Research Issues in Quantum Networks for Entanglement Distribution

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Outline

- qubits and entanglements
- entanglement networks
- research issues and research challenges
  - state information
  - route diversity
- summary

Elementary quantum 101

- bit has only two values: 0,1
- physically represented by two state device

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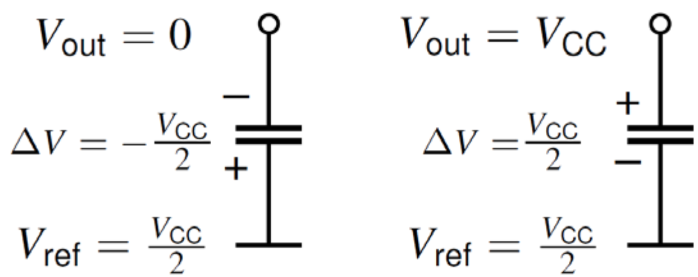
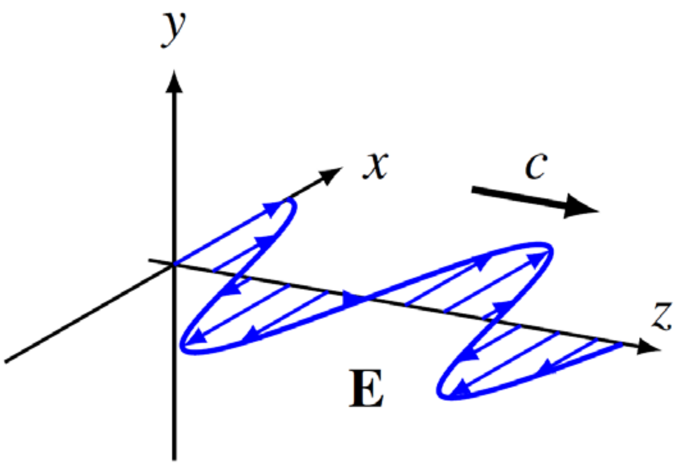
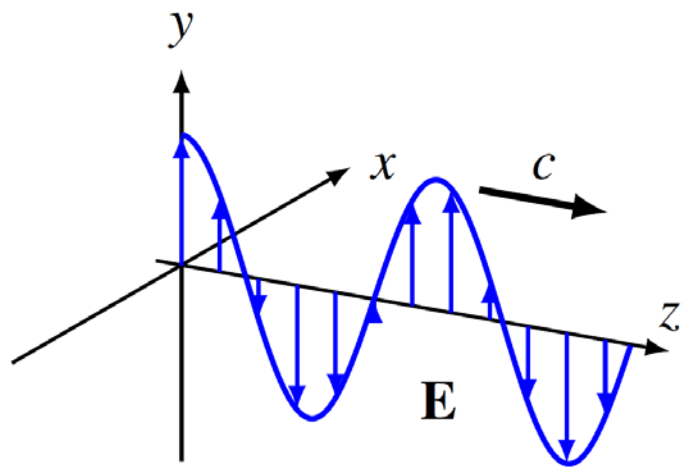


LOW HIGH

Quantum bits

- qubit - two-state quantum-mechanical system
- example: photon polarization





Horizontally polarized

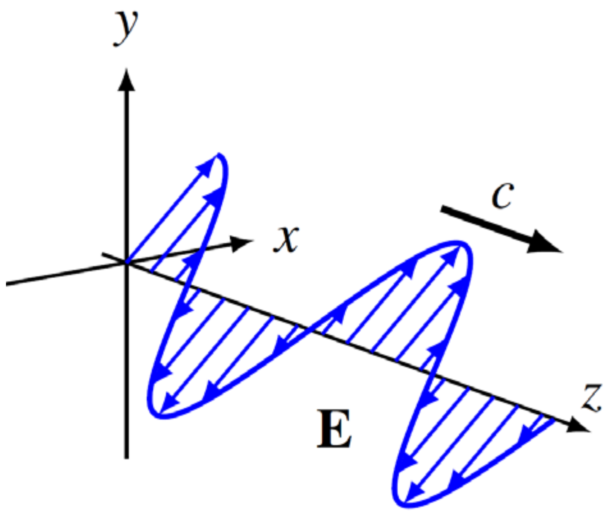
$$|x\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

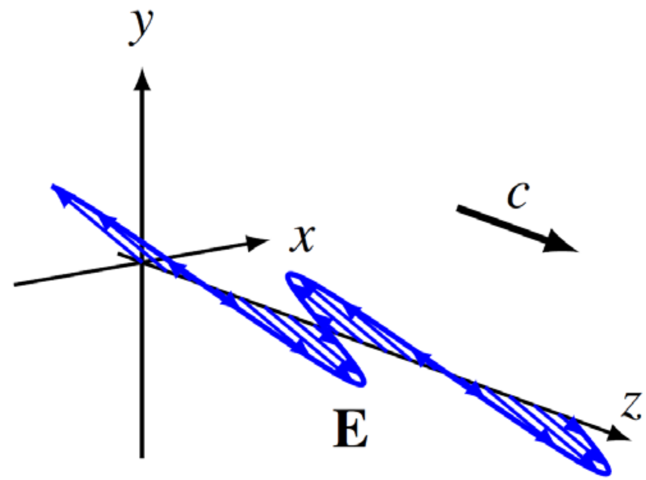
Vertically polarized

$$|y\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Superposition of states

$$|\phi\rangle = \alpha|x\rangle + \beta|y\rangle, \qquad \alpha^2 + \beta^2 = 1$$



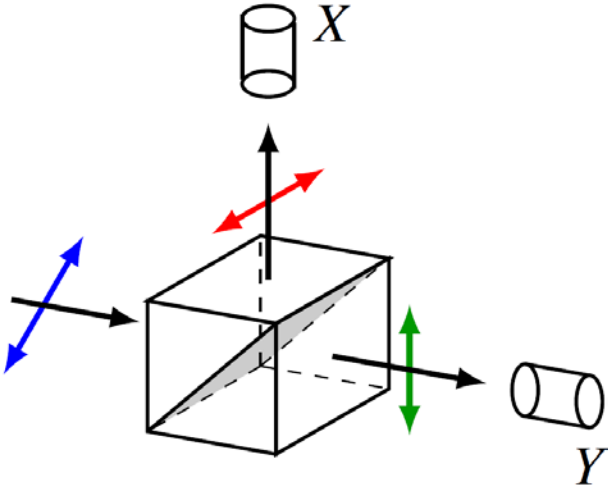


$$|+\rangle = \frac{1}{\sqrt{2}}(|x\rangle + |y\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \qquad |-\rangle = \frac{1}{\sqrt{2}}(|x\rangle - |y\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

Measurement

- uncountable number of states

$$|\phi\rangle = \alpha\,|x\rangle + \beta\,|y\rangle$$



- single photon: either X or Y goes off, not both
- repeat many times: $P(x) = \alpha^2, \;\; P(y) = \beta^2$

Two qubits

- four basis states, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

- Bell state (Einstein-Podolsky-Rosen(EPR) pair)

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Two qubit states

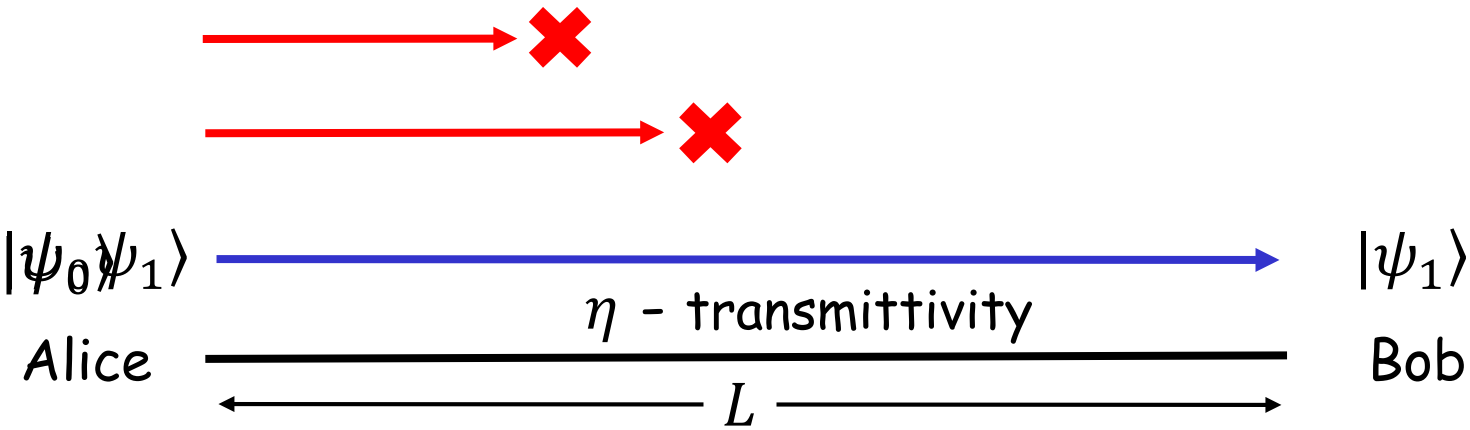
- Bell state (EPR pair)

