Self-tests of Physical Theories in Networks and their Implications for the Foundations of Quantum Theory

based on joint works with

Marc-Olivier Renou, David Trillo, Le Phuc Thinh, Armin Tavakoli, Nicolas Gisin, Antonio Acín, Miguel Navascués

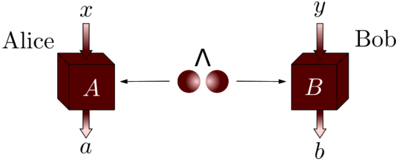
and with

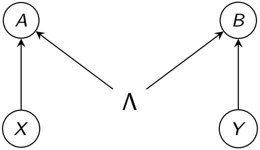
Roger Colbeck

IQI Vienna

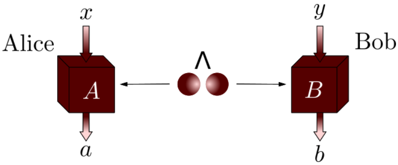
ÖAW

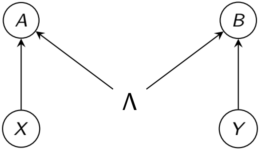
Bell Tests: Ruling out Local Hidden Variable Models by means of a Network

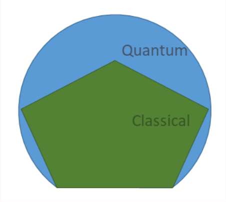




Bell Tests: Ruling out Local Hidden Variable Models by means of a Network



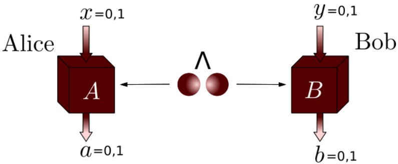


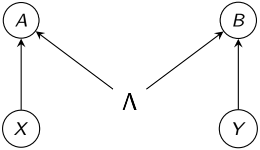


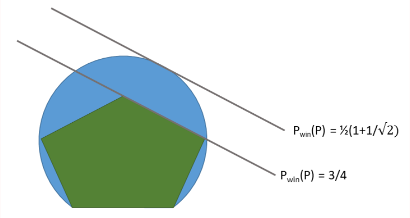
$P(ab|xy)$

$$P_C(ab|xy) = \sum_\lambda P(a|x,\lambda)P(b|y,\lambda)P(\lambda)$$
$$P_Q(ab|xy) = \mathrm{tr}(A_x^a \otimes B_y^b \rho_\Lambda)$$

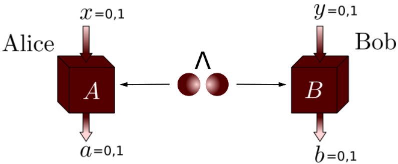
Bell Tests: Ruling out Local Hidden Variable Models by means of a Network

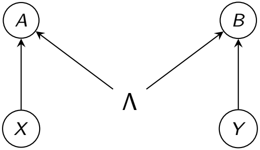


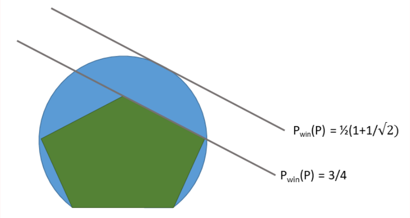




Bell Tests: Ruling out Local Hidden Variable Models by means of a Network







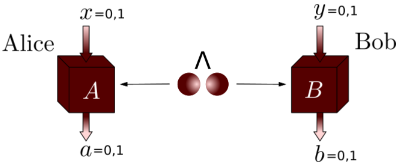
CHSH game with winning probability

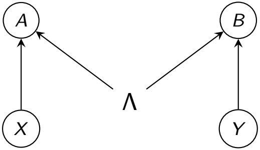
$$p_{\text{win}}(P) = \sum_{a,b,x,y} \frac{1}{4} P_{AB|xy}(a,b) Q(a,b,x,y)$$

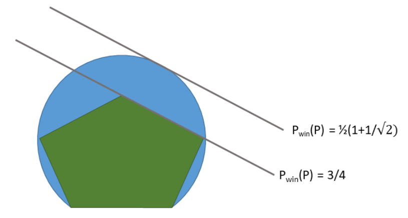
and winning condition

$$Q(a,b,x,y) = \delta(x \cdot y, a \oplus b).$$

Bell Tests: Ruling out Local Hidden Variable Models by means of a Network







CHSH game with winning probability

$$p_{\mathrm{win}}(P) = \sum_{a,b,x,y} \frac{1}{4} P_{AB|xy}(a,b) Q(a,b,x,y)$$

and winning condition

$$Q(a,b,x,y) = \delta(x \cdot y, a \oplus b).$$

- Optimal classical strategy: use Λ to prepare perfectly correlated outputs a, b.

