Lattice problems are (sort of) equivalent in all norms (and the mysterious wiggle)

Frederick Eisenbrand   Moritz Venzin

Divesh Aggarwal   Yanlin Chen   Rajendra Kumar

Zeyong Li   Noah Stephens-Davidowitz

Thomas Rothvoss   Moritz Venzin

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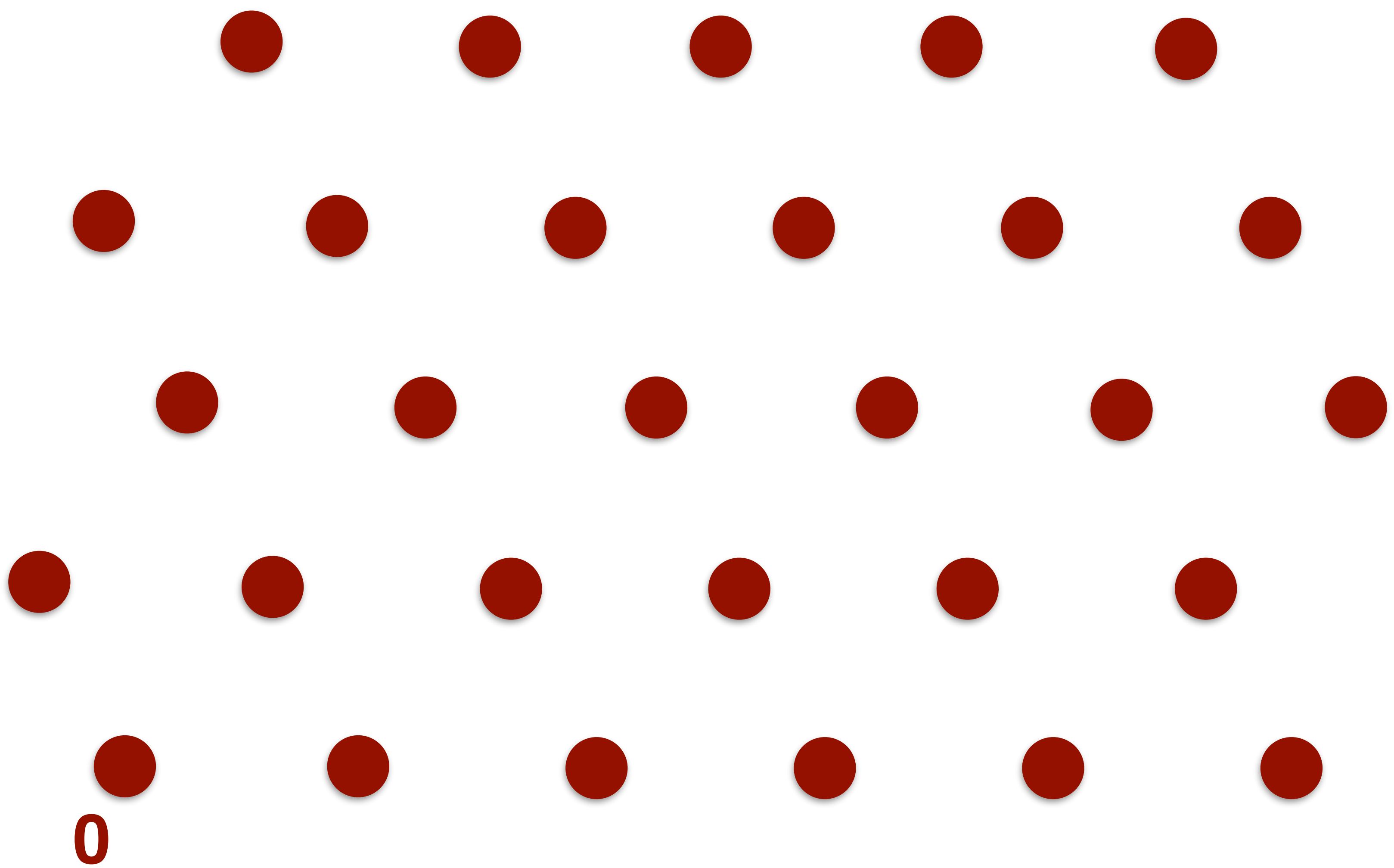

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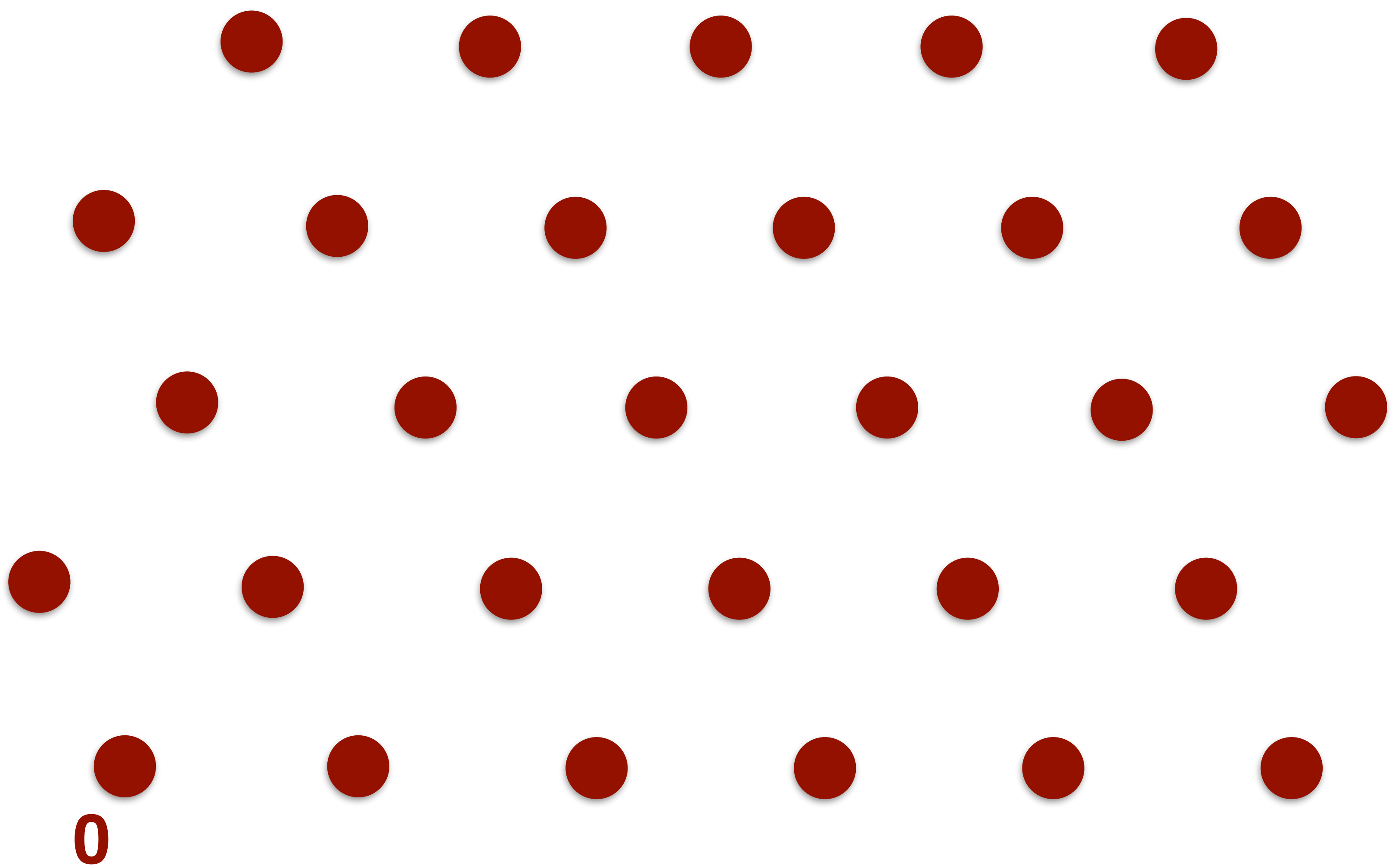

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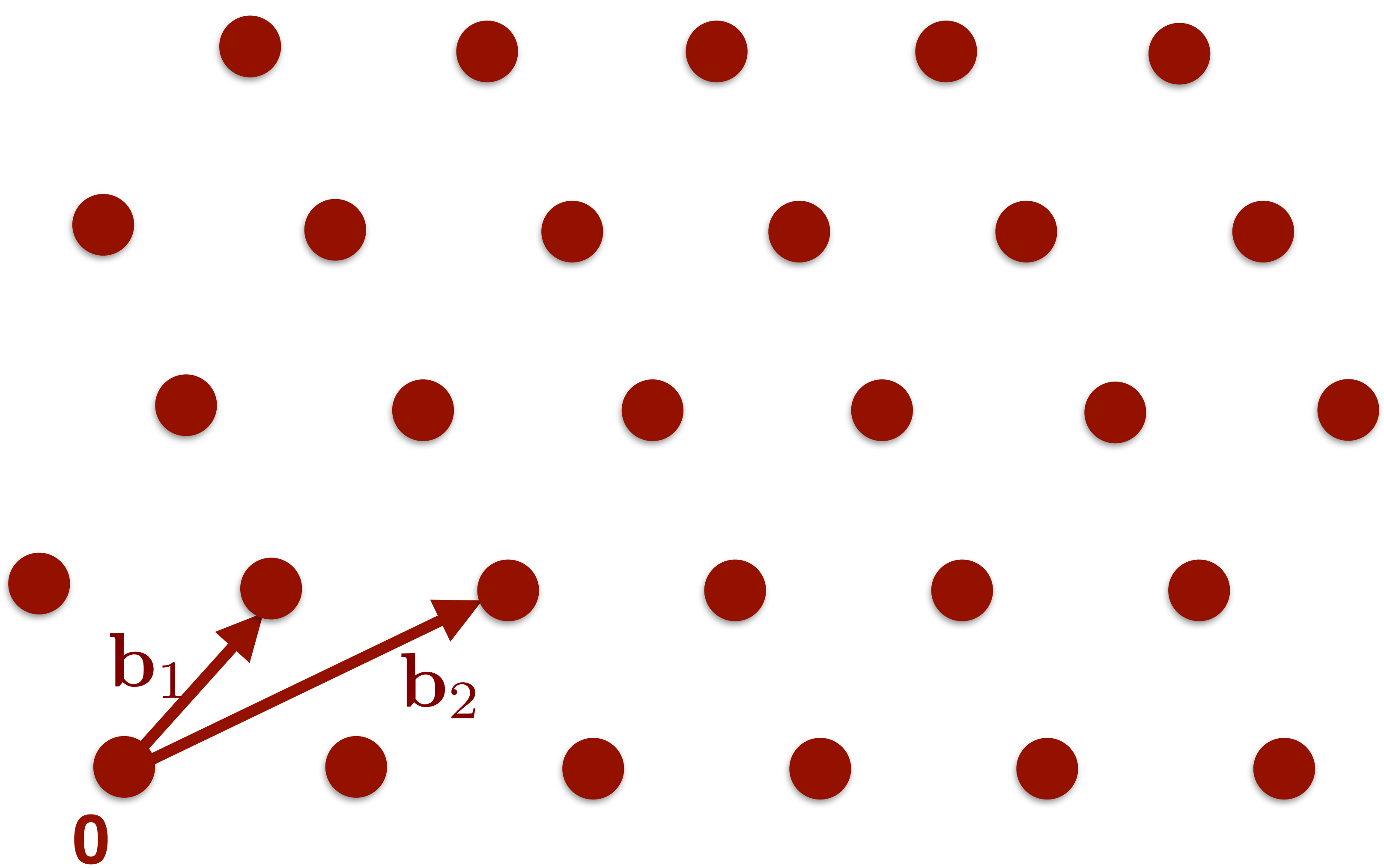

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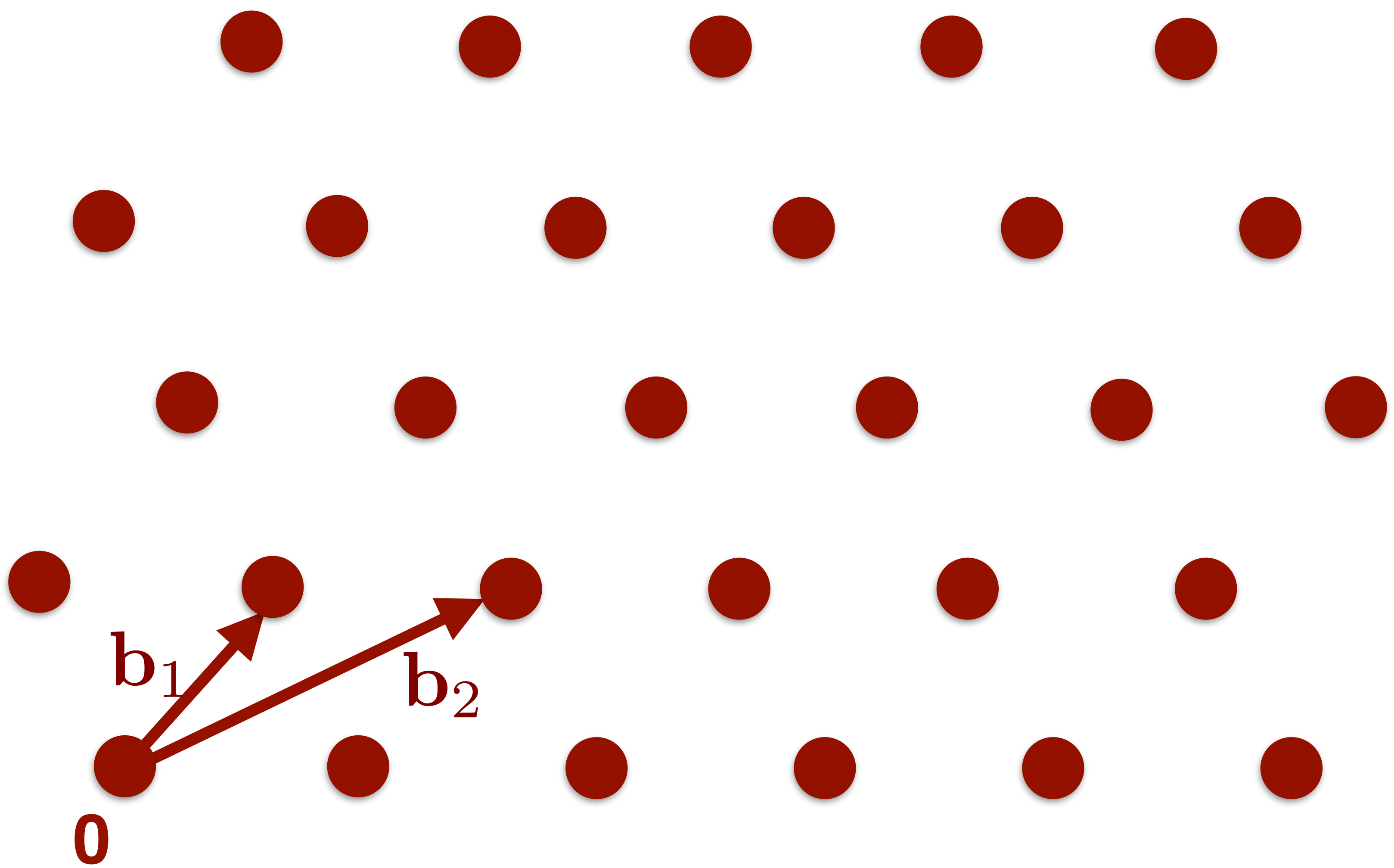

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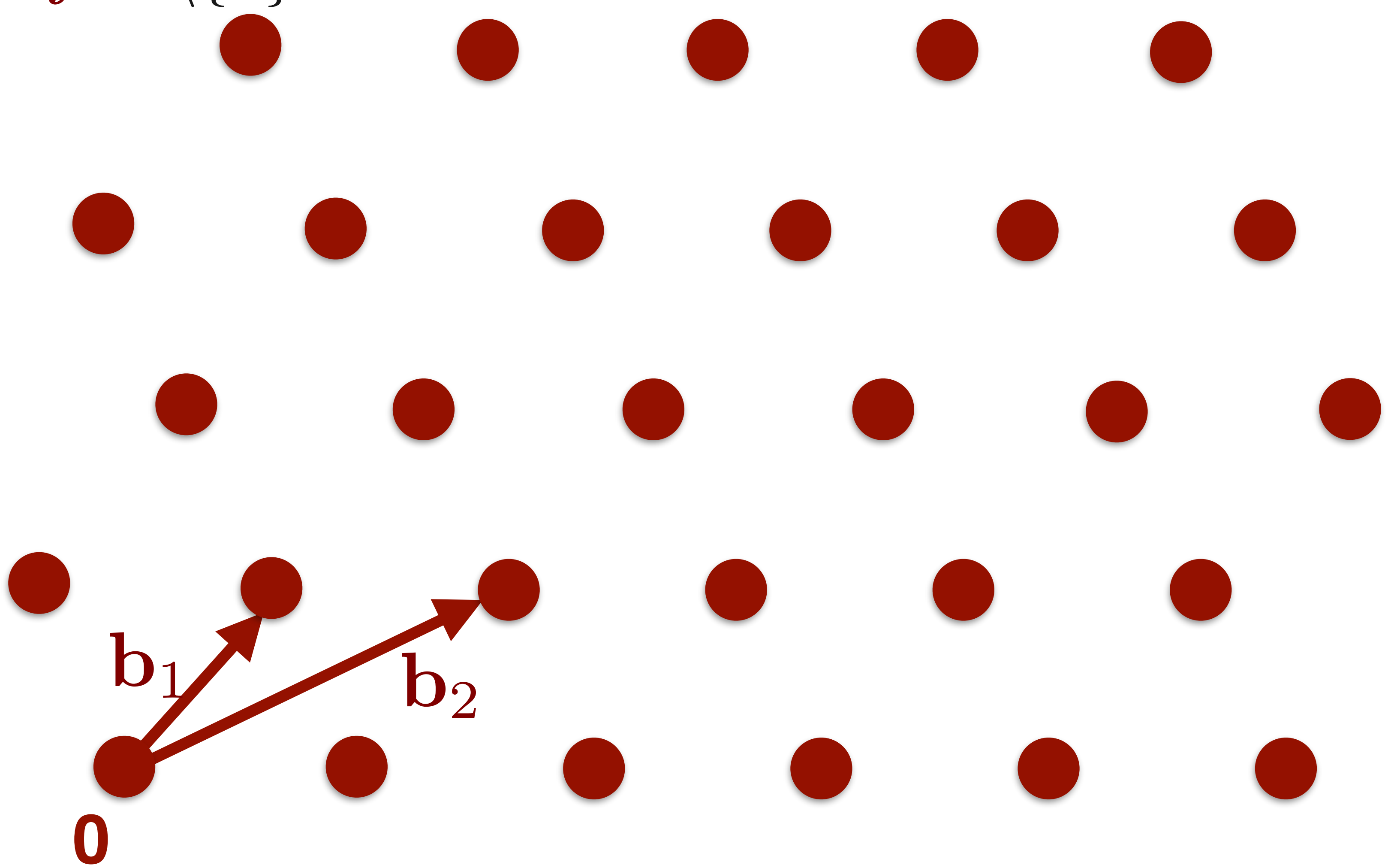

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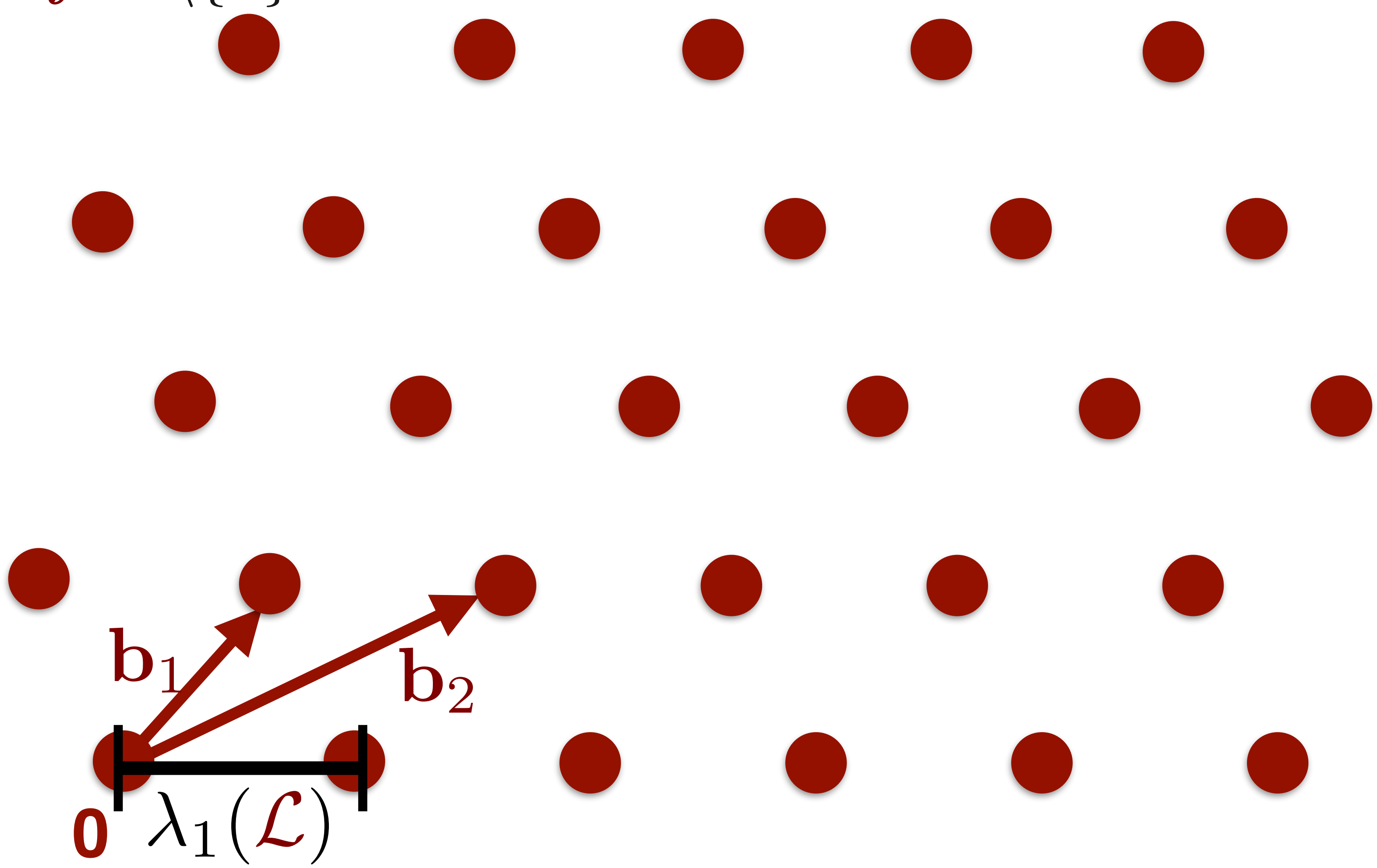

