Hodgkin-Huxley ,

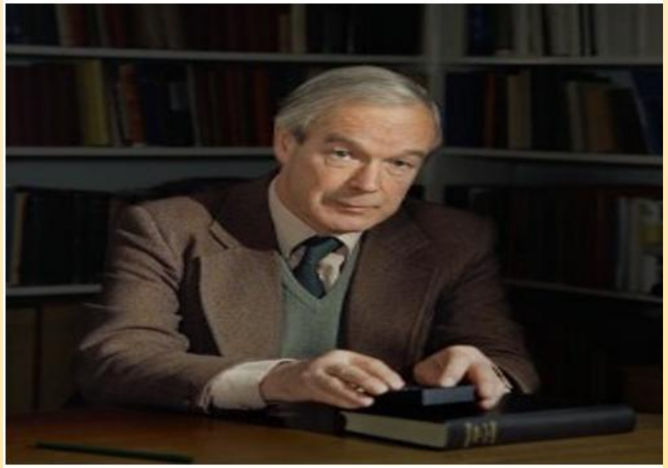
Memristor ,

and

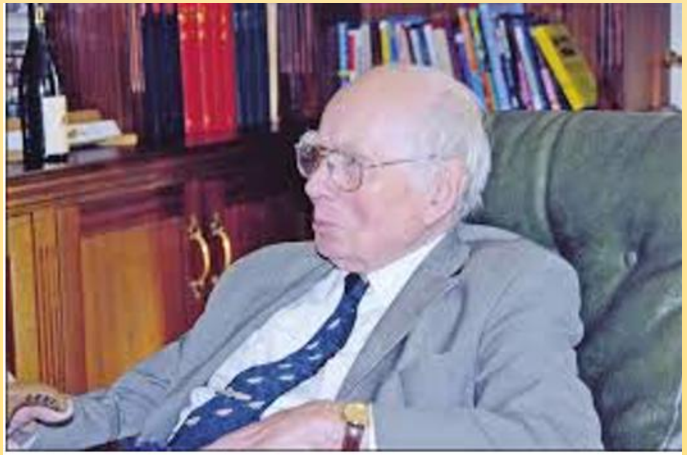
Edge of Chaos

Slide-2 b

Journal of Physiology, Vol 117, pp. 500-544, (1952)



Sir A. L. Hodgkin



Sir A. F. Huxley

1961 Nobel Prize in Physiology

A QUANTITATIVE DESCRIPTION OF MEMBRANE CURRENT AND ITS APPLICATION TO CONDUCTION AND EXCITATION IN NERVE*

■ A. L. HODGKIN and A. F. HUXLEY
Physiological Laboratory,
University of Cambridge,
Cambridge, U.K.

This article concludes a series of papers concerned with the flow of electric current through the surface membrane of a giant nerve fibre (Hodgkin *et al*., 1952, *J. Physiol*. **116**, 424–448; Hodgkin and Huxley, 1952, *J. Physiol*. **116**, 449–566). Its general object is to discuss the results of the preceding papers (Section 1), to put them into mathematical form (Section 2) and to show that they will account for conduction and excitation in quantitative terms (Sections 3–6).

1. Discussion of Experimental Results. The results described in the preceding papers suggest that the electrical behaviour of the membrane may be represented by the network shown in Fig. 1. Current can be carried through the

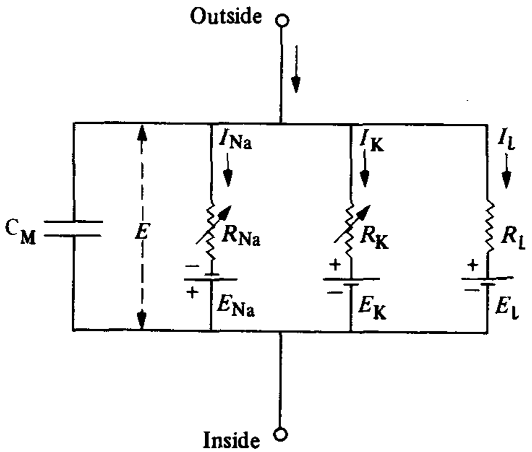


Figure 1. Electrical circuit representing membrane. $R_{Na}=1/g_{Na}$; $R_K=1/g_K$; $R_l=1/\bar{g}_l$. R_{Na} and R_K vary with time and membrane potential; the other components are constant.

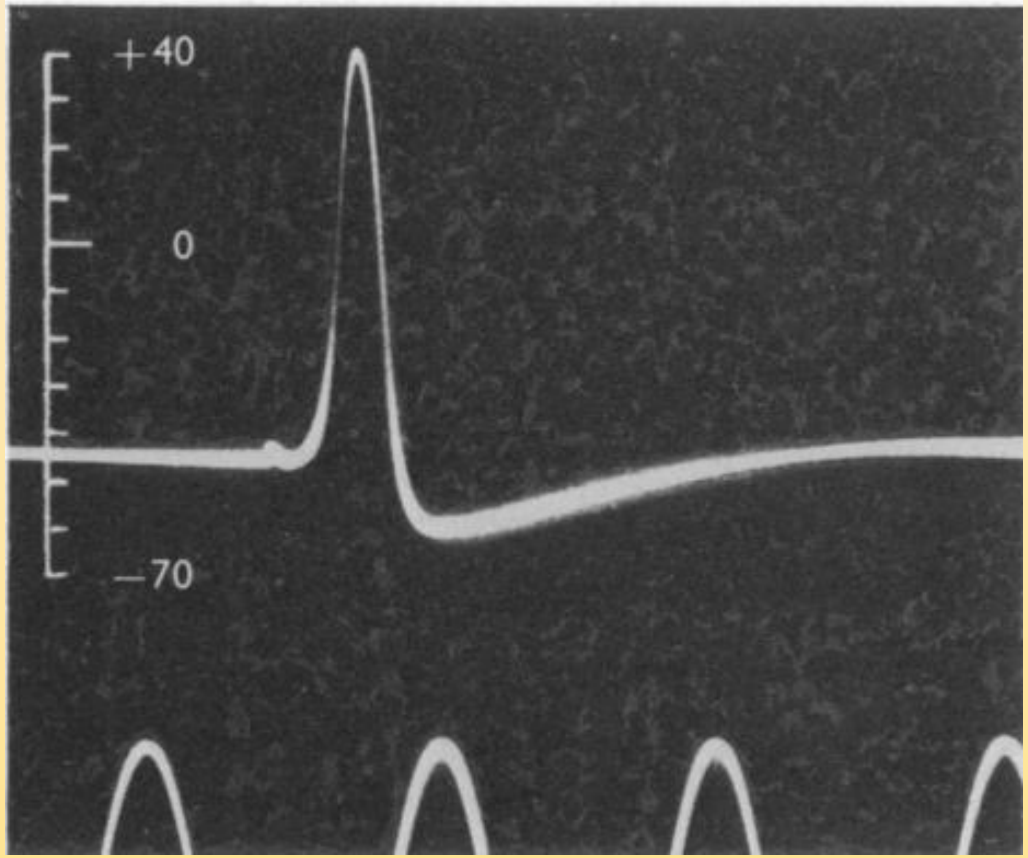
Hodgkin-Huxley Equations

$$\frac{dV}{dt} = -\frac{1}{C_M}\left[\bar{g}_K n^4 (V - E_K) + \bar{g}_{Na} m^3 h (V - E_{Na}) + \bar{g}_l (V - E_l) - I\right]$$

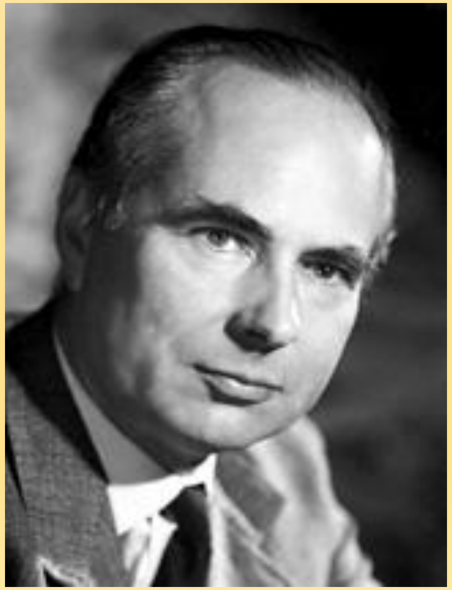
$$\frac{dn}{dt} = \left[\frac{0.01\,(V + 10)}{\left(\exp\frac{V + 10}{10} - 1\right)}(1 - n)\right] - \left[0.125\,\exp\left(\frac{V}{80}\right) n\right]$$

$$\frac{dm}{dt} = \left[\frac{0.1\,(V + 25)}{\left(\exp\frac{V + 25}{10} - 1\right)}(1 - m)\right] - \left[4\exp\left(\frac{V}{18}\right) m\right]$$

$$\frac{dh}{dt} = \left[0.07\exp\left(\frac{V}{20}\right)(1 - h)\right] - \left[\frac{1}{\left(\exp\frac{V + 30}{10} + 1\right)} h\right]$$



Alan Hodgkin



Andrew Huxley

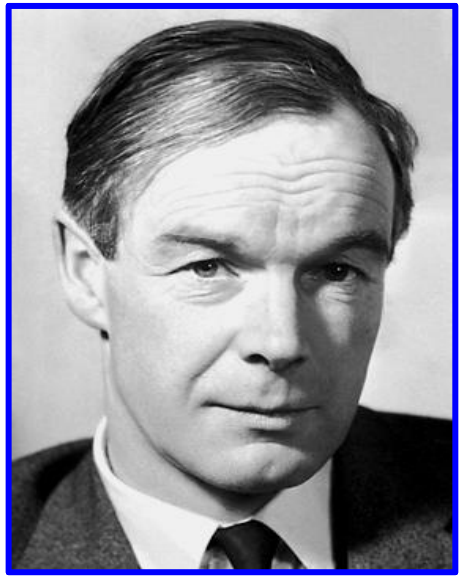
A. L. Hodgkin and A. F. Huxley

Action Potentials Recorded from inside the Nerve Fibre,

Nature, Lond. 144, 710-711, 1939

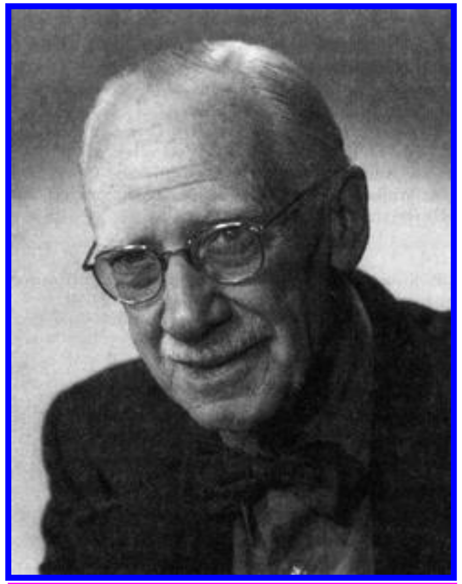
Anomalous ***Inductance and Diodes***

in the Brain



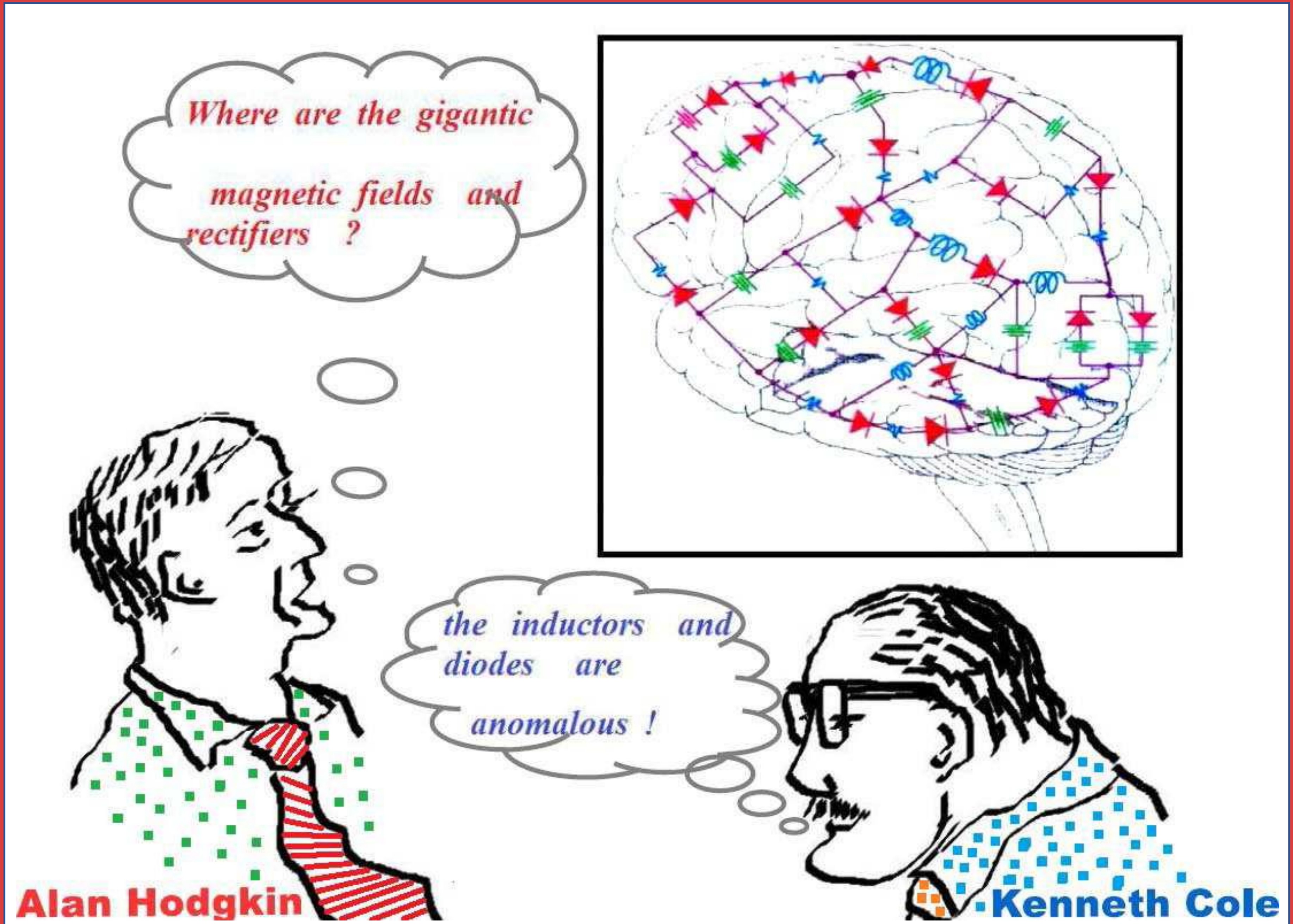
Alan L. Hodgkin

Hodgkin and Cole
have independently
measured from the
squid axon membrane
gigantic inductances
and ***DC V-I Curves***
that resemble
rectifying diodes.



Kenneth S. Cole

Po_19-189

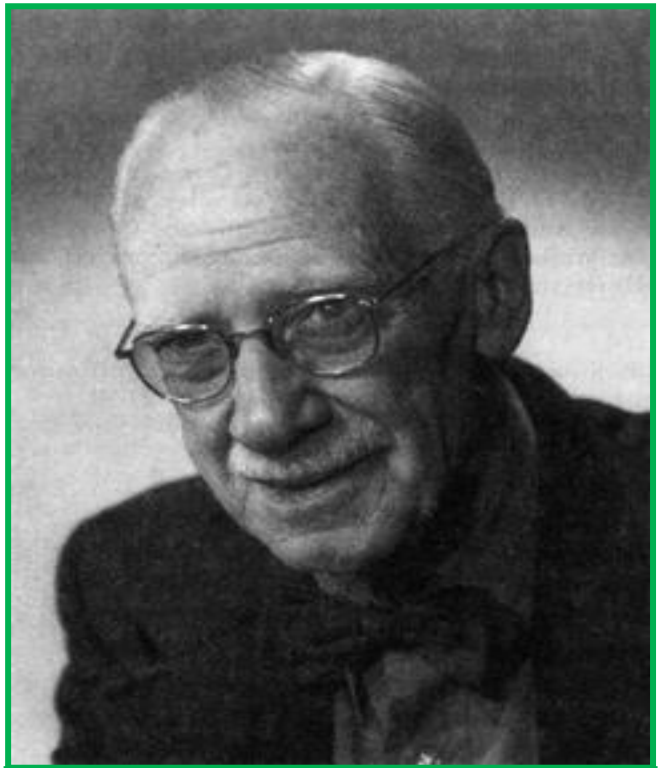

Where are the gigantic
magnetic fields and
rectifiers ?
the inductors and
diodes are
anomalous !
Alan Hodgkin
Kenneth Cole

Po_19-192

Po_19-

Po_19-191

The suggestion of an inductive reactance anywhere in the system was shocking to the point of being unbelievable.



Kenneth Cole

Po_19-190

Hodgkin-Huxley Mystery

Although the Hodgkin-Huxley Equations have been around for more than 70 years, we still do not understand the complex nonlinear dynamics responsible for the generation of the ***action potentials*** (***spikes***).

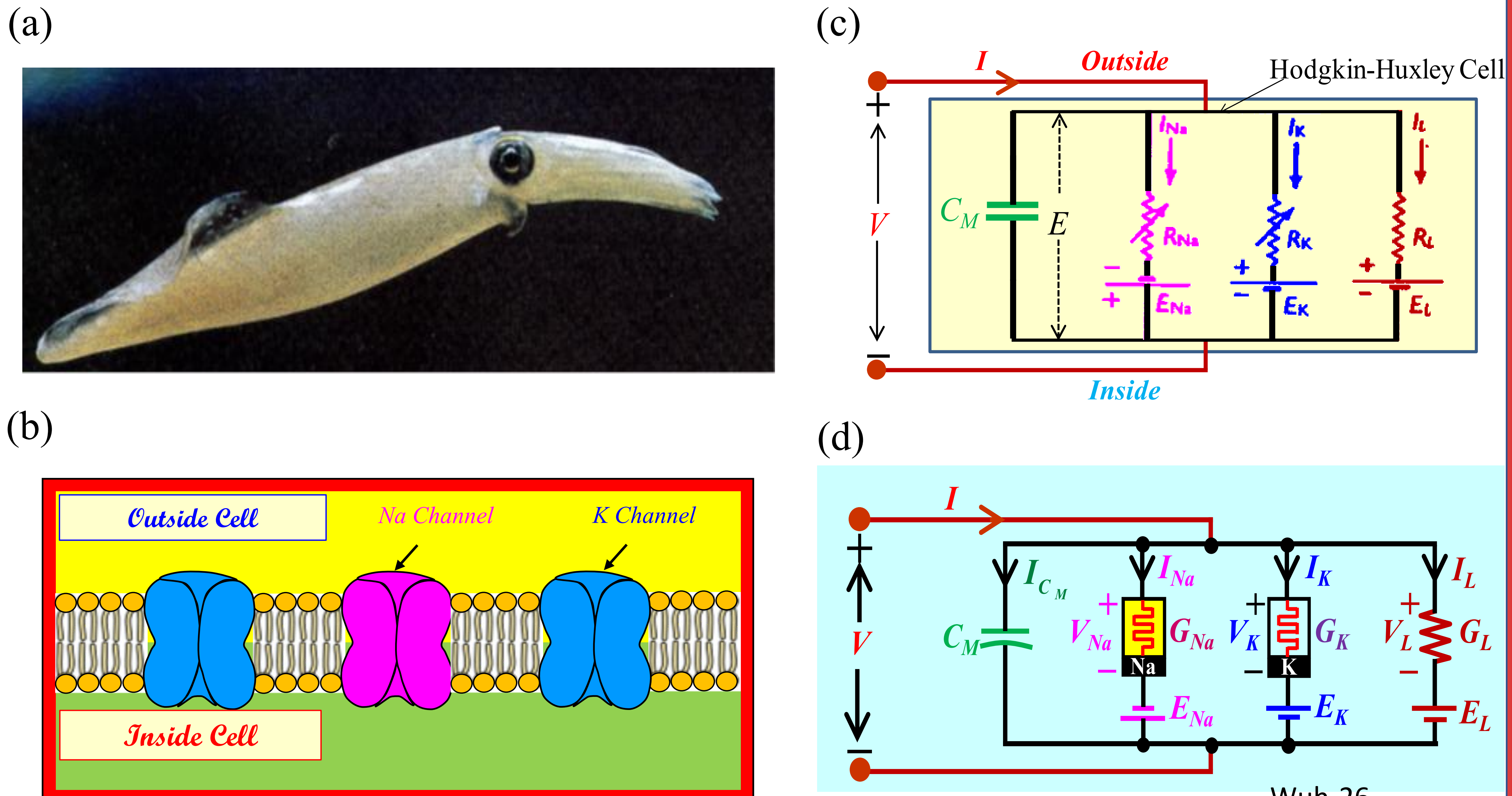
SM-12

Hodgkin's Blunder

Hodgkin had struggled in vain searching for a *physical* interpretation of the *squid axon inductance*. He failed because he had mistaken the *axon* for a *time-varying conductance*, when in fact it has a simple explanation if the *Potassium* and *Sodium ion channels* are identified as *memristors*.

A. L. Hodgkin,
"The ionic basis of electrical activity in nerve and muscle,"
Biological Review, Vol. 26, pp. 339-409, 1951

Po_19-193


(a)
(c)
I
Outside
Hodgkin-Huxley Cell
+
V
−
C_M
E
I_{Na}
R_{Na}
E_{Na}
I_K
R_K
E_K
I_L
R_L
E_L
Inside
(b)
Outside Cell
Na Channel
K Channel
Inside Cell
(d)
I
V
I_{C_M}
C_M
I_{Na}
V_{Na}
G_{Na}
Na
E_{Na}
I_K
V_K
G_K
K
E_K
I_L
V_L
G_L
E_L

Wuh-26

International Journal of Bifurcation and Chaos, Vol. 22, No. 4
(2012) 1250098 (49 pages)