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

- measuring first qubit yields 0,1
  - if 1, measuring second qubit yields 1
  - if 0, measuring second qubit yields 0

- other powerful measurement correlations
- basis of quantum computing, quantum key distribution, quantum sensing

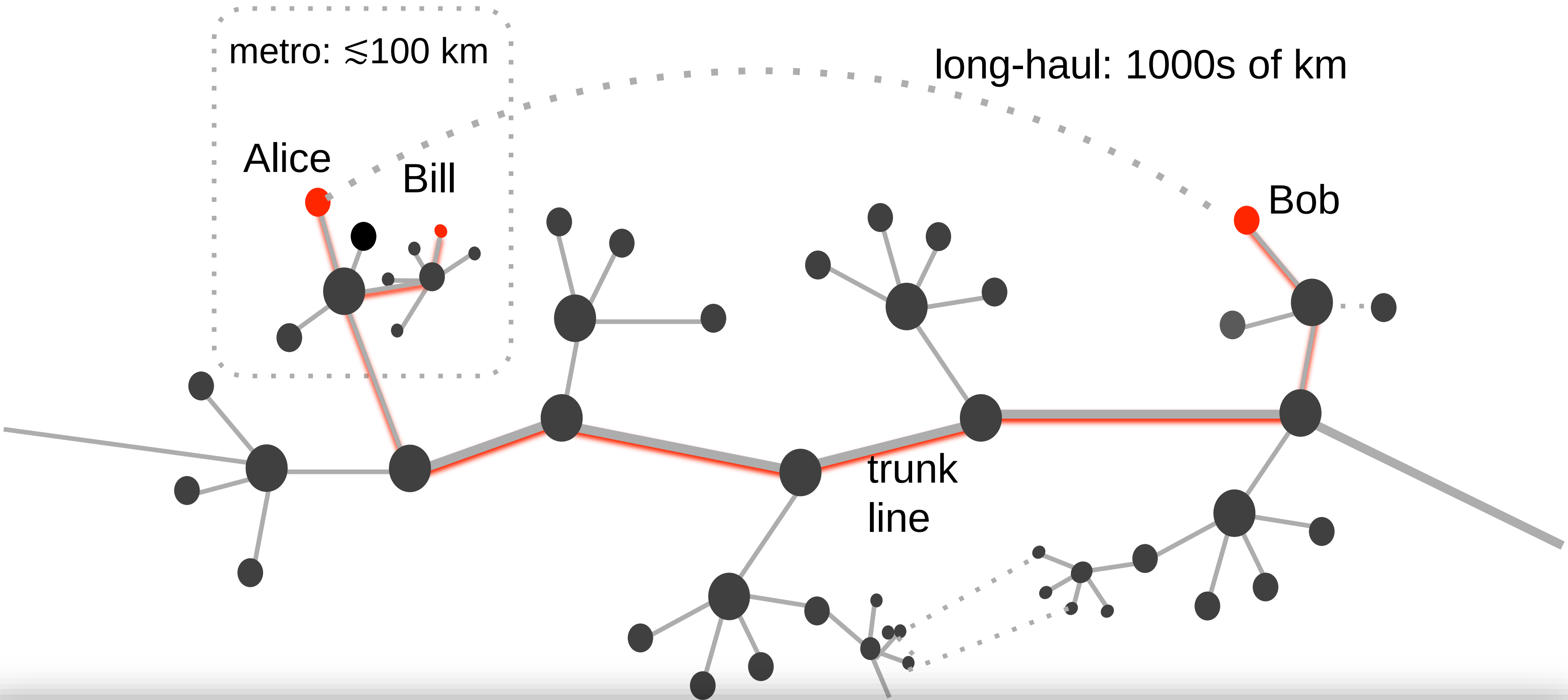
Long distance entanglement



$R \approx 1.44\eta$ bits/mode when $\eta \ll 1$

$\eta = e^{-\alpha L}$ in fiber

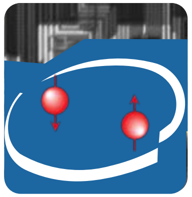
Quantum Networks



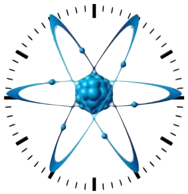
Quantum Network Applications

Secure Comm,

Quantum foundations



Distributed quantum computing

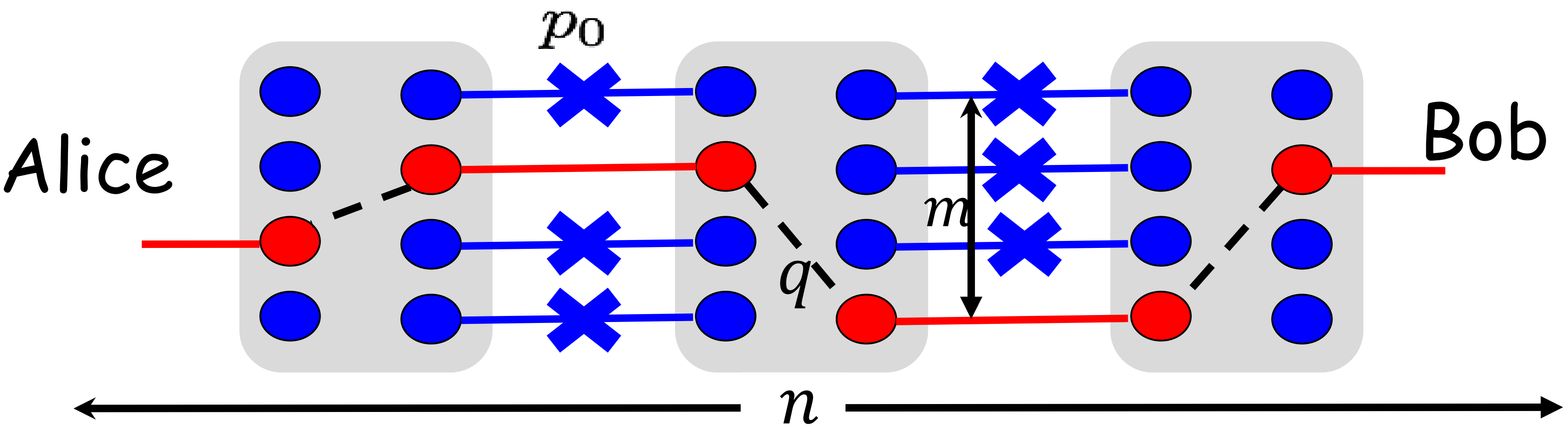


Sensing Timing, GPS, ..

