Device-independent protocols from computational assumptions

Tony Metger (ETH Zürich)

Self-testing: arXiv:2001.09161, with Thomas Vidick

DIQKD: arXiv:2010.04175, with Rotem Arnon-Friedman, Andrea Coladangelo, and Yfke Dulek

Outline

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↑ replaces Bell inequality violation

Device-independent QKD

Ekert, Quantum cryptography based on Bell's theorem, PRL 67, 661 (1991).
Mayers & Yao, Quantum cryptography with imperfect apparatus, FOCS 1998.

Device-independent QKD

Eve

Alice

Bob

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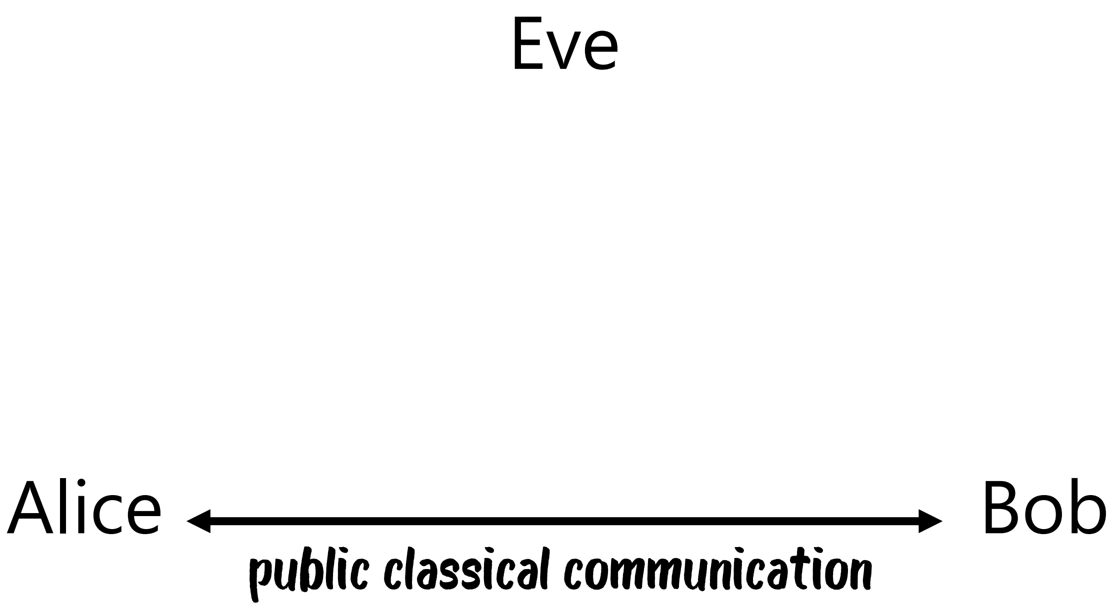
Device-independent QKD



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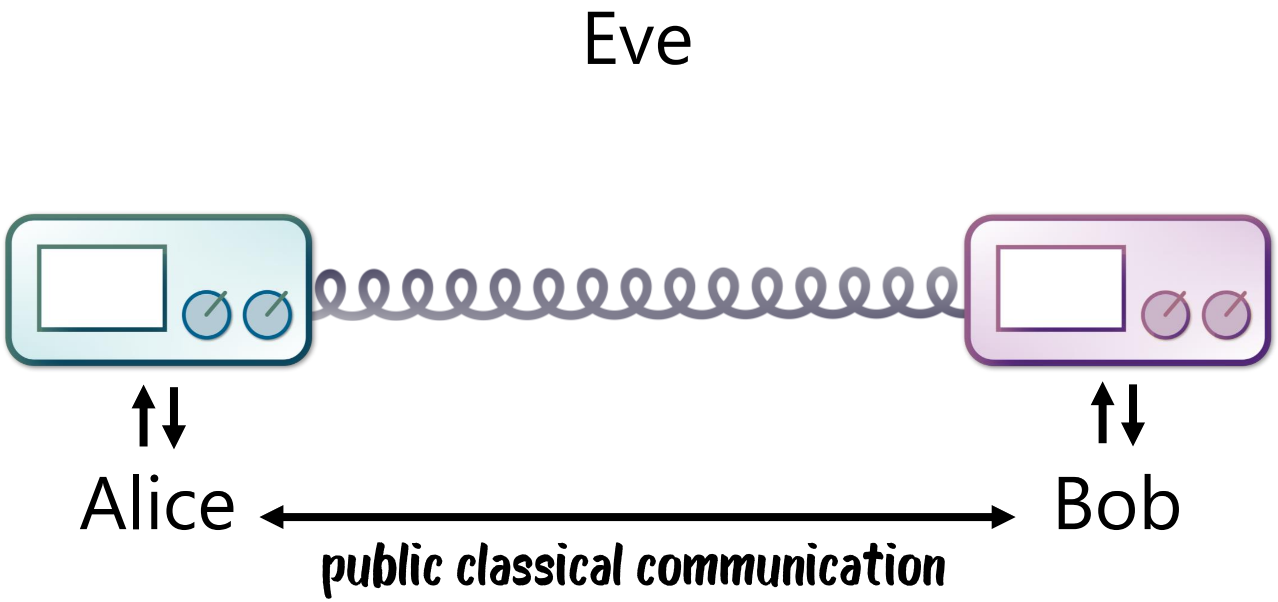

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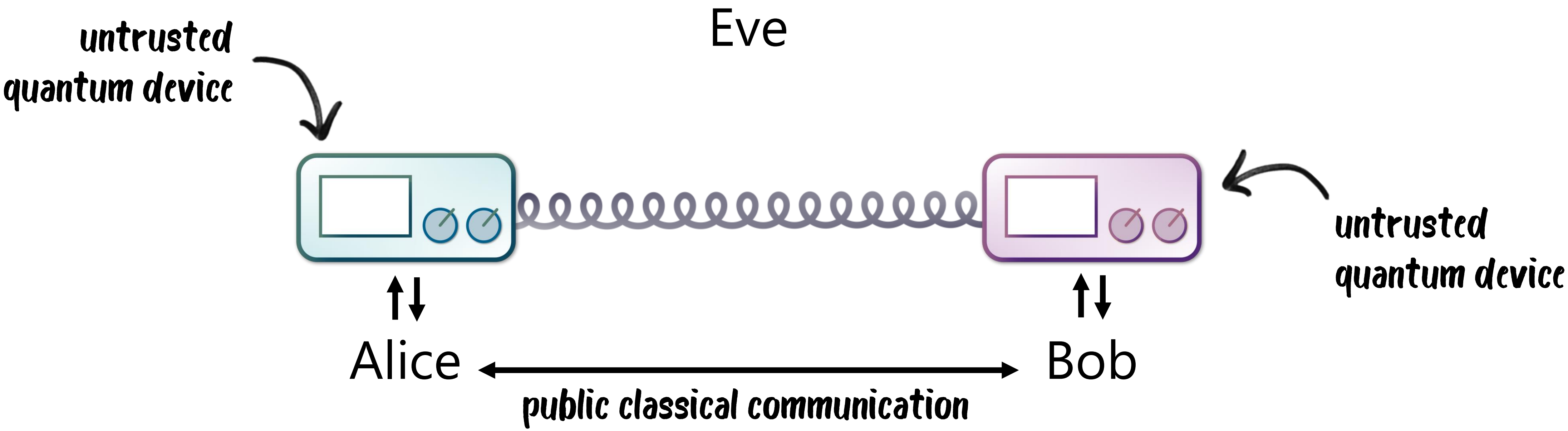

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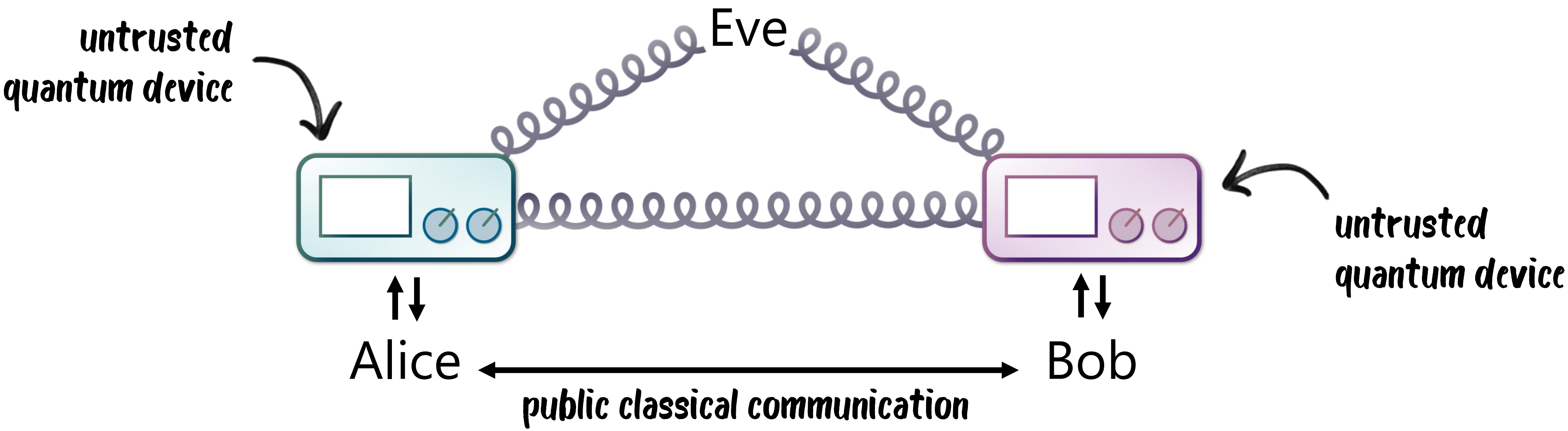

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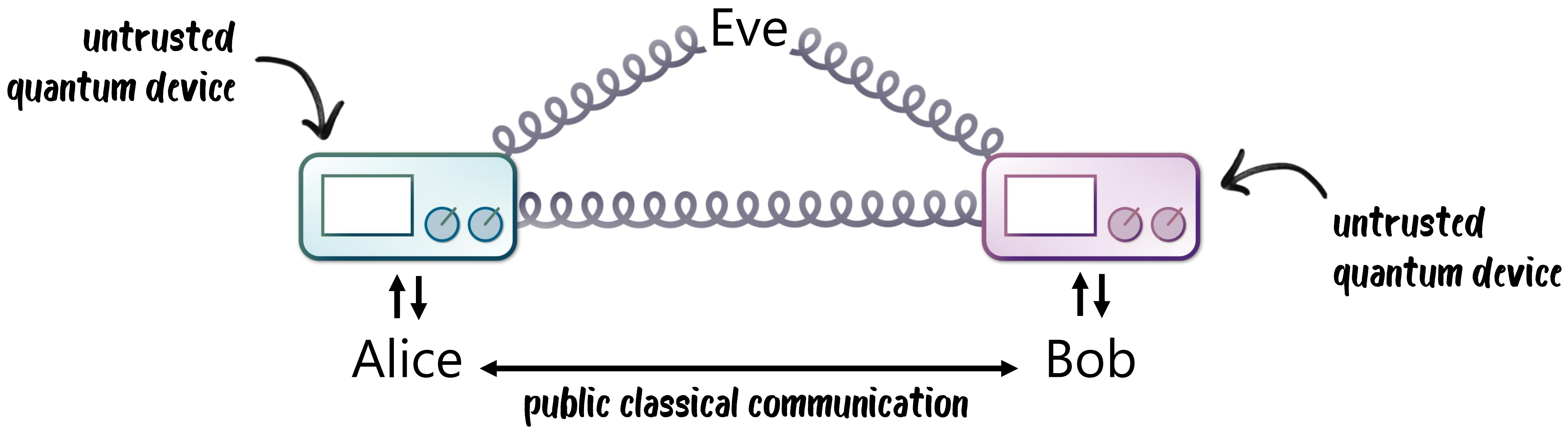

Bell inequality
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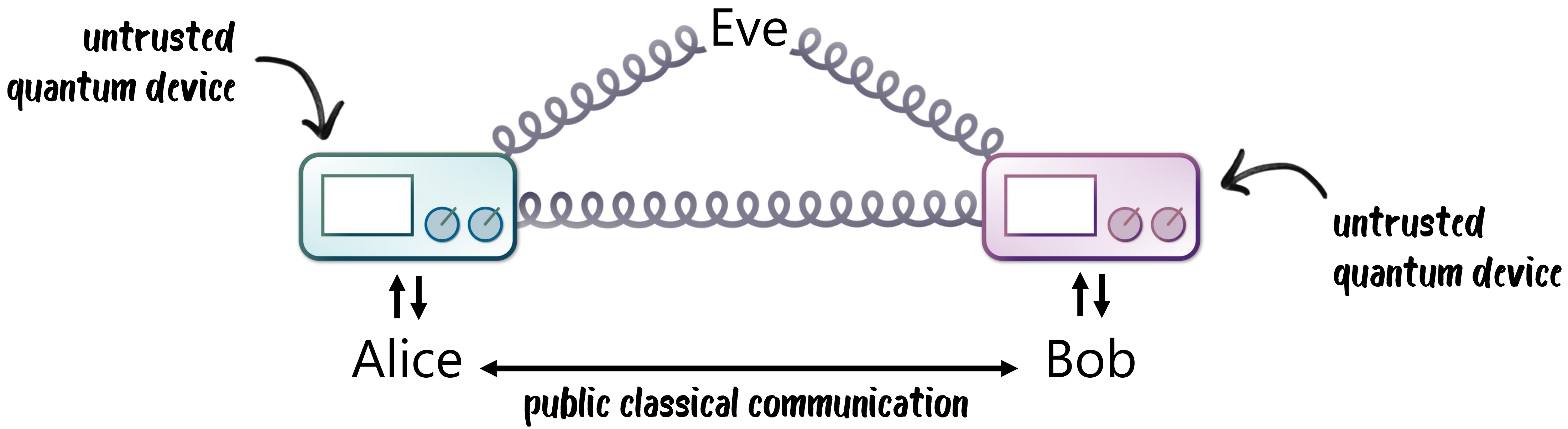

Bell inequality violation → Statement about devices' states and measurements

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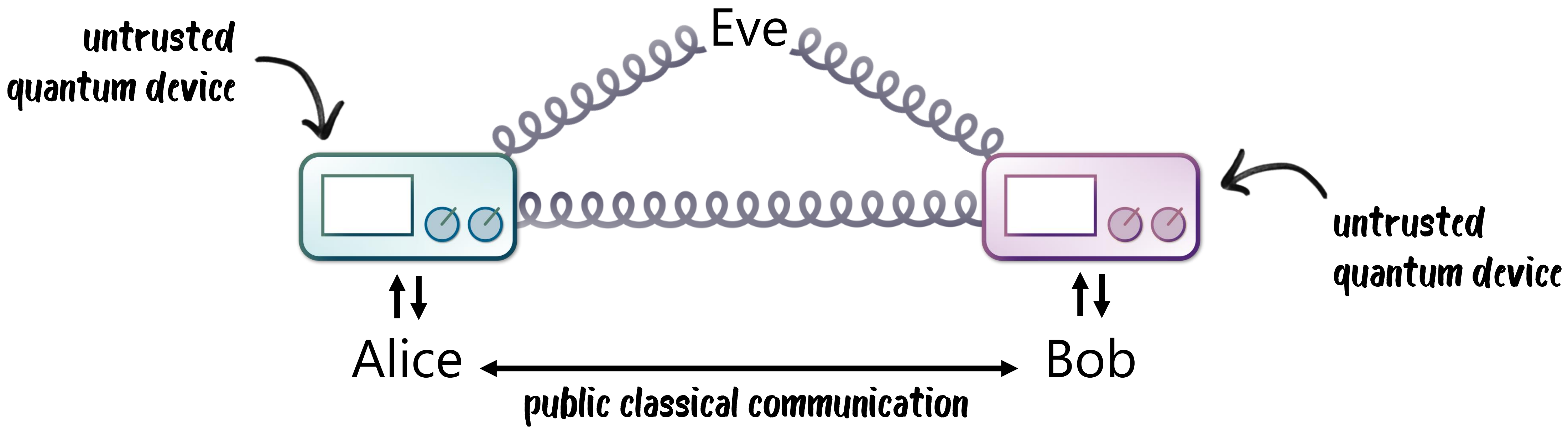
Device-independent QKD



Bell inequality violation → Statement about devices' states and measurements → **Information-theoretic security** of final key

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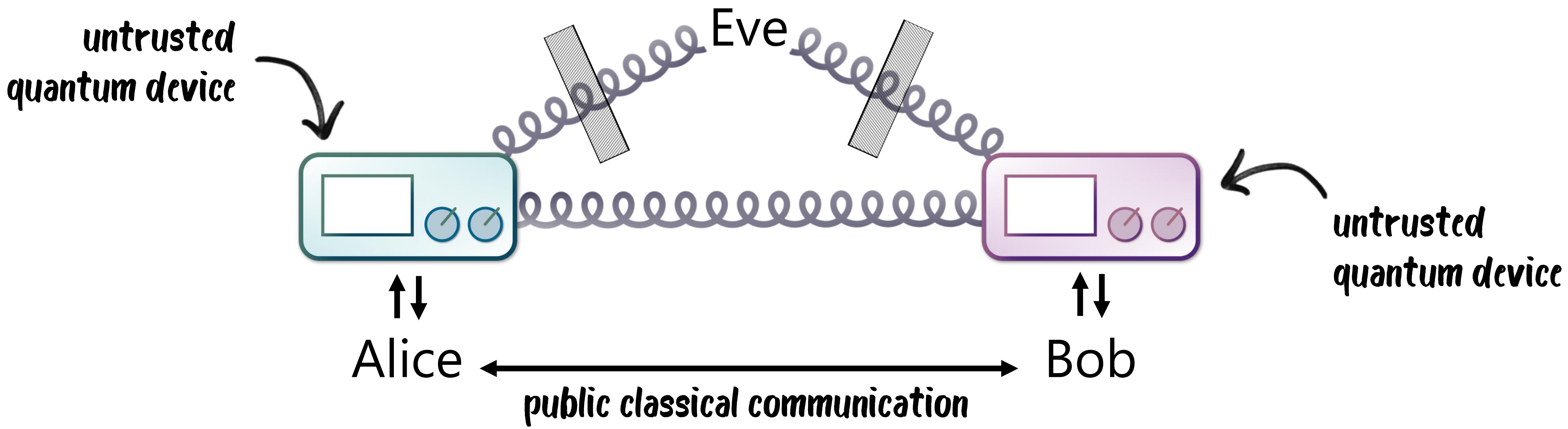

