BitGC: Garbling with 1 Bit per Gate

Xiao Wang

Two Routes to Garbling Programs



(1) Use a hash function (e.g. a random oracle)

1.5k is unavoidable in MiniCrypt!!

(2) Take a stronger assumption (e.g. RLWE, DCR)



Later Today

Outline

- BitGC: A garbling scheme of size 1 bit per Boolean gate
 - Hanlin Liu, Xiao Wang, Kang Yang, Yu Yu
 - Circular-secure Ring LWE
 - Appeared in Eurocrypt 2025
- Ongoing progress in pushing BitGC's concrete efficiency
 - LWYY + Wenhao Zhang, Wenjie Lu, and Chenkai Weng
 - 10 ms per gate for single-thread CPU (with some caveat)

Related Work

- To appear Crypto 2025
 - "Authenticated BitGC for Act Yang, Yu
 - "A Unified Framework for Suc Sharing" by Ishai, Li, Lin
 - "Silent Circuit Relinearisation Garbled Circuits from DCR"
- To appear FOCS 2025
 - "Succinct Homomorphic MA Li, Lin,



$$(A^0, A^1)$$

Why Classical Garbled Circuit is So Big?

$$(B^0, B^1)$$

$$(C^0,C^1)$$

AND

Garbler

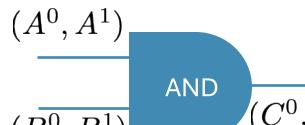
$$(A^0, A^1), (B^0, B^1) (C^0, C^1)$$

 $ig(extstyle extstyle extstyle extstyle extstyle extstyle (A^{v_a}, B^{v_b})$

Encryptions of C's under (A,B)'s

Needs to contain information of C

 C^{v_c}



A Different Way to Think About GC



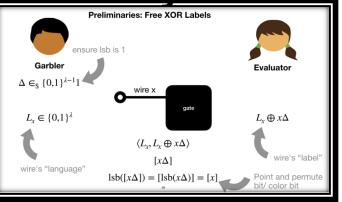
$$A^1 = A^0 \oplus \Delta$$

 $A^v = A^0 \oplus v \cdot \Delta$

[]: secret sharing

Garbler

$$(A \mathcal{U}_{A} \Delta) (B \mathcal{V}_{B} \Delta) C^{0}, C^{1})$$



Evaluator

$$(A[v_a A] [v_b \Delta]$$

Encryptions of output shares under input shares

$$[(v_a \wedge v_b)\Delta]$$

Only needs to contain information of v_a, v_b

$$[(v_a \wedge v_b)\Delta]$$

$$(A^0,A^1)$$
 AND $\overline{(C^0,C^1)}$

"Blueprint"

$$A^1 = A^0 \oplus \Delta$$

$$[v_a\Delta] [v_b\Delta]$$

Evaluator

$$[v_a\Delta]$$
 $[v_b\Delta]$

Encrypted truth table bits

Garbler cannot obtain it non-interactively

$$[v_a \wedge v_b) \cdot \Delta$$

$$[(v_a \wedge v_b)\Delta]$$

Our solution: distributively evaluate encrypted truth table directly

$$[(v_a \wedge v_b)\Delta]$$

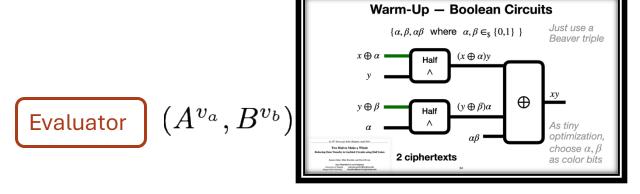
Some Technical Details!

$$(A^0,A^1)$$
 AND (C^0,C^1)