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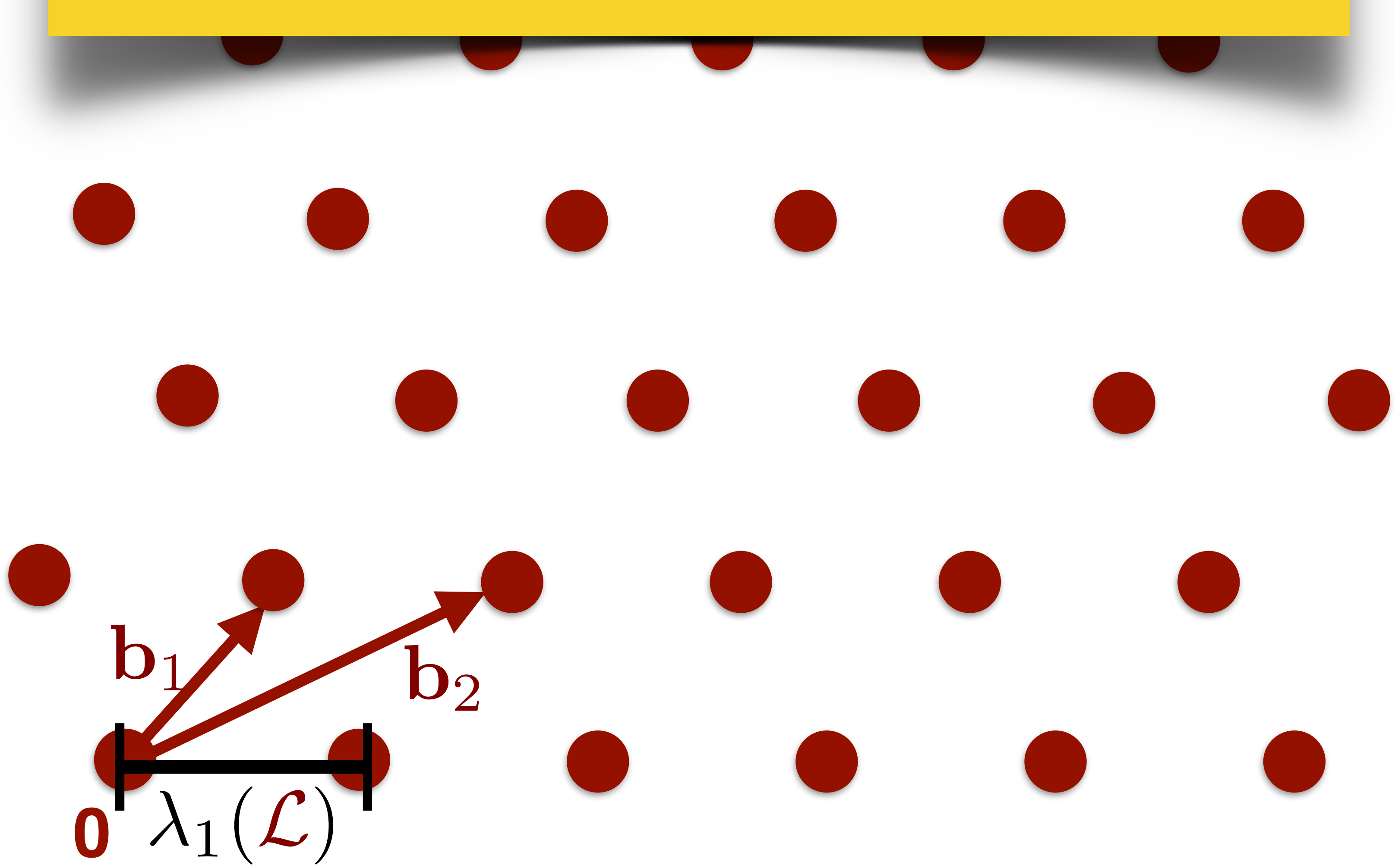

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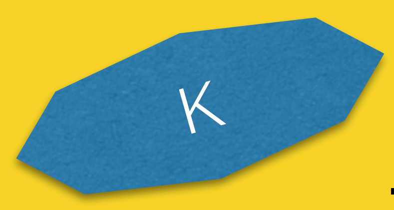
K is a symmetric convex body 

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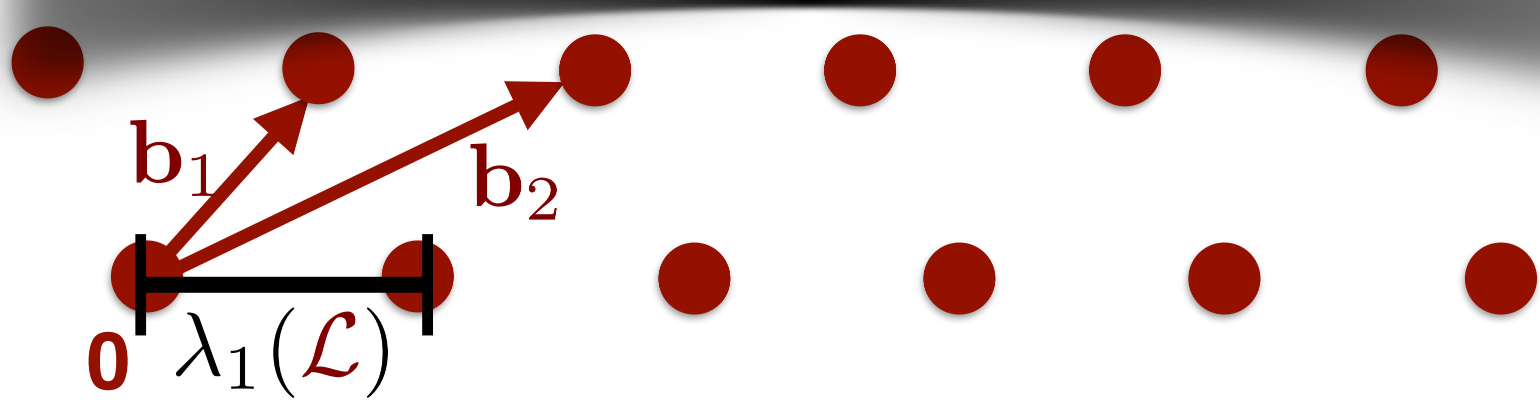

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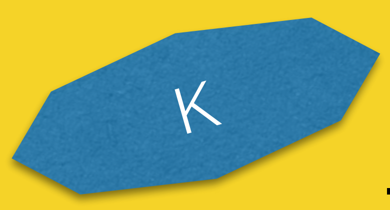
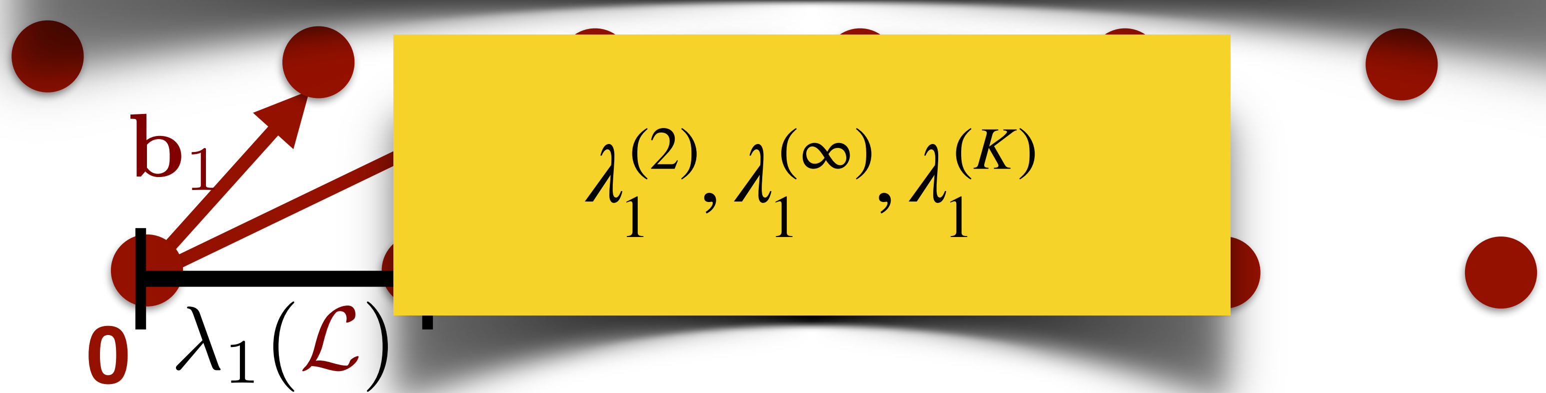
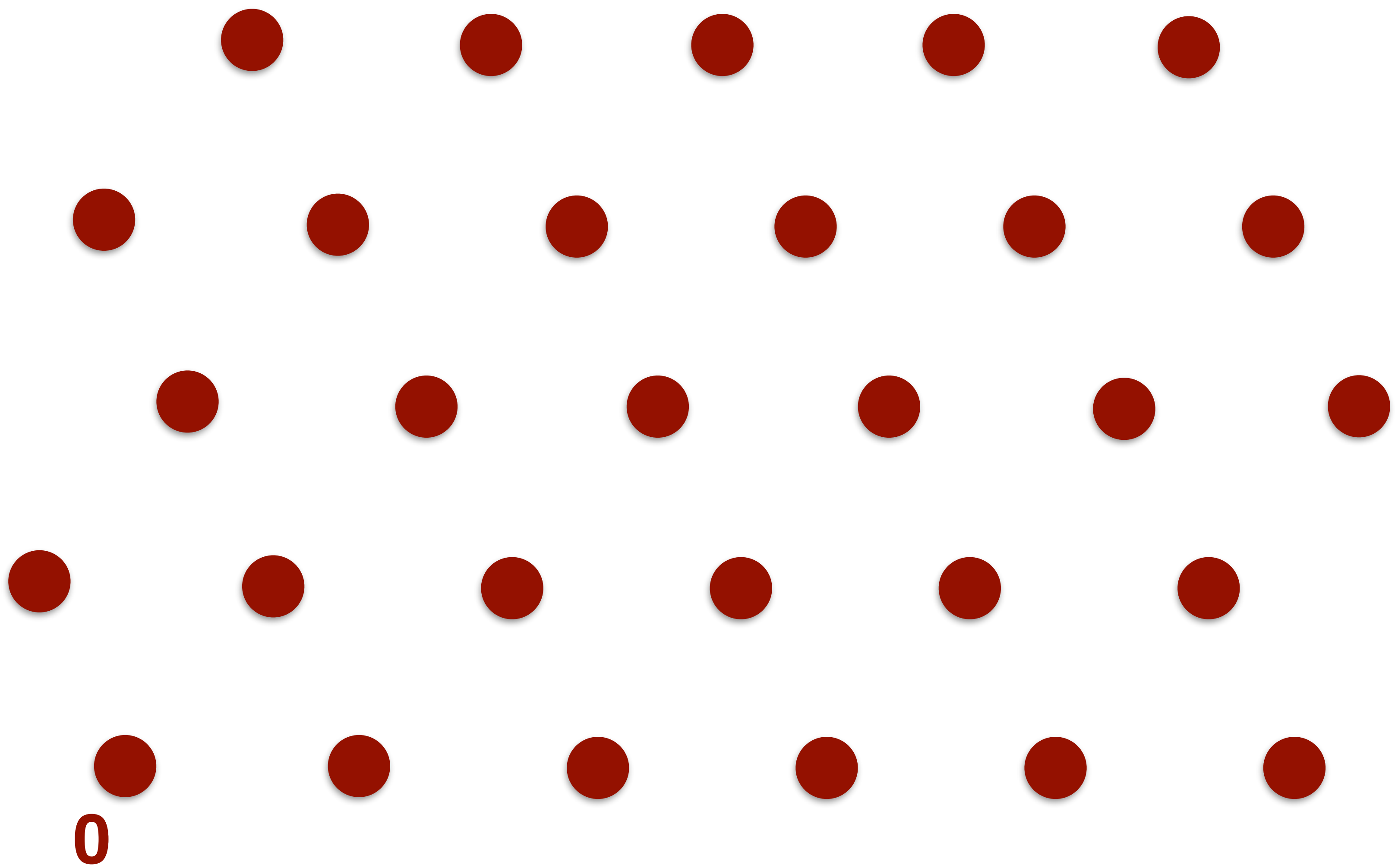
K is a symmetric convex body .

$$\lambda_1^{(2)}, \lambda_1^{(\infty)}, \lambda_1^{(K)}$$



Shortest Vector Problem (SVP)



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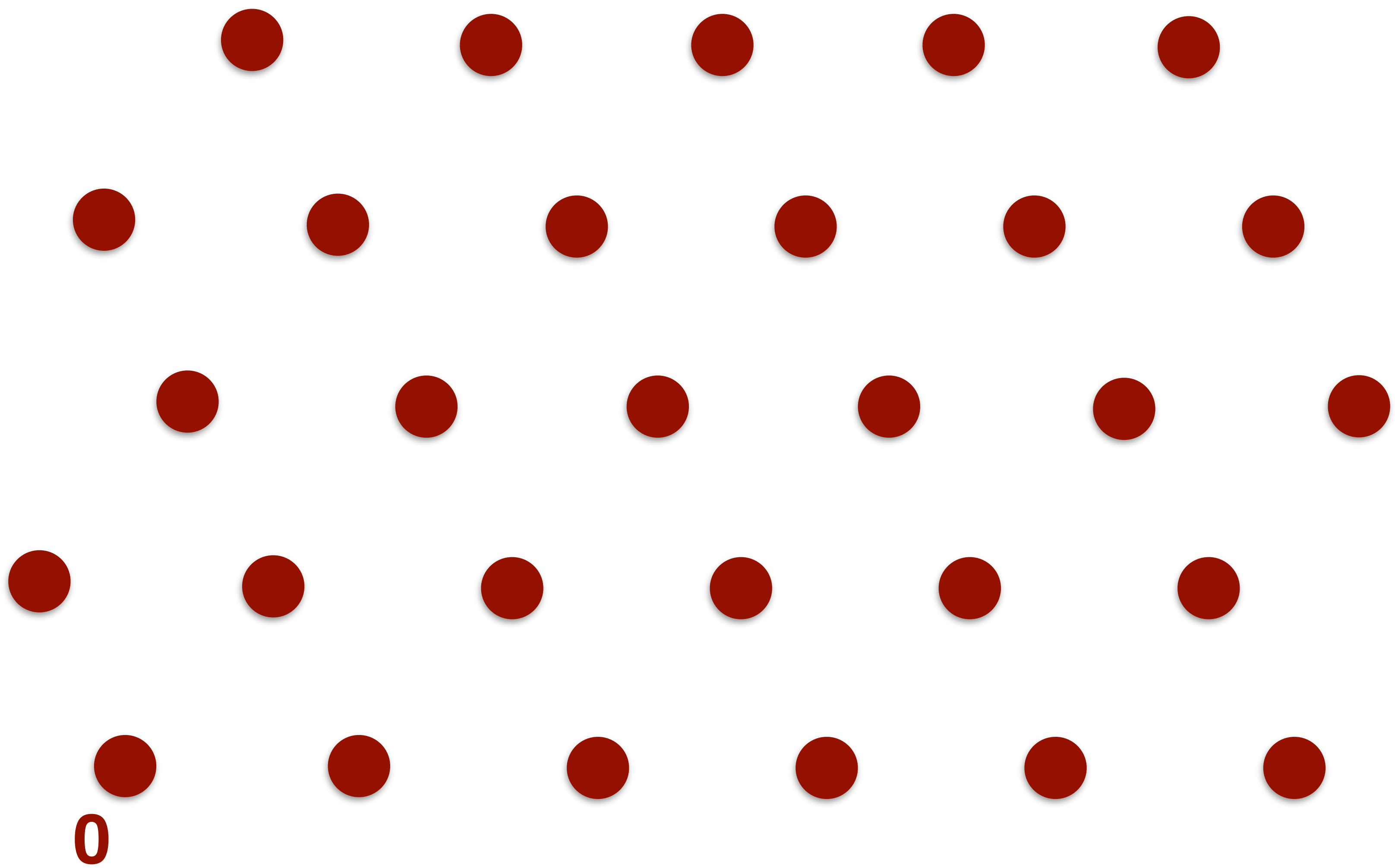

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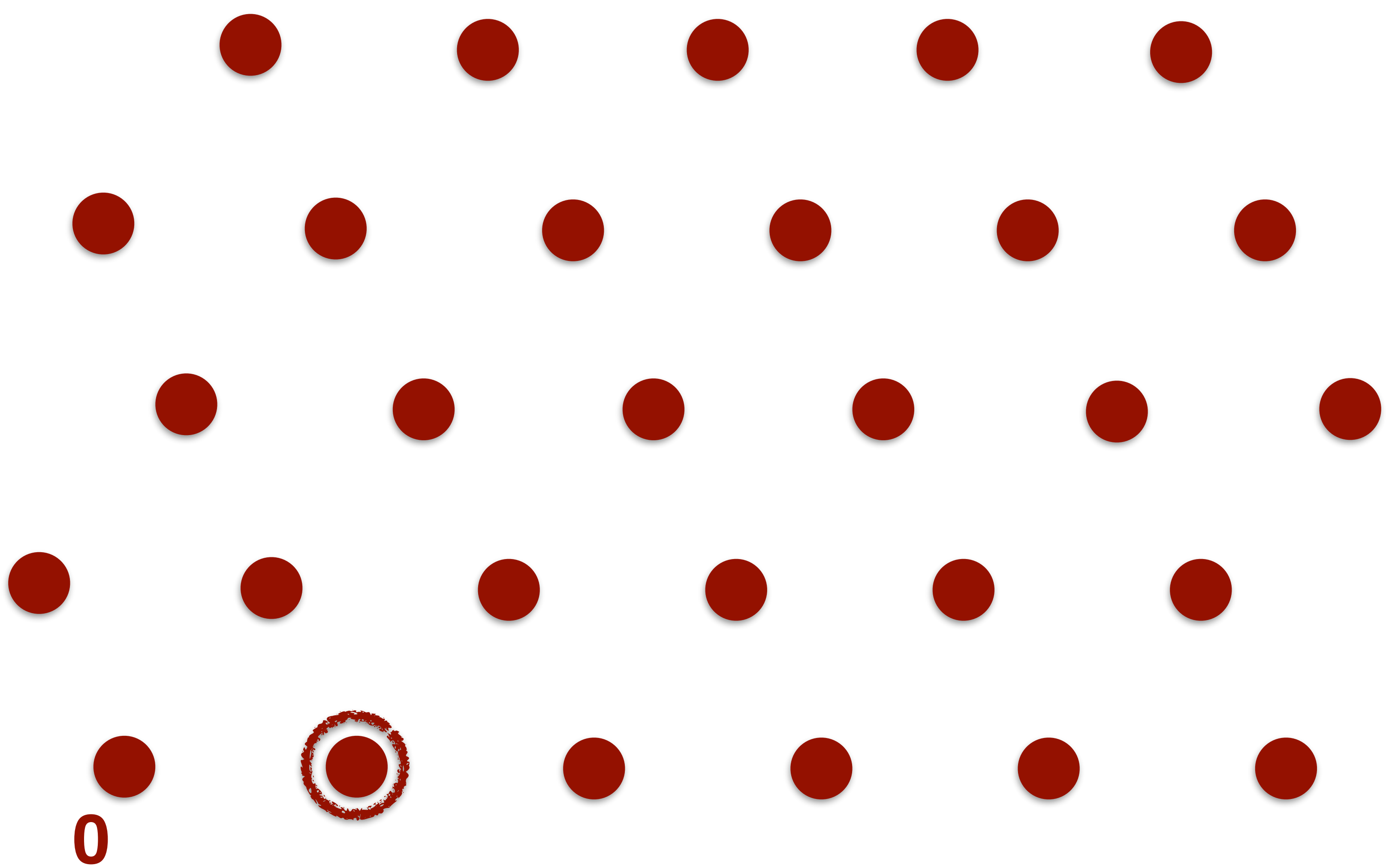