NEURONS ARE POISED NEAR THE EDGE OF CHAOS

Leon Chua, Valery Sbitnev, Hyonsuk Kim

Crisis in Circuit Theory

Pre-1970 Definitions *of the*

3 Basic Circuit Elements

Capacitors, Resistors, and Inductors

give unrealistic or incorrect circuit solutions

when the elements are

time-varying or nonlinear

Po_19-26

To Recover from

the perfect storm

Capacitors, Resistors, Inductors

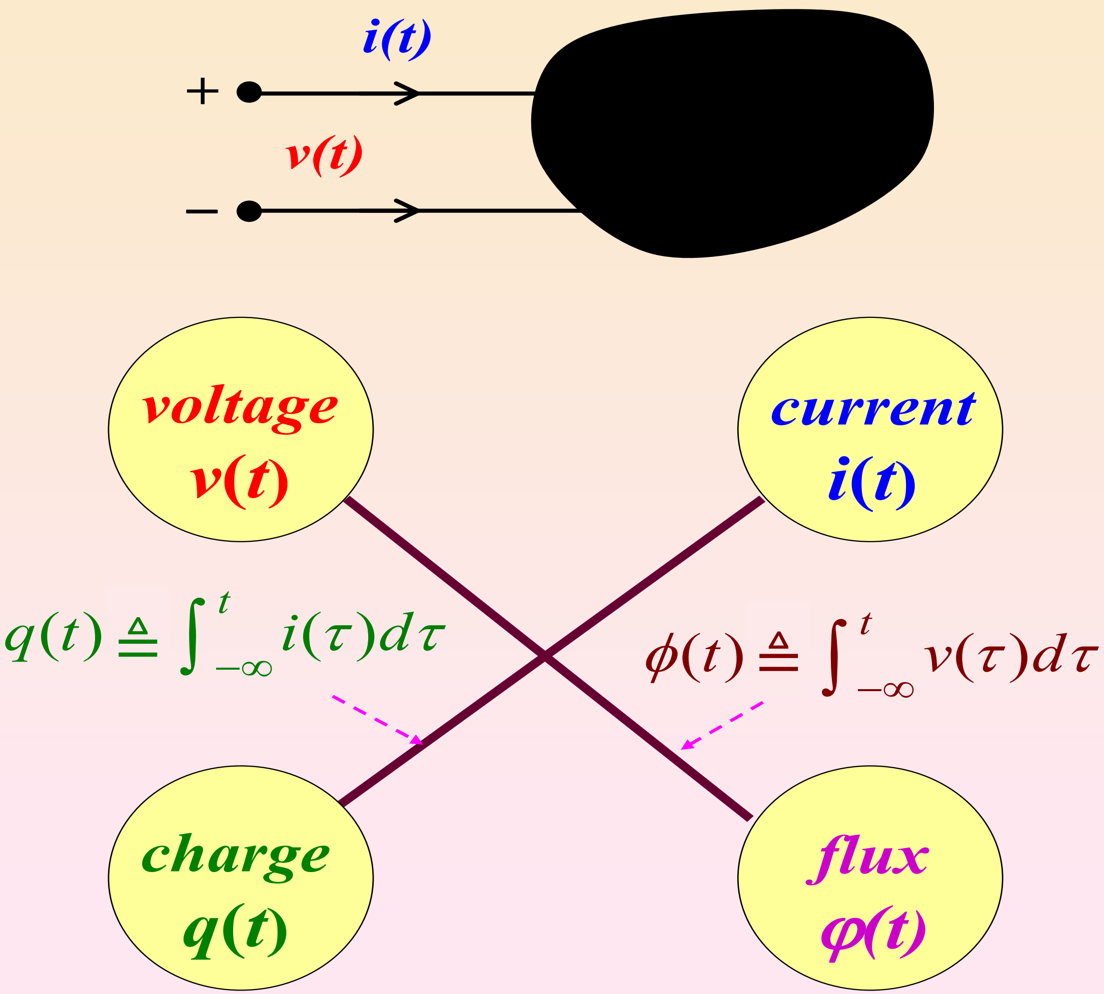
must be

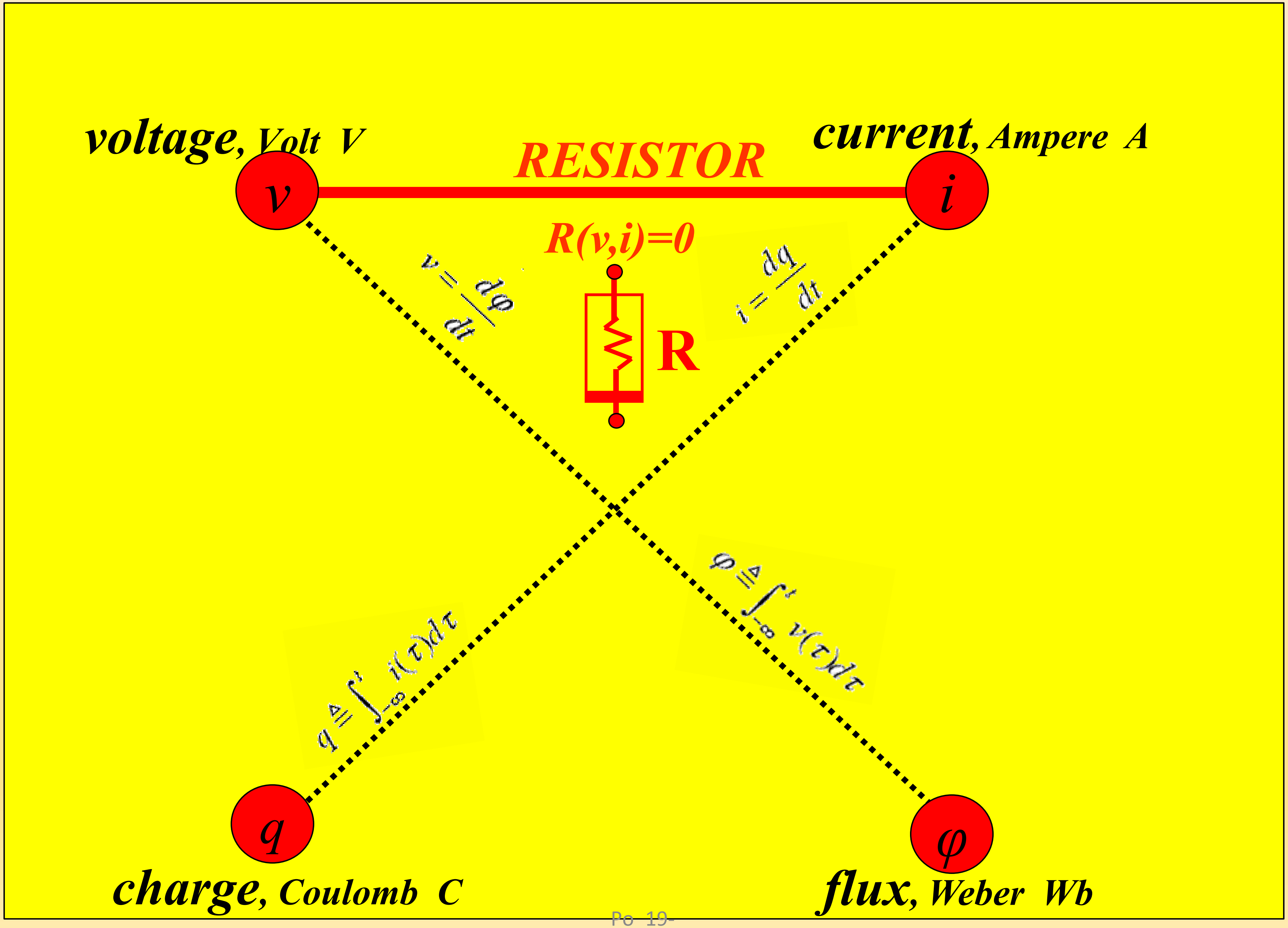
redefined

via an

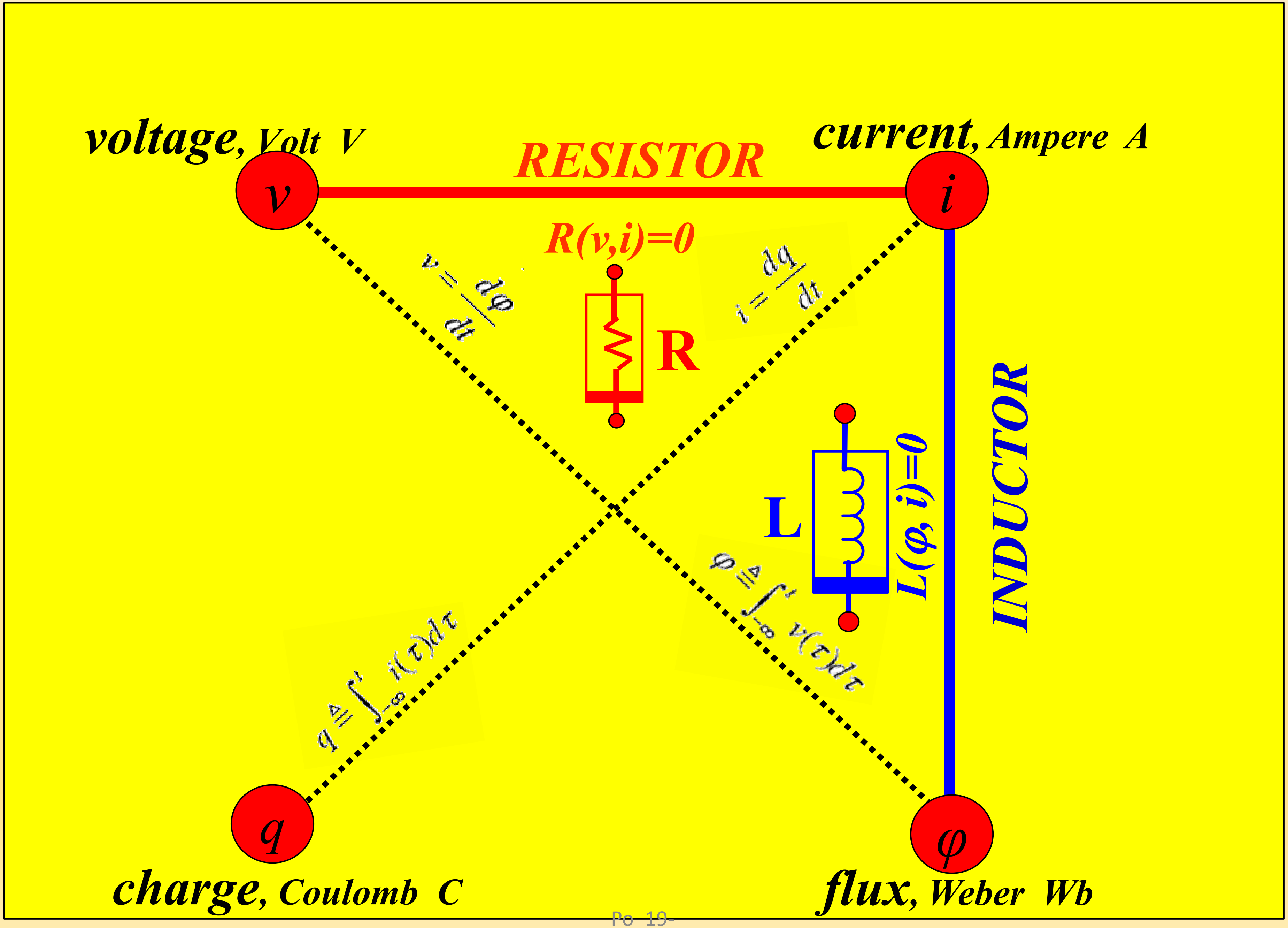
AXIOMATIC APPROACH Po_19-30

Four Basic Circuit Variables



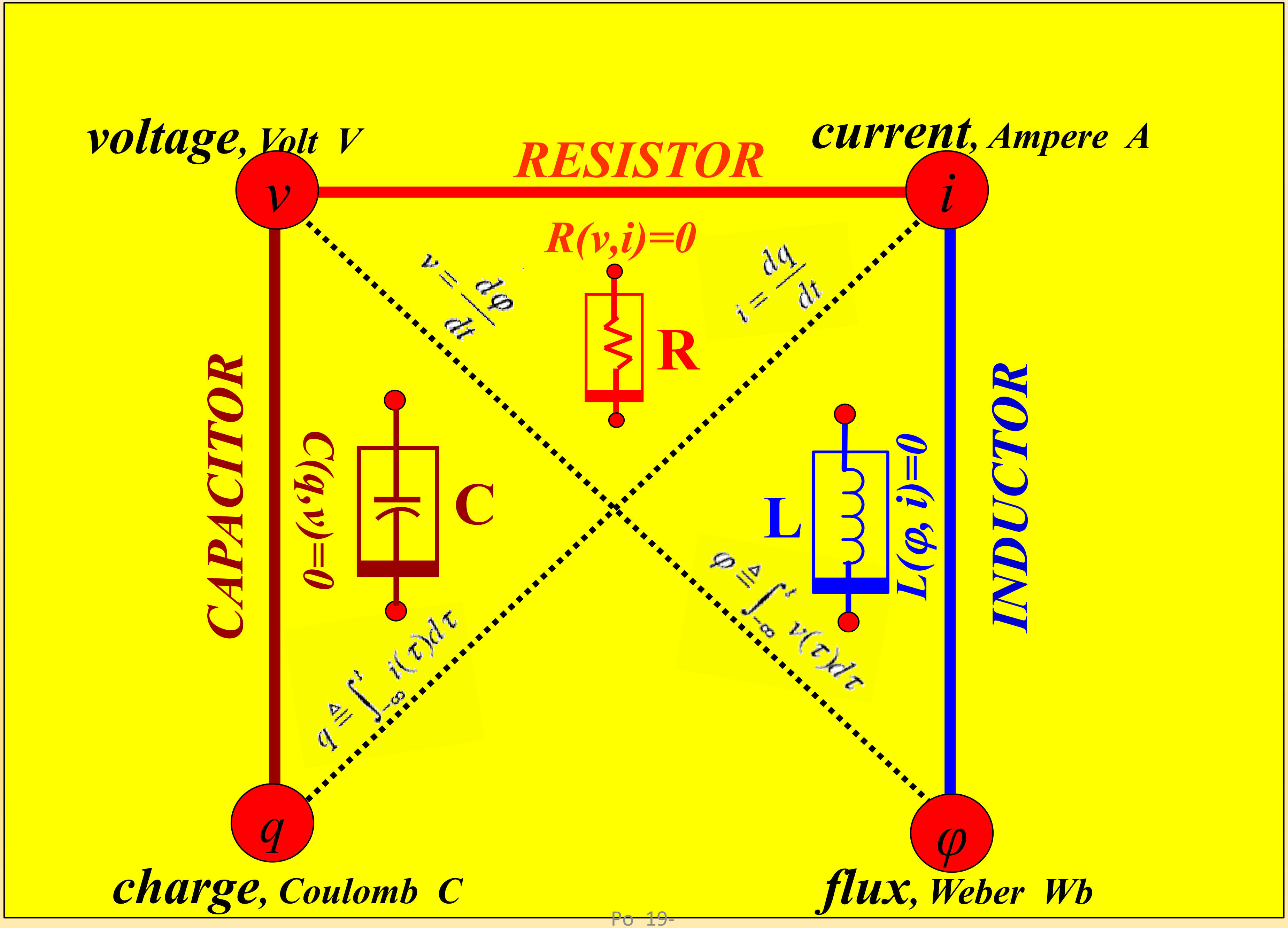

voltage, Volt V
RESISTOR
current, Ampere A
v
i
R(v,i)=0
R
q
φ
charge, Coulomb C
flux, Weber Wb

Po_19-35



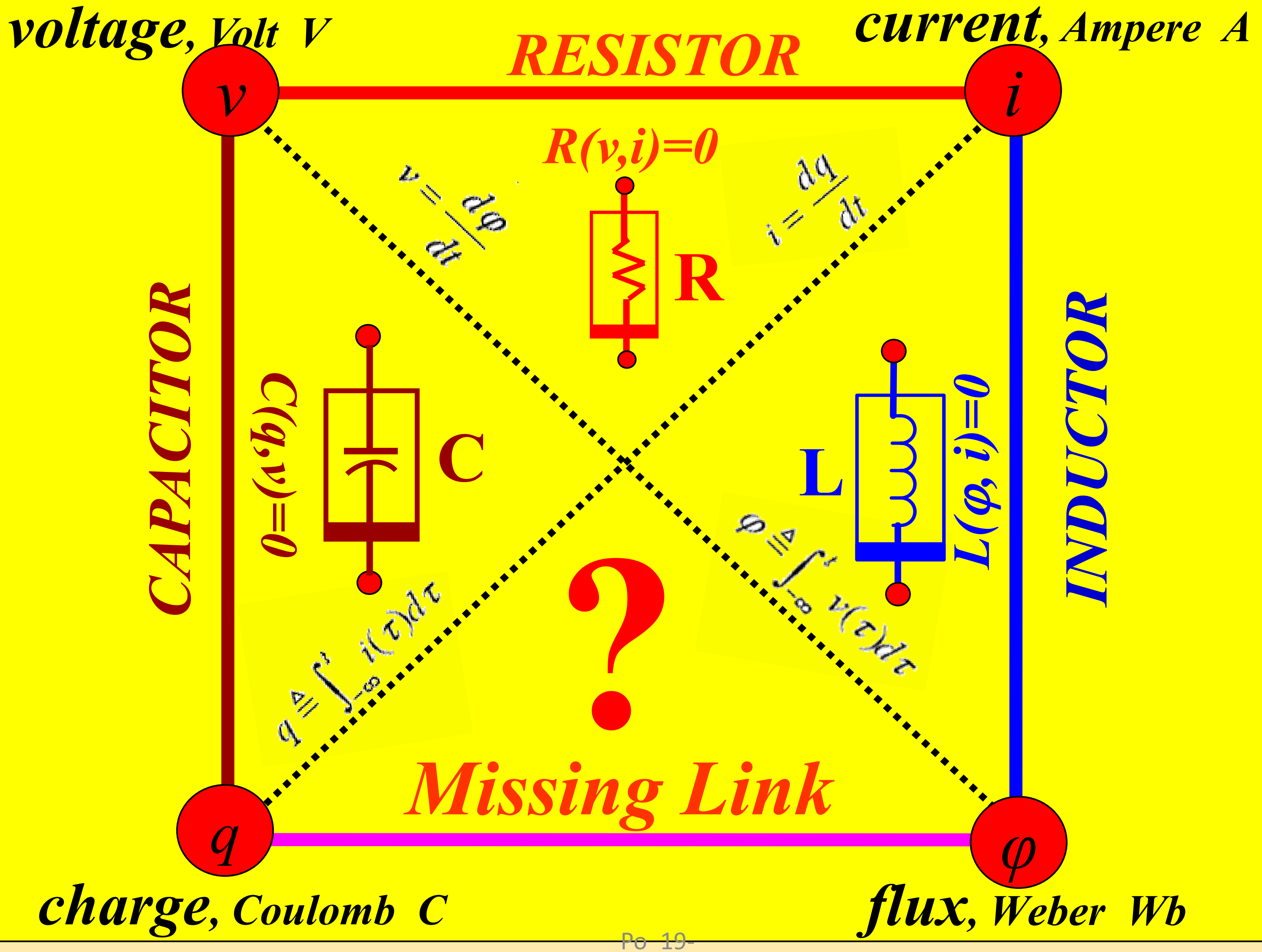
voltage, Volt V
RESISTOR
current, Ampere A
v
i
R(v,i)=0
R
L
L(φ, i)=0
INDUCTOR
q
φ
charge, Coulomb C
flux, Weber Wb

Po_19-36


voltage, Volt V
current, Ampere A
RESISTOR
v
i
R(v,i)=0
R
v = dφ/dt
i = dq/dt
CAPACITOR
C(q,v)=0
C
L
L(φ,i)=0
INDUCTOR
q ≜ ∫_{-∞}^{t} i(τ)dτ
φ ≜ ∫_{-∞}^{t} v(τ)dτ
q
φ
charge, Coulomb C
flux, Weber Wb
Po_19-

Po_19-37

4 Basic Circuit Elements

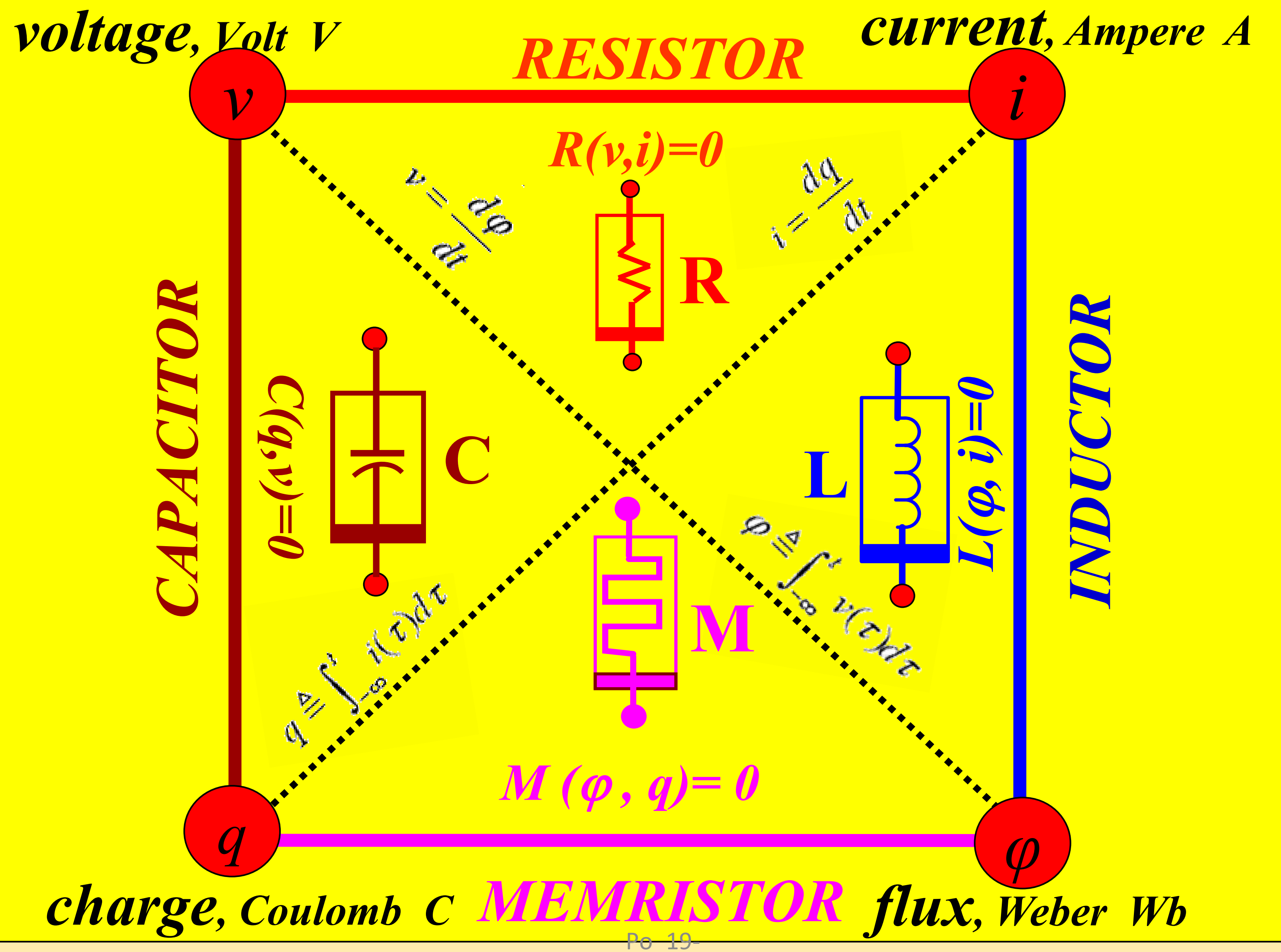


Po_19-38

The missing circuit element is the memristor !

Po_19-39

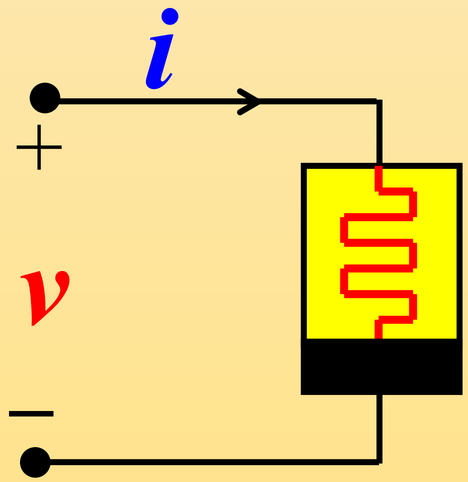
4 Basic Circuit Elements



What is a *MEMRISTOR* ?

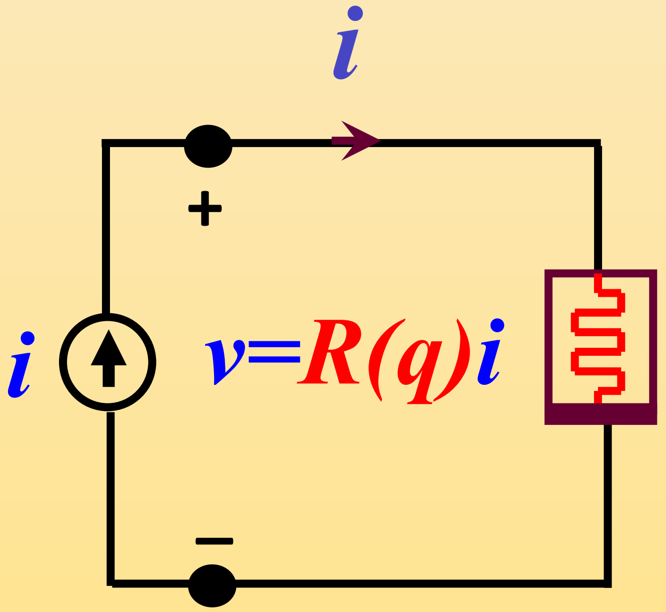
A **MEMRISTOR** is a 2-terminal device whose resistance can be tuned to some desired value by applying a voltage signal, and which **remembers** its resistance after the voltage source is disconnected.

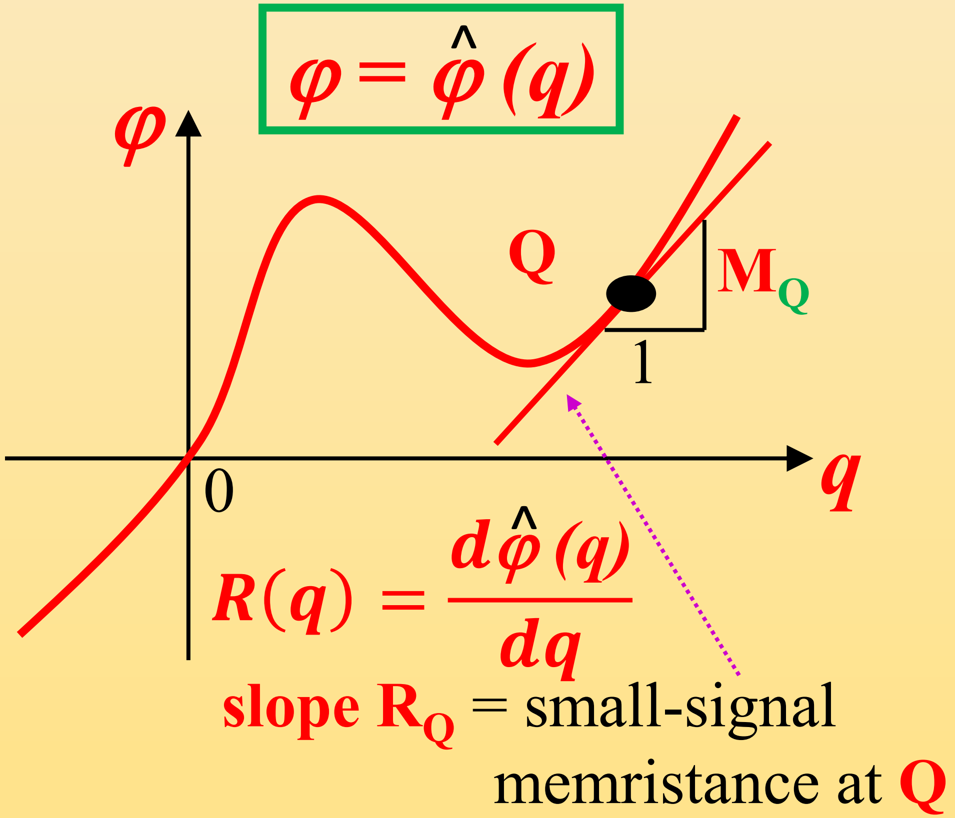
It remembers its past!



Po_19-12

What is a Memristor ?





At any point **Q** the memristor obeys the **Ohm's Law**:

$$v = R(q)\, i$$

SM-15

Memristor

is

not

a **switch !**

Memristor

is

a

continuum

Dalian 23

Memristor

is an acronym for

Memory Resistor

Po_19-42

Two *memristors* suffice to compute all Boolean functions

From:

E. Lehtonen, J.H. Poikonen, and M. Laiho

Electronic Letters, vol. 3, pp. 239, 2010.

SM-20

IEEE TRANSACTIONS ON CIRCUIT THEORY, VOL. CT-18, NO. 5, SEPTEMBER 1971 507

Memristor—The Missing Circuit Element

LEON O. CHUA, SENIOR MEMBER, IEEE

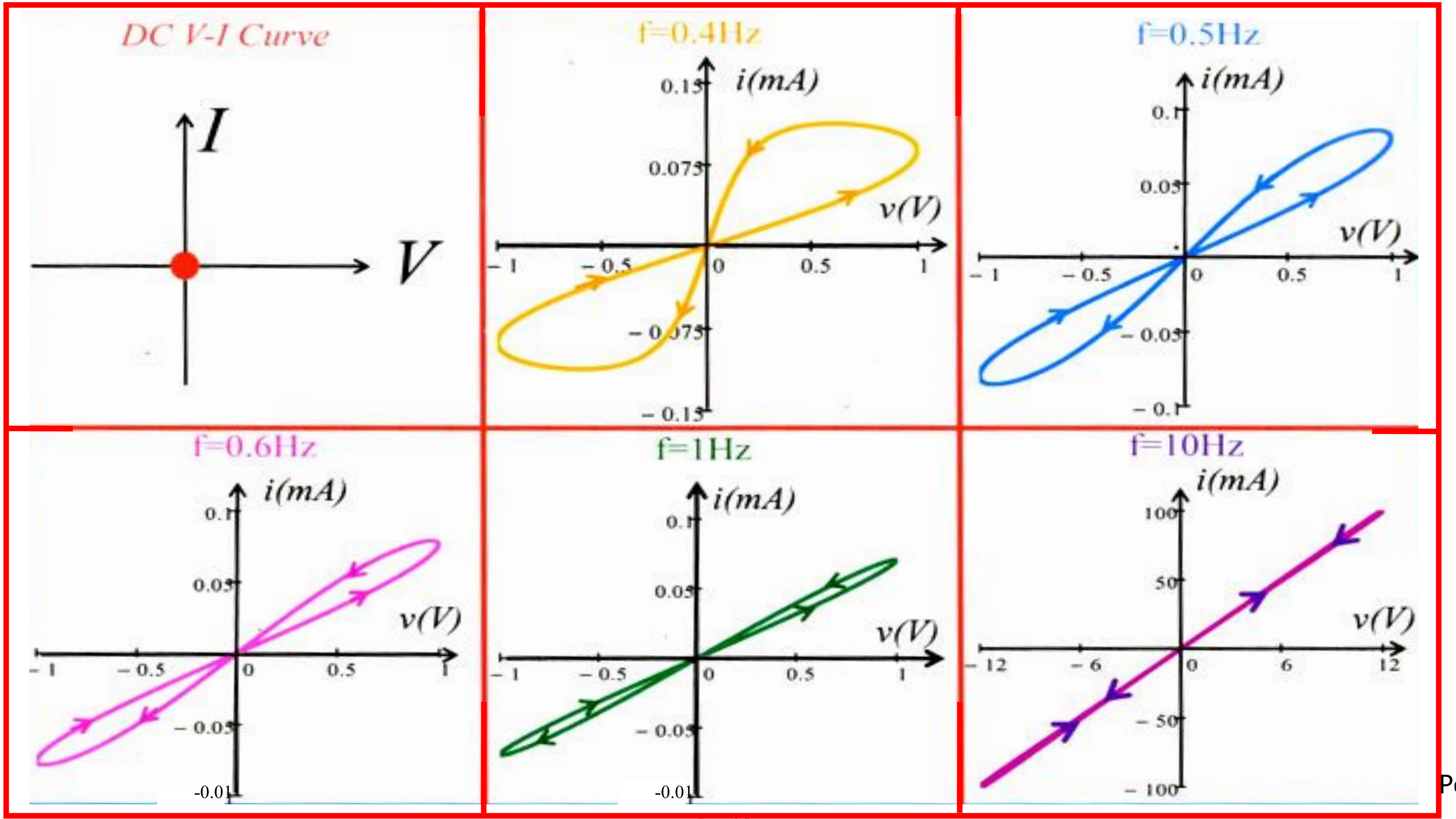
From:
Leon O. Chua,
"Memristor - The Missing Circuit Element,"
IEEE Transaction on Circuit Theory,
Vol. CT-18, No. 5, pp. 507-519, 1971

Nature **453**, 80–83 (2008). https://doi.org/10.1038/nature06932

The Missing Memristor Found

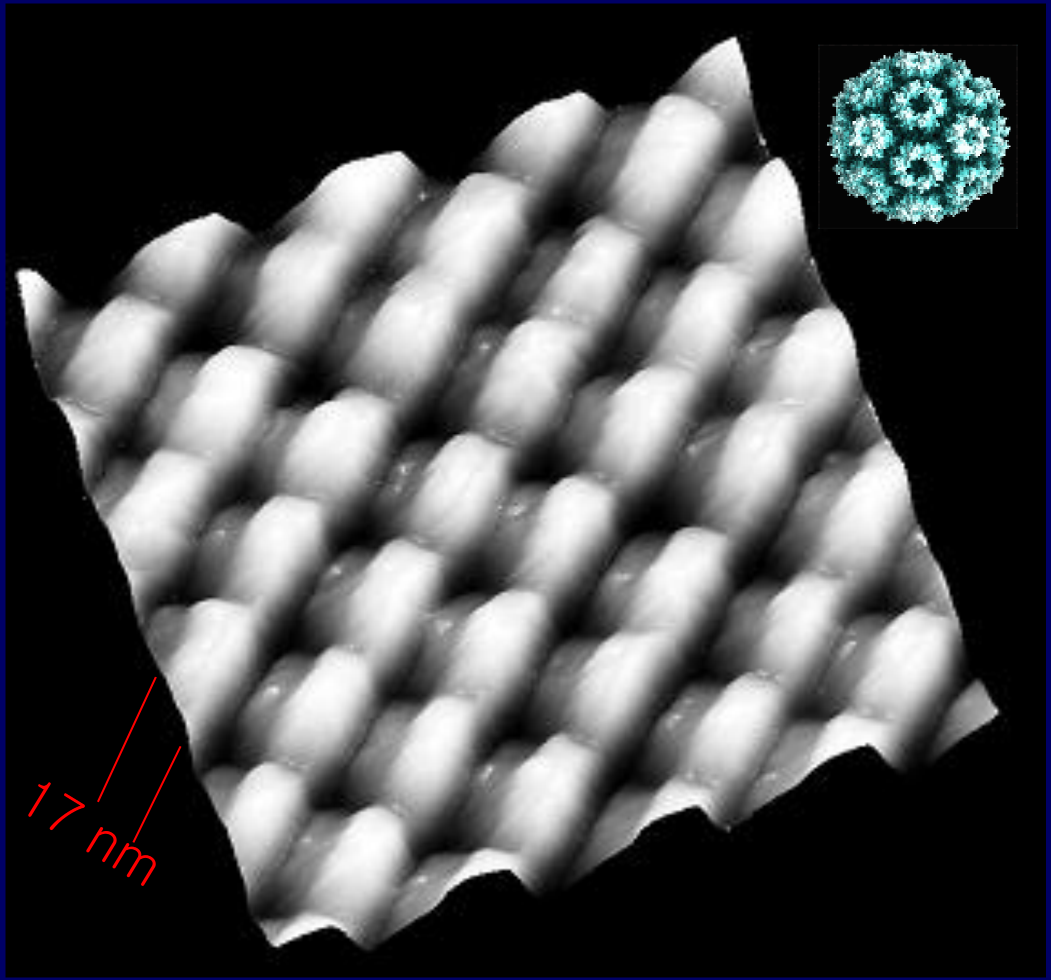
Dmitri B. Strukov, Gregory S. Snider, Duncan R. Stewart and R. Stanley Williams

HP Memristor



Po_19-49

AFM image of memristor crossbar at 17nm

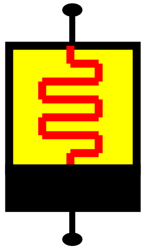


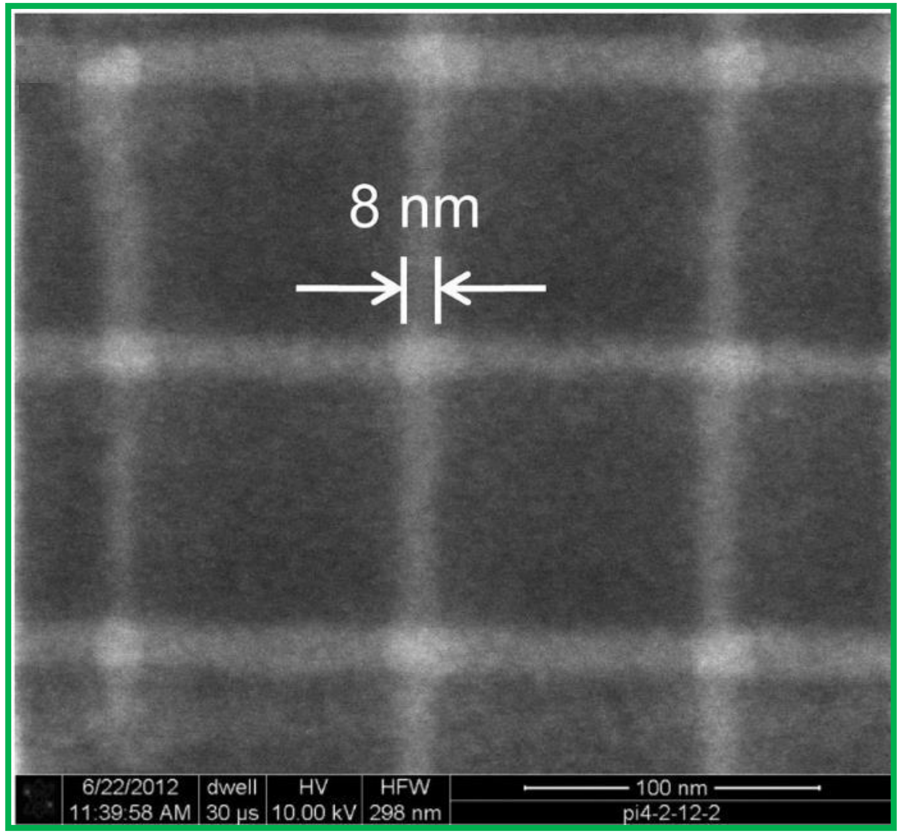
The smallest virus:
~25 nm in dia

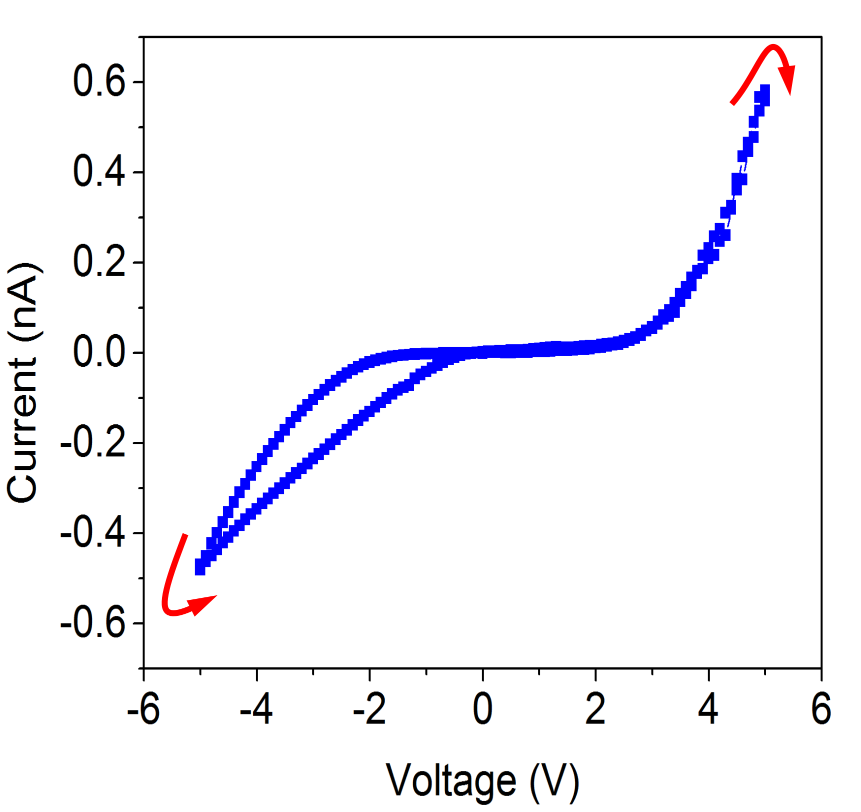
Fabricated by
Imprint Lithography –
Can be stacked for
Higher area density