Undis-covered

app's

Repeaters, e2e entanglements

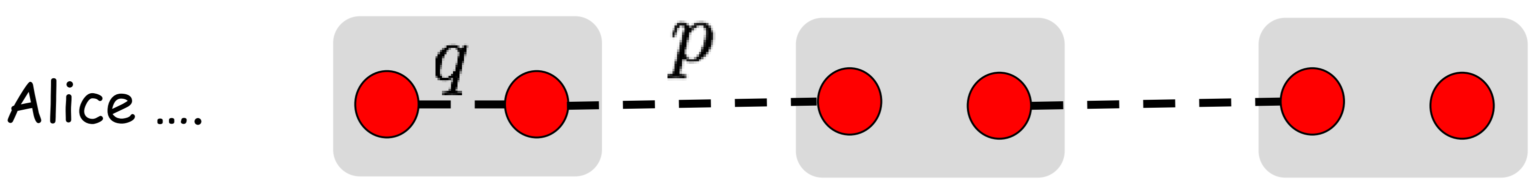


Phase 1: link entanglements

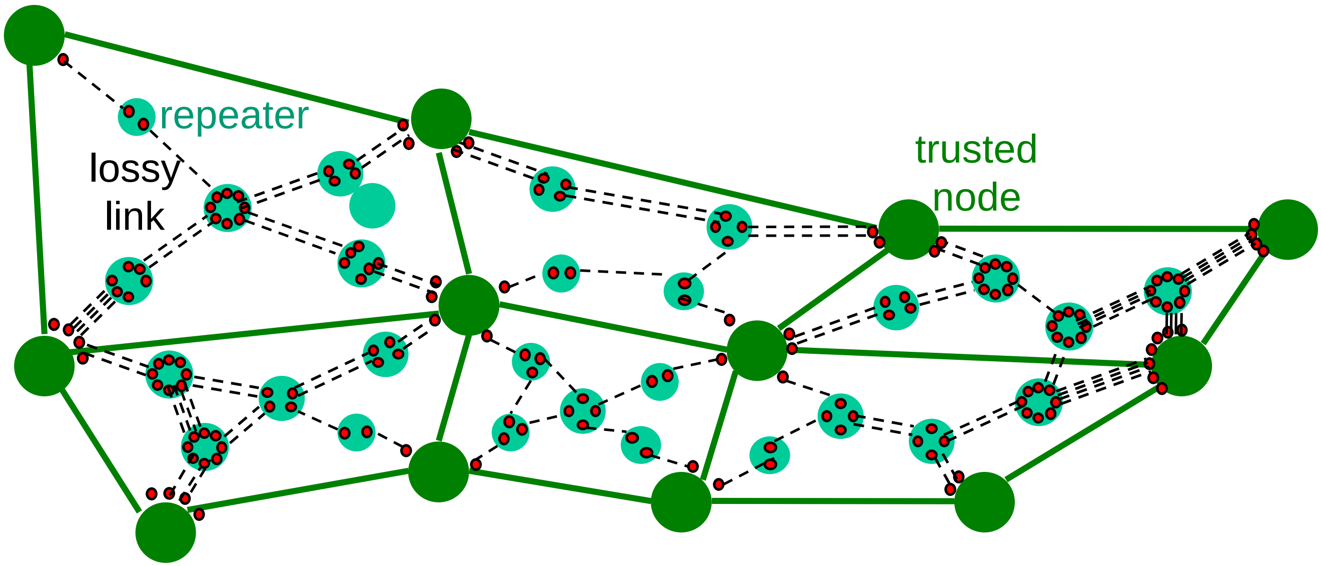
$$p = 1 - (1 - p_0)^m$$

Phase 2: splice links together

Bell state measurements, q – prob.of success



Quantum entanglement network



- can connect multiple users
- multiple paths per user pair

Challenges: performance, control

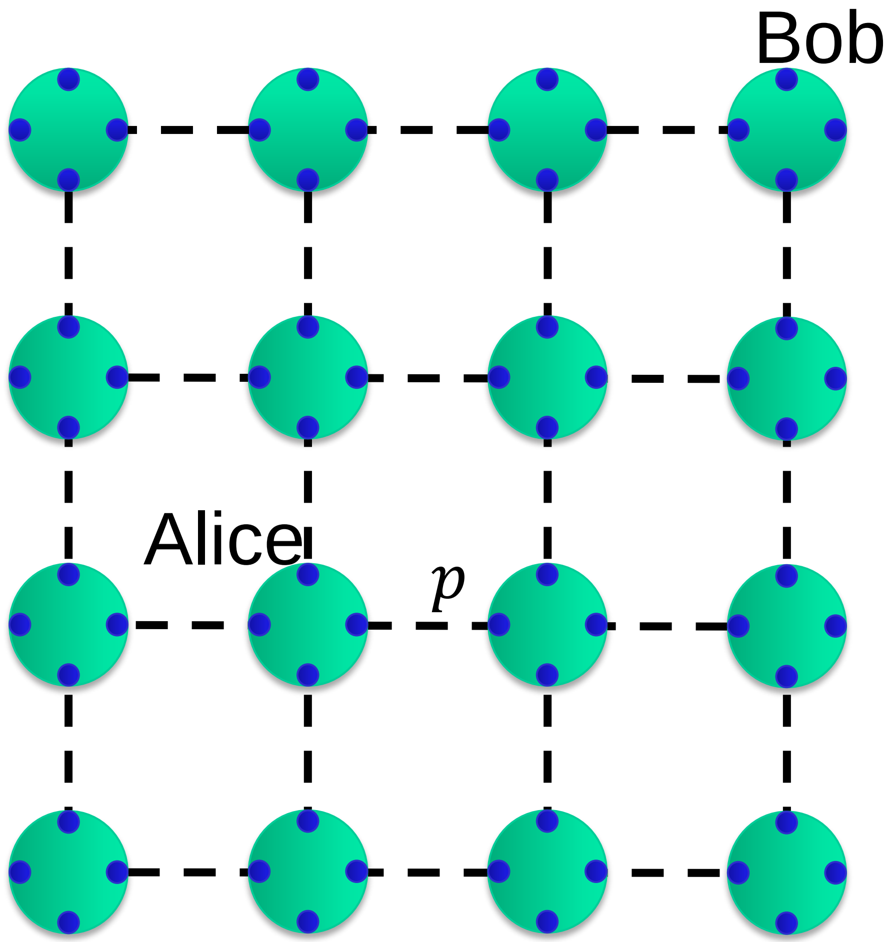
- given pairs of users, capacity region?
- resource allocation schemes?
- stateless vs stateful control?
- static routing vs opportunistic routing?
- latency models?

State information, Path diversity

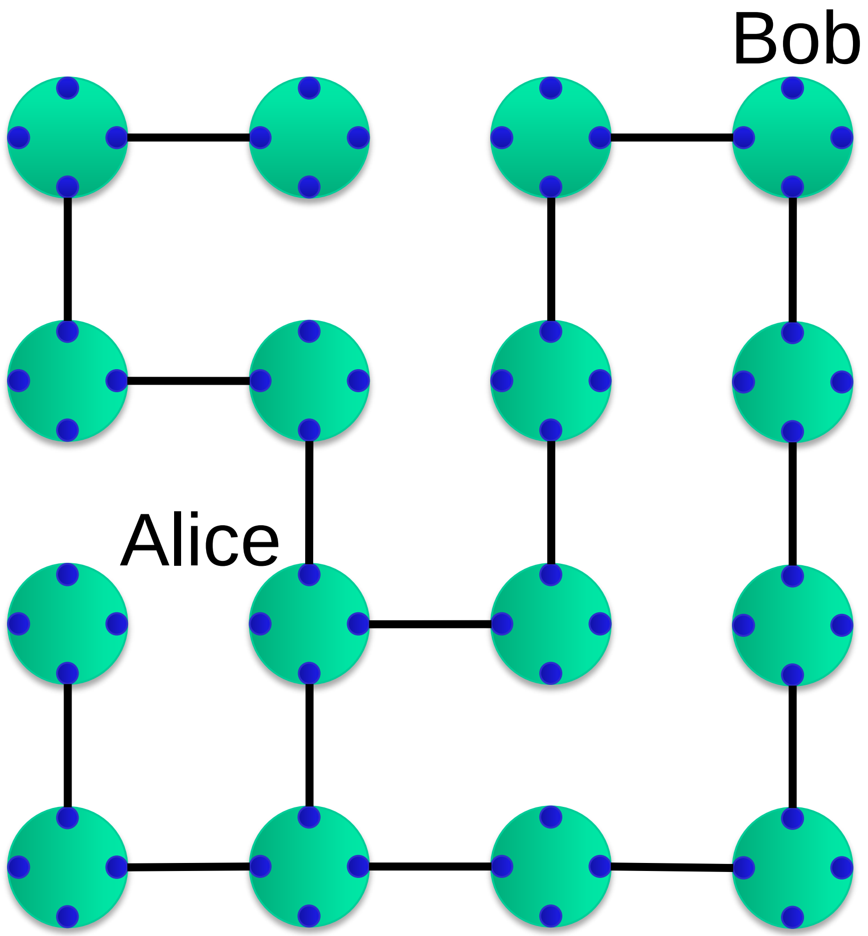
- grid network
- single mode per link
- one memory per repeater per link per mode
- one pair of end-to-end communicating nodes

1- Pant, Mihir, Hari Krovi, Don Towsley, Leandros Tassiulas, Liang Jiang, Prithwish Basu, Dirk Englund, and Saikat Guha. "Routing entanglement in the quantum internet." arXiv preprint arXiv:1708.07142 (2017).