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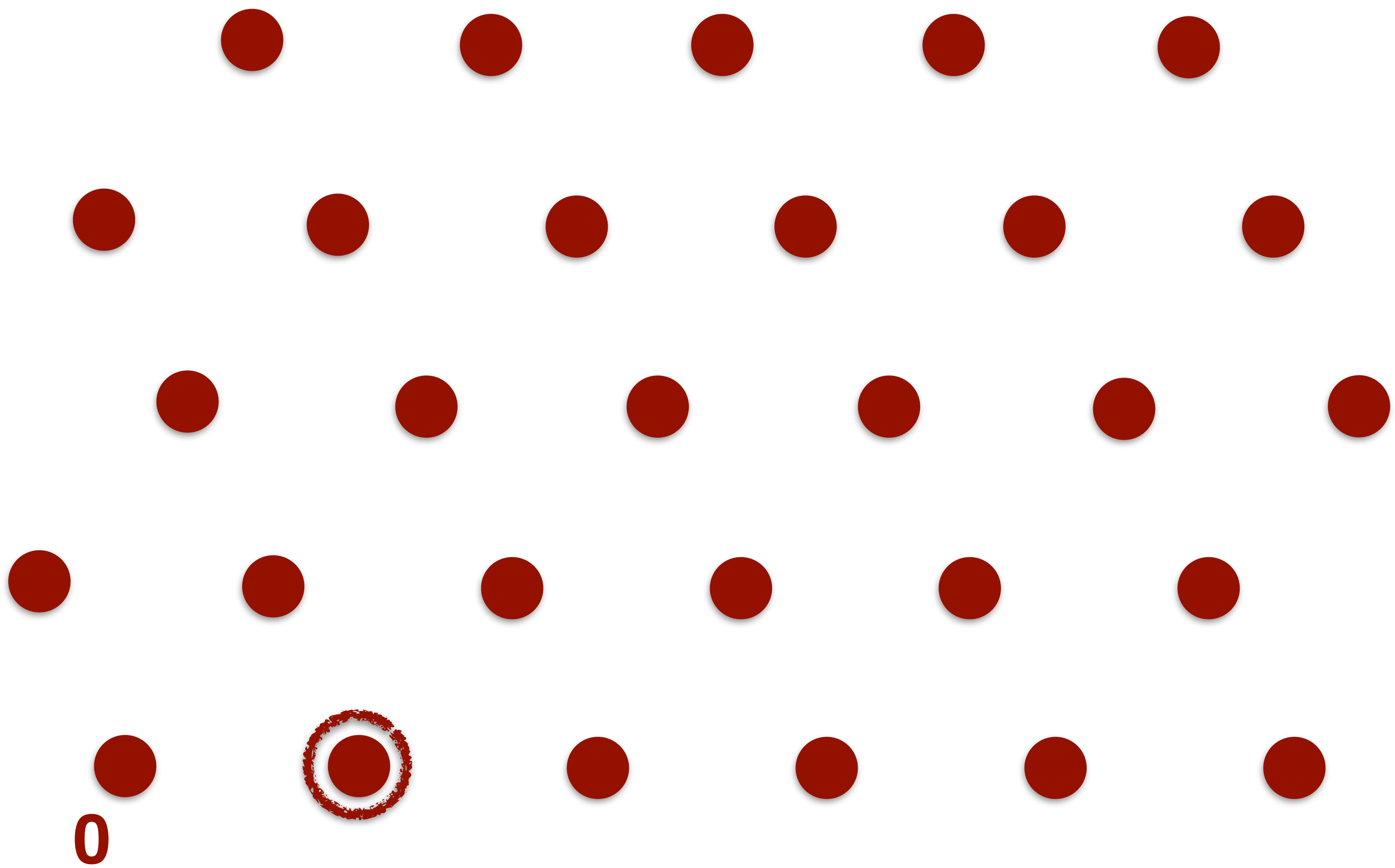

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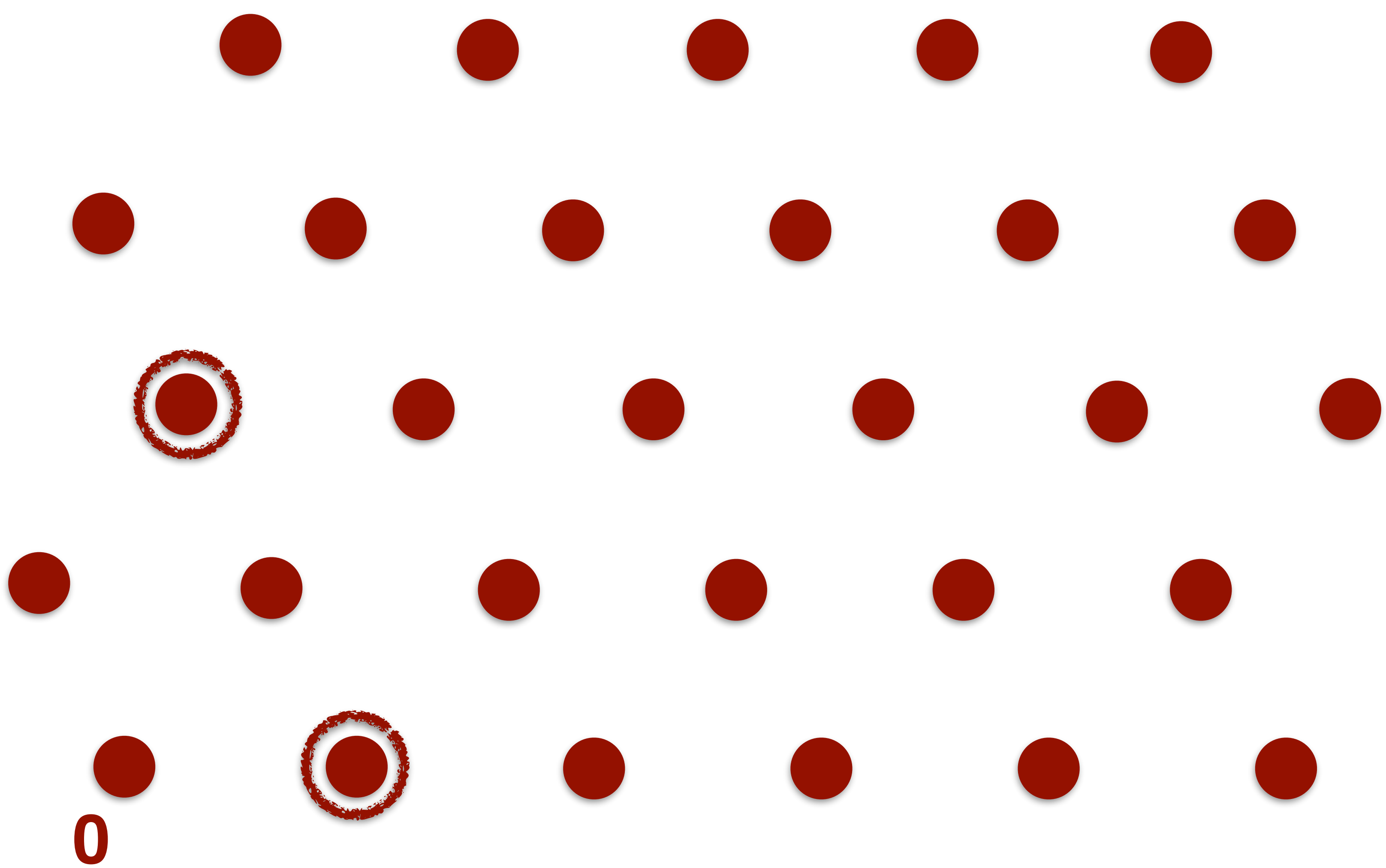

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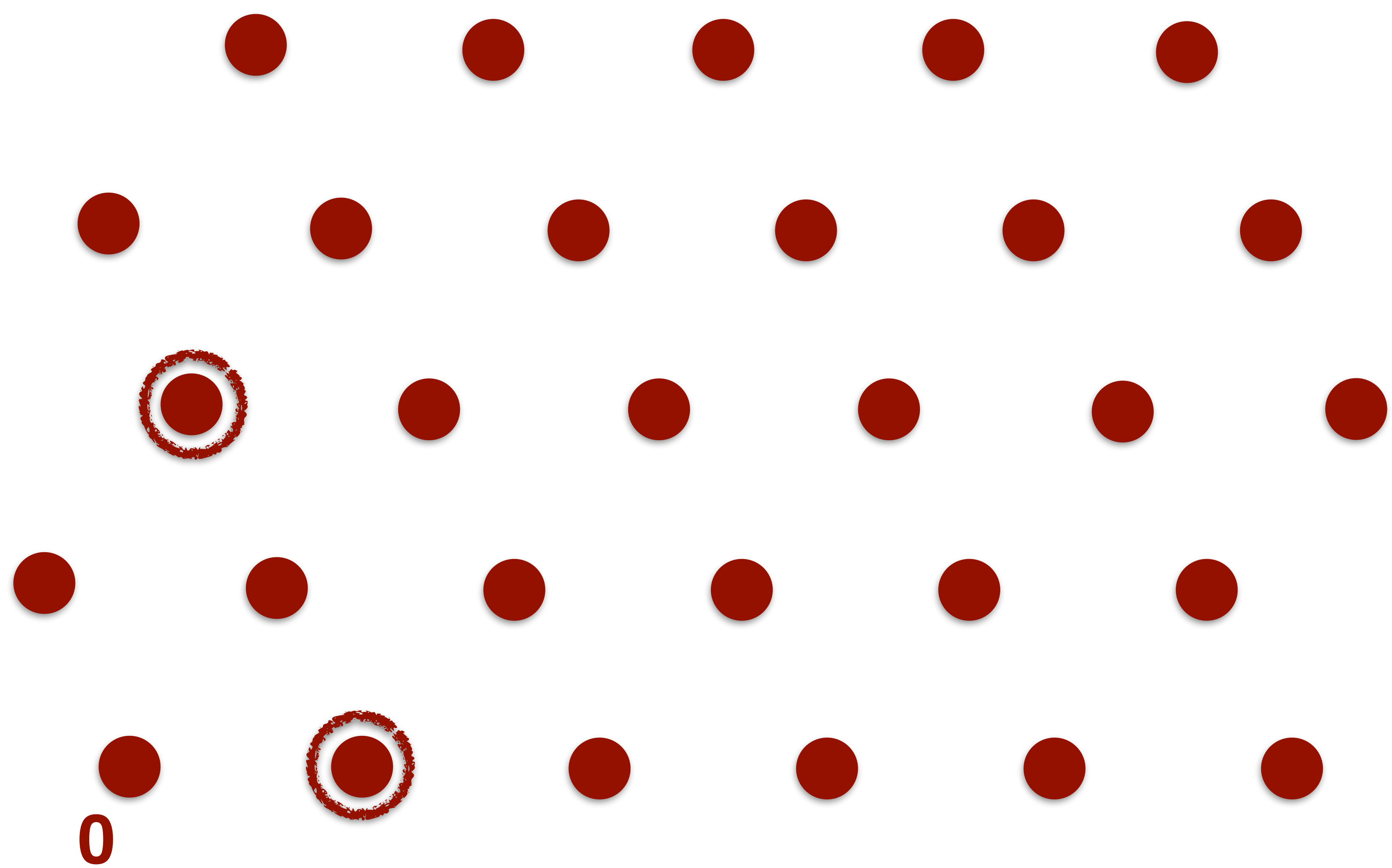

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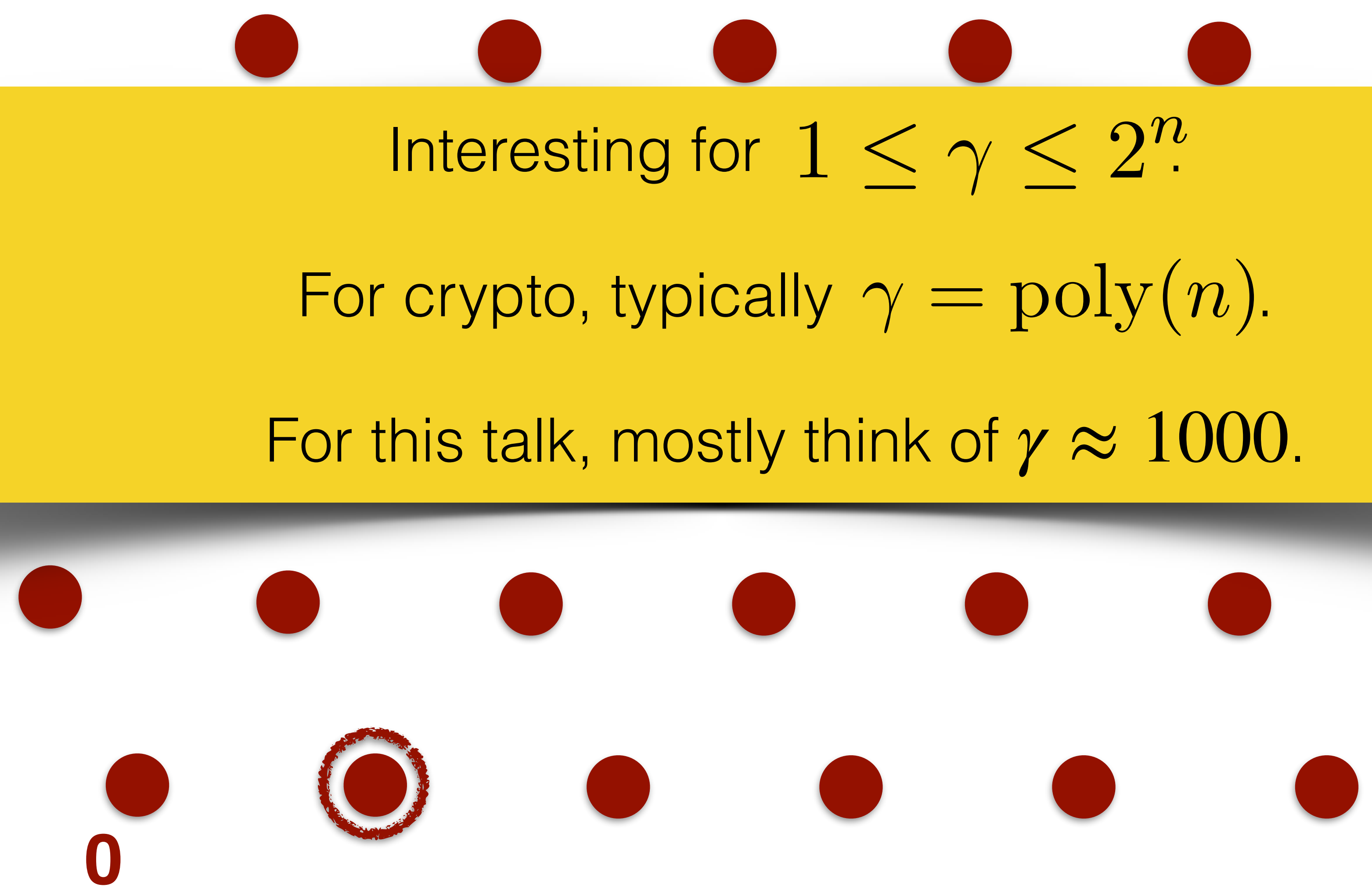
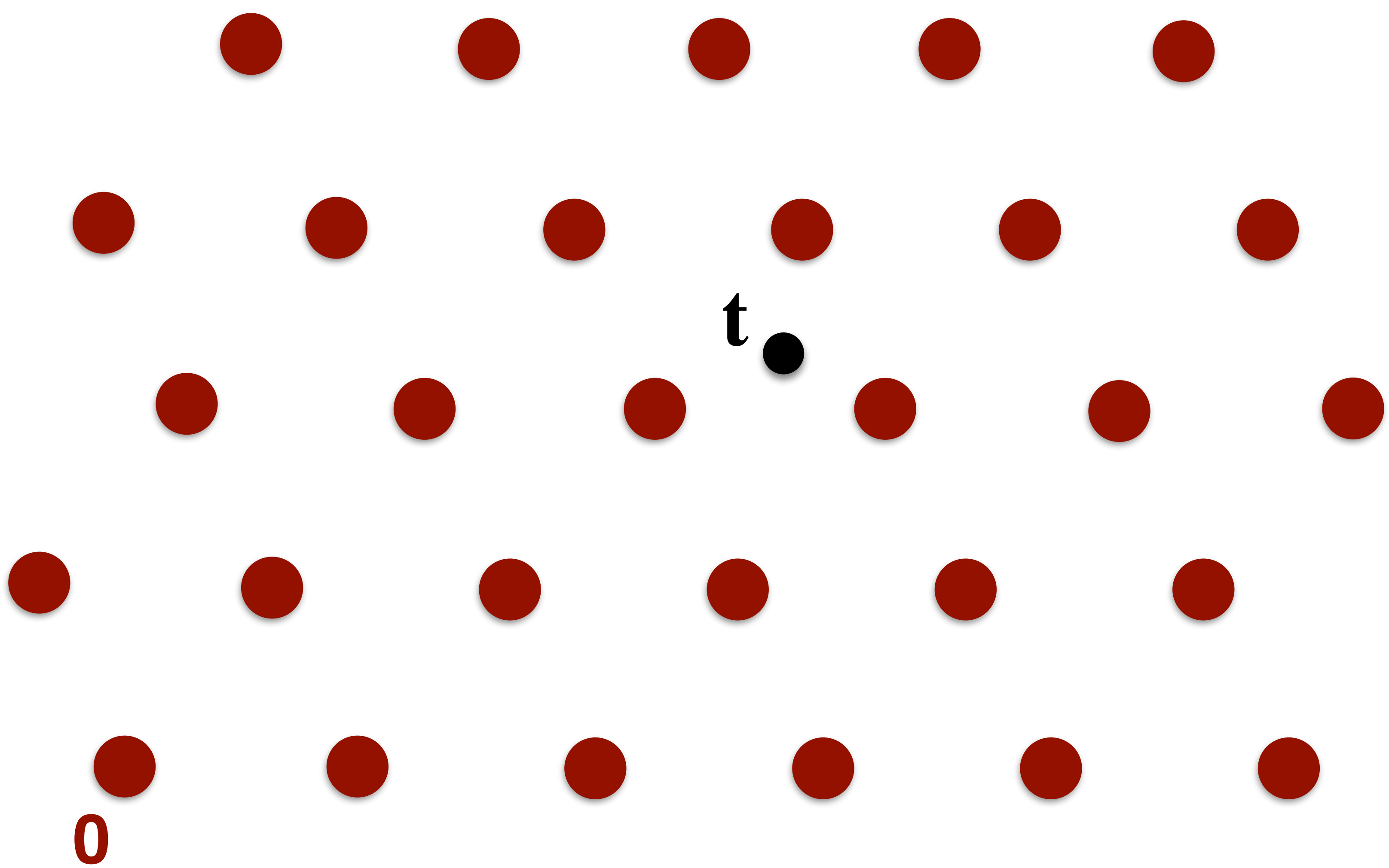
Interesting for $1 \leq \gamma \leq 2^n$.

For crypto, typically $\gamma = \mathrm{poly}(n)$.

For this talk, mostly think of $\gamma \approx 1000$.