(cell density : 100 Gbit/cm^2)

An 8 nm Memristor







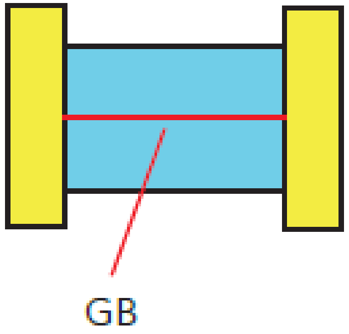
From:
S. Pi, P. Lin, Q. Xia, "Cross point arrays of 8 nm × 8 nm memristive devices fabricated with nano imprint lithography", J. Vac. Sci. Technol. B 31, 06FA02-1 - 06FA02-6, 2013

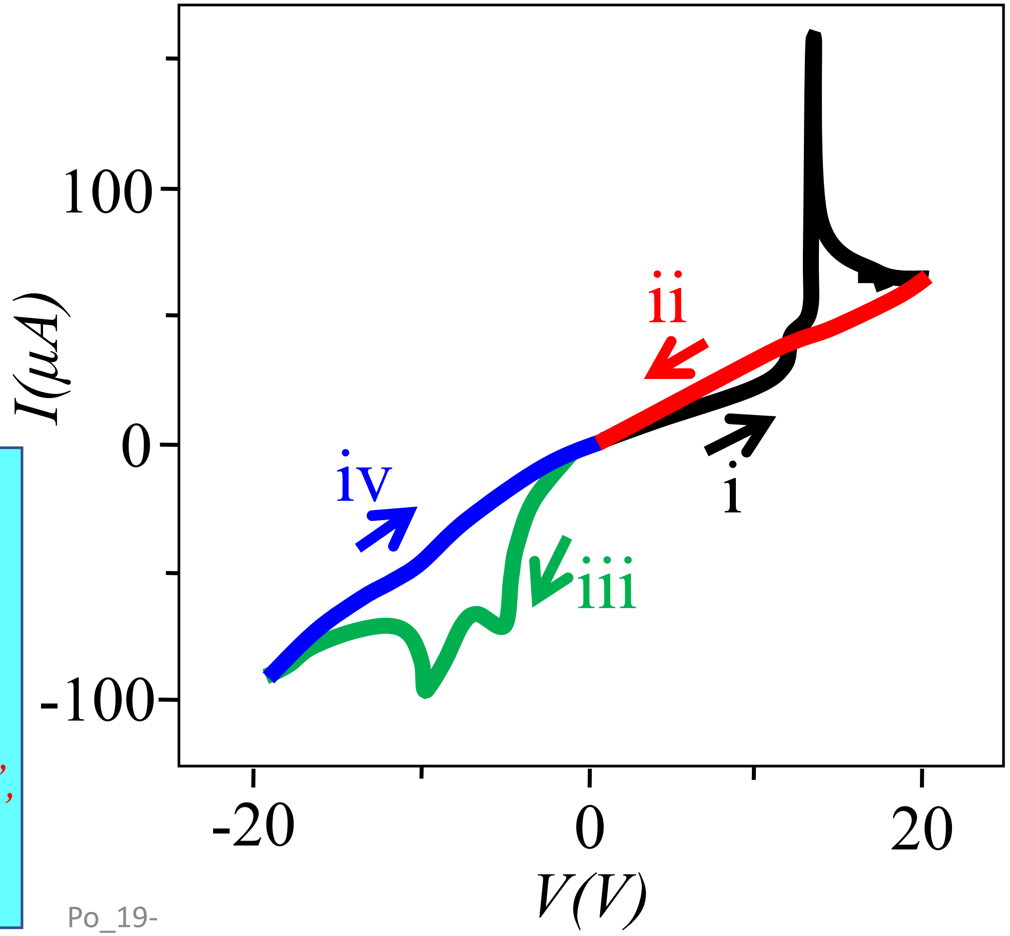
Po_19-

Memristor

made from

A single Layer of the Molecule MoS_2



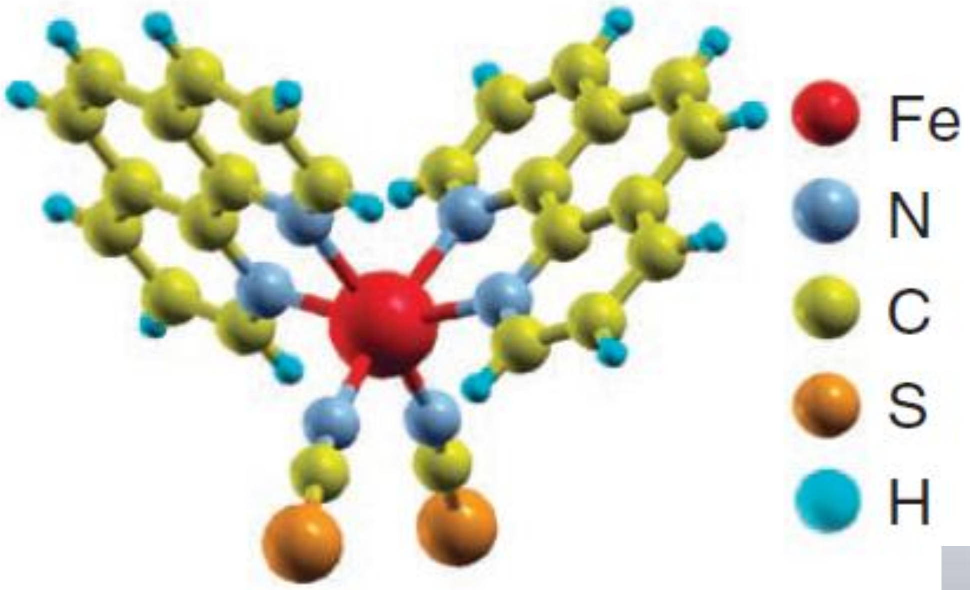


From:
V. K. Sangwan, D. Jariwala, I. S. Kim, K. S. Chen, T. J. Marks, L. J. Lauhon, M. C. Hersam, "Gate-tunable memristive phenomena mediated by grain boundaries in single-layer MoS_2", Nature Nanotechnology 10, p. 403-406, 2015

Po_19-

Po_19-52

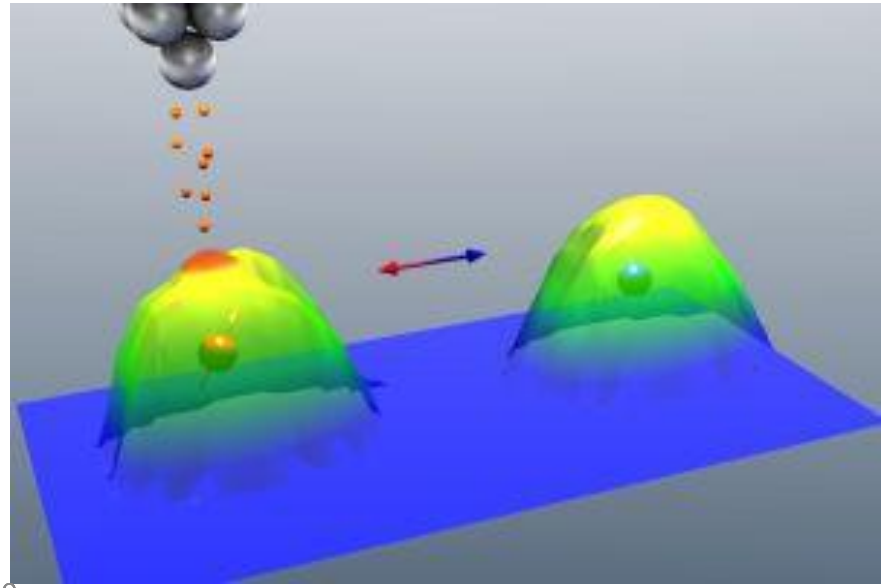
One-Molecule Memristor



Toshio Miyamachi et al.
" Robust spin crossover and memristance across a single molecule "
Nature Communications ***3***, 938, p.1-6, 2012

Nature: Molecule Changes Magnetism and Conductance

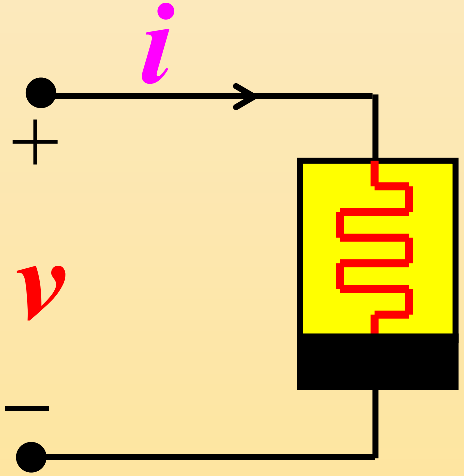
KIT Press Release No.116, July 2012
Karlsruhe Institute of Technology.



Karlsruhe Institute of Technology

Experimental Definition of the

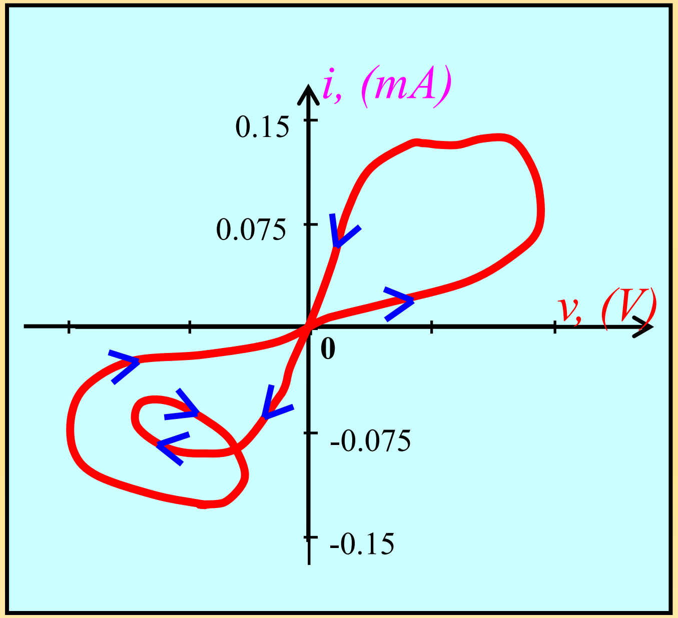
Memristor



If it's Pinched,

it's a

Memristor



Po_19-55

Some **Virtues** of ***MEMRISTORS*** ?

- ***Memory for free (i.e., no battery is needed to retain its memory)***
- ***Very High Speed (nano seconds)***
- ***Very Low Power (nano watts)***
- ***Very Small (sub-nanometer)***
- ***Great Data Retention***
- ***Great Endurance***
- ***Compatibility with CMOS Technology***

Po_19-13

Memristors Can Be Made from Many Different Materials

- RRAM *Memristors* (metal oxides Tio2, TaOx, etc.)
- Polymeric *Memristors* (conducting polymers)
- Ferroelectric *Memristors* (Ferroelectric films)
- Manganite *Memristors* (Perovskite manganite)
- Spintronic *Memristors* (spin-transfer torque magnetic layers)

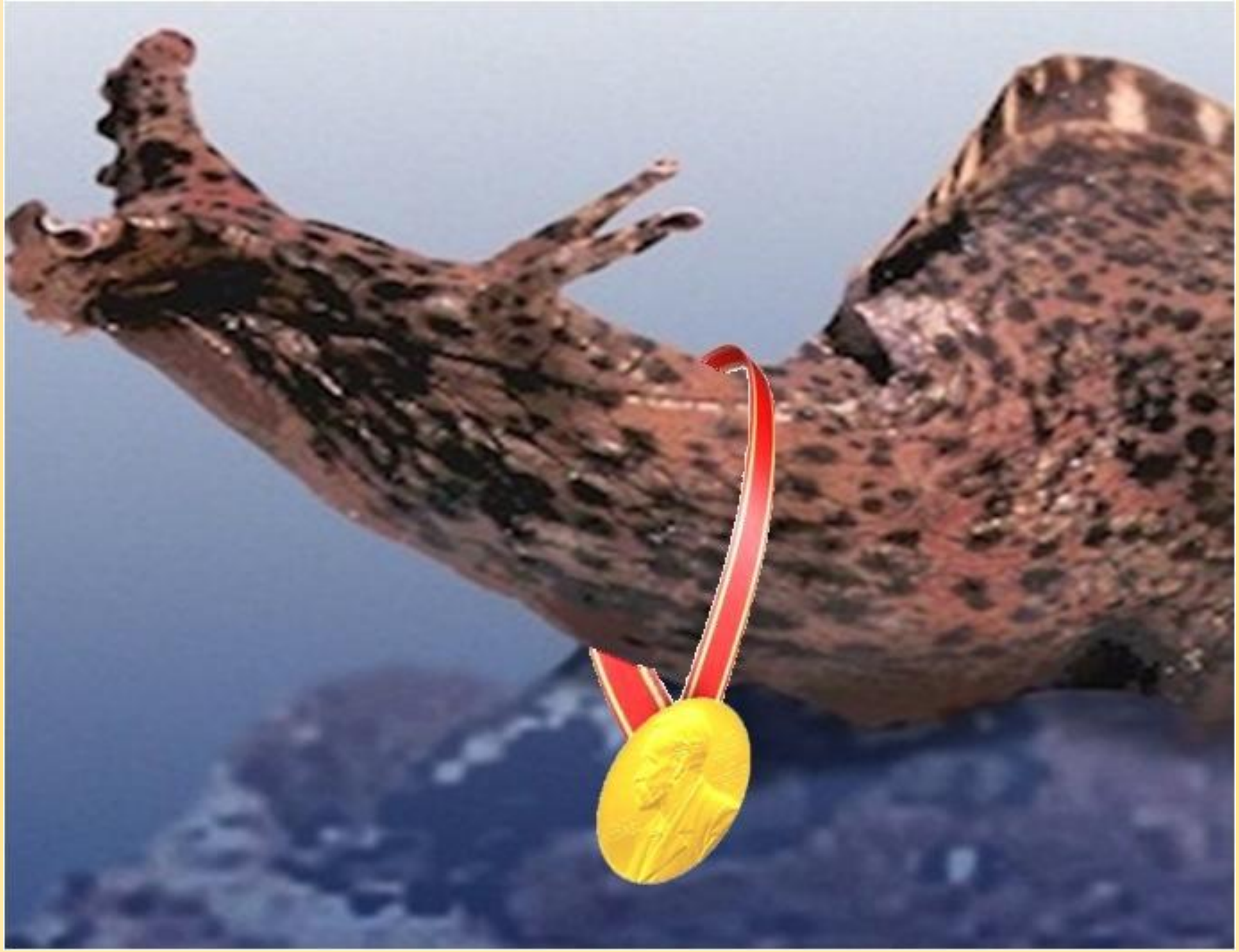
Po_19-58

Memristors fabrication

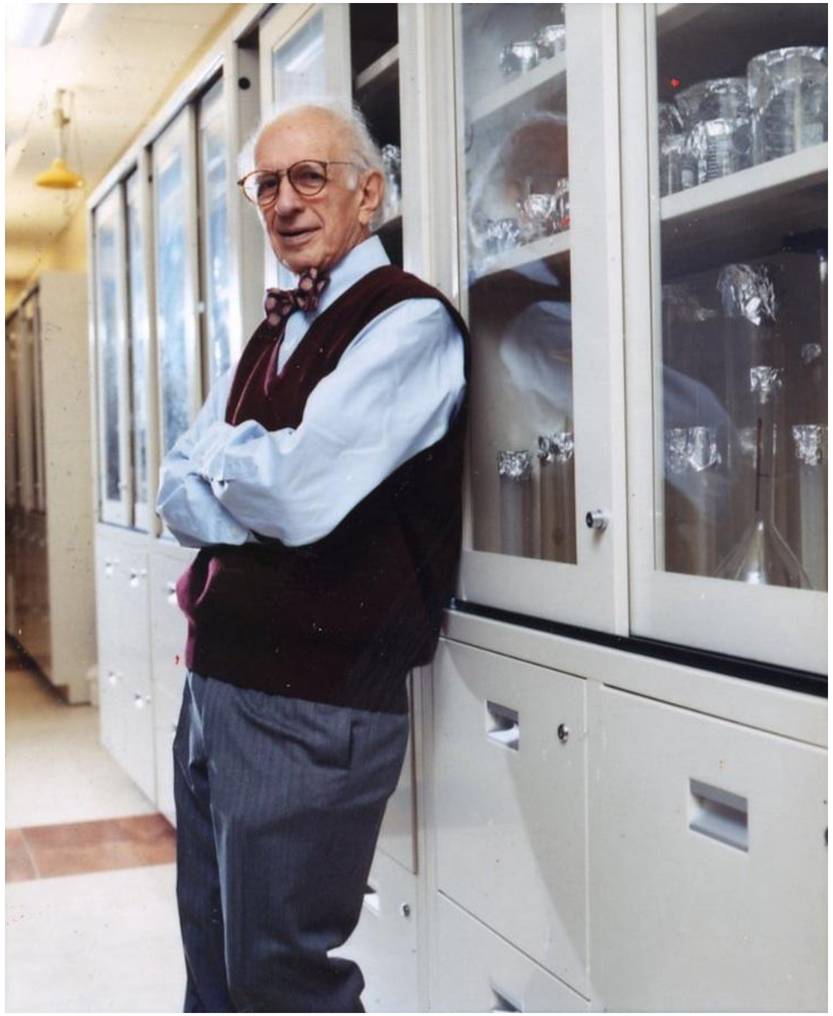
Everything **switches!**

Po_19-59

Aplysia honored with a Nobel Prize Medal

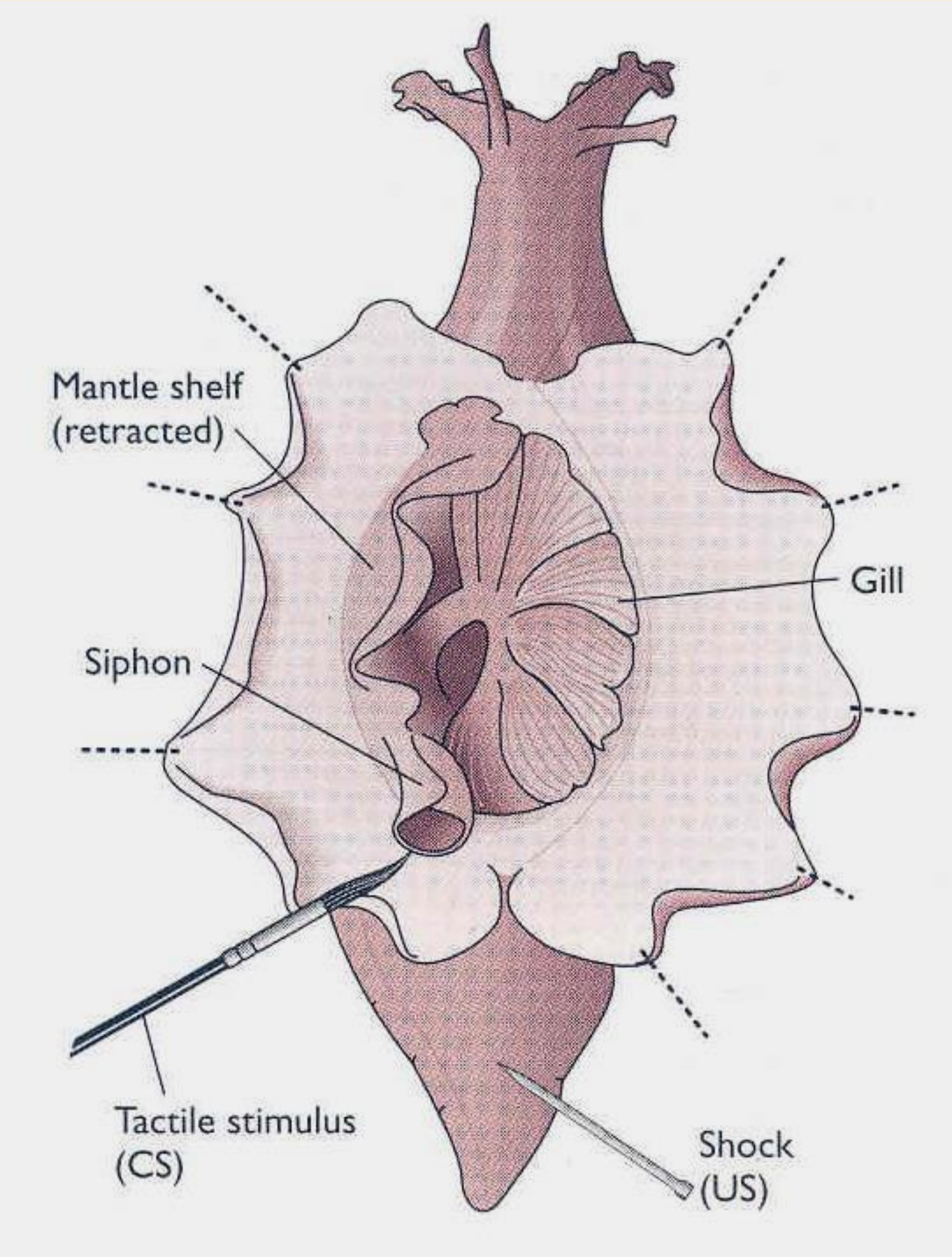


SM-6



Eric R. Kandel

Nobel Prize in Physiology or Medicine, 2000.

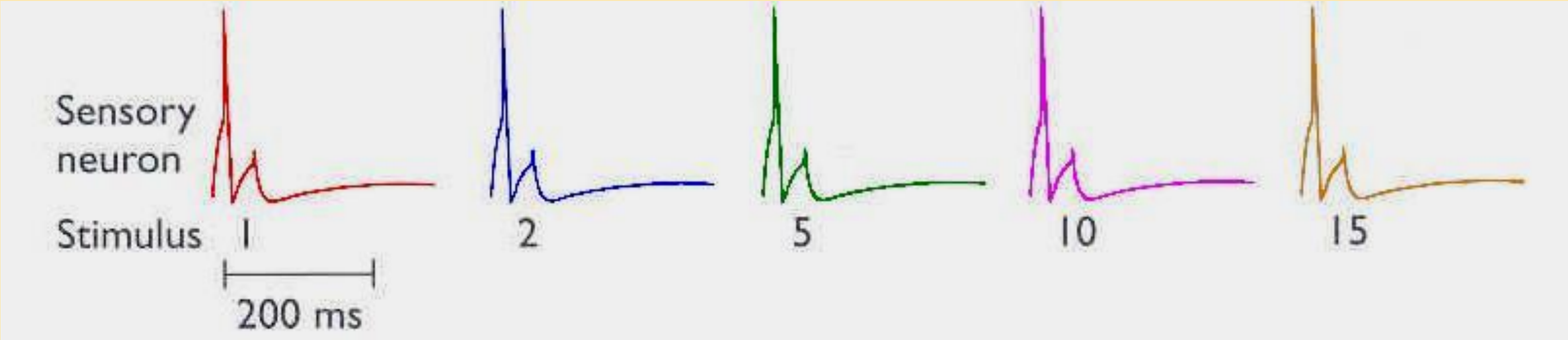


Mantle shelf
(retracted)
Gill
Siphon
Tactile stimulus
(CS)
Shock
(US)

SM-14

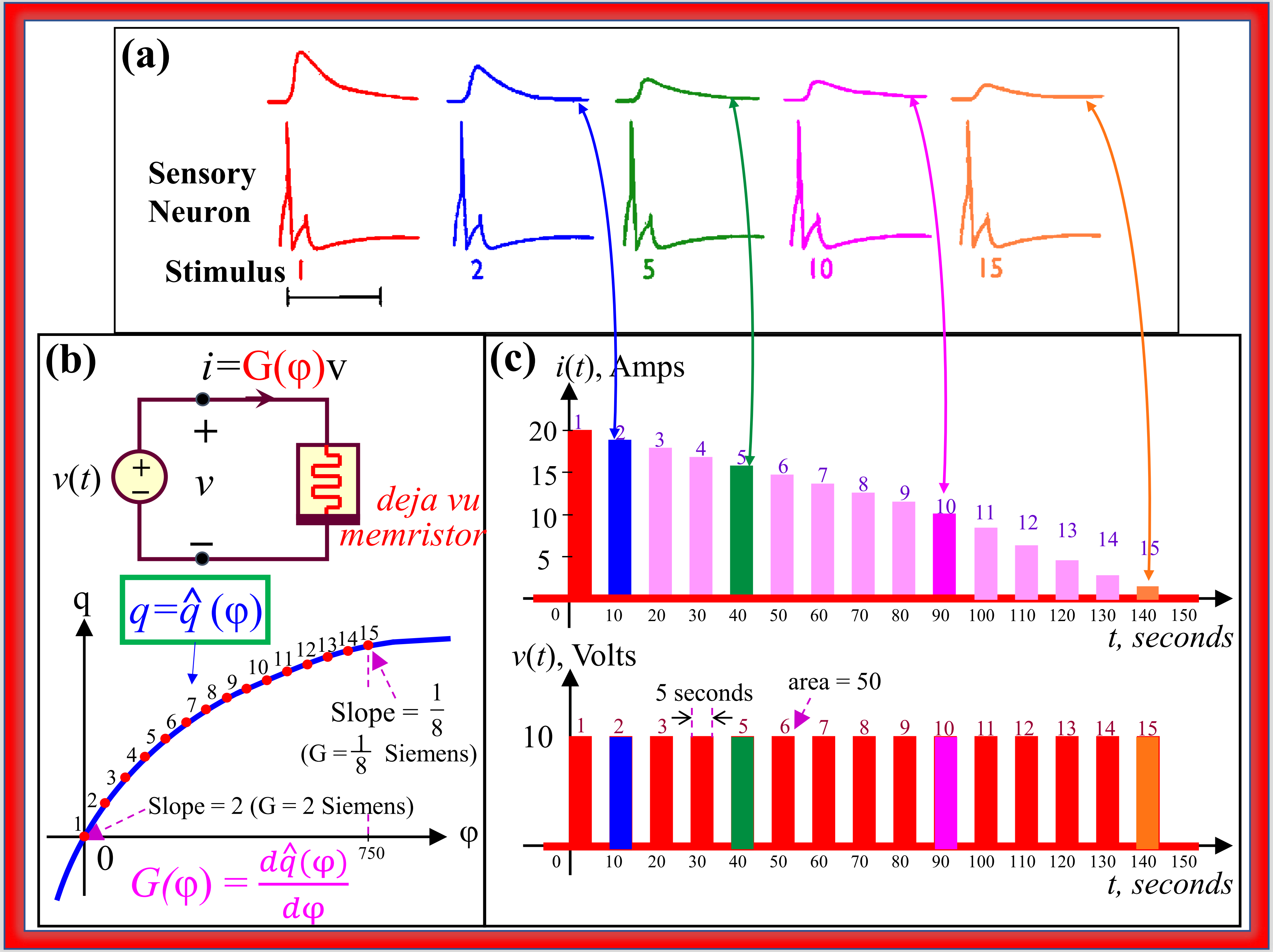
Example of Déjà vu Response of the Aplysia