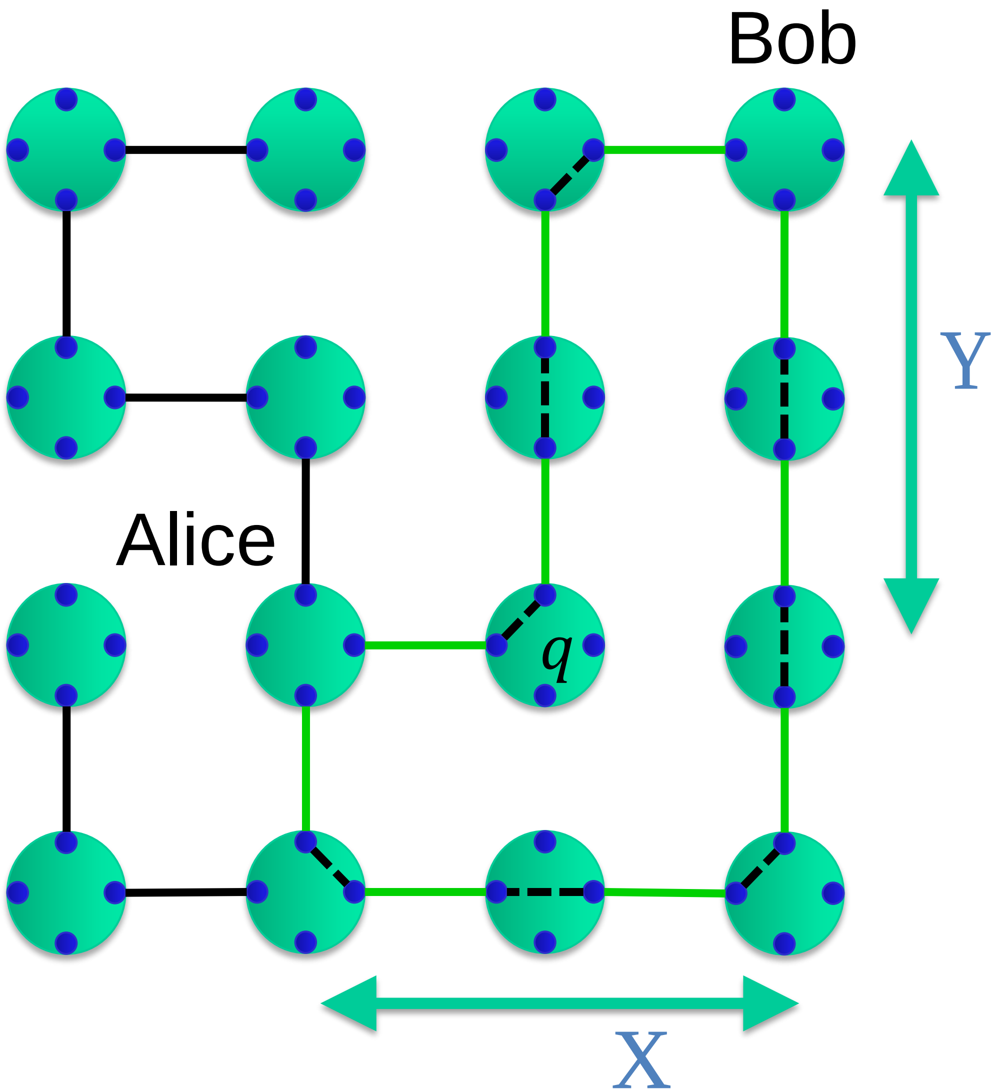
Grid Network



Grid Network - Phase 1



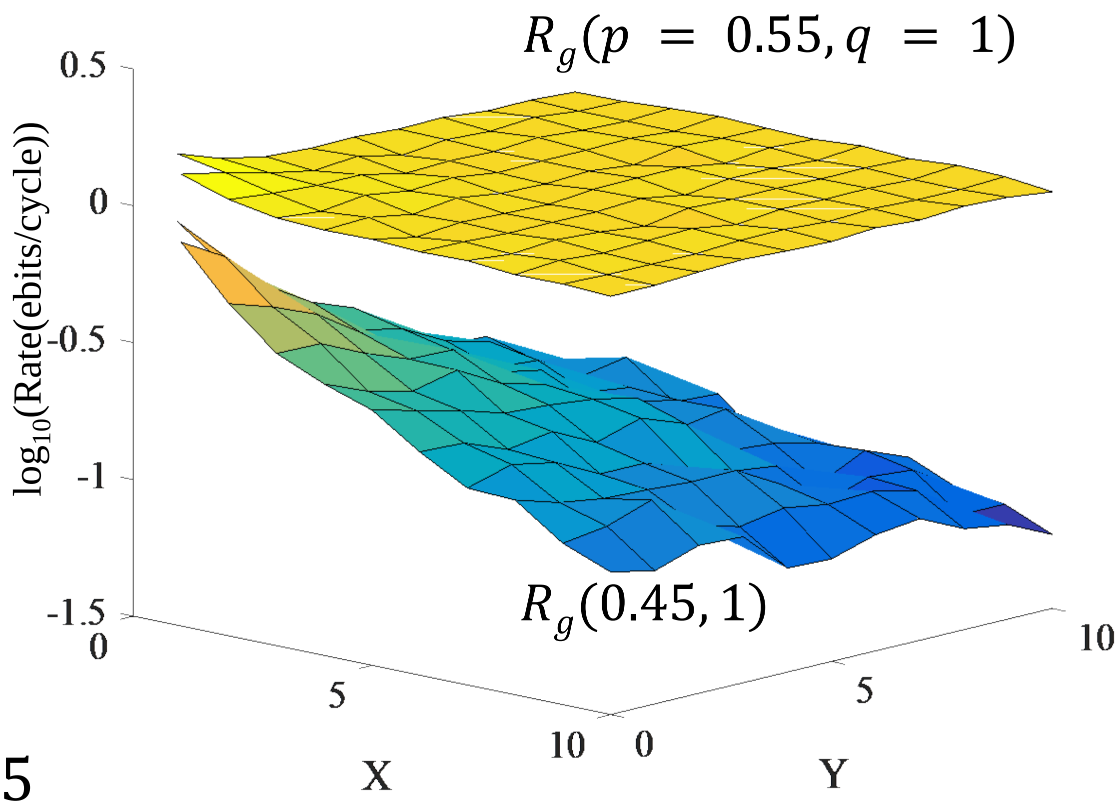
Grid Network – Phase 2



Rate dependence on p

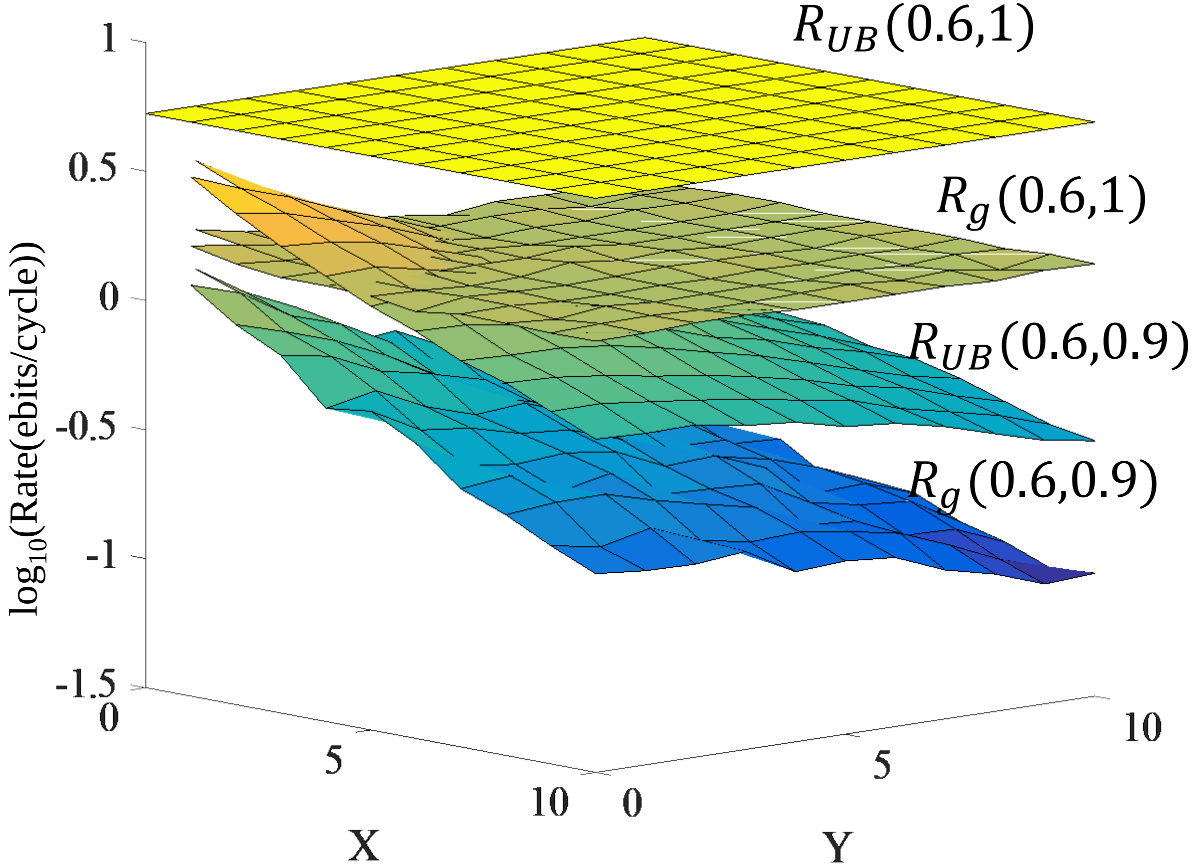
- greedy shortest path algorithm
  - find shortest path
  - next shortest path
  - …
- requires global information
- $R_g(p, q)$ – entanglement rate

