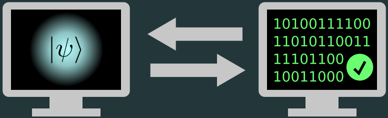

Classical verification of quantum computational advantage

Gregory D. Kahanamoku-Meyer

July 14, 2021

arXiv:2104.00687

Theory collaborators:

Norman Yao (UCB)
Umesh Vazirani (UCB)
Soonwon Choi (UCB -> MIT)

SIMONS INSTITUTE for the Theory of Computing

Berkeley UNIVERSITY OF CALIFORNIA

“Black-box” proofs of quantumness

Efficiently-verifiable test that only quantum computers can pass.

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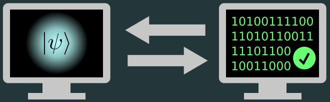
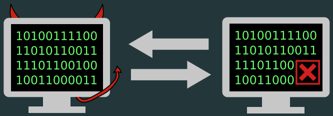
For polynomially-bounded classical verifier:



Completeness

$\exists$ BQP prover s.t. Verifier accepts w.p. $> 2/3$



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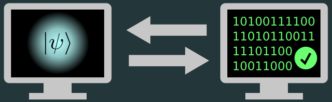

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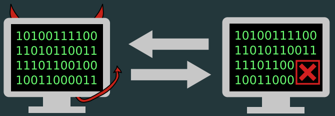

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Fully classical verifier (and comms.),

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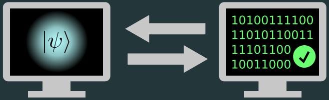

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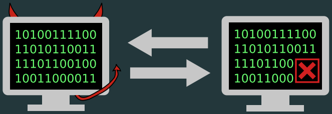

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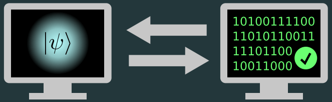

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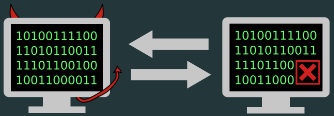

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Disprove null hypothesis that prover is classical!

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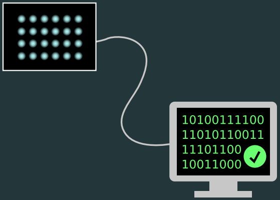

Local: powerfully refute the extended Church-Turing thesis

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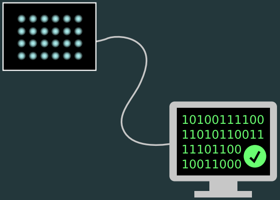

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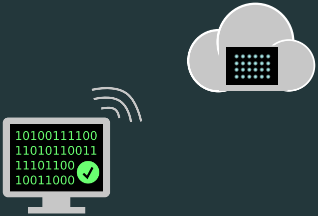

Remote: validate an untrusted quantum cloud service

NISQ verifiable quantum advantage

Trivial solution: Shor's algorithm

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NISQ: Noisy Intermediate-Scale Quantum devices

Sampling problems
e.g. random circuits, Boson sampling, ...
✓ NISQ feasible
✗ Efficiently verifiable

Number theory problems
e.g. factoring, discrete logarithm, ...
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✓ Efficiently verifiable

add structure

make less costly

???
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Adding structure to sampling problems

Generically: seems hard.

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Adding structure opens opportunities for classical cheating

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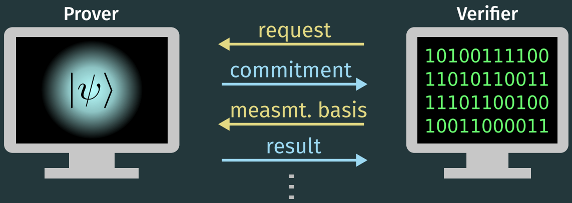
Interactive proofs of quantumness



Round 1: Prover commits to a specific quantum state

Round 2+: Verifier asks for measurement in specific basis

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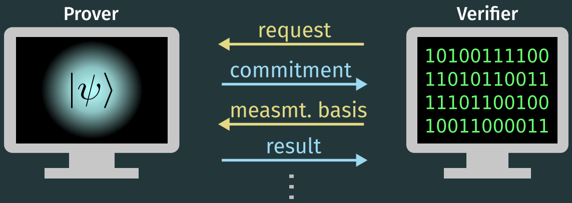

Round 1: Prover commits to a specific quantum state

Round 2+: Verifier asks for measurement in specific basis

By randomizing choice of basis and repeating interaction, can ensure prover would respond correctly in *any* basis

Brakerski, Christiano, Mahadev, Vidick, Vazirani '18 (arXiv:1804.00640).

