Happy Retirement, Jean!

An illustrious career of achievements

- Beautiful results in many areas
  - When flows in Queueing Networks are Poisson
  - When delays are independent (non-overtaking)
  - “Delay” explanation for geometric steady state under feedback
  - Quick simulation for networks based on large deviations
  - Very elegant use of coupling to simplify proofs
    - Short probabilistic proof of our threshold result
  - Multiarmed bandits: Simple interchange argument proof
  - Very nice development of CSMA
  - Seminal result on max weight solution for input-queued switches
  - Decoupling bandwidth in the days of ATM
  - Network security
  - Economics of networks
  - Network games

A role model for genuine research values

- High standard for theoretical results
- Role model for research of the highest caliber

- Papers and results are shorn of all adornment
- Explain the central idea in the simplest way
- No sentence is wasted – no sentence can be ignored

- And, in the same style, several succinct books

- Exemplary understatement and modesty

- And a serial entrepreneur (!)

- Has continually held aloft high standards to emulate for research and dissemination

Incentive compatibility in stochastic dynamic systems

Ke Ma and P. R. Kumar

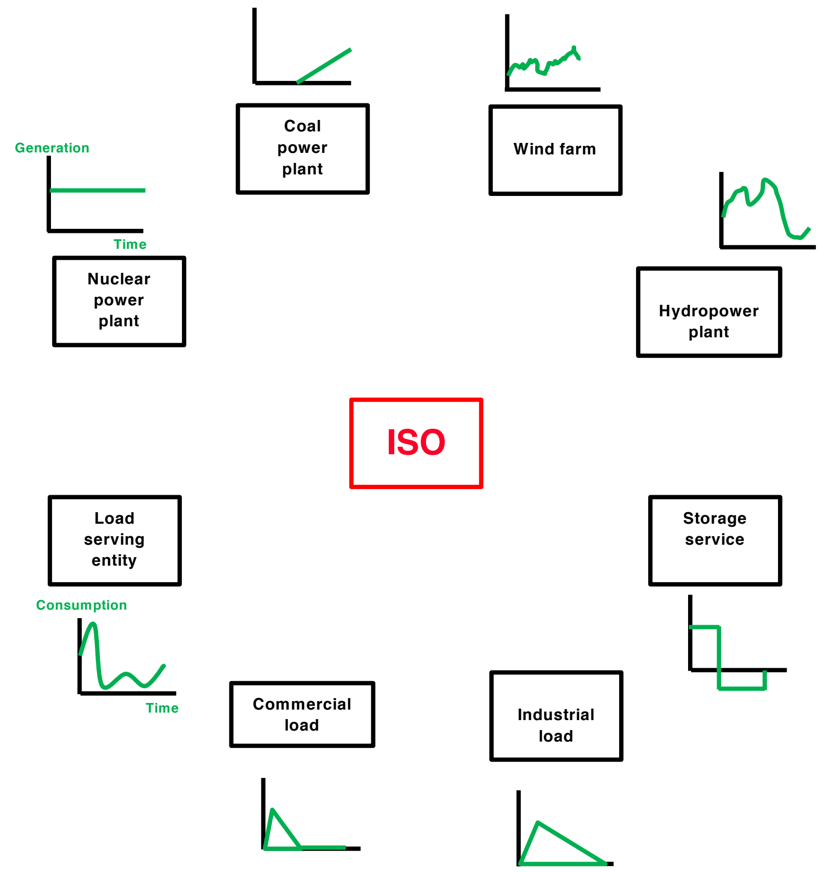
Dept. of Electrical and Computer Engineering
Texas A&M University

Email: prk.tamu@gmail.com
Web: http://cesg.tamu.edu/faculty/p-r-kumar/

“Next wave in Networking Research”
Festschrift for Jean Walrand
Simons Institute
University of California, Berkeley
September 7, 2018

The Independent System Operator (ISO) Problem

- Many generators and loads
- Different dynamics, uncertainties, and costs/rewards
- Yet power generation and consumption should in balance at all times
- There is a System Operator
- Job of System Operator is to
  - Maintain power balance
  - Maximize Social Welfare
  - Budget Balance (no need for subsidy)
  - Individual Rationality (agents will actively participate in the mechanism)
  - Charge Fair Price: "Lagrange Optimality"
- *Without* knowing details of generators/loads
- ISO can ask them, but they can lie (Enron)
- How can System Operator operate?