Note: when $q = 1$, 2-D grid percolates at $p > 0.5$

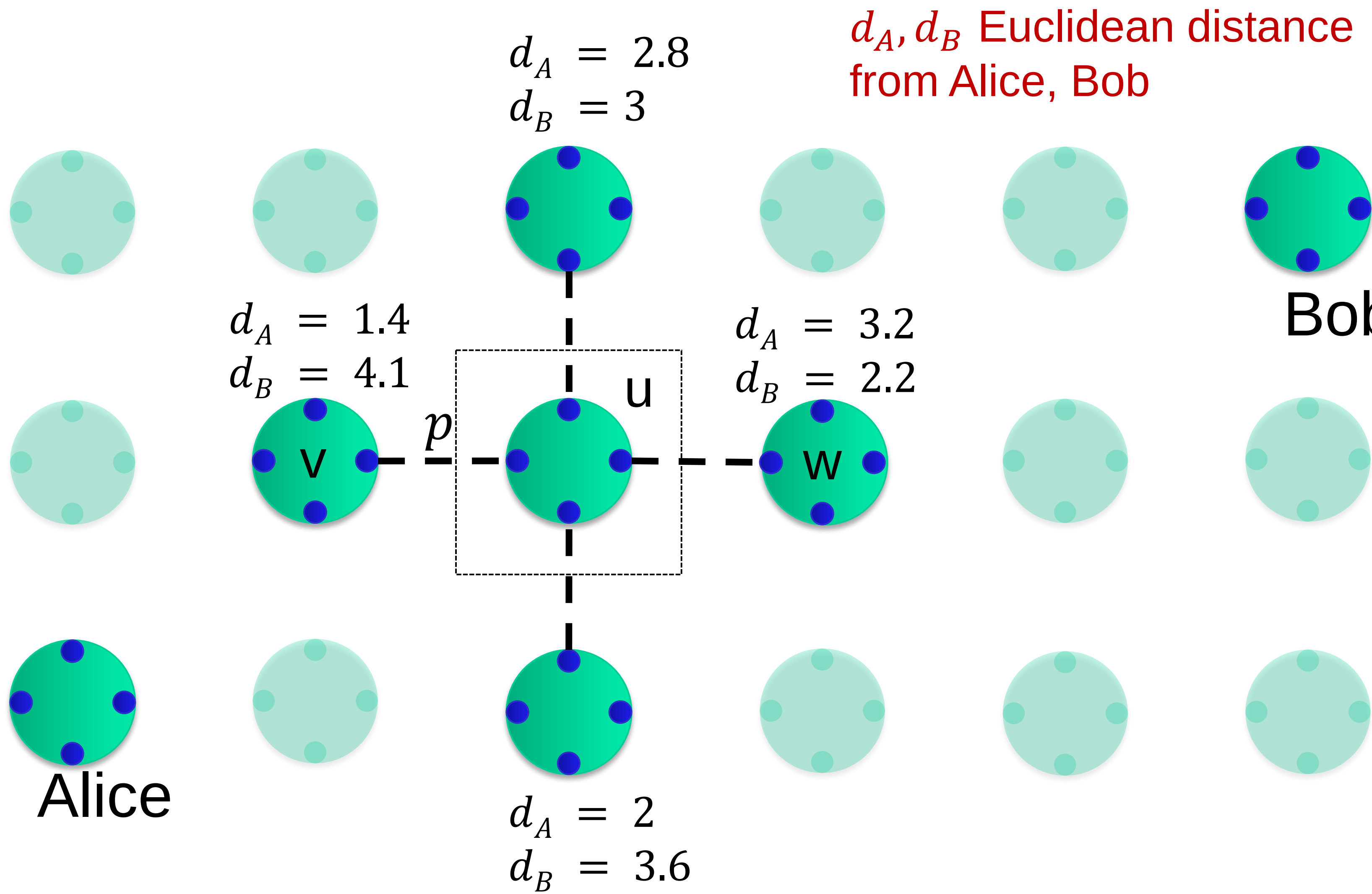


When every repeater has global state information

- $R_{UB}(p,q)$ – upperbound
- $q = 1$, max flow
  - achievable with global information
- $q < 1$,
  $R_{UB} = 4 \times R_g$



Routing entanglement flows with local state information



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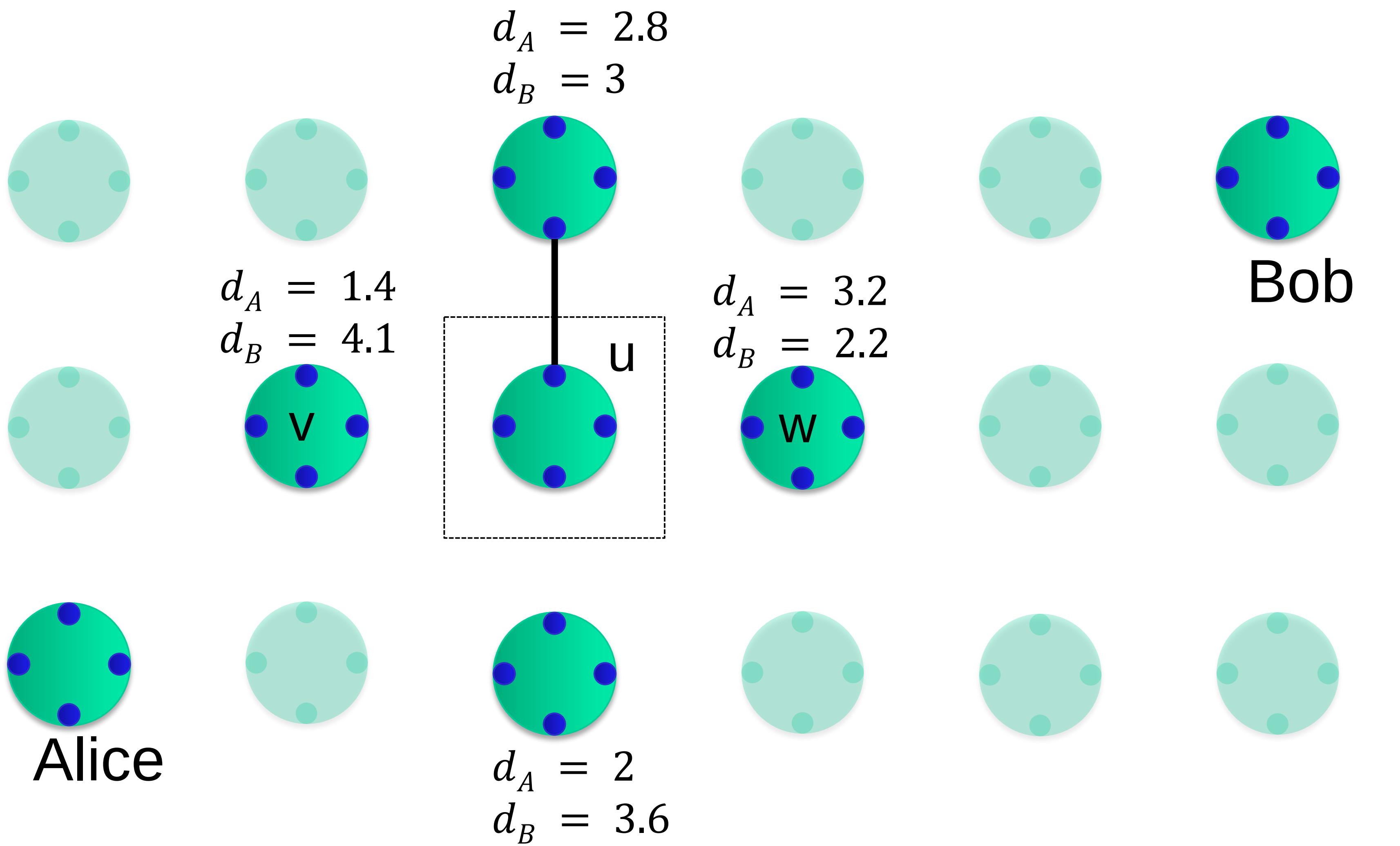