Can be extended to verify arbitrary quantum computations! (arXiv:1804.01082)

State commitment (round 1): trapdoor claw-free functions

How does the prover commit to a state?

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Evaluate f on uniform superposition $\sum_x \lvert x\rangle \lvert f(x)\rangle$	$\xleftarrow{\quad f \quad}$	Pick 2-to-1 function f
Measure 2nd register as y	$\xrightarrow{\quad y \quad}$	Store y as commitment

Prover has committed to the state $(|x_0\rangle + |x_1\rangle)\,|y\rangle$

LWE protocol

Prover		Verifier
		
Evaluate f on uniform superposition: $\sum_x \|x\rangle \|f(x)\rangle$	$\xleftarrow{\quad f \quad}$	Pick trapdoor claw-free function f
Measure 2nd register as y	$\xrightarrow{\quad y \quad}$	Compute x_0, x_1 from y using trapdoor

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Measure qubits of $|x_0\rangle + |x_1\rangle$ in given basis	$\xrightarrow{\text{result}}$	Validate result against x_0, x_1

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		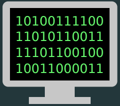
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Learning-with-Errors TCF has adaptive hardcore bit

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Trapdoor claw-free functions

TCF	Trapdoor	Claw-free	Adaptive hard-core bit
LWE [1]	✓	✓	✓
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Can we remove AHCB in the standard model?

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Replace Hadamard basis measurement with “1-player CHSH”

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Interactive measurement: computational Bell test

Replace Hadamard basis measurement with two-step process: "condense" x_0, x_1 into a single qubit, and then do a "Bell test."

$\vdots \qquad \vdots \qquad \vdots$

$\|x_0\rangle \|x_0 \cdot r\rangle + \|x_1\rangle \|x_1 \cdot r\rangle$	$\xleftarrow{\quad r \quad}$	Pick random bitstring r
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Cryptographic secret (here) $\Leftrightarrow$ Non-communication (Bell test)

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Measure qubit in basis	$\xleftarrow{\text{basis}}$	Pick $(Z + X)$ or $(Z - X)$ basis
	$\xrightarrow{\text{result}}$	Validate against r, x_0, x_1, d

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Computational Bell test: classical bound

Run protocol many times, collect statistics.

p_s: Success rate for standard basis measurement.

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Note: Let $p_s = 1$. Then for p_{CHSH}:

Classical bound 75%, ideal quantum $\sim 85\%$. Same as regular CHSH!

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

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    - Need to implement public-key crypto. on a superposition

Partial measurements in the lab

Trapped Ion Quantum Information lab at U. Maryland

Working on demonstration of protocols in trapped ions!

Prof. Christopher Munroe

Dr. Daiwei Zhu

Dr. Crystal Noel

and others!

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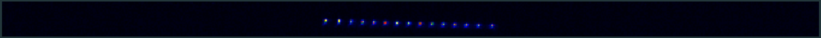

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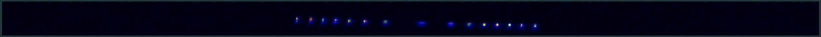

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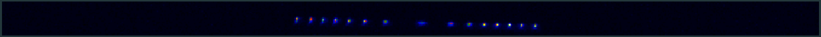

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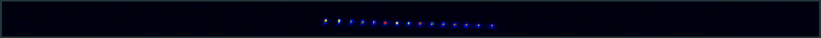

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When we generate $\sum_x |x\rangle\,|f(x)\rangle$, **add redundancy to $f(x)$, for bit flip error detection!**

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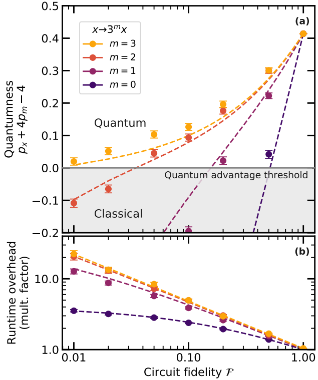

Numerical results for $x^2 \bmod N$ with $\log N = 512$ bits.

Here: make transformation $x^2 \bmod N \Rightarrow (kx)^2 \bmod k^2 N$

Improving circuit sizes

Most demanding step in all these protocols: evaluating TCF

$$\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$$

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Getting rid of adaptive hardcore bit helps!

$x^2 \bmod N$ and Ring-LWE have classical circuits as fast as $\mathcal{O}(n \log n)$...