Bell Tests: Ruling out Local Hidden Variable Models by means of a Network

LETTER

doi:10.1038/nature15759

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen[1,2], H. Bernien[1,2]†, A. E. Dréau[1,2], A. Reiserer[1,2], N. Kalb[1,2], M. S. Blok[1,2], J. Ruitenberg[1,2], R. F. L. Vermeulen[1,2], R. N. Schouten[1,2], C. Abellán[3], W. Amaya[3], V. Pruneri[3,4], M. W. Mitchell[3,4], M. Markham[5], D. J. Twitchen[5], D. Elkouss[1], S. Wehner[1], T. H. Taminiau[1,2] & R. Hanson[1,2]

More than 50 years ago[1], John Bell proved that no theory of nature that obeys locality and realism[2] can reproduce all the predictions of quantum theory: in any local-realist theory, the correlations

sufficiently separated such that locality prevents communication between the boxes during a trial, then the following inequality holds under local realism:

Bell Tests: Ruling out Local Hidden Variable Models by means of a Network

LETTER

doi:10.1038/nature15759

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen[1,2], H. Bernien[1,2]†, A. E. Dréau[1,2], A. Reiserer[1,2], N. Kalb[1,2], M. S. Blok[1,2], J. Ruitenberg[1,2], R. F. L. Vermeulen[1,2],
R. N. Sch
S. Wehne

More than
that obeys
quantum

PRL 115, 250401 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons

Marissa Giustina,[1,2,*] Marijn A. M. Versteegh,[1,2] Sören Wengerowsky,[1,2] Johannes Handsteiner,[1,2] Armin Hochrainer,[1,2] Kevin Phelan,[1] Fabian Steinlechner,[1] Johannes Kofler,[3] Jan-Åke Larsson,[4] Carlos Abellán,[5] Waldimar Amaya,[5] Valerio Pruneri,[5,6] Morgan W. Mitchell,[5,6] Jörn Beyer,[7] Thomas Gerrits,[8] Adriana E. Lita,[8] Lynden K. Shalm,[8] Sae Woo Nam,[8] Thomas Scheidl,[1,2] Rupert Ursin,[1] Bernhard Wittmann,[1,2] and Anton Zeilinger[1,2,†]

[1]*Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, Boltzmanngasse 3, Vienna 1090, Austria*
[2]*Quantum Optics, Quantum Nanophysics and Quantum Information, Faculty of Physics, University of Vienna, Boltzmanngasse 5, Vienna 1090, Austria*
[3]*Max-Planck-Institute of Quantum Optics, Hans-Kopfermann-Straße 1, 85748 Garching, Germany*
[4]*Institutionen för Systemteknik, Linköpings Universitet, 581 83 Linköping, Sweden*
[5]*ICFO – Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels, Barcelona, Spain*
[6]*ICREA – Institució Catalana de Recerca i Estudis Avançats, 08015 Barcelona, Spain*
[7]*Physikalisch-Technische Bundesanstalt, Abbestraße 1, 10587 Berlin, Germany*
[8]*National Institute of Standards and Technology (NIST), 325 Broadway, Boulder, Colorado 80305, USA*

(Received 10 November 2015; published 16 December 2015)

LETTER

doi:10.1038/nature15759

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen[1,2], H. Bernien[1,2]†, A. E. Dréau[1,2], A. Reiserer[1,2], N. Kalb[1,2], M. S. Blok[1,2], J. Ruitenberg[1,2], R. F. L. Vermeulen[1,2],
R. N. Sch
S. Wehne

More than
that obeys
quantum

PRL 115, 250401 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons

Marissa Giustina,[1,2,*] Marijn A. M. Versteegh,[1,2] Sören Wengerowsky,[1,2] Johannes Handsteiner,[1,2] Armin Hochrainer,[1,2]

Spain

PRL 115, 250402 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Strong Loophole-Free Test of Local Realism*

Lynden K. Shalm,[1,†] Evan Meyer-Scott,[2] Bradley G. Christensen,[3] Peter Bierhorst,[1] Michael A. Wayne,[3,4] Martin J. Stevens,[1] Thomas Gerrits,[1] Scott Glancy,[1] Deny R. Hamel,[5] Michael S. Allman,[1] Kevin J. Coakley,[1] Shellee D. Dyer,[1] Carson Hodge,[1] Adriana E. Lita,[1] Varun B. Verma,[1] Camilla Lambrocco,[1] Edward Tortorici,[1] Alan L. Migdall,[4,6] Yanbao Zhang,[2] Daniel R. Kumor,[3] William H. Farr,[7] Francesco Marsili,[7] Matthew D. Shaw,[7] Jeffrey A. Stern,[7] Carlos Abellán,[8] Waldimar Amaya,[8] Valerio Pruneri,[8,9] Thomas Jennewein,[2,10] Morgan W. Mitchell,[8,9] Paul G. Kwiat,[3] Joshua C. Bienfang,[4,6] Richard P. Mirin,[1] Emanuel Knill,[1] and Sae Woo Nam[1,‡]