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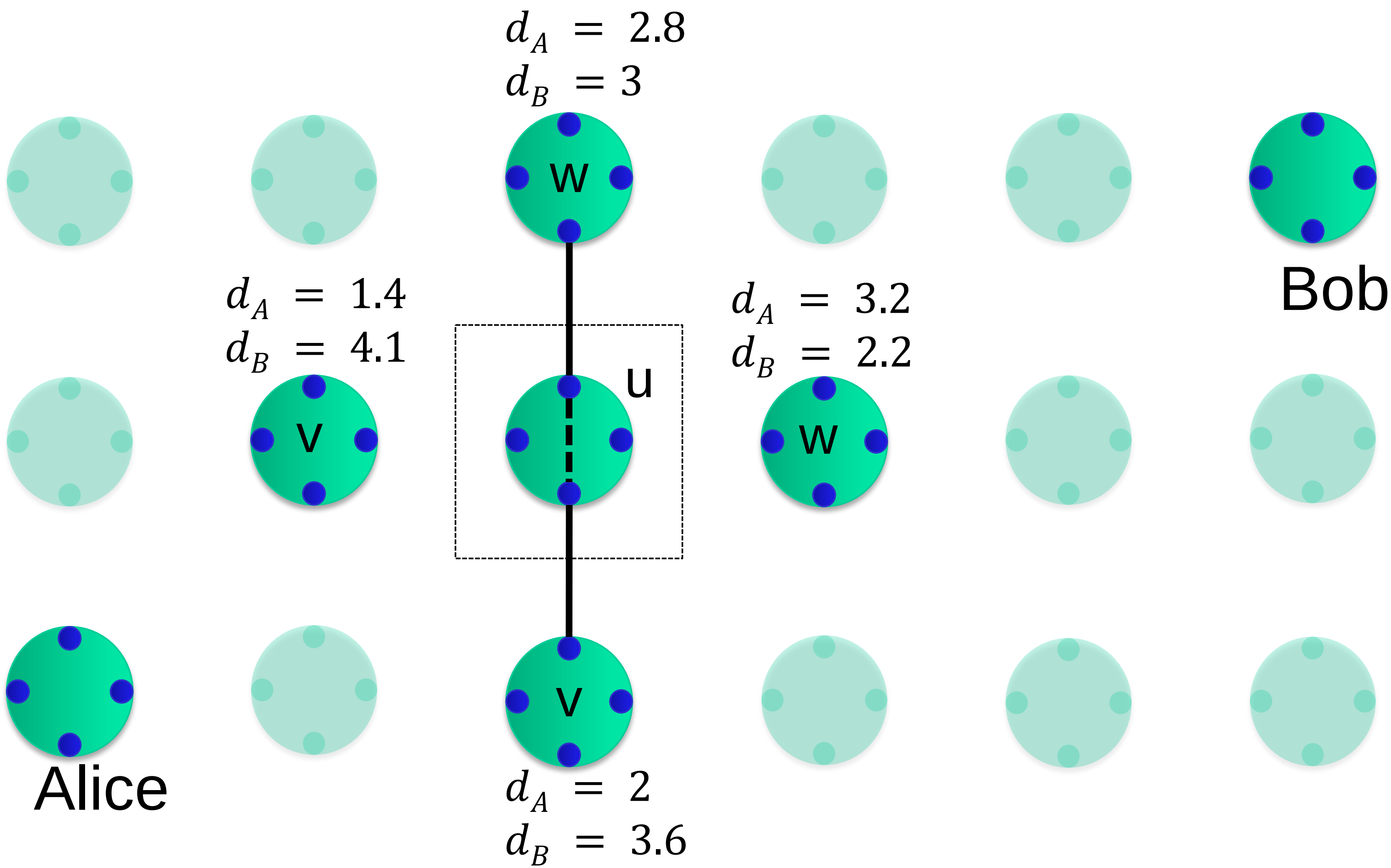

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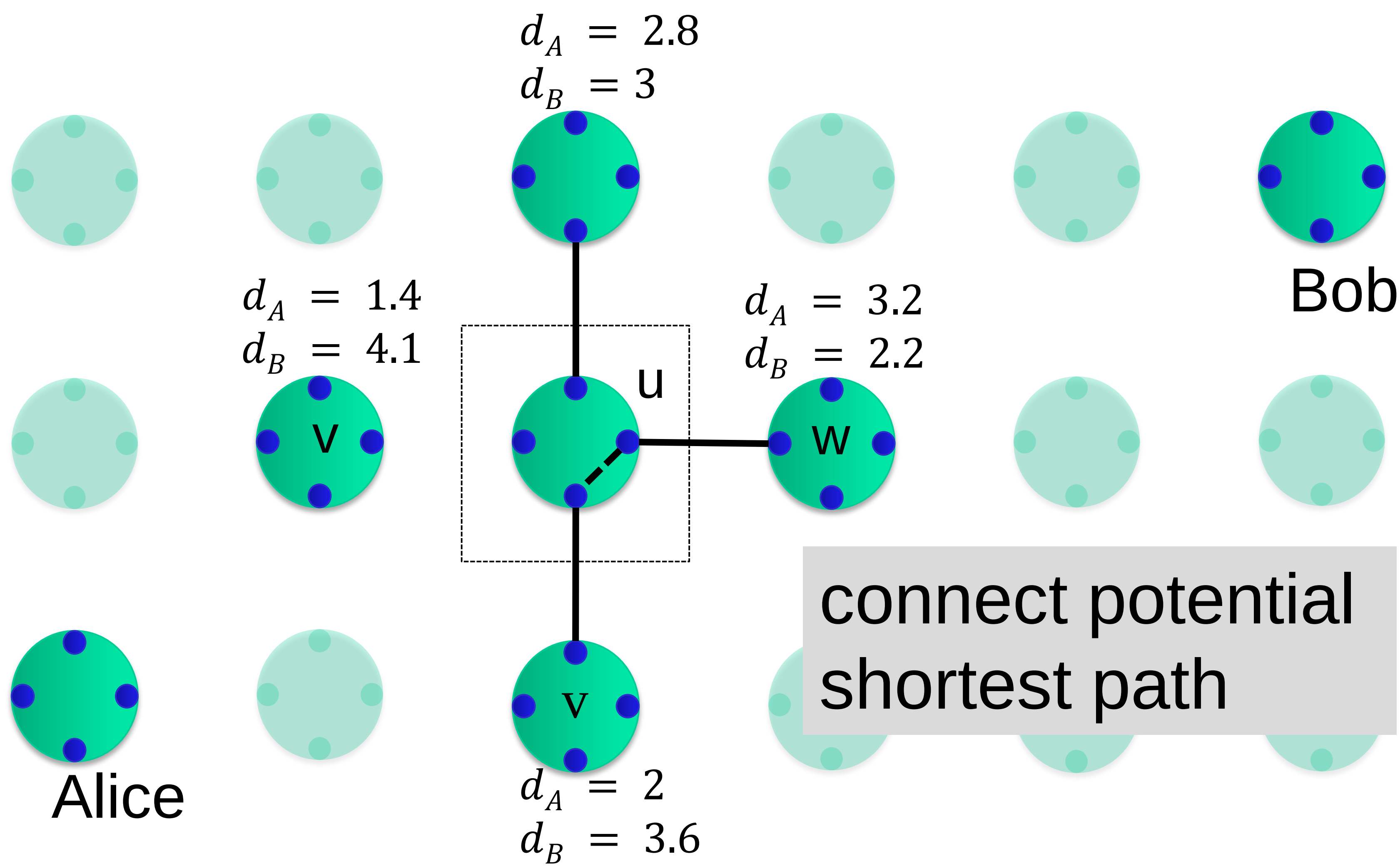
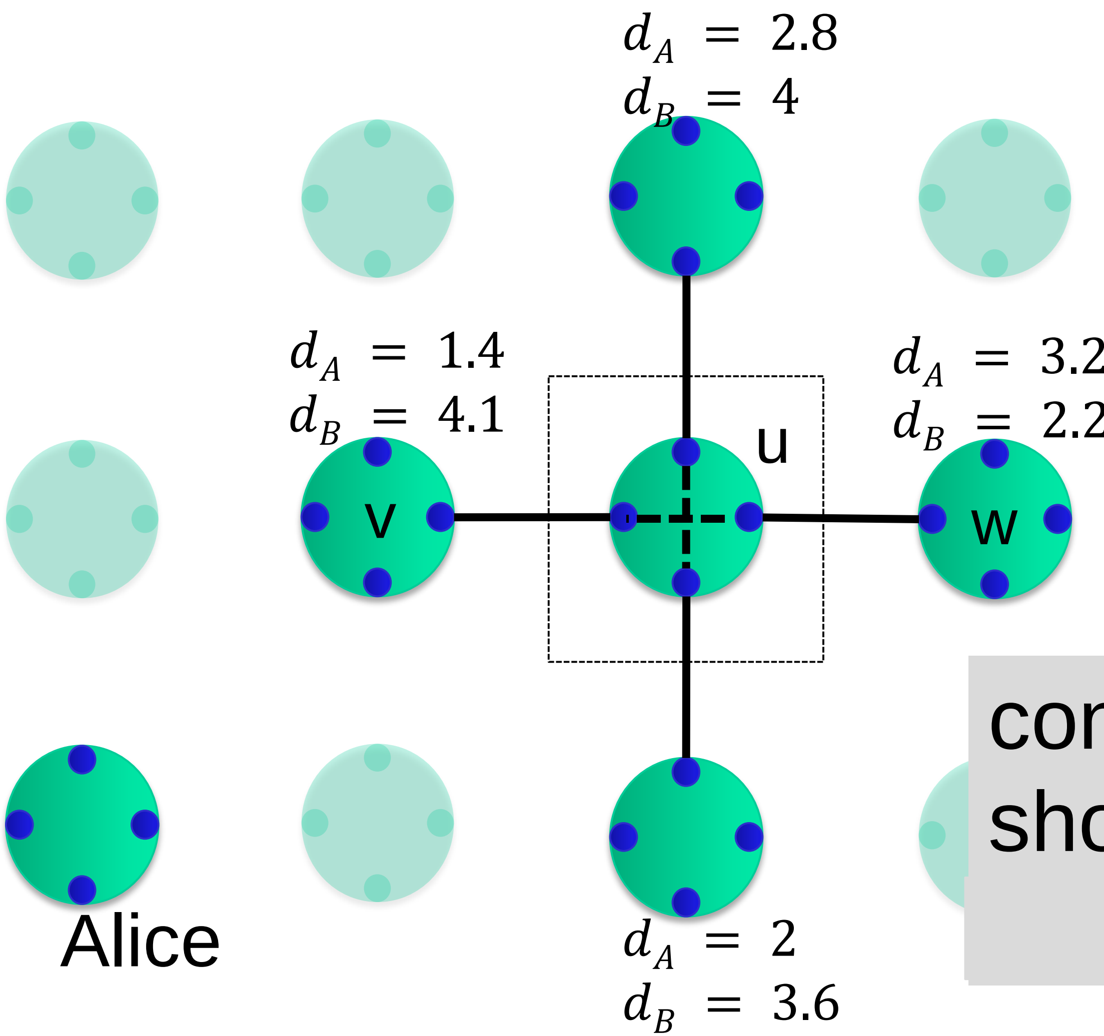