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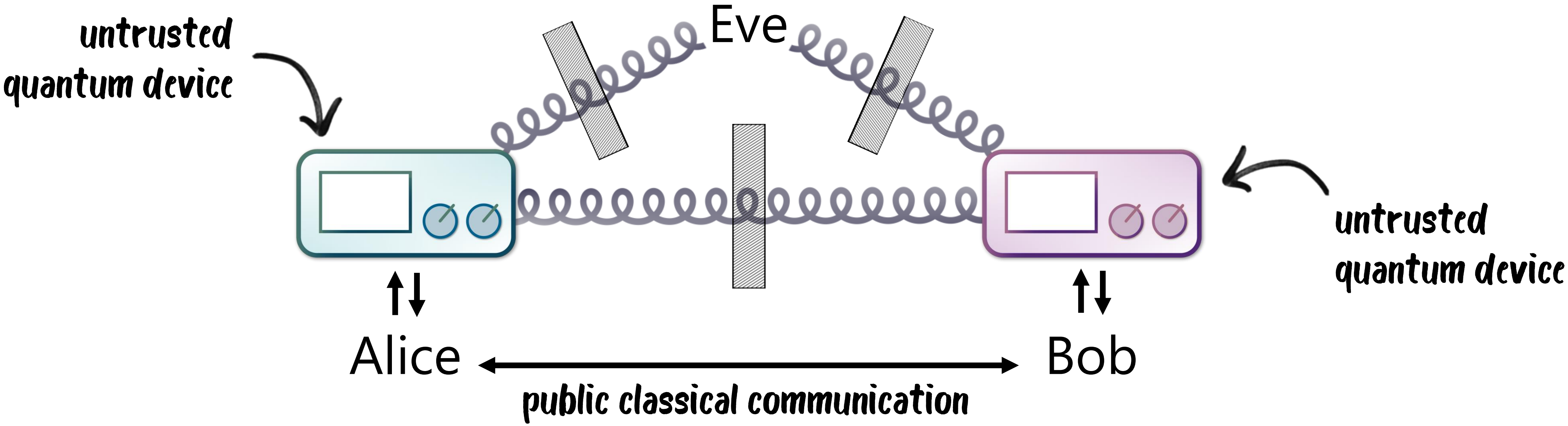

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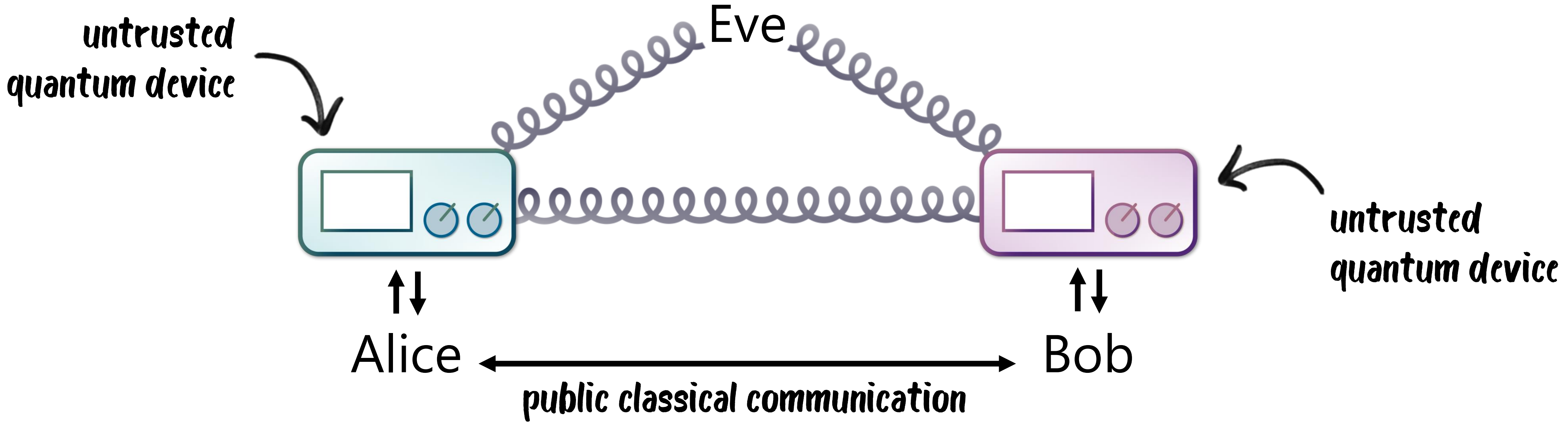

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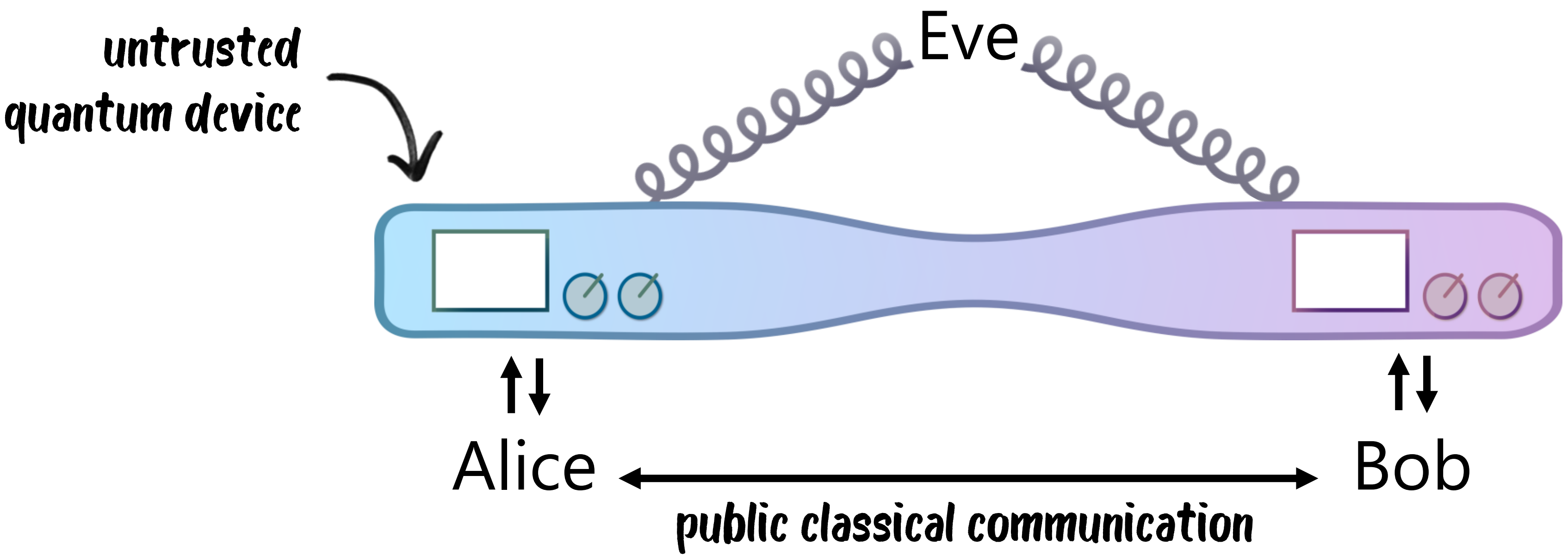

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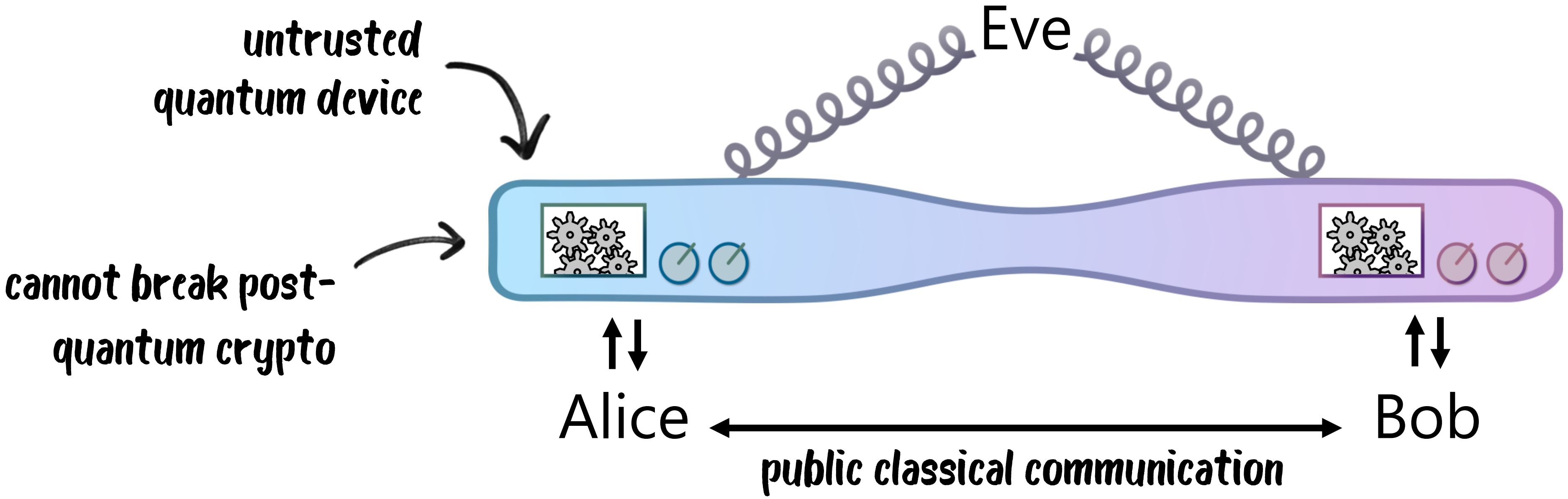
Computational DIQKD setting



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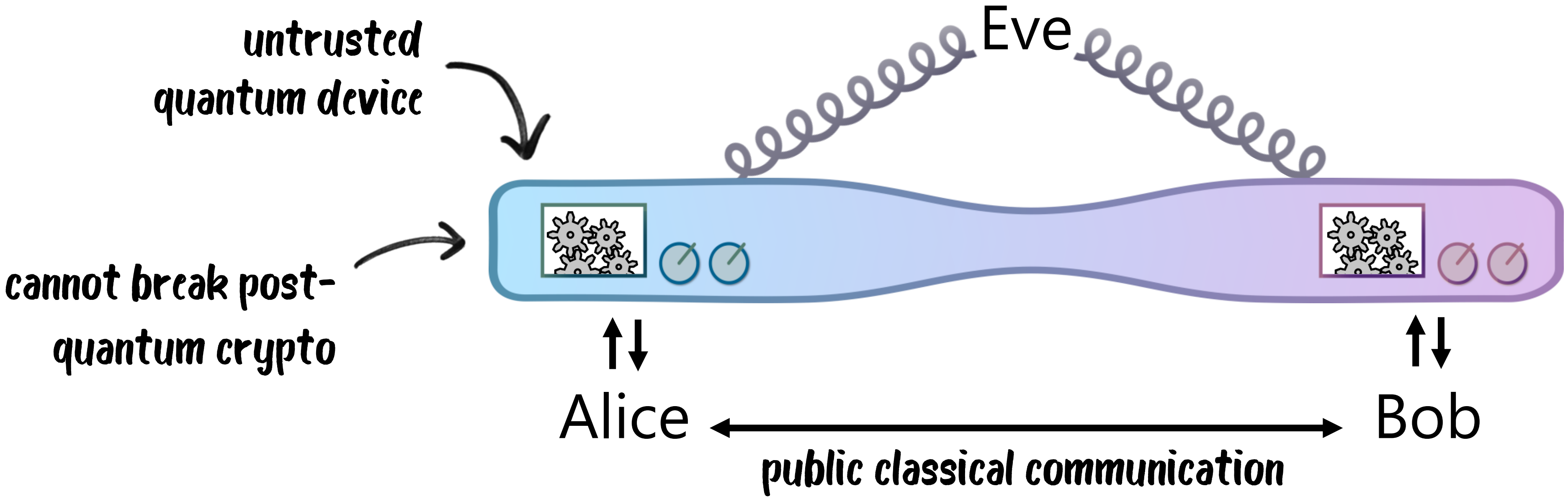

Interactive protocol with **computational assumption** → Statement about devices' states and measurements → **Information-theoretic security** of final key

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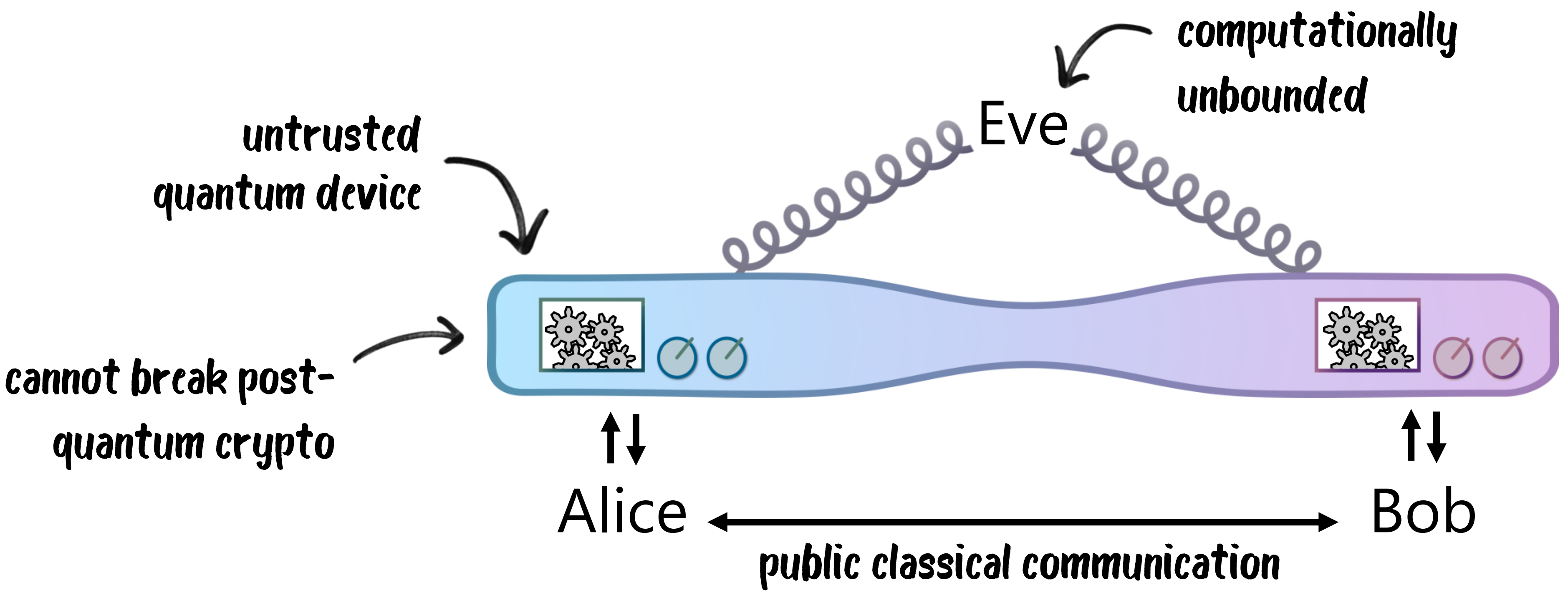

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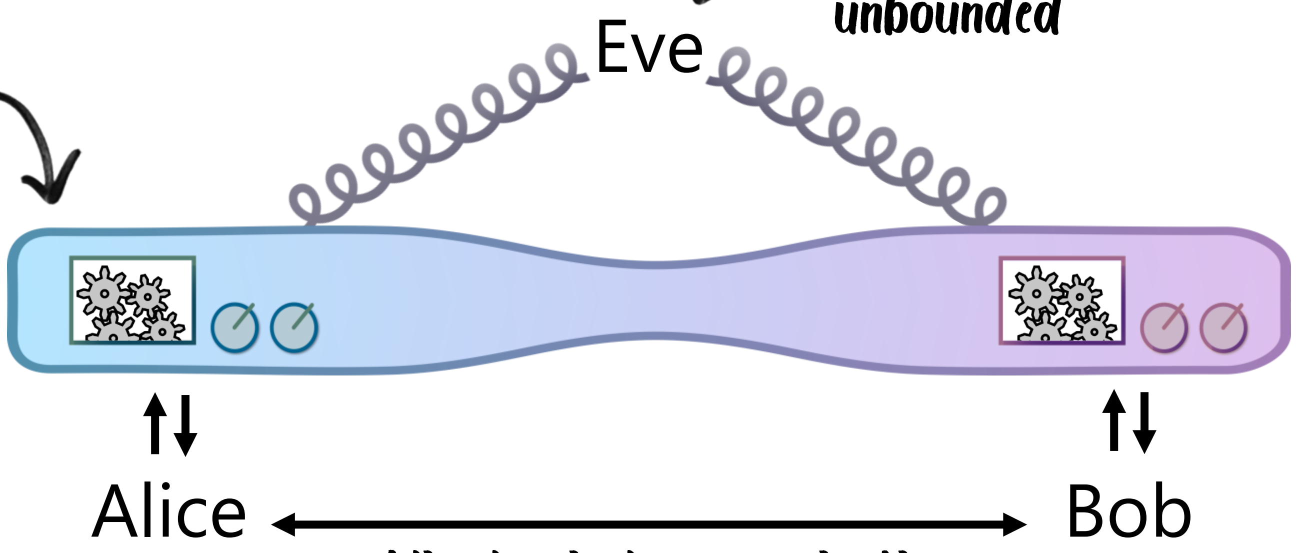
Computational DIQKD setting

computationally unbounded

untrusted quantum device

cannot break post-quantum crypto



public classical communication

Interactive protocol with **computational assumption** → Statement about devices' states and measurements → **Information-theoretic security** of final key

required only for duration of the protocol

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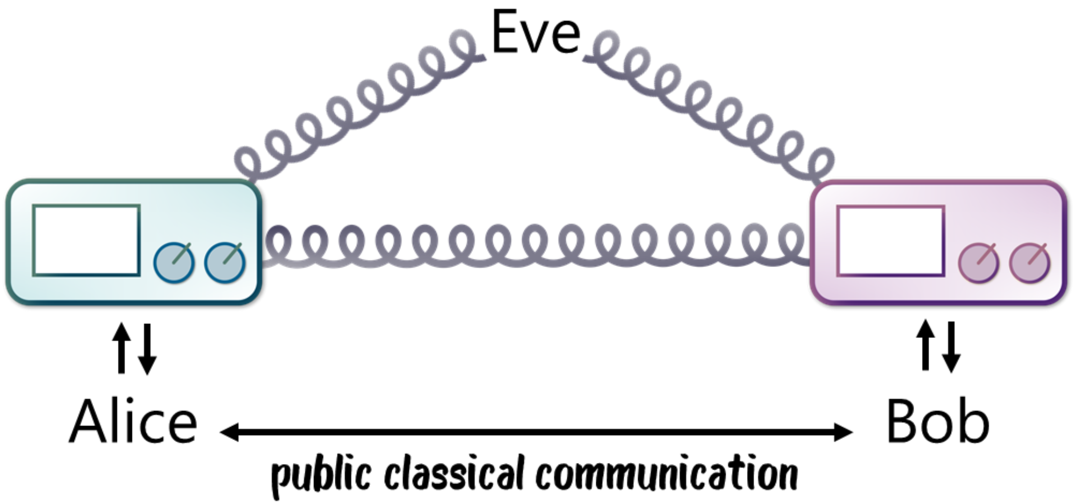
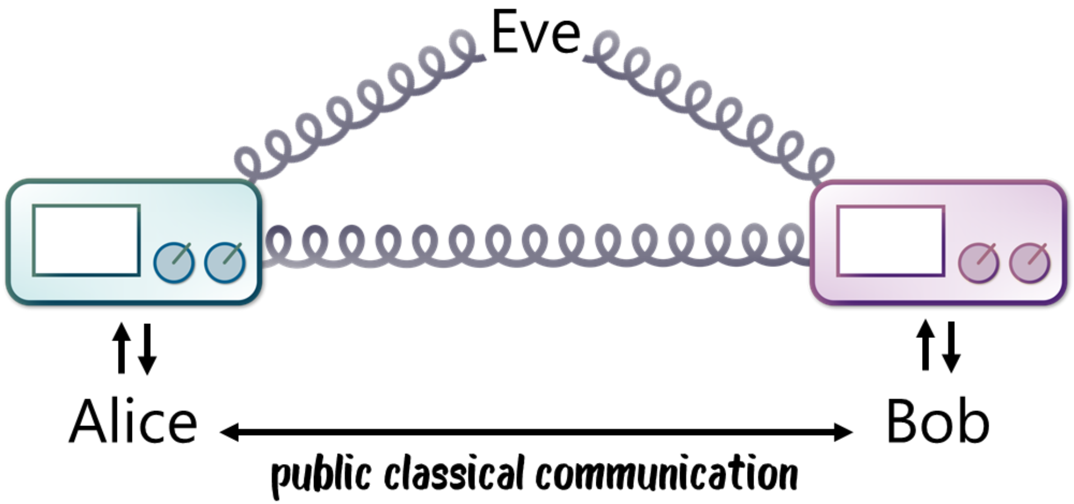

