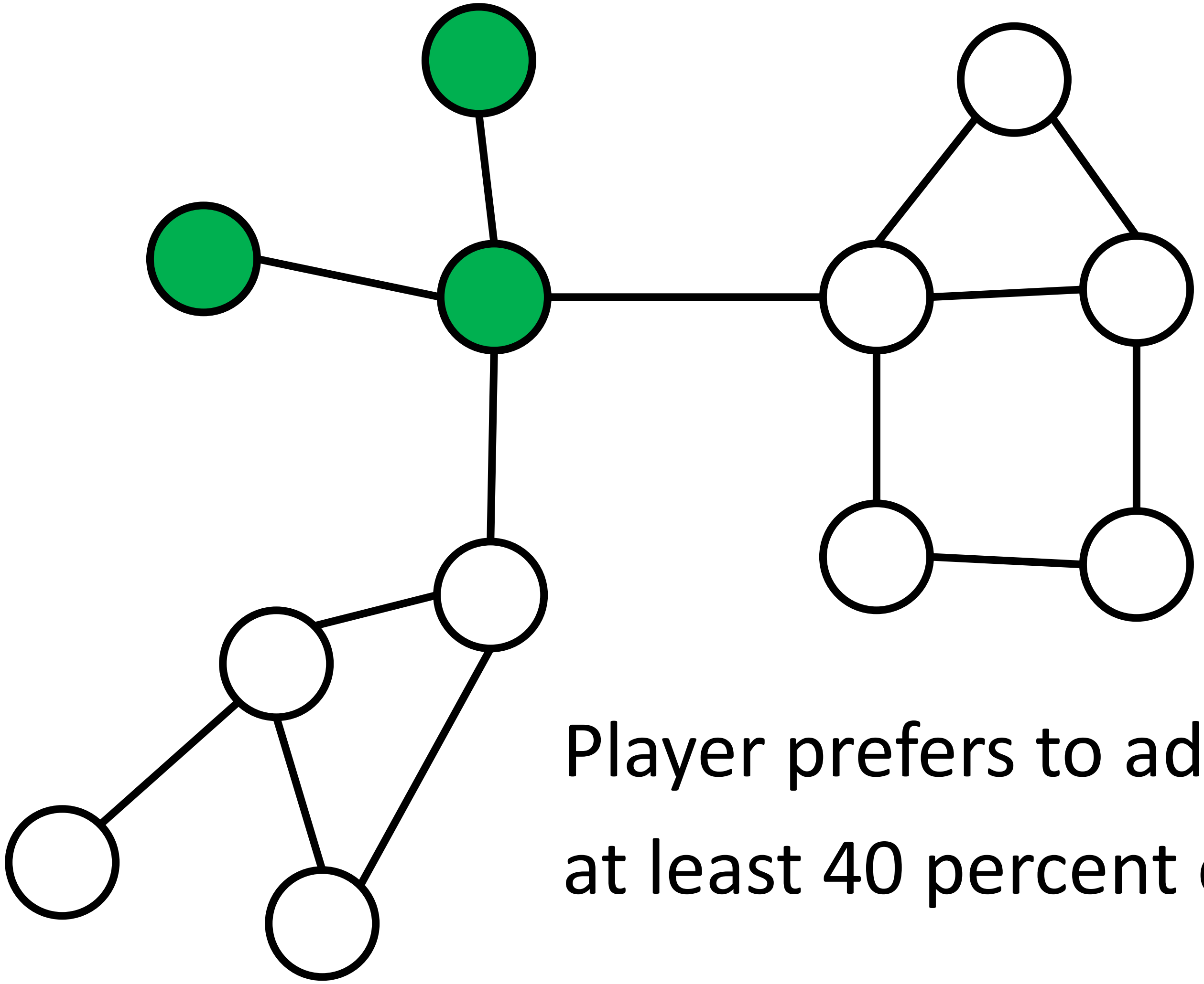
Example:





Player prefers to adopt new technology if at least 40 percent of neighbors do

Example:



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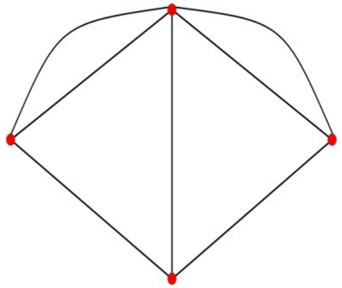

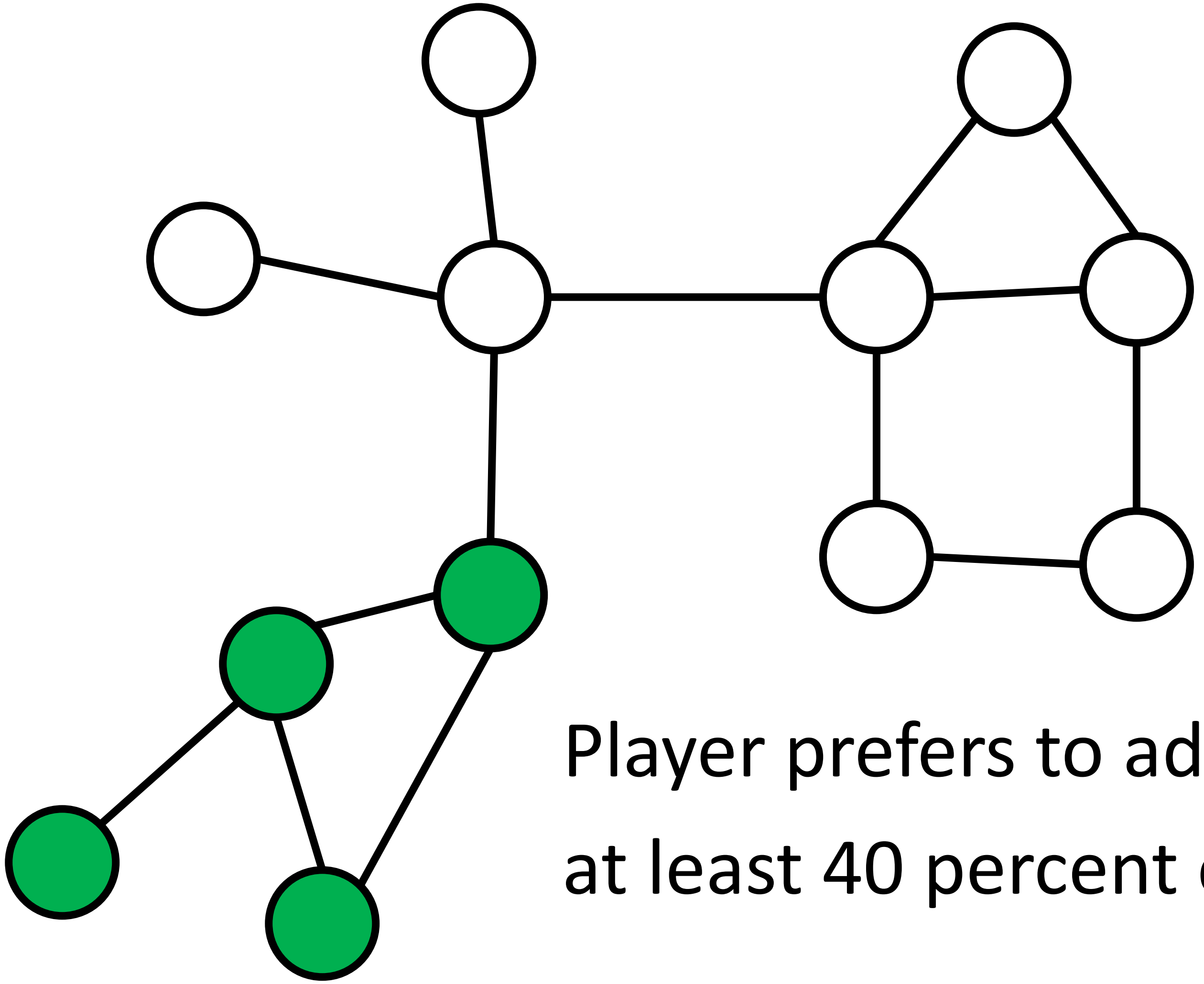

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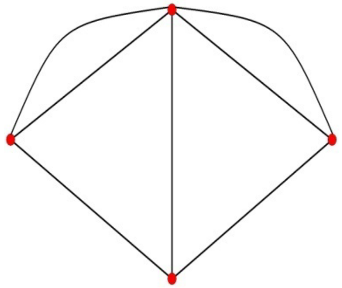
Example: Complements

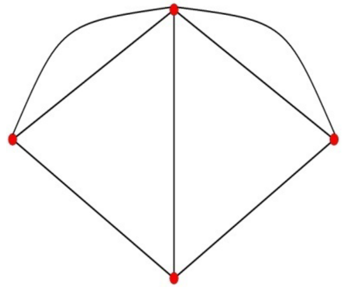


- agent i is willing to choose 1 if and only if at least t neighbors do:

- Payoff action 0: $u_{d_i}(0, m_{N_i}) = 0$

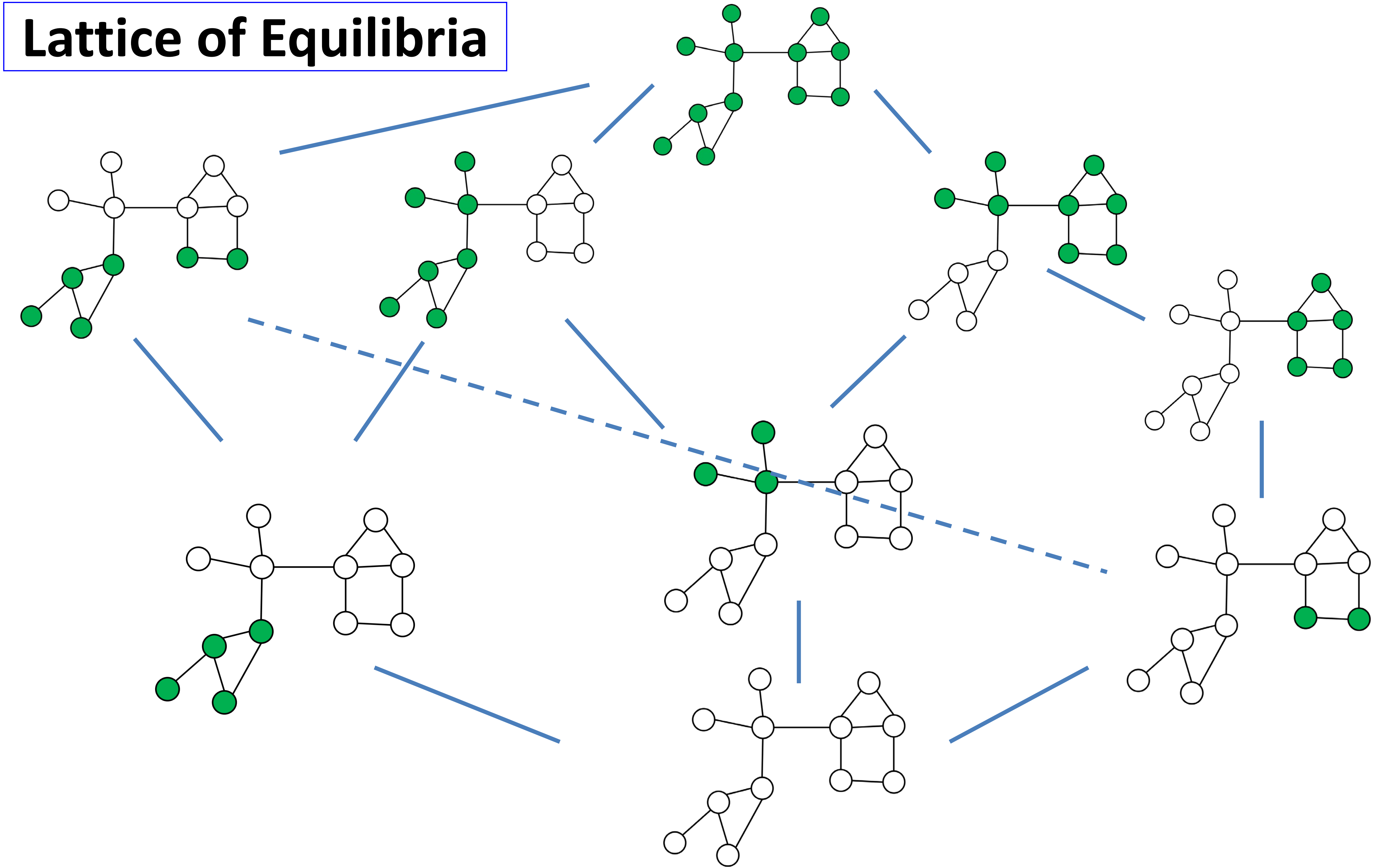
- Payoff action 1: $u_{d_i}(1, m_{N_i}) = \frac{m_{N_i}}{d_i} - .4$

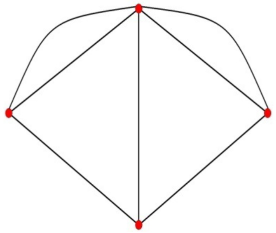
A Nash Equilibrium



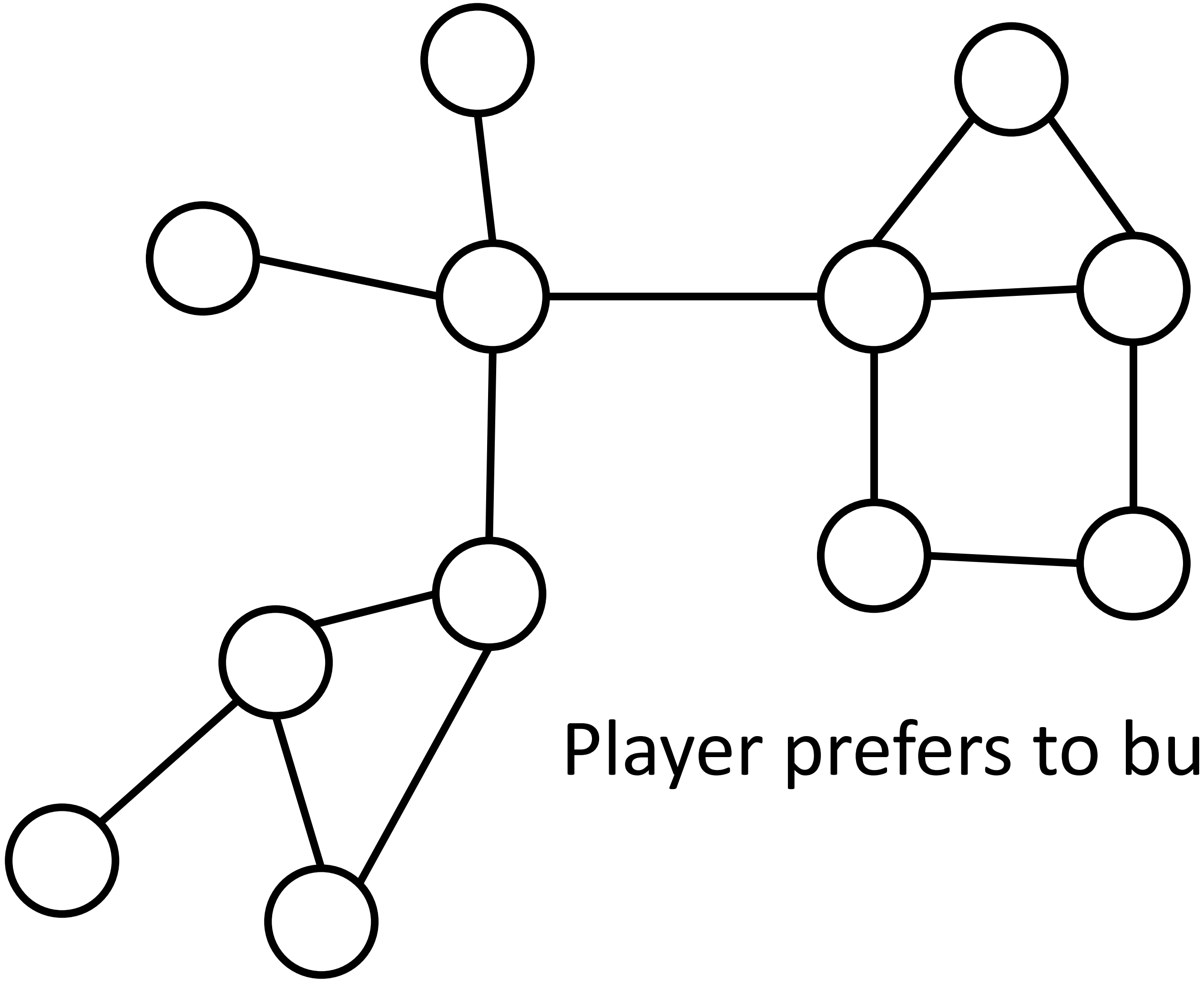
- A pure strategy Nash equilibrium on a network
  - Specify a choice for each person x_i in X_i
  - Nobody should want to change their action given what their friends are doing: $u_i(x_i, x_{-i}) \geq u_i(x_i', x_{-i})$ for all i, x_i'

Lattice of Equilibria





Another Example: Best-Shot Public Goods



Player prefers to buy if no neighbors do

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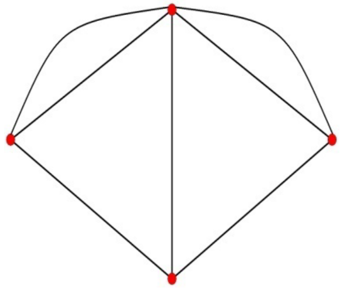

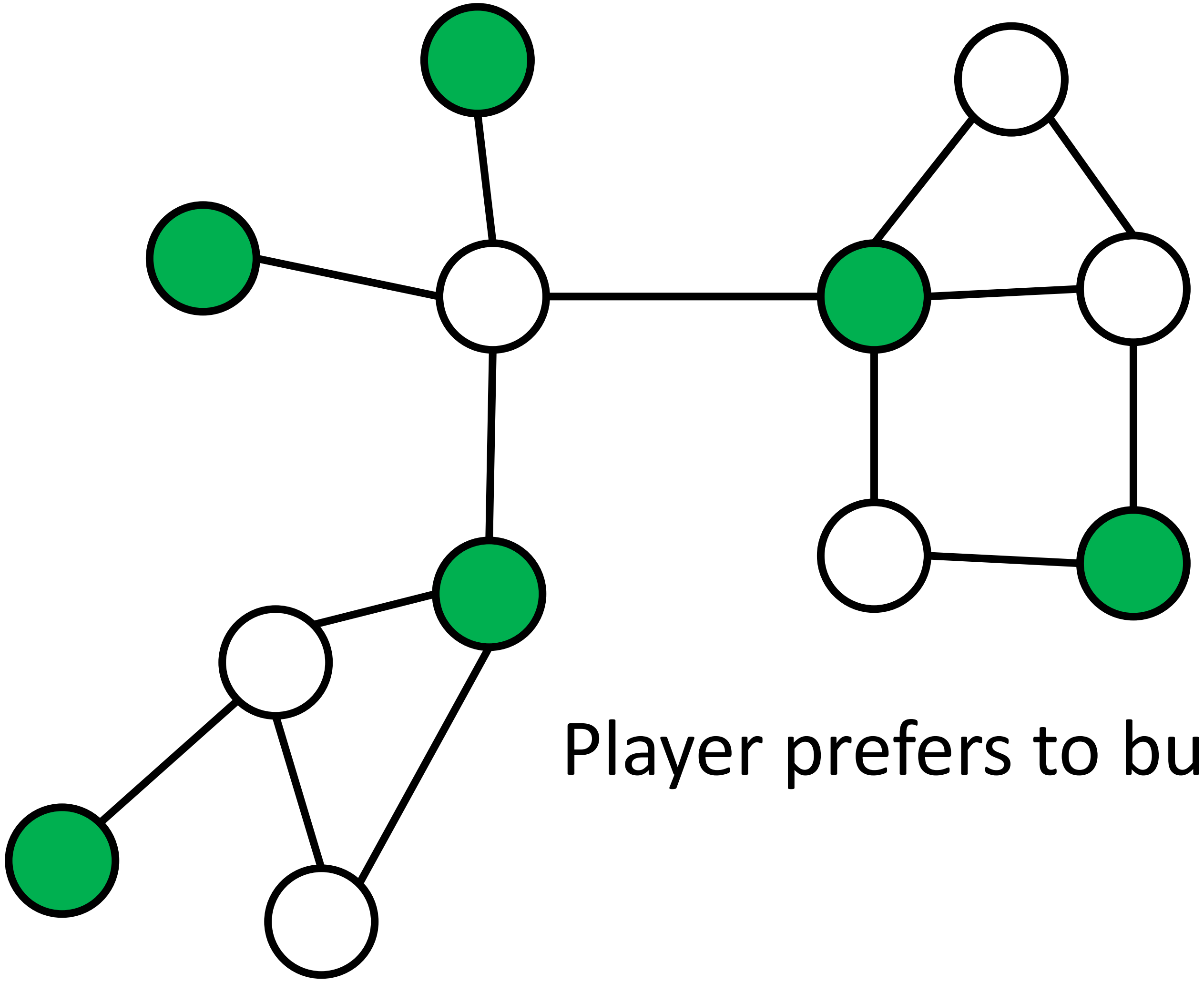

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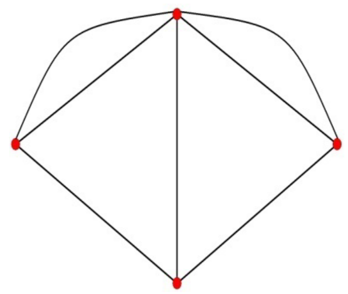

