Pseudorandom Obfuscation

And Applications

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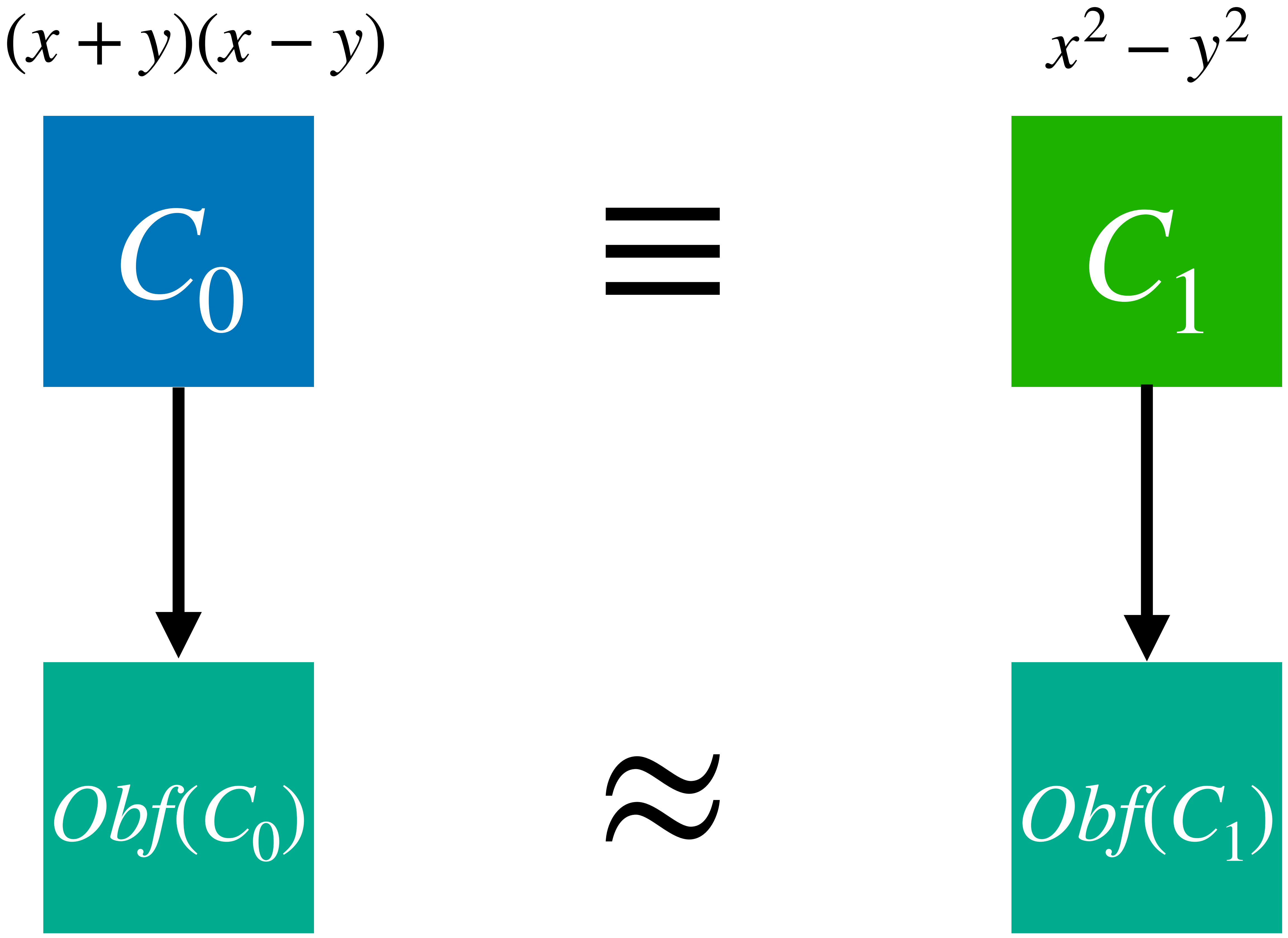
Indistinguishability Obfuscation
[BGI+01,GGH+13]



- Currently only candidates from standard assumptions follows the **[JLS20]** mould
- Critically relies on pairings for compactness (structure!)
- **[BDGM20]** paradigm: Only heuristic instantiations

iO: Swiss Army knife of Cryptography

(subexponential) iO is "crypto complete"

- iO + OWF
  - All the standard Stuff and more
  - PKE, Short Sigs, Perfect NIZKs (non-adaptive SNARGs), OT, Deniable Enc **[SW'14]**
  - FHE **[CLTV'15]**
  - WE **[GGHRSW'14]**
  - …
- iO + SSB
  - More advanced stuff
  - adaptive BE **[Zha'14]**
  - Succinct Garbling **[KLW'15]**
  - …

Or is it?
Limits of iO/VBB

- Doubly Efficient Private Information Retrieval
  - Constructions from Ring-LWE **[LMW'23]**
  - Black-box Separated from VBB **[LMW'25]**
- iO/VBB seems to hit a wall even with (seemingly) simpler tasks:
  - Pseudorandom Codes **[GGW'25, DMS'25]**
  - PKE with pseudorandom keys and ciphertexts