[1]*National Institute of Standards and Technology, 325 Broadway, Boulder, Colorado 80305, USA*
[2]*Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, 200 University Avenue West, Waterloo, Ontario, Canada, N2L 3G1*
[3]*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[4]*National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA*

Bell Tests: Ruling out Local Hidden Variable Models by means of a Network

LETTER

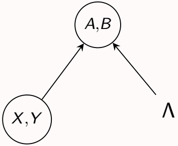
doi:10.1038/nature15759

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen[1,2], H. Bernien[1,2]†, A. E. Dréau[1,2], A. Reiserer[1,2], N. Kalb[1,2], M. S. Blok[1,2], J. Ruitenberg[1,2], R. F. L. Vermeulen[1,2],
R. N. Sch
S. Wehne

PRL 115, 250401 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

The network is crucial!



$P(ab|xy)$

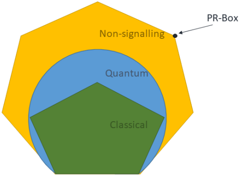
[1]National Institute of Standards and Technology, 325 Broadway, Boulder, Colorado 80305, USA
[2]Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, 200 University Avenue West, Waterloo, Ontario, Canada, N2L 3G1
[3]Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA
[4]National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA

Physical Principles Underlying Quantum Theory?

- Requiring no superluminal signals is not sufficient for singling out quantum correlations.

$$\sum_a P(ab|xy) = \sum_a P(ab|x'y) \quad \forall\ x, x', b, y,$$

$$\sum_b P(ab|xy) = \sum_b P(ab|xy') \quad \forall\ a, x, y, y'$$



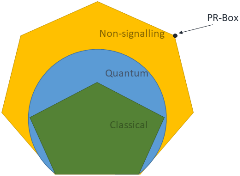
Popescu & Rohrlich, Foundations of Physics 24, 379, 1994.

Physical Principles Underlying Quantum Theory?

- Requiring no superluminal signals is not sufficient for singling out quantum correlations.

$$\sum_a P(ab|xy) = \sum_a P(ab|x'y) \quad \forall\, x, x', b, y,$$

$$\sum_b P(ab|xy) = \sum_b P(ab|xy') \quad \forall\, a, x, y, y'$$



- Various (information-theoretic) physical principles towards recovering quantum correlations.

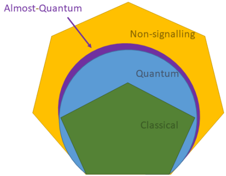
Popescu & Rohrlich, Foundations of Physics 24, 379, 1994.

Physical Principles Underlying Quantum Theory?

- Requiring no superluminal signals is not sufficient for singling out quantum correlations.

$$\sum_a P(ab|xy) = \sum_a P(ab|x'y) \quad \forall\, x, x', b, y,$$

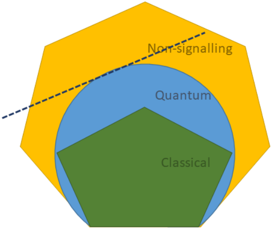
$$\sum_b P(ab|xy) = \sum_b P(ab|xy') \quad \forall\, a, x, y, y'$$

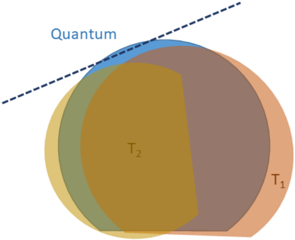


- Various (information-theoretic) physical principles towards recovering quantum correlations.

Popescu & Rohrlich, Foundations of Physics 24, 379, 1994.
Navascués et al., Nature communications 6, 6288, 2015.

Networks Enable New Approaches to Singling out Quantum Theory





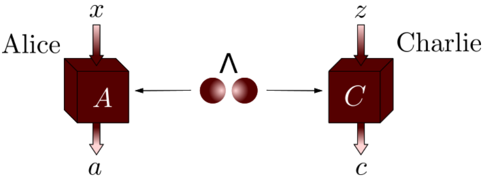
Goal: find a network and information processing task where quantum correlations are (uniquely) extremal.

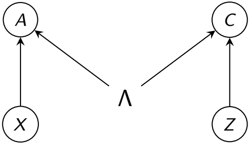
⟶ Rule out all other generalised probabilistic theories experimentally.

⟶ Point to a physical principle underlying quantum theory.

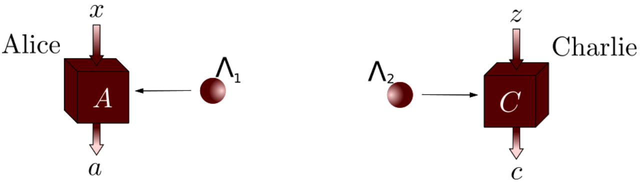
MW, R. Colbeck, PRL 125, 060406 (2020) and PRA 102, 022203 (2020).