Closest Vector Problem (CVP)



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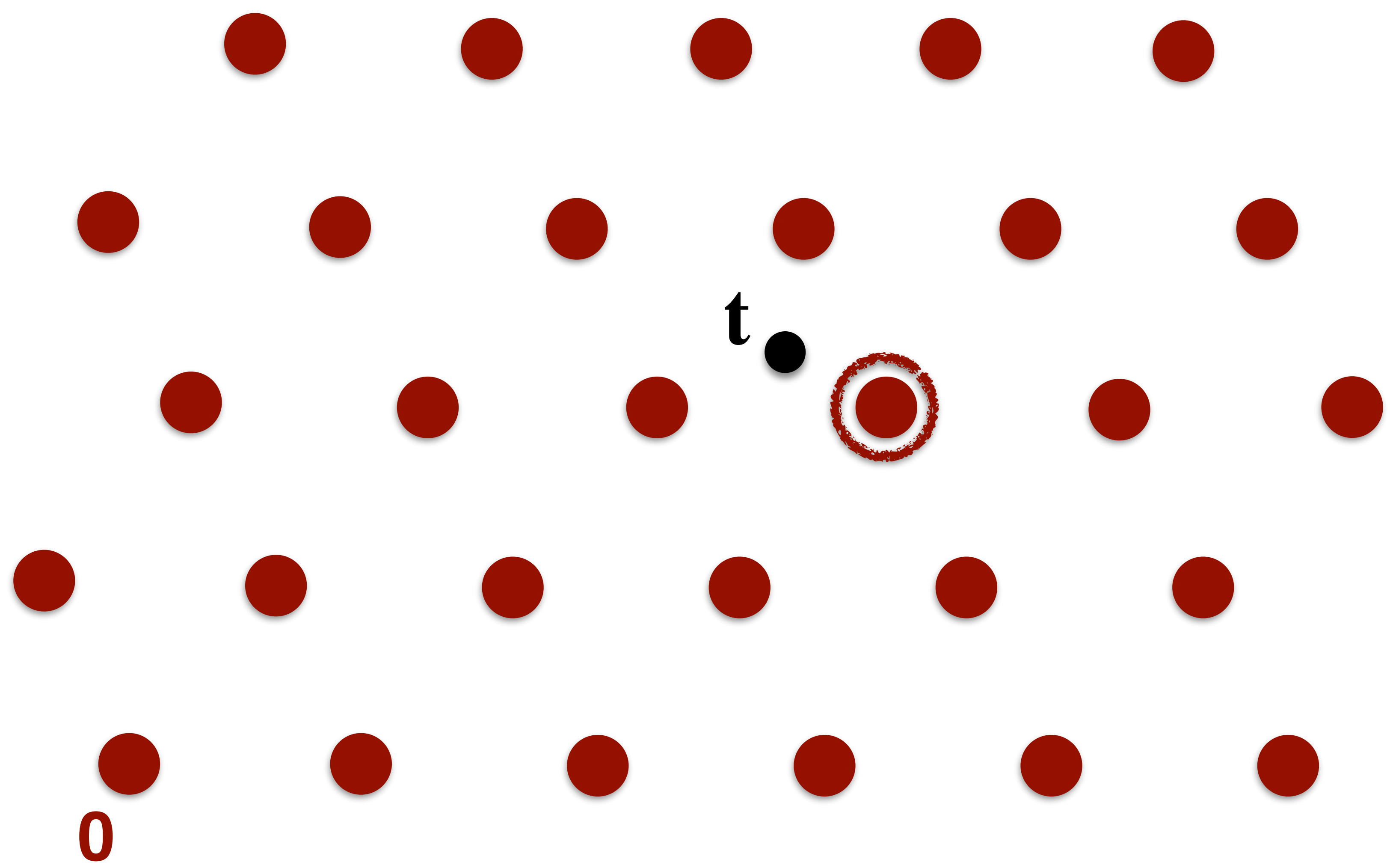

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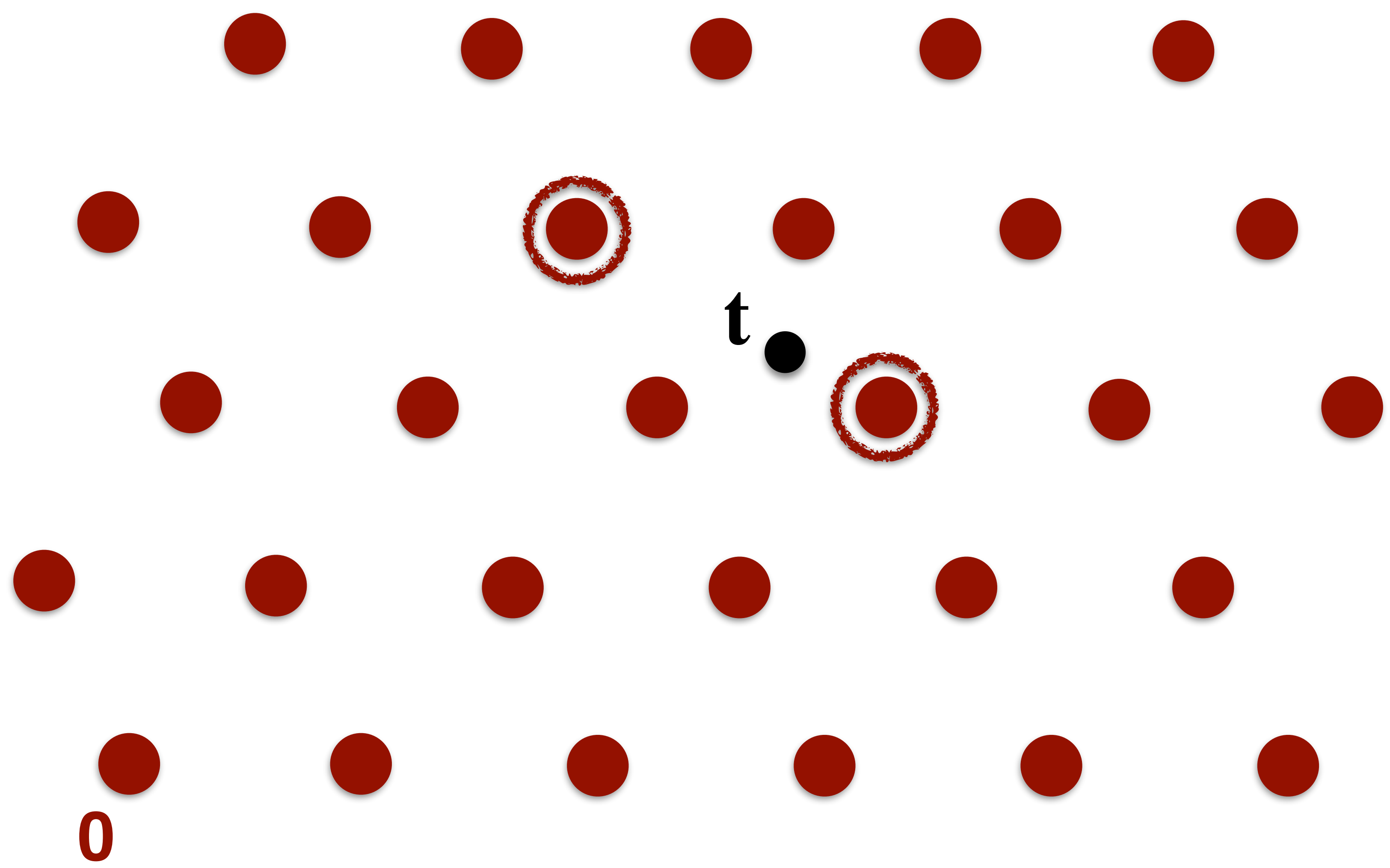

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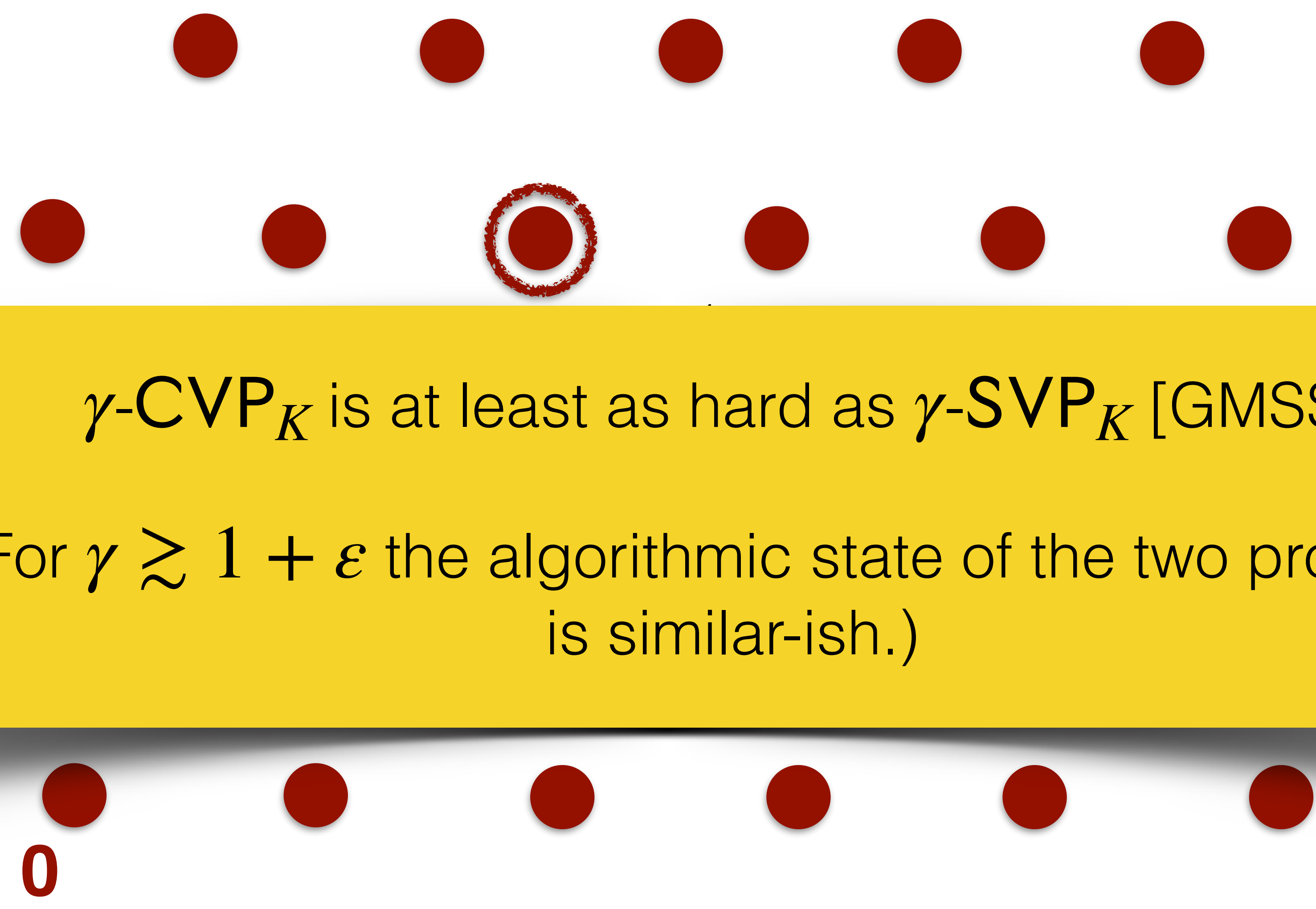

γ-CVP$_K$ is at least as hard as γ-SVP$_K$ [GMSS].

(For $\gamma \gtrsim 1 + \varepsilon$ the algorithmic state of the two problems is similar-ish.)

The World Before May 7, 2020

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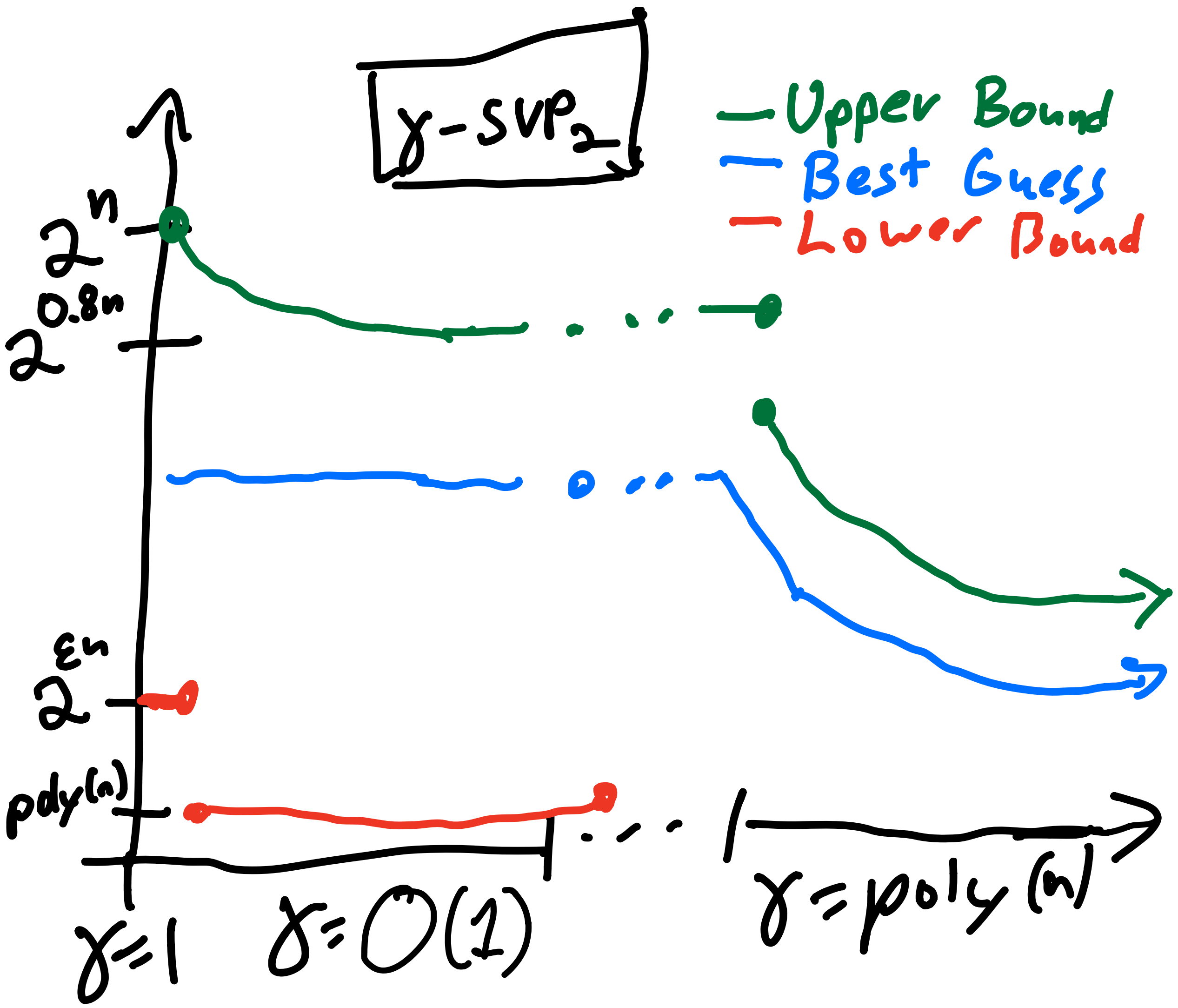

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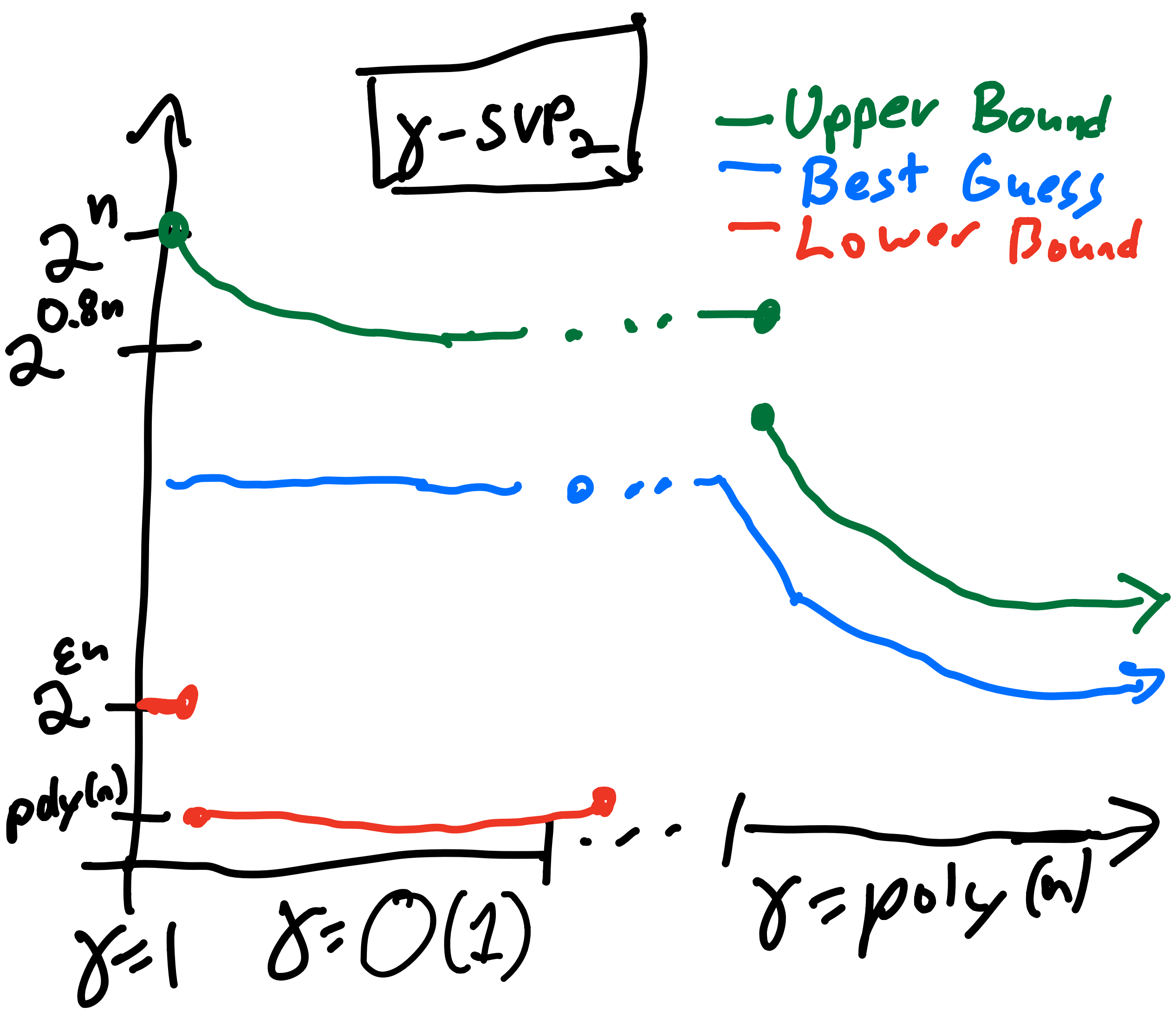

(See [ALS21, Table 1].)

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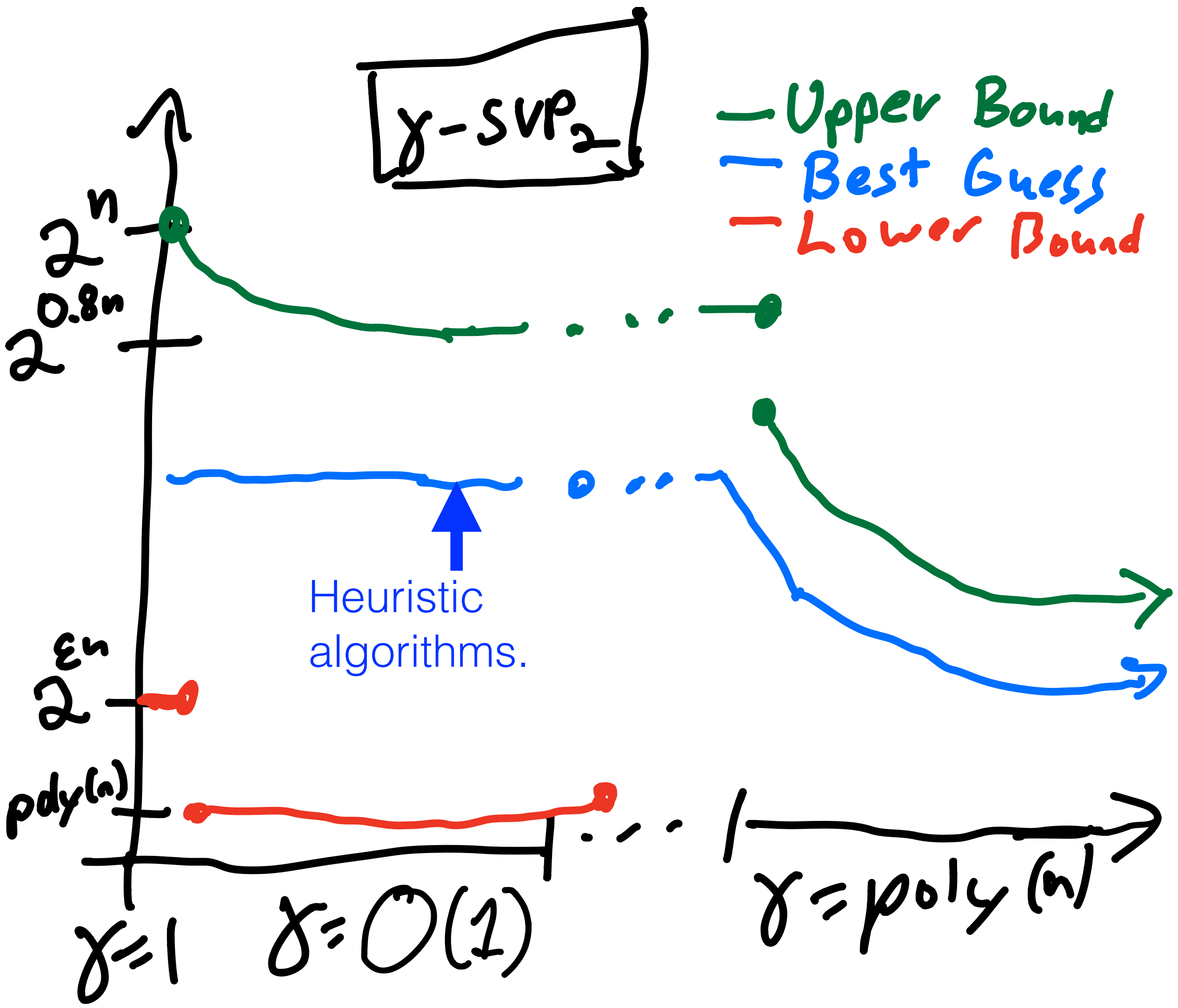

