Analog quantum simulations

What we have, what we would like to have, and the potential for seeing quantum advantages

Jens Eisert, Freie Universität Berlin
Challenges in Quantum Computation Workshop, Berkeley, June 2018
Analog quantum simulations

What we have, what we would like to have, and the potential for seeing quantum advantages

- What is an analog quantum simulator? What are relevant problems?
When and in what sense can we hope quantum simulators to provide a speedup over classical computers?
- 20-50 qubit quantum devices
- Noisy intermediate scale quantum computers

When and in what sense can we hope quantum simulators to provide a speedup over classical computers?
Analog(ue) quantum simulators
- Address interesting physics problems
- Not BQP-complete, what is computational power?
- Error correction/fault tolerance unavailable
- Robustness?
Quantum simulators should solve problems inaccessible to classical computers.

When can it be claimed that a system has been successfully simulated? Testable advantage?
Analog quantum simulators
Analog quantum simulators

- “Analog”, rather than discrete
- Probing questions in physics (not so much quantum chemistry)

- System size $n$
- Local Hamiltonians with some levels of control
- Noise levels
- Classes of preparations and measurements
Analog quantum simulators

- Cold atoms in optical lattices most advanced
  - Global control over $n \sim 10^5$ sites (1D-3D)
  - Bosons and fermions
  - Some tuneability
  - Time-of-flight and in-situ measurements

- Towards programmable potentials

Parsons, Mazurenko, Chiu, Ji, Greif, Greiner, Science, 353, 1253 (2016)
Analog quantum simulators

- Trapped ions

- Optical microtraps

- Polaritonic/photonic architectures

- $n \leq 53$
- Universal control
- Some global gates easier than others
- Tomographically complete measurements

Zhang, Pagano, Hess, Kyprianidis, Becker, Kaplan, Gorshkov, Gong, Monroe 551, 601 (2017)


- $n \sim 50 \times 50$, long-ranged Ising
- Large, but intrisically open and noisy
Analog quantum simulators

- **Trapped ions**
  - Universal control
  - Some global gates easier than others
  - Tomographically complete measurements

  Zhang, Pagano, Hess, Kyprianidis, Becker, Kaplan, Gorshkov, Gong, Monroe 551, 601 (2017)

- **Cold atoms in Rydberg states**
  - Programmable


- **Polaritonic/photonic architectures**
  - Large, but intrinsically open and noisy

What can they probe?
What can they probe?

- **Time-dependent** problems ("quenches")
  \[ \rho(t) = e^{-itH} \rho e^{itH} \]

- E.g. probe **equilibration and thermalisation**

- **Dynamical phase transitions**
What can they probe?

- **Time-dependent** problems ("quenches")

- Imbalance as function of time for $|\psi(0)\rangle = |0, 1, \ldots, 0, 1\rangle$
  under Bose-Hubbard Hamiltonian (MPQ)

![Graph showing imbalance as function of time](image)

Best available classical tensor network simulation, bond dimension 5000

What can they probe?

- **Slow parameter variations** (reminiscent of adiabatic quantum algorithms)
  - E.g., Kibble-Zurek dynamics (1D-2D)
  - Probing scaling laws of correlations

**What can they probe?**

- **Ground state** problems
  - Hubbard model, probing high-Tc superconductivity
  - Cooled to create a magnetic state with long-range order

- Many-body localization (1D-2D)
  - Debated in 2D

Mazurenko, Chiu, Ji, Parsons, Kanász-Nagy, Schmidt, Grusdt, Demler, Greif, Greiner, Nature 545, 462 (2017)
Esslinger, Ann Rev Cond Mat Phys 1, 129 2010

Schreiber, Hodgman, Bordia, Lüschen, Fischer, Vosk, Altman, Schneider, Bloch, Science 349, 842 (2015)
What can they probe?

- Many-body localization (1D-2D)
  - Debated in 2D
  - Quantum simulators
    - Existing quantum simulators outperform state-of-the-art algorithms on classical supercomputers
  - Cleverer simulation method?
Intermediate problems

To be safe against “lack of imagination”, we must prove the hardness of the task in a complexity-theoretic sense.
Super-polynomial quantum advantages?
Complexity-theoretic quantum advantages

- **Aim:** Find some problem with strong evidence for quantum advantage

- **Boson sampling**
  Aaronson, Arkhipov, Th Comp 9, 143 (2013)

Sampling from a distribution close in $l_1$ norm to boson sampling distribution is "computationally hard" with high probability if the unitary $U$ is chosen from Haar measure and $m$ increases sufficiently fast with $n$ ($m \in \Omega(n^5)$)

- **IQP and random universal circuits**
  Bremner, Jozsa, Shepherd, arXiv:1005.1407
  Boixo, Isakov, Smelzanski, Babbush, Ding, Jiang, Bremner, Martinis, Neven, Nature Physics 14, 595-600 (2018)