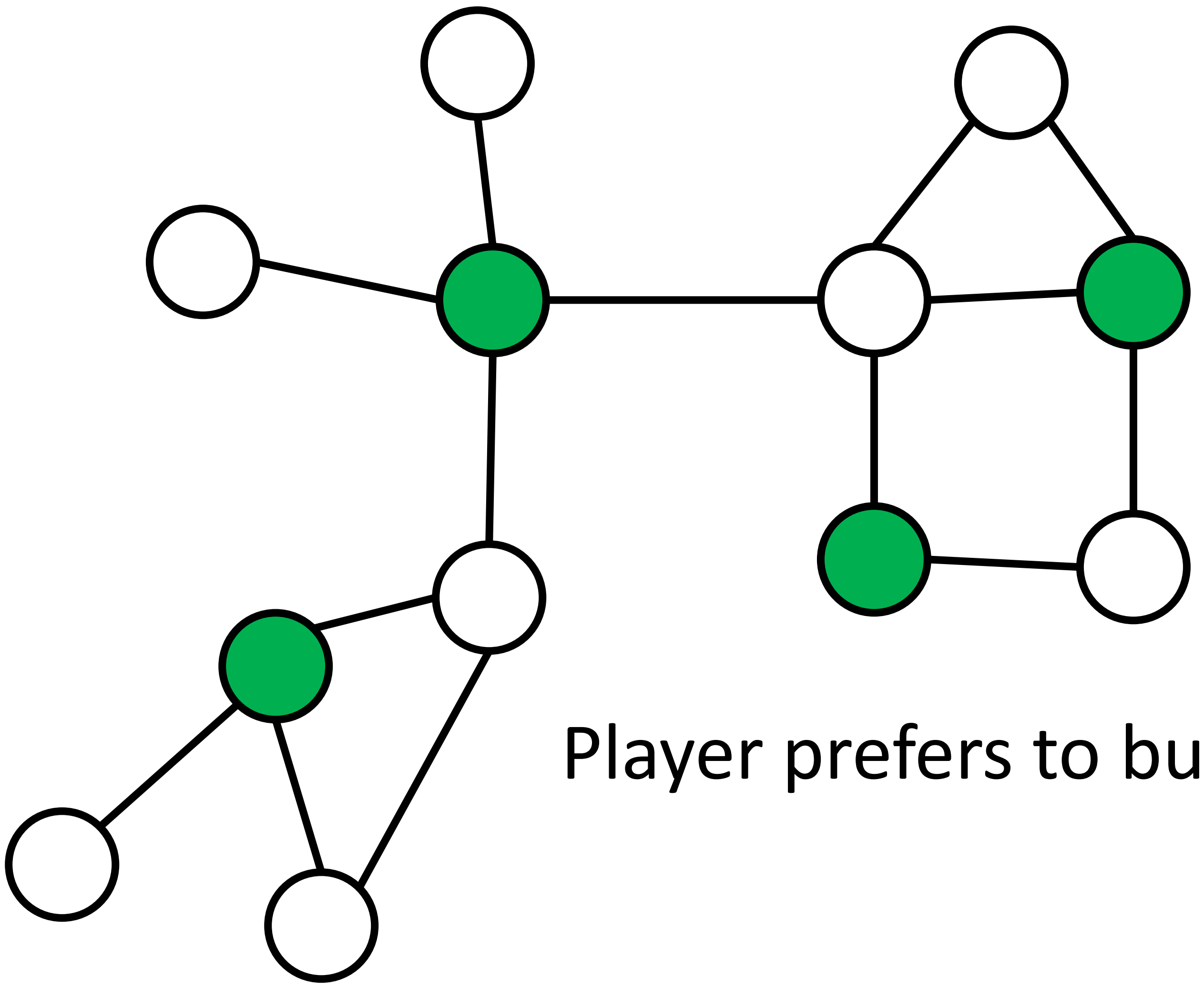

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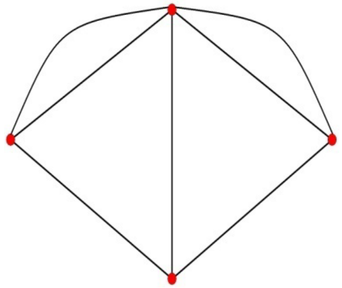
Example: Best-Shot

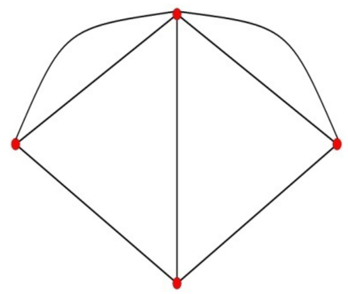


- agent i is willing to choose 1 if and only if no neighbors do:

- Payoff action 0: $u_{d_i}(0, m_{N_i}) = 1$ if $m_{N_i} > 0$
  $= 0$ if $m_{N_i} = 0$

- Payoff action 1: $u_{d_i}(1, m_{N_i}) = 1 - c$

Example: Best-Shot Maximal Independent Sets

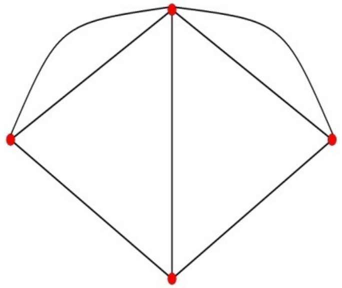


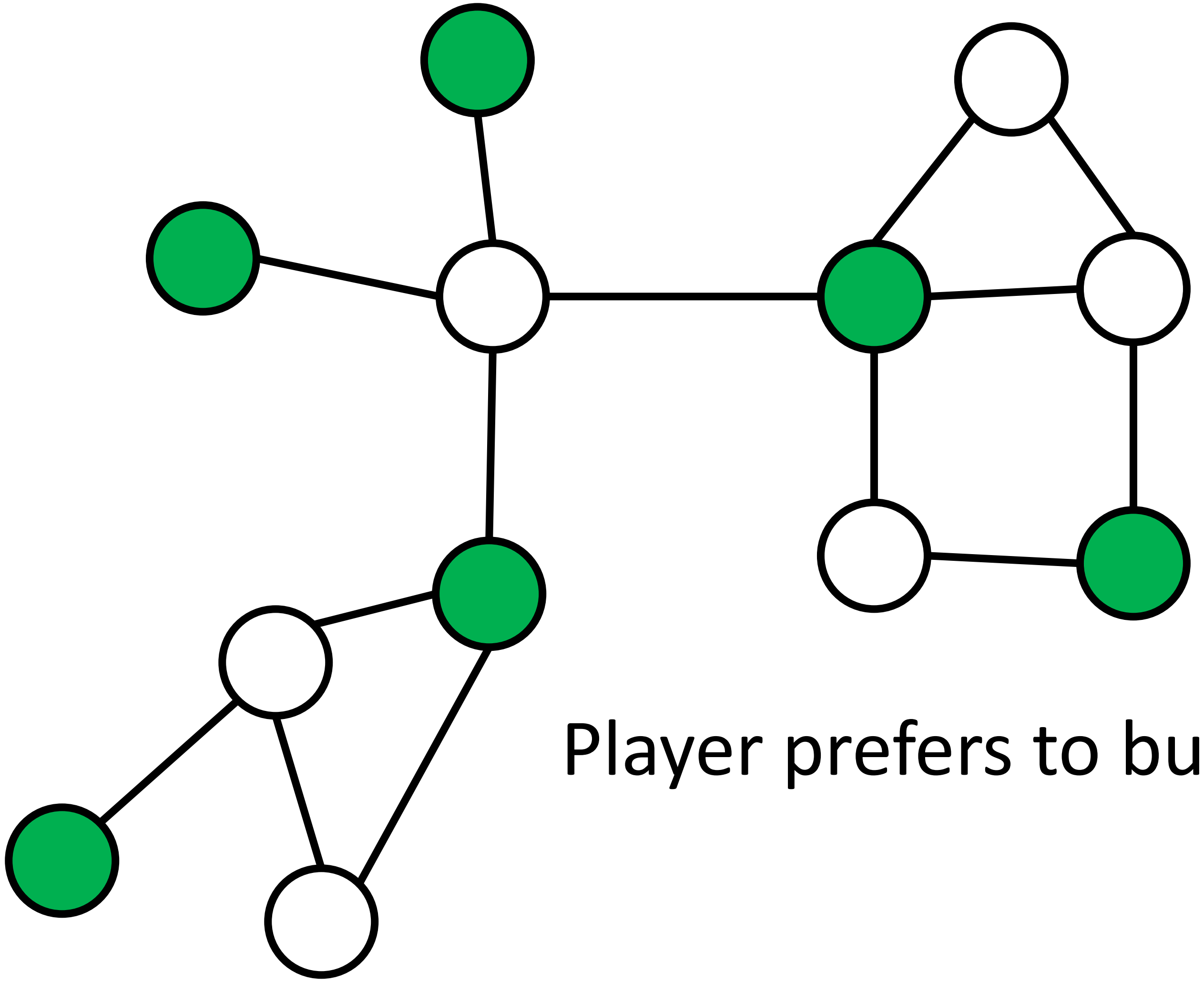
- Independent Set: a set S of nodes such that no two nodes in S are linked,