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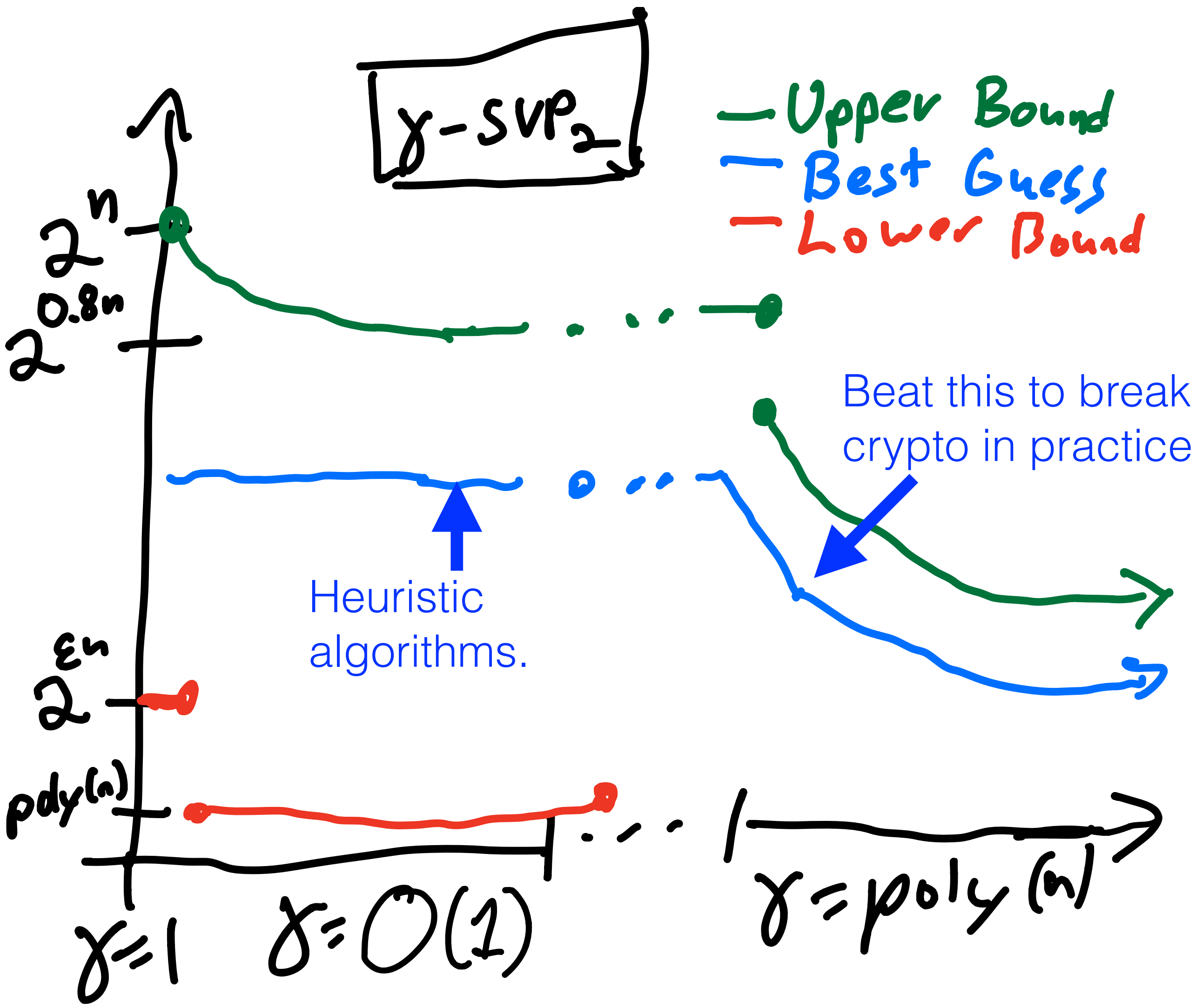

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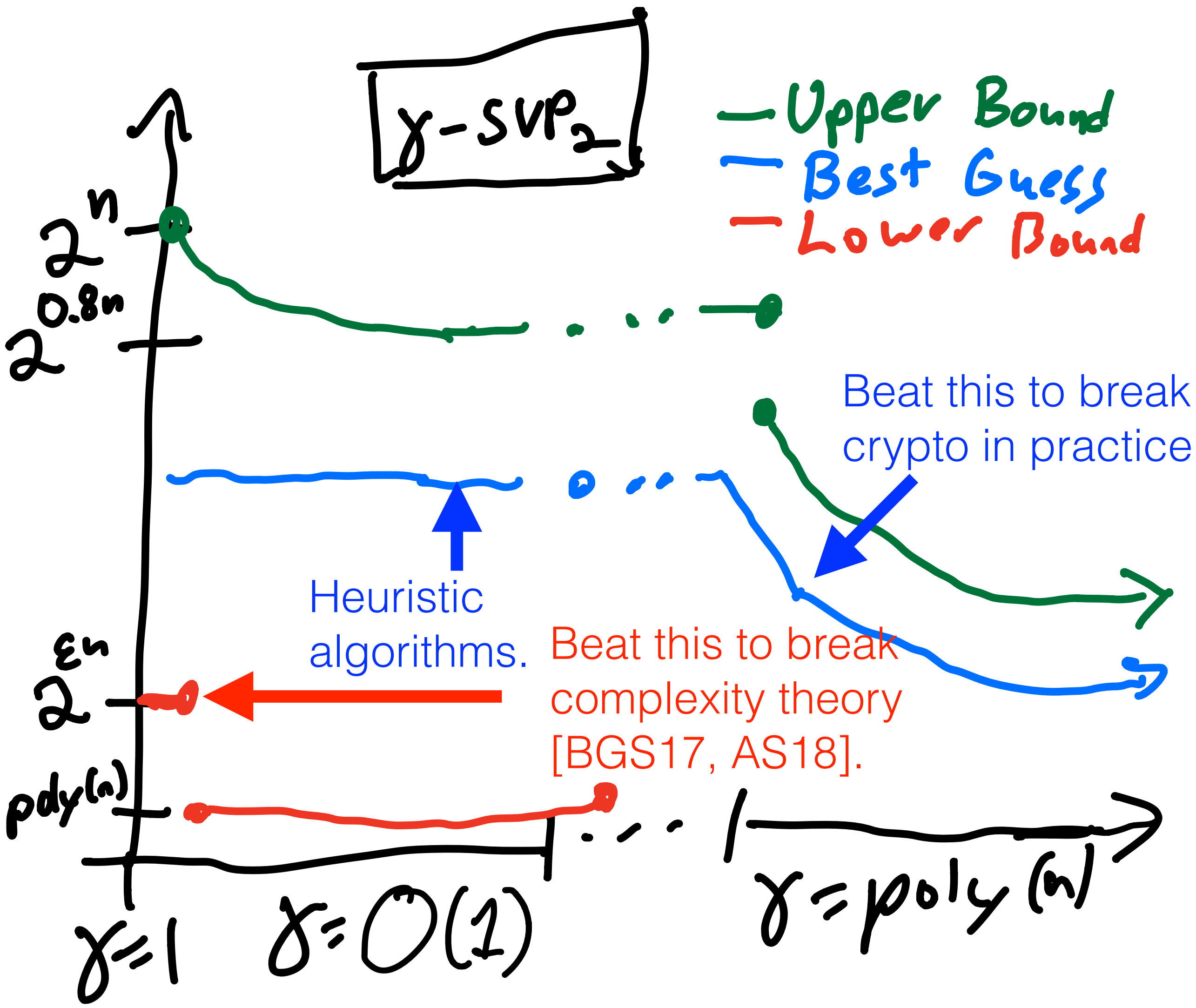

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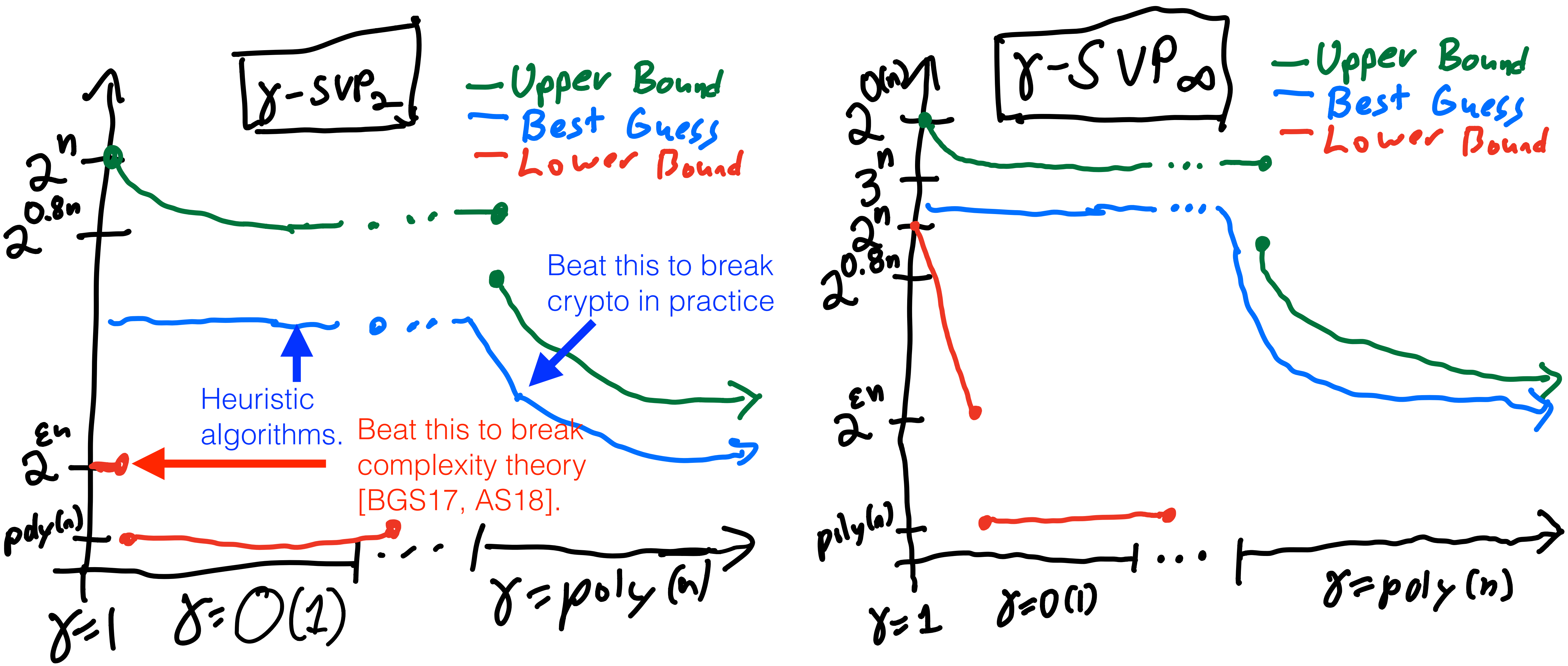

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The World Before May 7, 2020

List of Open Problems

- ♦ 1. Tight fine-grained hardness for exact CVP in the Euclidean norm?
- ♦ 2. Hardness for polynomial approximation factor?
- ♦ 3. Tight fine-grained hardness for exact SVP in the Euclidean norm?
- ● 4. Better algorithms in L_p norms?
- ■ 5. Better understanding of locally dense lattices?
- ■ 6. ***NP***-hardness of $n^{1/\log \log n}$-SVP?
- ■/♦ 7. Fine-grained hardness of approximation? (CVP/SVP)
- ■ 8. Upper bound between $n^{1/\log \log n}$ and $\sqrt{n/\log n}$?

● Easy

■ Medium

♦ Hard

Noah Stephens-Davidowitz | Lattice Problems

MAY 7, 2020!!

MAY 7, 2020!!

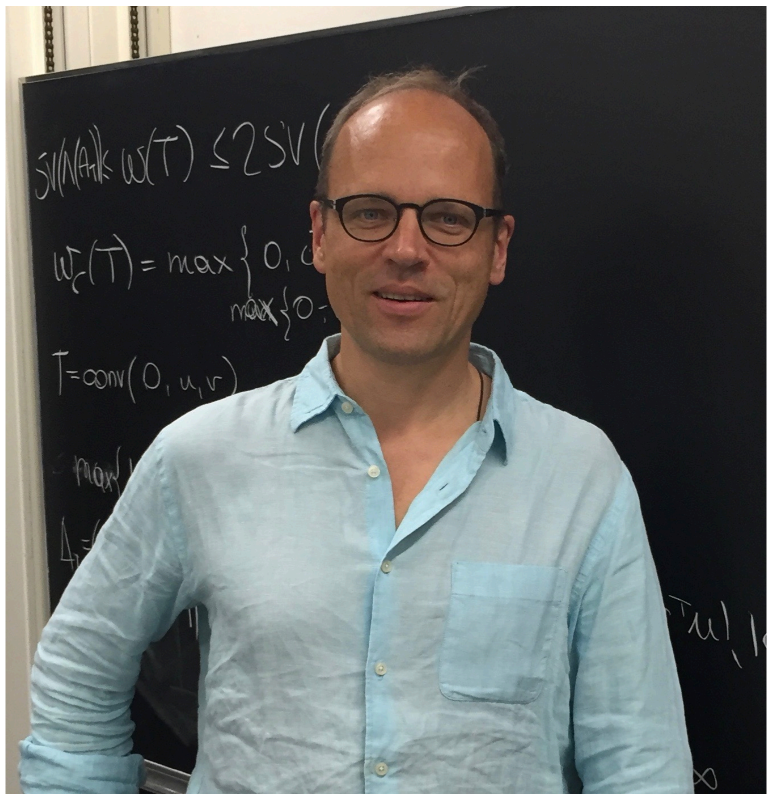
Faster algorithms for SVP_p/CVP_p (question at Simons) Inbox ×

Venzin Moritz Andreas moritz.venzin@epfl.ch via gmail.com — Thu, May 7, 2020, 4:28 PM

to noahsd@gmail.com

Dear Noah

Eisenbrand and Venzin



Friedrich Eisenbrand

Moritz Venzin

Approximate CVP_p in time $2^{0.802\,n}$

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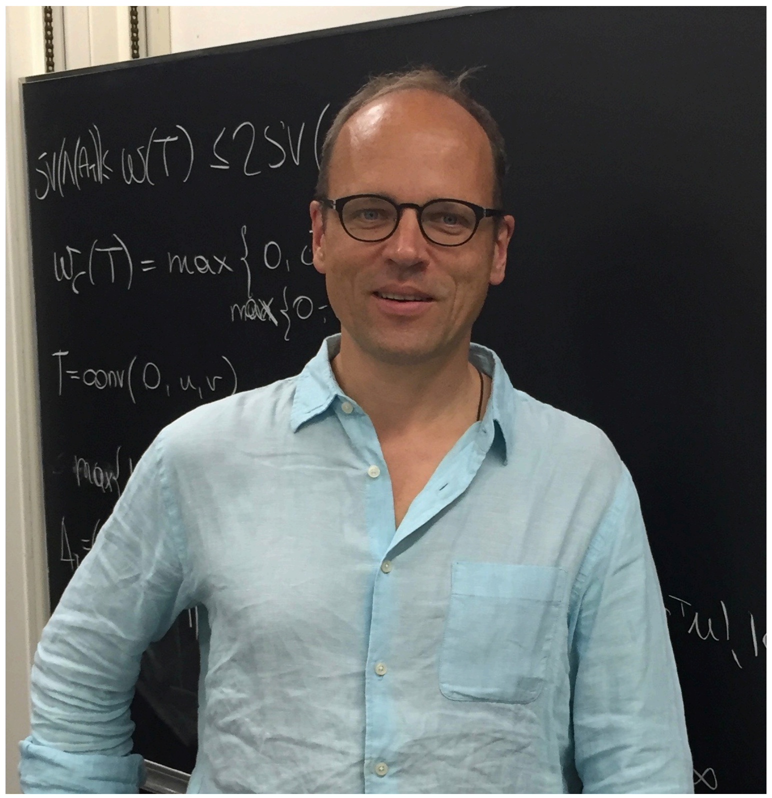

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Best known running time for $O(1)$-SVP_2 [LWXZ11, WLW15, AUV19]

The World After May 7, 2020

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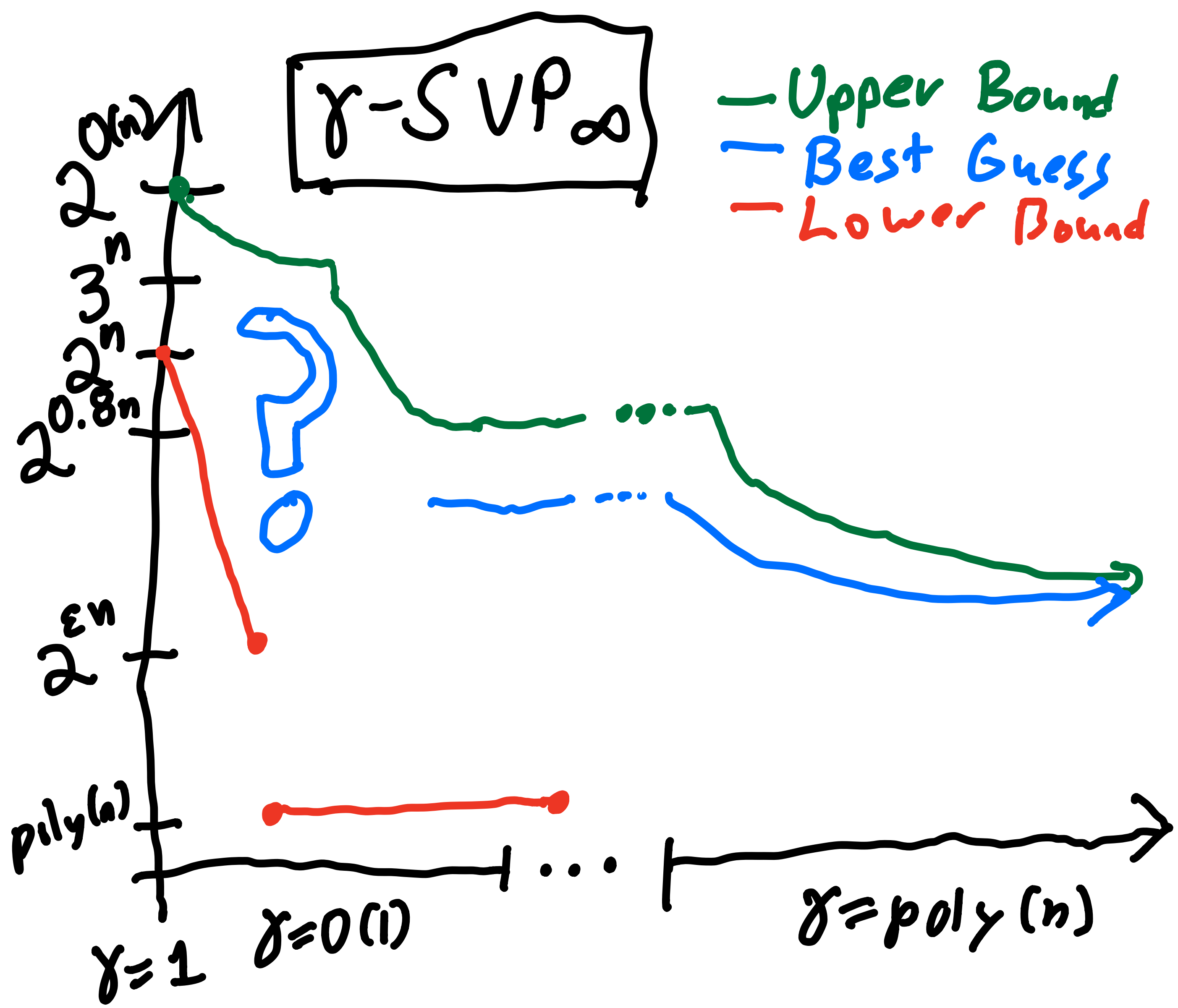
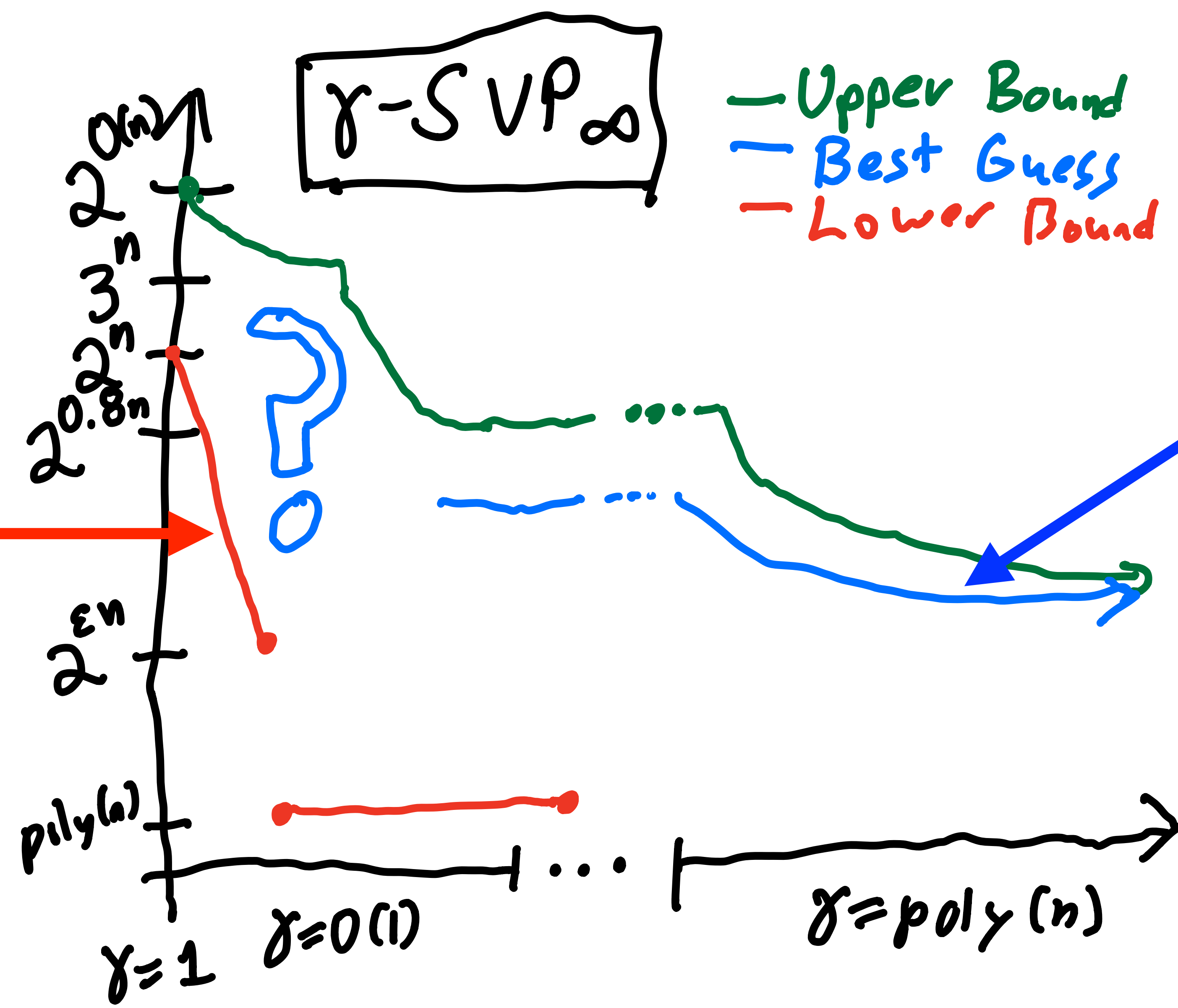


The World After May 7, 2020

Algorithms below this line break SETH [BGS17].