Pseudorandom Encryption

iO and pseudorandomness

		key		ciphertext	
SKE	K	uniform ✓	$r, PRF_K(r) \oplus m$	pseudorandom ✓	
ElGamal	g, h	uniform ✓	$g^r, h^r \cdot m$	pseudorandom under DDH ✓	
[SW'14]	$iO((r,m) \mapsto PRG(r), PRF_K(PRG(r)) \oplus m)$	**not pseudorandom** ✗	$PRG(r), PRF_K(PRG(r)) \oplus m$	pseudorandom ✓	

Even if F computes unstructured function, Obf(F) has structure

Compressing pseudorandom objects

- Pseudorandom keys and ciphertext ⇝ Equivocality (e.g. non-committing Enc)
- iO-based schemes: Structure needs to go somewhere (e.g. into CRS **[CPR17]**)
- Does obfuscation need to expose discernible structure?
- Or can it be pseudorandom?
- Advanced encryption schemes from **evasive LWE [Wee'22,Tsabary'22,VWW'22]** seemingly achieving this feat
- **Are there (achievable) notions of obfuscation which are not at odds with pseudorandomess?**

Rest of the Talk

- Pseudorandom Obfuscation
- Applications
  - FHE
  - Succinct Garbling
  - Succinct WE
- Counterexample & Alternative Notions
- Construction from evasive LWE via BDGM blueprint
- Natural Counterexamples against evasive LWE

Pseudorandom Obfuscation

Pseudorandom Obfuscation

- Can obfuscation $Obf(C)$ be unstructured?
- Can $Obf(C)$ be pseudorandom?
- $Obf(C)$ exposes $TT(C)$
  - **Minimum requirement:** C computes pseudorandom function!
  - Need to consider distributions of circuits
- How does one evaluate a random program?
- Need to fix “machine model” which interprets arbitrary strings as programs
  - $\rightarrow$ some form of universal circuit

Pseudorandom Obfuscation

Strongest Notion: Double Pseudorandomness

Precondition

$TT(C) \approx U$ given aux(C)

$$\Downarrow$$

Postcondition

$PRO(C) \approx u$ given aux(C)