Eve
Alice
Bob
public classical communication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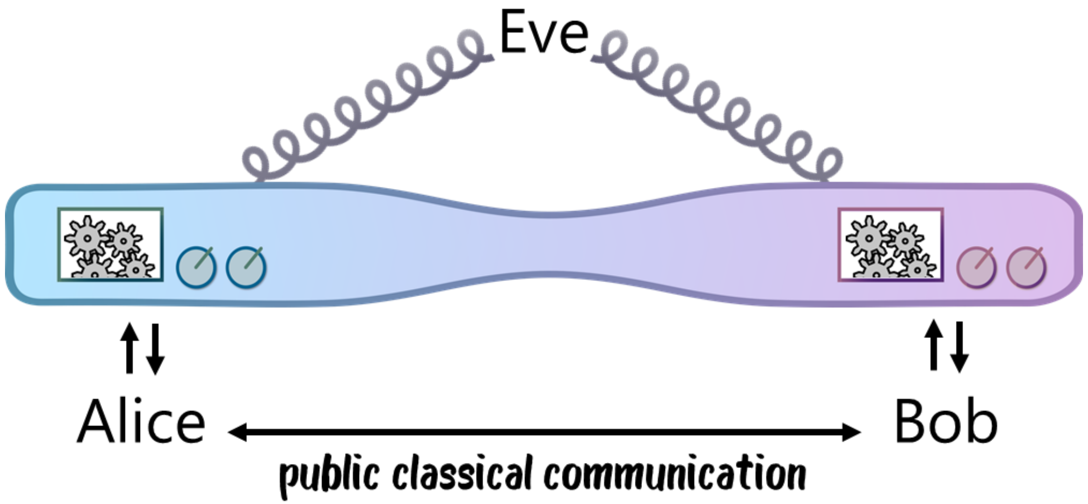
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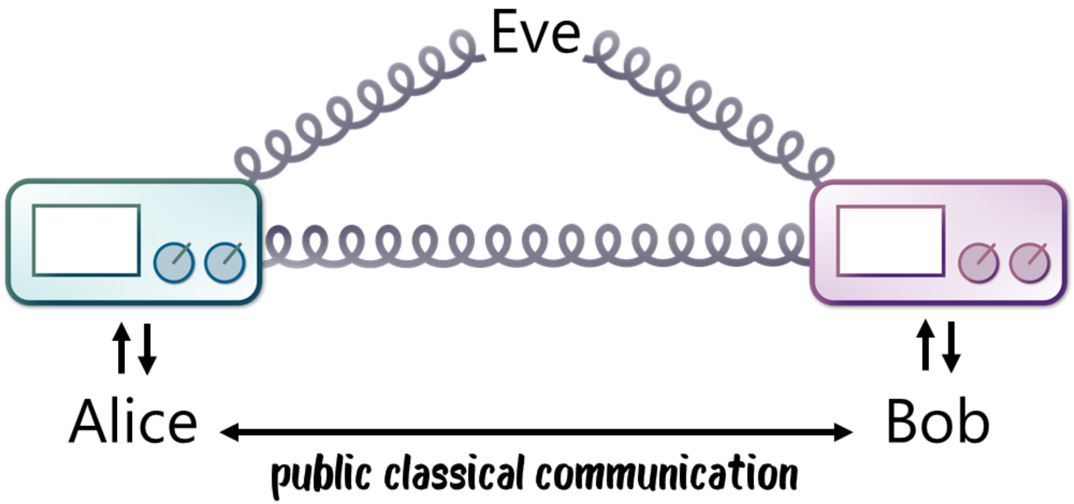
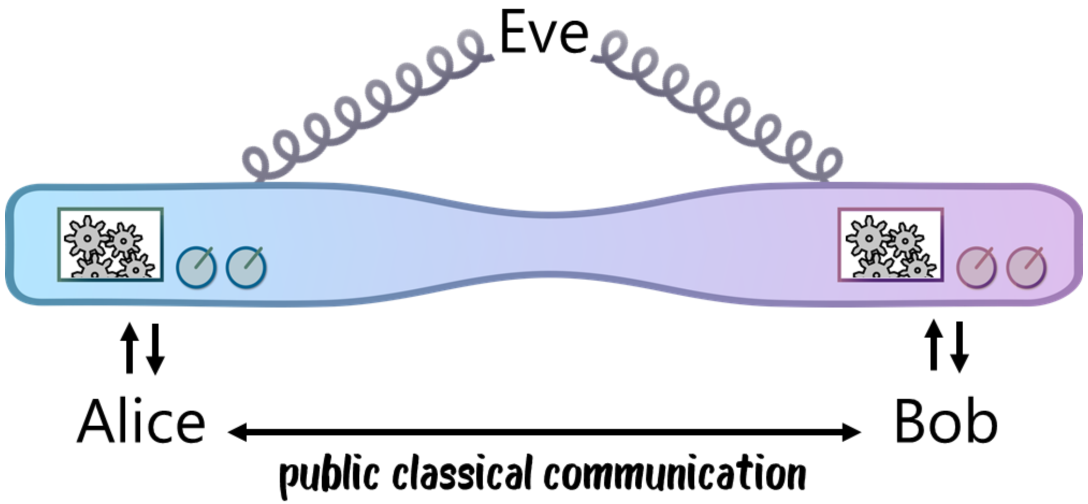
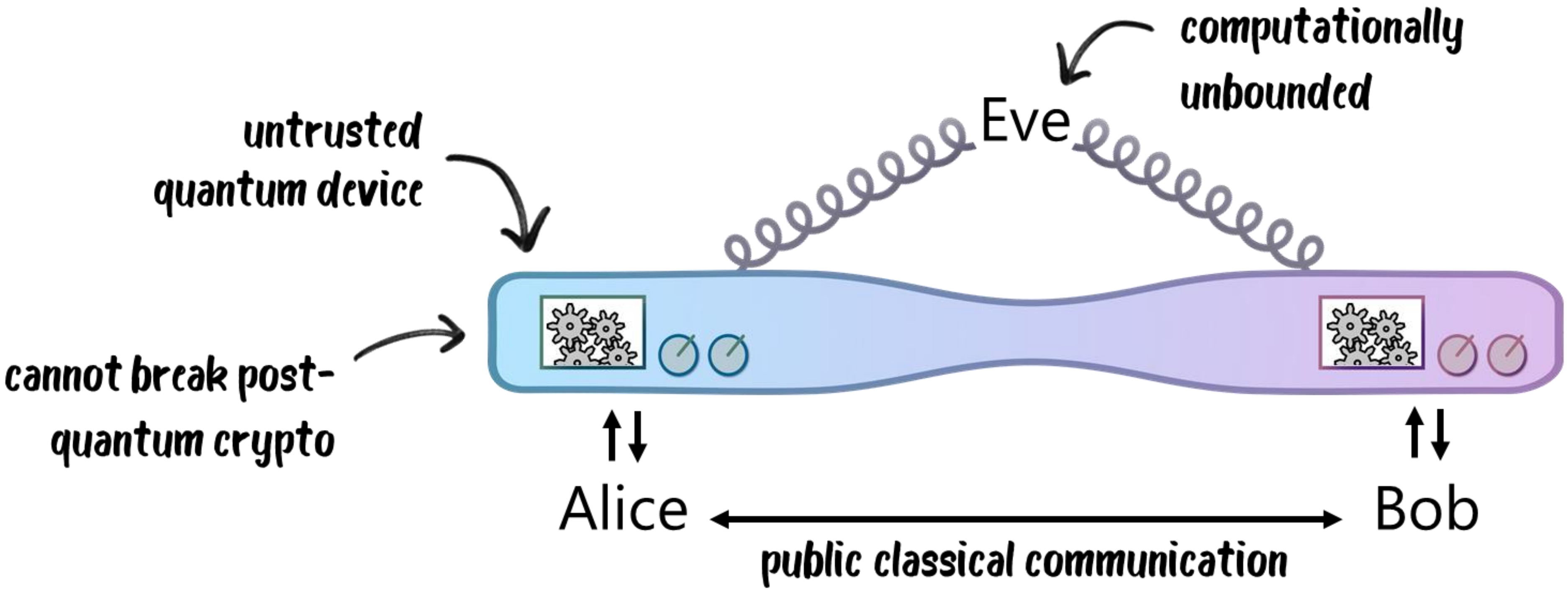




Extra requirement: **honest** devices should be able to succeed in the protocol with pre-shared EPR pairs and local operations

Computational DIQKD setting



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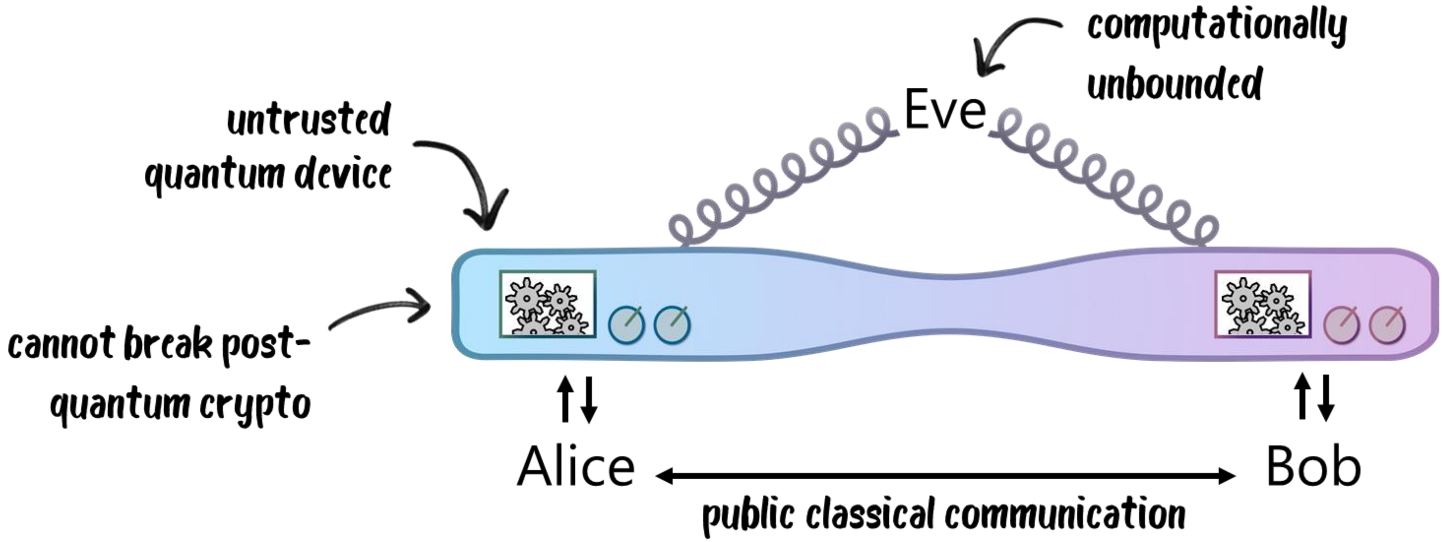

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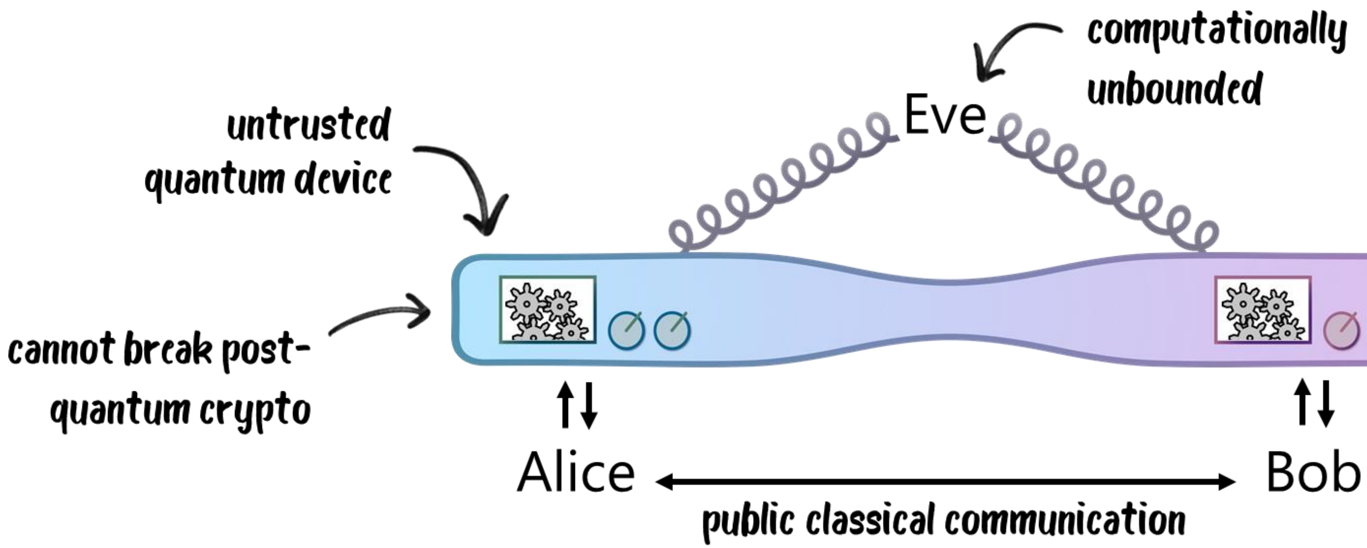

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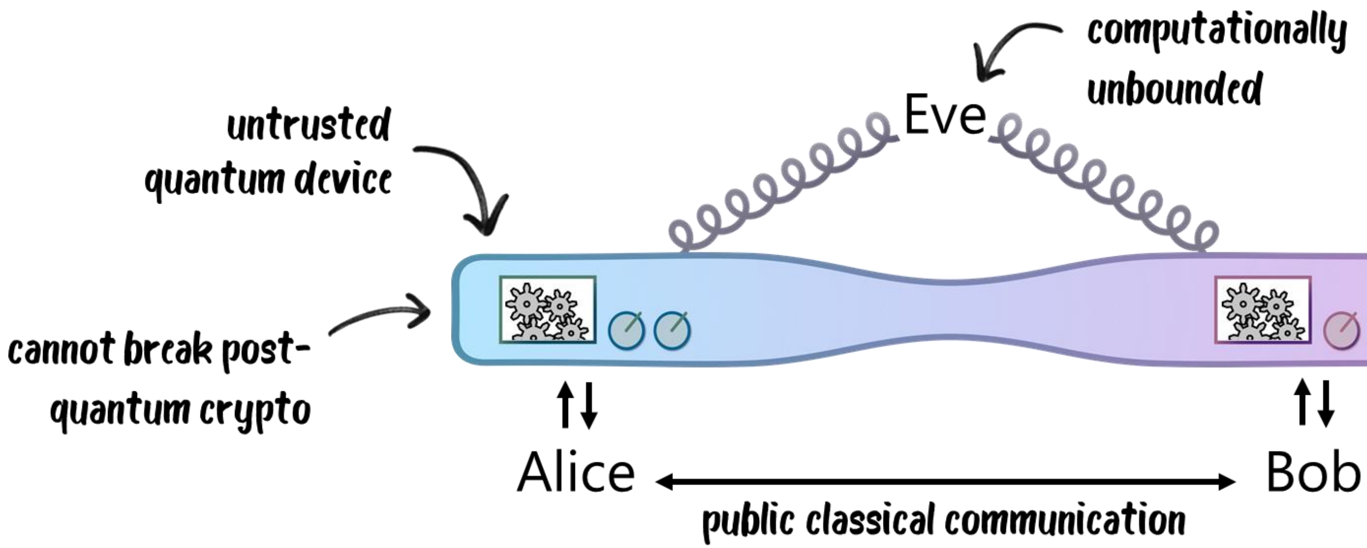
Computational self-testing protocol



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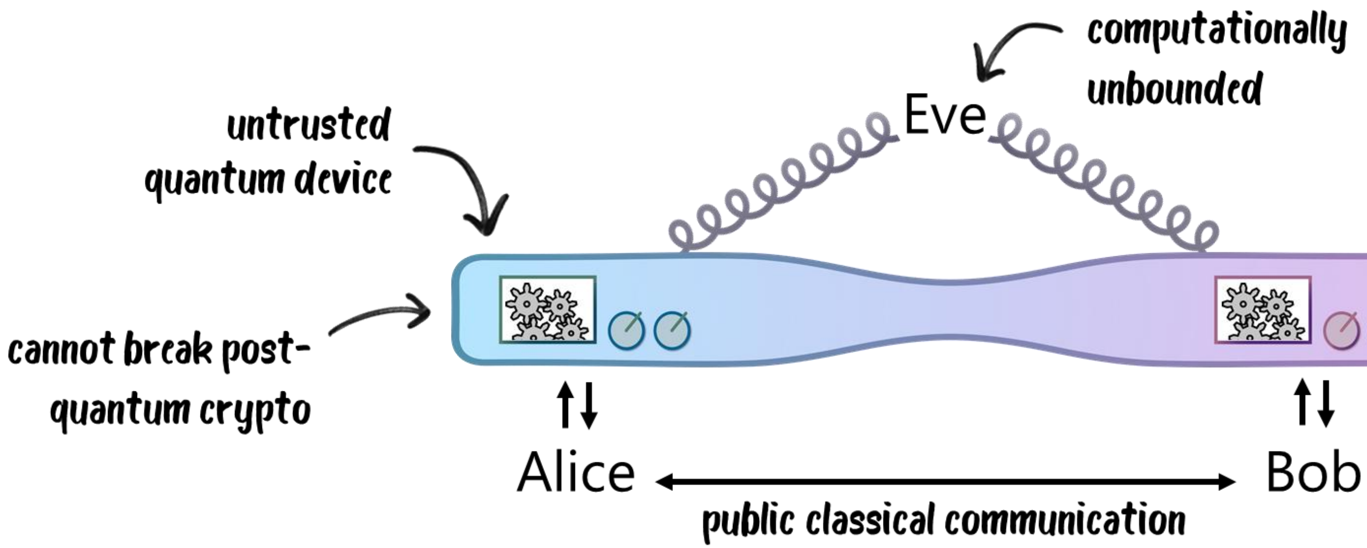
Computational self-testing protocol