Excitation

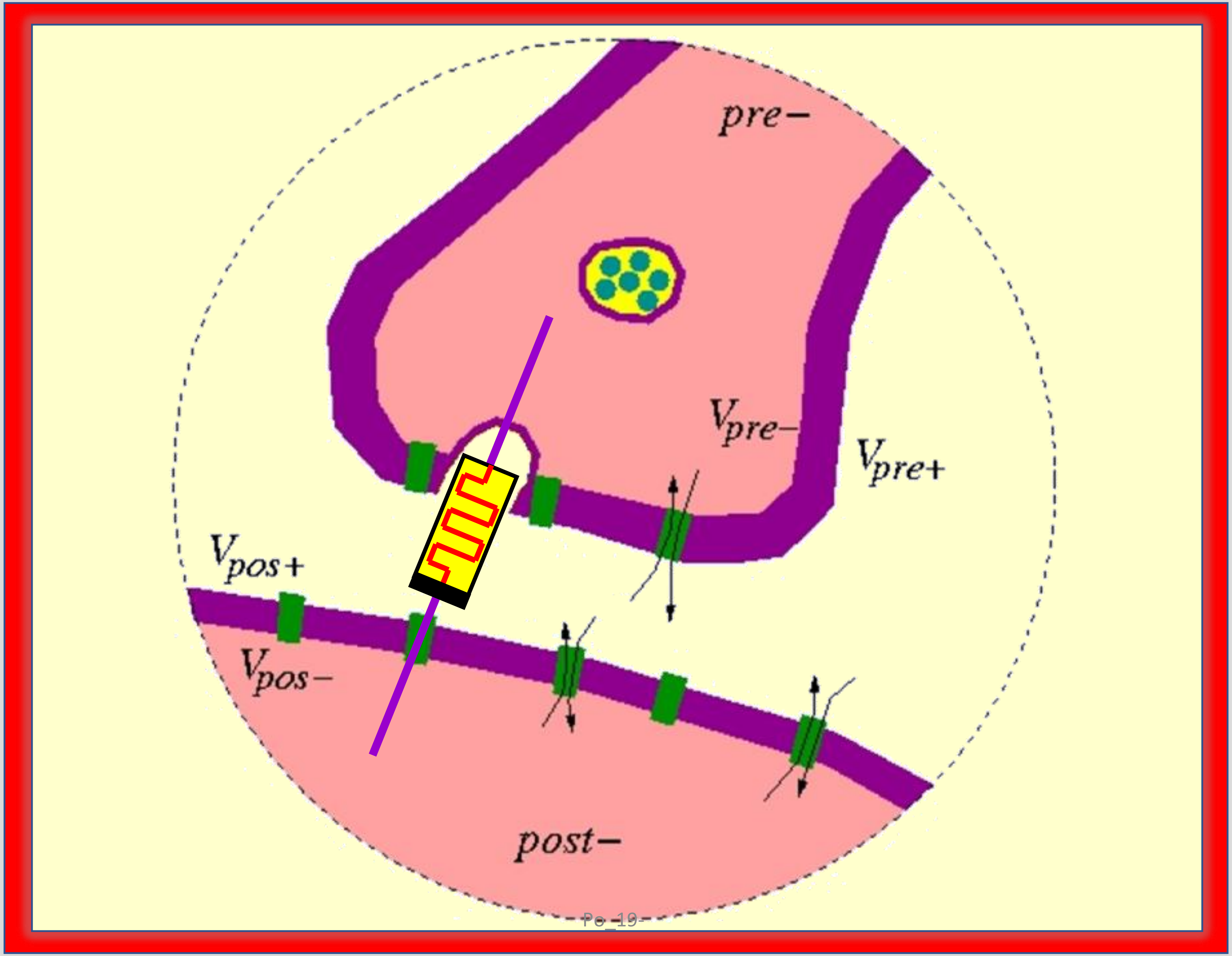


Response

SM-18



(a)
Sensory
Neuron
Stimulus
1
2
5
10
15
(b)
i=G(φ)v
v(t)
+
−
v
deja vu
memristor
q
q=q̂ (φ)
Slope = 1/8
(G = 1/8 Siemens)
Slope = 2 (G = 2 Siemens)
φ
0
750
G(φ) = dq̂(φ)/dφ
(c)
i(t), Amps
t, seconds
v(t), Volts
5 seconds
area = 50
10
t, seconds


pre−
Vpre−
Vpre+
Vpos+
Vpos−
post−

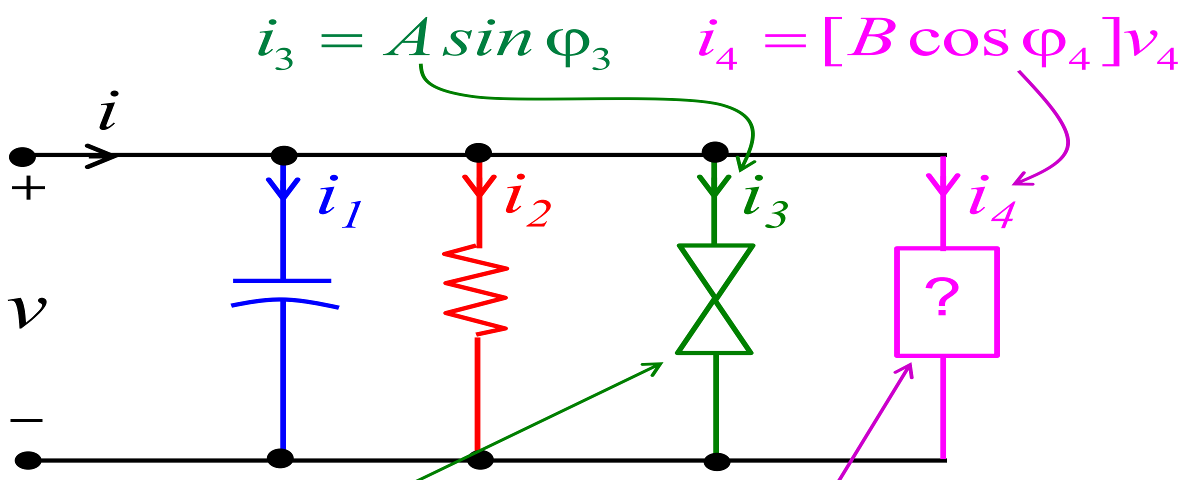
Synapses

are

Memristors!

Brian Josephson
1973 Nobel Prize in Physics:

JOSEPHSON JUNCTION CIRCUIT MODEL



2-terminal element to model the Josephson Pair-tunneling current

2-teminal element to model the Quasi-Particle Pair interference current

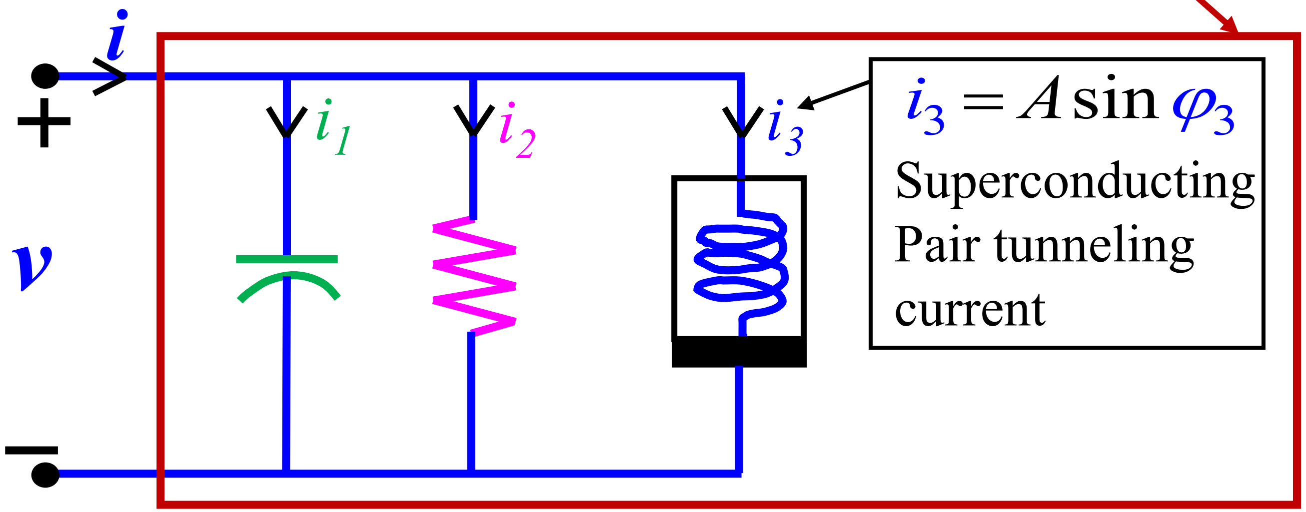
B.D. Josephson

Nobel Prize in Physics

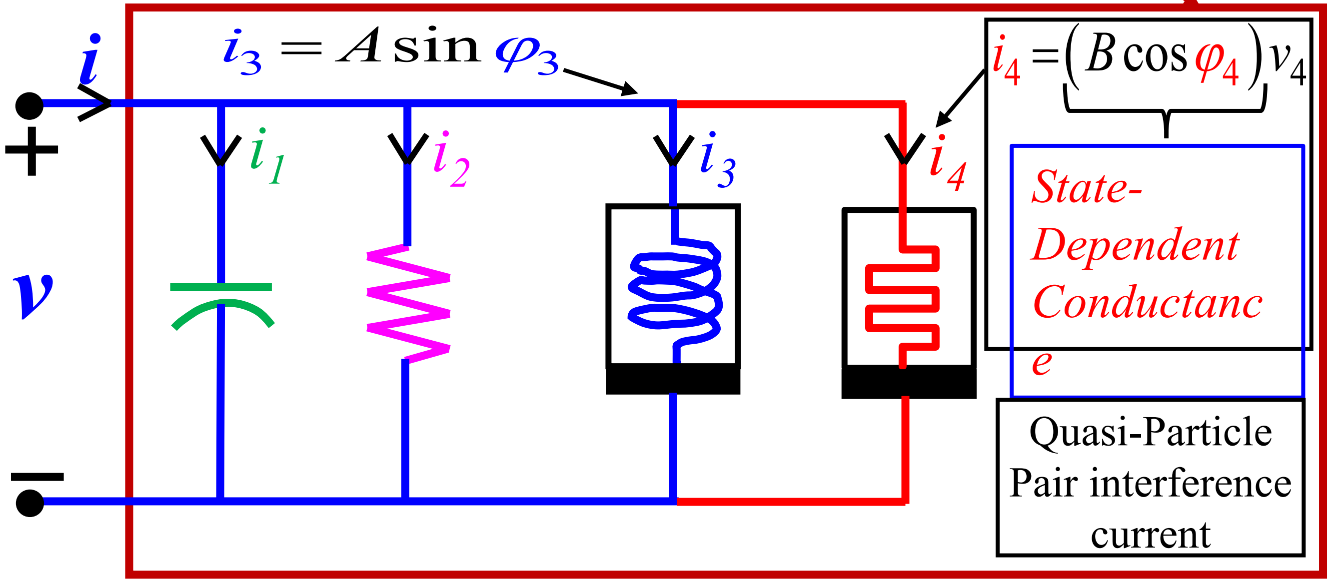
(1973)

Po_19-92

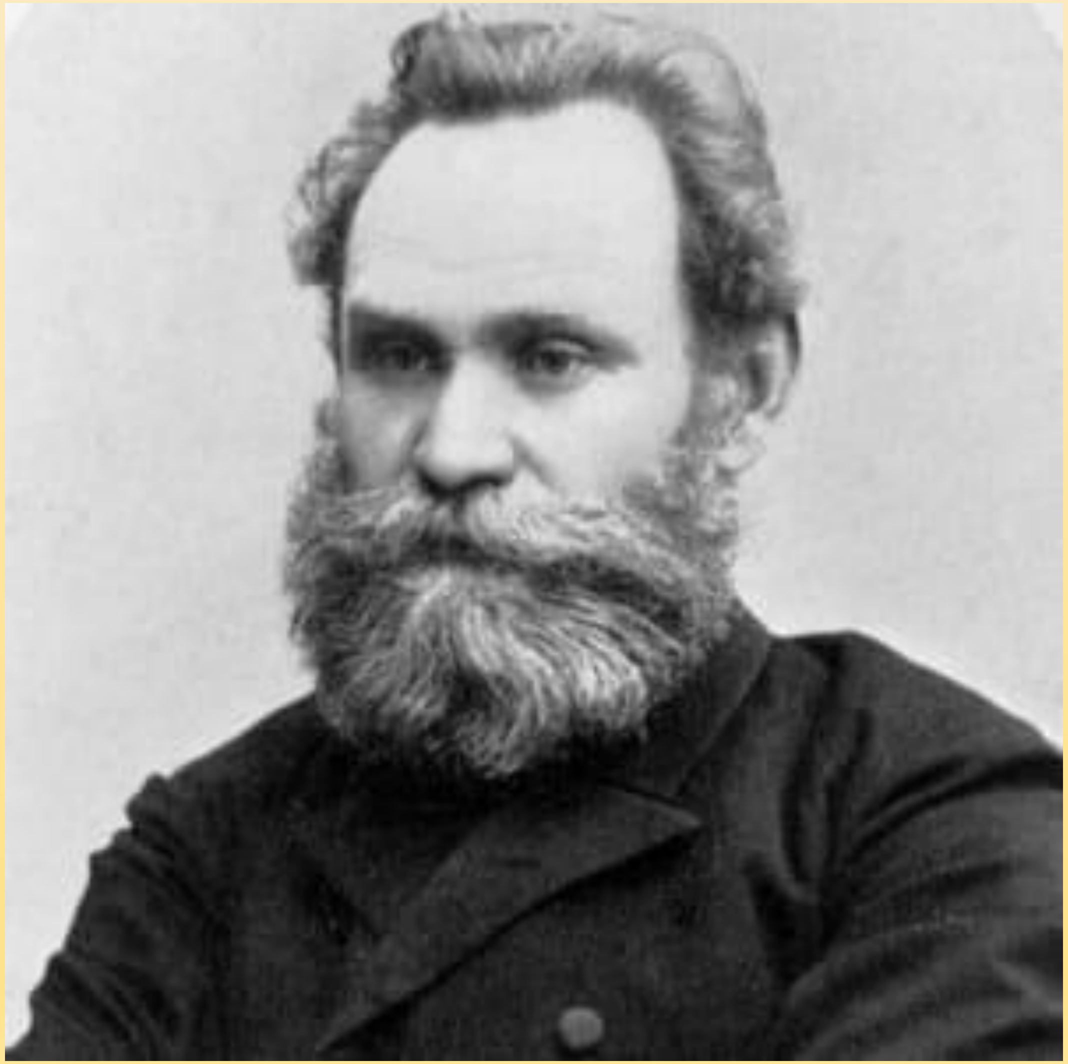
Original Josephson Junction Circuit Model [1962]



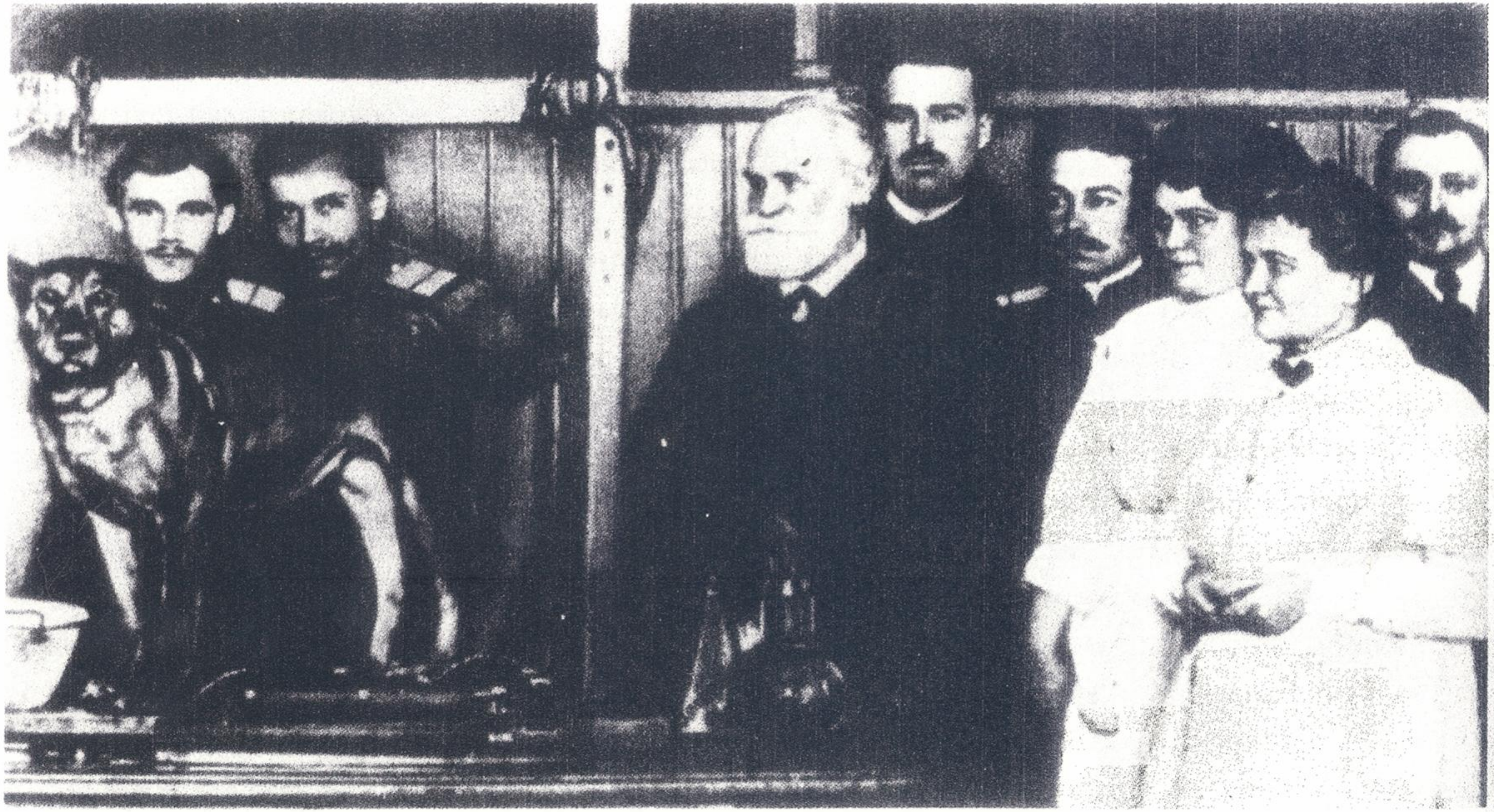
Improved Josephson Junction Circuit Model



1904 Nobel Prize in Physiology or Medicine

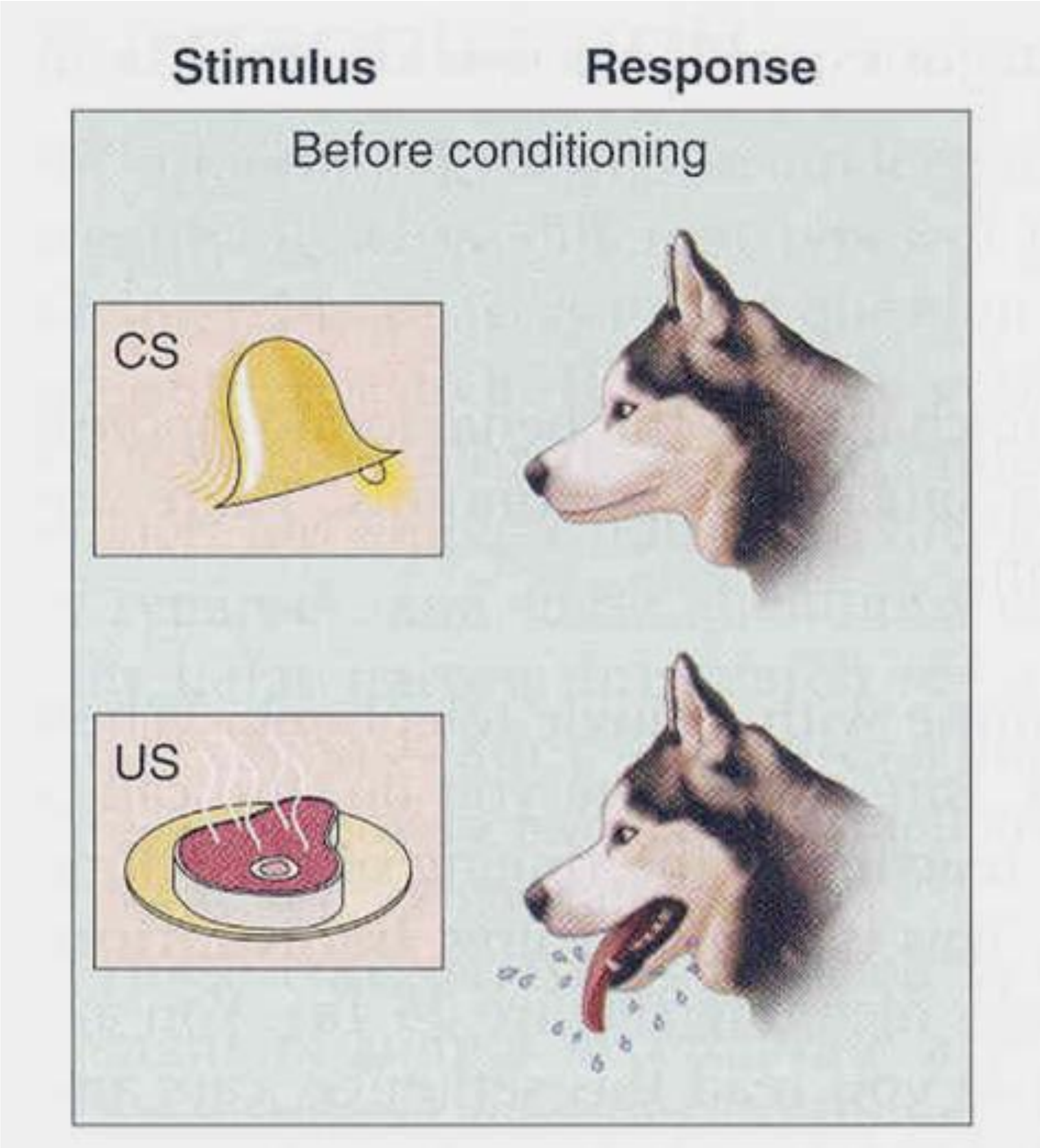


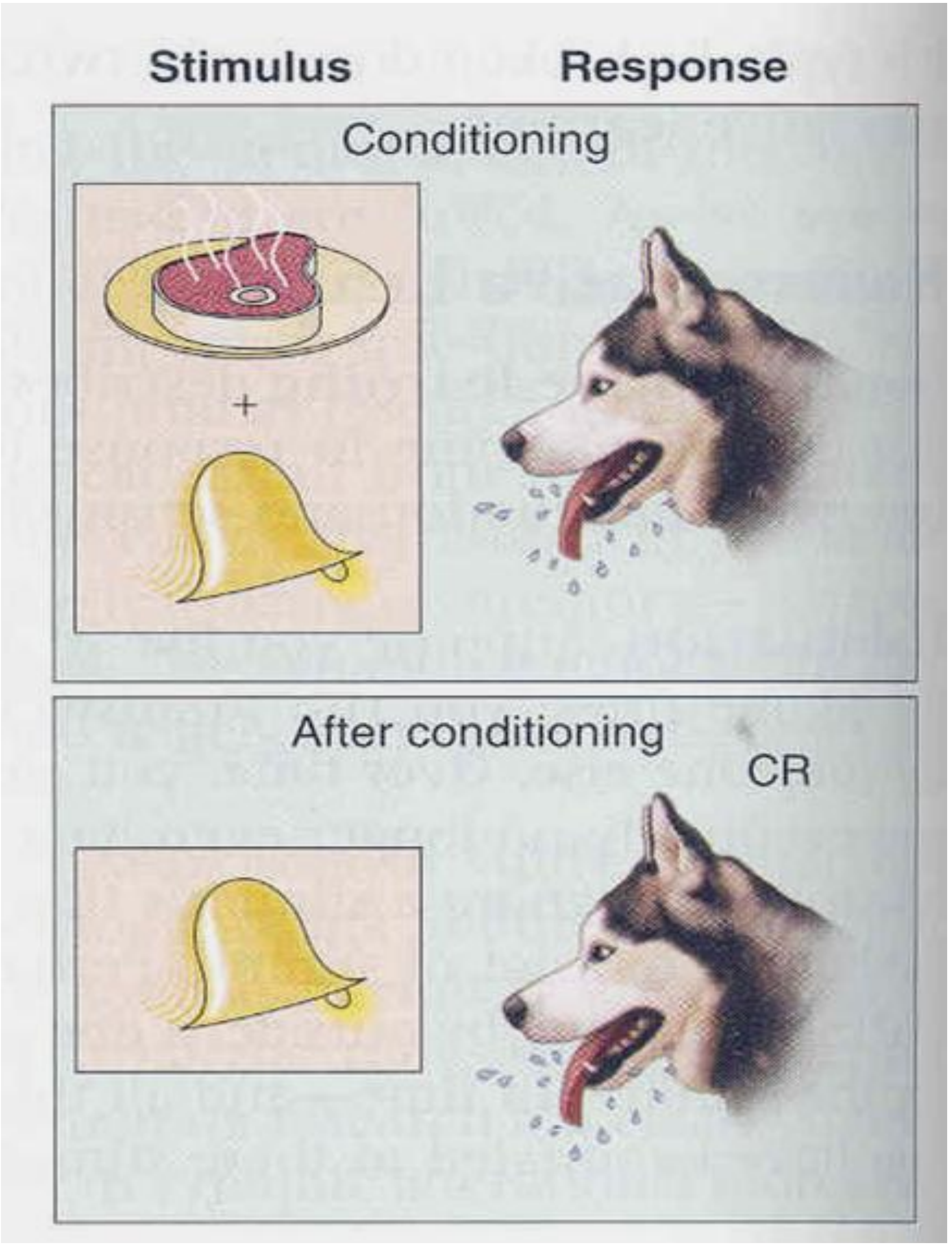
Ivan Petrovich Pavlov (1849-1936)



Pavlov, assistants, visitors, and dog. Pavlovian (classical) conditioning is the simplest associative learning paradigm.

Po_19-121


Stimulus
Response
Before conditioning
CS
US



Po_19-123

Associative Memory and Learning

Associative memory and learning is defined as the ability to learn and remember the relationship between unrelated experiences or events.

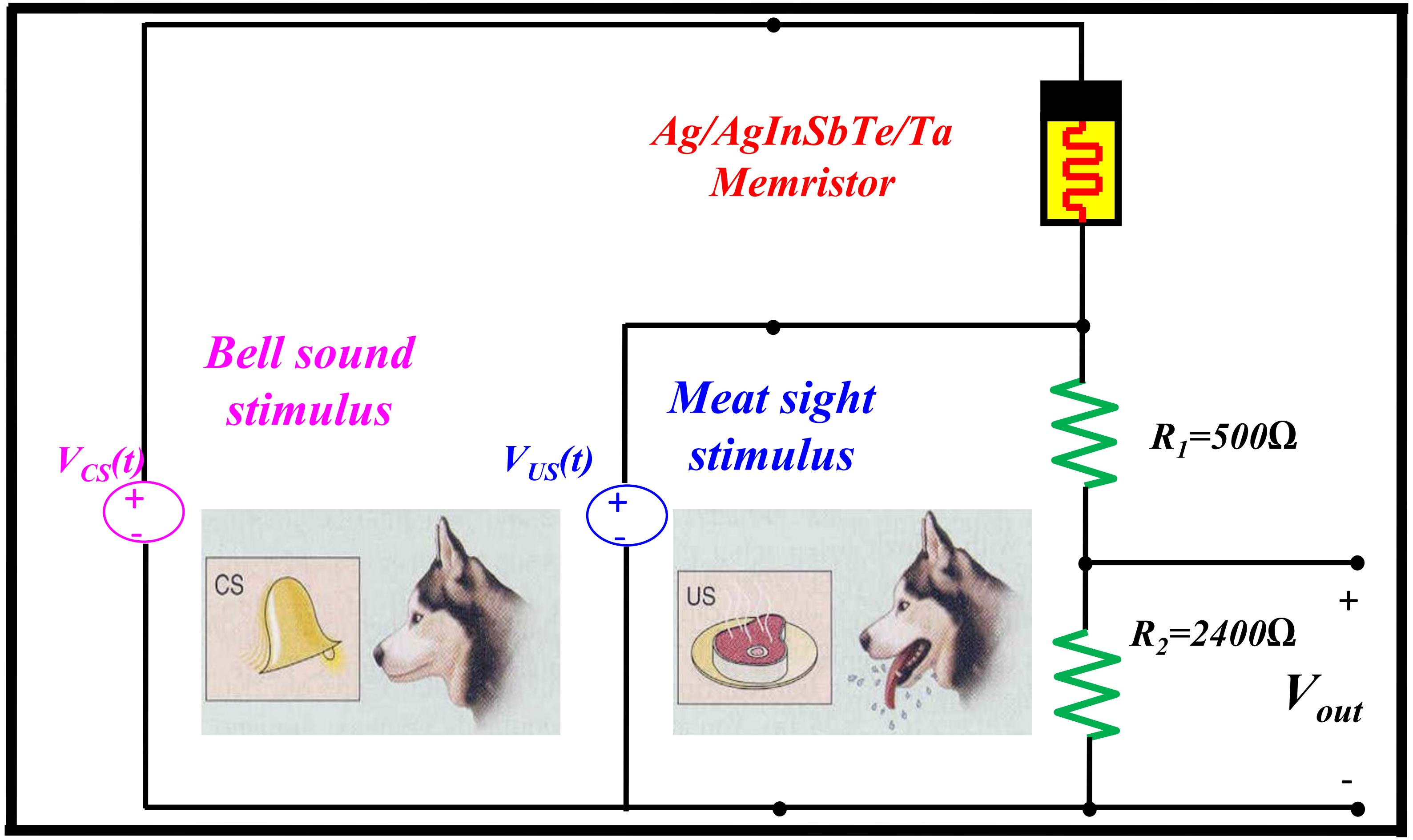
Associative Learning

In 350 BC,

Aristotle had already suggested that learning involves an association of ideas.