- Maximal: every node in N is either in S or linked to a node in S

- Equilibria: Adopters = a maximal independent set

Another Example: Best-Shot Public Goods





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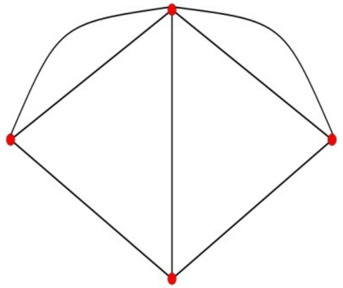
Games on Networks - Outline



- Basic Definitions
- Examples
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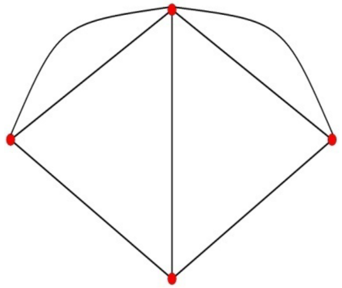
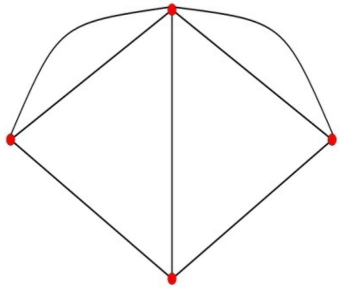
Externalities:



- Others' behaviors affect one's **utility/welfare**

- Others' behaviors affect one's ***decisions, actions, consumptions, opinions...***
  – others' actions affect the ***relative*** payoffs to one's behaviors

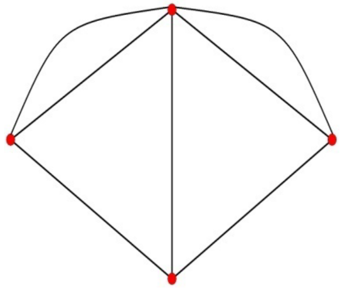
Strategic Substitutes



- b = benefit of a book/etc
- c = cost b > c

	Friend buys	Friend does not
Buy	b - c	b - c
Not	b	0

Strategic Substitutes



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externality

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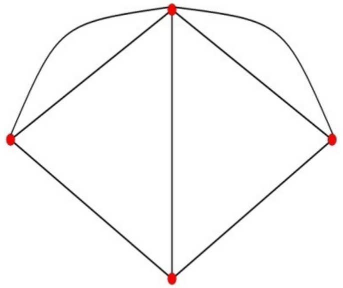
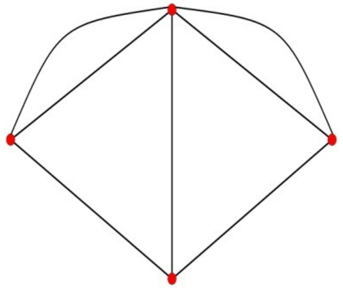


- Others’ behaviors affect one’s **utility/welfare**

- Game theory: others’ behaviors affect one’s ***decisions, actions, consumptions, opinions…***
  - others’ actions affect the ***relative*** payoffs to one’s behaviors

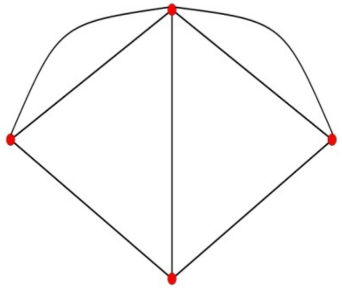
Strategic Substitutes



- b = benefit of a book/cd/etc.
- c = cost b> c

	Friend has	Friend does not
Buy	b - c	**b - c**
	↓	↑
Not	**b**	0

Strategic Complements

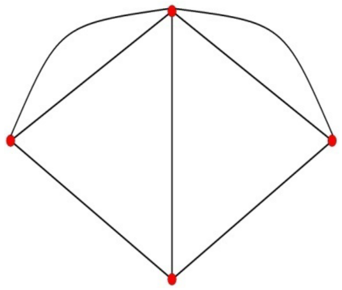


- b = benefit of playing game with friend
- c = cost of learning to play

	Friend plays	Friend do not
Play	b - c	- c
Not	0	0

externality (arrows point to "b" under Friend plays and to the empty place before "- c" under Friend do not)

Strategic Complements



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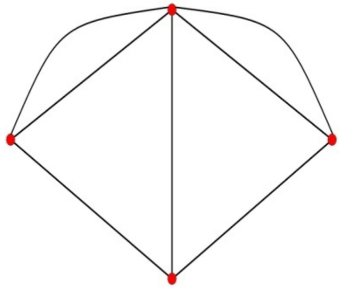
- c = cost of learning to play

	Friend plays	Friend do not
Play	**b - c**	- c
	↑	↓
Not	0	**0**

Strategic Complements

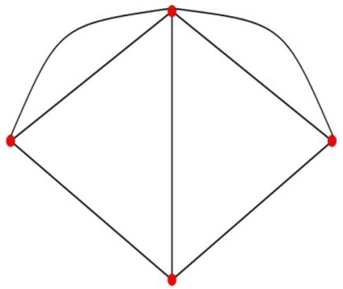


- with negative externality, e.g., doping:

	Competitor dopes	Competitor does not
Dope	- c	b - c
Not	- L	0

negative externality

Strategic Substitutes

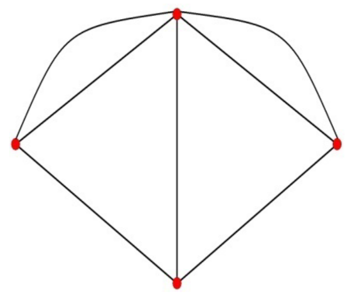


- with negative externality, e.g., congestion game:

	Other shows	Other not
Show	- c	b - c
Not	0	0

negative externality

Strategic Complements/Substitutes



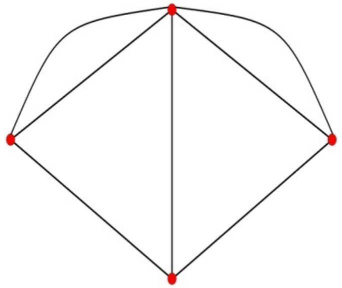
- strategic complements -- for all d, $m \geq m'$
  - Increasing differences:

$u_d(1,m) - u_d(0,m) \geq u_d(1,m') - u_d(0,m')$

- strategic substitutes -- for all d, $m \geq m'$
  - Decreasing differences:

$u_d(1,m) - u_d(0,m) \leq u_d(1,m') - u_d(0,m')$

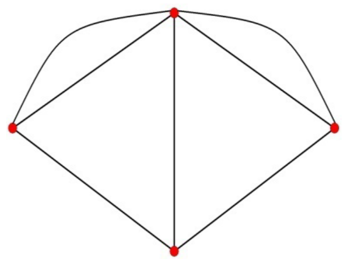
Strategic Complements/Substitutes



- **Complements:** Choice to take an action by my friends increases my relative payoff to taking that action (e.g., friend learns to play a video game)

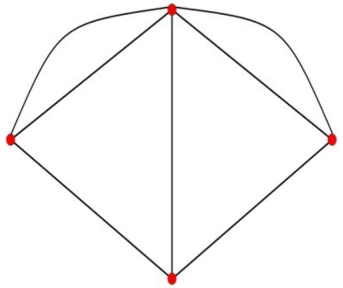
- **Substitutes:** Choice to take an action by my friends decreases my relative payoff to taking that action (e.g., roommate buys a stereo/fridge)

Examples



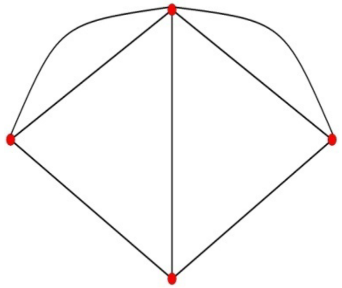
- Complements:
  - smoking & other behavior among teens, peers, ...
  - technology adoption – care about fraction others compatible...
  - educate/drop out work force
  - learn a language
  - corruption, crime
  - cheating, doping
- Substitutes
  - information gathering
  - local public goods (shareable products...)
  - competing firms (oligopoly with local markets)
  - vaccinations (near herd immunity)...

Useful Observation



- Complements: there is a threshold t(d), such that i prefers 1 if $m_{N_i} > t(d)$ and 0 if $m_{N_i} < t(d)$
- Substitutes: there is a threshold t(d), such that i prefers 1 if $m_{N_i} < t(d)$ and 0 if $m_{N_i} > t(d)$
- Can be indifferent at the threshold

Games on Networks - Outline



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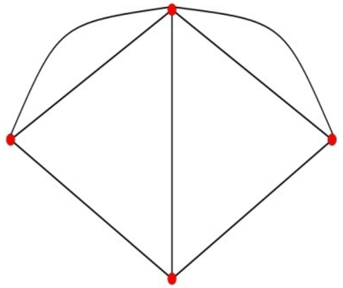
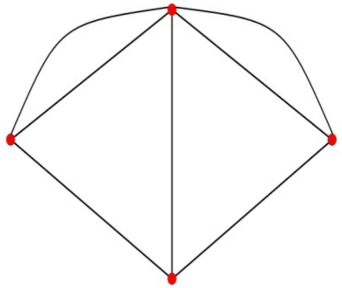
Equilibrium



- Nash equilibrium: Every player's action is optimal for that player given the actions of others
- Often look for pure strategy equilibria
- May require some mixing in case of substitutes

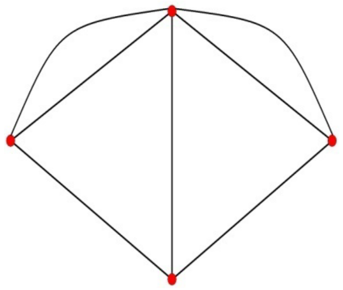
Proposition



In a game on a network of strategic complements where the individual strategy sets are complete lattices:

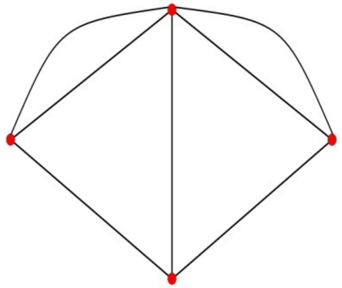
the set of pure strategy equilibria are a (nonempty) complete lattice.

Complete lattice



- Complete Lattice: for every set of equilibria X
  - there exists an equilibrium x’ such that x’ ≥ x for all x in X, and
  - there exists an equilibrium x’’ such that x’’ ≤ x for all x in X.