The mathematical problem

- N stochastic dynamic agents

$$x_i(t+1) = g_i(x_i(t), u_i(t), w_i(t)) \quad \text{for } i = 1, 2, \ldots, N$$

- Social planner's goal

$$Max\ \mathrm{E} \sum_{i=1}^{N} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$$

  – While maintaining $\sum_{i=1}^{N} u_i(t) = 0 \quad \text{for all } t = 0, 1, 2, \ldots, T$

- Without knowing dynamics g_i, utility functions F_i, or states $x_i(t)$

The strategic problem for the static deterministic case

- Agent i has a utility function $F_i(u_i)$, but can lie and bid $\hat{F}_i$

- If agent i is allocated u_i^* and is charged a payment p_i, then the net utility of agent i is $F_i(u_i^*) - p_i$

- The allocation is selected by the Social Planner to maximize Social Welfare

$$u^*(\hat{F}) = \arg\max_{u \in U} \sum_{i=1}^{N} \hat{F}_i(u_i)$$

- Payment p_i needs to be defined in a way that agent i internalizes the social externality

Static VCG Mechanism

- Vickrey-Clarke-Groves (VCG) payments

$$p_i(\hat{F}) = \sum_{j \neq i} \hat{F}_j(u^{(i)}) - \sum_{j \neq i} \hat{F}_j(u^*)$$

where $u^{(i)} = \arg \max_{u_{-i} \in U_{-i}} \sum_{j \neq i} \hat{F}_j(u_j)$

- Ensures
  - Incentive compatibility (Truth telling is dominant strategy)
  - Social efficiency (Maximizes social welfare)

- More general Groves payments is the *only* such mechanism (Green, Laffont and Holmstrom)

$$p_i(\hat{F}) = h_i(\hat{F}_{-i}) - \sum_{j \neq i} \hat{F}_j(u^*)$$

Deterministic dynamic VCG Mechanism vs. stochastic dynamic VCG Mechanism

- The standard VCG mechanism can be extended to *deterministic* dynamic systems
- The entire decision on the sequence of controls to be employed is taken at the initial time (open-loop solution)

- However, for *stochastic* dynamic systems, states of the system are *private* random variables
- The social welfare optimal allocation needs knowledge of the states of the private systems
- Hence, the Social Planner needs to additionally ensure that agents reveal "true" states at all times

Difficulty in extending to stochastic dynamic agents

- Suppose F_i and g_i are known to the social planner and agents bid their states as $\hat{x}_i(t)$
- Natural extension of the static VCG mechanism is to collect payment $p_i(t)$ from agent i at time t as:

$$p_i(t) = h_i(\hat{X}_{-i}(t)) - \mathbb{E} \sum_{j \neq i} \sum_{\tau = t}^{T-1} \left[F_j(\hat{x}_j(\tau), u_j^*(\tau)) \mid X(t) = \hat{X}(t) \right]$$

where $u^*(t)$ is the optimal solution to:

$$\max_{u(t) \in U} \mathbb{E} \sum_{i=1}^{N} \sum_{\tau = t}^{T-1} \left[F_i(x_i(t), u_i(t)) \mid X(t) = \hat{X}(t) \right]$$

- Then truth-telling of states by all agents forms a subgame perfect Nash equilibrium, but is not a dominant strategy
  - Bid $\hat{x}_j(t+1)$ need not be stochastically consistent with $\hat{x}_j(t)$

Yet another problem: Budget Balance and Individual Rationality

- Budget Balance if

$$\sum_i p_i \geq 0$$

- Individual Rationality if

$$F_i(u_i^*) - p_i \geq 0$$

- No mechanism can satisfy all the four properties (IC, EF, BB, IR) at the same time (Green and Laffont)
- Also, we want Fair Price: Price charged should be Lagrange Multiplier (in absence of strategic considerations)
- So what can Social Planner do?