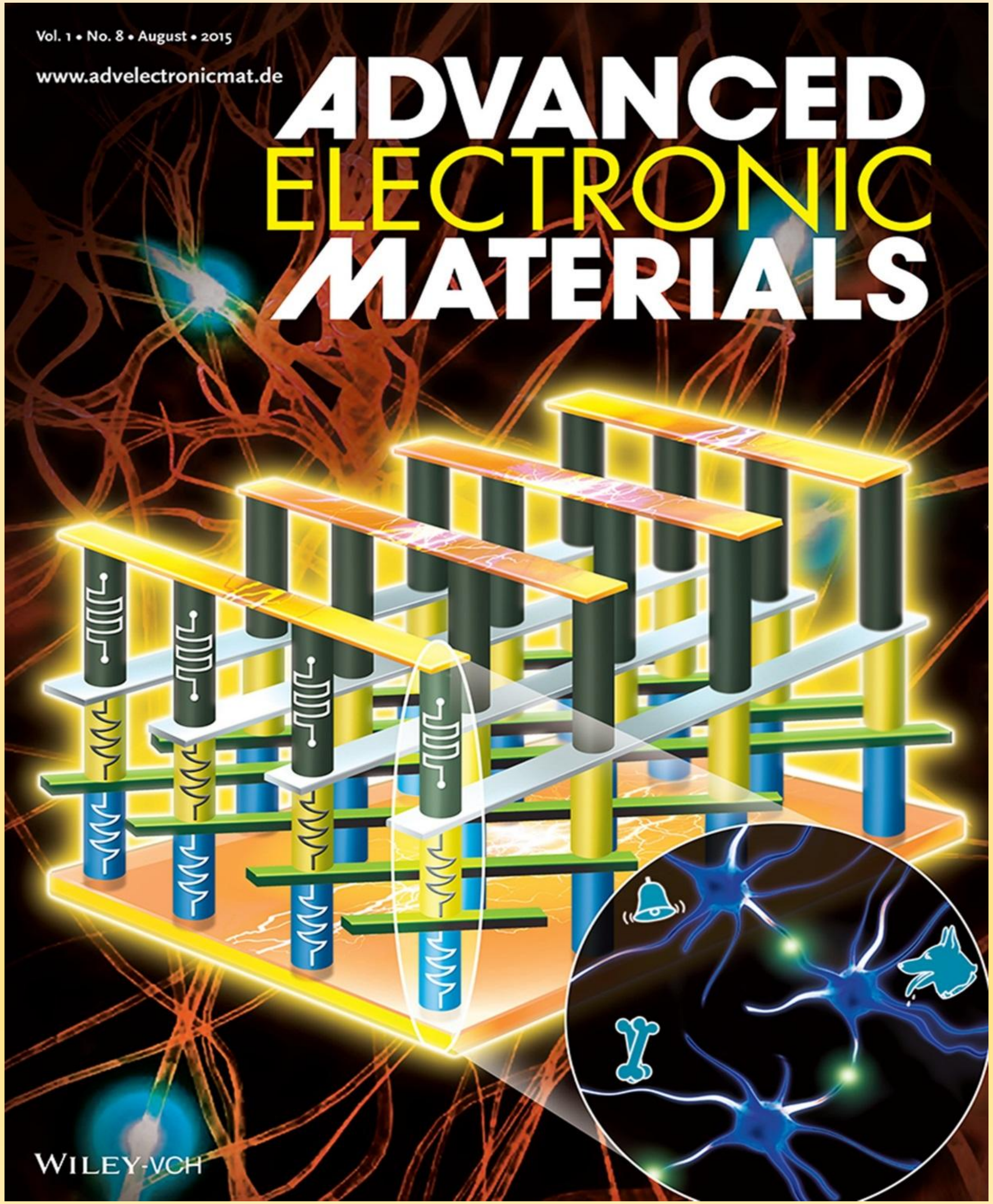
Emulating Pavlov's Dog Associative Learning Phenomenon

Memristor Circuit for Emulating Pavlov's Dog

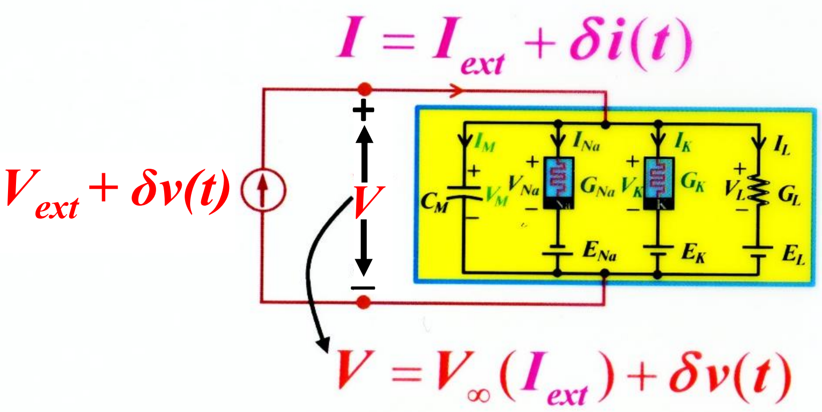


From:

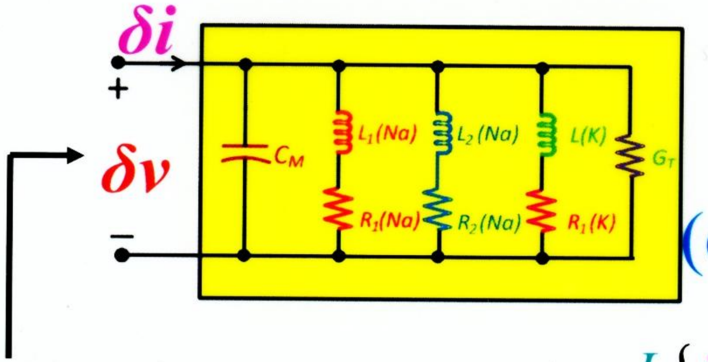
Y. Li, L. Xu, Y. P. Zhong, Y. X. Zhou, S. J. Zhong, Y. Z. Hu, L. O. Chua, X. S. Miao
Associative Learning with Temporal Contiguity in a Memristive Circuit for Large-Scale Neuromorphic Networks
Adv. Electronic Materials, 1500125, p.1-8, 2015.



Memristors are promising candidates for applications as artificial synapses in neuromorphic engineering. In article number 1500125, Xiang-Shui Miao and co-workers demonstrate ***associative learning*** and extinction functions in a compact circuit with only one **memristor** and two resistors. Based on the the spike-timing-dependent plasticity rule, learning with temporal contiguity is realized, consistent with biological behaviors.



Hodgkin-Huxley Axon Circuit



Small-Signal equivalent circuit at DC (equilibrium point) (V_{ext}, I_∞)

Admittance: $Y(s\ ;V_{ext}) \triangleq \dfrac{L\{\delta i(t)\}}{L\{\delta v(t)\}}$

Complexity function:

$$Y(s\ ;\ V_{ext}) = \frac{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

SM-16

Main Theorem:

Eigenvalues of the Hodgkin-Huxley 4×4 Jacobian matrix $J_{HH}(I_{ext})$ are identical to the zeros of the complexity function $Y(s; V_{ext})$, i.e. the solution of the scalar polynomial equation.

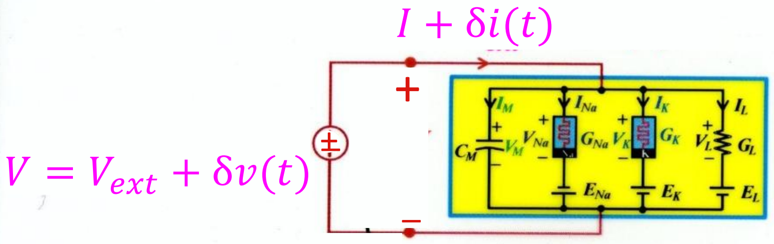
$$b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0 = 0$$

SM-23

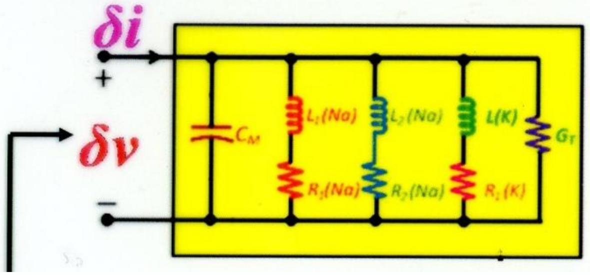
A Glimpse
at
Edge of Chaos

SM-22

Hodgkin-Huxley
Axon Circuit



Small-Signal equivalent
circuit at DC
(equilibrium point) (V_{ext}, I_∞)



Admittance: $Y(s;\, V_{ext}) \triangleq \dfrac{L\{\delta i(t)\}}{L\{\delta v(t)\}}$

Hodgkin-Huxley Edge of Chaos Criterion:

The ***Hodgkin-Huxley Neuron*** under synaptic input V_{ext} is on the ***edge of chaos*** if, and only if,

1. All zeros of $Y(s;\, V_{ext})$ are in the open left half plane,

   i. e. $Re\ z_k(V_{ext}) < 0 \quad k = 1,\ 2,\ 3,\ 4$

2. $Re\ Y(i\omega; V_{ext}) < 0$ for at least one frequency $\omega = \omega_{ext}$

Edge of Chaos is Testable !

Testing

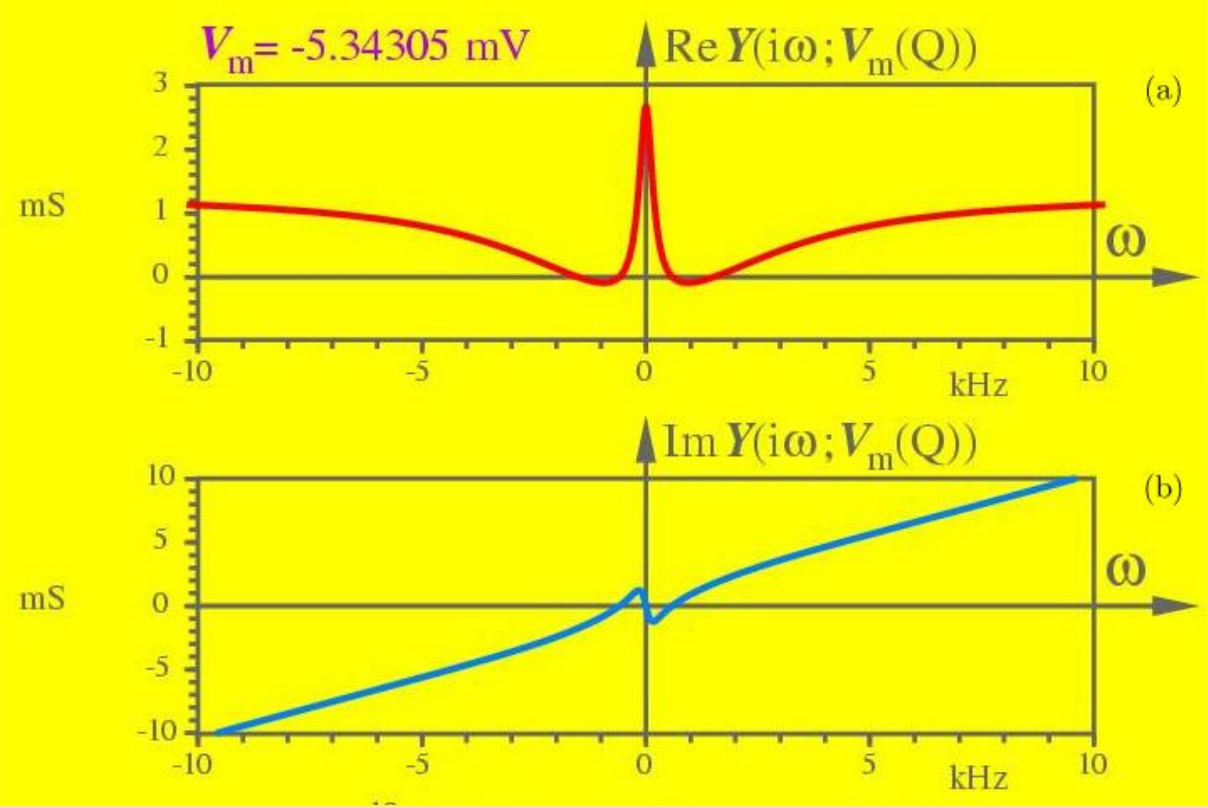
for

Edge of Chaos

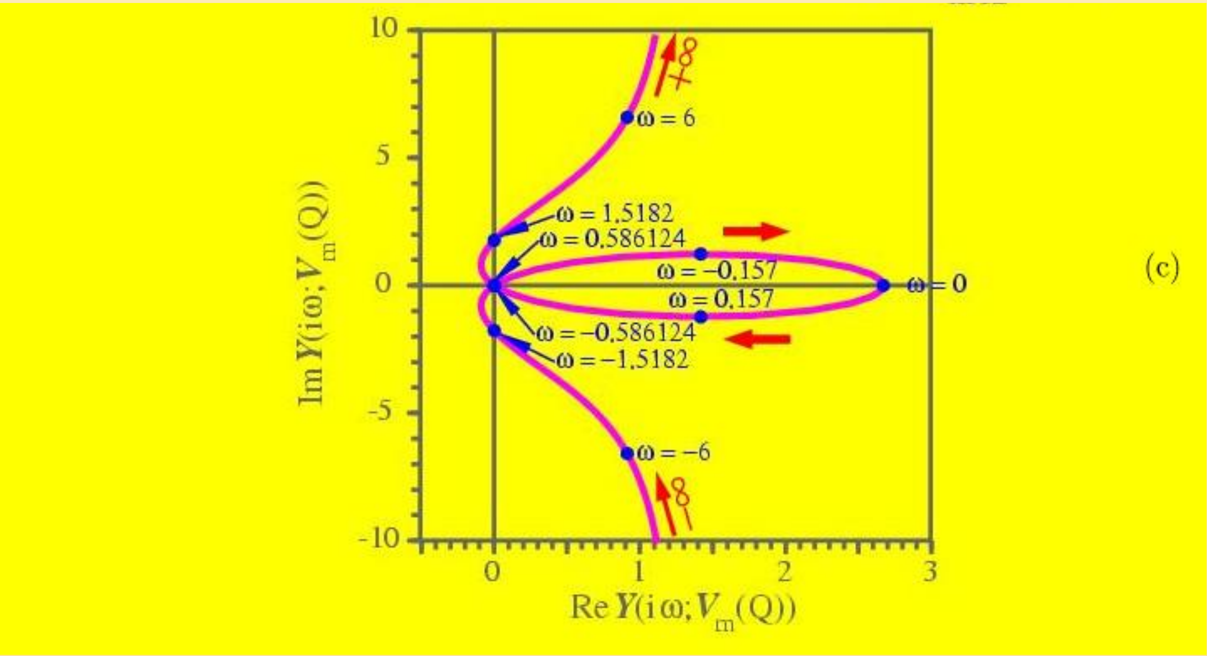
Involves only

Linear Algebra

5-82



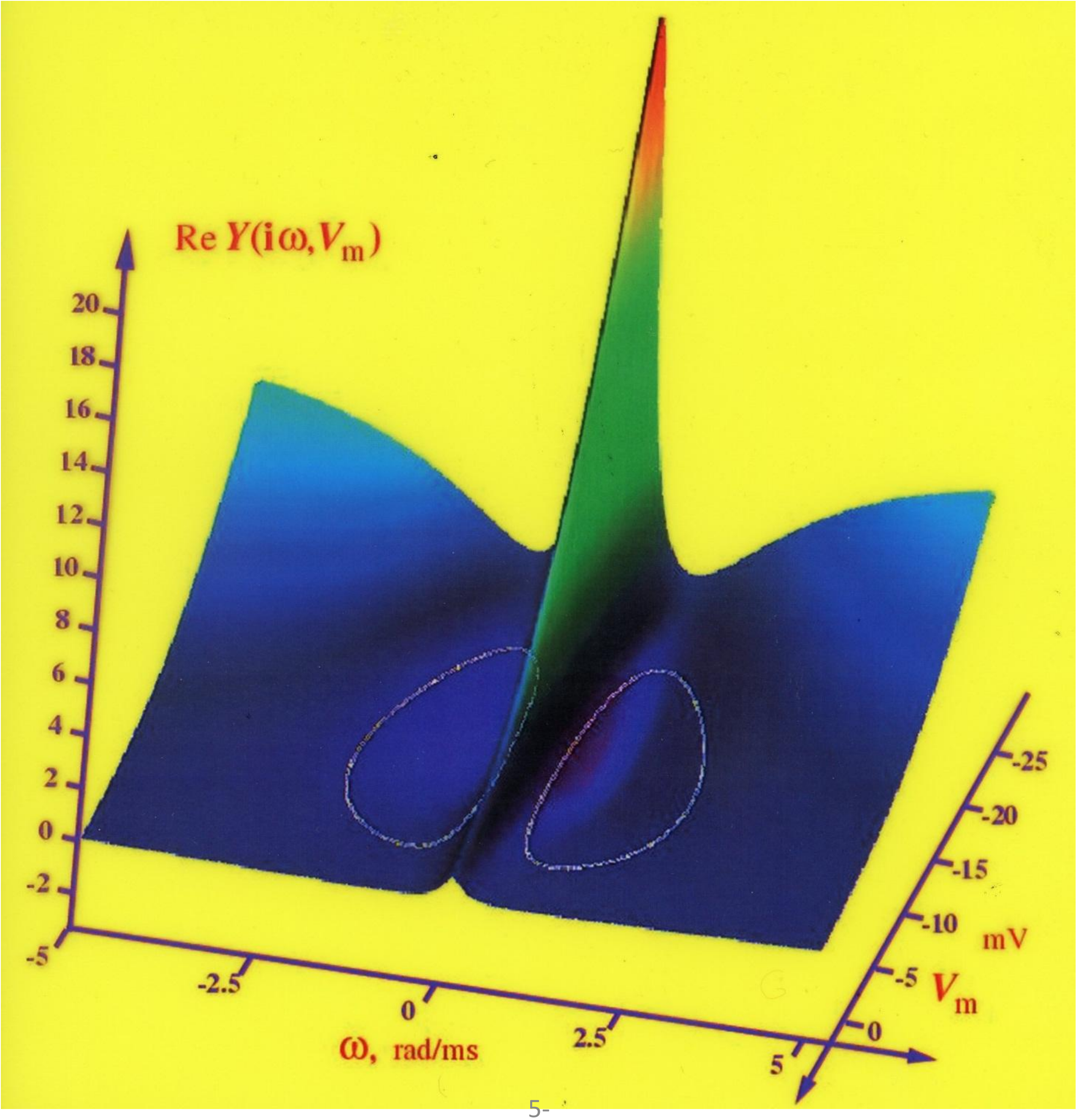
Nyquist Plot



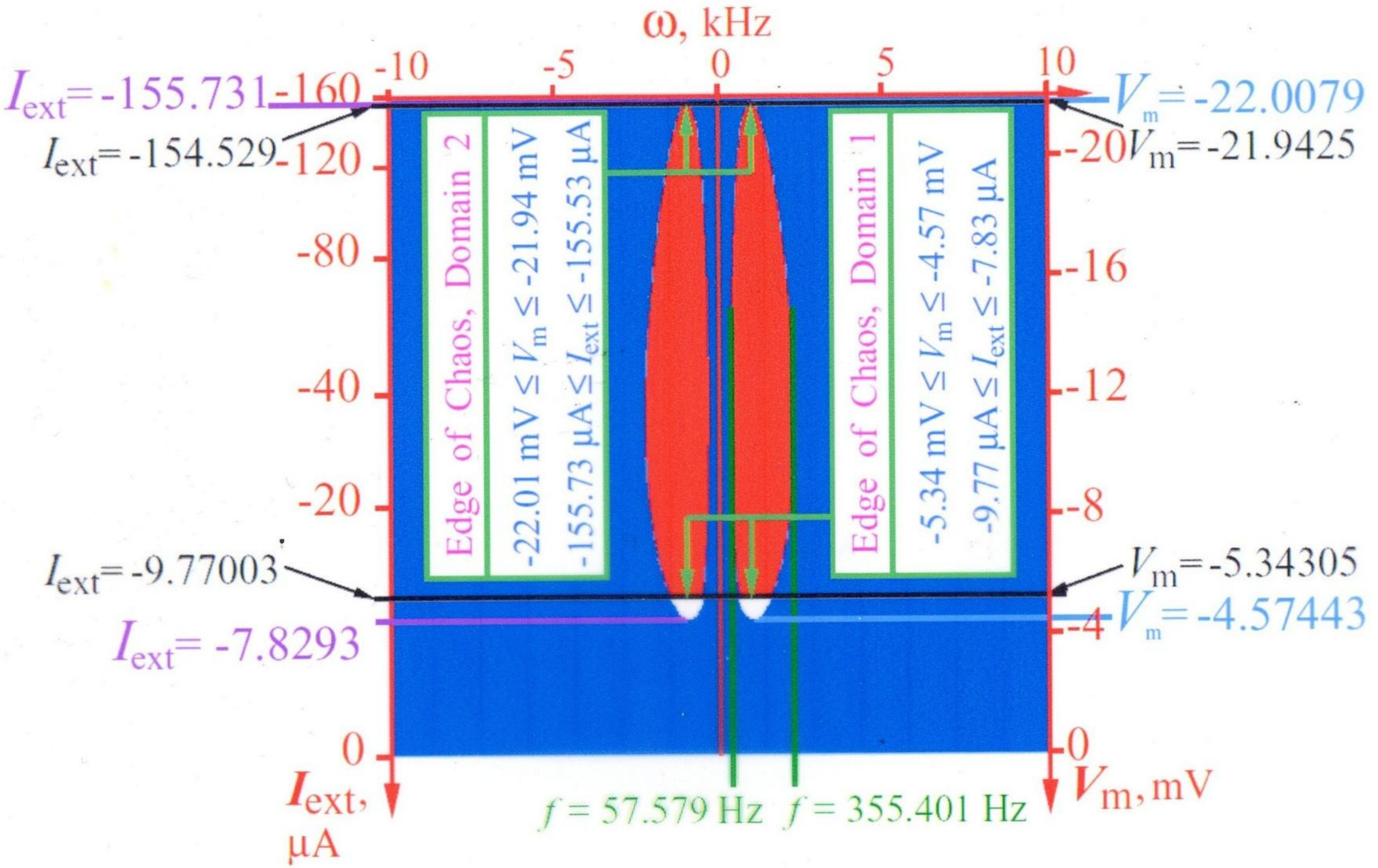
Test for ***Edge of Chaos***

Hodgkin Huxley Model is on ***Edge of Chaos*** at $V=V_m$

if its **Nyquist Plot** intersects the **imaginary axis Im Y(iω)**


Re $Y(i\omega, V_m)$
20
18
16
14
12
10
8
6
4
2
0
-2
-5
-2.5
0
2.5
5
ω, rad/ms
-25
-20
-15
-10
-5
0
mV
V_m

5-87



Range of Edge of Chaos :

$-9.77\mu A \leq I_{ext} \leq -7.83\ \mu A \Rightarrow \Delta I_{ext} = 1.94\ \mu A$

$-5.34\ mV \leq V_m \leq -4.57\ mV \Rightarrow \Delta V_m = 0.77\ mV$

5-89

Goldilocks Zone for Action Potential

$-9.77\mu A \leq I_{ext} \leq -7.83\ \mu A$

$-5.34\ mV \leq V_m \leq -4.57\ mV$

Window For Life to be Possible

$\Delta I < 2\ \mu A$

$\Delta V < 1\ mV$

Galvani's Irritability *Doctrine* is a manifestation of the *Edge of Chaos,* the *crown jewel* of the *Principle of Local Activity*

Galvani was well aware of the ***doctrine of "irritability,"*** one of the most important conceptual elaborations of **18th century physiology.**

According to ***Galvani***, in the framework of ***doctrine of irritability***, external electricity was acting as an ***excitatory stimulus*** (as a ***trigger*** we would say now) for the **contraction**, and ***not as the direct "efficient" cause of the observed phenomena***.

How could it be that such a tiny electrical force would produce muscle contraction if it were not setting in motion some ***internal force***, and **triggering** some extremely ***mobile principle*** existing in nerves which then ***excites the action*** of the nerve-muscular force?

UK-10

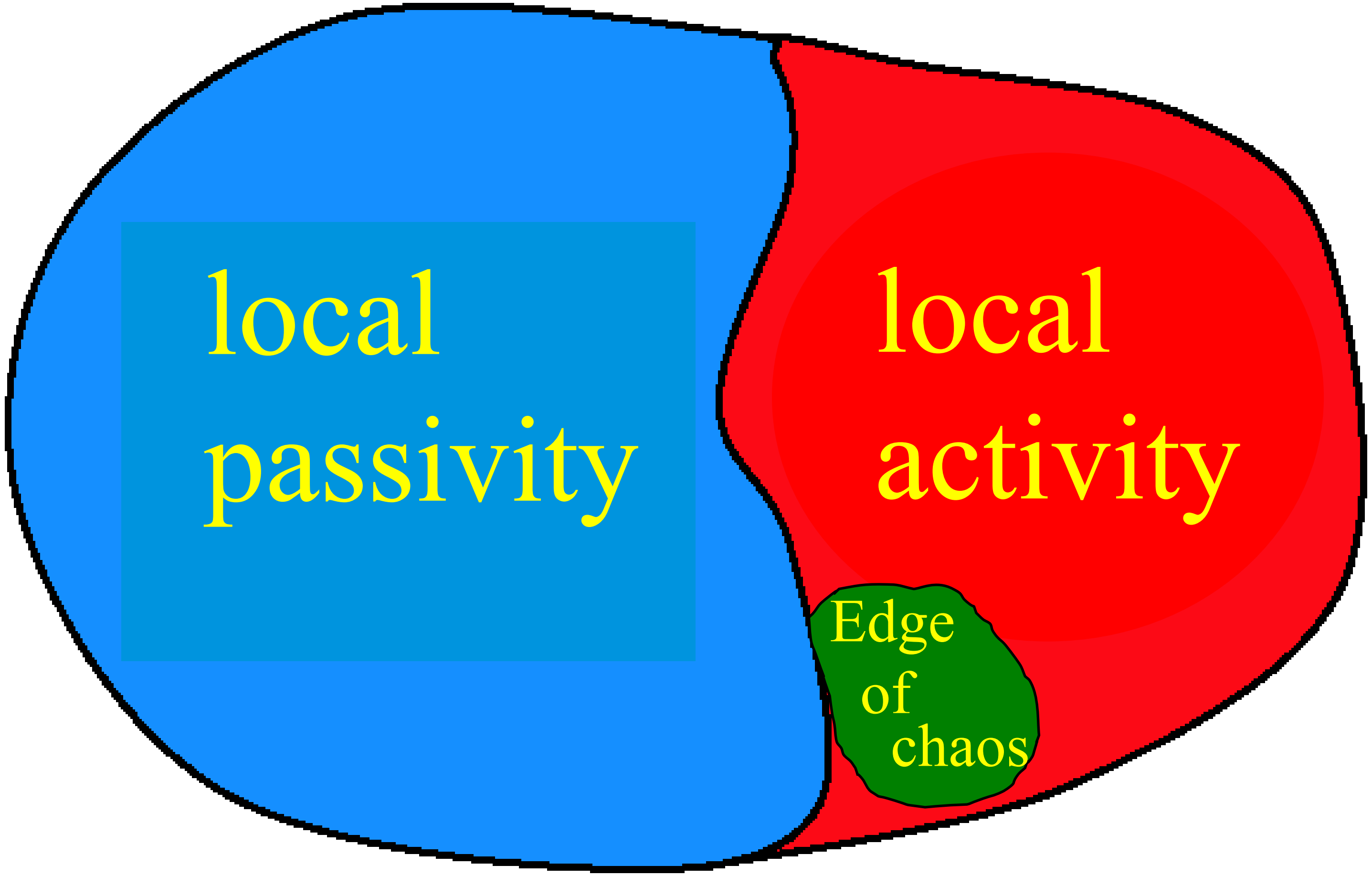
The Local Activity Principle

All 2-terminal electrical devices can be classified into either "*locally passive*", or "*locally active*". Occasionally, a tiny subset of "*locally active*" devices is blessed with an "*edge of chaos*" domain capable of exhibiting fascinating phenomena and attributes, such as "*learning*", "*artificial intelligence*", and even "*life*" itself.

Proof of the Principle of Local Activity requires non-trivial applications of the following mathematical results:

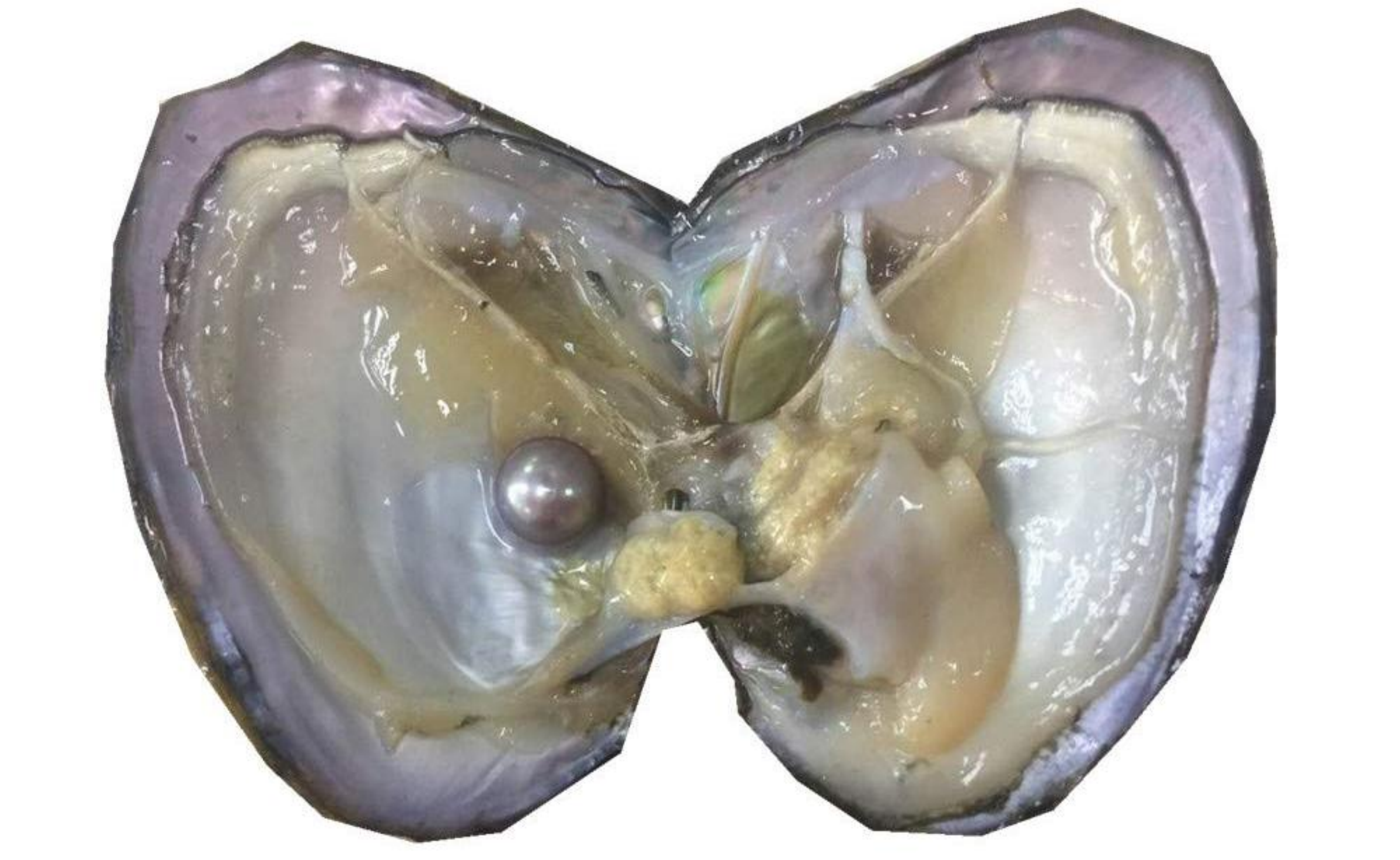
1. ***Nonlinear Monotone Operator in Function Space***
2. ***High-dimensional Analog of the Mean-Value Theorem***
3. ***LaSalle's Complete Stability Theorem***
4. ***Poisson's Integral on half-plane***
5. ***Theory of Positive-Real Function of a complex Variable***
6. ***Tellegen's Theorem***

Local Activity Principle



Mem-35

The "*Edge of Chaos*" is the "**Pearl**" amidst the "***locally active domain***"



Mem-36

Local Activity Theorem

A ***reaction-diffusion equation*** with one diffusion coefficient is ***Locally active*** at $Q \Leftrightarrow$ any one of the following conditions holds :

1. $Z_Q(s)$ ***has a pole in Re*** [s] > 0.
2. $Re\, Z_Q(i\omega) < 0$ ***for at least one*** $\omega = \omega_0$.
3. $Z_Q(s)$ ***has a simple pole*** $s = \omega_P$ ***on the imaginary axis i. e.,***

$$Z_Q(s) = \frac{Z_1(s)}{(s - i\omega_p)^m Z_2(s)}, \quad m = 1$$

and

$$\lim_{s \to i\omega_p} (s - i\omega_p)\, Z_Q(s) < 0.$$

UK-56

4. $Z_Q(s)$ ***has a multiple pole on the imaginary axis.***

Physical Meaning of Locally Active Cells

One can use a locally-active cell to amplify "small" signals at the expense of an external power supply.