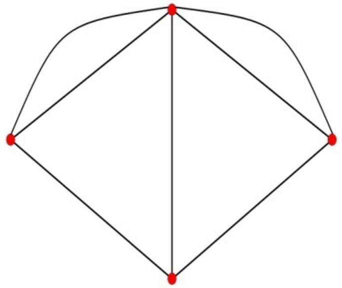
Contrast: Complements and Substitutes



- In a game of complements: pure strategy equilibria are a (nonempty) complete lattice

- In a game of strategic substitutes:
  - Best shot game: pure strategy equilibria exist and are related to maximal independent sets
  - Others: pure strategy may not exist, but mixed will (with finite action spaces, or appropriate measure spaces)
  - Equilibria often do not form a lattice

When can multiple actions be sustained:

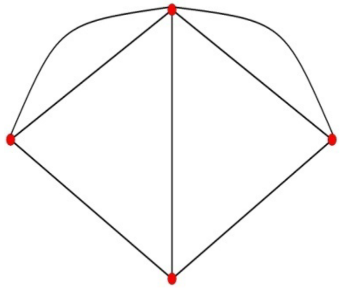


- Coordination game

- Care about fraction of neighbors taking action 1:

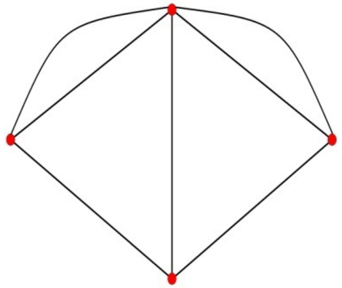
  prefer to take action 1 if fraction q or more take 1

Equilibrium Structure



- Let S be the group that take action 1
- Each i in S must have fraction of at least q neighbors in S
- Each i not in S must have a fraction of at least 1-q neighbors outside of S

Cohesion

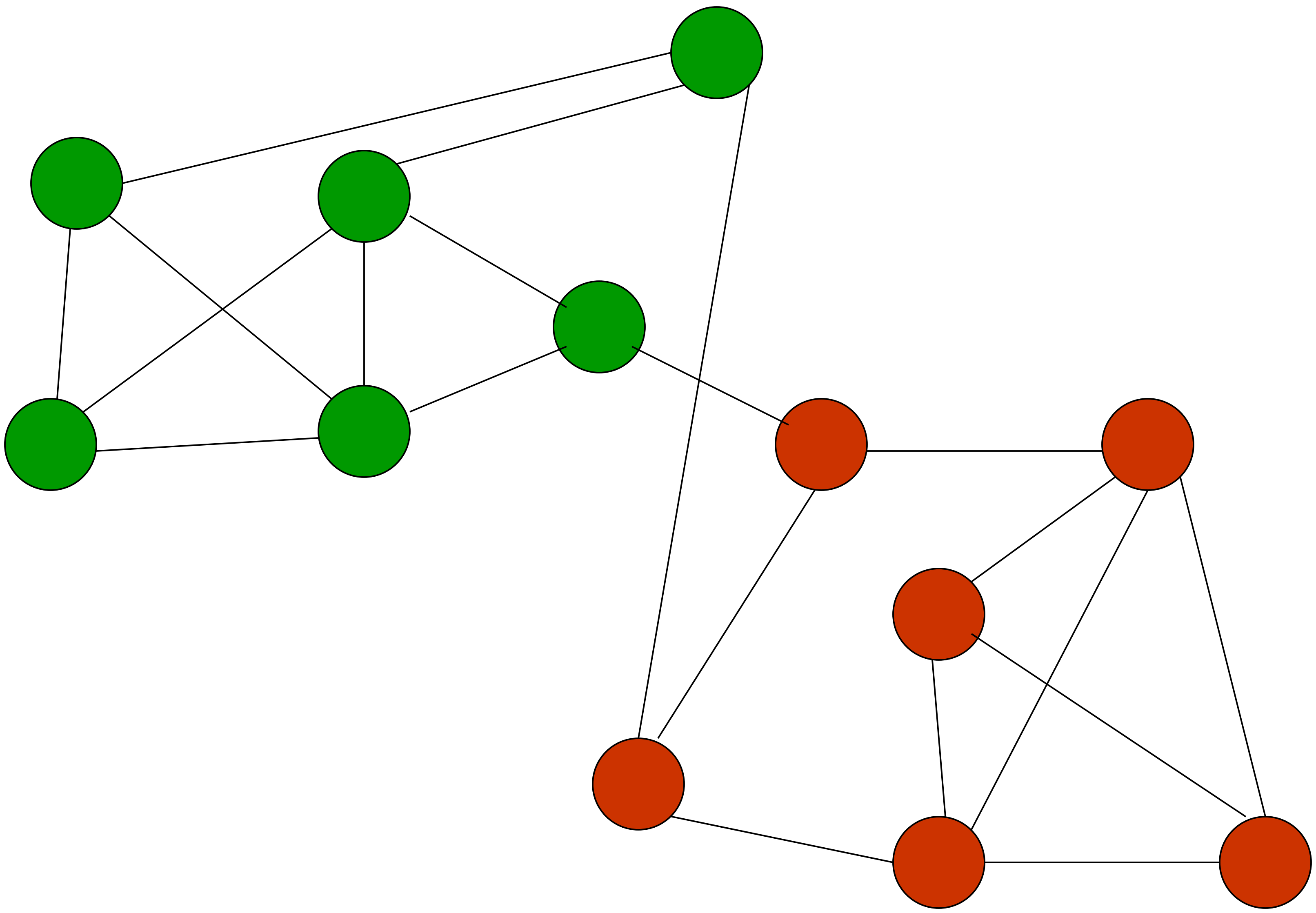


Morris 2000: A group S is r-cohesive relative to g if

$$\min_{i \in S} |N_i(g) \cap S| / d_i(g) \geq r$$

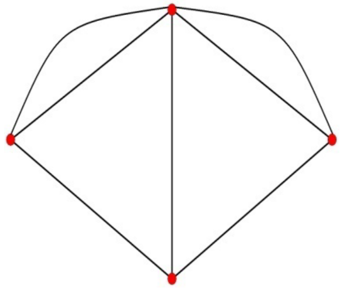
At least a fraction r of each member of S's neighbors are in S

Cohesiveness of S is $\min_{i \in S} |N_i(g) \cap S| / d_i(g)$



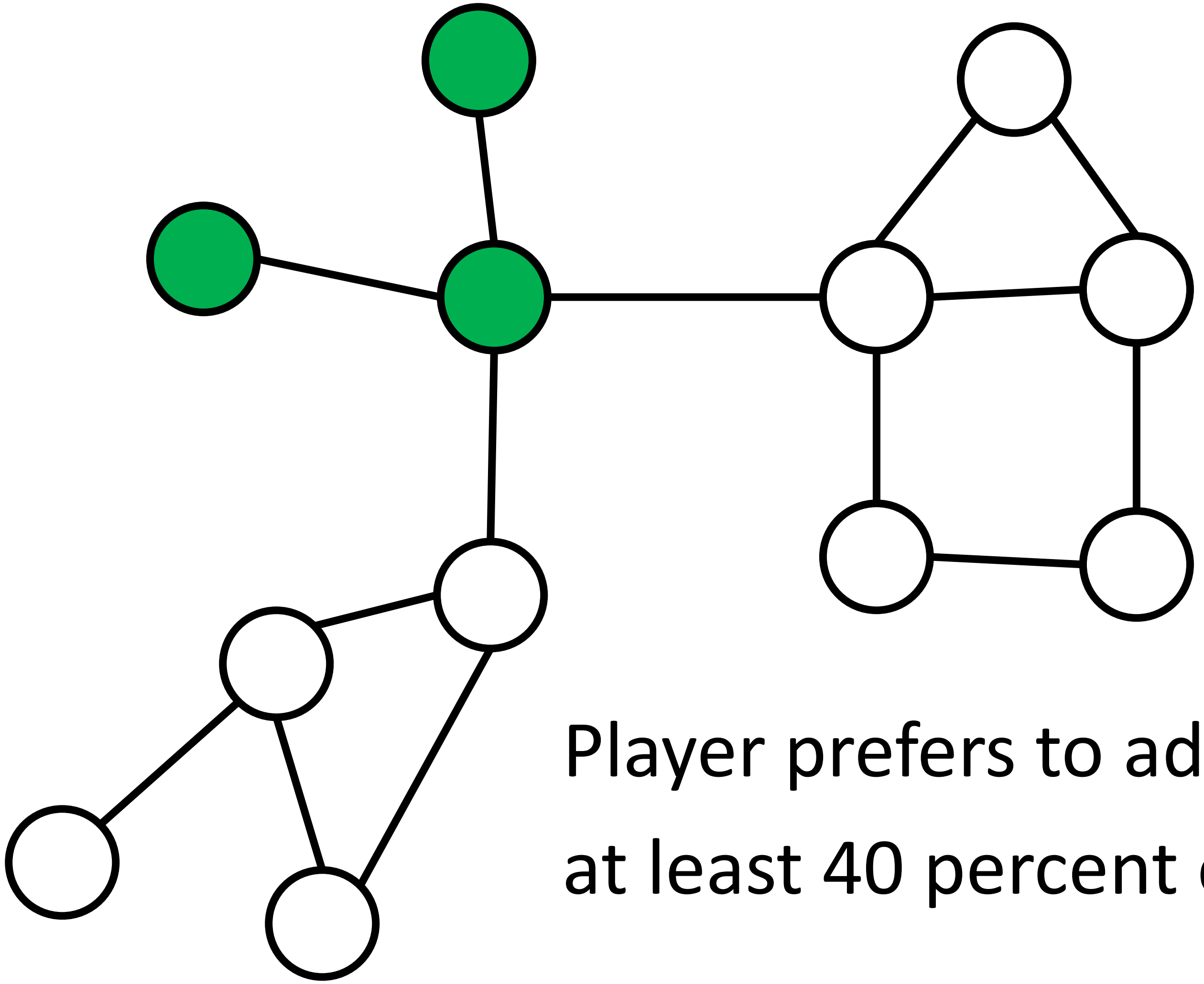
Both groups are 2/3 cohesive

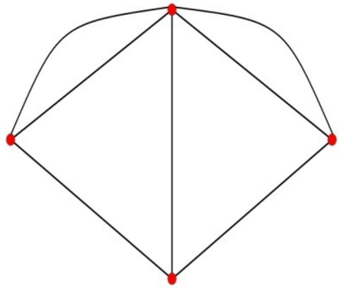
Equilibria where both strategies are played:



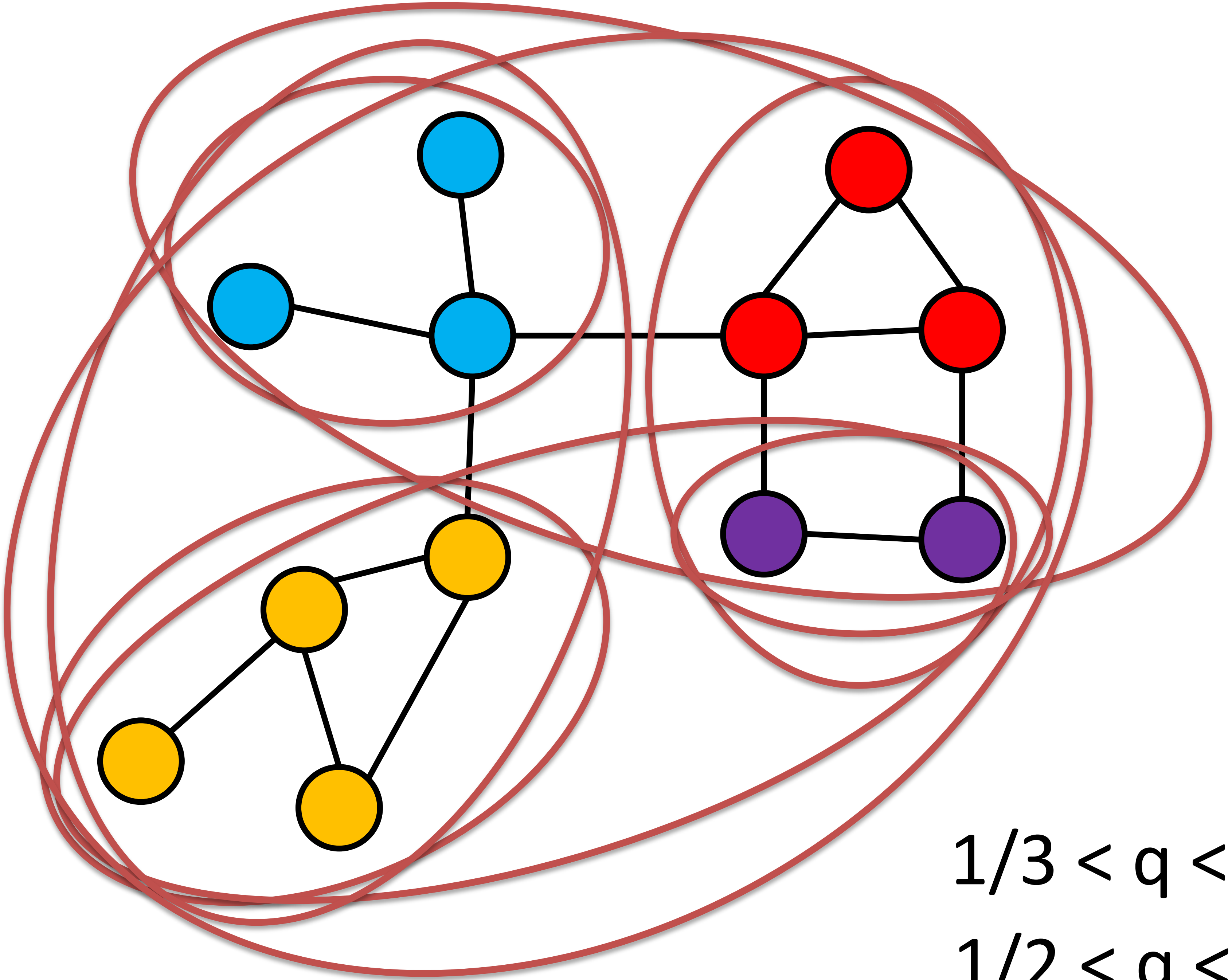
There exists a pure strategy equilibrium where both actions are played if and only if there is a group S that is at least q cohesive and such that its complement is at least 1-q cohesive.

Example:



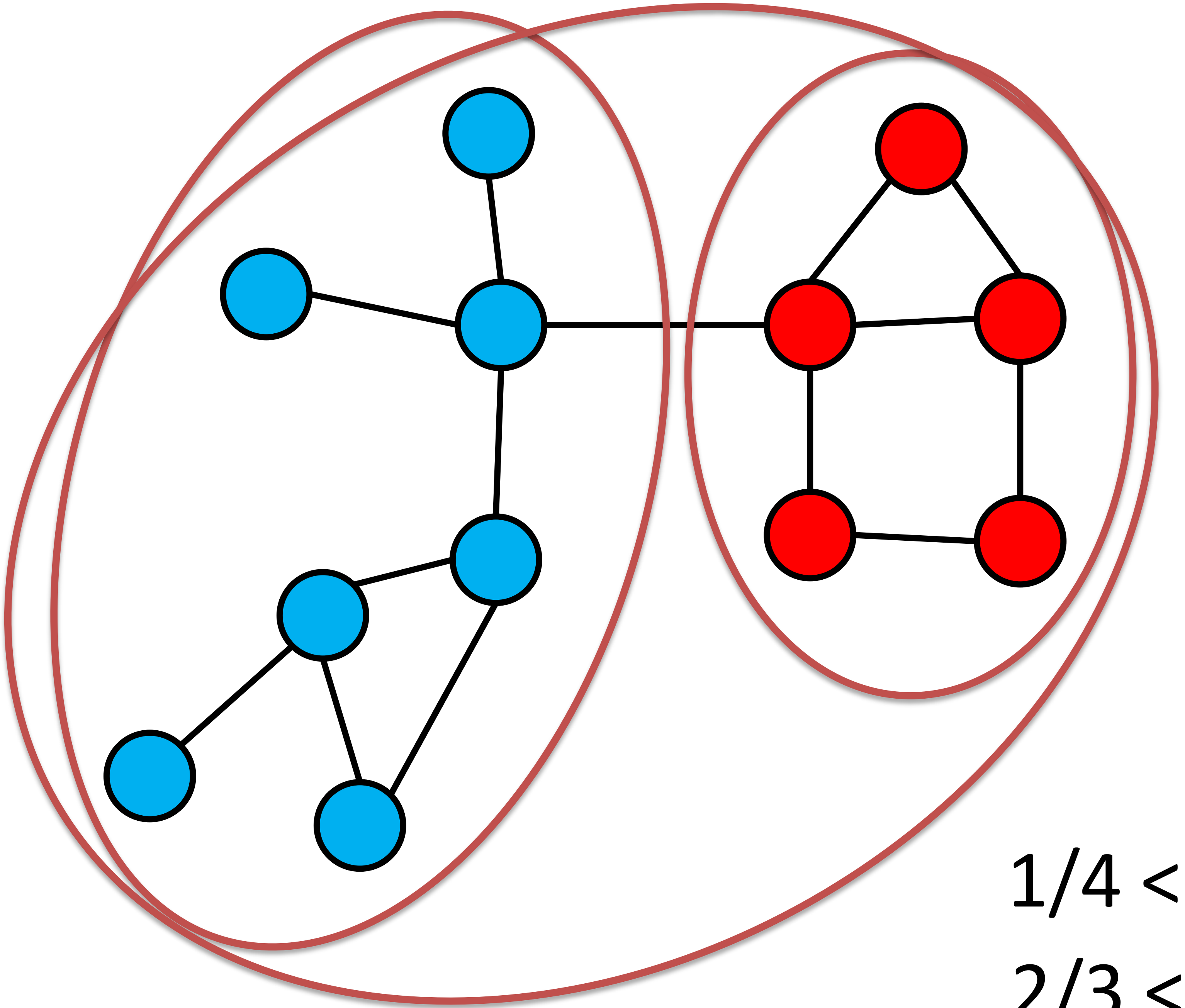


Player prefers to adopt new technology if at least 40 percent of neighbors do



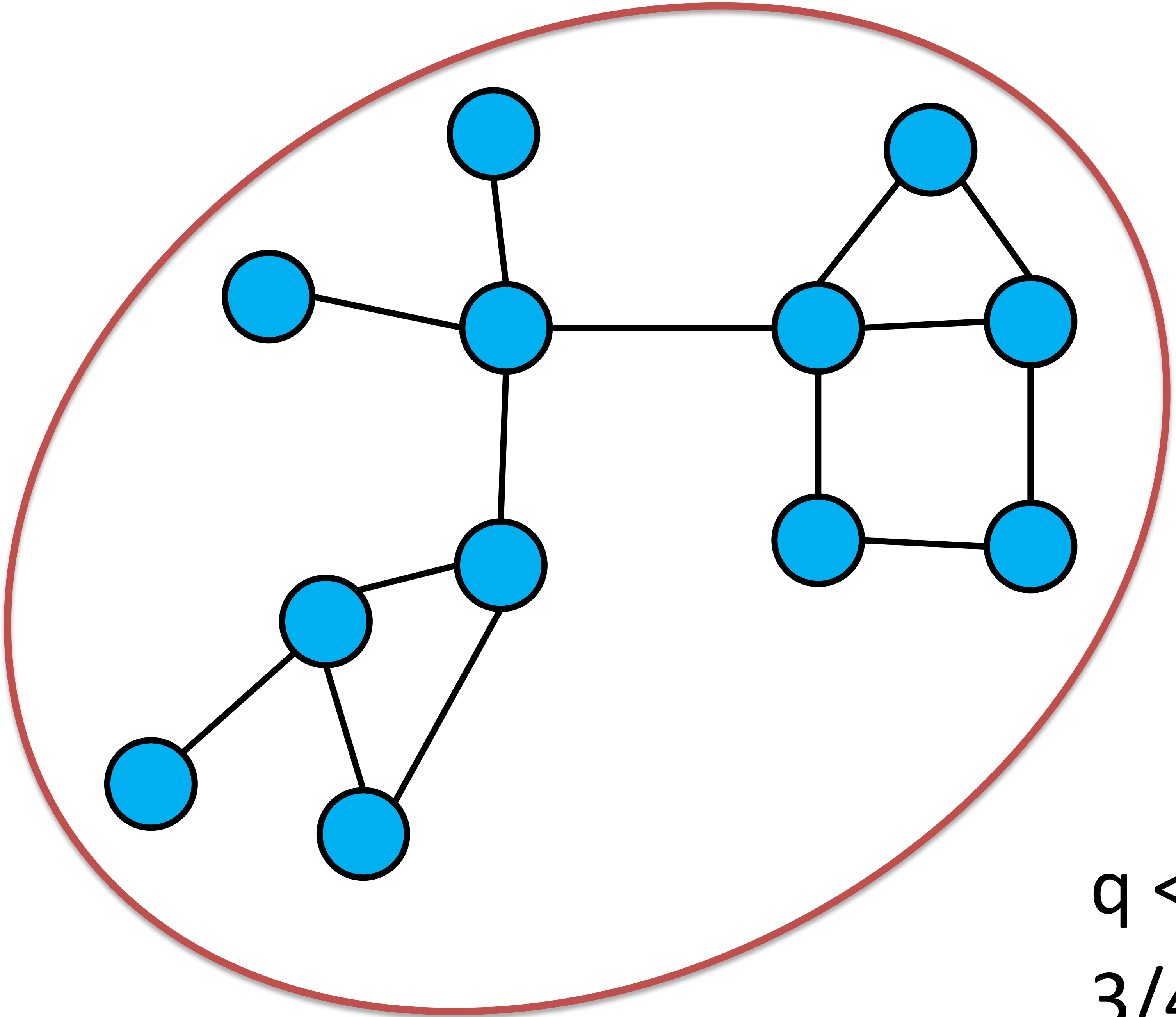
1/3 < q < 1/2

1/2 < q < 2/3



$1/4 < q < 1/3$

$2/3 < q < 3/4$



$q < 1/4$

$3/4 < q$

Equilibria in Large SBM

Growing block models: blocks b in B(n)

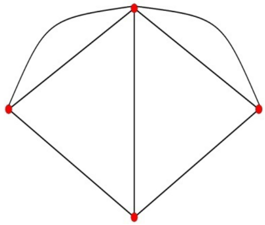
Probability of linking nodes from blocks b, b' is $p_{bb'}(n)$

expected degree of node in b to nodes in b' $d_{bb'}(n)$

overall expected degree of node in b $d_b(n)$

($> (1 + \varepsilon) \log(n)$ for all b, n)

Equilibria in Large SBM



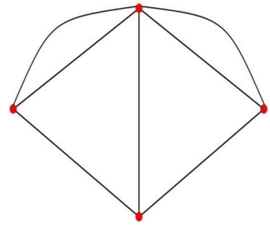
Convergent growing block models:

$|B(n)| = k$ for large k

$d_{bb'}(n)/d_b(n)$ converges for all b, b'

Weakly homophilous: $d_{bb}(n)/d_b(n) > d_{b'b}(n)/d_{b'}(n) + \varepsilon$

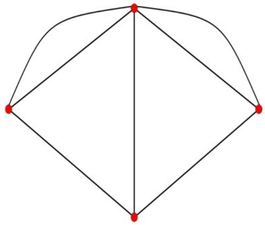
Theorem:
Equilibria in Large SBM (J-S22)



Consider a growing sequence of stochastic block networks.

- Any sequence of sets of adopters that are equilibria for some open set of q, are a superset of the blocks with a probability going to 1.

- If the sequence of block models is convergent and weakly homophilous, then there exists some open set of q, for which any given block is an equilibrium for those q with a probability going to 1.



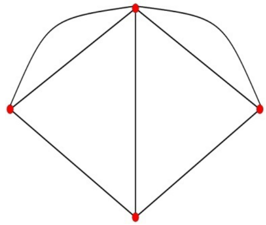
Theorem:
Equilibria in Large SBM (J-S22)

Proof ideas

Thm by McDiarmid, Skerman 2018 – modularity of G(n,p) goes to 0

Relate modularity to equilibrium structure: if equilibrium splits some block, then modularity of that block has lower bound.

Other results



- Community structures: equilibria define groups of people whose behaviors are always tied, communities differ based on behavior (q)

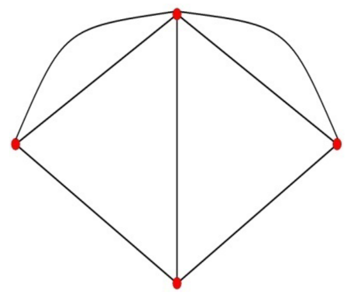
- Seeding: communities help for seeding

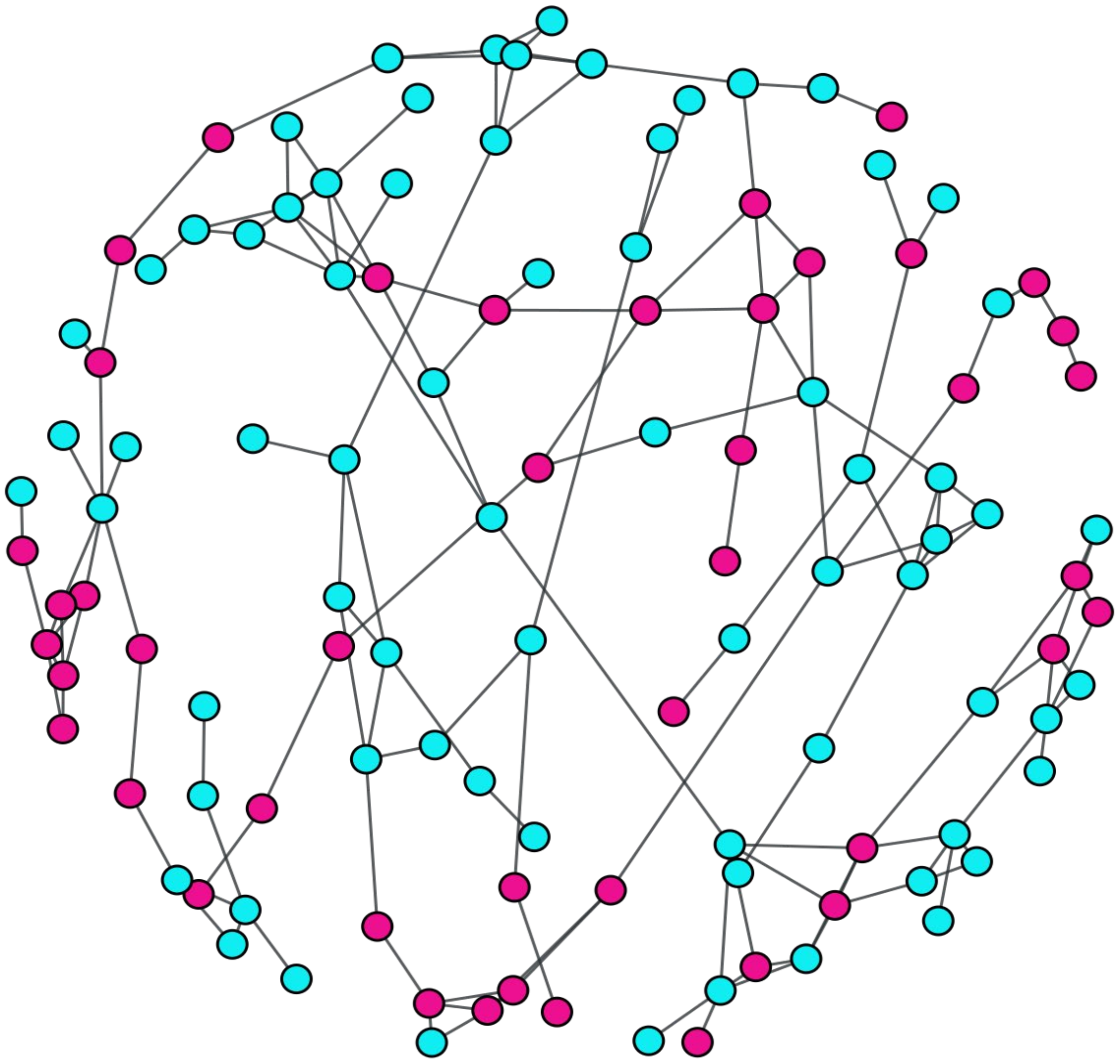
- Complex contagion differs from simple: clustering needed for diffusion

- Equilibria can be ordered by degree distributions in random networks

(Bayesian games, mean field games, graphon games)

Estimate q from data…





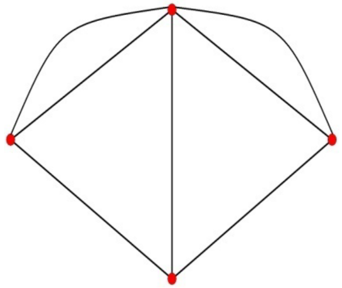
12th grade smoking, add health data

Estimate q presuming equilibrium: q=.39

mis-predict 29% of behavior

Jackson-Storms 2019

Intensity of an Action



Each person chooses a level of behavior x_i:

level of criminal activity

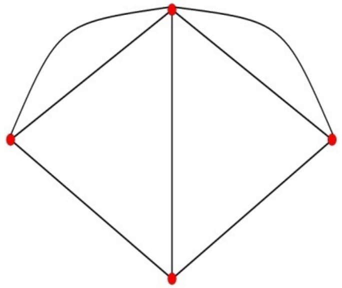
how fast drive

how long stay in school

how much study

effort spent legislating

A Linear-Quadratic Model



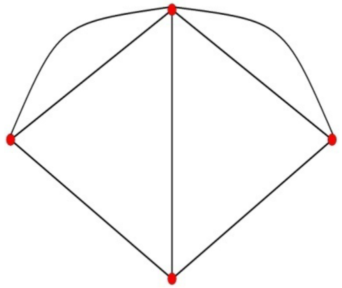
g_{ij} intensity of connection from i to j: how much i is influenced by what j does

can be weighted and directed

$u_i(x_i,x_{-i}) = a\, x_i - c\, x_i^2/2 + b\, \Sigma_j\, g_{ij}\, x_i\, x_j$

Ballester, Calvo-Armengol and Zenou (2006)