Is there any hope?

LQG Agents

- The workhorse of system modeling

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t) + w_i(t)$$

$$w_i(t) \sim N(0, \Sigma_i), \text{ i.i.d., } x_i(t) \sim N(0, P_i)$$

$$Max \sum_{t=0}^{T-1} x_i^T(t) Q_i x_i(t) + \sum_{t=0}^{T-1} u_i^T(t) R_i u_i(t)$$

- Denote system comprised of all agents by

$X = (x_1, x_2, \ldots, x_N)$ and $U = (u_1, u_2, \ldots, u_N)$

Idea of Incentive Compatible and Social Welfare LQG

- At each time s, random disturbance $w_i(s)$ occurs at each agent
- ISO charges VCG payment taking into account the effect of all the disturbances at time s on future states, and balancing power at all times in the future
- Due to superposition of linear systems, future states can be written as the sum of the effects of all past disturbances
- The quadratic nature of cost renders the additional interaction between past and present as a product
- These can be shown in expectation to be zero
- This yields ex ante results

Layered VCG Mechanism

- Will ask agents to bid their state noise $W(s-1)$
- Let $X(s,s) := W(s-1)$ and propagate state forward

$$X(s,t) := AX(s,t-1) + BU(s,t-1),\ 0 \le s \le t-1$$

- Trajectory resulting from the disturbance $W(s-1)$ at time s
- Use superposition to decompose state of system as:

$$X(t) := \sum_{s=0}^{t} X(s,t),\ 0 \le t \le T-1$$

- Suppose that $U(s,t)$ is the adjustment made at time s to allocation at time t Social Planner due to disturbance at time s
- Commensurately decompose

$$U(t) := \sum_{s=0}^{t} U(s,t),\ 0 \le t \le T-1$$

Random social welfare

- The *random* social welfare can be decomposed in terms of $X(s,t)$'s and $U(s,t)$'s as $RSW = \sum_{s=0}^{T-1} L_s$

where
$$L_s := \sum_{t=s}^{T-1} \Big[X^T(s,t) Q X(s,t) + U^T(s,t) R U(s,t)$$
$$+2\left(\sum_{\tau=0}^{s-1} X(\tau,t)\right) Q X(s,t) + 2\left(\sum_{\tau=0}^{s-1} U(\tau,t)\right) R U(s,t)\Big],$$

- There are certain cross terms involving $X(s,t)$ and $X(\tau,t)$
  - Can only be eliminated in expectation
  - Results are "ex ante" (in expected sense) rather than "ex post" (almost surely)

Layered VCG Mechanism

- Agent i bids $\hat{x}_i(s,s)$ at time s
- The social planner solves the problem:

$$\max_{U(s,t)\in U} L_s$$

subject to $\hat{X}(s,t) = A\hat{X}(s,t-1) + BU(s,t-1)$

- Social planner collects payment $p_i(s)$ from agent i at s

equal to random cost to system of Agent i's noise at time s

$$p_i(s) := h_i(\hat{X}_{-i}(s,s)) - \sum_{j\neq i}\sum_{t=s}^{T-1}\left[q_j\hat{x}_j^2(s,t) + r_j u_j^{*2}(s,t)\right.$$

$$\left. +2q_j\left(\sum_{\tau=0}^{s-1}\hat{x}_j(\tau,t)\right)\hat{x}_j(s,t) + 2r_j\left(\sum_{\tau=0}^{s-1}u_j(\tau,t)\right)u_j^*(s,t)\right]$$

Rational Agents and Incentive Compatibility

- We need agents to be *rational*
- Agent i is rational at time $T-1$, if it adopts a dominant strategy, when there is a *unique* dominant strategy.
- An agent i is rational at time t if it adopts a dominant strategy at time t under the assumption that all agents including itself are rational at times $t+1, ..., T-1$, when there is a *unique* such dominant strategy
- *Theorem:*