For example, neurons in our brain maintain their local levels of organization by burning glucose. In fact, every "living" cell is a "molecular" amplifier.

UK-57

Fundamental New Result

Edge of Chaos Theorem

A stable equilibrium state Q can be destabilized by coupling it to a passive environment, if, and only if, Q is poised on the ***Edge of chaos***

Mem-32

Action Potential

is

Impossible

without

Edge of Chaos

MEM-31

Neurons

are poised near the

Edge of Chaos

Mem-63

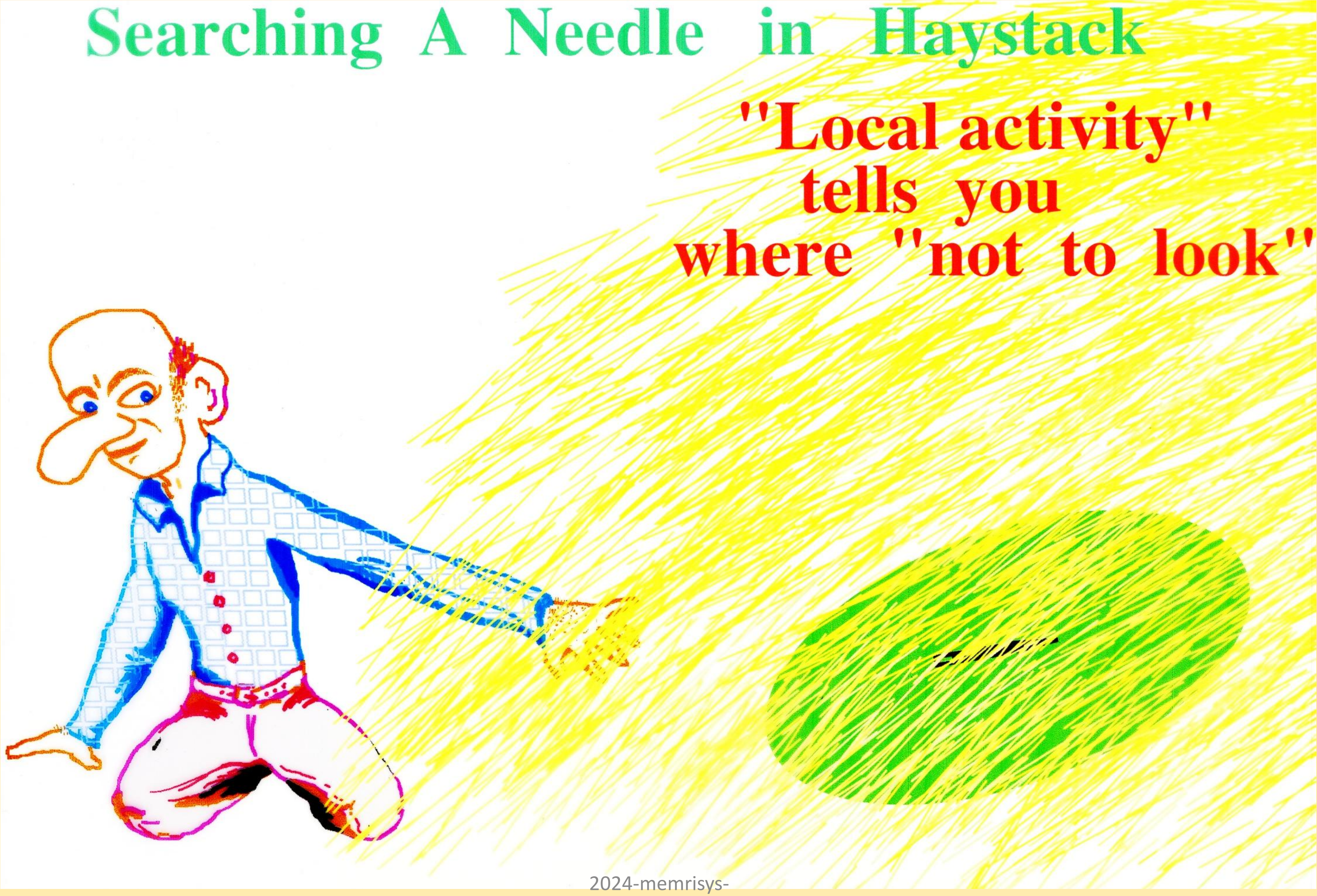
Local Activity

is the

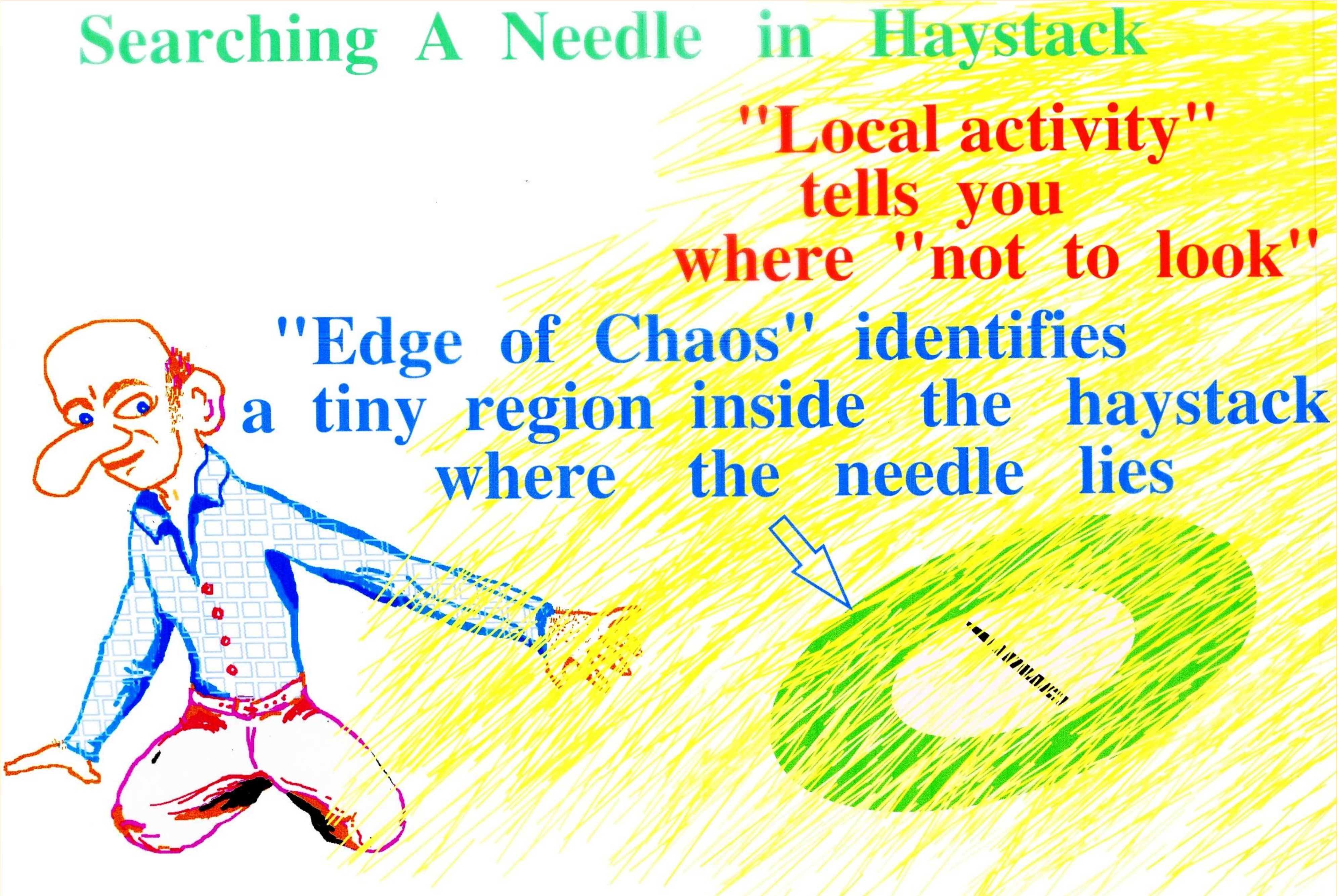
Origin of Complexity

Local Activity Test

UK-20


Searching A Needle in Haystack
"Local activity"
tells you
where "not to look"

UK-21


Searching A Needle in Haystack
"Local activity"
tells you
where "not to look"
"Edge of Chaos" identifies
a tiny region inside the haystack
where the needle lies

UK-22

Following Reversed Engineering Article has just appeared online in

NATURE ELECTRONICS

14 May, 2018

How We Predicted the Memristor

Leon O. Chua

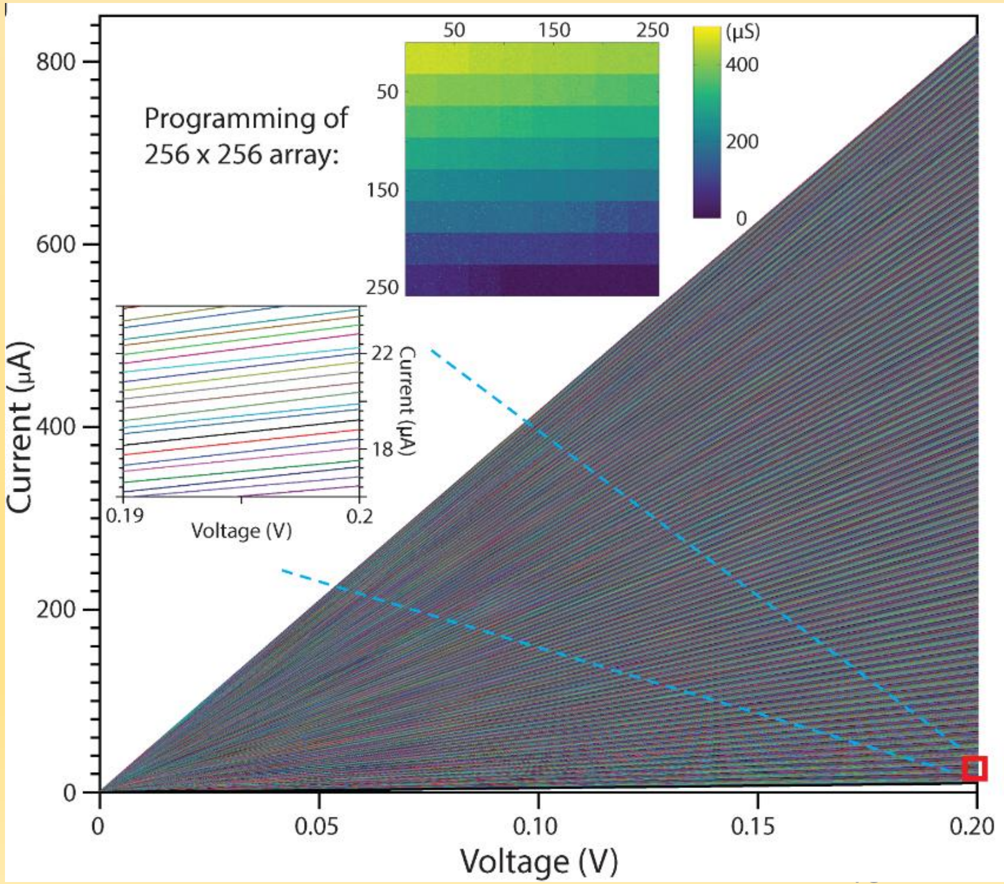
Po_19-19

Nature Vol. 615, No. 7954, pp.823-829, 2023

Thousands of conductance levels in memristors monolithically integrated on CMOS

Mingyi Rao, Hao Tang, Jiangbin Wu, Wenhao Song, Max Zhang, Wenbo Yin, Ye Zhuo, Fatemeh Kiani, Benjamin Chen, Xiangqi Jiang, Hefei Liu, Hung-Yu Chen, Rivu Midya, Fan Ye, Hao Jiang, Zhongrui Wang, Mingche Wu, Miao Hu, Han Wang, Qiangfei Xia, Ning Ge, Ju Li, J. Joshua Yang.

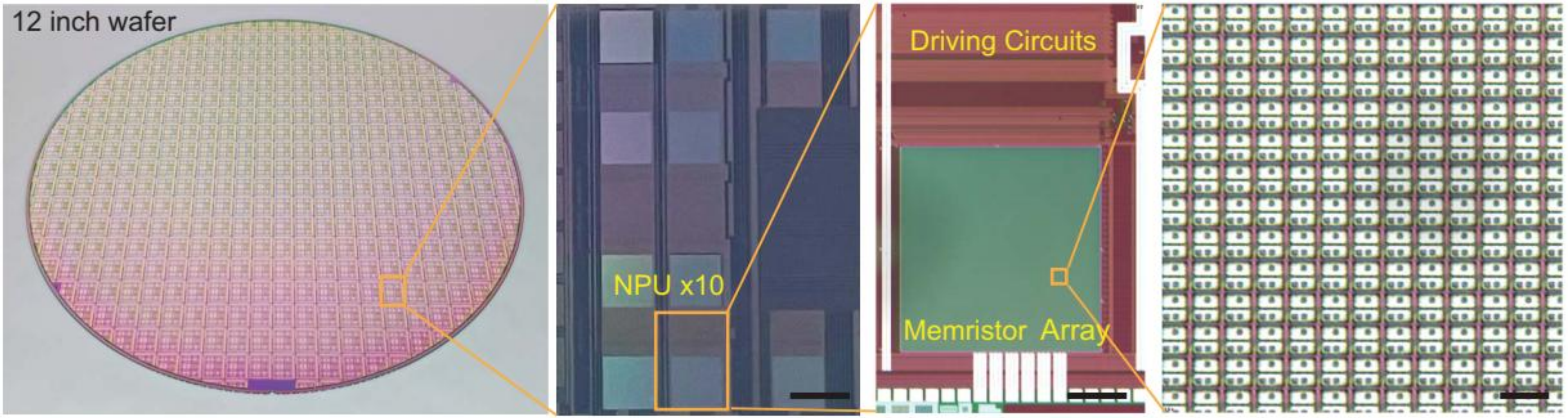
2048 resistance levels (11 bit/cell) in one memristor



Science: Vol. 383, No. 6685, 804-810, February 2024

Programming memristor arrays with arbitrarily high precision for analog computing

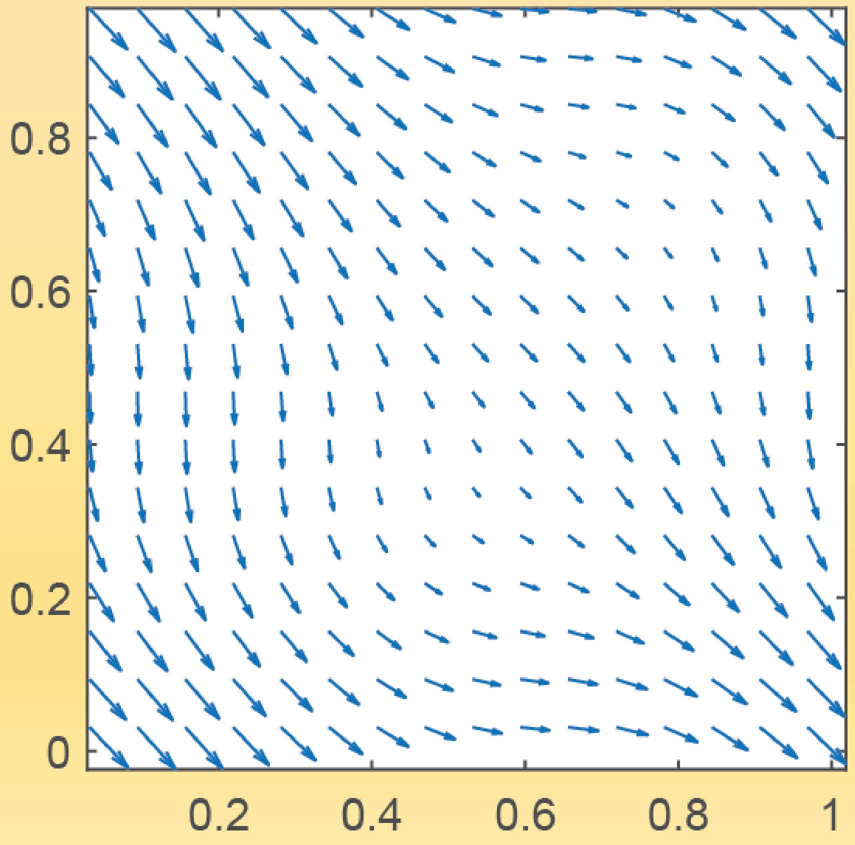
Wenhao Song, Mingyi Rao, Yunning Li, Can Li, Ye Zhuo, Fuxi Cai, Mingche Wu, Wenbo Yin, Zongze Li, Qiang Wei, Sangsoo Lee, Hengfang Zhu, Lei Gong, Mark Barnell, Qing Wu, Peter A. Beerel, Mike Shuo-Wei Chen, Ning Ge, Miao Hu, Qiangfei Xia, J. Joshua Yang



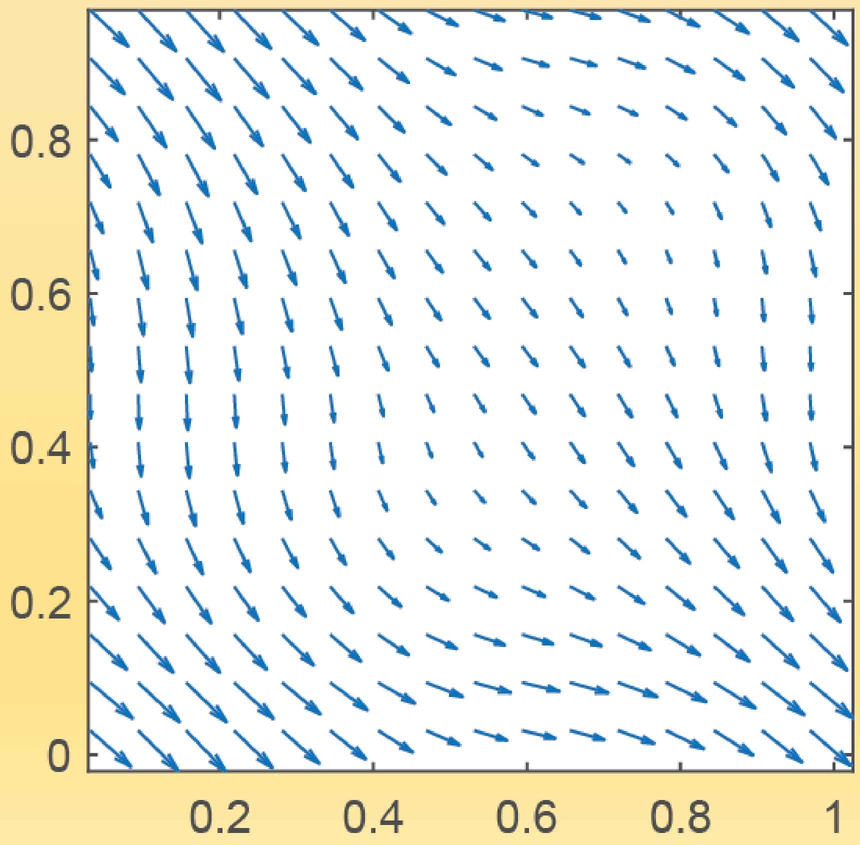
Programming Memristor Array to Solve PDE

Example: Navier-Stokes Equations

Solved by MATLAB



Solved by MEMRISTOR



J-C

Nature: Vol. 640, 17 April, 2025

The Growing Memristor Industry

https://doi.org/10.1038/s41586-025-08733-5

Received: 21 November 2023

Accepted: 3 February 2025

Published online: 16 April 2025

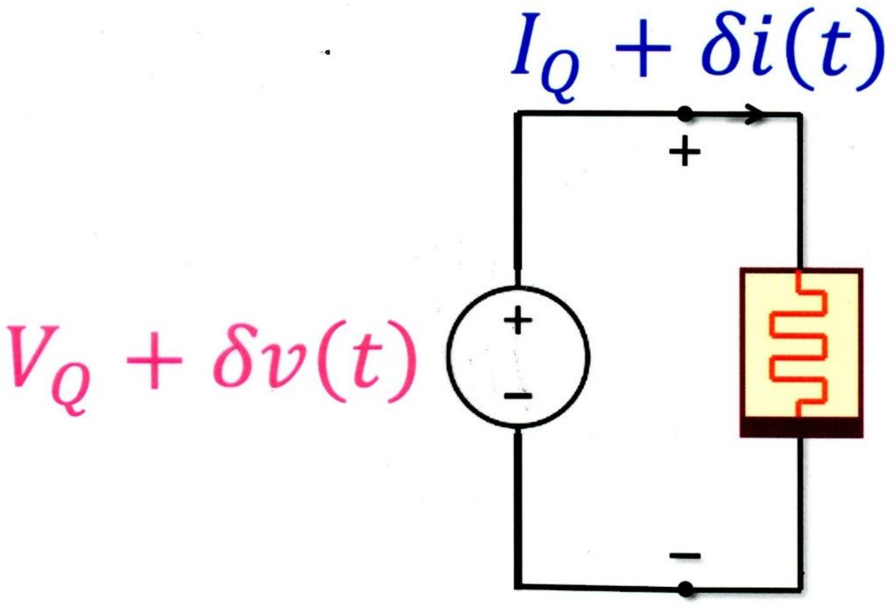
Check for updates

Mario Lanza[1,2,3,15] ✉, Sebastian Pazos[4,15], Fernando Aguirre[5], Abu Sebastian[6], Manuel Le Gallo[6], Syed M. Alam[7], Sumio Ikegawa[7], J. Joshua Yang[8], Elisa Vianello[9], Meng-Fan Chang[10], Gabriel Molas[11], Ishai Naveh[11], Daniele Ielmini[12], Ming Liu[13] & Juan B. Roldan[14]

The semiconductor industry is experiencing an accelerated transformation to overcome the scaling limits of the transistor and to adapt to new requirements in terms of data storage and computation, especially driven by artificial intelligence applications and the Internet of Things. In this process, new materials, devices, integration strategies and system architectures are being developed and optimized. Among them, memristive devices and circuits—memristors are two-terminal memory devices that can also mimic some basic bioelectronic functions—offer a potential approach to create more compact, energy-efficient or better-performing systems. The memristor industry is growing quickly, raising abundant capital investment, creating new jobs and placing advanced products in the market. Here we analyse the status and prospects of the memristor industry, focusing on memristor-based products that are already commercially available, prototypes with a high technological readiness level that might affect the market in the near future, and discuss obstacles and pathways to their implementation.

SM-10

Locally Active Memristor



Definition:

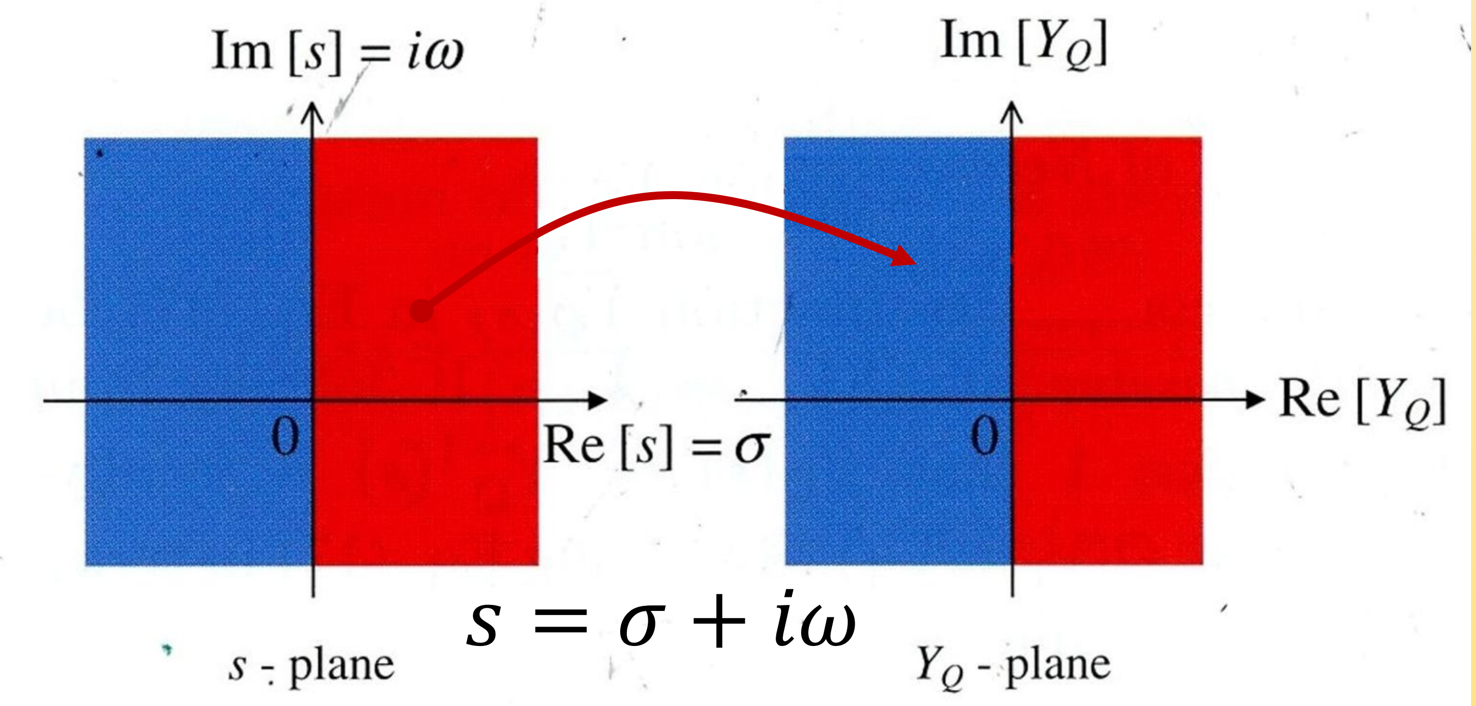
A **memristor** is ***locally active*** at an Equilibrium Point $Q(V_Q, I_Q)$ *if* there exists an **admissible small-signal pair** $(\delta v(t), \delta i(t))$ such that

$$\int_0^T \delta v(\tau)\delta i(\tau)d\tau < 0$$

at some **finite** time T.

2024-
memrisys-54

Not-Positive Real Function implies N is *locally-Active*



Small-Signal Admittance

$$Y_Q(s) = \frac{I(s)}{V(s)}$$

at DC operating point Q

2024-memrisys-71

World War II hero Alan Turing's saved papers sell for record price at UK auction

TU-1

A treasure trove of wartime codebreaker Alan Turing's scientific papers has sold for more than three times their expected sale price... All after being saved from the shredder.

A collection of rare scientific papers written by mathematician, computer scientist and Second World War codebreaker Alan Turing has sold for a record £465,400 (€544,400) at auction in Lichfield, UK.

The documents were discovered in a loft at a property in Bermondsey, London, and were almost destroyed during a house clearance.

Some of the rare items that went under the hammer included a personal signed copy of Turing's 1938 PhD dissertation, "Systems of Logic Based on Ordinals", which sold for £110,500 (€129,200), as well as his paper "On Computable Numbers" - also known as "Turing's Proof" - which introduced the world to the idea of a universal computing machine in 1936.

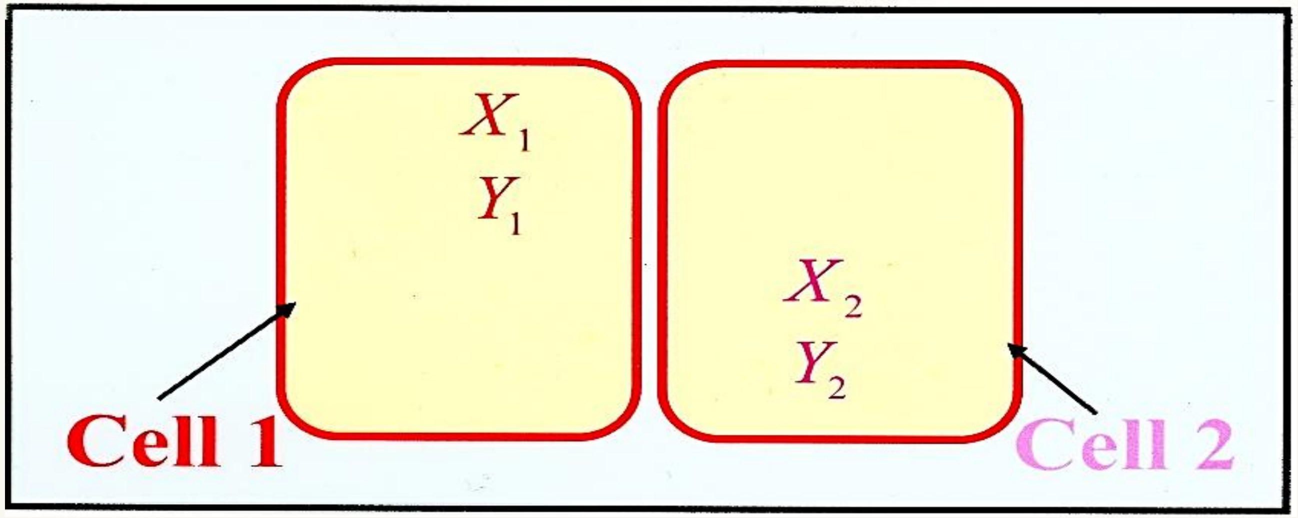
The collection also included "The Chemical Basis Of Morphogenesis", which sold for £19,500 (€22,800). Dating from 1952, it is Turing's last major published work.

TU-2a

What is TURING INSTABILITY ?