Masked bits	Input labels	Outbit bit	Output labels	Garbled table
(0,0) $(1,0)$	$\begin{array}{ c } \hline (A^{\pi_a},B^{\pi_b}) \\ (A^{\overline{\pi_a}},B^{\pi_b}) \end{array}$	$egin{array}{ c c c c c c c c c c c c c c c c c c c$	$egin{array}{c} C^{oldsymbol{z}_{0,0}} \ C^{oldsymbol{z}_{1,0}} \end{array}$	$oxed{ au_0 = H(A^{\pi_a}, B^{\pi_b}) \oplus C^{z_{0,0}} \ au_1 = H(A^{\overline{\pi_a}}, B^{\pi_b}) \oplus C^{z_{1,0}} }$
(0,1) $(1,1)$	$(A^{\pi_a},B^{\overline{\pi_b}}) \ (A^{\overline{\pi_a}},B^{\overline{\pi_b}})$	$egin{aligned} z_{0,1} &= \pi_a \wedge \overline{\pi_b} \ z_{1,1} &= \overline{\pi_a} \wedge \overline{\pi_b} \end{aligned}$		$ au_2 = H(A^{\pi_a}, B^{\overline{\pi_b}}) \oplus C^{z_{0,1}} \ au_3 = H(A^{\overline{\pi_a}}, B^{\overline{\pi_b}}) \oplus C^{z_{1,1}}$

Garbler $(A^0, A^1), (B^0, B^1)$

Uniformly pick (C^0, C^1)



$$C^{v_a \wedge v_b} = \mathsf{Eval}(A^{v_a}, B^{v_b}, au_{0,1,2,3})$$

Masked bits	Input labels	Outbit bit	Output labels	Garbled table
(0,0) $(1,0)$ $(0,1)$ $(1,1)$	$(A^{\pi_a}, B^{\pi_b}) \ (A^{\overline{\pi_a}}, B^{\overline{\pi_b}}) \ (A^{\pi_a}, B^{\overline{\pi_b}}) \ (A^{\overline{\pi_a}}, B^{\overline{\pi_b}})$	$z_{0,0} = \pi_a \wedge \pi_b$ $z_{1,0} = \overline{\pi_a} \wedge \pi_b$ $z_{0,1} = \pi_a \wedge \overline{\pi_b}$ $z_{1,1} = \overline{\pi_a} \wedge \overline{\pi_b}$	$C^{oldsymbol{z}_{1,0}} \ C^{oldsymbol{z}_{0,1}}$	$ \begin{array}{c c} \tau_0 = H(A^{\pi_a}, B^{\pi_b}) \oplus C^{z_{0,0}} \\ \tau_1 = H(A^{\overline{\pi_a}}, B^{\pi_b}) \oplus C^{z_{1,0}} \\ \tau_2 = H(A^{\pi_a}, B^{\overline{\pi_b}}) \oplus C^{z_{0,1}} \\ \tau_3 = H(A^{\overline{\pi_a}}, B^{\overline{\pi_b}}) \oplus C^{z_{1,1}} \end{array} $

$C^{z_{i,j}}\oplus C^{z_{0,0}}=(z_{i,j}\oplus z_{0,0})\cdot \Delta$

Assuming RO is homomorphic:

$$H(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}) \oplus H(A^{\pi_a}, B^{\pi_b}) = i \cdot H(\Delta, 0) \oplus j \cdot H(0, \Delta) \oplus ij \cdot H(\Delta, \Delta)$$



Garbled table

$$0$$

$$\tau_{1} = H(\Delta, 0) \oplus \underbrace{(z_{1,0} \oplus z_{0,0})} \cdot \Delta$$

$$\tau_{2} = H(0, \Delta) \oplus \underbrace{(z_{0,1} \oplus z_{0,0})} \cdot \Delta$$

$$\tau_{3} = H(\Delta, \Delta) \oplus \underbrace{(z_{1,1} \oplus z_{1,0} \oplus z_{0,1} \oplus z_{0,0})} \cdot \Delta$$

Encrypted Truth Table Bits

Garbled table

$$egin{aligned} 0 \ au_1 &= H(A^{\overline{\pi_a}}, B^{\pi_b}) \oplus H(A^{\pi_a}, B^{\pi_b}) \oplus (z_{1,0} \oplus z_{0,0}) \cdot \Delta \ au_2 &= H(A^{\pi_a}, B^{\overline{\pi_b}}) \oplus H(A^{\pi_a}, B^{\pi_b}) \oplus (z_{0,1} \oplus z_{0,0}) \cdot \Delta \ au_3 &= H(A^{\overline{\pi_a}}, B^{\overline{\pi_b}}) \oplus H(A^{\pi_a}, B^{\pi_b}) \oplus (z_{1,1} \oplus z_{0,0}) \cdot \Delta \end{aligned}$$

So Far...