Device must have prepared EPR pair and measured single qubits in computational or Hadamard basis



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Certified entropy of device's measurement outcomes conditioned on side information

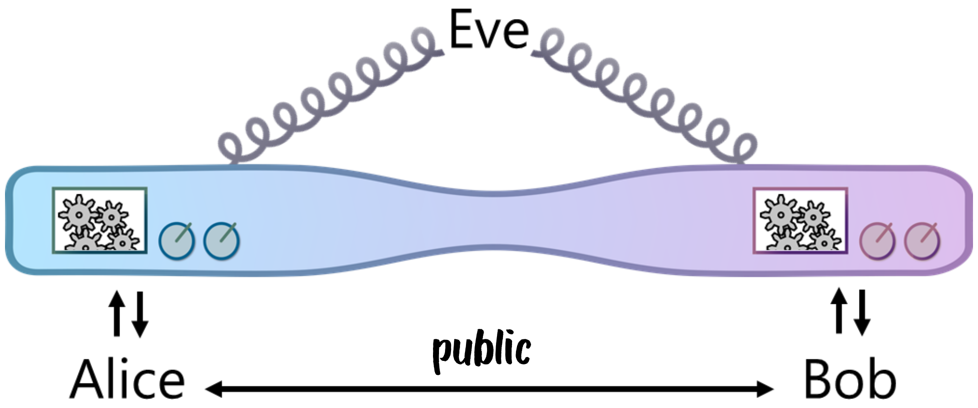
Devetak & Winter, Distillation of secret key and entanglement from quantum states, Proc. R. Soc. A. 461207–235 (2005)

Computational self-testing protocol

Computational self-testing

Classical **interactive protocol** run by Alice and Bob

Device can win or lose



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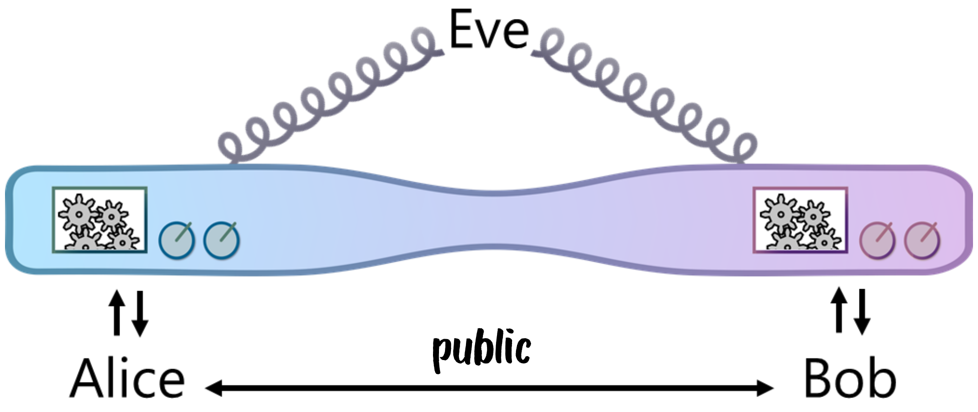

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If a **computationally bounded device** wins with probability (close to) 1:

Computational self-testing

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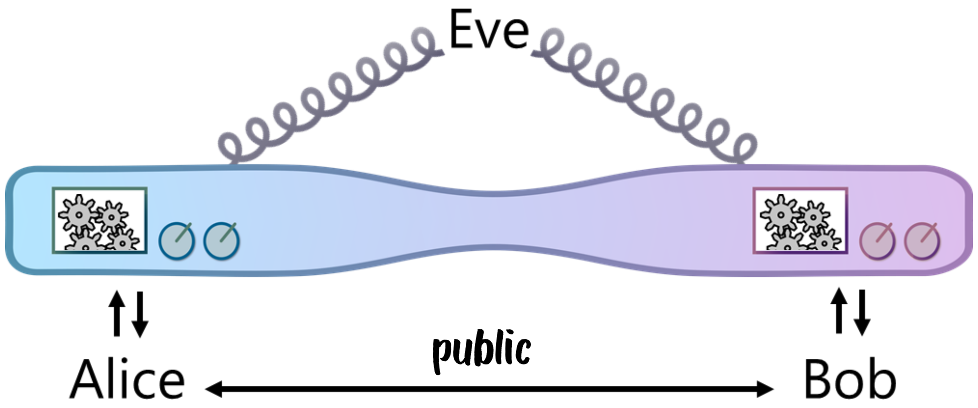

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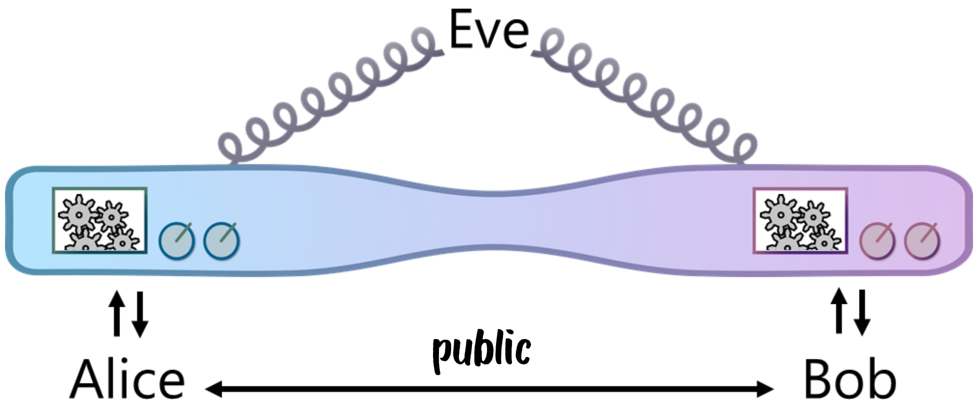

If a **computationally bounded device** wins with probability (close to) 1:

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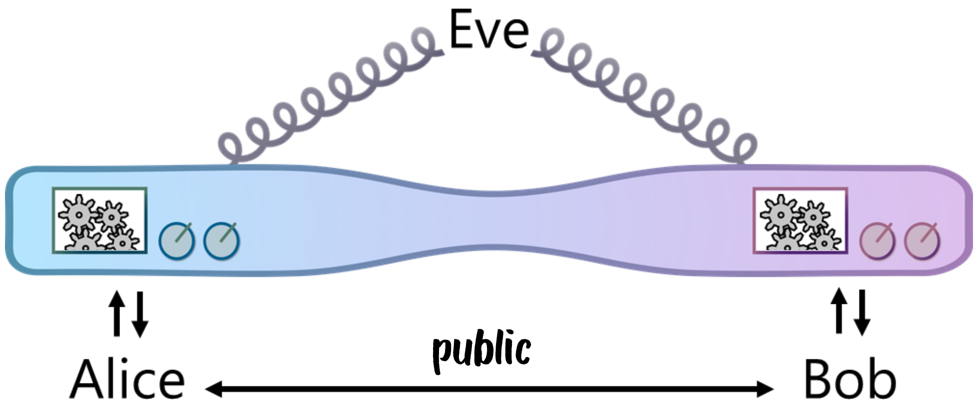

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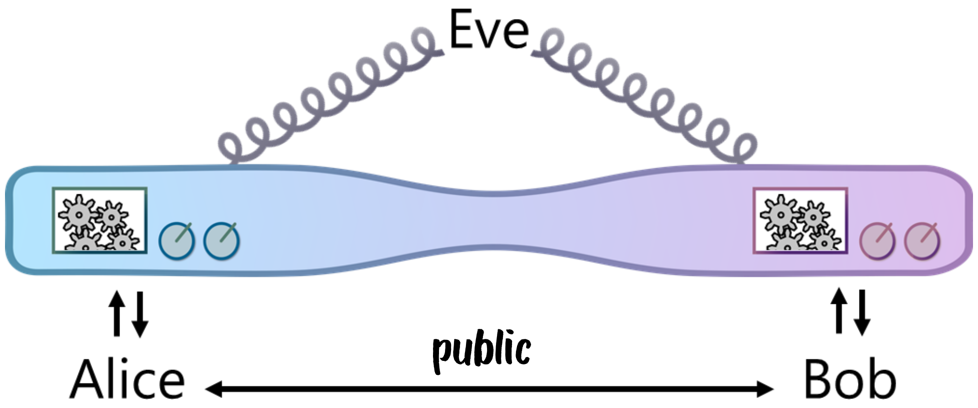

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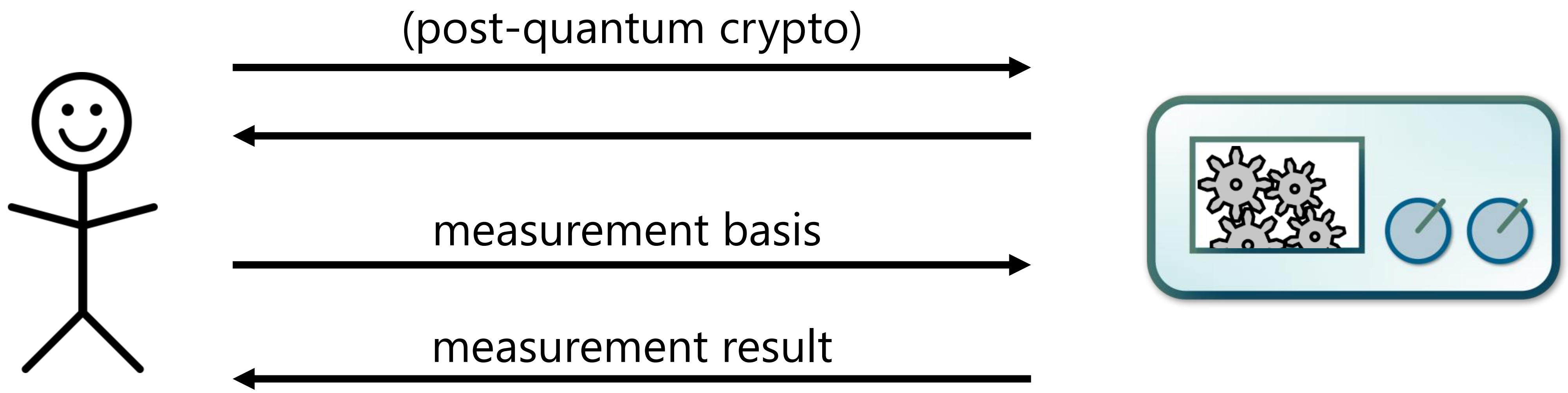
Remote state preparation [GV’19]

Given: set of **single-qubit** states $\{|\psi_i\rangle\} = \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$



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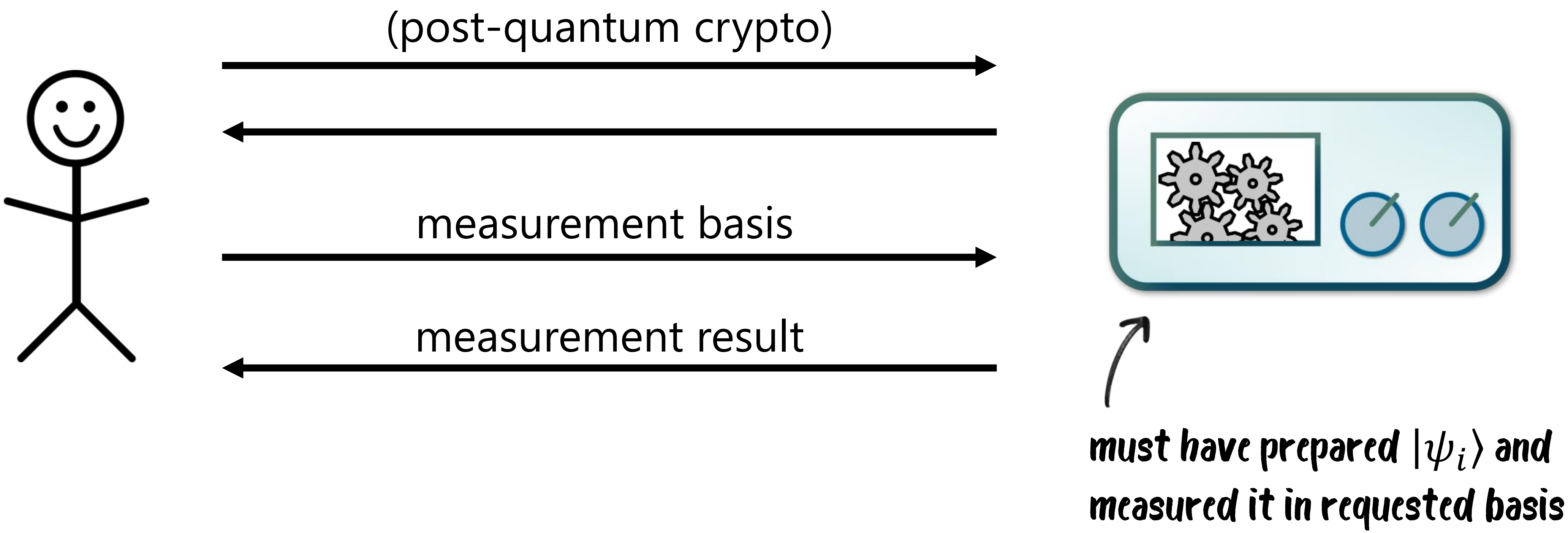

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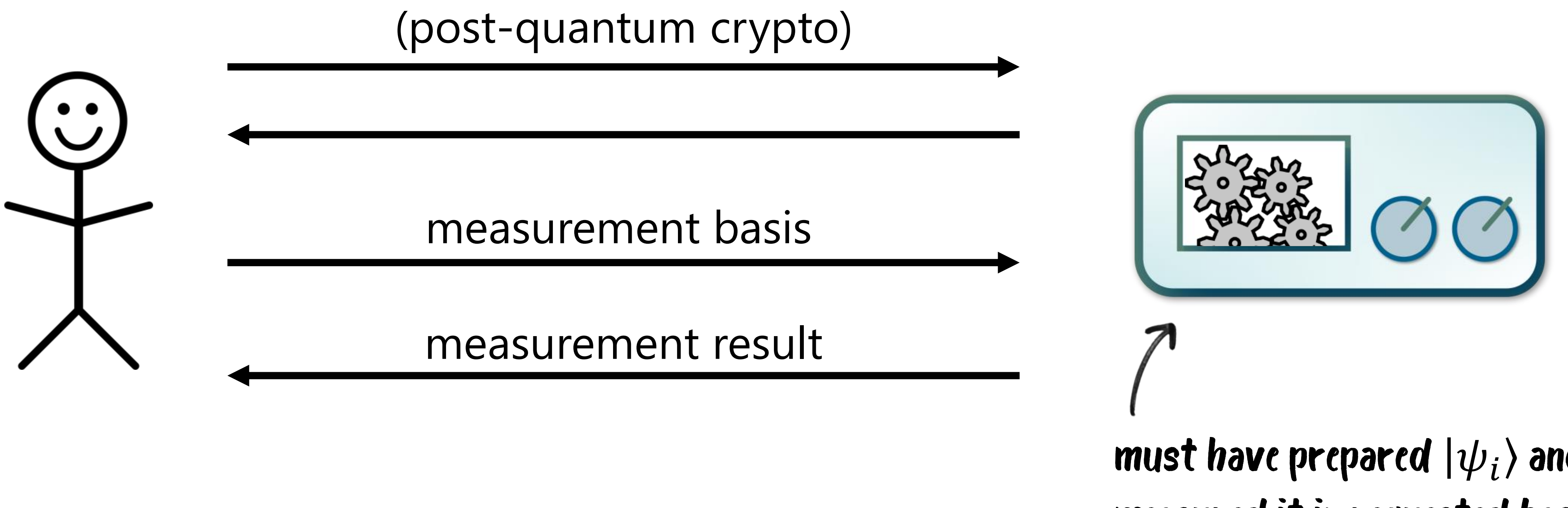

knows which state $|\psi_i\rangle$ the device has prepared

must have prepared $|\psi_i\rangle$ and measured it in requested basis

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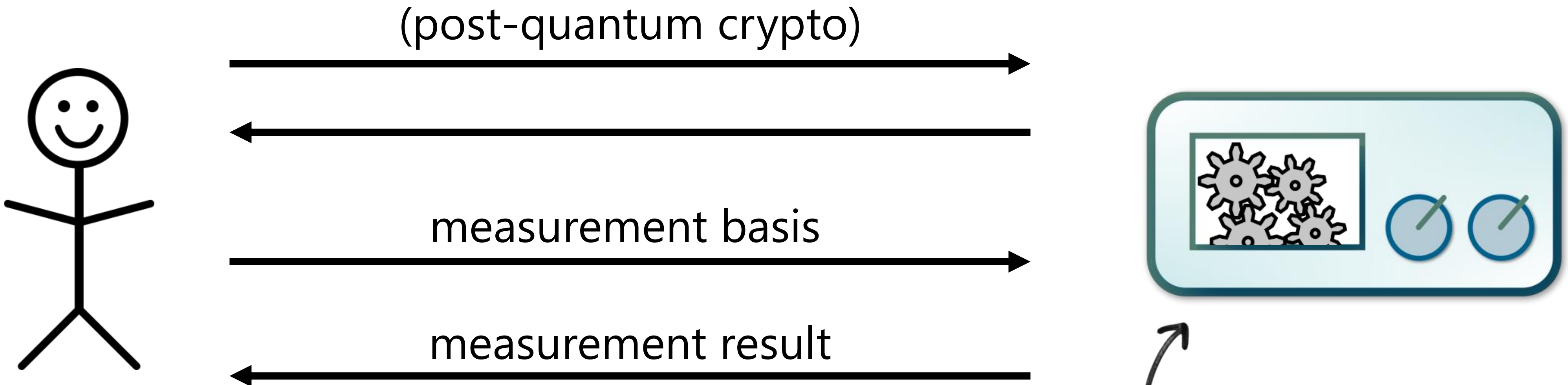

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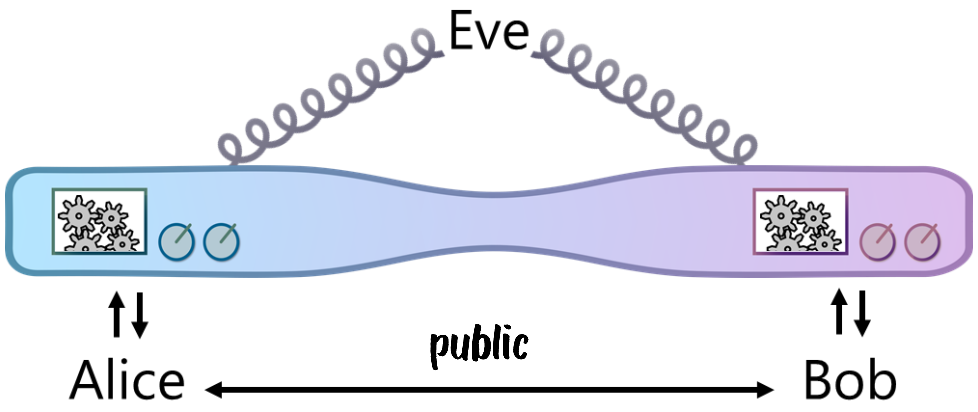
Main challenges for self-testing EPR states