Looking at the Bilocal Network through the Adaptive CHSH Game

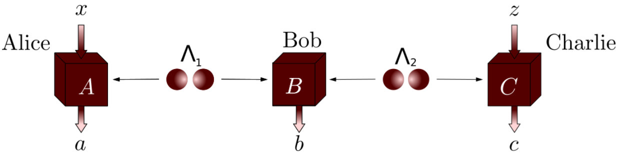




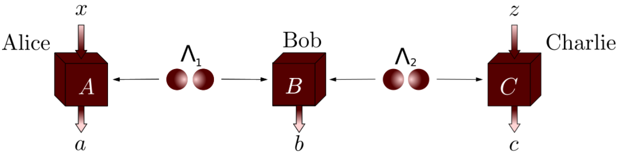
Looking at the Bilocal Network through the Adaptive CHSH Game

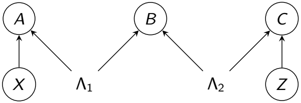


Looking at the Bilocal Network through the Adaptive CHSH Game

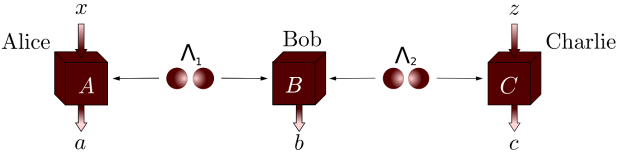


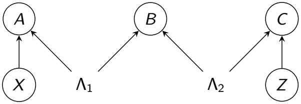
Looking at the Bilocal Network through the Adaptive CHSH Game





Looking at the Bilocal Network through the Adaptive CHSH Game





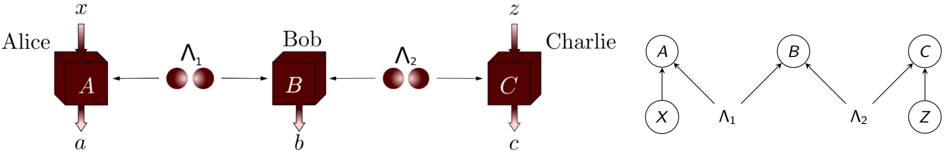
- Adaptive CHSH game with winning probability

$$p_{\mathrm{win}}(P) = \sum_{a,b,c,x,z} \frac{1}{4} P_{ABC|xz}(a,b,c) Q(a,b,c,x,z)$$

and $Q(a,b,c,x,z) = 1$ iff

B	condition for A and C
$b = (0,0)$	$(x \oplus 1) \cdot z = a \oplus c$
$b = (0,1)$	$(x \oplus 1) \cdot (z \oplus 1) \oplus 1 = a \oplus c$
$b = (1,0)$	$(x \oplus 1) \cdot (z \oplus 1) = a \oplus c$
$b = (1,1)$	$(x \oplus 1) \cdot z \oplus 1 = a \oplus c$

Looking at the Bilocal Network through the Adaptive CHSH Game



- Adaptive CHSH game with winning probability

$$p_{\mathrm{win}}(P) = \sum_{a,b,c,x,z} \frac{1}{4} P_{ABC|xz}(a,b,c) Q(a,b,c,x,z)$$

and $Q(a,b,c,x,z) = 1$ iff

B	condition for A and C
$b = (0,0)$	$(x \oplus 1) \cdot z = a \oplus c$
$b = (0,1)$	$(x \oplus 1) \cdot (z \oplus 1) \oplus 1 = a \oplus c$
$b = (1,0)$	$(x \oplus 1) \cdot (z \oplus 1) = a \oplus c$
$b = (1,1)$	$(x \oplus 1) \cdot z \oplus 1 = a \oplus c$

- Optimal quantum strategy at $p_{win}(P) = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$.

MW, R. Colbeck, PRL 125, 060406 (2020) and PRA 102, 022203 (2020).

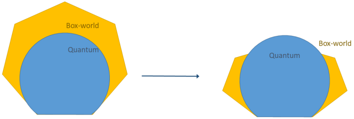
Results Enabled by the Bilocal Network

- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on system composition.[1]
- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on single system state spaces.[1]

[1] MW, R. Colbeck, PRL 125, 060406 (2020) and PRA 102, 022203 (2020).

Results Enabled by the Bilocal Network

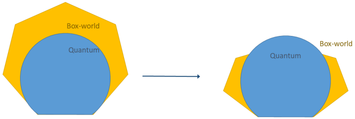
- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on system composition.[1]
- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on single system state spaces.[1]



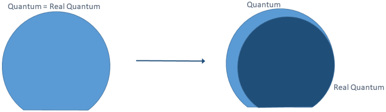
[1] MW, R. Colbeck, PRL 125, 060406 (2020) and PRA 102, 022203 (2020).