If only there is a way to send ciphertext cheaply and a way to instantiate homomorphic RO!

Garbler

$$(A^0, A^1), (B^0, B^1)$$

Evaluator
$$(A^{v_a}, B^{v_b})$$

$$au_1 = H(\Delta,0) \oplus (z_{1,0} \oplus z_{0,0}) \cdot \Delta \ au_2 = H(0,\Delta) \oplus (z_{0,1} \oplus z_{0,0}) \cdot \Delta \ au_3 = H(\Delta,\Delta) \oplus (z_{1,1} \oplus z_{1,0} \oplus z_{0,1} \oplus z_{0,0}) \cdot \Delta$$

$$C^{z_{0,0}}\stackrel{\mathsf{def}}{=} H(A^{\pi_a},B^{\pi_b})$$

Evaluate to obtain $C^{v_a \wedge v_b}$

Transmitting Encrypted Bits is Cheap

To send an encryption of bit x

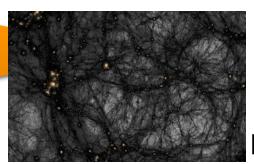
Bitstring labels not compatible with SWHE ciphertext!

[[seed]]

```
 = PRG([[seed]])
```

$$d = x \oplus r_i$$

Need a PRG that we can evaluate in HE without bootstrapping



$$[[x]] = [[ri]] \bigoplus$$

[TCC:BIPSW18,...]

Using Polynomial Rings as Wire Labels

$$\mathbb{Z}_p[X]/(X^n+1)$$

Garbling based on bit-strings

Garbling based on ring elements

$$A^1 \stackrel{\mathsf{def}}{=} A^0 \oplus \Delta \ A^v = A^0 \oplus v \cdot \Delta$$

$$A^{1} \stackrel{\text{def}}{=} A^{0} + (-1)^{\pi_{a}} \cdot \Delta$$
$$A^{v} = A^{0} + (-1)^{\pi_{a}} \cdot v \cdot \Delta$$

$$A^v = A^{\pi_a} \oplus (\pi_a \oplus v) \cdot \Delta$$

$$A^v = A^{\pi_a} + (\pi_a \oplus v) \cdot \Delta$$

$$A^v = A^0 + (-1)^{\pi_a} \cdot v$$

$ au_1 = H(\Delta,0) \oplus (z_{1,0} \oplus z_{0,0}) \cdot \Delta$
$ au_2 = H(0,\Delta) \oplus (z_{0,1} \oplus z_{0,0}) \cdot \Delta$
$ au_3 = H(\Delta,\Delta) \oplus (z_{1,1} \oplus z_{1,0} \oplus z_{0,1} \oplus z_{0,0}) \cdot \Delta$

Masked bits	Input labels	Outbit bit	Output labels	Garbled table
(0,0) $(1,0)$ $(0,1)$ $(1,1)$	$ \begin{array}{c c} (A^{\pi a}, B^{\pi b}) \\ (A^{\overline{\pi} a}, B^{\overline{\pi} b}) \\ (A^{\pi a}, B^{\overline{\pi} b}) \\ (A^{\overline{\pi} a}, B^{\overline{\pi} b}) \end{array} $	$egin{aligned} z_{0,0} &= \pi_a \wedge \pi_b \ z_{1,0} &= \overline{\pi_a} \wedge \pi_b \ z_{0,1} &= \pi_a \wedge \overline{\pi_b} \ z_{1,1} &= \overline{\pi_a} \wedge \overline{\pi_b} \end{aligned}$	$C^{z_0,1}$	$egin{array}{ c c c c c c c c c c c c c c c c c c c$

Garbler

 $(A^0, A^1), (B^0, B^1)$

$$C^{z_{0,0}}\stackrel{\mathsf{def}}{=} \overline{\mathsf{Eval}}(A^{\pi_a},B^{\pi_b}, au_{1,2,3})$$

Evaluator (A^{v_a}, B^{v_b})

$$C^{v_a\wedge v_b}:= \overline{\mathsf{Eval}}(A^{v_a}, B^{v_b}, au_{1,2,3})$$

If only there is a way to send ciphertext cheaply and a way to instantiate homomorphic RO!

