INTERFERENCE QUEUING NETWORKS
Wireless Spatial Birth-Death Processes

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En l'honneur de Jean Walrand

The Next Wave in Networking Research
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Structure of the Lecture

- **Background and Motivation**
- **Wireless Birth-Death Processes**
  with A. Sankararaman, IEEE Tr. IT, 63(6) 2017
  1. Stability, 2. Clustering, 3. Quantitative results
- **Interference Queuing Networks**
  with S. Foss & A. Sankararaman, arXiv 1710.09797
  1. Stability, 2. Minimal solution, 3. Initial condition

Motivations in Wireless Networks

- **Lack of understanding and analysis of**

  Space-time interactions

  – **Static spatial setting well understood: Stochastic Geometry [FB, Blaszczyszyn 01]**

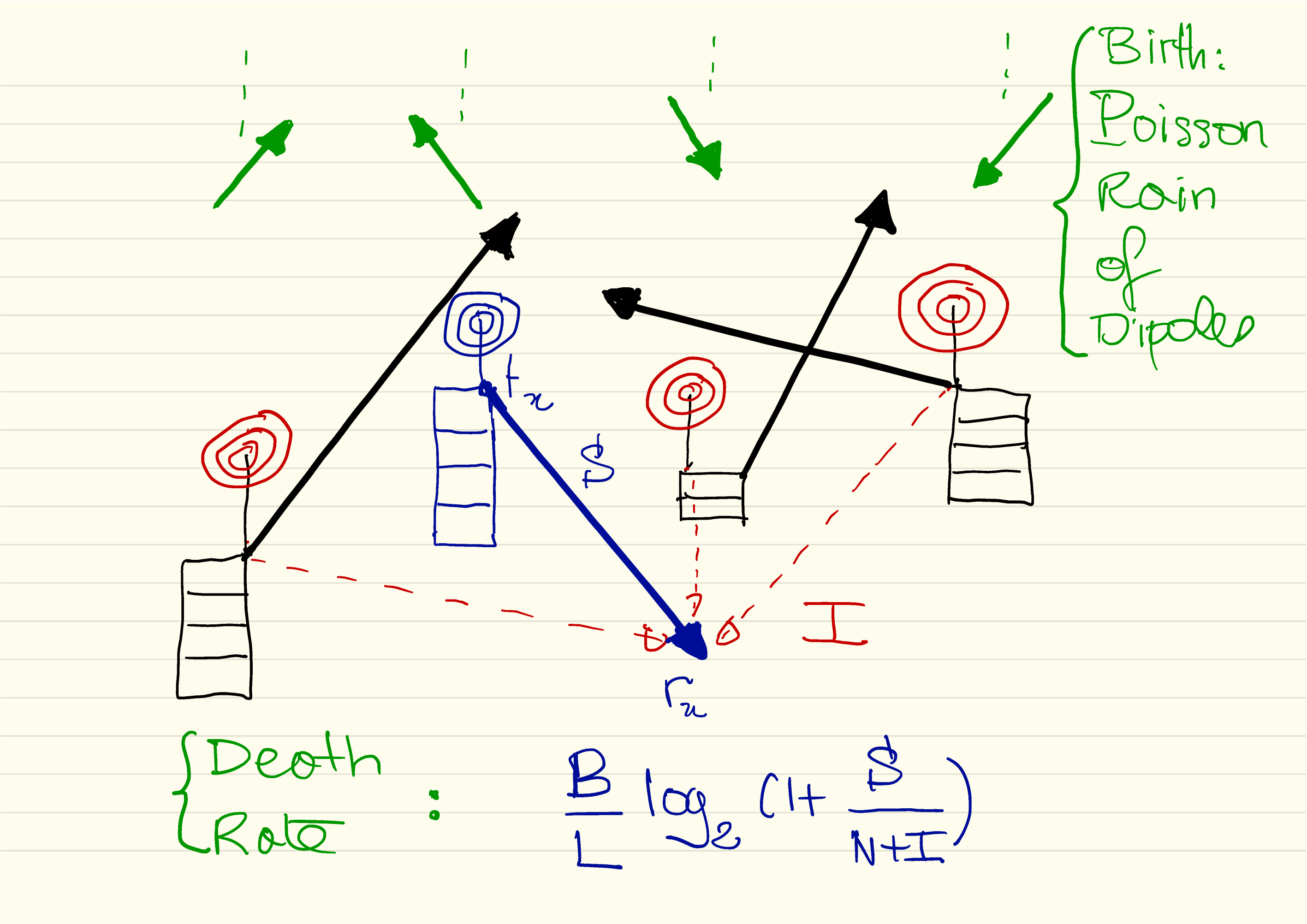
  – **Churn taken into account in flow-based queuing [Bonald, Proutiere 06], [Shakkottai, De Veciana 07] [Jiang, Walrand 09]**