- **Ising-type interactions (but, period 56 of unit cell)**
Aim: Find some problem with strong evidence for quantum advantage

Verification and testing? Black-box verification seems out of question
Hamiltonian quantum simulation architectures

- **Aim:** Find some problem with strong evidence for quantum advantage

- **Challenging prescription:** Is it possible to scale it up to provably hard regimes, in an architecture close to a quantum simulation?
Hamiltonian quantum simulation architectures

- **Aim:** Find some problem with strong evidence for quantum advantage

  **Combine benefits of both worlds**

  - Hamiltonian quench architecture
  - **Low periodicity** of the interaction Hamiltonian (NN or NNN)
  - **Hardness proofs** with $l_1$-norm error (under some assumptions)

Hangleiter, Bermejo-Vega, Schwarz, Eisert, Quantum 2, 65 (2018)
Hamiltonian quantum simulation architectures

**Aim:** Find some problem with strong evidence for quantum advantage

Combine benefits of both worlds

Random

Quasi-periodic

Translationally invariant

Bermejo-Vega, Hangleiter, Schwarz, Raussendorf, Eisert, Phys Rev X 8, 021010 (2018)
Hangleiter, Bermejo-Vega, Schwarz, Eisert, Quantum 2, 65 (2018)
Simple Ising models

- Prepare $N$ qubits in $n \times m$ square lattice in product

$$|\psi_\beta\rangle = \bigotimes_{i,j=1}^{n,m} (|0\rangle + e^{i\beta_{i,j}} |1\rangle)$$

with $\beta_{i,j} \in \{0, \pi/4\}$, \{●, ●\} i.i.d. randomly

- Quench to $H = \sum_{(i,j)} H_{i,j}$

- Measure all qubits in $\{0, \pi/2\}$

- Reminscent of disordered optical lattices

- Controlled coherent collisions long realized

- Single-site addressing possible (within limits)


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  with $\beta_{i,j} \in \{0, \pi/4\}$, \{\(\bullet\), \(\bullet\)\} i.i.d. randomly

- **Quench** to $H = \sum_{(i,j)\in E} Z_i Z_j + \frac{\pi}{4} \sum_{i\in V} Z_i$ and evolve under $U = e^{iH}$

- **Controlled coherent collisions long realized**


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  Hangleiter, Bermejo-Vega, Schwarz, Eisert, Quantum 2, 65 (2018)
Simple Ising models

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- **Measure** all qubits in $X$-basis

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Theorem (Hardness of classical sampling):
Assuming three highly plausible complexity-theoretic conjectures are true a classical computer cannot efficiently sample from the outcome distribution of our scheme up to constant error in $l_1$ distance.
- It is \#P-hard to approximate the outcome distribution

- Polynomial hierarchy (similar P \neq NP)
- Average-case complexity
  - Bouland, Fefferman, Nirkhe, Vazirani, arXiv:1803.04402
- Anti-concentration
  - Hangleiter, Bermejo-Vega, Schwarz, Eisert, Quantum 2, 65 (2018)
  - Mann, Bremner, arXiv:1711.00686

- Universal quantum circuit for postBQP

- Relate quench architecture to post-selected measurement-based quantum computing

- Relate hardness of computing probabilities to hardness of sampling with additive errors

\[ U \] Additive error \( \epsilon \)
\[ U' \]
\[ x \] Stockmeyer theorem
\[ s_{U'}(x) \]
Theorem (Hardness of classical sampling):
Assuming three highly plausible complexity-theoretic conjectures are true a classical computer cannot efficiently sample from the outcome distribution of our scheme up to constant error in $l_1$ distance.
This quantum simulation is intractable for classical computers
Verifiable quantum devices showing a quantum advantage

- One can with $\theta(N)$ many measurements detect closeness in $l_1$-norm!
- Ground state of fictitious frustration-free Hamiltonian
- Much simpler than fault tolerance

Bermejo-Vega, Hangleiter, Schwarz, Raussendorf, Eisert, Phys Rev X 8, 021010 (2018)
Hangleiter, Bermejo-Vega, Schwarz, Eisert, Quantum 2, 65 (2018)
Hangleiter, Kliesch, Schwarz, Eisert, Quantum Sci Technol 2, 015004 (2017)
Verifiable quantum devices showing a quantum advantage

- **Common prejudice**: In order to be able to verify a quantum simulation, one needs to be able to efficiently simulate it.
Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
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- Hope for **feasible** quantum simulators with **superpolynomial speedup**
Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
- Hope for feasible quantum simulators with superpolynomial speedup
- Not fault tolerant, but can be certified: Bell test for quantum computing - even if simulators exhibit quantum computational speedup

- Closer to physically more interesting schemes?
- More structured problems, optimization?
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- Robustness of quantum simulators? Readout?

Gluza, Schweigler, Krumnow, Rauer, Schmiedmayer, Eisert, in preparation
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- Space time trade offs?

(Mick Bremner)
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Thanks for your attention!