$$\begin{aligned}
0 \\
\tau_1 &= [(-1)^{\pi_c} \cdot (z_{1,0} - z_{0,0})] \\
\tau_2 &= [(-1)^{\pi_c} \cdot (z_{0,1} - z_{0,0})] \\
\tau_3 &= [(-1)^{\pi_c} \cdot (z_{1,1} - z_{0,1} - z_{1,0} + z_{0,0})]
\end{aligned}$$

$$A^1 \stackrel{\mathsf{def}}{=} A^0 + (-1)^{\pi_a} \cdot \Delta \ A^v = A^0 + (-1)^{\pi_a} \cdot v \cdot \Delta \ A^v = A^{\pi_a} + (\pi_a \oplus v) \cdot \Delta$$

Instantiating Homomorphic RO

Distributed "Evaluation":

$$\mathsf{Eval}(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}, au_{1,2,3}) - \mathsf{Eval}(A^{\pi_a}, B^{\pi_b}, au_{1,2,3}) = i \cdot \mathsf{Dec}(\Delta, au_1) + j \cdot \mathsf{Dec}(\Delta, au_2) + i \cdot j \cdot \mathsf{Dec}(\Delta, au_3)$$

Assume $\operatorname{Dec}(\Delta,\llbracket m \rrbracket) = m \cdot \Delta$

Homomorphic RO:

$$H(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}) - H(A^{\pi_a}, B^{\pi_b}) = i \cdot H(\Delta, 0) + j \cdot H(0, \Delta) + i \cdot j \cdot H(\Delta, \Delta)$$

$$\mathsf{Eval}(A^{\pi_a \oplus \imath}, B^{\pi_b \oplus \jmath}, au_{1,2,3}) - \mathsf{Eval}(A^{\pi_a}, B^{\pi_b}, au_{1,2,3}) = (-1)^{\pi_c} \cdot (z_{i,j} - z_{0,0}) \cdot \Delta$$

4^1	def =	$A^0 + (-1)^{\pi_a} \cdot \Delta$
		$A^0 + (-1)^{\pi_a} \cdot v \cdot \Delta$
i۳	=	$A^{\pi_a} + (\pi_a \oplus v) \cdot \Delta$

Masked bits Input labels O	Outbit bit Output labels	Garbled table
$ \begin{array}{c cccc} (0,0) & & (A^{\pi_a},B^{\pi_b}) & z_{0,0} \\ (1,0) & & (A^{\overline{\pi_a}},B^{\pi_b}) & z_{1,0} \\ (0,1) & & (A^{\overline{\pi_a}},B^{\overline{\pi_b}}) & z_{0,1} \\ (1,1) & & (A^{\overline{\pi_a}},B^{\overline{\pi_b}}) & z_{1,1} \end{array} $	$C^{z_0,0}$ $C^{z_0,0}$ $C^{z_0,0}$ $C^{z_0,0}$ $C^{z_0,0}$ $C^{z_0,0}$ $C^{z_0,0}$ $C^{z_0,0}$ $C^{z_0,1}$ $C^{z_0,1}$ $C^{z_0,1}$	$egin{array}{ c c c c c c c c c c c c c c c c c c c$

Garbler

$$(A^0, A^1), (B^0, B^1)$$

Define

$$C^{z_{0,0}}\stackrel{\mathsf{def}}{=} \mathsf{Eval}(A^{\pi_a},B^{\pi_b}, au_{1,2,3})$$

Evaluator

$$(A^{v_a},B^{v_b})$$

$$\mathsf{Eval}(A^{v_a}, B^{v_b}, au_{1,2,3})$$

$$\begin{split} &= \mathsf{Eval}(A^{\pi_a \oplus (\pi_a \oplus v_a)}, B^{\pi_b \oplus (\pi_b \oplus v_b)}, \tau_{1,2,3}) \\ &= C^{z_{0,0}} + (-1)^{\pi_c} \cdot (z_{\pi_a \oplus v_a, \pi_b \oplus v_b} - z_{0,0}) \cdot \Delta \\ &= C^0 + (-1)^{\pi_c} \cdot z_{\pi_a \oplus v_a, \pi_b \oplus v_b} \cdot \Delta \\ &= C^{z_{\pi_a \oplus v_a, \pi_b \oplus v_b}} \\ &= C^{v_a \wedge v_b} \end{split}$$

$$egin{aligned} \mathsf{Eval}(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}, au_{1,2,3}) - \mathsf{Eval}(A^{\pi_a}, B^{\pi_b}, au_{1,2,3}) \ &= (-1)^{\pi_c} \cdot (z_{i,j} - z_{0,0}) \cdot \Delta \end{aligned}$$