Results Enabled by the Bilocal Network

- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on system composition.[1]
- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on single system state spaces.[1]



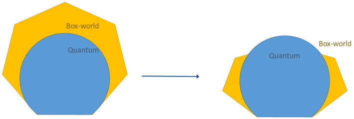
- Bilocal experiment to rule out quantum theory over real Hilbert spaces.[2]



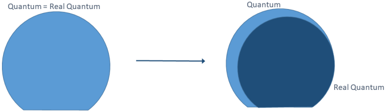
[1] MW, R. Colbeck, PRL 125, 060406 (2020) and PRA 102, 022203 (2020).
[2] M.O. Renou, D. Trillo, MW, T. P. Le, A. Tavakoli, N. Gisin, A. Acín, M. Navascués, Nature 600, (2021).

Results Enabled by the Bilocal Network

- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on system composition.[1]
- Proposed experiment to rule out various "exotic" generalised probabilistic theories by adaptive CHSH game relying on single system state spaces.[1]



- Bilocal experiment to rule out quantum theory over real Hilbert spaces.[2]



[1] MW, R. Colbeck, PRL 125, 060406 (2020) and PRA 102, 022203 (2020).
[2] M.O. Renou, D. Trillo, MW, T. P. Le, A. Tavakoli, N. Gisin, A. Acín, M. Navascués, Nature 600, (2021).

Complex Quantum Theory vs. Real Quantum Theory

- **States:** of a system A,

$$\mathcal{S}_A = \left\{ \rho \in \mathcal{B}(\mathcal{H}^A_{\mathbb{C}}) \mid \rho \geq 0, \text{tr}(\rho) = 1 \right\},$$

for a complex Hilbert space $\mathcal{H}^A_{\mathbb{C}}$.

- **Composition:** of independent systems $\rho_A \in \mathcal{S}_A$ and $\rho_B \in \mathcal{S}_B$,

$$\rho_{AB} = \rho_A \otimes \rho_B,$$

more generally,

$$\mathcal{S}_{AB} = \left\{ \rho \in \mathcal{B}(\mathcal{H}^A_{\mathbb{C}} \otimes \mathcal{H}^B_{\mathbb{C}}) | \rho \geq 0, \text{tr}(\rho) = 1 \right\}.$$

- **Evolution:** of a state ρ_{AB} is unitary

$$\rho'_{AB} = U_{AB} \rho_{AB} U^{\dagger}_{AB}.$$

- **Measurement:** x on AB, given by an observable $A_x = \sum_a a A^a_x$, with projectors A^a_x such that $\sum_a A^a_x = \mathbb{I}_{AB}$. The probability to observe a on ρ_{AB} is

$$P(a|x) = \text{tr}(A^a_x \rho_{AB}).$$

- **States:** of a system A,

$$\mathcal{S}_A = \left\{ \rho \in \mathcal{B}(\mathcal{H}^A_{\mathbb{R}}) \mid \rho \geq 0, \text{tr}(\rho) = 1 \right\},$$

for a real Hilbert space $\mathcal{H}^A_{\mathbb{R}}$.

- **Composition:** of independent systems $\rho_A \in \mathcal{S}_A$ and $\rho_B \in \mathcal{S}_B$,

$$\rho_{AB} = \rho_A \otimes \rho_B,$$

more generally,

$$\mathcal{S}_{AB} = \left\{ \rho \in \mathcal{B}(\mathcal{H}^A_{\mathbb{R}} \otimes \mathcal{H}^B_{\mathbb{R}}) | \rho \geq 0, \text{tr}(\rho) = 1 \right\}.$$

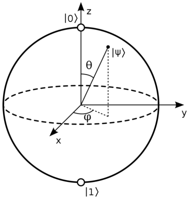
- **Evolution:** of a state ρ_{AB} is orthogonal

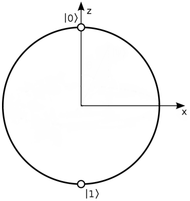
$$\rho'_{AB} = U_{AB} \rho_{AB} U^{T}_{AB}.$$

- **Measurement:** x on AB, given by a real observable $\tilde{A}_x = \sum_a a \tilde{A}^a_x$, with projectors $\tilde{A}^a_x$ such that $\sum_a \tilde{A}^a_x = \mathbb{I}_{AB}$. The probability to observe a on ρ_{AB} is

$$P(a|x) = \text{tr}(\tilde{A}^a_x \rho_{AB}).$$

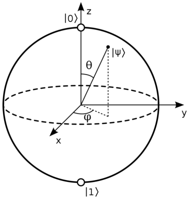
Real and Complex Quantum Theory have Different Properties

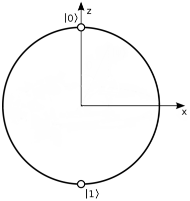




- Qubit: $\rho = \frac{1}{2}\left(\mathcal{I} + x\sigma_x + y\sigma_y + z\sigma_z\right)$.
- Rebit: $\rho = \frac{1}{2}\left(\mathcal{I} + x\sigma_x + z\sigma_z\right)$.