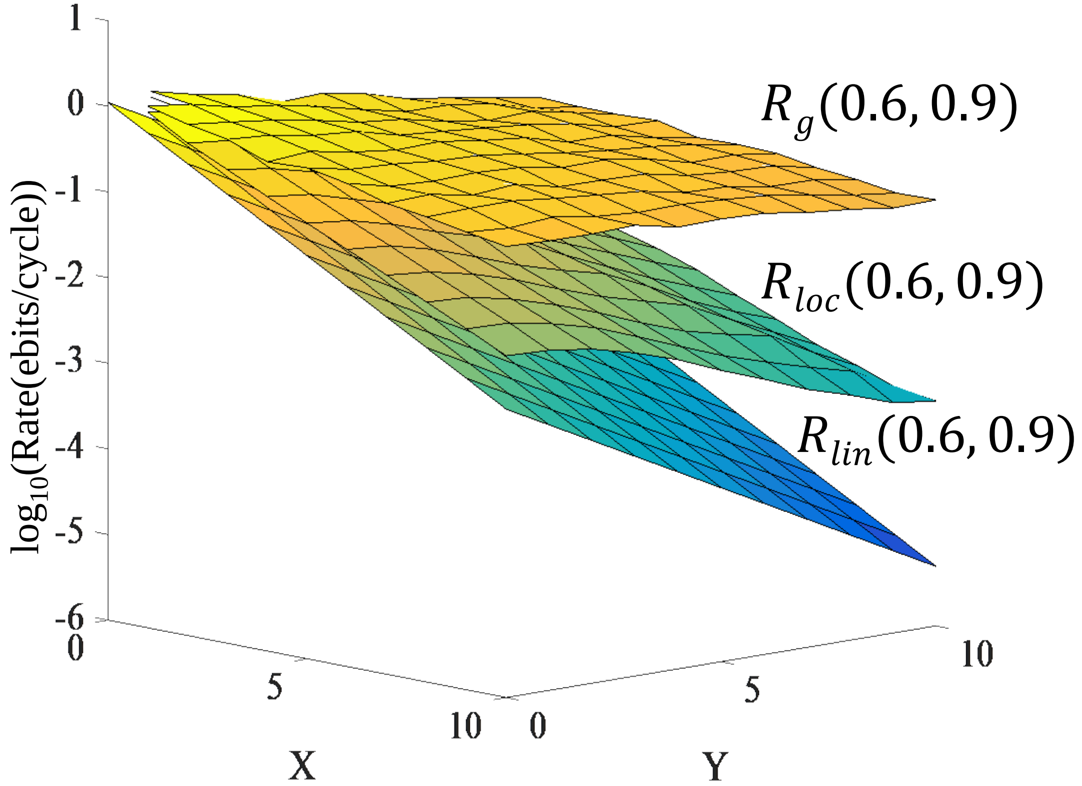
Routing entanglement flows with local state information