Recap

 $(A^0,A^1),\,(B^0,B^1)$ $\llbracket \pi_a
rbracket, \llbracket \pi_b
rbracket$ $\llbracket r_c
rbracket$ $[A^{v_a},B^{v_b}) \ \llbracket \pi_a
rbracket, \llbracket \pi_b
rbracket, \llbracket r_c
rbracket,
rbracket$ Garbler Evaluator Compute π_c $d = r_c \oplus \pi_c$ au_1, au_2, au_3 au_1, au_2, au_3 $\mathsf{Eval}(A^{v_a}, B^{v_b}, \tau_{1,2,3}) = C^{v_a \wedge v_b}$ $C^{z_{0,0}}\stackrel{\mathsf{def}}{=} \mathsf{Eval}(A^{\pi_a},B^{\pi_b}, au_{1,2,3})$ C^{v_c} $\llbracket \pi_c
rbracket$ (C^0,C^1) $[\![\pi_c]\!]$

Properties that we need

Homomorphic evaluation of a PRG and 2 more levels to assemble

Encrypted truth table bits

$$egin{aligned} \mathsf{-Dec}(\Delta,\llbracket m
rbracket] &= m\cdot\Delta \ \mathsf{Eval}(A^{\pi_a\oplus i},B^{\pi_b\oplus j}, au_{1,2,3}) - \mathsf{Eval}(A^{\pi_a},B^{\pi_b}, au_{1,2,3}) \ &= i\cdot\mathsf{Dec}(\Delta, au_1) + j\cdot\mathsf{Dec}(\Delta, au_2) + i\cdot j\cdot\mathsf{Dec}(\Delta, au_3) \end{aligned}$$

GSW OR

Any regev-like scheme + send an encryption of Δ , invoking circularity

$$\begin{aligned} & \mathsf{Eval}(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}, \tau_{1,2,3}) - \mathsf{Eval}(A^{\pi_a}, B^{\pi_b}, \tau_{1,2,3}) \\ &= \underbrace{i \cdot \mathsf{Dec}(\Delta, \tau_1)} + j \cdot \mathsf{Dec}(\Delta, \tau_2) + \underbrace{i \cdot j \cdot \mathsf{Dec}(\Delta, \tau_3)} \end{aligned}$$

$$A^{1} \stackrel{\text{def}}{=} A^{0} + (-1)^{\pi_{a}} \cdot \Delta$$

$$A^{v} = A^{0} + (-1)^{\pi_{a}} \cdot v \cdot \Delta$$

$$A^{v} = A^{\pi_{a}} + (\pi_{a} \oplus v) \cdot \Delta$$

$$egin{aligned} \mathsf{Dec}(A^{\pi_a \oplus i}, au_1) - \mathsf{Dec}(A^{\pi_a}, au) \ &= \mathsf{Dec}(A^{\pi_a} + i \cdot \Delta, au_1) - \mathsf{Dec}(A^{\pi_a}, au) \ &= i \cdot \mathsf{Dec}(\Delta, au_1) \end{aligned}$$

$$\mathsf{Eval}(A,B,\tau_{1,2,3}) = \mathsf{Dec}(A,\tau_1) + \mathsf{Dec}(B,\tau_2) + \mathsf{Something}(A,B,\tau_3)$$

Near Linear Decryption [BoyleKohlScholl19]

$$\mathsf{Dec}(X + \mathsf{sk}, \llbracket m \rrbracket) = \mathsf{Dec}(X, \llbracket m \rrbracket) + \mathsf{Dec}(\mathsf{sk}, \llbracket m \rrbracket)$$

$$[\![m]\!] = (a, a \cdot \mathsf{sk} + e + m \cdot t)$$

Plaintext space

Noise budget

Requirement:



Gap to allow lifting lemma "mod p can be dropped"