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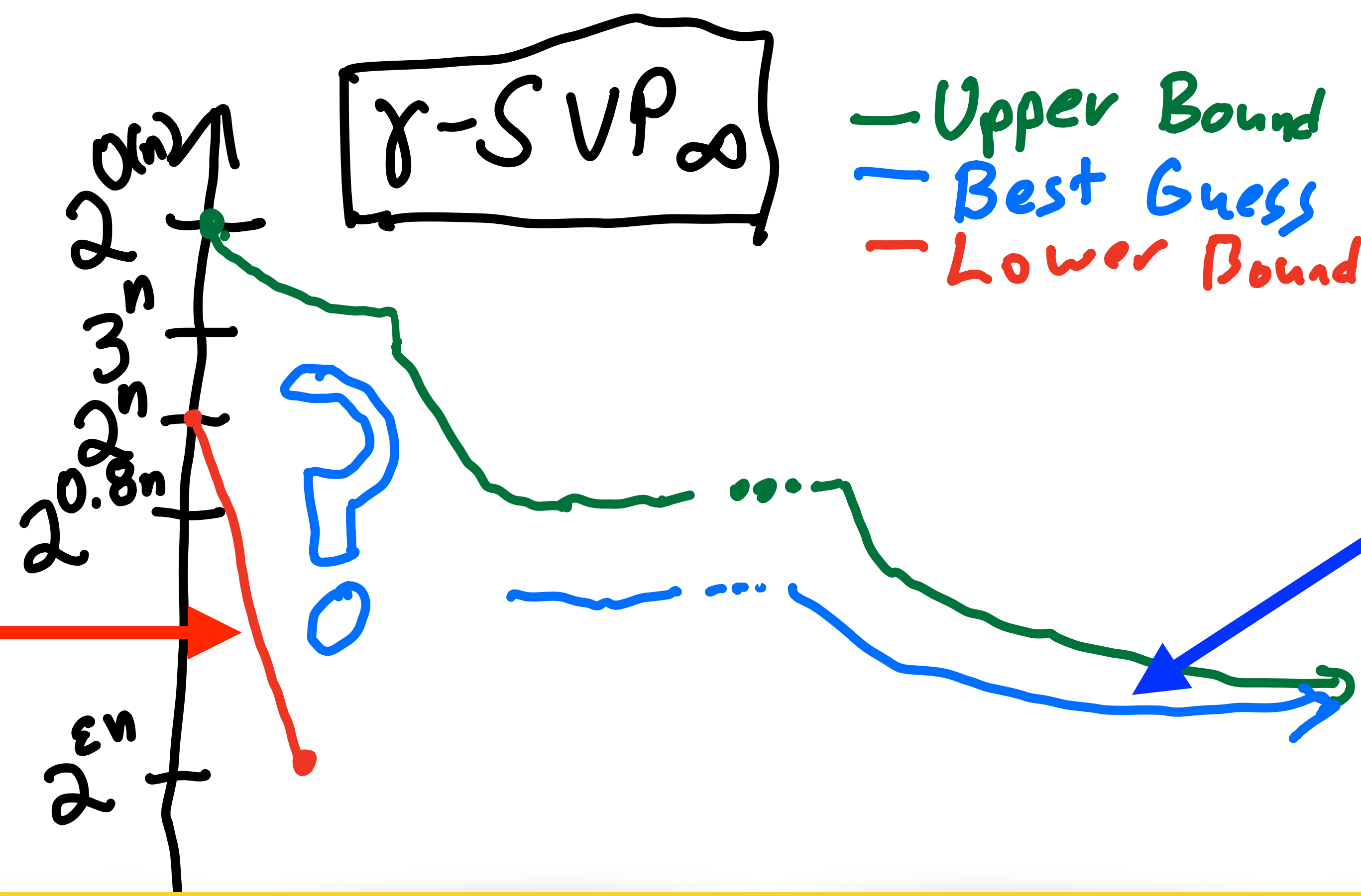

Algorithms below this line break crypto in practice.

Possible resolutions:

1. A strangely wiggly line.
2. SETH is false.
3. Lattice-based crypto is <u>much</u> less secure than we think.

Eisenbrand and Venzin (for ℓ_∞)

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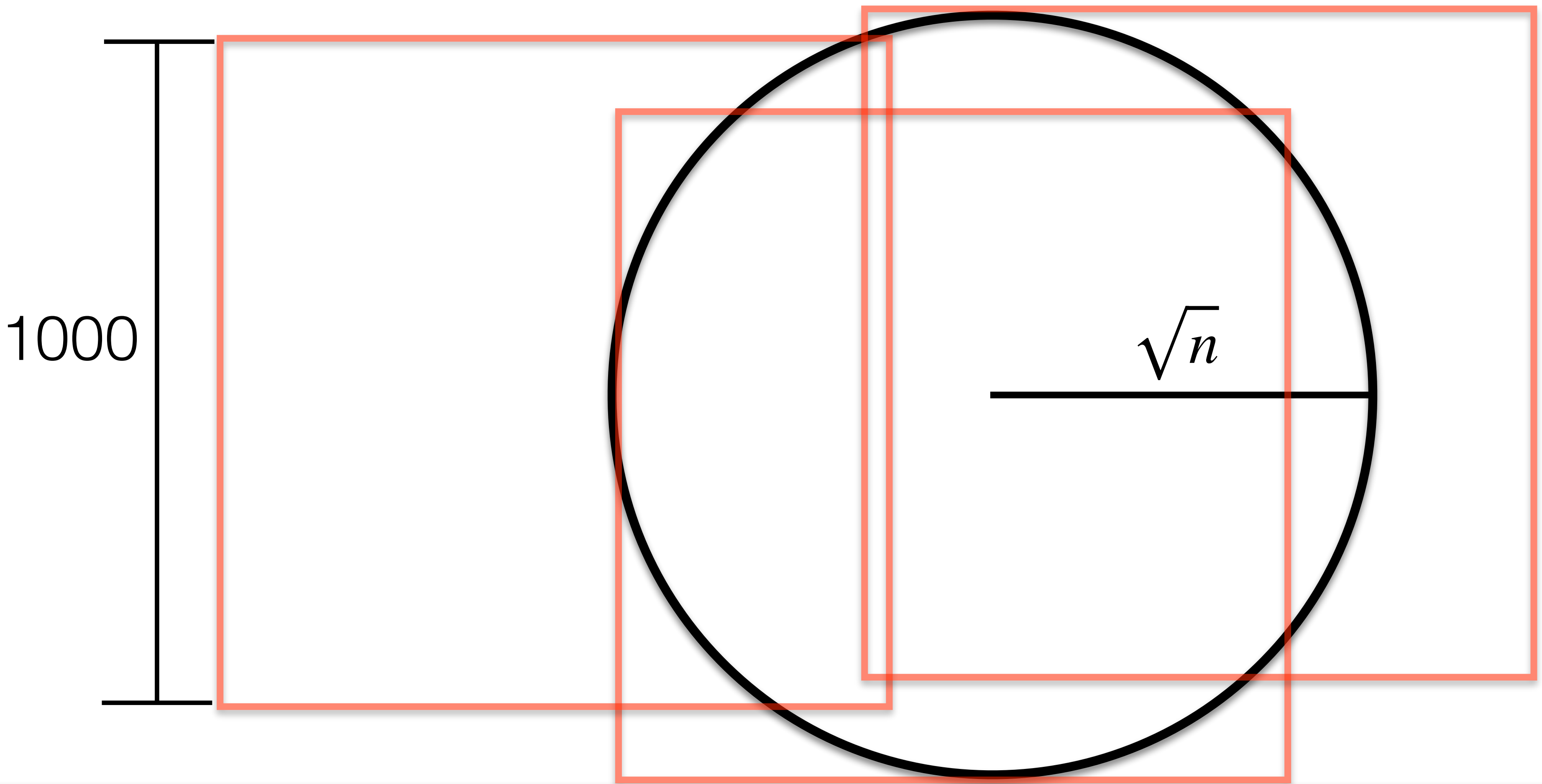
Can cover the $\sqrt{n}B_2$ by $2^{n/10}$ cubes $500B_\infty$.



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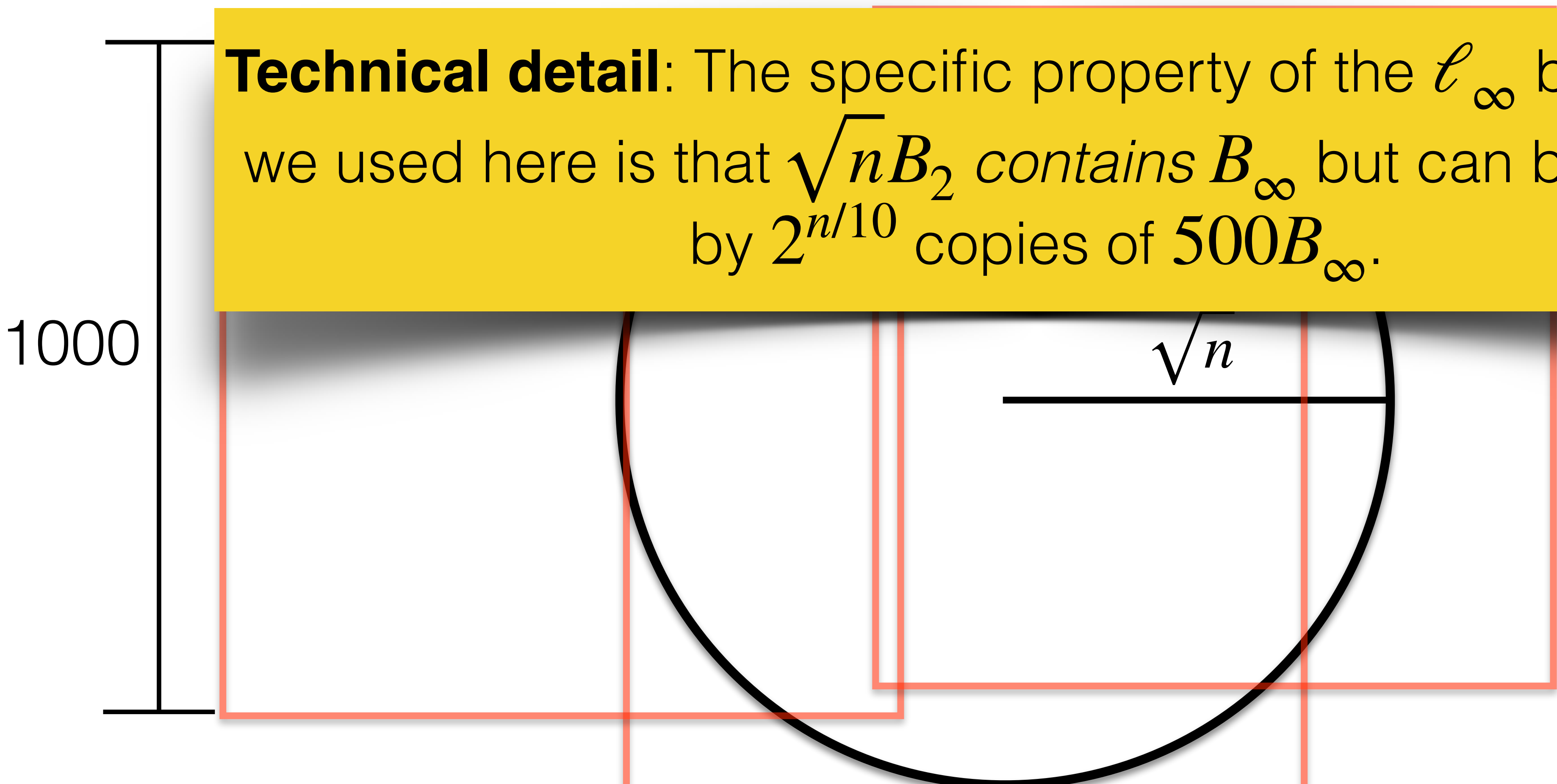

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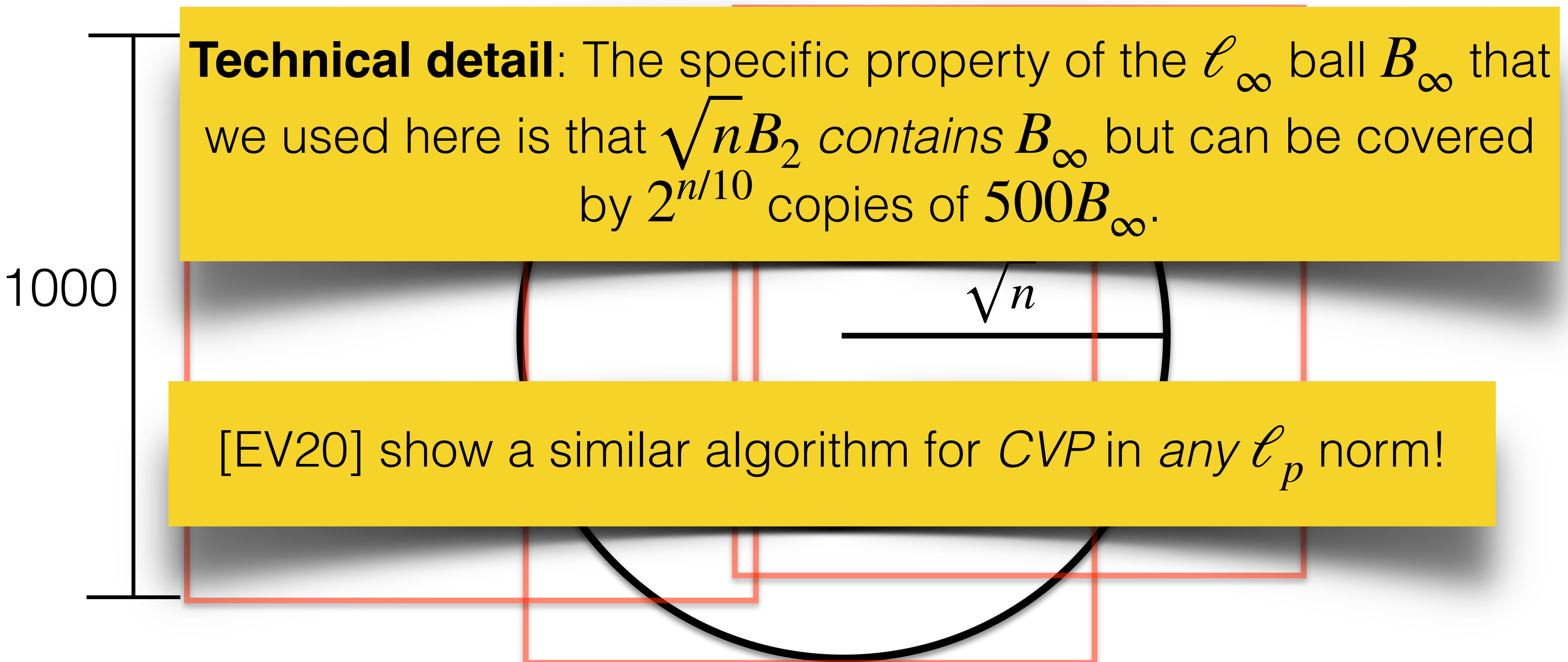

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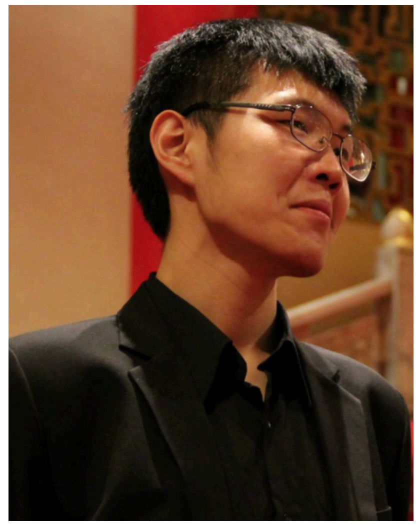
[EV20] show a similar algorithm for *CVP* in *any* ℓ_p norm!

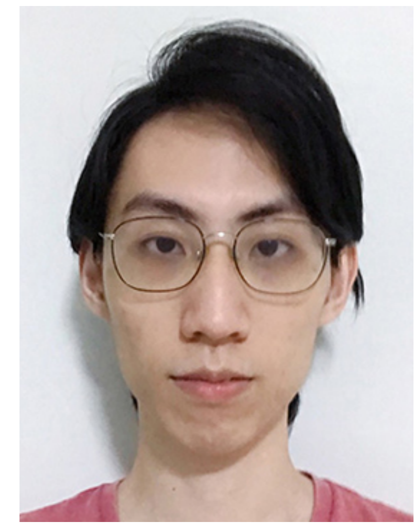
[ACKLS '21]

Divesh Aggarwal



Yanlin Chen

Rajendra Kumar



Zeyong Li

NSD

Dimension-Preserving Reductions Between SVP and CVP in Different p-Norms

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[ACKLS '21]

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Any γ-SVP/CVP algorithm can be converted into an algorithm that samples "random lattice points" with bounded norm/distance. (Key word: sparsification.)

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For any $q \geq p$, a $2^{\varepsilon n}$-time dimension- and rank-preserving reduction from

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For any $q \geq p$, a $2^{\varepsilon n}$-time dimension- and rank-preserving reduction from

1. $O_\varepsilon(\gamma)$-SVP_q to γ-SVP_p.

[ACKLS '21]

Any γ-SVP/CVP algorithm can be converted into an algorithm that samples "random lattice points" with bounded norm/distance. (Key word: sparsification.)

For any $q \geq p$, a $2^{\varepsilon n}$-time dimension- and rank-preserving reduction from

1. $O_\varepsilon(\gamma)$-SVP$_q$ to γ-SVP$_p$.
2. $O_\varepsilon(\gamma)$-CVP$_p$ to γ-CVP$_q$.