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Improving circuit sizes

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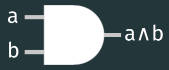
Protocol allows us to make circuits irreversible!

Technique: taking out the garbage

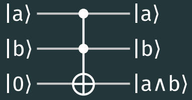
Goal: $\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$

When converting classical circuits to quantum:

Garbage bits: extra entangled outputs due to unitarity



Classical AND



Quantum AND (Toffoli)

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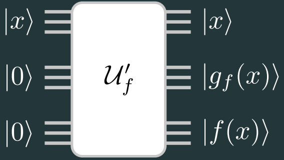

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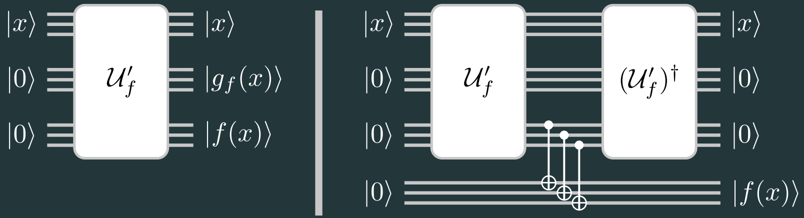

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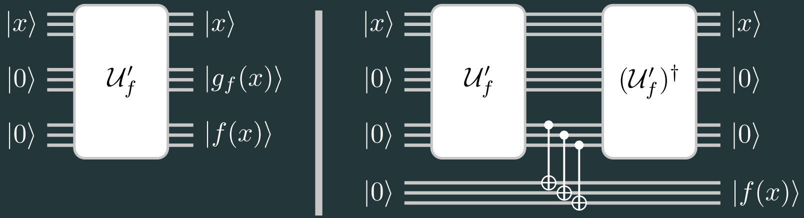

Lots of time and space overhead!

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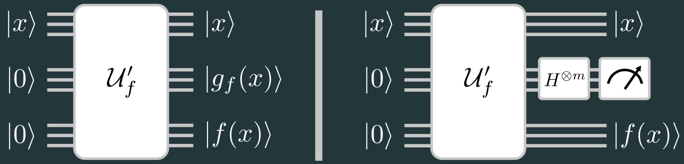

Can we “measure them away” instead?

Technique: taking out the garbage

Measure garbage bits $g_f(x)$ in Hadamard basis, get some string h.
End up with state:

$$\sum_x (-1)^{h \cdot g_f(x)} |x\rangle |f(x)\rangle$$

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1024-bit $x^2 \mod N$ costs only 10^6 Toffoli gates.

Paths forward

Bottleneck: Evaluating TCF on quantum superposition

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“In the box” ideas (not necessarily bad):

- Find more efficient TCFs

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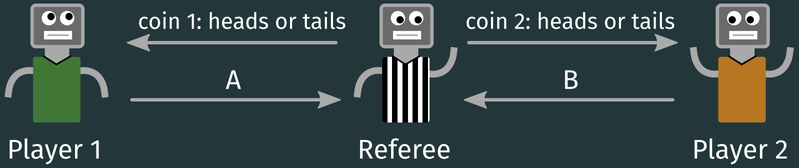
Way outside the box?

Backup!

The CHSH game (Bell test)

Two-player cooperative game.



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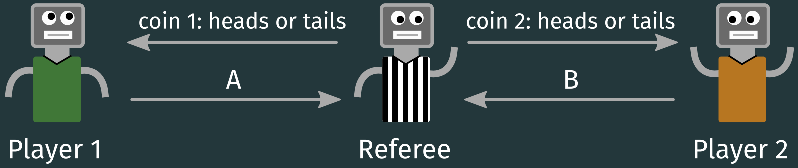

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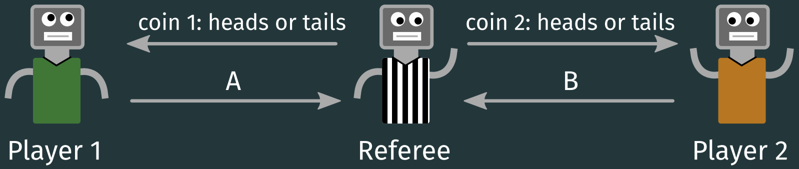

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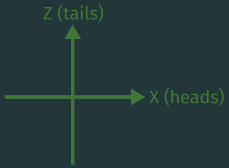