- **Contents of this lecture:**

  Models with such dynamics in stochastic geometry

I. Wireless Birth-Death Processes

- **Setting: Infrastructureless Wireless Network:**
  Ad-hoc Networks, D2D Networks, IoT
- **Statistical assumptions: Markov Models:**
  Poisson, Exponential
- **Mathematical tools:**
  Point processes, Fluid


Birth: Poisson Rain of Dipoles
t_x
S
I
r_x
Death Rate : $\frac{B}{L}\log_2(1+\frac{S}{N+I})$

Stochastic Network Model

- $\mathbf{S} = [-\mathbf{Q}, \mathbf{Q}] \times [-\mathbf{Q}, \mathbf{Q}]$: torus where the wireless links live
- **Links**: (Tx-Rx pairs)
- Links: **arrive** as a PPP on $\mathbb{R} \times \mathbf{S}$ with intensity λ:
  Prob. of a point arriving in space $\mathbf{dx}$ and time $\mathbf{dt}$: $\lambda \mathbf{dxdt}$
- Each Tx has an **i.i.d. exponential file size** of mean $\mathbf{L}$ bits to transmit to its Rx
- A point **exits** after the Tx finishes transmitting its file
- Φ_t: set of locations of links present at time t:

$$\Phi_t = \{x_1, \ldots, x_{N_t}\}, \quad x_i \in \mathbf{S}$$

Interference and Service Rate

- **Interference seen at point x due to configuration Φ**

$$\mathbf{I}(\mathbf{x}, \mathbf{\Phi}) = \sum_{\mathbf{x_i} \in \mathbf{\Phi} \neq \mathbf{x}} \mathbf{l}(||\mathbf{x} - \mathbf{x_i}||)$$

  – **Distance on the torus**

  – $\mathbf{l}(\cdot)$**:** $\mathbb{R}^+ \to \mathbb{R}^+$**: path loss function**

- **The speed of file transfer by link at x in configuration Φ is**

$$\mathbf{R}(\mathbf{x}, \mathbf{\Phi}) = \mathbf{B} \log_2 \left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{N} + \mathbf{I}(\mathbf{x}, \mathbf{\Phi})} \right)$$

- $\mathbf{B}, \mathbf{N}$ **Positive constants**

B& D Master Equation

- **A point born at x_p and time b_p with file-size L_p dies at time**

$$d_p = \inf \left\{ t > b_p : \int_{u=b_p}^{t} R(x_p, \Phi_u)du \geq L_p \right\}$$

- **Spatial Birth-Death Process**
  - **Arrivals from the Poisson Rain**
  - **Departures happen at file transfer completion**

Properties of the Dynamics

- **The statistical assumptions imply that Φ_t is a Markov Process on the set of simple counting measures on S**
- **Euclidean extension of the flow-level models of [Bonald, Proutiere 06], [Shakkottai, De Veciana 07]**

Questions

- **Existence and uniqueness of the stationary regimes of Φ_t**
- **Characterization of the stationary regime(s) if existence**

Main Stability Results

$$\mathbf{a} := \int_{\mathbf{x}\in\mathbf{S}} \mathbf{l}(||\mathbf{x}||)\mathbf{dx}$$

- **Theorem**
  - If $\lambda > \frac{\mathbf{B}}{\ln(\mathbf{2})\mathbf{La}}$, then Φ_t admits no stationary regime.
  - If $\lambda < \frac{\mathbf{B}}{\ln(\mathbf{2})\mathbf{La}}$, and $\mathbf{r} \to \mathbf{l}(\mathbf{r})$ bounded and monotone, then Φ_t admits a unique stationary regime
- Necessary condition by Palm calculus, Stochastic intensity
- Sufficient condition by fluid limit
- **Corollary**

  For the path-loss model $\mathbf{l}(\mathbf{r}) = \mathbf{r}^{-\alpha}$, $\alpha \geq \mathbf{2}$, for all $\lambda > \mathbf{0}$, and all mean file sizes, the process Φ_t admits no stationary-regime