Real and Complex Quantum Theory have Different Properties





- Qubit: $\rho = \frac{1}{2}\left(\mathcal{I} + x\sigma_x + y\sigma_y + z\sigma_z\right)$.
- Local Tomography: 2-qubit states characterised by local σ_x, σ_y, σ_z measurements on each qubit.

- Rebit: $\rho = \frac{1}{2}\left(\mathcal{I} + x\sigma_x + z\sigma_z\right)$.
- 2-rebit states not fully characterised σ_x, σ_z on separate rebits.

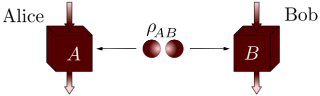
Example: for the *real* state $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ we have

$$|\psi^+\rangle\langle\psi^+| = \frac{1}{2}\left(\mathcal{I} + \sigma_x\otimes\sigma_x - \sigma_y\otimes\sigma_y + \sigma_z\otimes\sigma_z\right).$$

$\longrightarrow$ No Local tomography !

Real Simulations of Quantum Theory in Multi-Party Scenarios

Quantum correlations from local measurements

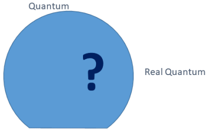


$$P(a,b|x,y) = \mathrm{tr}\left(A_x^a \otimes B_y^b \rho_{AB}\right).$$

Real Simulations of Quantum Theory in Multi-Party Scenarios

Quantum correlations from local measurements

$$P(a,b|x,y) = \mathrm{tr}\left(A_x^a \otimes B_y^b \rho_{AB}\right).$$

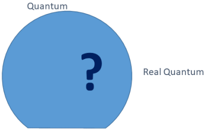


E. C. G. Stuckelberg, Helvetica Physica Acta, 33 (1960).
M. McKague, M. Mosca, N. Gisin, PRL 102, 020505 (2009).

Real Simulations of Quantum Theory in Multi-Party Scenarios

Quantum correlations from local measurements

$$P(a, b|x, y) = \mathrm{tr}\left(A_x^a \otimes B_y^b \rho_{AB}\right).$$



Real simulation on a larger Hilbert space preserves locality of measurements

$$P(a, b|x, y) = \mathrm{tr}(\tilde{A}_x^a \otimes \tilde{B}_y^b \tilde{\rho}_{AB})$$

using the real states and measurements (using $|\pm i\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm i\,|1\rangle)$)

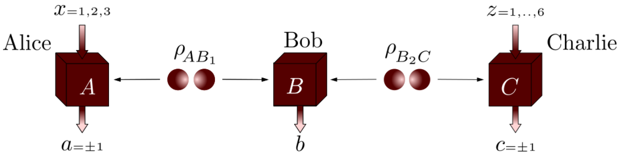
$$\tilde{\rho}_{AB} = \frac{1}{2}\left(\rho_{AB} \otimes |i\rangle\langle i|_{A'B'}^{\otimes 2} + \rho_{AB}^{*} \otimes |{-i}\rangle\langle {-i}|_{A'B'}^{\otimes 2}\right)$$

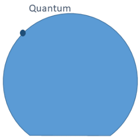
$$\tilde{A}_x^a = A_x^a \otimes |i\rangle\langle i|_{A'} + (A_x^a)^* \otimes |{-i}\rangle\langle {-i}|_{A'}$$

$$\tilde{B}_y^b = B_y^b \otimes |i\rangle\langle i|_{B'} + (B_y^b)^* \otimes |{-i}\rangle\langle {-i}|_{B'}.$$

E. C. G. Stuckelberg, Helvetica Physica Acta, 33 (1960).
M. McKague, M. Mosca, N. Gisin, PRL 102, 020505 (2009).

Candidate Complex Quantum Distribution





Candidate Distribution:

$$\bar{P}(a,b,c|x,z) = \text{tr}\left(A_x^a \otimes B^b \otimes C_z^c (|\psi^+\rangle\langle\psi^+|_{AB_1} \otimes |\psi^+\rangle\langle\psi^+|_{B_2C})\right),$$

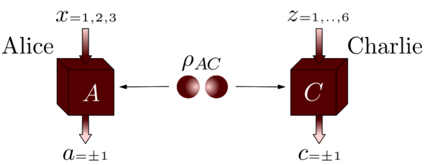
in terms of a Bell state measurement B and the observables

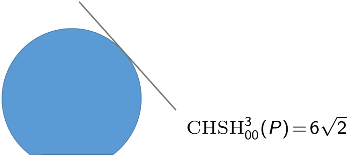
$$A_1 = \sigma_Z,\ A_2 = \sigma_X,\ A_3 = \sigma_Y,$$
$$C_1 = D_{zx},\ C_2 = E_{zx},\ C_3 = D_{zy},\ C_4 = E_{zy},\ C_5 = D_{xy},\ C_6 = E_{xy}.$$

where $D_{ij} = \frac{\sigma_i + \sigma_j}{\sqrt{2}}$, $E_{ij} = \frac{\sigma_i - \sigma_j}{\sqrt{2}}$.