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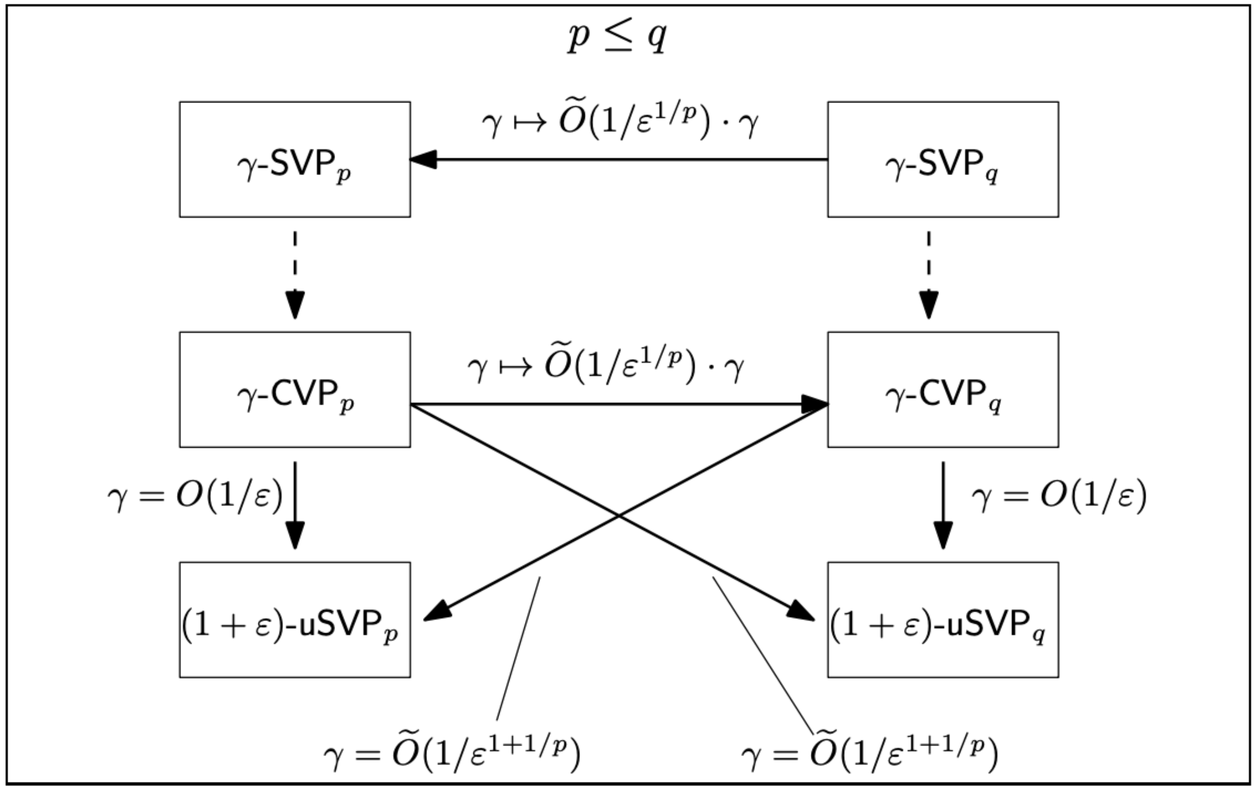
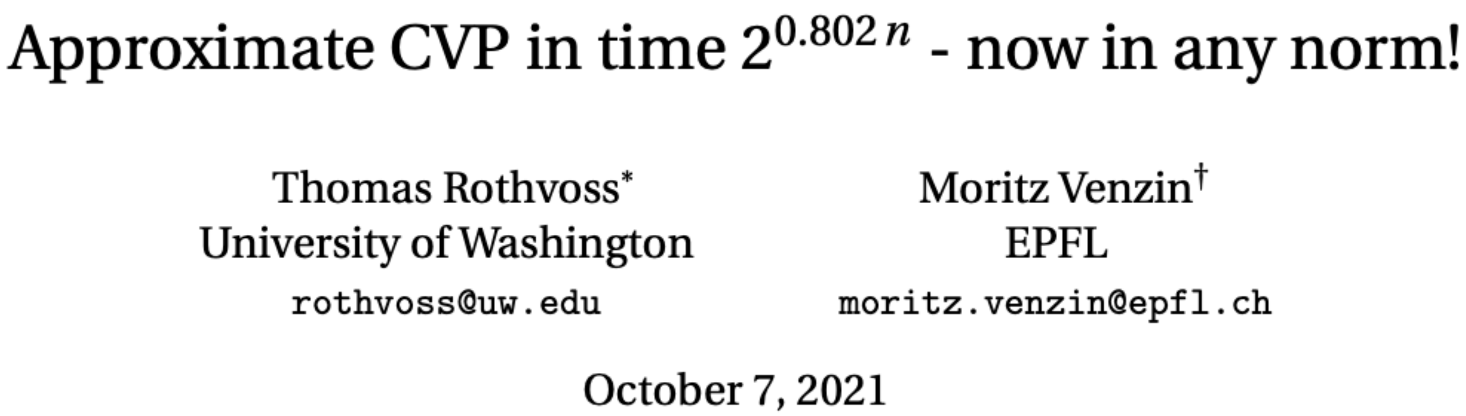


Rothvoss and Venzin

Thomas Rothvoss

Moritz Venzin



Rothvoss and Venzin

Theorem. *There is a* $2^{\varepsilon n}$*-time dimension-preserving reduction from* $O_\varepsilon(\gamma)$*-approximate* SVP_K *to* γ-CVP_2 *for any norm* K*.*

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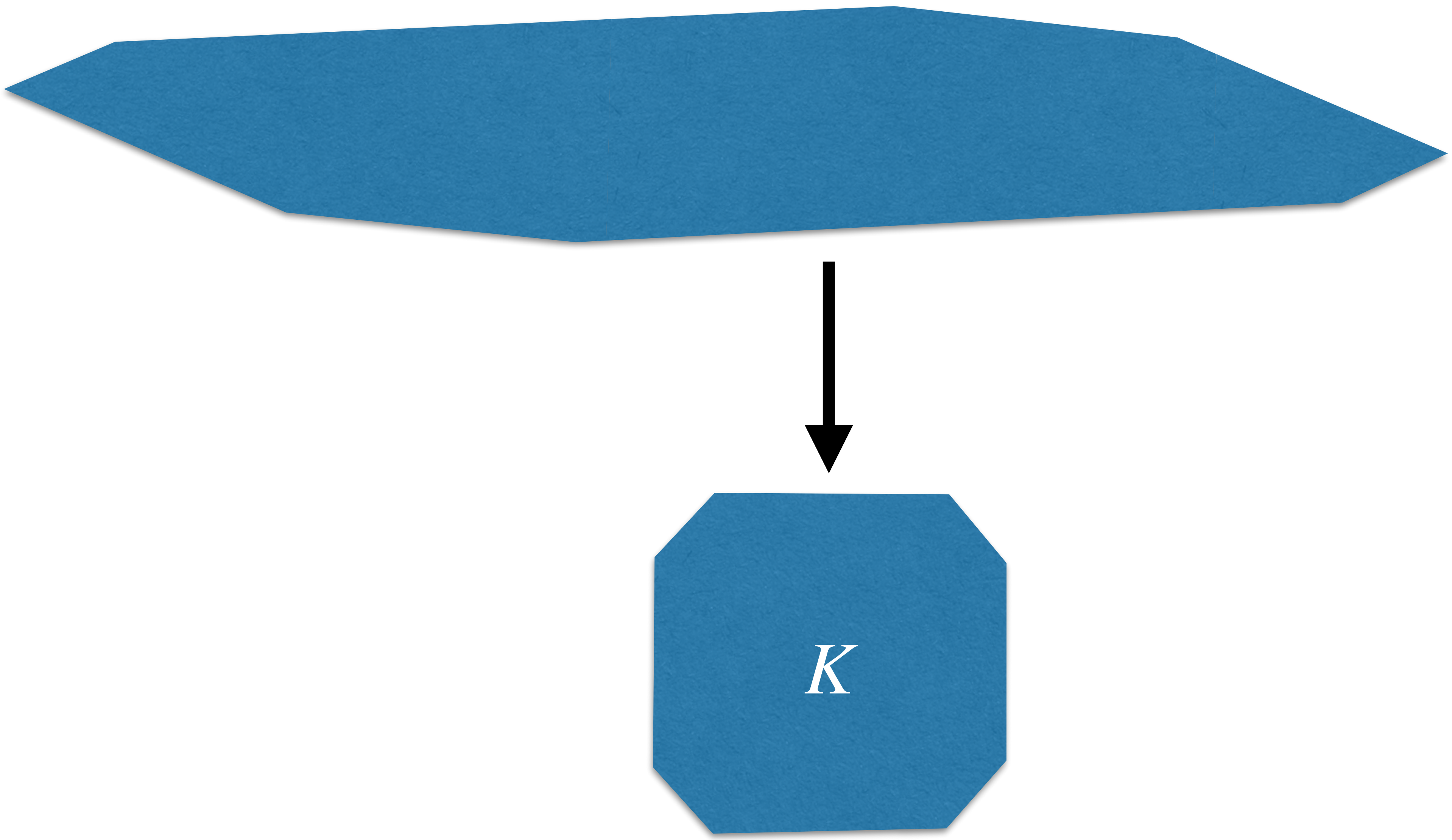
Theorem. *There is a* $2^{\varepsilon n}$*-time dimension-preserving reduction from* $O_\varepsilon(\gamma)$*-approximate* SVP_K *to* γ-CVP_2 *for any norm* K*.*

Theorem. *There is a* $2^{0.802n+o(n)}$*-time algorithm for* $O(1)$-CVP_K *for any* K*.*

Rothvoss and Venzin

Step 0: Apply a linear transformation to K so that it "looks roughly like the scaled ℓ_2 ball $B_2/20$."



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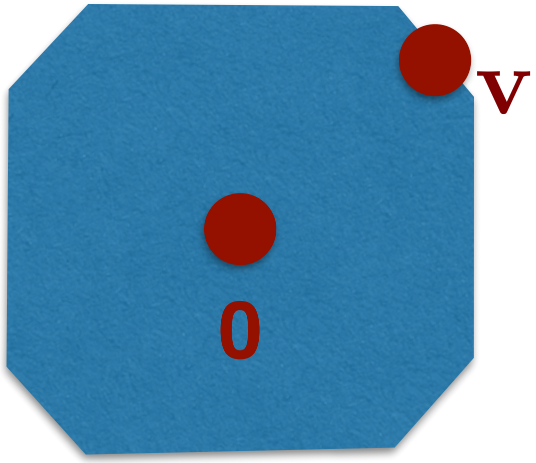
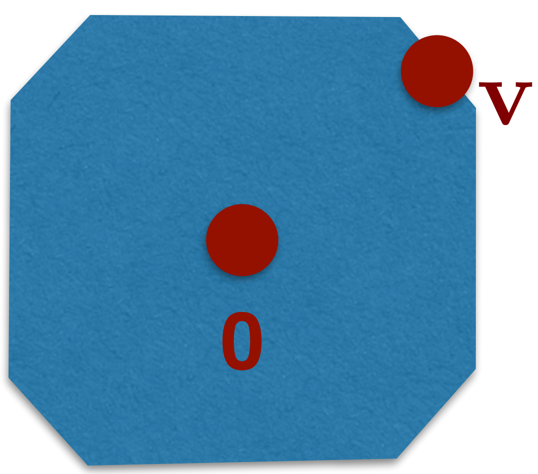


Rothvoss and Venzin

Step 1: Sample $\mathbf{t} \sim K + B_2$.

Pray that $\|\mathbf{t} - \mathbf{v}\|_2 \leq 1$ for a K-shortest non-zero vector.



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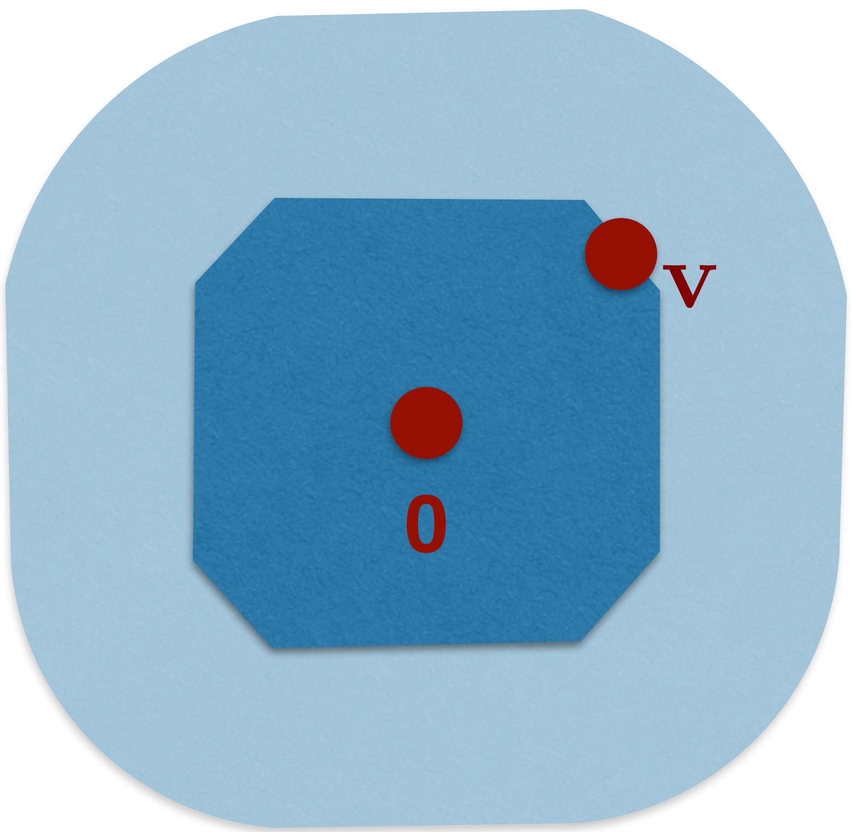

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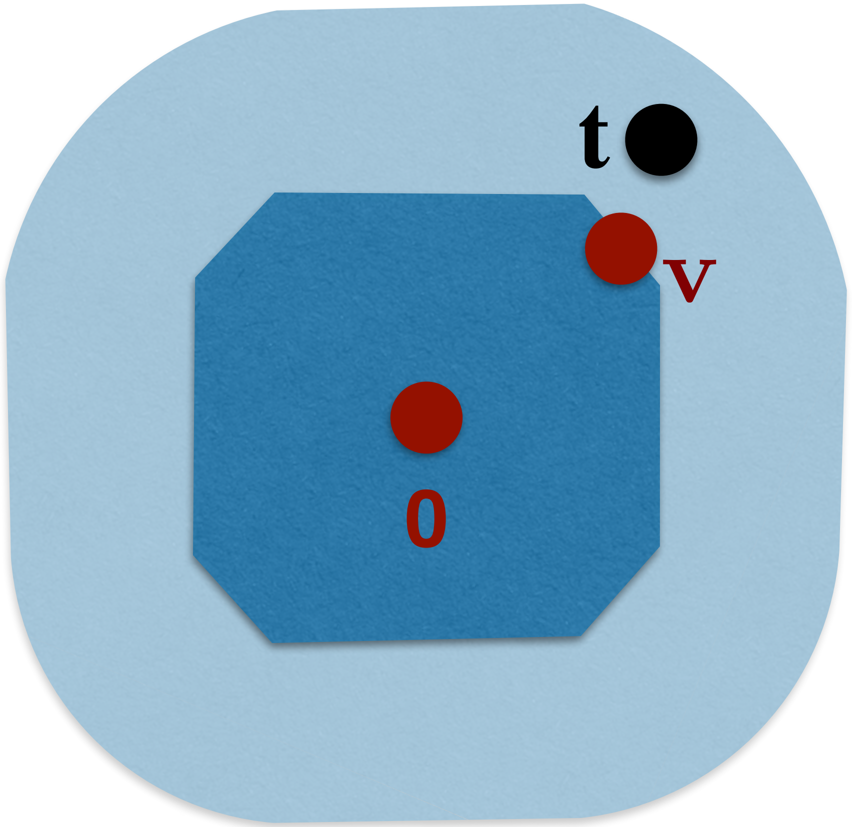

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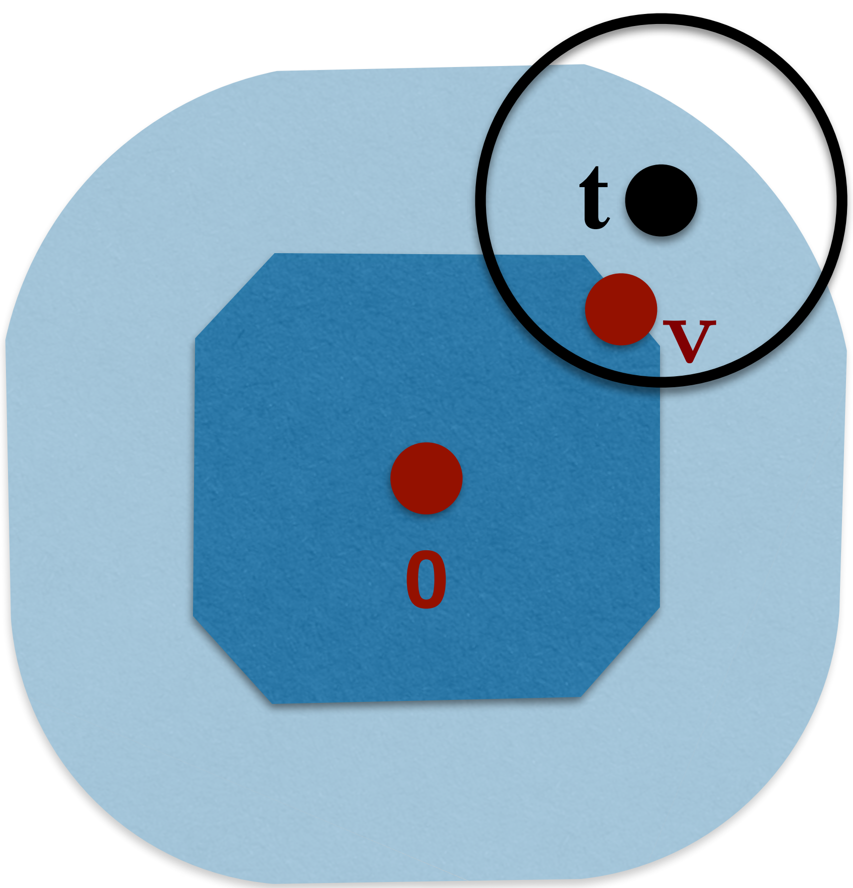

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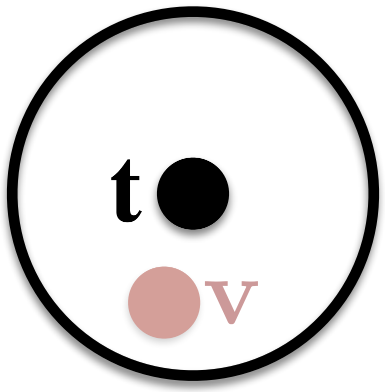
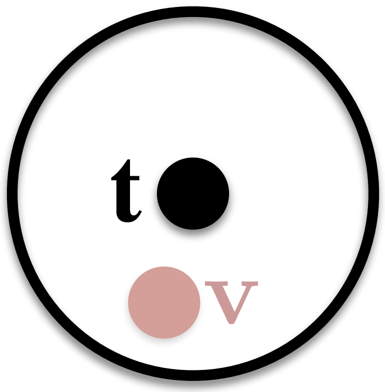


Rothvoss and Venzin

Step 2: Given $\mathbf{t} \in \mathbb{R}^n$ with $\|\mathbf{t} - \mathbf{v}\|_2 \leq 1$ for a K-shortest non-zero vector $\mathbf{v}$, use γ-CVP$_2$ oracle to find many "random" samples from $\mathbf{y}_1, \ldots, \mathbf{y}_N \in \mathcal{L} \cap (\gamma B_2 + \mathbf{t})$.