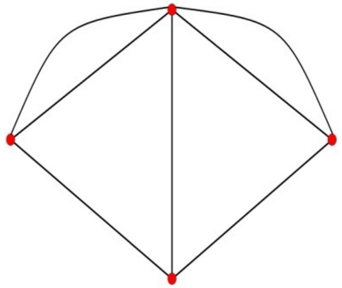
A Linear-Quadratic Model



$$u_i(x_i,x_{-i}) = a\,x_i - c\,x_i^2/2 + b\,\Sigma_j\, g_{ij}\, x_i\, x_j$$

the direct benefit of x_i

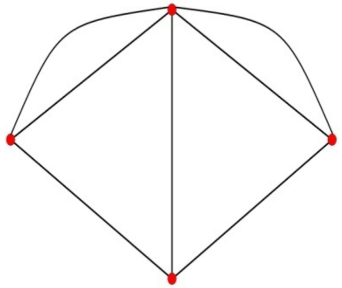
A Linear-Quadratic Model



$$u_i(x_i, x_{-i}) = a\, x_i - c\, x_i^2/2 + b\, \Sigma_j\, g_{ij}\, x_i\, x_j$$

the cost of x_i
convex – higher
marginal costs as
increase x_i

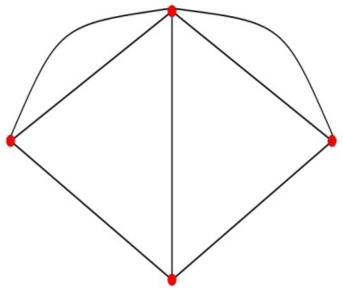
A Linear-Quadratic Model



$$u_i(x_i, x_{-i}) = a\, x_i - c\, x_i^2/2 + b\, \Sigma_j\, g_{ij}\, x_i\, x_j$$

interaction effect:
the higher x_j and the higher g_{ij}, the more i benefits from increasing x_i

A Linear-Quadratic Model

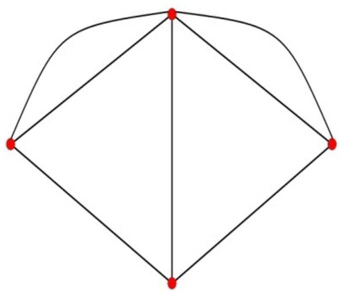


$u_i(x_i,x_{-i}) = a\, x_i - c\, x_i^2/2 + b\, \Sigma_j\, g_{ij}\, x_i\, x_j$

Maximize this function

the best response of x_i to x_{-i}:

A Linear-Quadratic Model



$u_i(x_i,x_{-i}) = a\, x_i - c\, x_i^2/2 + b\, \Sigma_j\, g_{ij}\, x_i\, x_j$

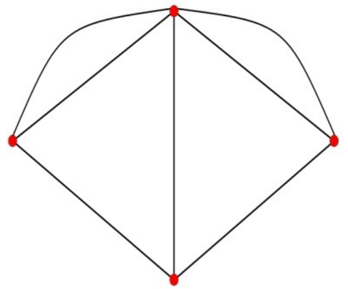
Maximize this function

the best response of x_i to x_{-i}:

$$a - c\, x_i + b\, \Sigma_j\, g_{ij}\, x_j = 0$$

$$x_i = (a + b\, \Sigma_j\, g_{ij}\, x_j)/c$$

A Linear-Quadratic Model

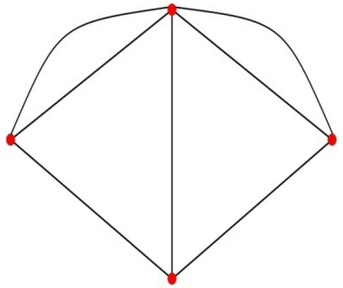


$x_i = (a + b \Sigma_j \, g_{ij} \, x_j)/c$

in matrix form: $\mathbf{x} = \mathbf{A} + \mathbf{G}\,\mathbf{x}$

where $\mathbf{A} = (a/c, \ldots, a/c)$, $\mathbf{G}_{ij} = b\, g_{ij}/c$

A Linear-Quadratic Model



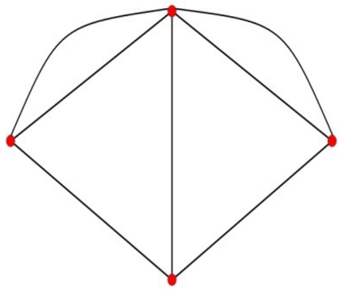
$\mathbf{x} = \mathbf{A} + \mathbf{G}\,\mathbf{x}$

or $\mathbf{x} = \mathbf{A} + \mathbf{G}\,(\mathbf{A} + \mathbf{G}\,(\mathbf{A} + \mathbf{G}\ldots)))\ = \ \Sigma_{k\geq 0}\ \mathbf{G}^{k}\,\mathbf{A}$

or $\mathbf{x} = (\mathbf{I} - \mathbf{G})^{-1}\,\mathbf{A}$ if invertible

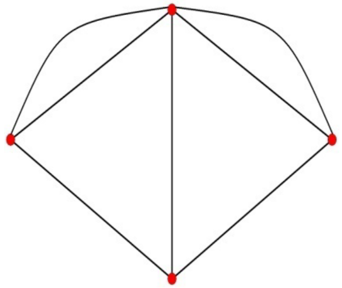
(or if a=0, then $\mathbf{x}=\mathbf{G}\mathbf{x}$, so unit eigenvector)

A Linear-Quadratic Model



- Actions are related to network structure:
  - higher neighbors' actions, higher own action
  - higher own action, higher neighbors actions
  - feedback – for solution need b/c to be small and/or g_{ij}'s to be small (need $\Sigma_{k \geq 0}$ $\mathbf{G}^k$ to converge)

A Linear-Quadratic Model



- Relation to centrality measures:

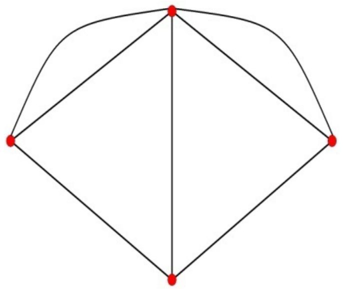
$$\mathbf{x} = \Sigma_{k\geq 0}\ \mathbf{G}^k \mathbf{A} = \Sigma_{k\geq 0}\ \mathbf{G}^k \mathbf{1}\ (a/c) = (\mathbf{1} + \Sigma_{k\geq 1}\ \mathbf{G}^k \mathbf{1})(a/c)$$

Katz-Bonacich centrality:

$$B(\mathbf{g}) = \Sigma_{k\geq 1}\ \mathbf{g}^k \mathbf{1}$$

So, $\mathbf{x} = (\mathbf{1} + B(\mathbf{G}))(a/c)$

A Linear-Quadratic Model

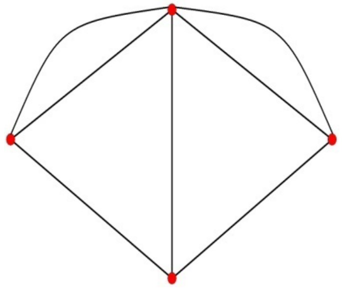


- Natural feedback, actions relate to the total feedback from various positions

- Capture network in tractable manner

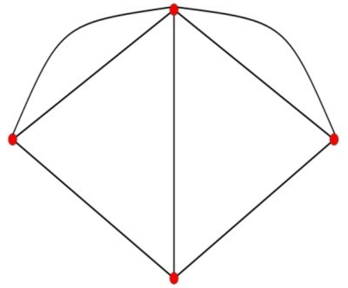
- Centrality: relative number of weighted influences going from one node to another

Applications of Model:



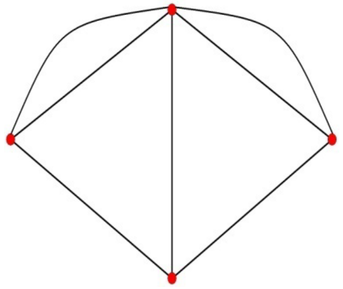
- criminal behavior, delinquency (Patacchini, Zenou 12)
- study habits (Calvo, Patacchini, Zenou 09)
- political efforts, party divisions (Canen, Jackson, Trebbi 22)
- corporate control (Vitali et al 11, Larcker et al 13)
- drug trafficking (Dell 15)
- friendship paradox and teen behavior (Jackson 19)

Application to Student Performance



- Calvo-Armengol, Patacchini, Zenou (2009) applied this to Add Health data
- Let x_i be how hard a student studies
- Measure this by academic performance (a factor analysis of survey answers and grades)
- Estimate b/c, see how much centrality matters in determining academic performance (w controls)

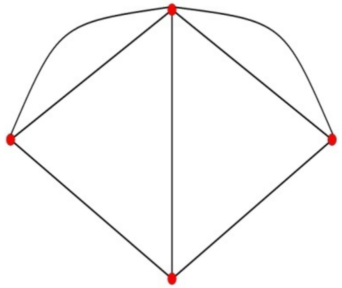
Estimates



- Estimate b/c to be .55

- Find a SD increase in Bonacich centrality increases performance by 7%

Games on Networks

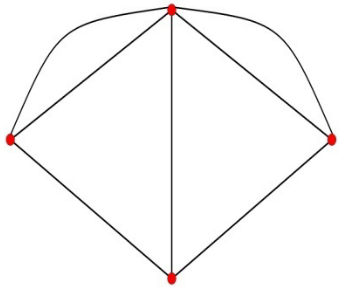


- *Many* applications

- Externalities make the analysis important – individual incentives and societal welfare diverge

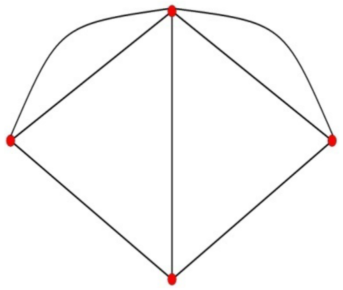
- Networks have systematic features that matter in ways that can be quantified

Network Formation Models



- Random models
  - *How* networks form
- Game theoretic/strategic models:
  - *Why* specific networks form
    - prefer to connect *because* someone is well connected
    - high clustering because lower cost for nearby connections
    - small worlds because value to bridges
  - Welfare analyses, inefficiencies, externalities, *policies*…

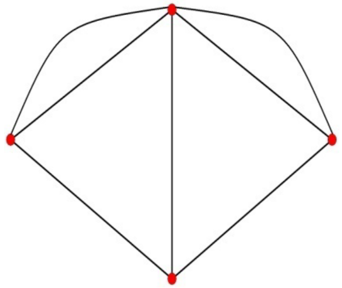
Questions:



- Which networks are formed by the people/nodes?

- Which networks are best for society?

An `Economic’ Analysis:



- Choose connections
  - benefits from connections
  - costs to maintaining relationships
  - time limits…

- Care about direct friendships, but also about indirect friendships
  - follow someone on media because they are connected…

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