Research Issues in Quantum Networks for Entanglement Distribution

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Outline

- qubits and entanglements
- entanglement networks
- research issues and research challenges
  - state information
  - route diversity
- summary
Elementary quantum 101

- bit has only two values: 0, 1
- physically represented by two state device

\[ V_{\text{out}} = 0 \]
\[ \Delta V = -\frac{V_{\text{CC}}}{2} \]
\[ V_{\text{ref}} = \frac{V_{\text{CC}}}{2} \]

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Quantum bits

- qubit - two-state quantum-mechanical system
- example: photon polarization

Horizontally polarized: $|x\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Vertically polarized: $|y\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Superposition of states

\[ |\phi\rangle = \alpha |x\rangle + \beta |y\rangle, \quad \alpha^2 + \beta^2 = 1 \]

\[ |+\rangle = \frac{1}{\sqrt{2}} (|x\rangle + |y\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]

\[ |-\rangle = \frac{1}{\sqrt{2}} (|x\rangle - |y\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \]
Measurement

- uncountable number of states

- single photon: either $X$ or $Y$ goes off, not both
- repeat many times: $P(x) = \alpha^2$, $P(y) = \beta^2$
Two qubits

- four basis states, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

- Bell state (Einstein-Podolsky-Rosen (EPR) pair)

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
Two qubit states

- Bell state (EPR pair)

\[ \frac{|00\rangle + |11\rangle}{\sqrt{2}} \]

- measuring first qubit yields 0, 1
  - if 1, measuring second qubit yields 1
  - if 0, measuring second qubit yields 0

- other powerful measurement correlations
- basis of quantum computing, quantum key distribution, quantum sensing
Long distance entanglement

\[ R \approx 1.44\eta \text{ bits/mode when } \eta \ll 1 \]

\[ \eta = e^{-\alpha L} \text{ in fiber} \]
Quantum Networks

metro: $\leq 100$ km

long-haul: 1000s of km

Quantum Network Applications

Secure Comm, Quantum foundations

Distributed quantum computing

Sensing, Timing, GPS, ..

Undiscovered app’s
Repeaters, e2e entanglements

Phase 1: link entanglements

\[ p = 1 - (1 - p_0)^m \]

Phase 2: splice links together

Bell state measurements, \( q \) – prob. of success
Quantum entanglement network

- can connect multiple users
- multiple paths per user pair
Challenges: performance, control

- given pairs of users, capacity region?
- resource allocation schemes?
- stateless vs stateful control?
- static routing vs opportunistic routing?
- latency models?
State information, Path diversity

- grid network
- single mode per link
- one memory per repeater per link per mode
- one pair of end-to-end communicating nodes

Grid Network
Grid Network - Phase 1
Grid Network – Phase 2
Rate dependence on $p$

- greedy shortest path algorithm
  - find shortest path
  - next shortest path
  - ...
- requires global information
- $R_g(p, q)$ — entanglement rate

Note: when $q = 1$, 2-D grid percolates at $p > 0.5$
When every repeater has global state information

- $R_{UB}(p, q) \quad$ upperbound
- $q = 1$, max flow
  - achievable with global information
- $q < 1$, $R_{UB} = 4 \times R_g$
Routing entanglement flows with local state information

\[ d_A = 2.8 \]
\[ d_B = 3 \]

\[ d_A = 1.4 \]
\[ d_B = 4.1 \]

\[ d_A = 3.2 \]
\[ d_B = 2.2 \]

\[ d_A = 2 \]
\[ d_B = 3.6 \]

Euclidean distance from Alice, Bob
Routing entanglement flows with local state information

\[ d_A = 2.8 \]
\[ d_B = 3 \]

\[ d_A = 1.4 \]
\[ d_B = 4.1 \]

\[ d_A = 3.2 \]
\[ d_B = 2.2 \]

Alice

Bob

v w
Routing entanglement flows with local state information

\[
d_A = 2.8 \\
d_B = 3 \\
\]

\[
d_A = 3.2 \\
d_B = 2.2 \\
\]

\[
d_A = 1.4 \\
d_B = 4.1 \\
\]

\[
d_A = 2 \\
d_B = 3.6 \\
\]
Routing entanglement flows with local state information

Alice

Bob

$d_A = 2.8$
$d_B = 3$

$d_A = 2$
$d_B = 3.6$

$d_A = 1.4$
$d_B = 4.1$

$d_A = 3.2$
$d_B = 2.2$

connect potential shortest path
Routing entanglement flows with local state information

Connect potential shortest path

\[
\begin{align*}
    d_A &= 2.8 \\
    d_B &= 4 \\
    d_A &= 3.2 \\
    d_B &= 2.2 \\
    d_A &= 1.4 \\
    d_B &= 4.1 \\
    d_A &= 2 \\
    d_B &= 3.6
\end{align*}
\]
When every repeater only has local state information

- $R_{loc}(p, q)$ – rate using local rule
- $R_{lin}(p, q)$ - rate using single static path of same distance
  - no diversity
Multi-flow routing

Local Rule based on Flow 1

Local Rule based on Flow 2

Alice 1

Bob 1

Alice 2

Bob 2

multi-flow spatial division

multi-flow time-share

single-flow time-share

$R_1$ vs. $R_2$ graph
Multi-flow routing

\[ \theta \]

Alice 1

Bob 1

Alice 2

Bob 2

\[ R_1 \]

\[ R_2 \]

D

C

multi-flow spatial division

multi-flow time-share

single-flow time-share

B

A

0 0.1 0.2 0.3 0.4 0.5 0.6

0 0.1 0.2 0.3 0.4 0.5 0.6
Open Questions

- rate-optimal protocol?
- effect of multiple modes, multiple memories?
- effect of coherence times, purification, etc.?
- 3+ qubit entanglements?
Conclusions

- quantum repeater networks achieve much larger rates than linear chains due to multi-path routing, even with only local information.

- multi-flow strategies that exploit spatial division can provide significant performance improvements in such networks.

- research on Q-networks in its infancy. Many exciting problems!
Happy retirement, Jean

How about a new hobby?

Design and analysis of quantum networks