Main Qualitative Result

- $\mathbf{\Phi}$ **stationary point-process on S with Palm distribution** $\mathbb{P}^{\mathbf{0}}$
- **Clustering**

  $\mathbf{\Phi}$ is clustered if for all bounded, positive, non-increasing functions $\mathbf{f}(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$, the shot-noise

$$\mathbf{F}(\mathbf{x}, \mathbf{\Phi}) := \sum_{\mathbf{y} \in \mathbf{\Phi} \setminus \{\mathbf{x}\}} \mathbf{f}(||\mathbf{y} - \mathbf{x}||)$$

  satisfies

$$\mathbb{E}^{\mathbf{0}}[\mathbf{F}(\mathbf{0}, \mathbf{\Phi})] \geq \mathbb{E}[\mathbf{F}(\mathbf{0}, \mathbf{\Phi})]$$

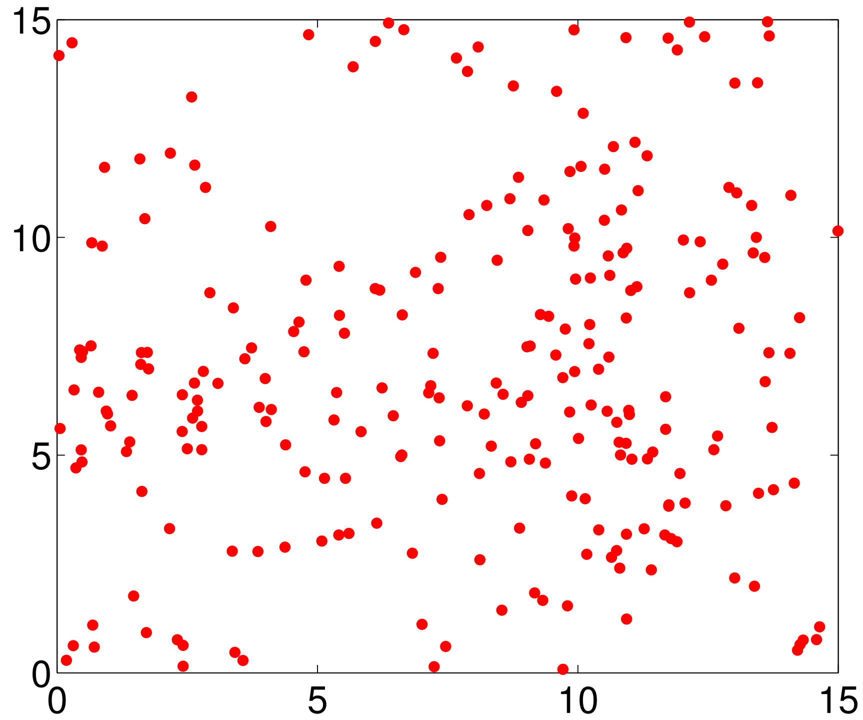
Main Qualitative Result (*continued*)

- **Theorem**
  The steady-state point process, when it exists, is clustered

- **Follows from Palm calculus + the FKG inequality**

- **Interpretation of the result**
  The steady-state interference measured at a uniformly randomly chosen point of is larger in mean than that at an uniformly random location of space.

- **Key Observation**
  - **Dynamics Shapes Geometry**
  - **Geometry Shapes Dynamics**



A sample of Φ when $\lambda = 0.99$ and $\mathbf{l}(\mathbf{r}) = (\mathbf{r} + \mathbf{1})^{-4}$.

Quantitative Results

- **Heuristics for the intensity of the steady-state process**
  1. **Poisson heuristic $\beta_{\mathbf{f}}$ - derived by neglecting clustering and assuming Poisson**
  2. **Second-order heuristic $\beta_{\mathbf{s}}$ based on a second-order cavity approximation of the dynamics**

Poisson Heuristic

- **Exact Rate Conservation Law:**

$$\lambda L = \beta \mathbb{E}_{\Phi}^{0}\left[\log_2\left(1+\frac{1}{N+I(0)}\right)\right].$$

Poisson Heur.: Largest solution to the fixed point equation:

$$\lambda L = \frac{\beta_f}{\ln(2)}\int_{z=0}^{\infty}\frac{e^{-Nz}(1-e^{-z})}{z}e^{-\beta_f\int_{x\in S}(1-e^{-zl(||x||)})dx}dz$$