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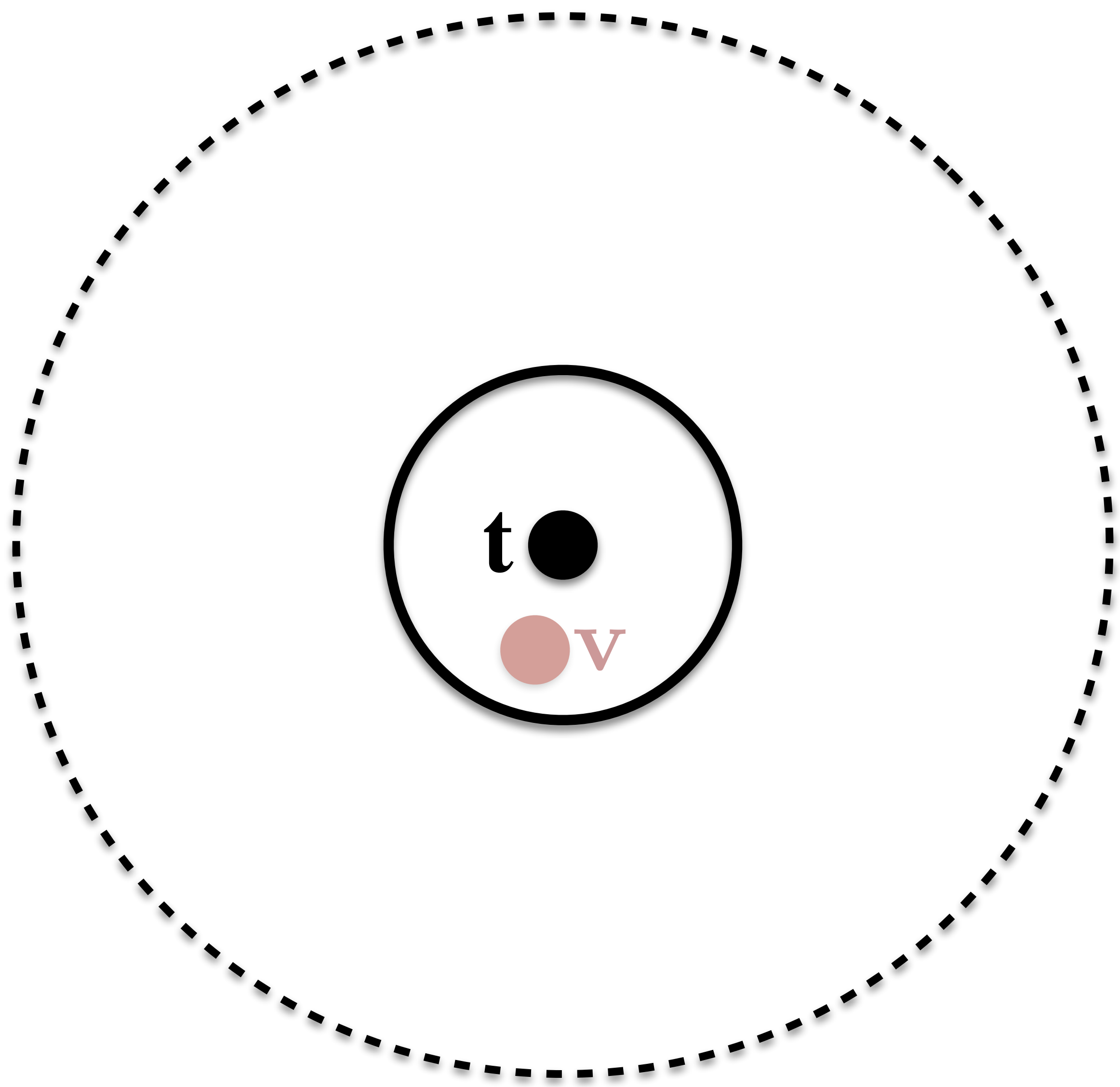

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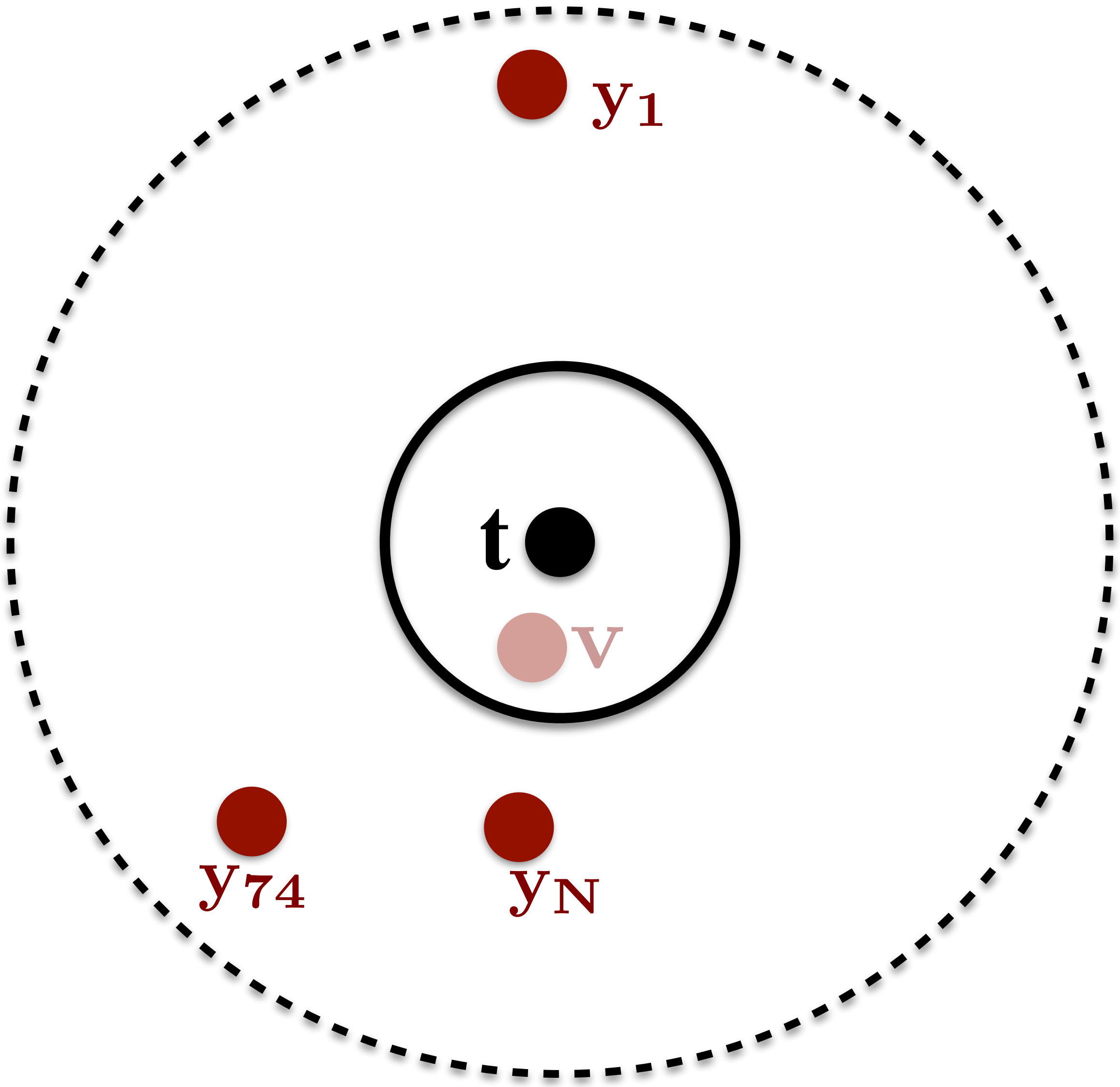

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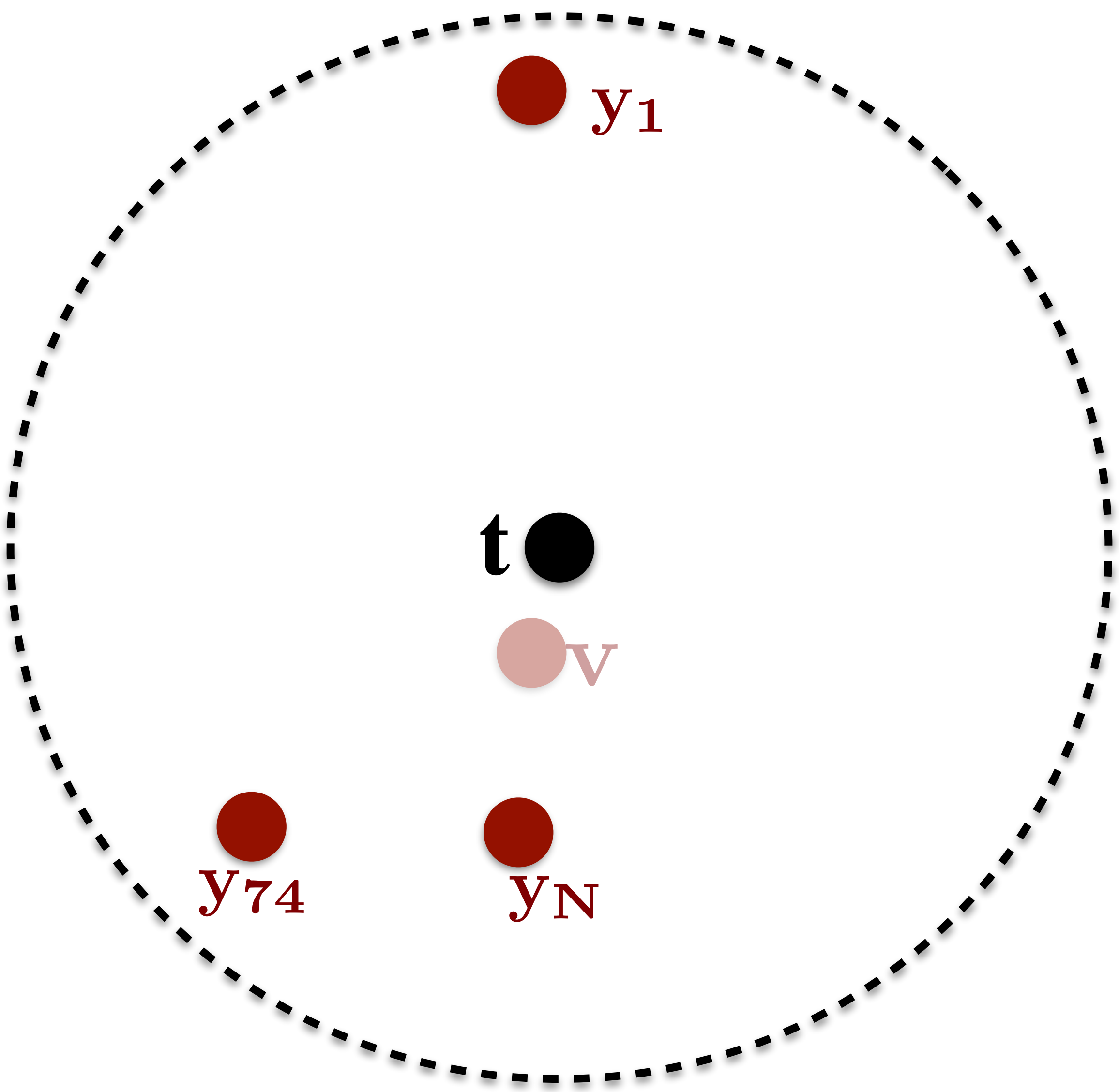

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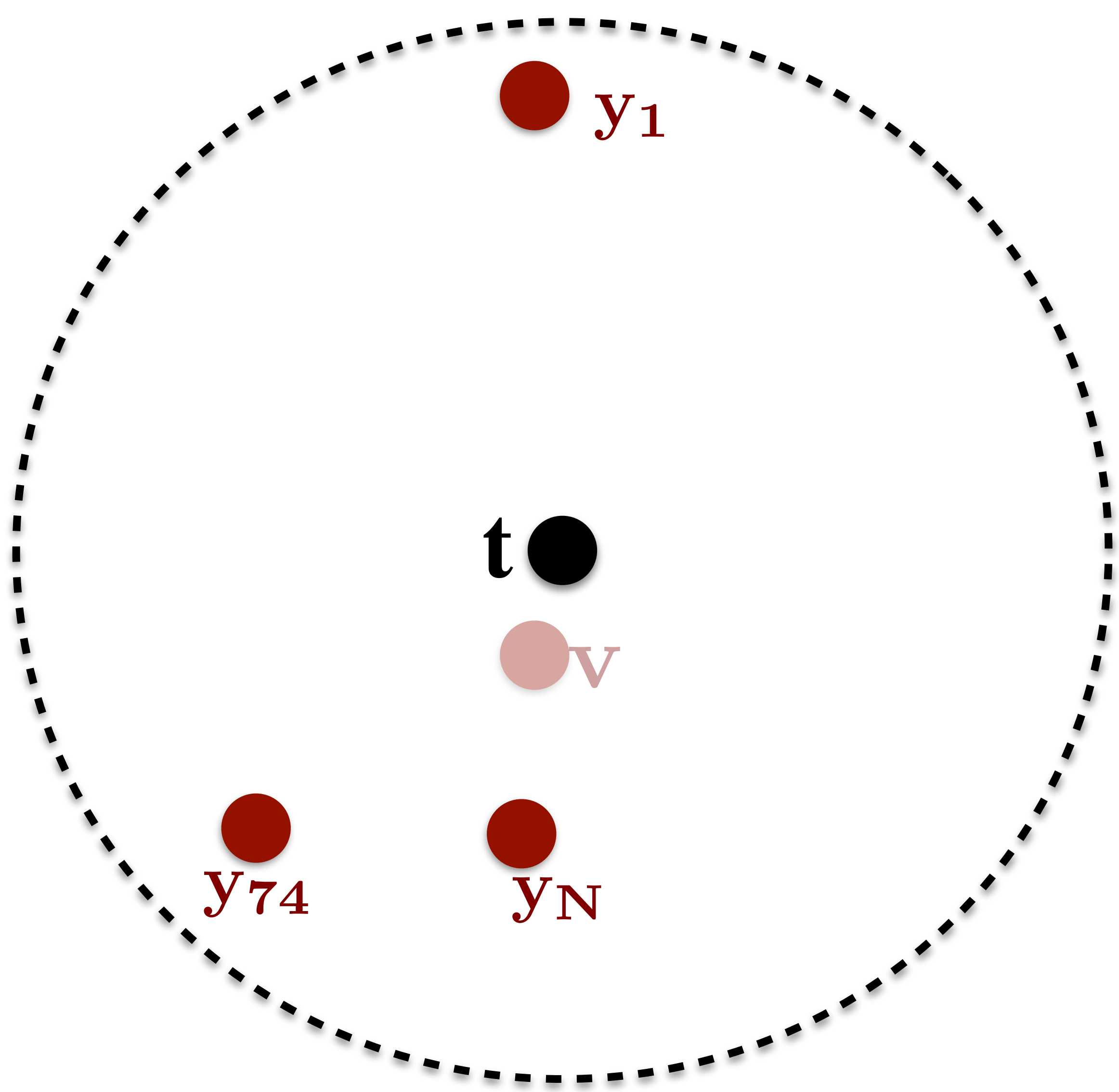
Step 3: Given a bunch of “random” lattice vectors $\mathbf{y}_1, \ldots, \mathbf{y}_N \in \mathcal{L} \cap (\gamma B_2 + \mathbf{t})$, output non-zero $\mathbf{y_i} - \mathbf{y_j}$ minimizing $\|\mathbf{y_i} - \mathbf{y_j}\|_K$ (or output $\mathbf{y_i}$ itself).



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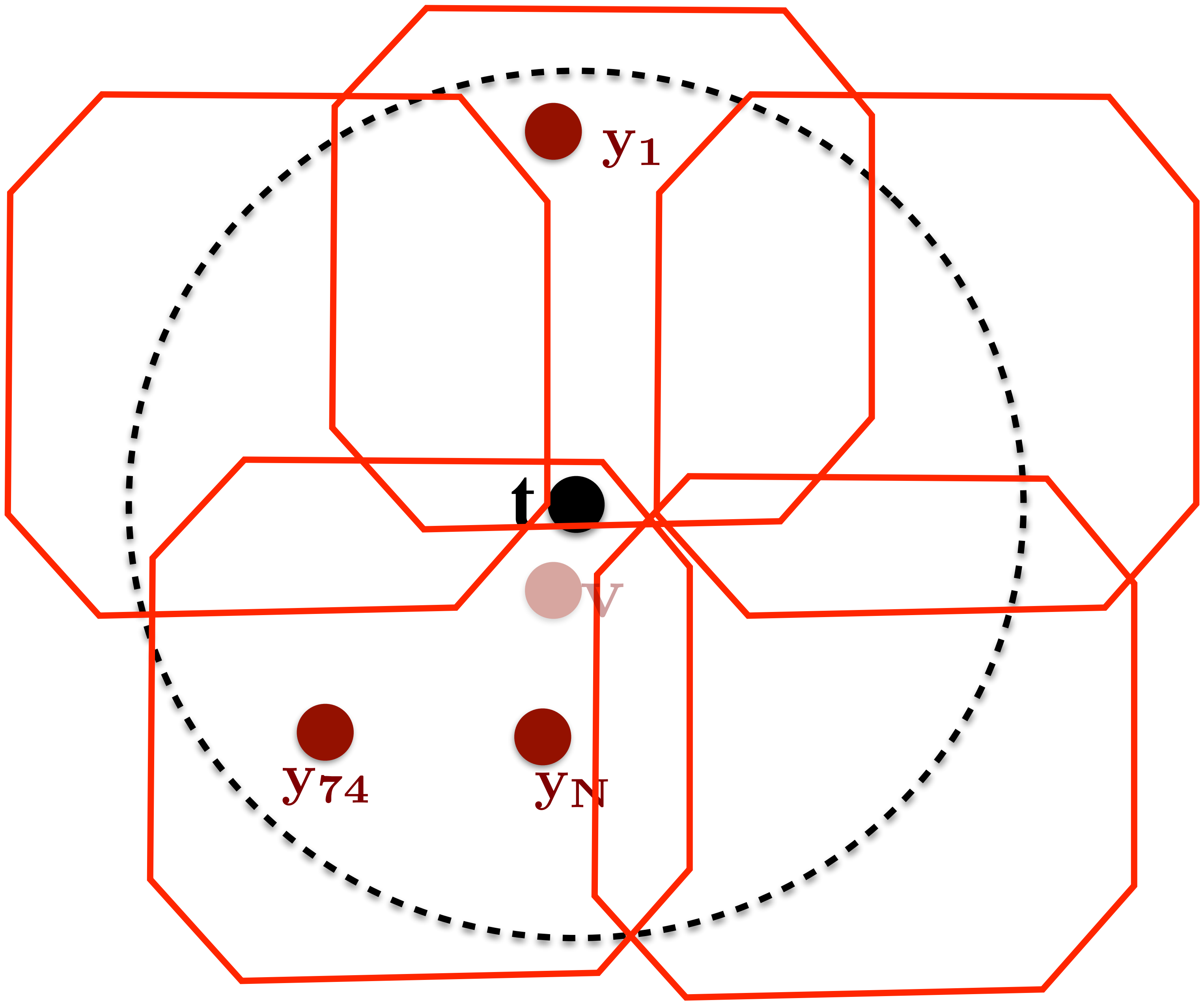

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Need for step 1: $\mathrm{vol}(K + B_2) \leq 2^{n/10}\mathrm{vol}(B_2)$.

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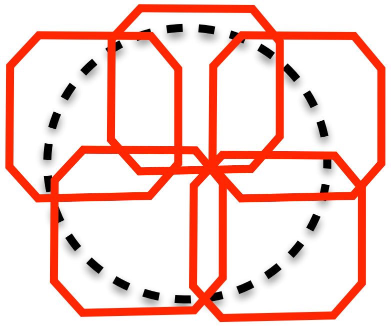
Need for step 3: B_2 can be covered by $2^{n/10}$ copies of $1000K$.



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If $K \approx B_2/20$, this works.

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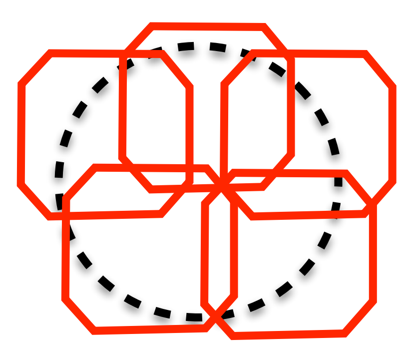

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Step 3: Given a bunch of random lattice vectors

Rothvoss and Venzin show how to find a linear transformation of any convex body so that the transformed body has these properties.

(Closely related to M-position. M = Milman)

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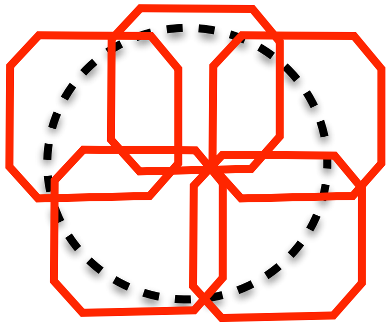
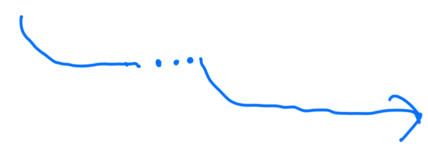


Summary

- The fastest algorithms for $O(1)$-CVP$_K$/$O(1)$-SVP$_K$ for any norm is $2^{0.802n}$!!
- We can reduce $O_\varepsilon(\gamma)$-SVP$_K$ to γ-CVP$_2$ in $2^{\varepsilon n}$ time for any K!!
- “Morally, lattice problems in any norm are equivalent up to a constant in the approximation factor!!”

Open Questions?

- ■ 1. Is there a dimension-preserving reduction from $O_\varepsilon(\gamma)$-SVP$_K$ in *any* norm to γ-SVP$_2$ in $2^{\varepsilon n}$ time? (Currently have to reduce to γ-CVP$_2$ or from γ-SVP$_p$ for $p \geq 2$.)
- ■ 2. What is the best running time for γ-SVP$_\infty$ for small constant γ?!!
  - What's going on with that wiggle? 
- ■ 3. More generally, what about γ-SVP$_K$??!
- ♦ 4. Is there a norm K for which sieving algorithms work particularly well…

● Easy

■ Medium

♦ Hard

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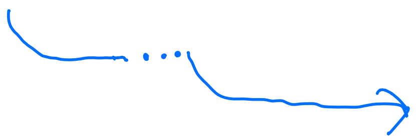

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