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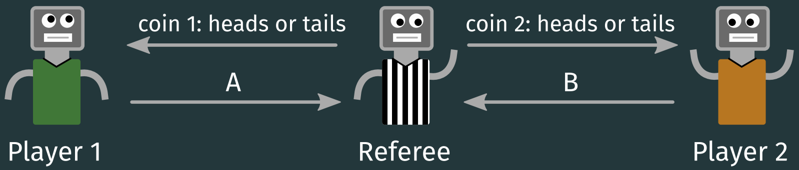

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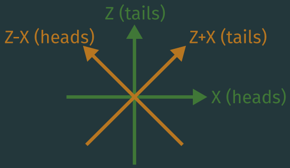

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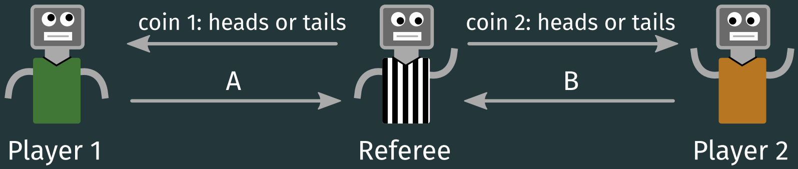

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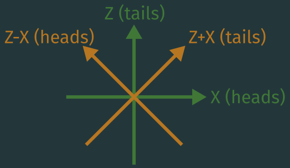

Quantum: $\cos^2(\pi/8) \approx 85\%$

Classical: 75%

Decisional Diffie-Hellman (DDH)

Problem (not TCF): Consider a group $\mathbb{G}$ of order N, with generator g. Given the tuple (g, g^a, g^b, g^c), determine if $c = ab$.

Elliptic curve crypto.: $\log N \sim 160$ bits is as hard as 1024 bit factoring!!

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How to build a TCF?

Trapdoor [Peikert, Waters '08; Freeman et al. '10]: linear algebra in the exponent

Claw-free [GDKM et al. '21 (arXiv:2104.00687)]: collisions in linear algebra in the exponent!

Full protocol

Prover (quantum)

Verifier (classical)

Prover (quantum)	Message	Verifier (classical)
Round 1	$\xleftarrow{f_i}$	1. Sample $(f_i, t) \leftarrow \mathsf{Gen}(1^n)$
2. Generate state $\sum_{x=0}^{N/2} \lvert x\rangle_{\mathsf{x}} \lvert f_i(x)\rangle_{\mathsf{y}}$		
3. Measure y register, yielding bitstring y State is now $(\lvert x_0\rangle + \lvert x_1\rangle)_{\mathsf{x}} \lvert y\rangle_{\mathsf{y}}$; y register can be discarded	$\xrightarrow{y}$	4. Using trapdoor t compute x_0 and x_1
	$\xleftarrow{\text{choice}}$	5. Randomly choose to request a preimage or continue
If preimage requested:		
6a. Projectively measure x register, yielding x	$\xrightarrow{x}$	7a. If $x \in \{x_0, x_1\}$ return Accept
Otherwise, continue:		
Round 2	$\xleftarrow{r}$	6b. Choose random bitstring r
7b. Add one ancilla b; use CNOTs to compute $\lvert r \cdot x_0\rangle_{\mathsf{b}} \lvert x_0\rangle_{\mathsf{x}} + \lvert r \cdot x_1\rangle_{\mathsf{b}} \lvert x_1\rangle_{\mathsf{x}}$ where $r \cdot x$ is bitwise inner product		
8b. Measure x register in Hadamard basis, yielding a string d. Discard x, state is now $\lvert\psi\rangle_{\mathsf{b}} \in \{\lvert 0\rangle, \lvert 1\rangle, \lvert +\rangle, \lvert -\rangle\}$	$\xrightarrow{d}$	9b. Using r, x_0, x_1, d, determine $\lvert\psi\rangle_{\mathsf{b}}$
Round 3	$\xleftarrow{m}$	10b. Choose random $m \in \{\frac{\pi}{4}, -\frac{\pi}{4}\}$
11b. Measure ancilla b in the rotated basis $\left\{\begin{matrix} \cos\left(\frac{m}{2}\right)\lvert 0\rangle + \sin\left(\frac{m}{2}\right)\lvert 1\rangle \\ \cos\left(\frac{m}{2}\right)\lvert 1\rangle - \sin\left(\frac{m}{2}\right)\lvert 0\rangle \end{matrix}\right\}$, yielding a bit b	$\xrightarrow{b}$	11b. If b was likely given $\lvert\psi\rangle_{\mathsf{b}}$ return Accept