2 Identical Uncoupled Cells

dynamics inside



dynamics inside

Cell 1

$$\dot{X}_1 = (5X_1 - 6Y_1 + 1)$$
$$\dot{Y}_1 = (6X_1 - 7Y_1 + 1)$$

Cell 2

$$\dot{X}_2 = (5X_2 - 6Y_2 + 1)$$
$$\dot{Y}_2 = (6X_2 - 7Y_2 + 1)$$

Equilibrium Solution of Cell 1:

$$X_1 = 1, \quad Y_1 = 1$$

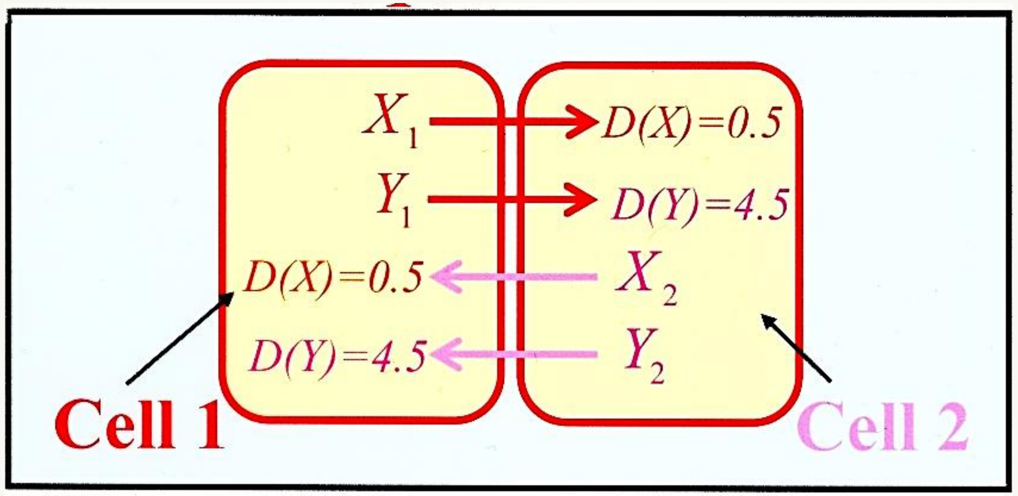
Equilibrium Solution of Cell 2:

$$X_2 = 1, \quad Y_2 = 1$$

TURING INSTABILITY

Emerges from the Reaction-Diffusion Equations of 2 coupled identical cells

(X_1, Y_1) diffuses to Cell 2



(X_2, Y_2) diffuses to Cell 1

Diffusion is Dissipation !

Cell 1

$$\dot{X}_1 = (5X_1 - 6Y_1 + 1) + 0.5(X_2 - X_1)$$

$$\dot{Y}_1 = (6X_1 - 7Y_1 + 1) + 4.5(Y_2 - Y_1)$$

Cell 2

$$\dot{X}_2 = (5X_2 - 6Y_2 + 1) + 0.5(X_1 - X_2)$$

$$\dot{Y}_2 = (6X_2 - 7Y_2 + 1) + 4.5(Y_1 - Y_2)$$

TURING's Two-Cell Reaction-Diffusion Equations

$$\underbrace{\begin{bmatrix} 5 & -6 & 0 & 0 \\ 6 & -7 & 0 & 0 \\ 0 & 0 & 5 & -6 \\ 0 & 0 & 6 & -7 \end{bmatrix}}_{\mathbf{G_1}} + \underbrace{\begin{bmatrix} -0.5 & 0 & 0.5 & 0 \\ 0 & -4.5 & 0 & 4.5 \\ 0.5 & 0 & -0.5 & 0 \\ 0 & 4.5 & 0 & -4.5 \end{bmatrix}}_{\mathbf{G_2}} = \underbrace{\begin{bmatrix} 4.5 & -6 & 0.5 & 0 \\ 6 & -11.5 & 0 & 4.5 \\ 0.5 & 0 & 4.5 & -6 \\ 0 & 4.5 & 6 & -11.5 \end{bmatrix}}_{\mathbf{G = G_1 + G_2}}$$

Eigen Values of G_1

G_1 Stable Matrix

$$\lambda_1(\mathbf{G}_1) = -1$$
$$\lambda_2(\mathbf{G}_1) = -1$$
$$\lambda_3(\mathbf{G}_1) = -1$$
$$\lambda_4(\mathbf{G}_1) = -1$$

Eigen Values of G_2

G_2 Stable Matrix

$$\lambda_1(\mathbf{G}_2) = -9$$
$$\lambda_2(\mathbf{G}_2) = -1$$
$$\lambda_3(\mathbf{G}_2) = 0$$
$$\lambda_4(\mathbf{G}_2) = 0$$

Alpha-3a

A FOLK Theorem

Most engineers and mathematicians who are not specialists in matrix algebra believe in the following proposition:

Adding 2 stable matrices gives another stable matrix.

mem-13

TURING Instability: A shocking Phenomenon!

TURING's Two-Cell Reaction-Diffusion Equations

$$\underbrace{\begin{bmatrix} 5 & -6 & 0 & 0 \\ 6 & -7 & 0 & 0 \\ 0 & 0 & 5 & -6 \\ 0 & 0 & 6 & -7 \end{bmatrix}}_{\mathbf{G_1}} + \underbrace{\begin{bmatrix} -0.5 & 0 & 0.5 & 0 \\ 0 & -4.5 & 0 & 4.5 \\ 0.5 & 0 & -0.5 & 0 \\ 0 & 4.5 & 0 & -4.5 \end{bmatrix}}_{\mathbf{G_2}} = \underbrace{\begin{bmatrix} 4.5 & -6 & 0.5 & 0 \\ 6 & -11.5 & 0 & 4.5 \\ 0.5 & 0 & 4.5 & -6 \\ 0 & 4.5 & 6 & -11.5 \end{bmatrix}}_{\mathbf{G = G_1 + G_2}}$$

Eigen Values of G_1

G_1 Stable Matrix

$$\lambda_1(\mathbf{G}_1) = -1 \quad \lambda_2(\mathbf{G}_1) = -1 \quad \lambda_3(\mathbf{G}_1) = -1 \quad \lambda_4(\mathbf{G}_1) = -1$$

Eigen Values of G_2

G_2 Stable Matrix

$$\lambda_1(\mathbf{G}_2) = -9 \quad \lambda_2(\mathbf{G}_2) = -1 \quad \lambda_3(\mathbf{G}_2) = 0 \quad \lambda_4(\mathbf{G}_2) = 0$$

Eigen Values of G_3

G Unstable Matrix

$$\lambda_1(\mathbf{G}) = -14 \quad \lambda_2(\mathbf{G}) = -1 \quad \lambda_3(\mathbf{G}) = -1 \quad \lambda_4(\mathbf{G}) = +2$$

Alpha-3

What **Special Property** must G_1 possess to make **destabilization** possible ?

Mem-16

Various forms of ***Turing's equations***, or ***reaction-diffusion equations*** have appeared in one form or another in many works and fields. However, any sort of systematic under-standing or analysis seems far away. Before one can expect any general understanding, many examples will have to be thought through, both on the mathematical side and on the experimental side.

A MATHEMATICAL MODEL OF TWO CELLS VIA TURING'S EQUATION
BY
S. SMALE

Edge of Chaos is Sine Qua Non for Turing Instability

A. Ascoli *Senior Member, IEEE*, A.S. Demirkol, R. Tetzlaff *Senior Member, IEEE*, and L. Chua *Fellow, IEEE*

***Abstract*—Diffusion-driven instabilities with pattern formation may occur in a network of identical, regularly-spaced, and resistively-coupled cells if and only if the uncoupled cell is poised on a locally-active and stable operating point in the Edge of Chaos domain. This manuscript presents the simplest ever-reported two-cell neural network, combining together only 7 two-terminal components, namely 2 batteries, 3 resistors, and 2 volatile NbO_x memristive threshold switches from NaMLab, and subject to diffusion-driven instabilities with the concurrent emergence of Turing patterns. Very remarkably, this is the first time an homogeneous cellular medium, with no other dynamic element than 2 locally-active memristors, hence the attribute *all-memristor* coined to address it in this paper, is found to support complex phenomena. The destabilization of the homogeneous solution occurs in this second-order two-cell array if and only if the uncoupled cell circuit parameters are chosen from the Edge of Chaos domain. A deep circuit- and system-theoretic investigation, including linearization analysis and phase portrait investigation, provides a comprehensive picture for the local and global dynamics of the bio-inspired network, revealing how a theory-assisted approach may guide circuit design with inherently non-linear memristive devices.**

***Index Terms*—Smale Paradox, Turing Instability, Prigogine Symmetry-Breaking, Destabilization of the Homogeneous, Emergent Phenomena, Pattern Formation, Two-Cell Reaction-Diffusion System, Cellular Nonlinear/Neural/Nanoscale Network, Bio-Inspired Memristor Oscillator, NaMLab NbO_x Memristor, Threshold Switch, Theory of Local Activity, Edge of Chaos Principle**

implementation of computing paradigms, which could resolve the current limitations of purely-CMOS electronic systems. Moreover, some volatile devices, called threshold switches in the device physics community, are blessed with the extraordinary capability to act as local energy sources under suitable polarization, a property which earns them the attribute of *locally-active* [7]. Since, similarly as the sodium and potassium ion channels in neuronal axon membranes [8], devices of this kind feature a negative differential resistance (NDR) branch along their DC current-voltage characteristic, a signature for their small-signal amplification capability, their adoption in circuit design allows to design more reliable electronic neuron implementations [9], and to develop artificial cellular neural networks, which, processing information according to biological principles, enable the solution of challenging computational problems more efficiently than current software- or hardware-based solutions [10]. Recently [11], the adoption of locally-active NbO_x memristors [12] from NaMLab [13] has enabled to develop the simplest ever-reported reaction-diffusion network, employing only four dynamicas states to reproduce the counterintuitive phenomenon, which mesmerised the American luminary Stephen Smale in 1974 [14], as he reported how two biological cells, mathematically-dead on their own, were unexpectedly found to pulse indefinitely under diffusive coupling. As a follow-up study, this manuscript

2023-Mem-55

Turing's

instability via

dissipation phenomenon

originates from the

Edge of Chaos

Mem-17

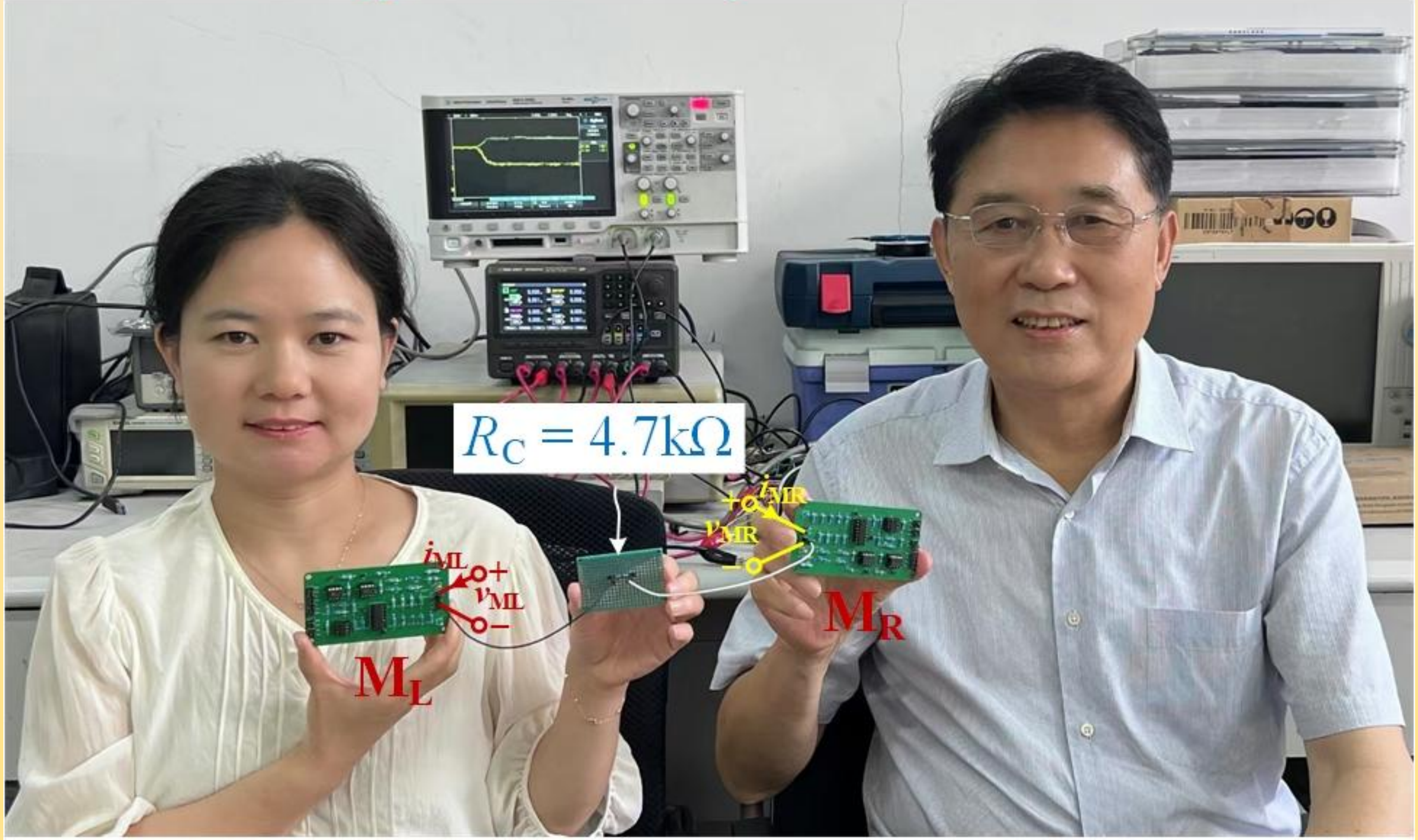
Chemists have pointed out to me that interpreting the reaction R to be an "**open system**" makes the model more acceptable.

A MATHEMATICAL MODEL OF TWO CELLS VIA TURING'S EQUATION
BY
S. SMALE

Chemists have pointed out to me that interpreting the reaction **R** to be an "**open system**" makes the model more acceptable.

In my Talk today, R is a
locally-active System
operating on
Edge of Chaos

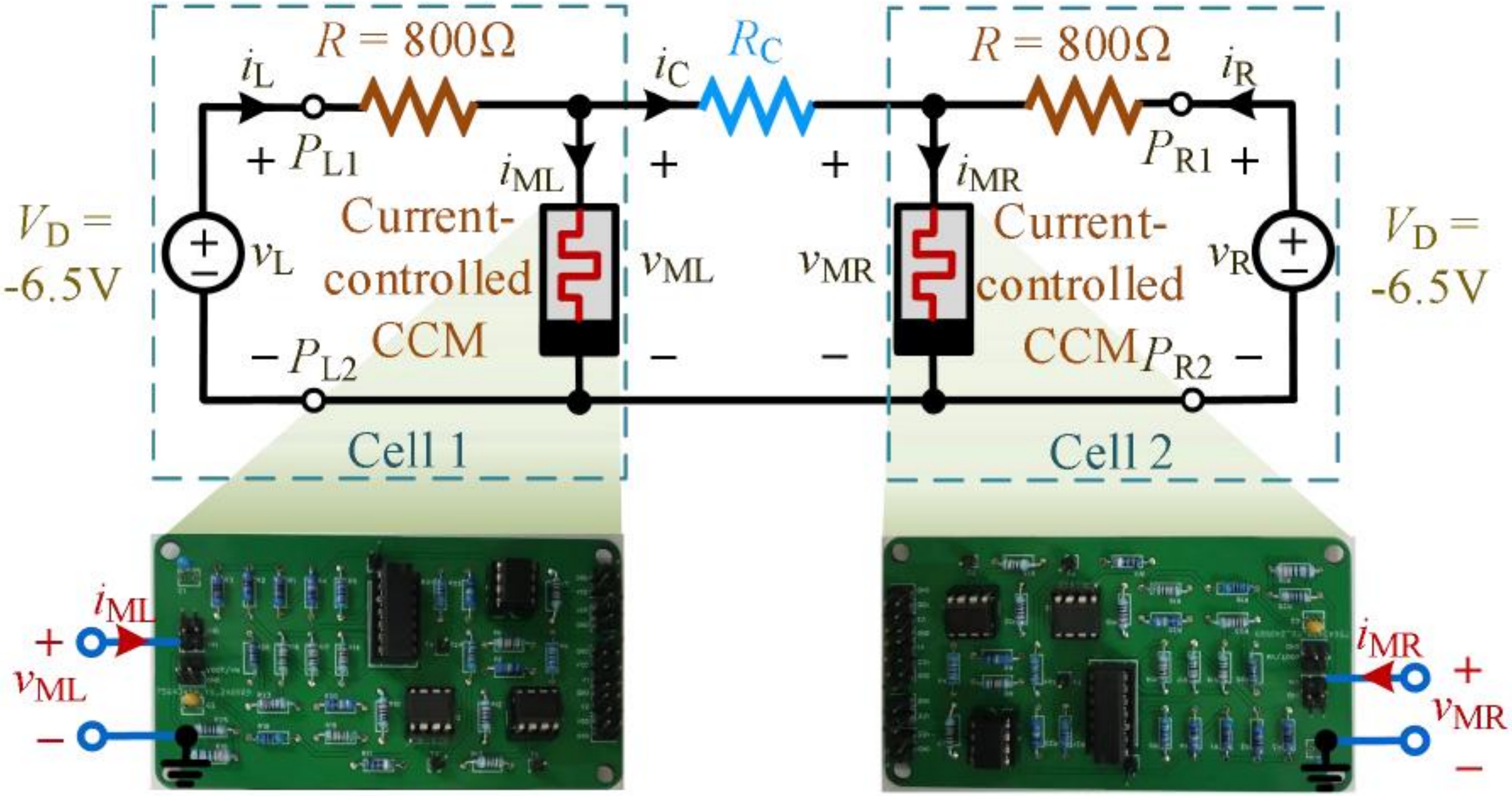
Turing Instability Demonstration



Guangyi Wang (right) and Peipei Jin (left)

W-5

Hardware Demonstration of Turing Instability



W-25

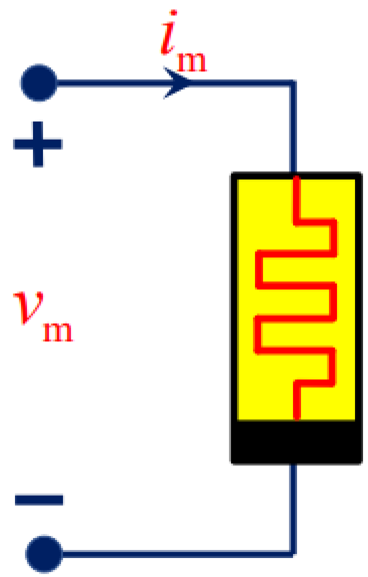
Current-Controlled Chua Corsage Memristor(CCM)

State-Dependent Ohm's Law $$v_m = \left(20x^2\right) i_m$$

State Equation $$\frac{dx}{dt} = h\left(x, i_m\right)$$

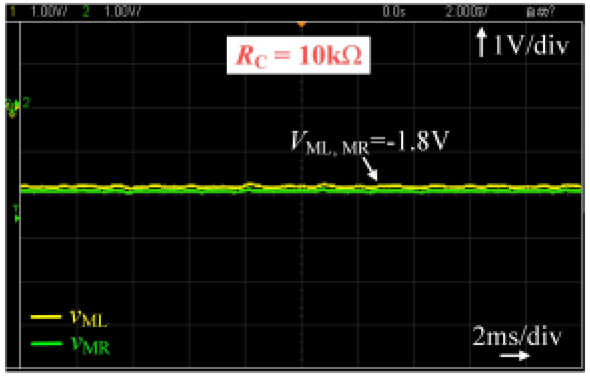
where

$$h\left(x, i_m\right) = 10^4 \left(30 - x + |x - 20| - |x - 40| + 10^3 i_m\right)$$

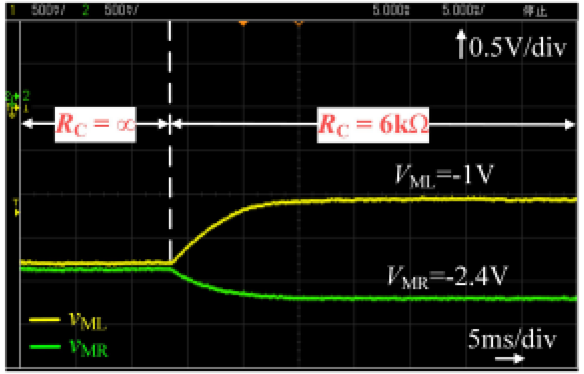


Experimental Results of Turing Instability

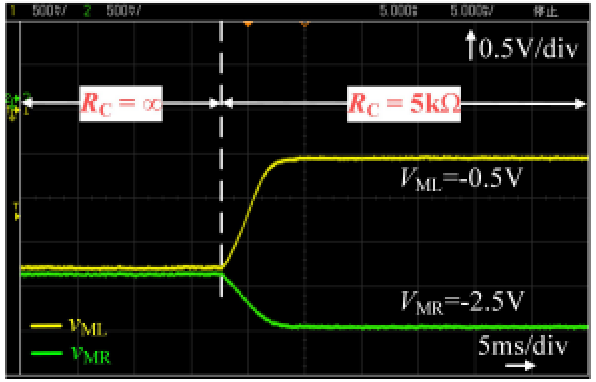
$R_C = 10\text{k}\Omega > 7.1\text{k}\Omega$



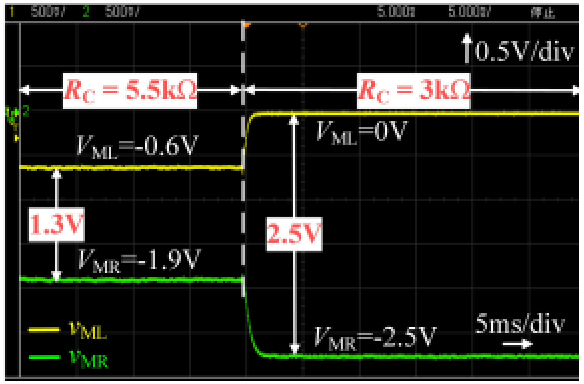
$R_C = 6\text{k}\Omega < 7.1\text{k}\Omega$



$R_C = 5\text{k}\Omega < 7.1\text{k}\Omega$



$R_C = 5.5\text{k}\Omega \rightarrow 3\text{k}\Omega$



Live hardware demonstration of Turing Instability. Experimentally measured memristor voltages $v_{ML}(t)$ and $v_{MR}(t)$ of the left and right cells in the coupled circuit with $R_C = 10\text{k}\Omega$, $R_C = \infty \rightarrow 6\text{k}\Omega$, $R_C = \infty \rightarrow 5\text{k}\Omega$. and $R_C = 5.5\text{k}\Omega \rightarrow 3\text{k}\Omega$.

W-27

There is a paradoxical aspect to the example. One has two dead (mathematically dead) cells interacting by a diffu-sion process which has a tendency in itself to equalize the concentrations. Yet in interaction, a state continues to pulse indefinitely.

A MATHEMATICAL MODEL OF TWO CELLS VIA TURING'S EQUATION
BY
S. SMALE

Edge of Chaos Theory Resolves Smale Paradox

Alon Ascoli, *Member, IEEE*, Ahmet Samil Demirkol, Ronald Tetzlaff, *Senior Member, IEEE*, and Leon Chua, *Life Fellow, IEEE*

***Abstract*—No isolated system may ever support complexity. Emergent phenomena may however appear in an open system, if, as established by the Edge of Chaos theory, some of its constitutive elements feature the capability to amplify infinitesimal fluctuations in energy, provided an external source supplies them with a sufficient amount of DC power, which is known to be a signature for locally-active behaviour. In particular, complex behaviours, including static and dynamic pattern formation, may emerge in arrays of identical diffusively-coupled cells, if and only if the basic unit is poised on a particular sub-domain of the Local Activity regime, referred to as Edge of Chaos, within which a quiet state hides in fact a high degree of excitability. Here we show, for the first time, that these counterintuitive phenomena may emerge in a basic memristor cellular neural network, consisting of two identical diffusively-coupled second-order cells. The proposed bio-inspired array represents the simplest ever-reported open system, which reproduces the shocking phenomenon, reported by Smale in 1974, when, while studying a model from cellular biology, he observed two identical reaction cells, "mathematically dead" on their own, pulsating together upon diffusive coupling. Impressively, the bio-inspired two-cell reaction-diffusion network contains only nine circuit elements, specifically two DC voltage sources, three linear resistors, two linear capacitors, and two functional niobium oxide (NbO) memristors from NaMLab. Applying the theory of Local Activity to an accurate model of the memristor oscillator, a comprehensive picture for its local and global dynamics may be drawn, providing a systematic method to tune the design parameters of the two-cell array to enable diffusion-driven instabilities therein.**

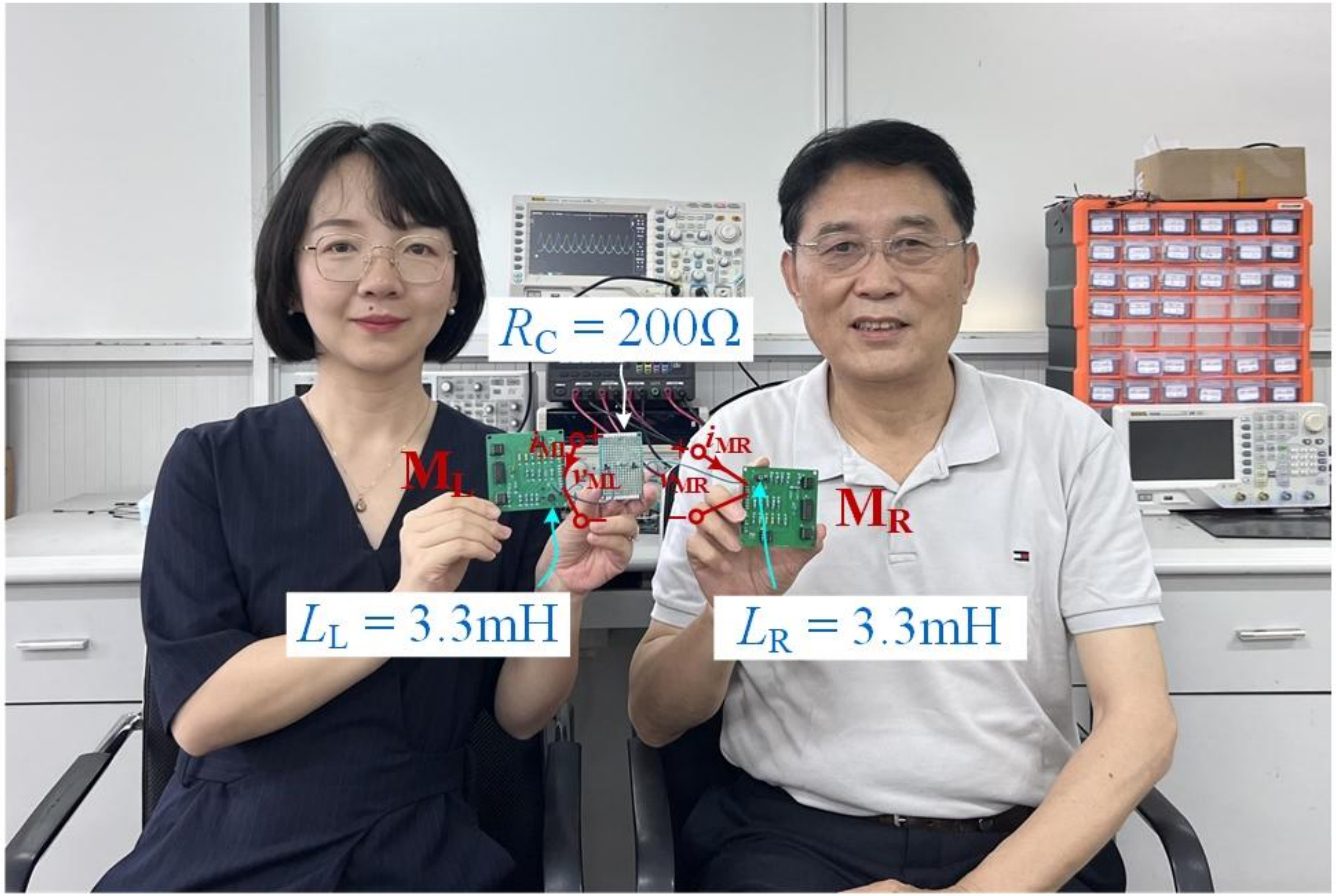
***Index Terms*—Smale paradox, turing instability, Prigogine symmetry-breaking, destabilization of the homogeneous, emergent phenomena, pattern formation, two-cell reaction-diffusion system, cellular nonlinear/neural/nanoscale network, bio-inspired memristor oscillator, NaMLab niobium oxide (NbO) memristor, theory of Local Activity, Edge of Chaos principle.**

lies within a particular sub-domain of the Local Activity regime, referred to as *Edge of Chaos*. A neuronal axon in either the brain [3] or the heart [4], for example, may be modelled as a one-dimensional array of identical diffusively-coupled Hodgkin-Huxley cells. The propagation of an all-or-nothing spike, known as action potential, through this transmission line – an emergent phenomenon "par excellence" [5] – may occur when the single Hodgkin-Huxley cell, hosting biological memristors, namely the sodium and potassium ion channels [6], which are capable to amplify infinitesimal fluctuations in energy under suitable bias conditions, is poised near the Edge of Chaos.