- Device should prepare two qubits and perform single-qubit measurements
  → Alice and Bob need to enforce **tensor product structure** on device's global space



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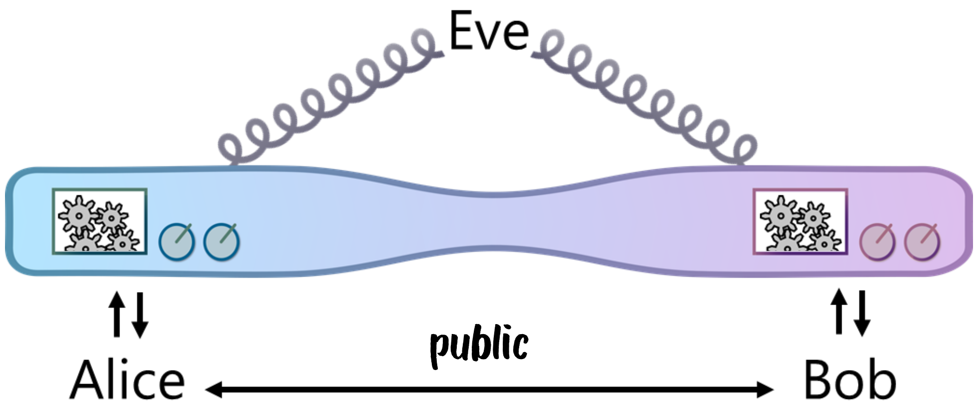

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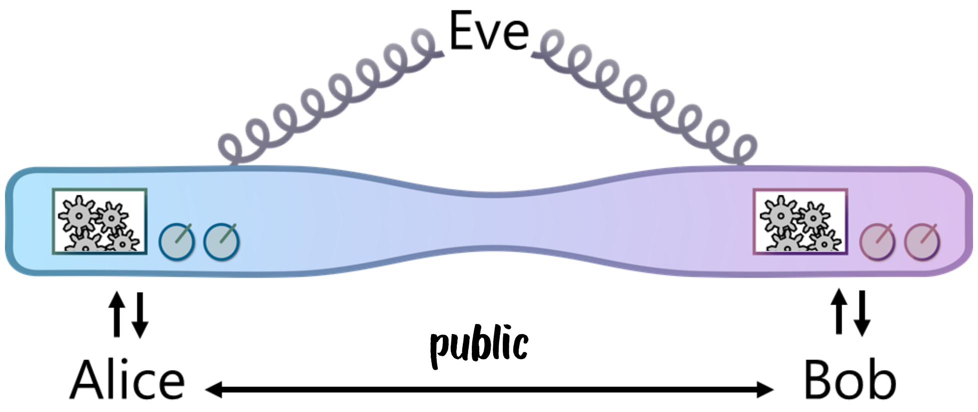
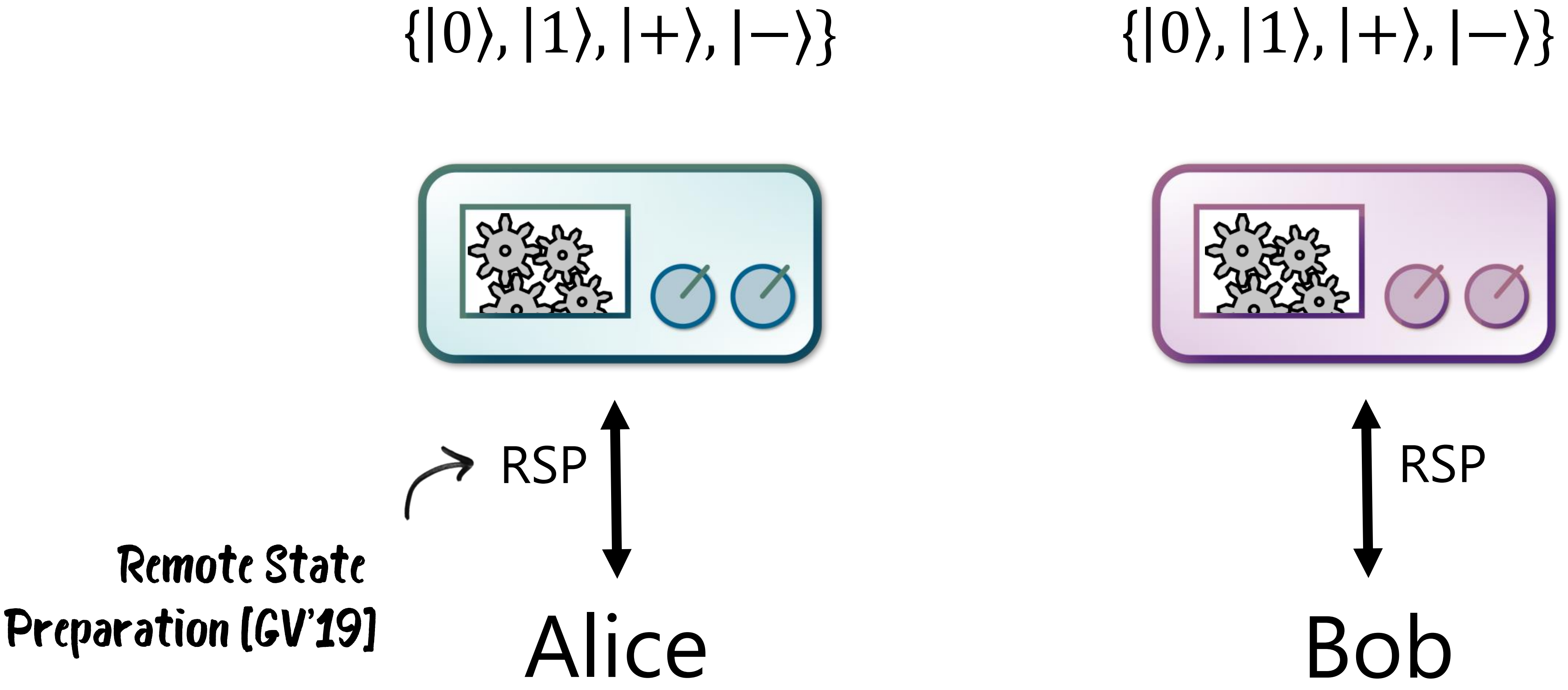


- Device should **entangle** qubits with respect to this tensor product structure

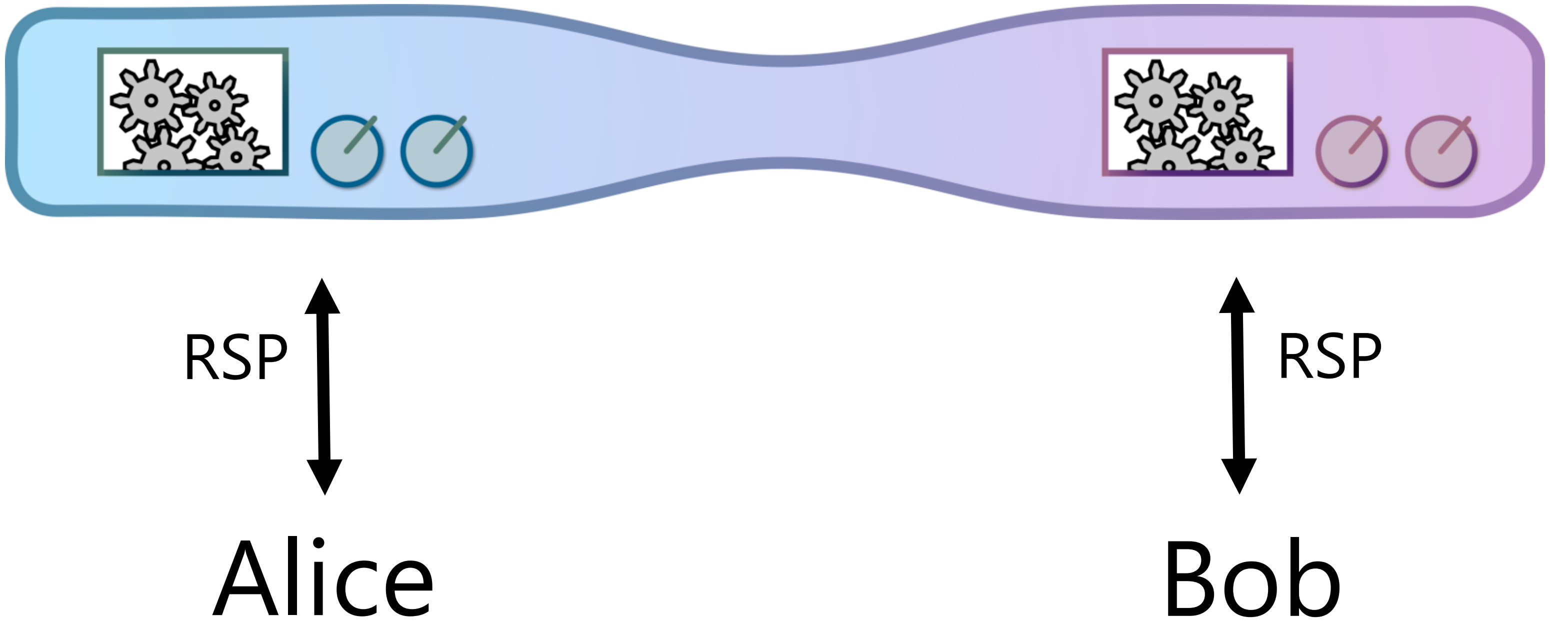
- **Honest** device should only have to use **local operations** and pre-shared EPR pairs

Remote state preparation with two isolated devices



Parallel implementation with single device

$$\{|0\rangle, |1\rangle, |+\rangle, |-\rangle\} \times \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$$



$$|0/1\rangle|0/1\rangle$$

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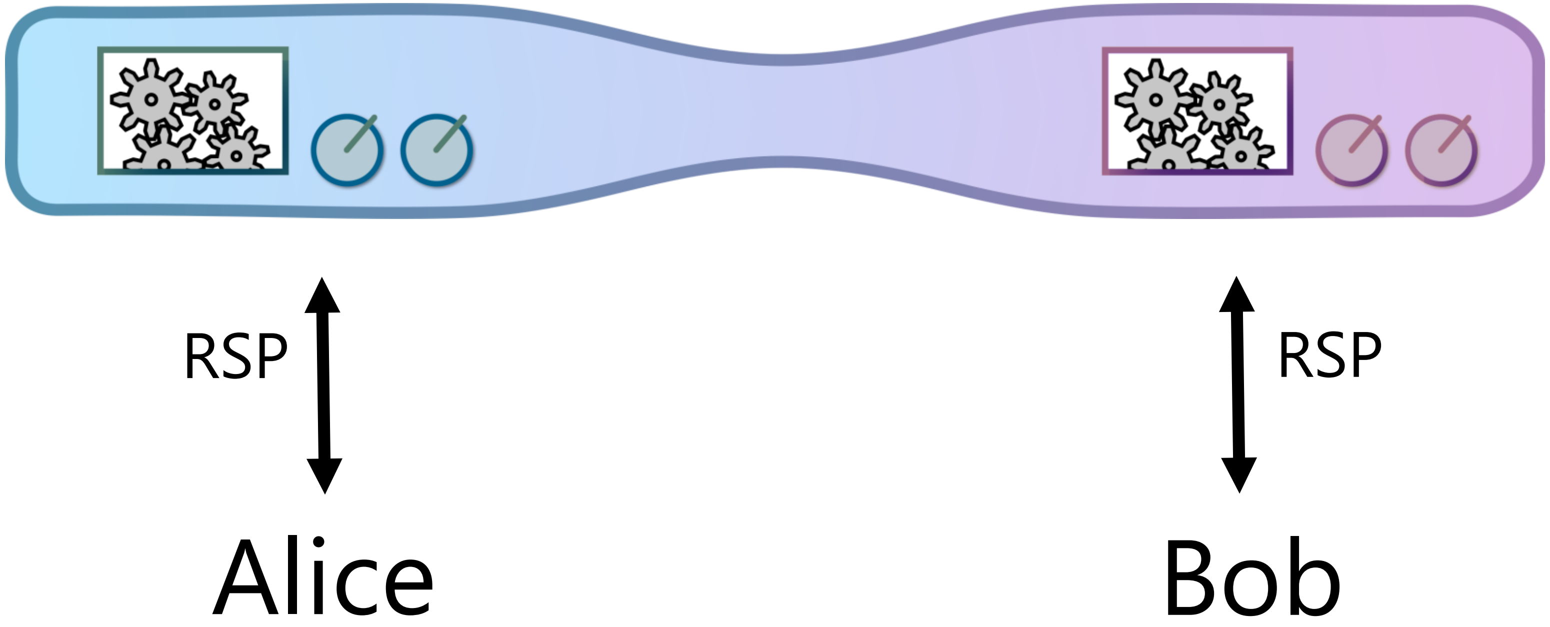

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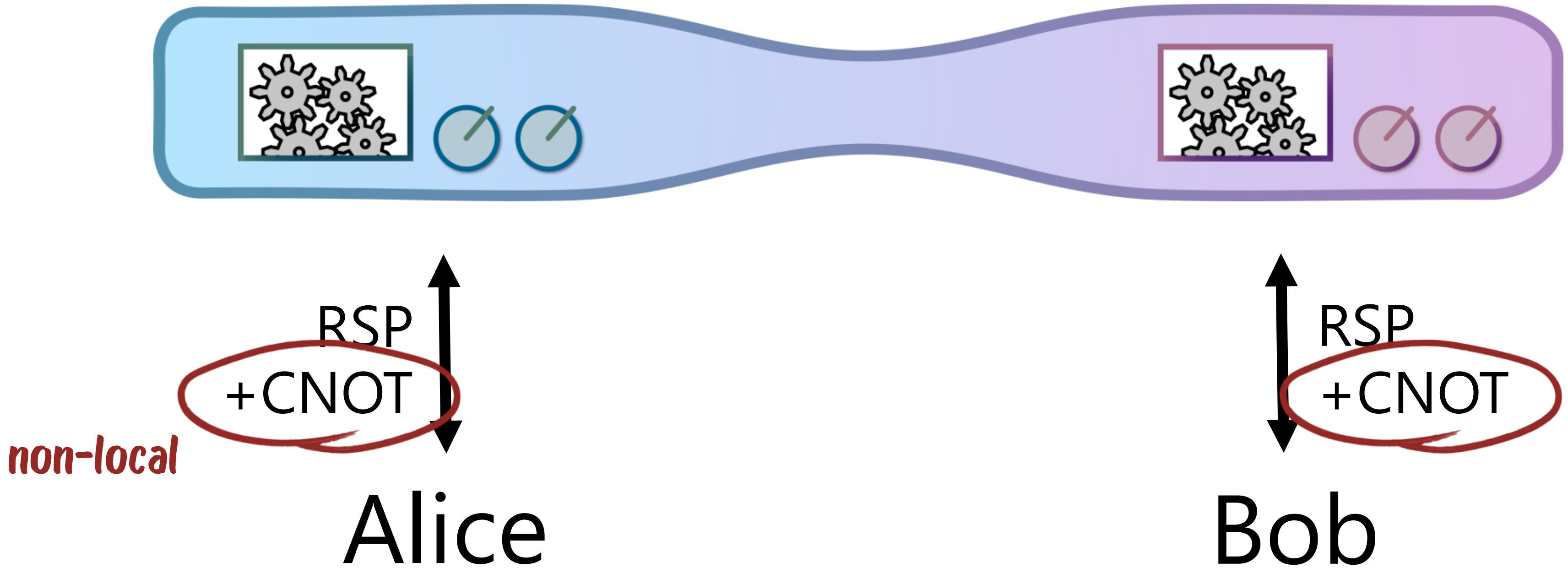

$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

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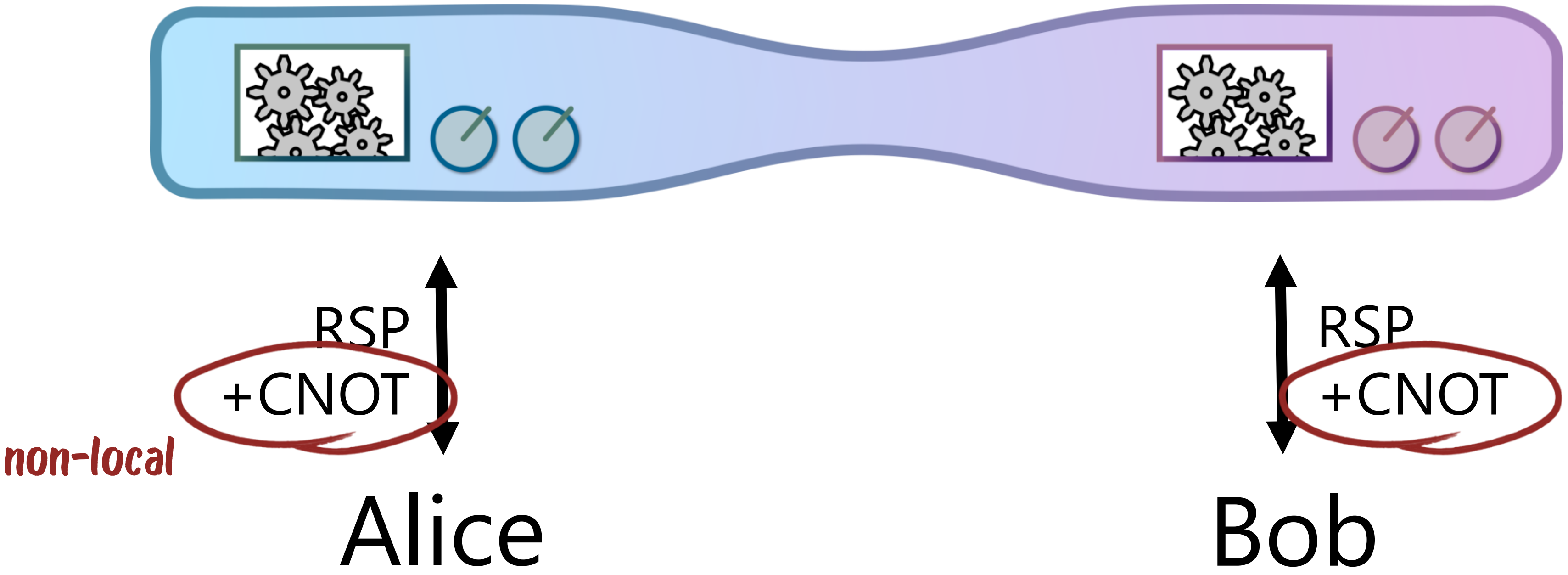
CNOT $\longmapsto$