Ignores the Palm effect and uses that if X, Y are non-negative and independent,

$$\mathbb{E}\left[\ln\left(1+\frac{X}{Y+a}\right)\right] = \int_{z=0}^{\infty}\frac{e^{-az}}{z}(1-\mathbb{E}[e^{-zX}])\mathbb{E}[e^{-zY}]dz.$$

- **The Poisson heuristic is tight in heavy and light traffic**

Second Order Heuristic

The intensity β_s is given by

$$\beta_{\mathbf{s}} = \frac{\lambda \mathbf{L}}{\mathbf{B} \log_{\mathbf{2}} \left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{N} + \mathbf{I_s}} \right)}$$

where $\mathbf{I_s}$ is the smallest solution of the fixed-point equation

$$\mathbf{I_s} = \lambda \mathbf{L} \int_{\mathbf{x} \in \mathbf{S}} \frac{\mathbf{l}(||\mathbf{x}||)}{\mathbf{B} \log_{\mathbf{2}} \left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{N} + \mathbf{I_s} + \mathbf{l}(||\mathbf{x}||)} \right)} \mathbf{dx}$$

Second Order Heuristic (*continued*)

- **Rationale** based on $\rho_2(\mathbf{x}, \mathbf{y})$: second moment measure of Φ
- **Rate Conservation for** ρ_2: when considering $\mathbf{I_s}$ as a constant

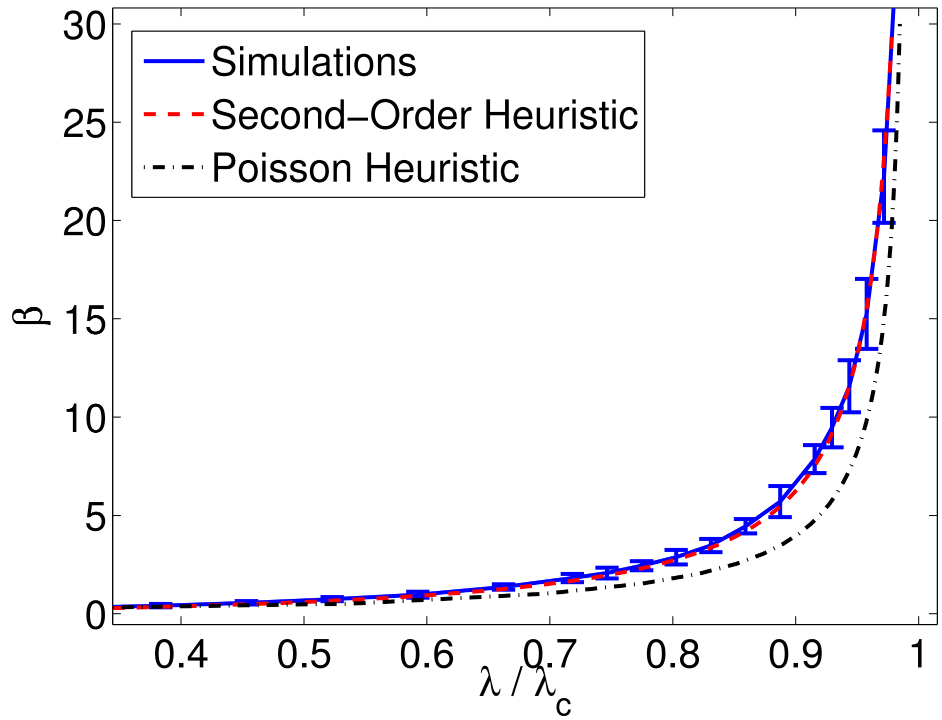
$$\rho_2(\mathbf{x}, \mathbf{y})\frac{\mathbf{1}}{\mathbf{L}}\mathbf{B}\log_2\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{N} + \mathbf{I_s} + \mathbf{l}(||\mathbf{x} - \mathbf{y}||)}\right) = \lambda\beta_{\mathbf{s}}$$