The upcoming end of Moore's era calls for the development of new materials, processing paradigms, and computing architectures, which may allow to keep the integrated circuit performance growth in the years to follow, despite further CMOS transistor size reduction will no longer be feasible. The disruptive memristive nano-technologies ([7], [8]) offer the key to open up unprecedented opportunities in circuit design. Besides their extensive adoption in crossbar configuration for storing data [9] or for accelerating machine learning algorithms [10], which is of great appeal to the electronics industry, especially for artificial intelligence applications [11], non-volatile (volatile) memristor physical realisations may also be employed to design plausible electronic implementations of biological synapses [12] (neurons [13], [14]). Gaining a deep insight into the nonlinear dynamics of these memristors [15] is a fundamental preliminary step toward the ultimate goal to synthesise neuromorphic circuits capable to resemble as close as possible biological neural networks [16]. A compre-

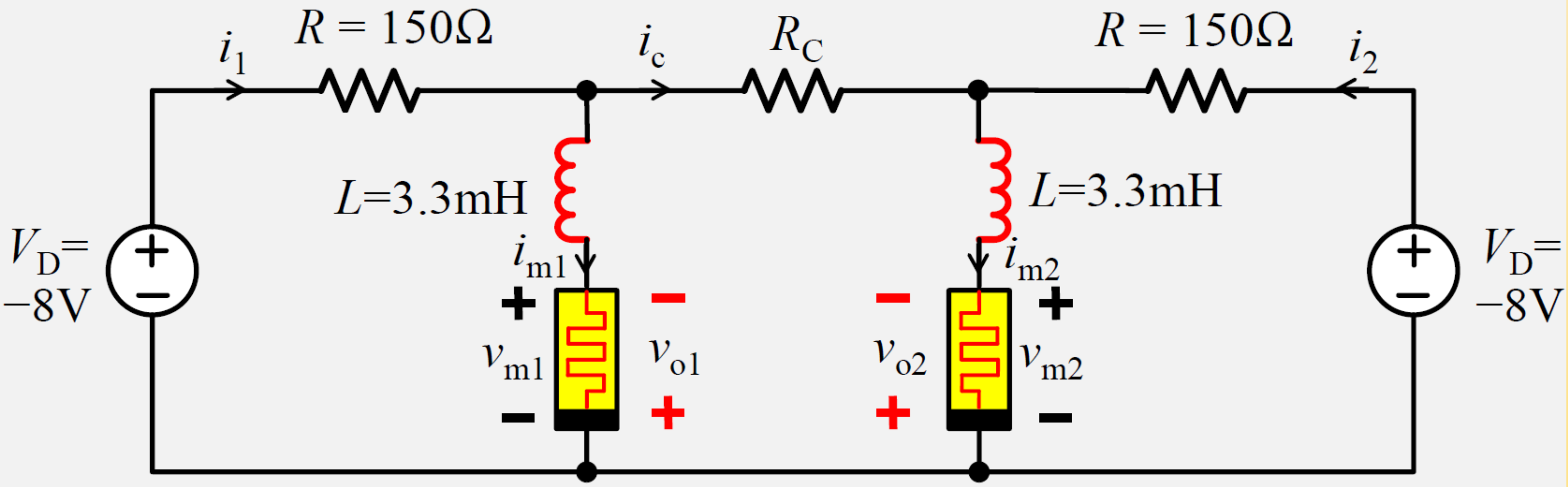
2023-Mem-77

Smale Paradox Demonstration

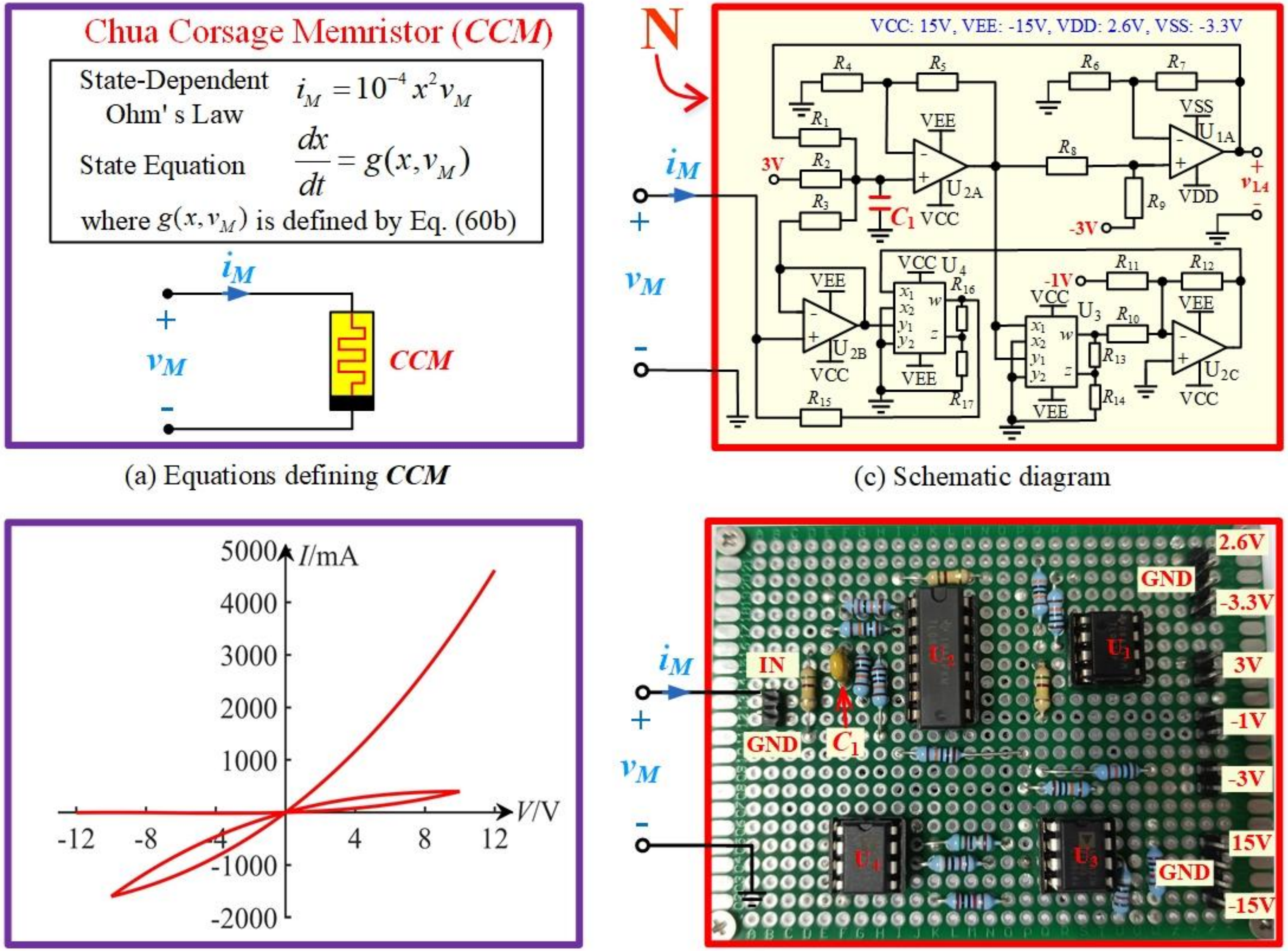


Guangyi Wang (right) and Yan Liang (left)

Coupled Circuit to Demonstrate Smale paradox



JW-3



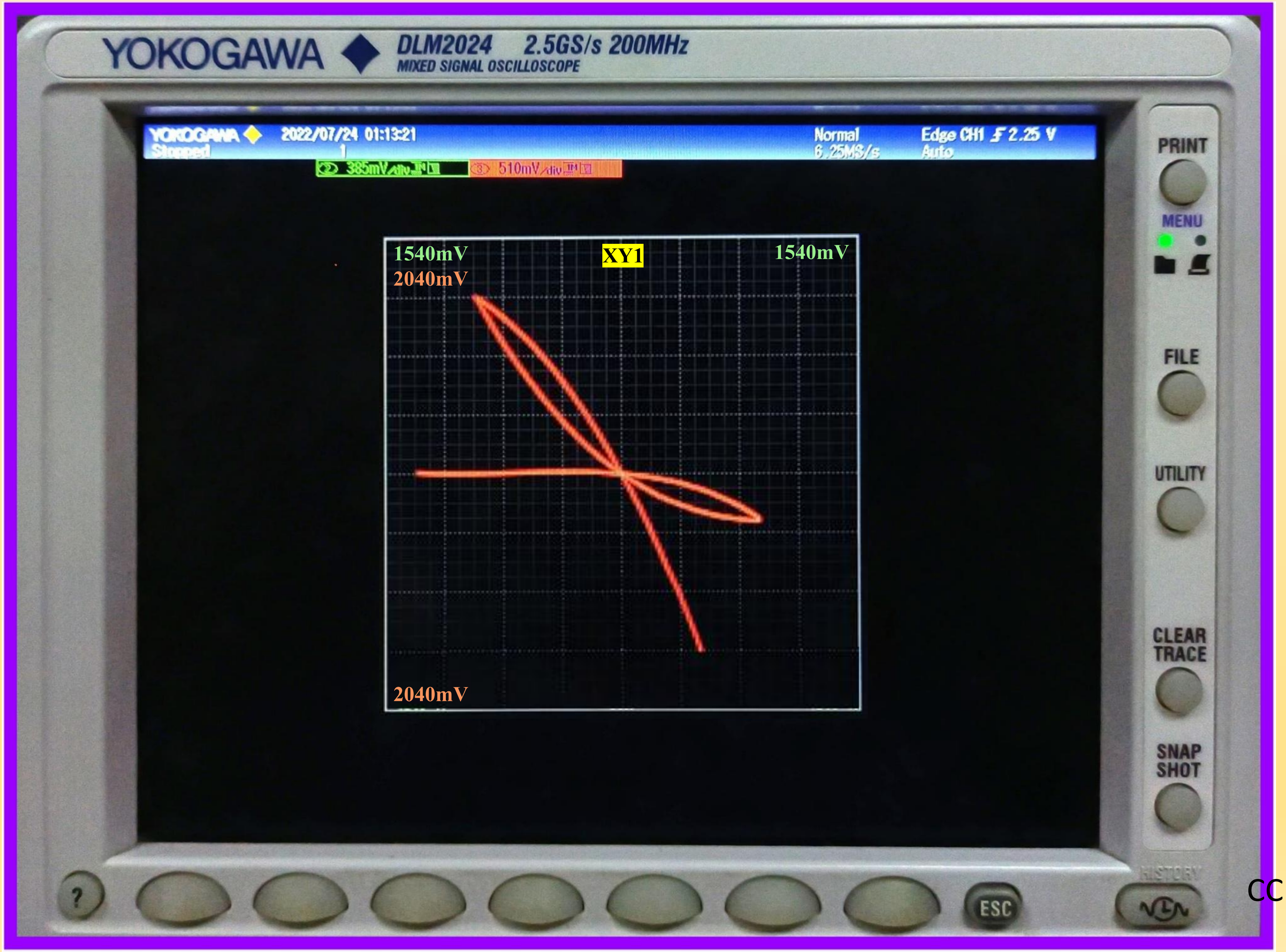
(a) Equations defining ***CCM***

(c) Schematic diagram

(b) DC *V-I* curve calculated from (a)

(d) Circuit board implementing (c)

CCM-IJBC-16


YOKOGAWA
DLM2024 2.5GS/s 200MHz
MIXED SIGNAL OSCILLOSCOPE
2022/07/24 01:13:21
Normal
6.25MS/s
Edge CH1 2.25 V
Auto
385mV/div
510mV/div
1540mV
XY1
1540mV
2040mV
2040mV
PRINT
MENU
FILE
UTILITY
CLEAR TRACE
SNAP SHOT
ESC

CCM-IJBC-20

Corsage Ribbon **decorating dress of** *Marie Antoinette*

2021-memrisys-82

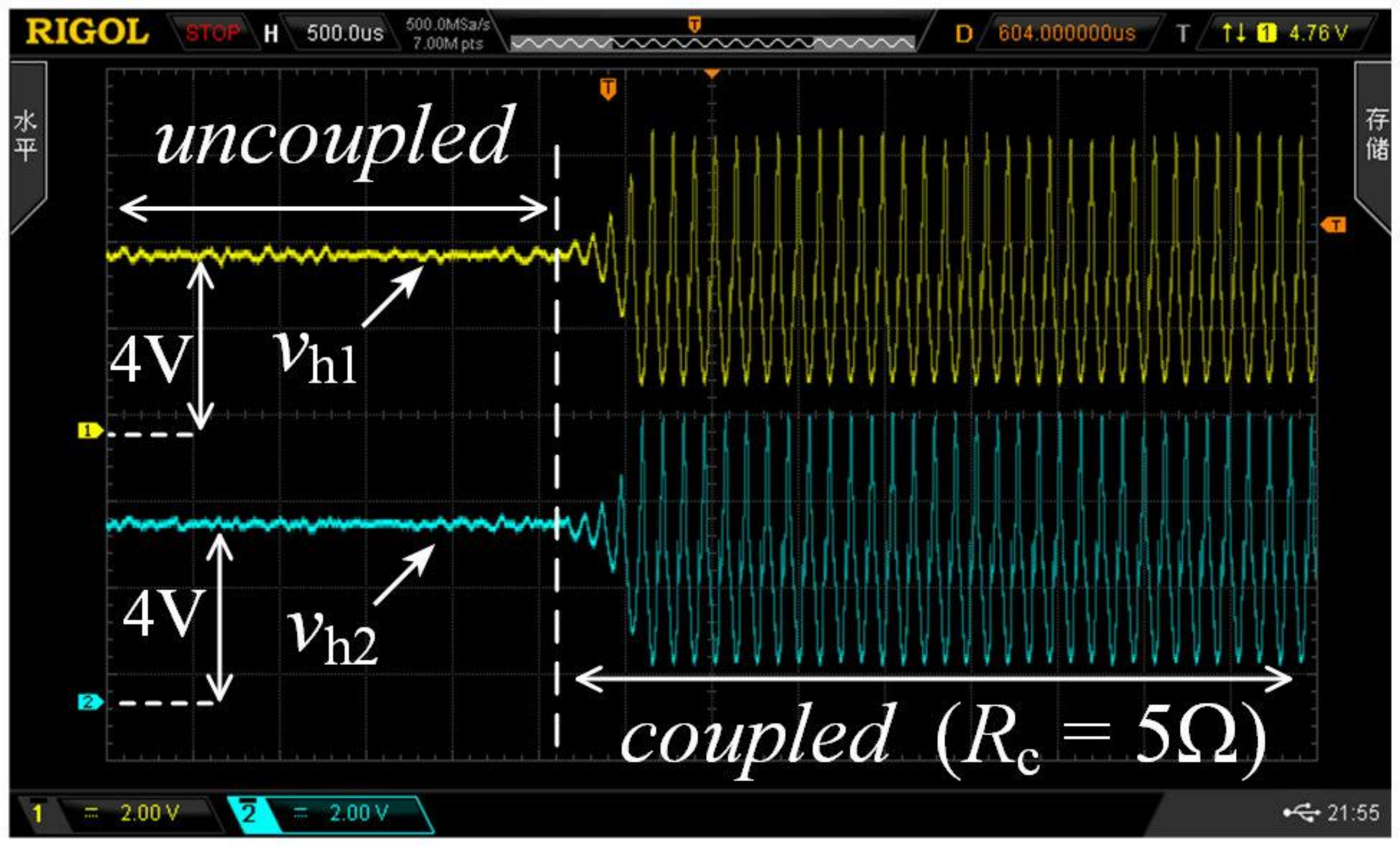
A Poor Man's Memristor

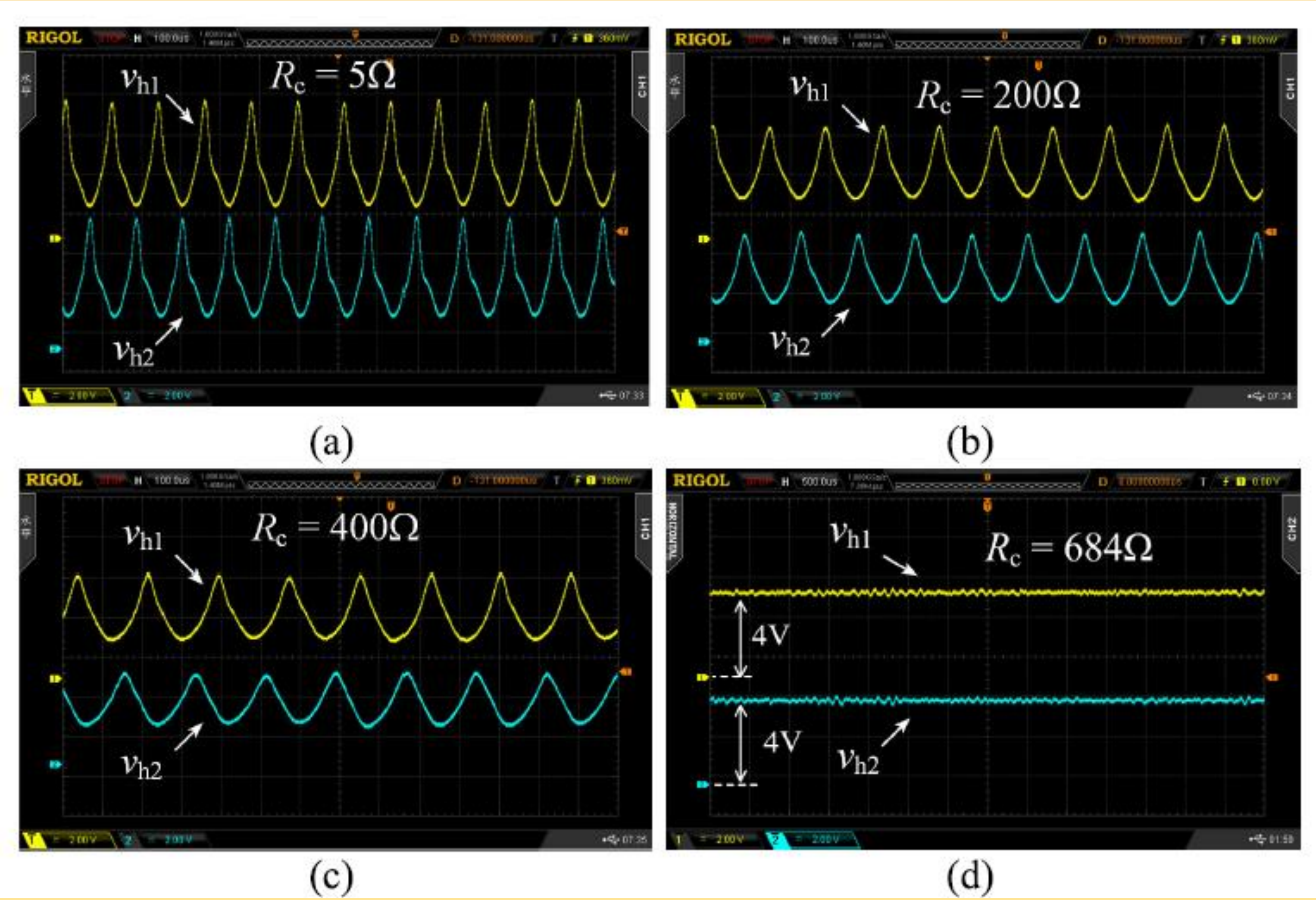
Chua Corsage Memristor

can be built
for less than

$10 !

Ind_4

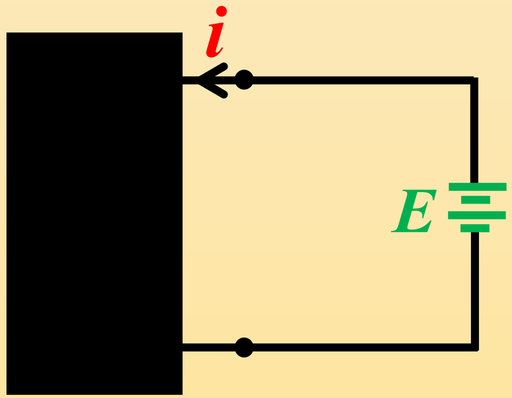

RIGOL
STOP
H 500.0us
500.0MSa/s
7.00M pts
D 604.000000us
T 4.76 V
水平
存储
uncoupled
4V
v_{h1}
4V
v_{h2}
coupled ($R_c = 5\Omega$)
1 2.00 V
2 2.00 V
21:55

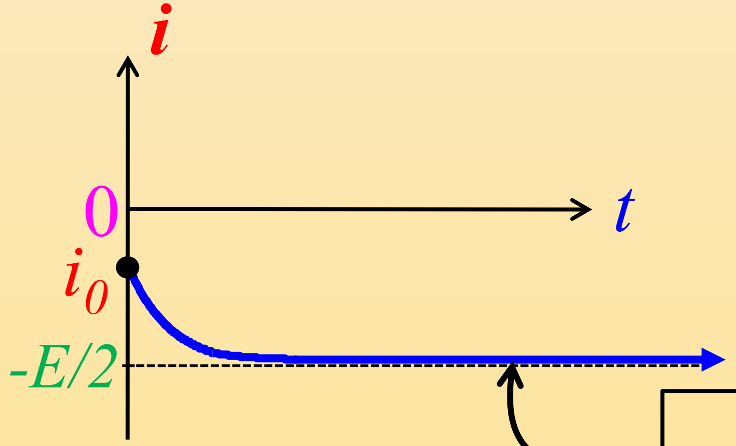

v_{h1}
$R_c = 5\Omega$
v_{h2}
(a)
v_{h1}
$R_c = 200\Omega$
v_{h2}
(b)
v_{h1}
$R_c = 400\Omega$
v_{h2}
(c)
v_{h1}
$R_c = 684\Omega$
4V
4V
v_{h2}
(d)

Chua’s Riddle

Slide 0

Chua's Riddle

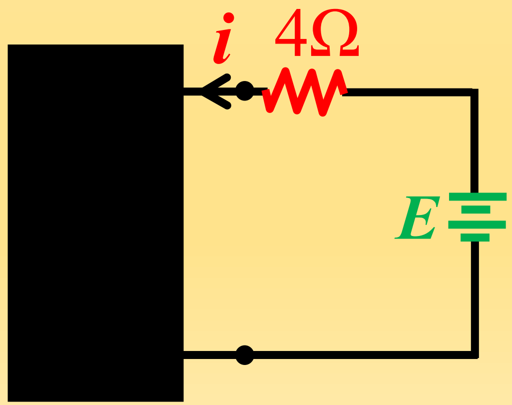


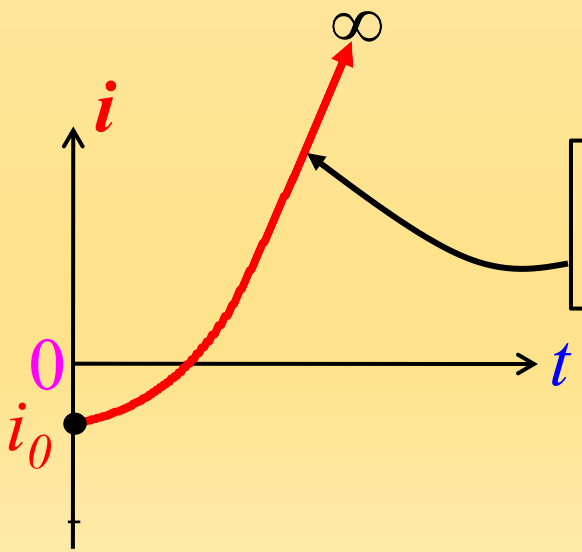


Exponential waveform

with time constant $\tau = 1$

$$i(t) = -\frac{E}{2} + \left[i_0 + \frac{E}{2} \right] e^{-t}$$





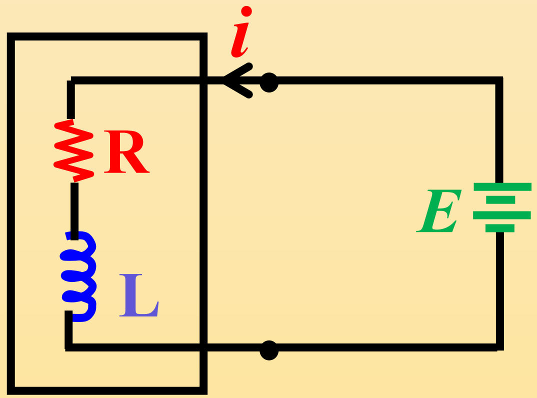
Exponential waveform

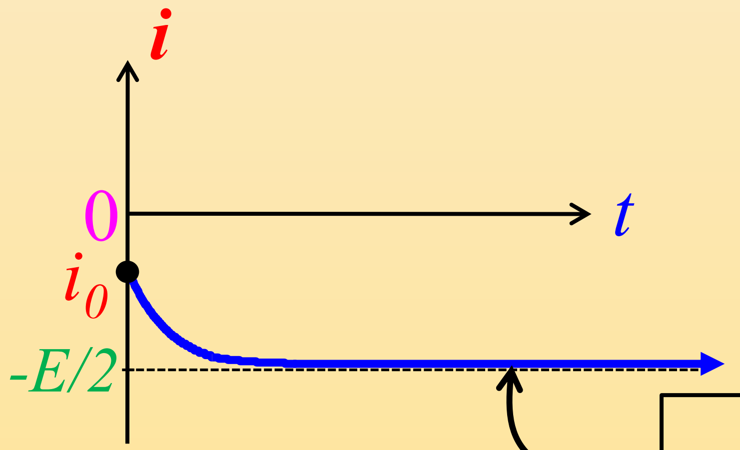
with time constant $\tau = -1$

$$i(t) = \frac{E}{2} + \left[i_0 - \frac{E}{2} \right] e^{t}$$

UK-3a

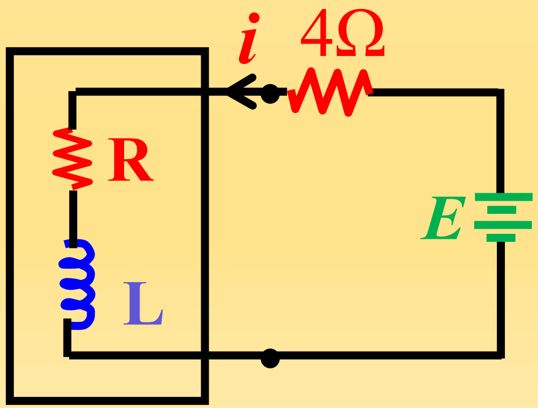
Chua's Riddle

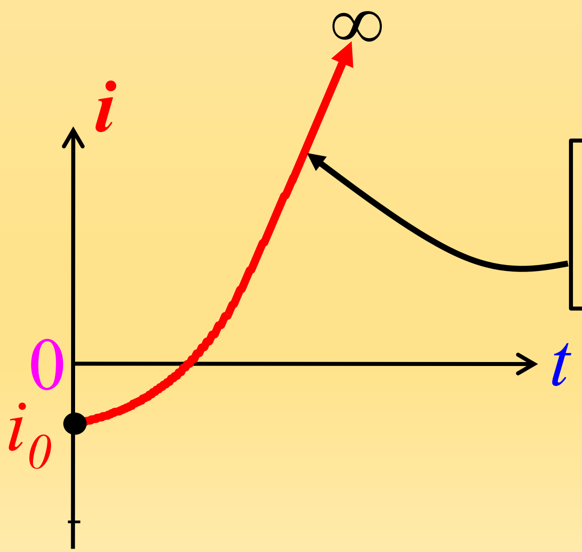




Exponential waveform with time constant $\tau = 1$

$$i(t) = -\frac{E}{2} + \left[i_0 + \frac{E}{2} \right] e^{-t}$$



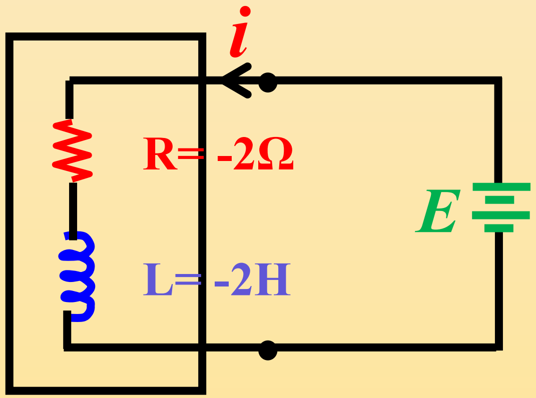


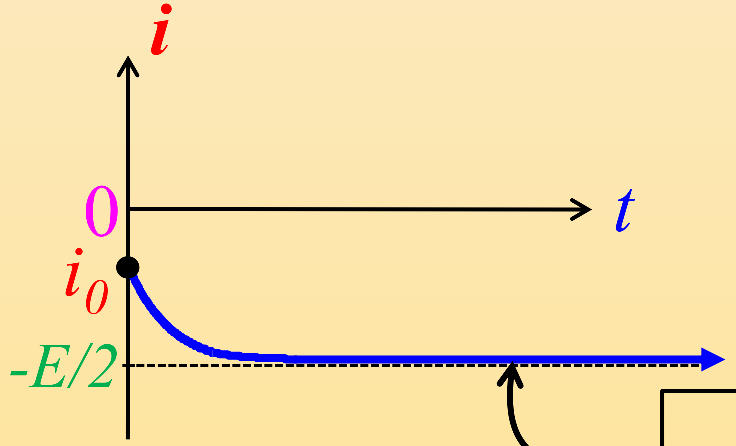
Exponential waveform with time constant $\tau = -1$

$$i(t) = \frac{E}{2} + \left[i_0 - \frac{E}{2} \right] e^{t}$$

UK-3b

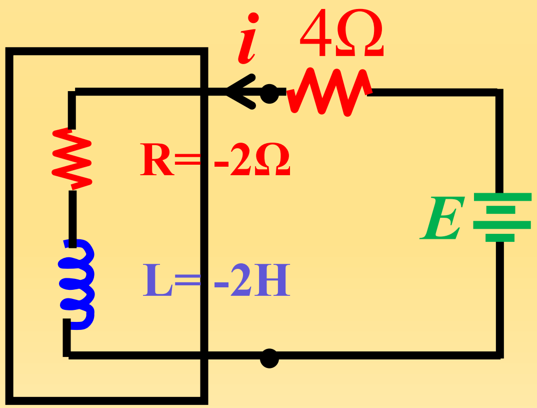
Chua's Riddle

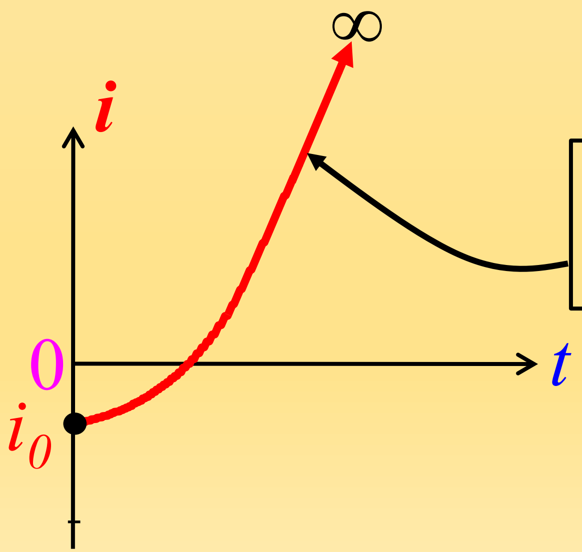




Exponential waveform with time constant $\tau = 1$

$$i(t) = -\frac{E}{2} + \left[i_0 + \frac{E}{2} \right] e^{-t}$$





Exponential waveform with time constant $\tau = -1$

$$i(t) = \frac{E}{2} + \left[i_0 - \frac{E}{2} \right] e^{t}$$

UK-3c

A New type of Computer

Geoffrey Hinton

"We are going to do what I call **mortal computation** where the knowledge that the system has learned and the hardware are inseparable. If we do that we can use very low power analog computation; you can have **trillion-way parallelism** ***using things like memristors***…"

Great great grandson of **George Boole**
Logician and Inventor of
Boolean Algebra

Geoffrey Hinton
Lecture presented at
NeurIPS, Dec. 2022

2024 Nobel Prize in Physics H-1