$$|0/1\rangle|0/1\rangle \qquad\qquad |0/1\rangle|0/1\rangle$$
$$|0/1\rangle|\pm\rangle \quad \xmapsto{\text{CNOT}} \quad |0/1\rangle|\pm\rangle$$
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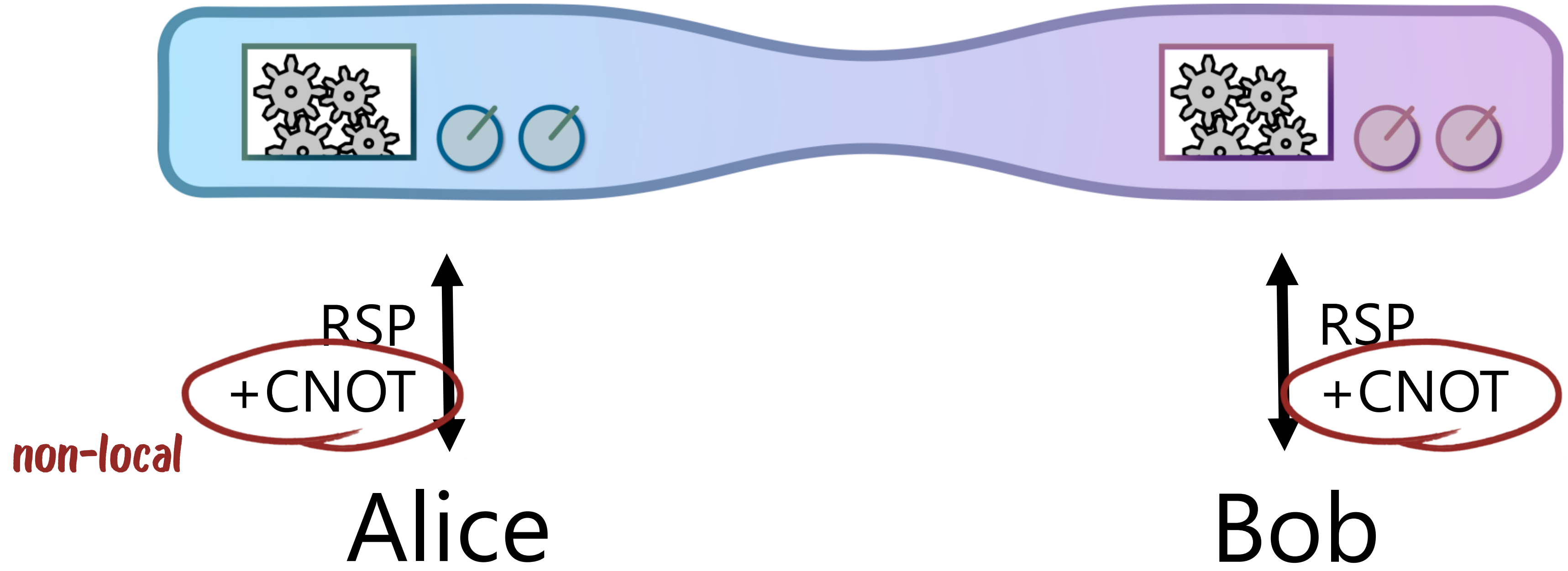
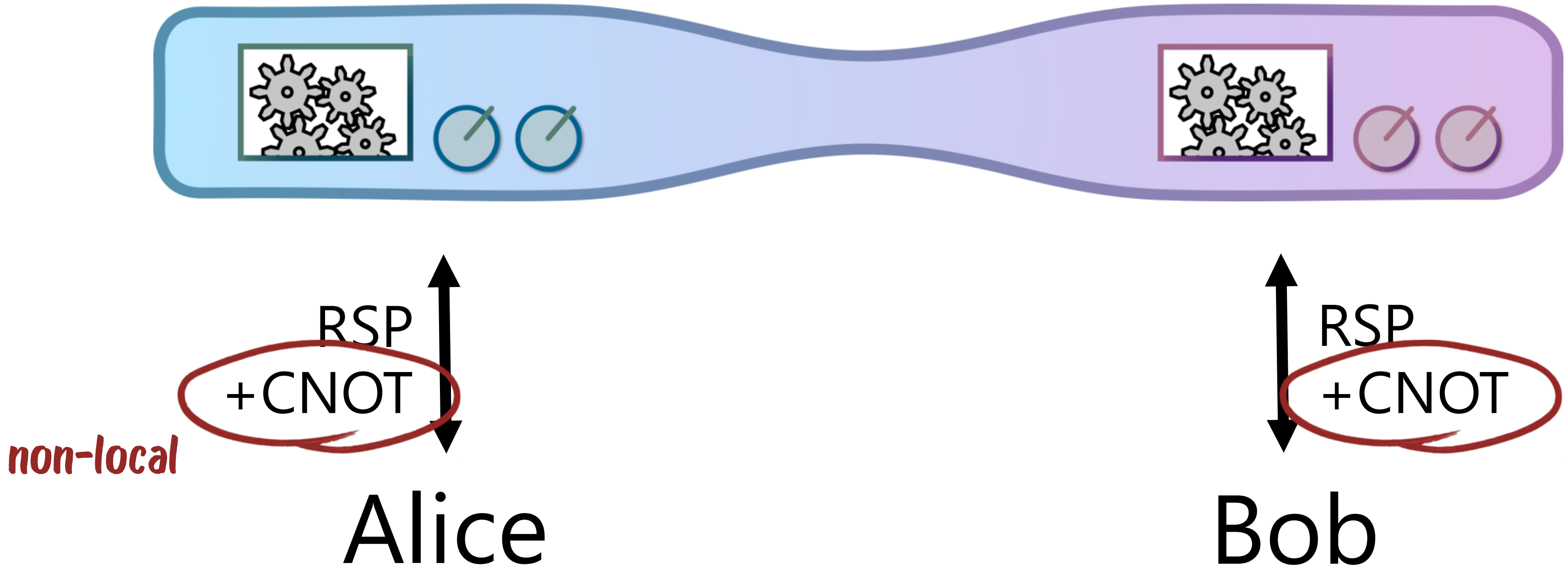
$$|\pm\rangle|0/1\rangle$$



$$|0/1\rangle|0/1\rangle \qquad\qquad |0/1\rangle|0/1\rangle$$
$$|0/1\rangle|\pm\rangle \quad \xmapsto{\text{CNOT}} \quad |0/1\rangle|\pm\rangle$$
$$|\pm\rangle|\pm\rangle \qquad\qquad |\pm\rangle|\pm\rangle$$

$$|\pm\rangle|0/1\rangle \qquad\qquad |00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$$

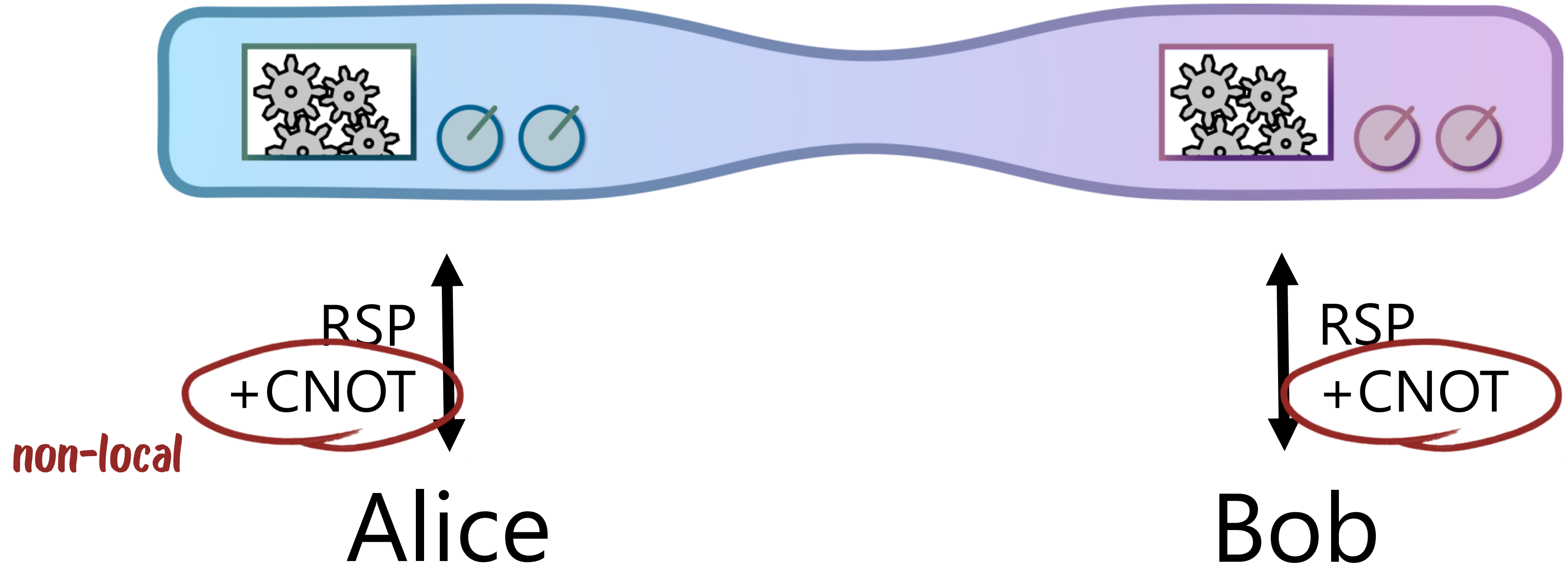


$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

CNOT $\longmapsto$

$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

$|\pm\rangle|0/1\rangle$ $\quad$ $|00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$



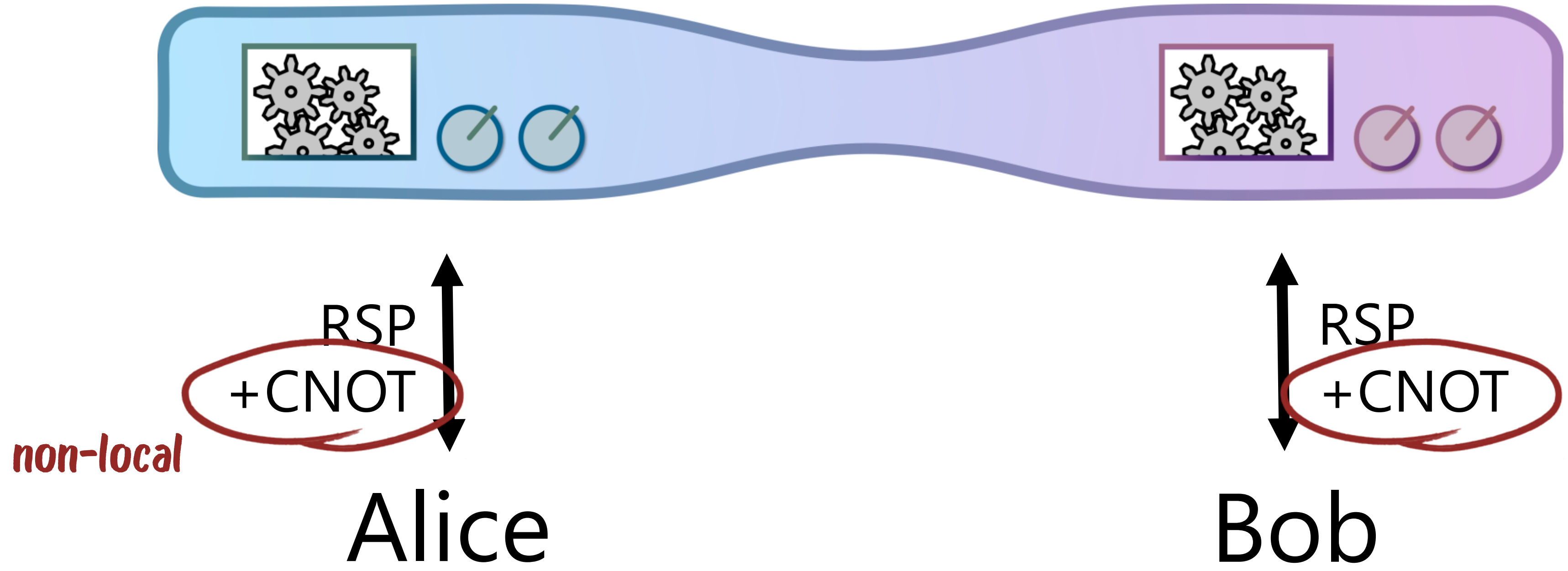
$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

CNOT $\longmapsto$

$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

} Certify **single-qubit** measurements

$|\pm\rangle|0/1\rangle$ $\qquad$ $|00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$



$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

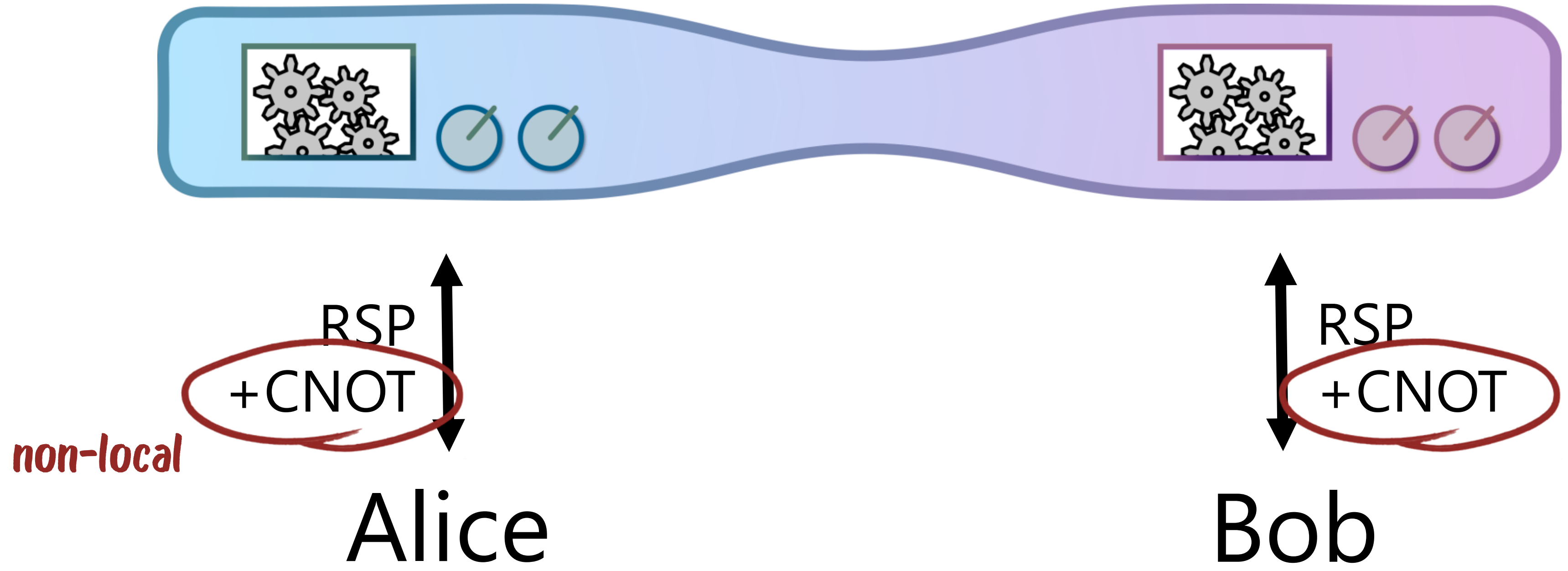
CNOT $\longmapsto$

$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

Certify **single-qubit** measurements

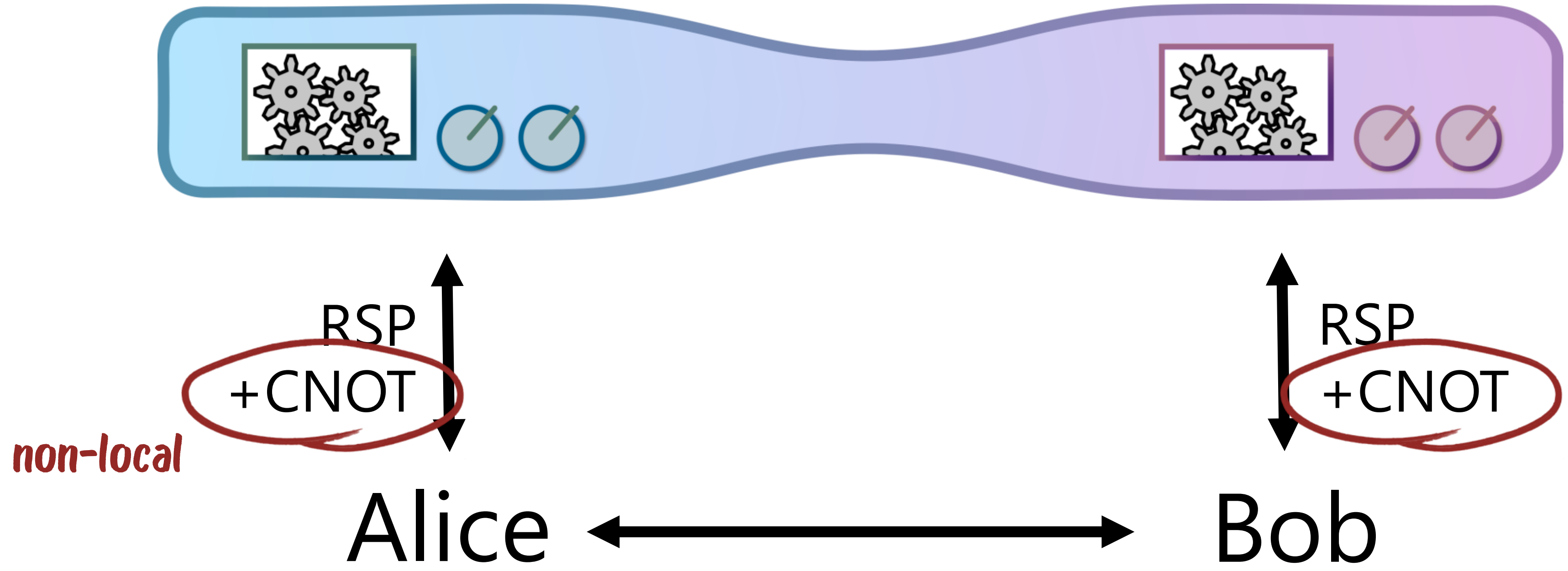
$|\pm\rangle|0/1\rangle$ $\quad$ $|00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$

Certify **Bell**-like correlations



$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$ $\xrightarrow{\text{CNOT}}$ $|0/1\rangle|0/1\rangle$, $|0/1\rangle|\pm\rangle$, $|\pm\rangle|\pm\rangle$ } Certify **single-qubit** measurements

$|\pm\rangle|0/1\rangle$ $\longmapsto$ $|00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$ Certify **Bell**-like correlations