Gap to allow rounding lemma "round can be added over q"

$$\begin{aligned} & \mathsf{Eval}(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}, \tau_{1,2,3}) - \mathsf{Eval}(A^{\pi_a}, B^{\pi_b}, \tau_{1,2,3}) \\ &= \underbrace{i \cdot \mathsf{Dec}(\Delta, \tau_1)} + j \cdot \mathsf{Dec}(\Delta, \tau_2) + \underbrace{i \cdot j \cdot \mathsf{Dec}(\Delta, \tau_3)} \end{aligned}$$

$$A^1 \stackrel{\mathsf{def}}{=} A^0 + (-1)^{\pi_a} \cdot \Delta \ A^v = A^0 + (-1)^{\pi_a} \cdot v \cdot \Delta \ A^v = A^{\pi_a} + (\pi_a \oplus v) \cdot \Delta$$

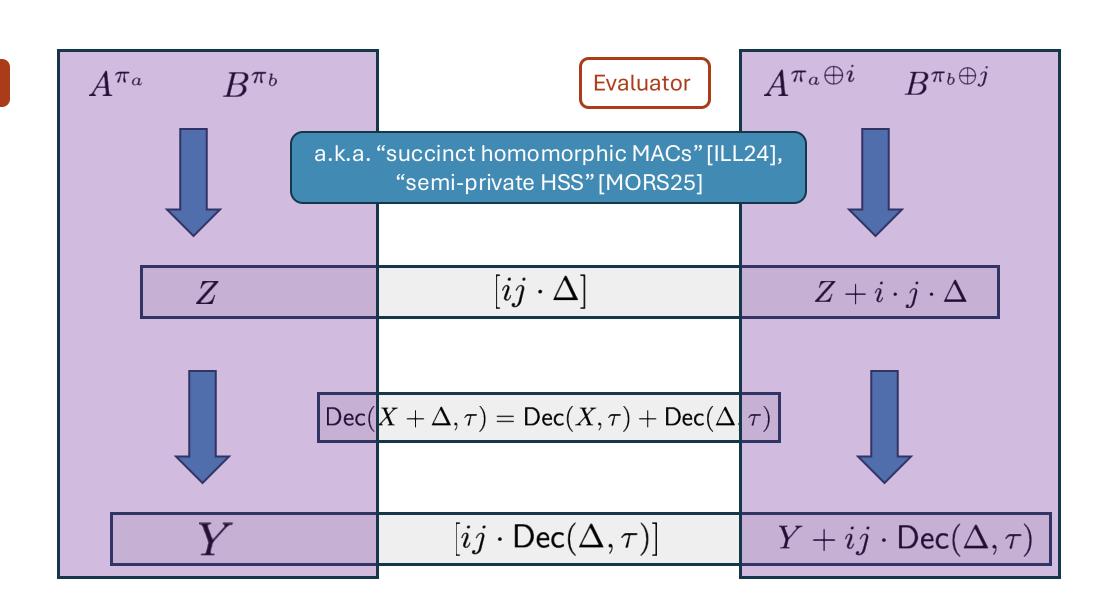
$$egin{aligned} \mathsf{Dec}(A^{\pi_a \oplus i}, au_1) - \mathsf{Dec}(A^{\pi_a}, au) \ &= \mathsf{Dec}(A^{\pi_a} + i \cdot \Delta, au_1) - \mathsf{Dec}(A^{\pi_a}, au) \ &= i \cdot \mathsf{Dec}(\Delta, au_1) \end{aligned}$$

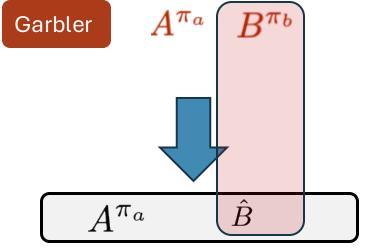
$$\mathsf{Eval}(A,B,\tau_{1,2,3}) = \mathsf{Dec}(A,\tau_1) + \mathsf{Dec}(B,\tau_2) + \mathsf{Something}(A,B,\tau_3)$$