M.O. Renou, D. Trillo, MW, T. P. Le, A. Tavakoli, N. Gisin, A. Acín, M. Navascués, Nature 600, (2021).

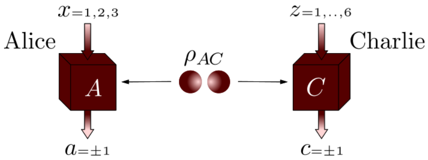
Intuition: Scenario that Requires Complex Quantum Measurements

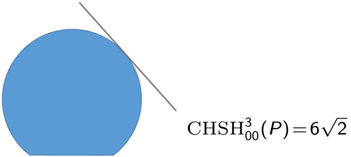




J. Bowles, I. Supić, D. Cavalcanti, A. Acín, PRA 98, 042336 (2018).

Intuition: Scenario that Requires Complex Quantum Measurements





- **Self-testing Pauli measurements:** If we have a quantum $P(a, c|x, z)$, such that

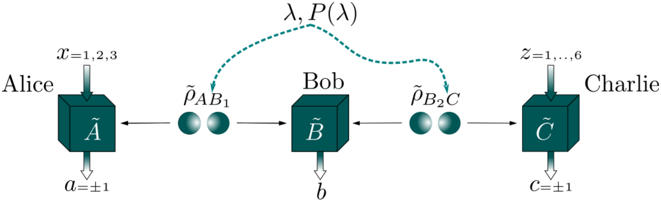
$$\mathrm{CHSH}^3_{00}(P) = \mathrm{CHSH}_{A_1,A_2,C_1,C_2}(P) + \mathrm{CHSH}_{A_1,A_3,C_3,C_4}(P) + \mathrm{CHSH}_{A_2,A_3,C_5,C_6}(P) = 6\sqrt{2},$$

then this self-tests $|\psi^+\rangle_{AC}$ and Alice's observables

$$A_1 = \sigma_Z, \ A_2 = \sigma_X, \ A_3 = \sigma_Y.$$

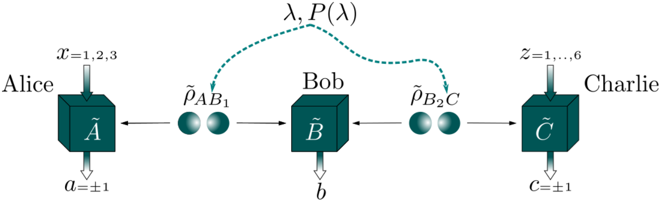
J. Bowles, I. Supić, D. Cavalcanti, A. Acín, PRA 98, 042336 (2018).

Result: Candidate Distribution Not Reproducible in Real Quantum Theory



M.O. Renou, D. Trillo, MW, T. P. Le, A. Tavakoli, N. Gisin, A. Acín, M. Navascués, Nature 600, (2021).

Result: Candidate Distribution Not Reproducible in Real Quantum Theory



Proposition: $\bar{P}$ does not admit a decomposition

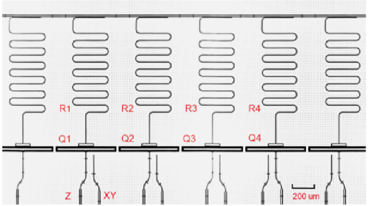
$$\bar{P}(a,b,c|x,z) = \sum_{\lambda} P(\lambda)\,\mathrm{tr}\left(\tilde{A}_x^a \otimes \tilde{B}^b \otimes \tilde{C}_z^c(\tilde{\rho}^{\lambda}_{AB_1} \otimes \tilde{\rho}^{\lambda}_{B_2C})\right)$$

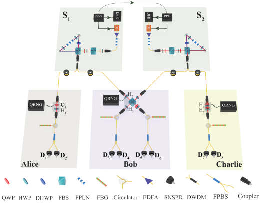
with real states $\tilde{\rho}^{\lambda}_{AB_1}, \tilde{\rho}^{\lambda}_{B_2C}$ and real measurements $\tilde{A}_x, \tilde{B}, \tilde{C}_z$ (of any dimension).

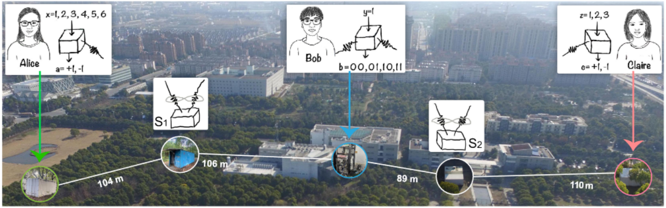
- The result can be made noise robust for an experimental test.