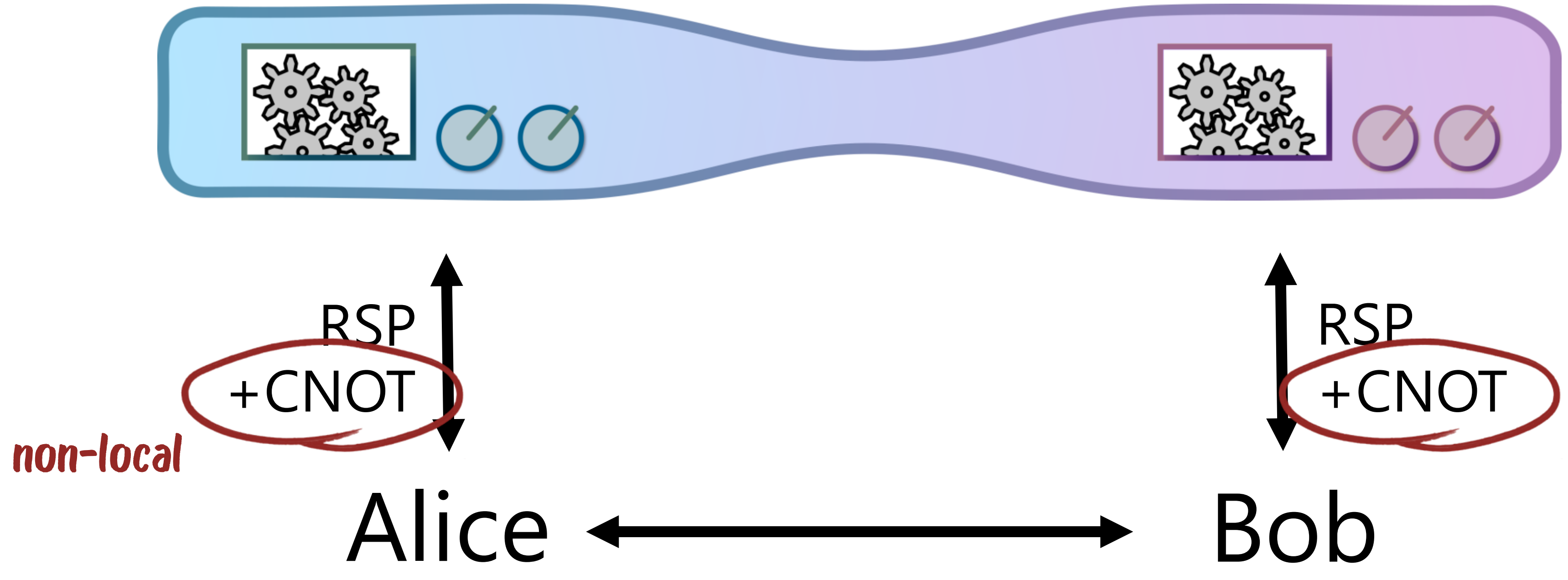


$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

Certify **single-qubit** measurements

$|00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$

Certify **Bell**-like correlations



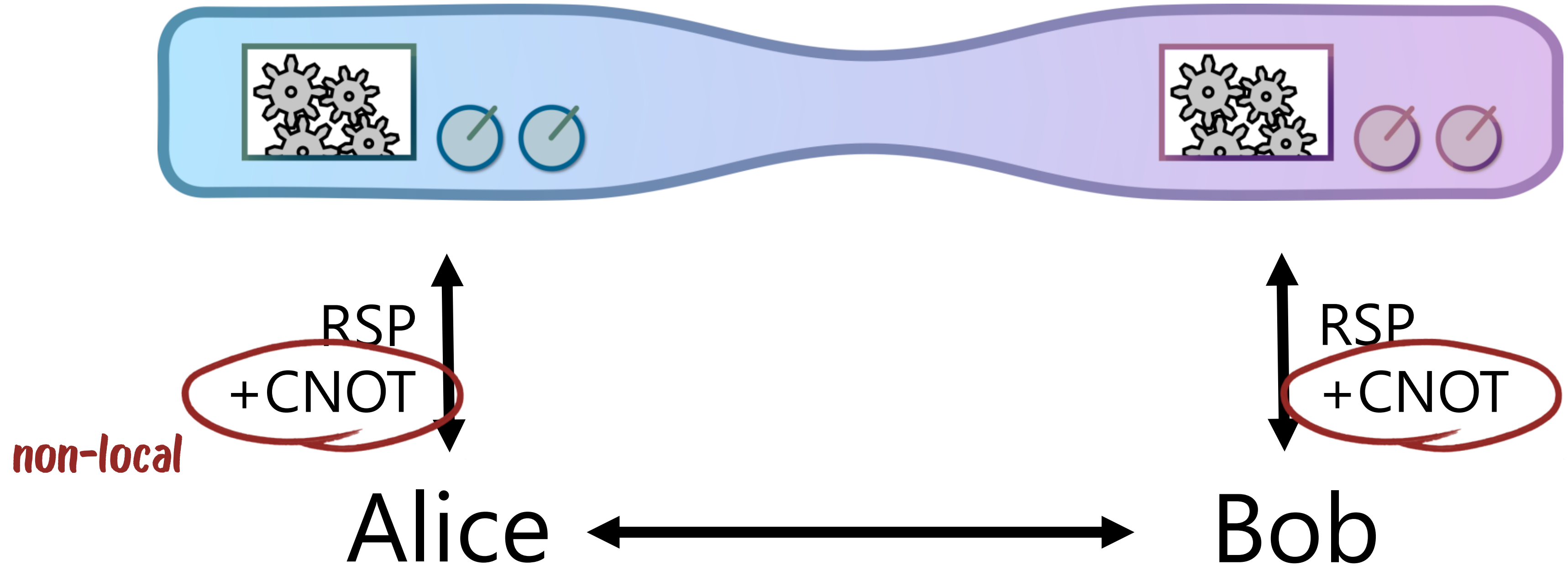
$|0/1\rangle|0/1\rangle$
$|0/1\rangle|\pm\rangle$
$|\pm\rangle|\pm\rangle$

} Certify **single-qubit** measurements

$|00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$ Certify **Bell**-like correlations

=> Blindness

Certify **single-qubit measurements** on **Bell state**



$$|0/1\rangle|0/1\rangle$$
$$|0/1\rangle|\pm\rangle$$
$$|\pm\rangle|\pm\rangle$$

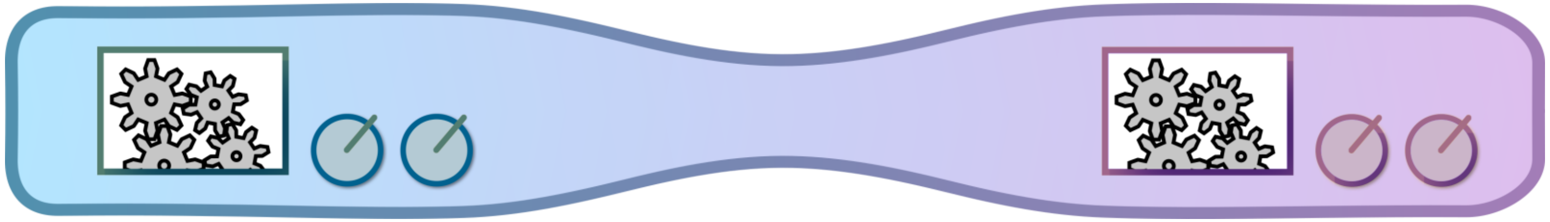
Certify **single-qubit** measurements

$$|00\rangle \pm |11\rangle, |01\rangle \pm |10\rangle$$

Certify **Bell**-like correlations

=> Blindness

Certify **single-qubit measurements** on **Bell state**



RSP +gate teleportation

RSP +gate teleportation

Alice ⟷ Bob

Gottesman & Chuang, Quantum Teleportation is a Universal Computational Primitive. Nature. 402: 390–393

(Incomplete) Genealogy

Proof of quantumness (1804.00640)

Verification of quantum computation (1804.01082)

(Incomplete) Genealogy

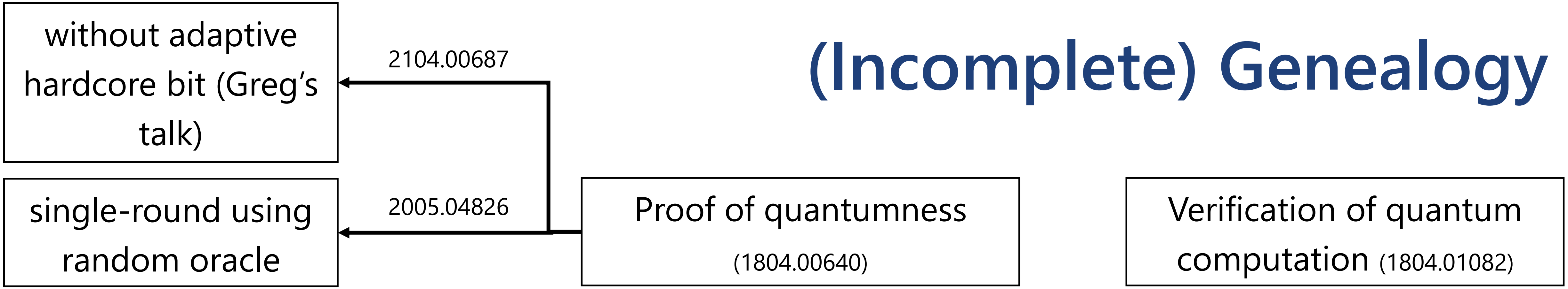
without adaptive hardcore bit (Greg's talk)

2104.00687

Proof of quantumness (1804.00640)

Verification of quantum computation (1804.01082)

(Incomplete) Genealogy



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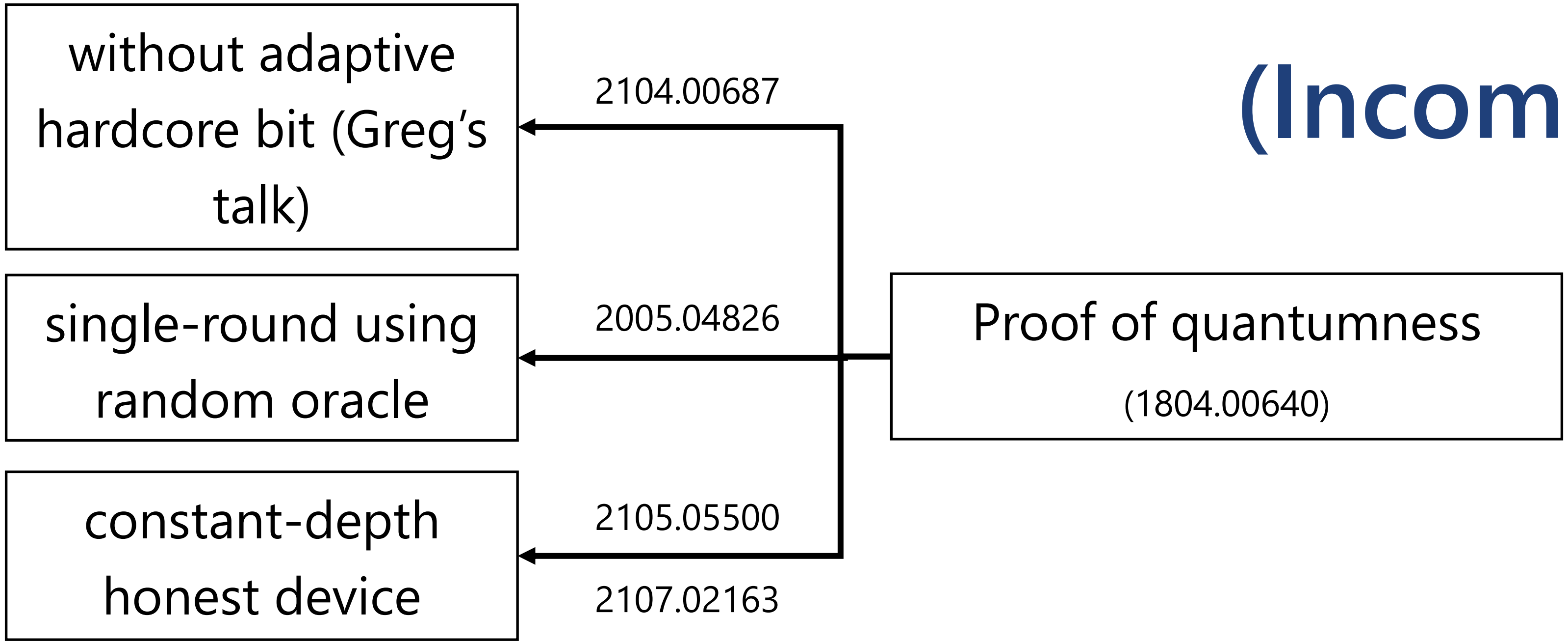
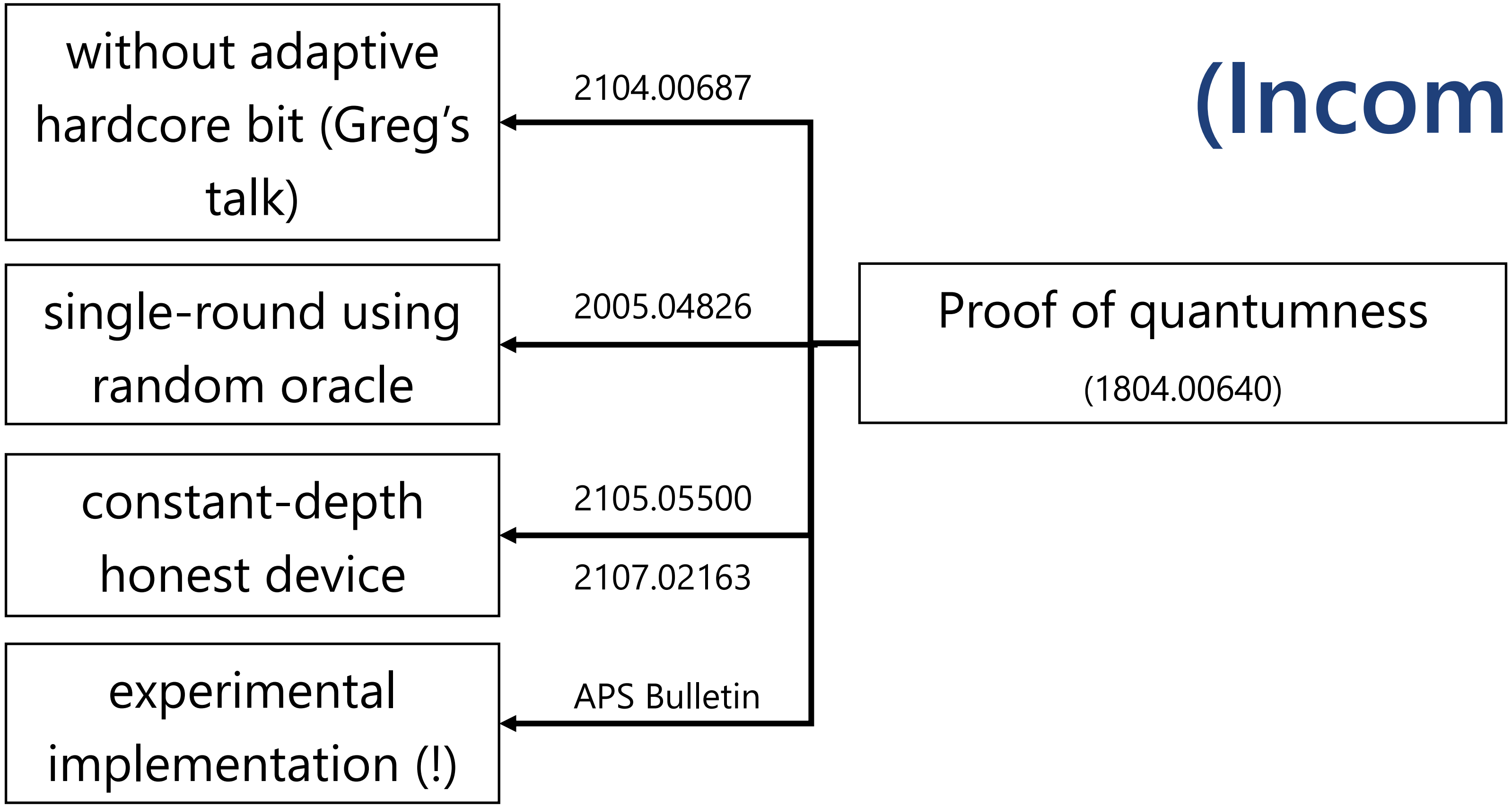


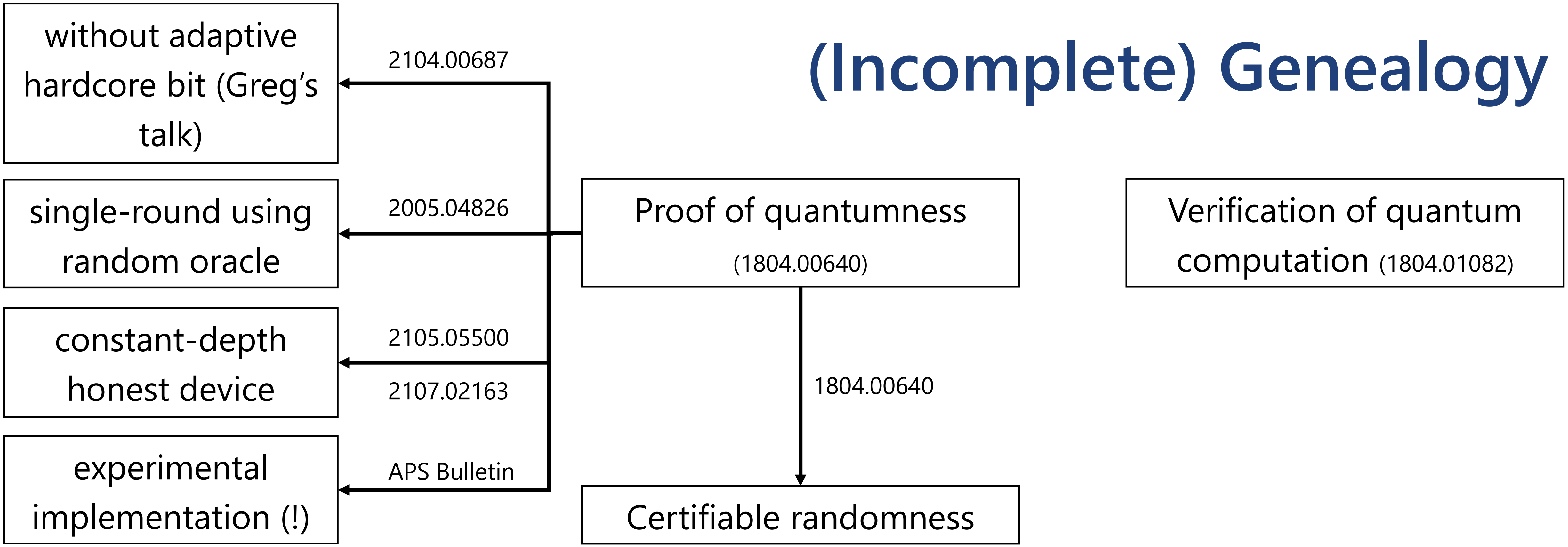
Verification of quantum computation (1804.01082)

(Incomplete) Genealogy



Verification of quantum computation (1804.01082)