- From the definition of second moment measure,

$$\mathbf{I_s} = \int_{\mathbf{x} \in \mathbf{S}} \mathbf{l}(||\mathbf{x}||)\frac{\rho_2(\mathbf{0}, \mathbf{x})}{\beta_{\mathbf{s}}}\mathbf{dx}$$

which gives the fixed point equation for $\mathbf{I_s}$

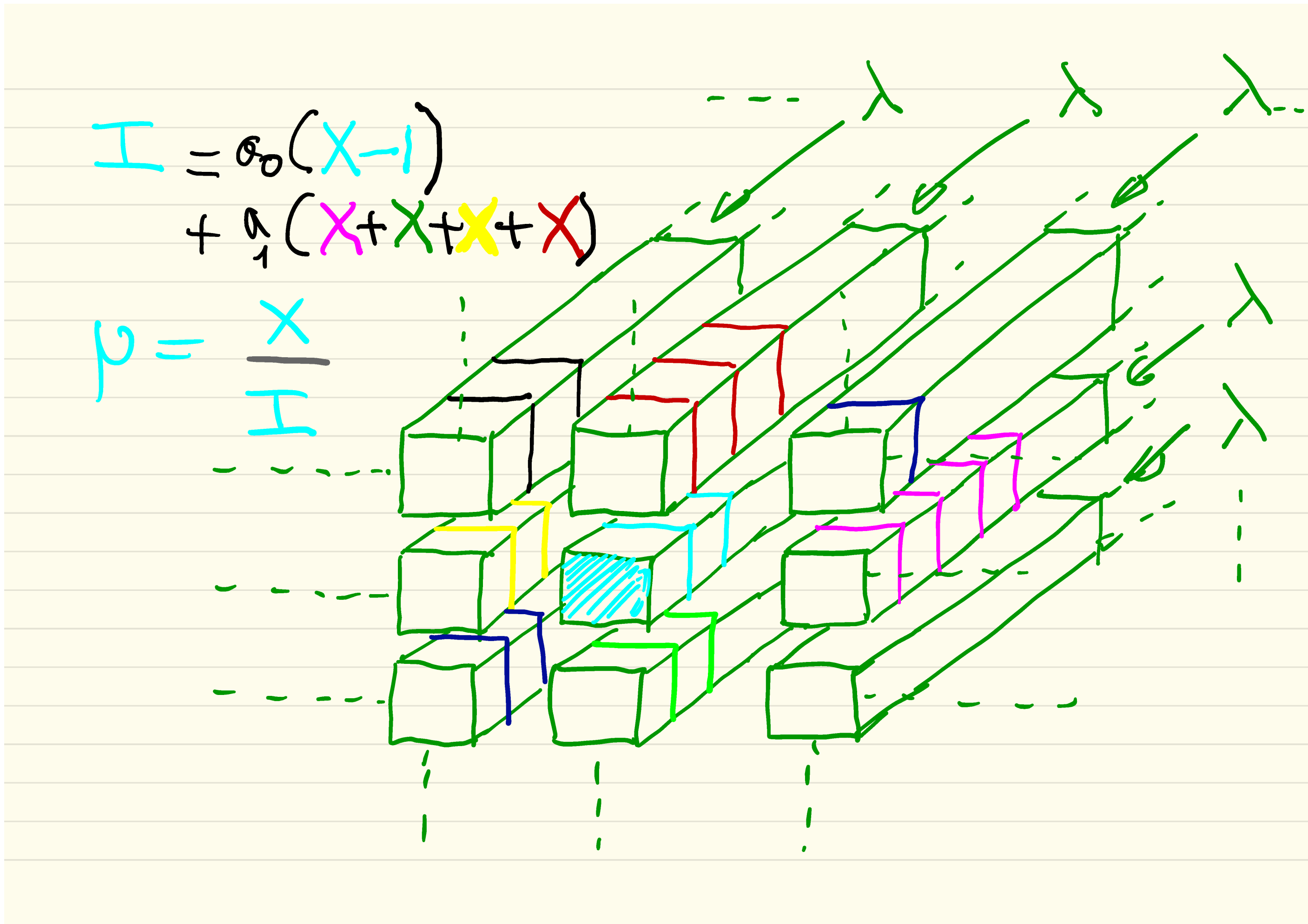
- The formula for $\beta_{\mathbf{s}}$ follows from **Rate Conservation for** $\rho_1 = \beta_{\mathbf{s}}$



95% **confidence interval when** $\mathbf{l(r) = (r+1)^{-4}}$

II. Interference Queuing Networks

- **Aim**: extension of dynamics to $\mathbb{R}^2$ (scalability)
- **Setting**
  - **Discretization**: queuing dynamics on a grid
  - **Low SINR**: linearization of the log
- **Mathematical Tools**
  - Interacting particle systems
  - Coupling from the past
  - Rate conservation principle