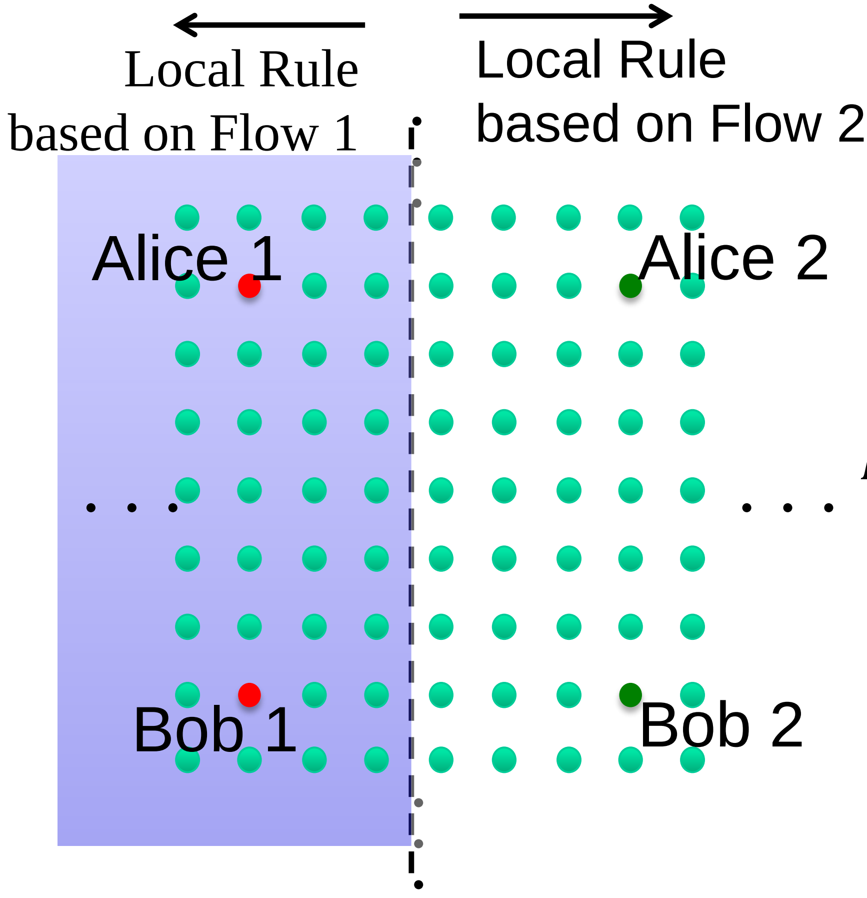
connect potential shortest path

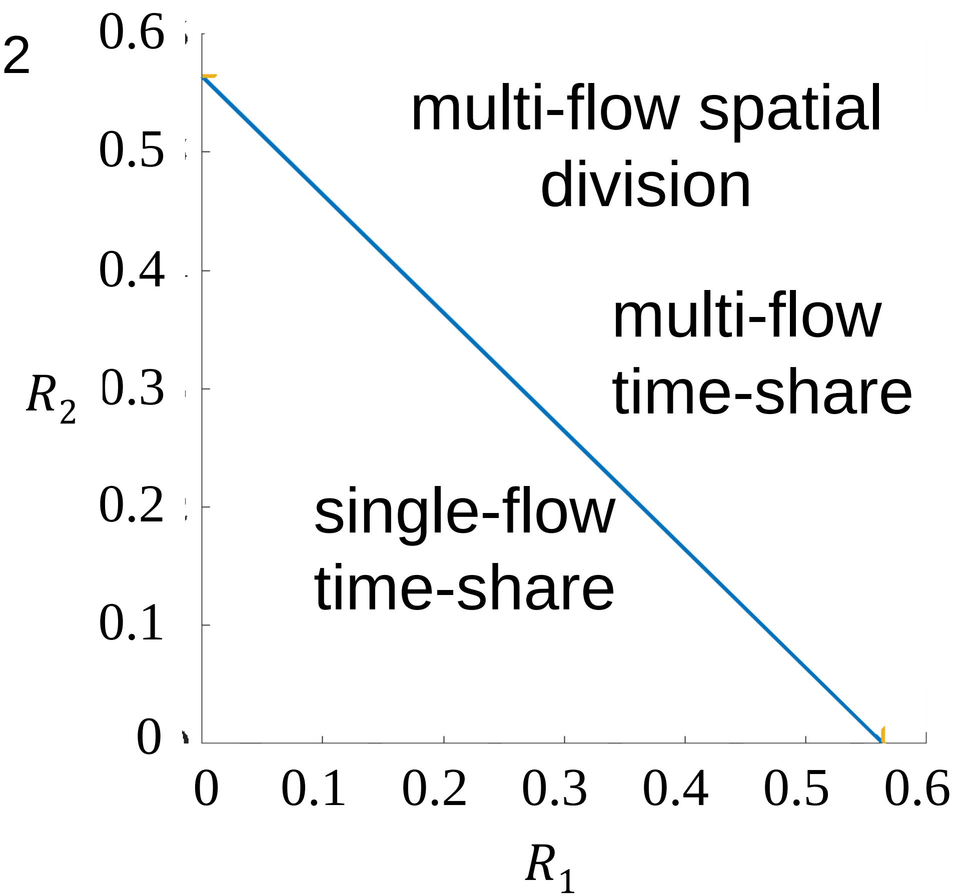
When every repeater only has local state information

- $R_{loc}(p,q)$ – rate using local rule
- $R_{lin}(p,q)$ - rate using single static path of same distance
  - no diversity

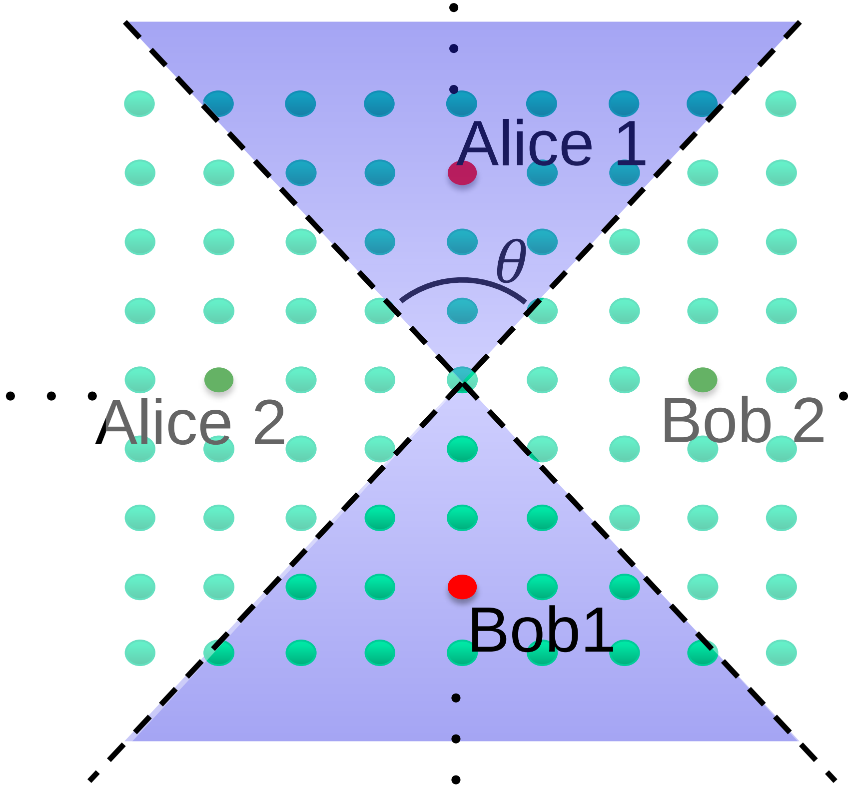


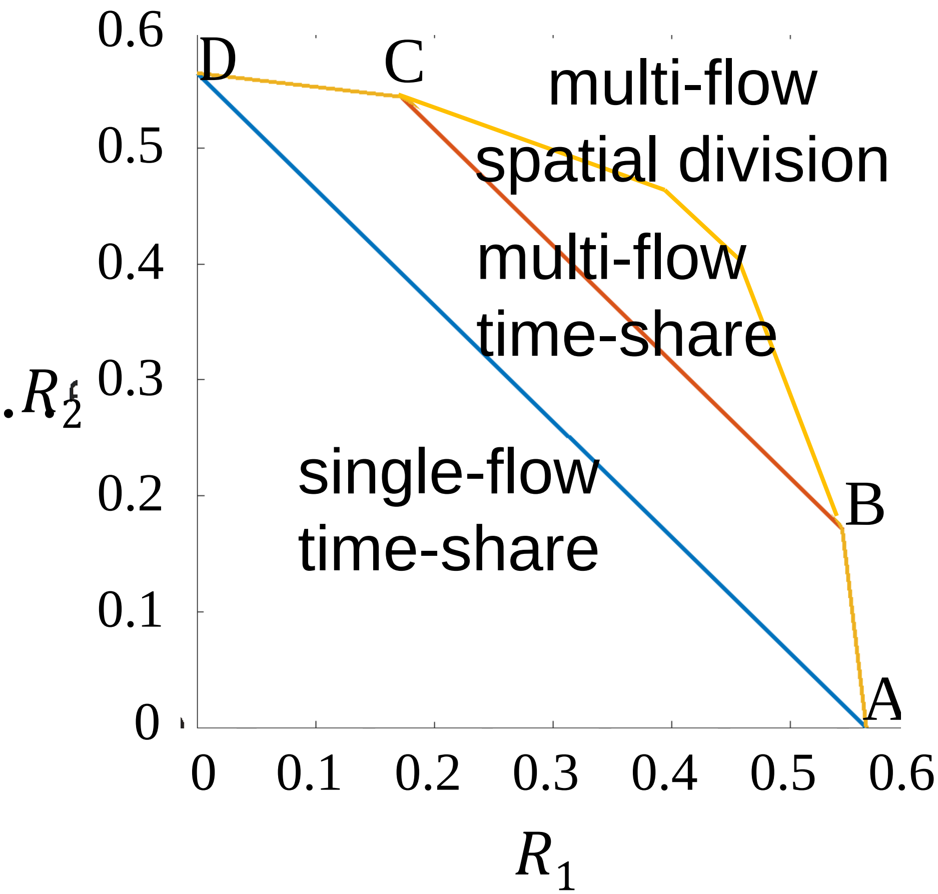
Multi-flow routing





Multi-flow routing





Open Questions

- rate-optimal protocol?
- effect of multiple modes, multiple memories?
- effect of coherence times, purification, etc.?
- 3+ qubit entanglements?

Conclusions

- quantum repeater networks achieve much larger rates than linear chains due to multi-path routing, even with only local information
- multi-flow strategies that exploit spatial division can provide significant performance improvements in such networks
- research on Q-networks in its infancy. Many exciting problems!

Happy retirement, Jean

How about a new hobby?

Design and analysis of quantum networks