(Incomplete) Genealogy



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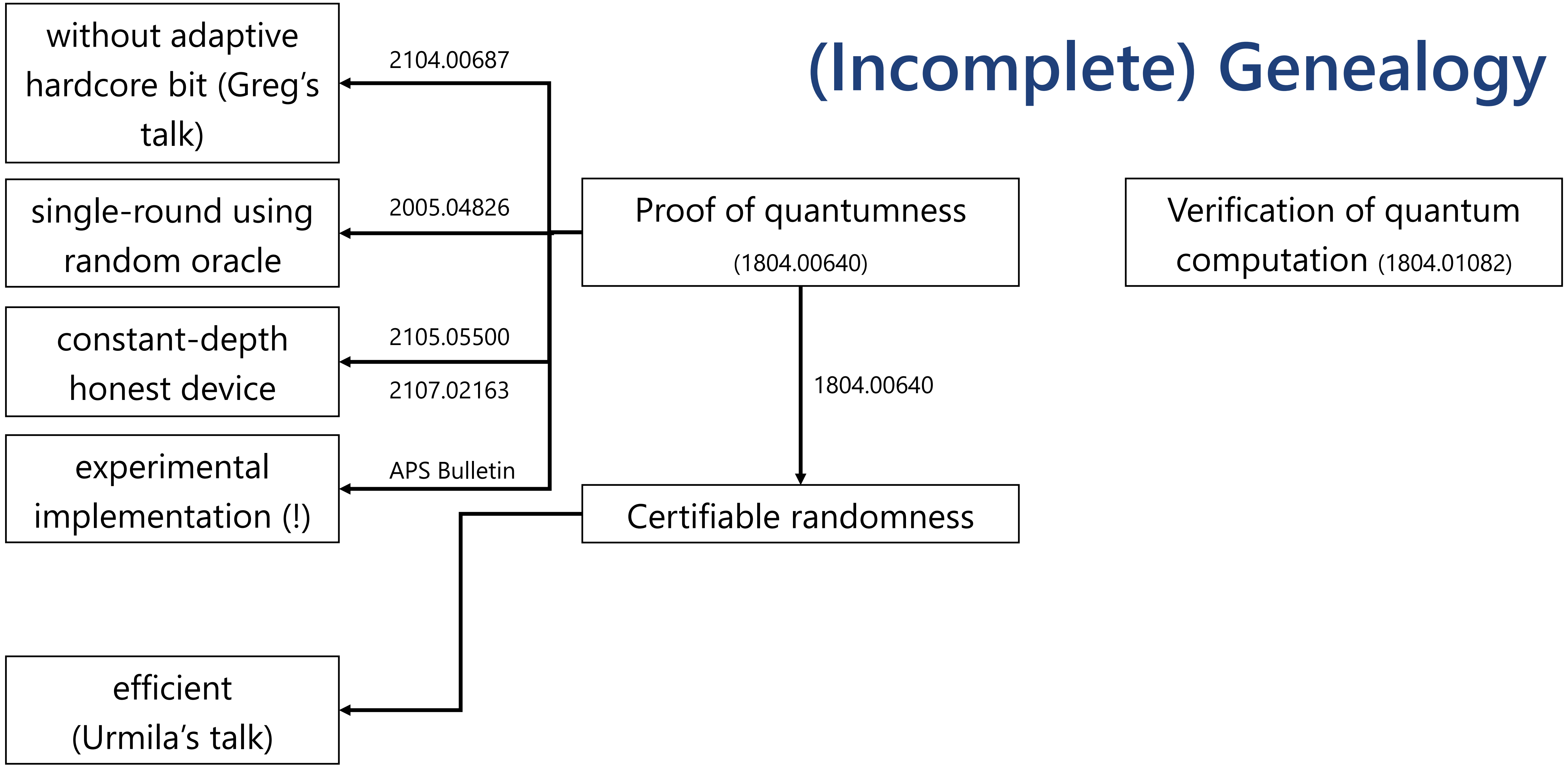

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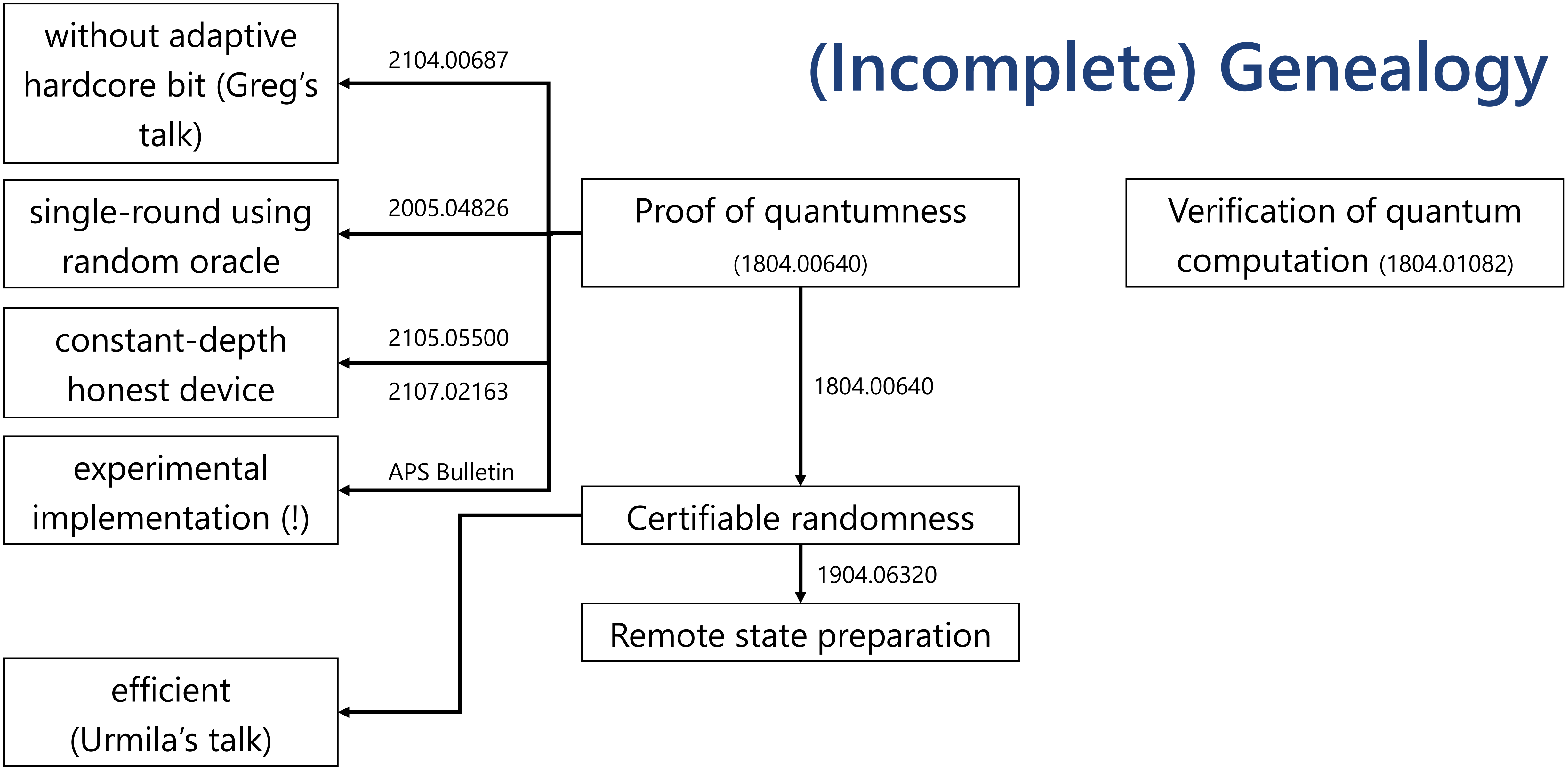

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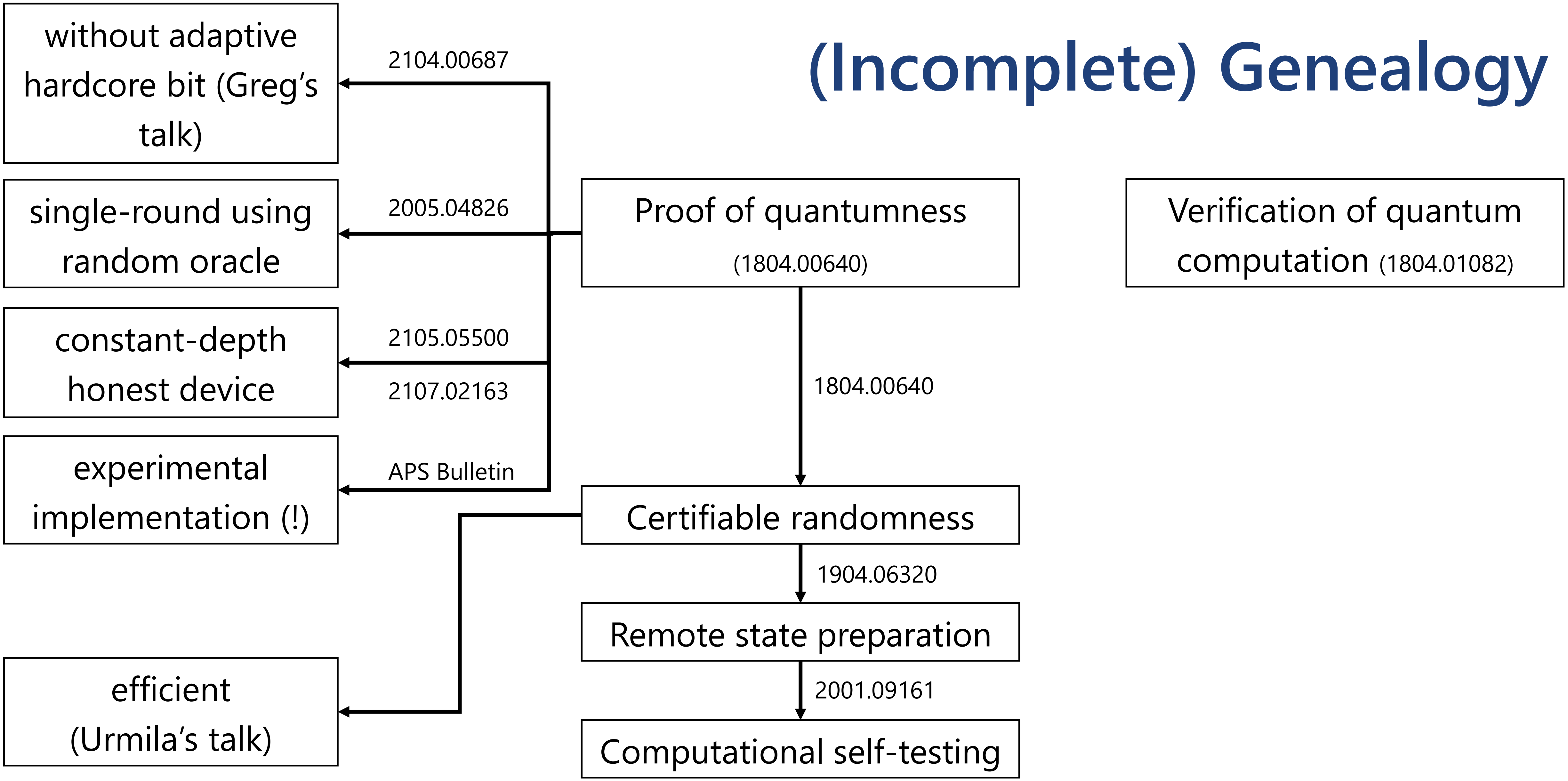

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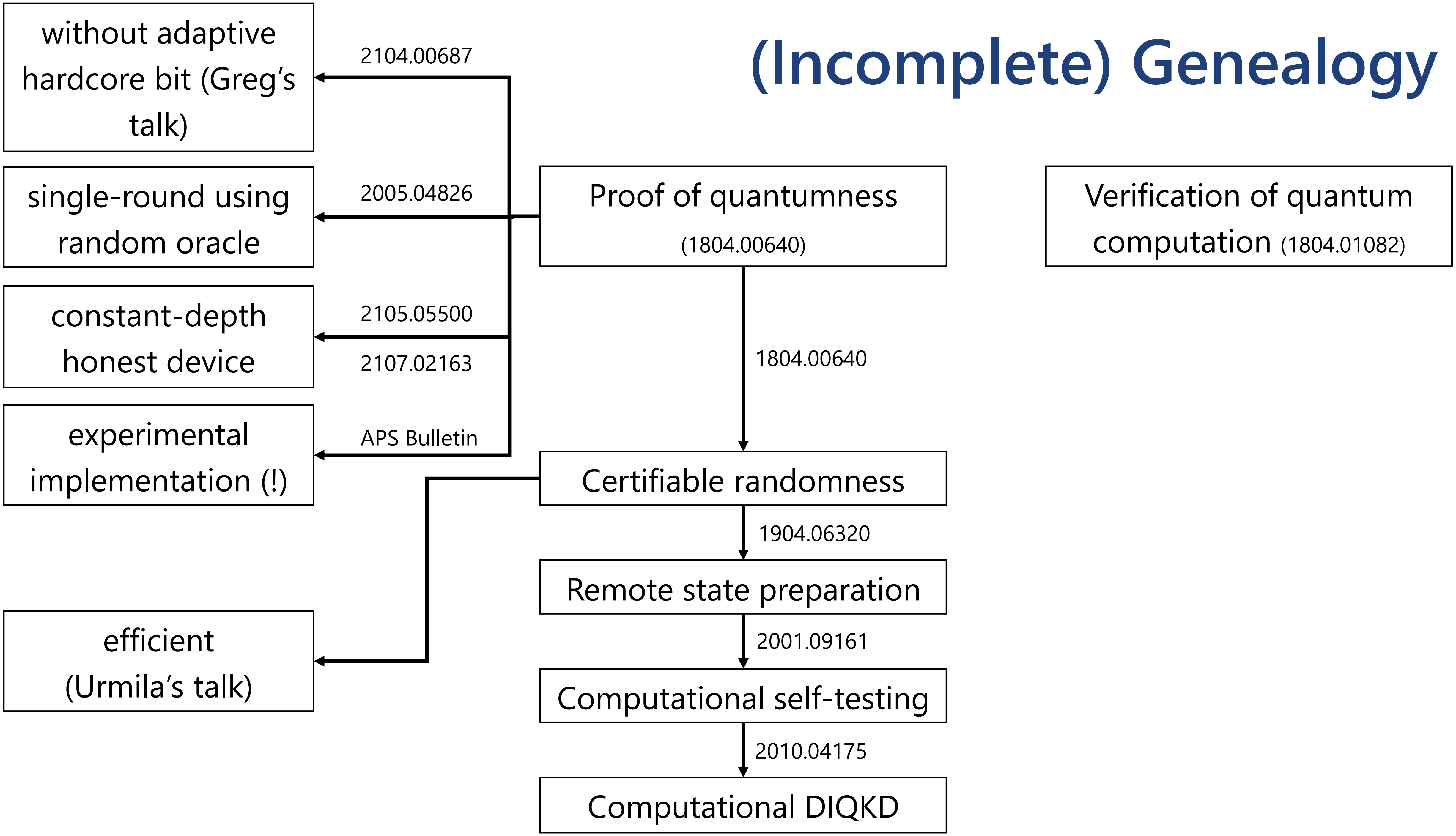

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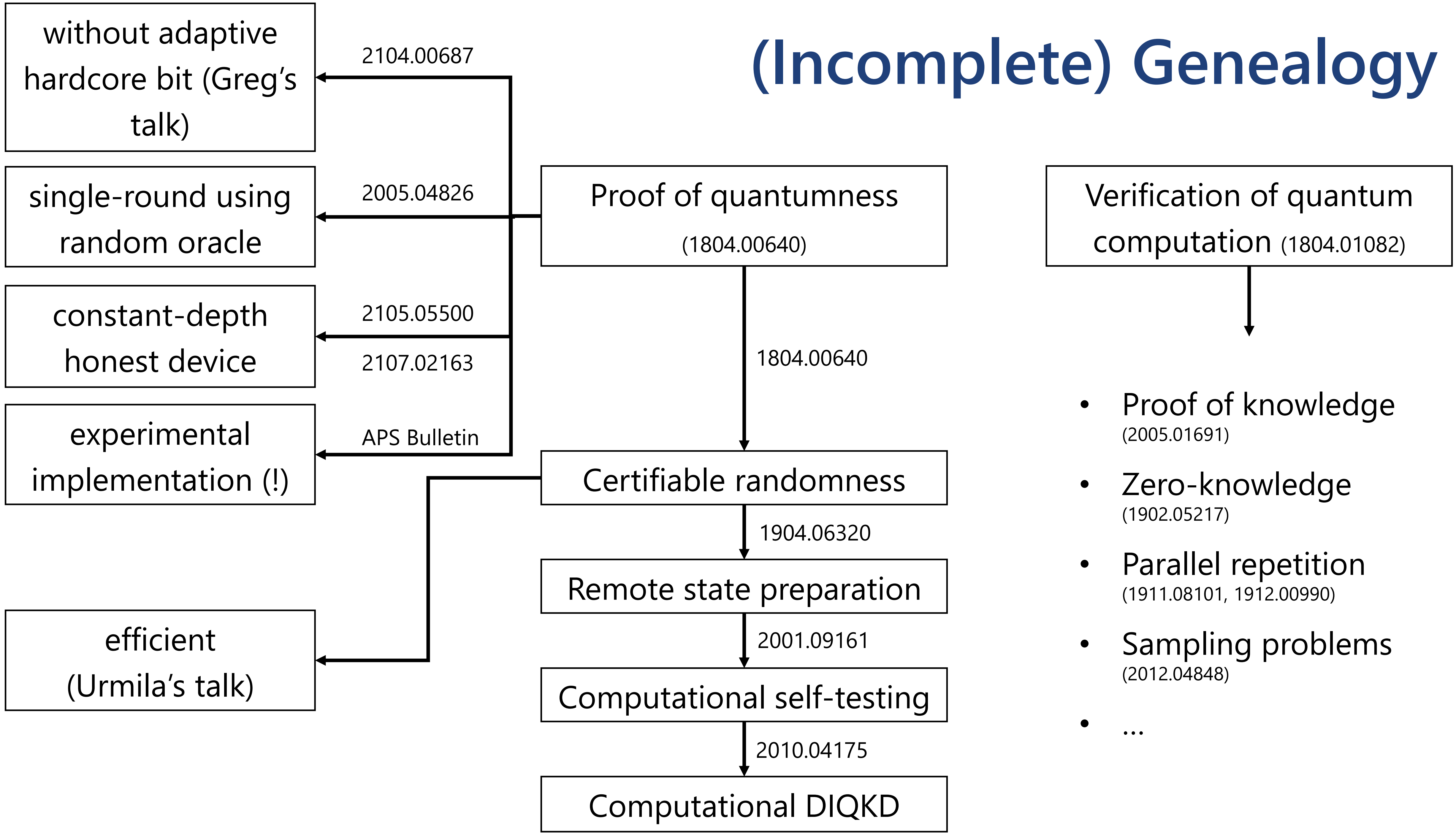

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