$I = a_0(X-1)$
$+ a_1(X+X+X+X)$
$\mu = \frac{X}{I}$
λ
λ
λ
λ
λ

Assumptions, Notation

- **Queue i at $\mathbf{i} \in \mathbb{Z}^{\mathrm{d}}$ has state $\mathbf{x_i(t)} \in \mathbb{N}$ at time t**
- **Arrivals to queues:**
  i.i.d. Poisson processes of rate $\lambda > 0$
- **Interference sequence:**
  $\{\mathbf{a_i}\}_{\mathrm{i}\in\mathbb{Z}^{\mathrm{d}}}$, non-negative, symmetric ($\mathrm{a_i} = \mathrm{a_{-i}}$), and irreducible
- **Service discipline:**
  generalized processor-sharing with rate of queue i at time t:

$$\frac{\mathbf{x_i(t)}}{\sum_{\mathrm{j}\in\mathbb{Z}^{\mathrm{d}}} \mathbf{a_j x_{i-j}(t)}}$$

Stability, Minimal Stationary Regime

- **Stability**: when starting the system empty at time 0, weak convergence of the state of any finite set of queues as $t \to \infty$
- **Theorem** If

$$\lambda < \frac{1}{\sum_{i \in \mathbb{Z}^d} a_i}$$

  – The network is stable

  – The weak limit is the minimal stationary regime

- **Proof**: CFP
- The stability condition $\lambda < \frac{1}{\sum_{i \in \mathbb{Z}^d} a_i}$ is sharp (current proof in special cases only)

Quantitative Properties of Minimal Stationary Regime

- **Theorem** The weak limit, when it exists, satisfies

$$\mathbb{E}[\mathbf{x_0}] = \frac{\lambda \mathbf{a_0}}{\mathbf{1} - \lambda \sum_{\mathbf{i} \in \mathbb{Z}^\mathbf{d}} \mathbf{a_i}}$$

  In addition its coordinates $(\mathbf{x_i})_{\mathbf{i} \in \mathbb{Z}^\mathbf{d}}$ are associated

- **Proof**: RCP
- **Association**: analogue of of clustering in the continuum
- **Remarkable fact**: closed form for the mean for this infinite-dimensional, non-reversible, non-asymptotically independent particle system

Uniqueness

- Below, assume that $\lambda < \frac{1}{\sum_{i \in \mathbb{Z}^d} a_i}$
- **Proposition** If $\mathbb{E}[x_0^2] < \infty$, then the minimal solution is the **unique** stationary solution with finite second moment
- **Proposition** If

$$\lambda < \frac{2}{3} \frac{1 + c}{\sum_{j \in \mathbb{Z}^d} a_j} \quad \text{where} \quad c = \frac{\sqrt{a_0^2 + a_0 \sum_{j \in \mathbb{Z}^d \setminus \{0\}} a_j} - a_0}{\sum_{j \in \mathbb{Z}^d \setminus \{0\}} a_j}$$

then $\mathbb{E}[x_0^2] < \infty$

Domain of Attraction of the Minimal Solution

- **Theorem** If $\lambda < \frac{2}{3}\frac{1+c}{\sum_{j\in\mathbb{Z}^d} a_j}$ and the initial condition satisfies

$$\sup_{i\in\mathbb{Z}^d} x_i(0) < \infty$$

then $\{x_i(\cdot)\}_{i\in\mathbb{Z}^d}$ **converges weakly to the minimal stationary solution**

- **Theorem** For $d = 1$, for all $\lambda > 0$, there exists

1. A deterministic sequence $(\alpha_i)_{i\in\mathbb{Z}}$ such that if $x_i(0) \geq \alpha_i$ for all $i \in \mathbb{Z}$, then $\lim_{t\to\infty} x_0(t) = \infty$ a.s.
2. A distribution ξ on $\mathbb{N}$ s.t. if $\{x_i(0)\}_{i\in\mathbb{Z}}$ is i.i.d. with marginal distr. ξ, then $\lim_{t\to\infty} x_0(t) = \infty$ a.s.

Summary

- A new basic representation of space-time interactions in wireless networks
- A generative model for clustering as assumed in simulation standards
- A new dynamic notion of capacity involving both queuing and IT
- First exact analytical results in the low SINR case and good heuristics in general
- A new particle system dynamics with closed form although no reversibility, no asymptotic independence

Thanks Jean

MERCI JEAN POUR NOS PREMIERS $\sim$ 40 ANS D'INTERACTIONS AMICALES ET SCIENTIFIQUES!