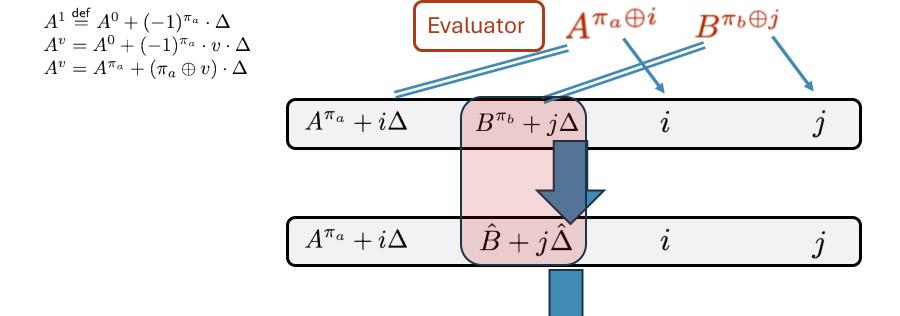
$$\widetilde{\mathsf{Dec}}(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}, au) - \widetilde{\mathsf{Dec}}(A^{\pi_a}, B^{\pi_b}, au) = i \cdot j \cdot \mathsf{Dec}(\Delta, au)$$

$$\widetilde{\mathsf{Dec}}(A^{\pi_a \oplus i}, B^{\pi_b \oplus j}, \tau) - \widetilde{\mathsf{Dec}}(A^{\pi_a}, B^{\pi_b}, \tau) = i \cdot j \cdot \mathsf{Dec}(\Delta, \tau)$$

Garbler







 $[(A^{\pi_a}+i\Delta)(\hat{B}+j\hat{\Delta}) \quad -i(\hat{B}+j\hat{\Delta}) \quad -j(A^{\pi_a}+i\Delta)]$



$$\mathbf{Z} = A^{\pi_a} \cdot \hat{B}$$

$$oxed{a\Delta} a\Delta \qquad a\hat{\Delta} \qquad a\Delta\hat{\Delta} + \Delta]$$



ij

$$Z + i \cdot j \cdot \Delta$$

BitGC (Being) Made Concretely Efficient

- Offline homomorphic PRG expansion:
 - ↑ Depth-5 BGV

 - ◆ Ciphertext unpacking to RLWE ciphertext
- Gate assembling:
 - 1 6 ring multiplications per gate
 - Additive homomorphism sufficient to assemble the gate
 - Increase size of AND to 6 bits and XOR to 2 bits

BitGC (Being) Made Concretely Efficient

CPU, single-thread

- Offline preprocessing: 1 7 ms per gate
- Garbling: 3 ms per gate

FHE and garbling are for different purposes but speed comparable to TFHE evaluation.

F1: A Fast and Programmable Accelerator for Fully Homomorphic Encryption (Extended Version)

Axel Feldmann^{1*}, Nikola Samardzic^{1*}, Aleksandar Krastev¹, Srini Devadas¹, Ron Dreslinski², Karim Eldefrawy³, Nicholas Genise³, Chris Peikert², Daniel Sanchez¹

¹ Massachusetts Institute of Technology
² University of Michigan
{axelf, nsamar, alexalex, devadas, sanchez}@csail.mit.edu
³ SRI International
{karim.eldefrawy, nicholas.genise}@sri.com

FPGA (estimated)

- Offline preprocessing: 0.1 0.4 us per gate
- Garbling: 0.2 us per gate

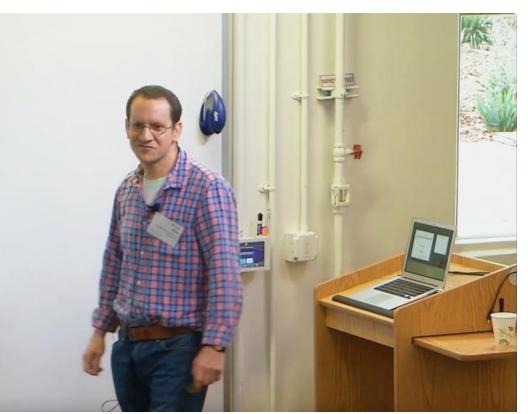
What 10 years cando? Efficient Garbling from a Fixed-Key Blockcipher

Fairplay — A Secure Two-Party Computation System

Dahlia Malkhi¹, Noam Nisan¹, Benny Pinkas², and Yaron Sella¹ ~0.1 s/gate USENIX 2004







Cryptography Boot Camp 2015

BitGC 2025 0.01 s/gate

BitGCrazy 2035 ?? s/gate