M.O. Renou, D. Trillo, MW, T. P. Le, A. Tavakoli, N. Gisin, A. Acín, M. Navascués, Nature 600, (2021).

Experimental Implementations







M.-C. Chen et al. "Ruling out real-number description of quantum mechanics", PRL 128, 040403 (2022).
Z.-Da. Li et al. "Testing real quantum theory in an optical quantum network", PRL 128, 040402 (2022).
D. Wu et al. "Experimental refutation of real-valued quantum mechanics under strict locality conditions", arXiv:2201.04177.

Summary and Open Questions

- Causal networks enable the design of experiments to single out quantum theory among various generalised probabilistic theories (including more non-local ones).
- Real and Complex Quantum Theory lead to different experimental predictions in scenarios where multiple states are independently prepared, which led to the experimental refutation of Real Quantum Theory (with loopholes).

Summary and Open Questions

- Causal networks enable the design of experiments to single out quantum theory among various generalised probabilistic theories (including more non-local ones).
- Real and Complex Quantum Theory lead to different experimental predictions in scenarios where multiple states are independently prepared, which led to the experimental refutation of Real Quantum Theory (with loopholes).
- Gaps to other potential contenders of quantum theory. Is there a network and task where quantum theory is uniquely optimal?
- New axiomatic frameworks recovering quantum theory?
- Other applications of causal networks in quantum foundations?

Thank you for your attention!

Main Idea of the Proof

- Applying real local isometries $U_{A\to AA'A''}$ and $V_{C\to CC'C''}$ to $\sum_\lambda P(\lambda)\tilde{\rho}^\lambda_{AB_1}\otimes\tilde{\rho}^\lambda_{B_2C}$ leads to a state

$$\tilde{\rho}_{A'C'} = \sum_\lambda P(\lambda)\,\mathrm{tr}_{AA''CC''}\left(U\tilde{\rho}^\lambda_A U^\dagger \otimes V\tilde{\rho}^\lambda_C V^\dagger\right).$$

[Such states satisfy $\tilde{\rho}^{T_{C'}}_{A'C'} = \tilde{\rho}_{A'C'}$.]

Main Idea of the Proof

- Applying real local isometries $U_{A\to AA'A''}$ and $V_{C\to CC'C''}$ to $\sum_\lambda P(\lambda)\tilde{\rho}^\lambda_{AB_1}\otimes\tilde{\rho}^\lambda_{B_2C}$ leads to a state

$$\tilde{\rho}_{A'C'} = \sum_\lambda P(\lambda)\,\mathrm{tr}_{AA''CC''}\left(U\tilde{\rho}^\lambda_A U^\dagger \otimes V\tilde{\rho}^\lambda_C V^\dagger\right).$$

[Such states satisfy $\tilde{\rho}^{T_{C'}}_{A'C'} = \tilde{\rho}_{A'C'}$.]

- We show that if we find a real strategy to recover $\bar{P}$ there are specific real isometries $\tilde{U}_{A\to AA'A''}$, $\tilde{V}_{C\to CC'C''}$ such that

$$\tilde{\rho}_{A'C'} = \frac{|i\rangle\langle i|^{\otimes 2} + |-i\rangle\langle -i|^{\otimes 2}}{2}$$

.
The state $\tilde{\rho}_{A'C'}$ is not of the above form, which we see from $\tilde{\rho}^{T_{C'}}_{A'C'} \neq \tilde{\rho}_{A'C'}$.

Main Idea of the Proof

- Applying real local isometries $U_{A\to AA'A''}$ and $V_{C\to CC'C''}$ to $\sum_\lambda P(\lambda)\tilde{\rho}^{\lambda}_{AB_1}\otimes\tilde{\rho}^{\lambda}_{B_2C}$ leads to a state

$$\tilde{\rho}_{A'C'} = \sum_{\lambda} P(\lambda)\,\mathrm{tr}_{AA''CC''}\left(U\tilde{\rho}^{\lambda}_{A}U^{\dagger}\otimes V\tilde{\rho}^{\lambda}_{C}V^{\dagger}\right).$$

  [Such states satisfy $\tilde{\rho}^{T_{C'}}_{A'C'} = \tilde{\rho}_{A'C'}$.]

- We show that if we find a real strategy to recover $\bar{P}$ there are specific real isometries $\tilde{U}_{A\to AA'A''}$, $\tilde{V}_{C\to CC'C''}$ such that

$$\tilde{\rho}_{A'C'} = \frac{|i\rangle\langle i|^{\otimes 2} + |-i\rangle\langle -i|^{\otimes 2}}{2}$$

  .

  The state $\tilde{\rho}_{A'C'}$ is not of the above form, which we see from $\tilde{\rho}^{T_{C'}}_{A'C'} \neq \tilde{\rho}_{A'C'}$.

$\longrightarrow$ Contradiction!