Truth-telling of state, i.e., $\hat{x}_i(s,s) = w_i(s-1)$, *for* $0 \leq s \leq T-1$ *is a dominant strategy for the layered VCG mechanism, if system parameters* Q, R, A, B *are truthfully known, and agents are rational.*

- There is a counterexample if system parameters unknown

Scaled VCG Mechanism for Budget Balance and Individual Rationality

- How to ensure that the layered VCG mechanism is BB and IR?
- Solution: Inflate (or deflate) the first term in the standard VCG mechanism by a *constant* factor c

$$p_i(\hat{F}) = c \cdot \sum_{j \neq i} \hat{F}_j(\boldsymbol{u}^{(i)}) - \sum_{j \neq i} \hat{F}_j(\boldsymbol{u}^*)$$

- Scaled VCG mechanism (SVCG)
- Want to adjust c to achieve BB and IR
- But if c is chosen as a function of the utility bids $\hat{F}$, then incentive compatibility is lost, since the first term is not allowed to depend on $\hat{F}_i$ in the Groves mechanism
- However, under a *Market Power Balance condition* there is a *range of values* of $c \in [\underline{c}, \overline{c}]$ that ensures BB and IR for a given system
- Through repeated long-term interactions, the social planner may be able to learn at least a subset of this range of values

Market Power Balance (MPB) condition

- Market Power Balance (MPB) condition:
  - Consider optimal solution with Agent i excluded

    $H_i := \mathbb{E}\sum_{t=0}^{T-1}[X^{(i)T}(t)Q^{(i)}X^{(i)}(t) + U^{(i)T}(t)R^{(i)}U^{(i)}(t)]$

    $H_{max} := \max_i H_i$
- We say the outcome satisfies MPB if $(N-1)H_{max} \leq \sum_i H_i$
  - Influence of excluding one agent on social welfare is not too great
- *Theorem: If the socially optimal outcome satisfies MPB, there exists a range of* $c \in [\underline{c}, \overline{c}]$ *such that the SVCG mechanism satisfies IC, EF, BB and IR at the same time*
- MPB provides an economic justification for load aggregators as entities that guarantee achievement of social welfare maximization

Asymptotic Lagrange Optimality

- *Lagrange Optimality*

  For constrained optimization problem with $\sum_i u_i(t) = 0$ if optimal solution (λ^*, u^*) is unique, then mechanism is Lagrange optimal if payment $p_i = \lambda^* u_i^*$

- Theorem:

  If $\underline{a} \le |a_i| \le \bar{a}, \underline{b} \le |b_i| \le \bar{b}, \underline{q} \le q_i \le \bar{q} < 0, \ \underline{r} \le r_i \le \bar{r} < 0$

  and MPB condition holds, then the range $[\underline{c}^N, \overline{c}^N]$ satisfies

  1. $\lim_{N\to\infty} \underline{c}^N = 1, \lim_{N\to\infty} \bar{c}^N = 1$
  2. *Asymptotic* Lagrange Optimality

$$\lim_{N\to\infty} \mathbb{E} \sum_{t=0}^{T-1} \left[\lambda^N(t) u_i^N(t) - p_i^N(t)\right] = 0$$

Some thoughts

- Today's bidding is essentially static
  - Does not allow dynamic optimization of uncertain resources
  - May be wasteful of resources
- Need Stochastic Dynamic Bidding Scheme
- Must satisfy IC, SW Optimality, BB, IR, Correct price payment (LO)
- Large LQG systems can be easily solved today by ISO
- So can LQG become the workhorse of dynamic bidding?

  Precedent: ap^2+bp+c
- Tuning just one parameter c can achieve all this
  - Prior knowledge acquired through repeated daily interaction, enough to tune "c"?
- Does Market Power Balance condition for Social Welfare maximization provide justification for Load Aggregators?
- Ke Ma is investigating these issues at his new job in PNNL

Happy retirement, Jean!

Wishing you the very best and continued success in everything!