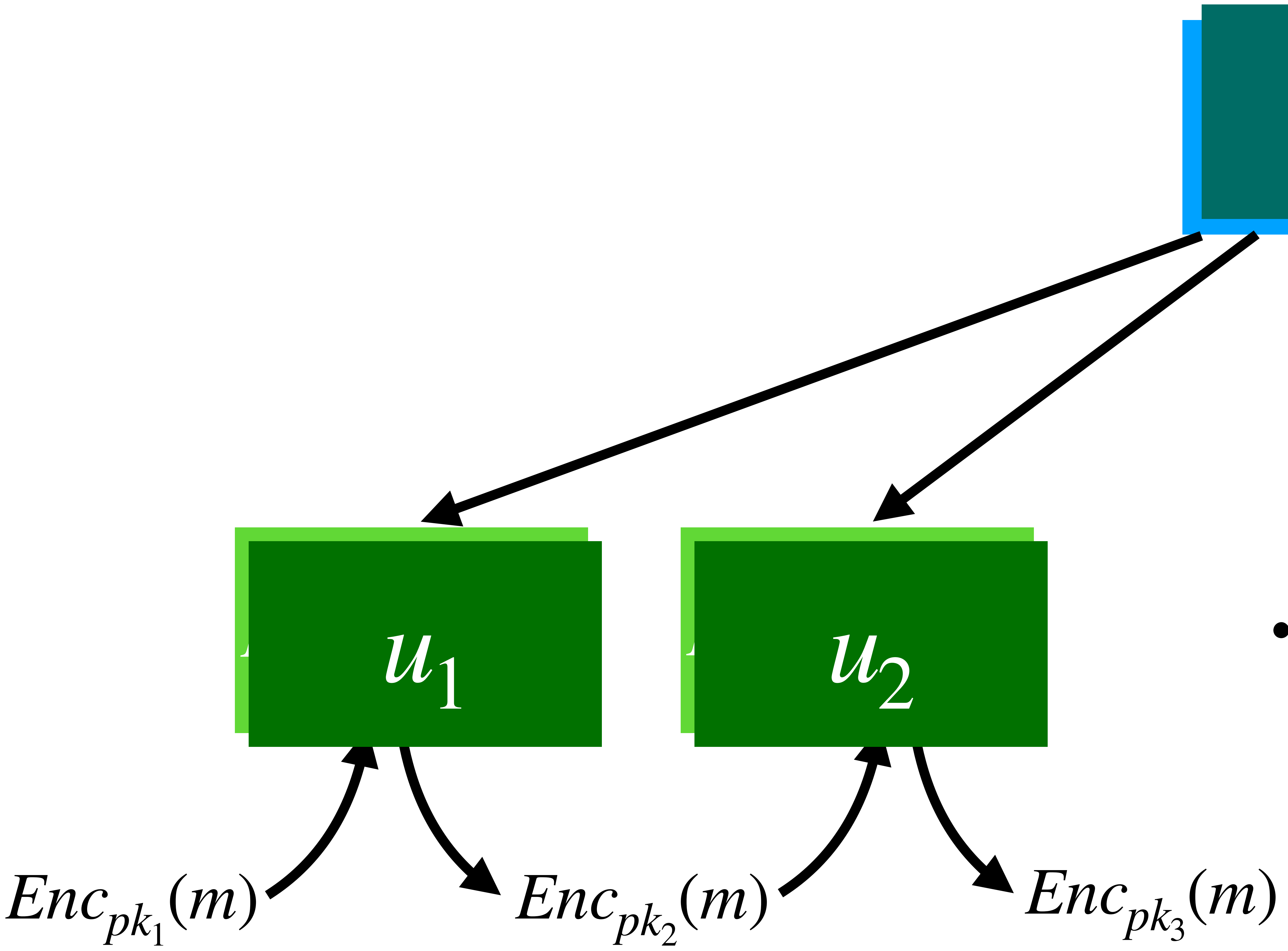
xPRO: $|PRO(C)| = |TT(C)|^{1-\epsilon}$

Applications

Fully Homomorphic Encryption

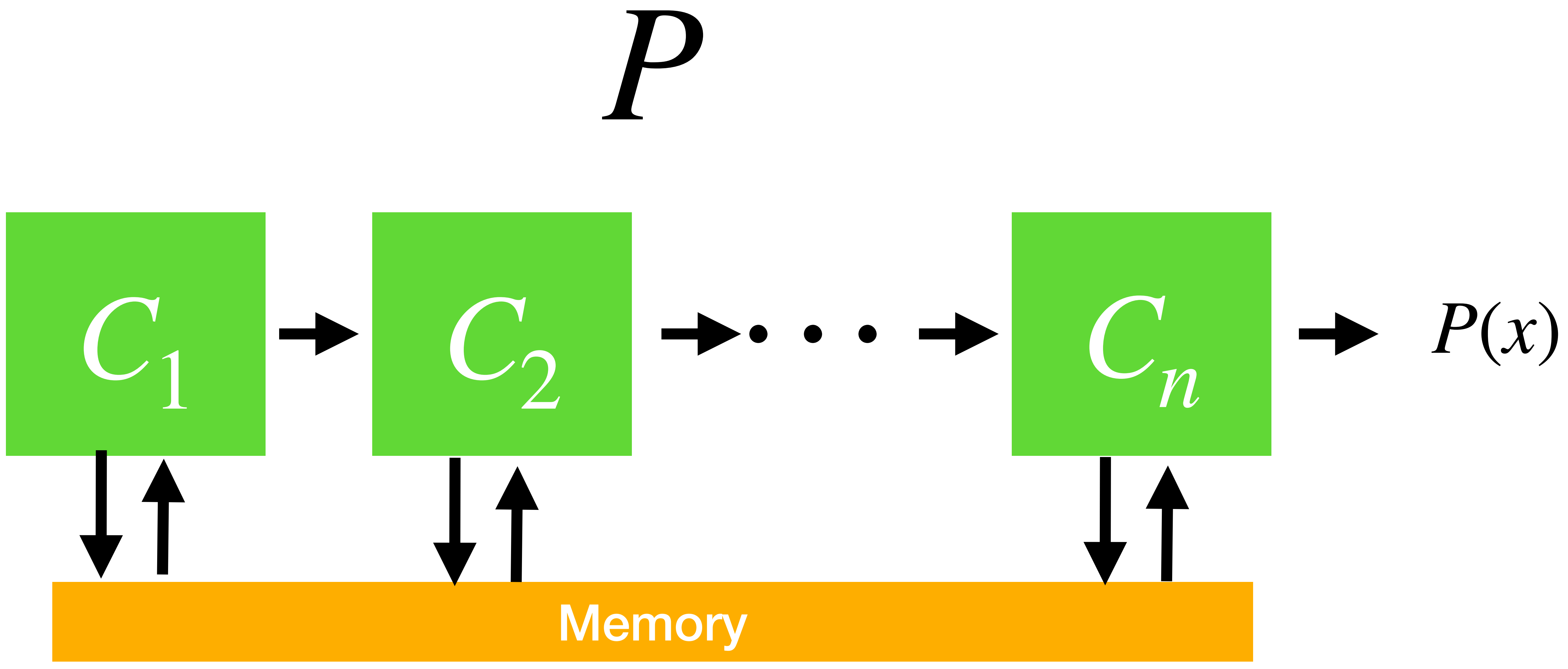
a la [CLTV'15]

- Levelled FHE: pk contains key chain
- Assume FHE with pseudorandom ciphertexts
- Long key chain: $N = 2^\lambda$
- Compute recryption keys locally



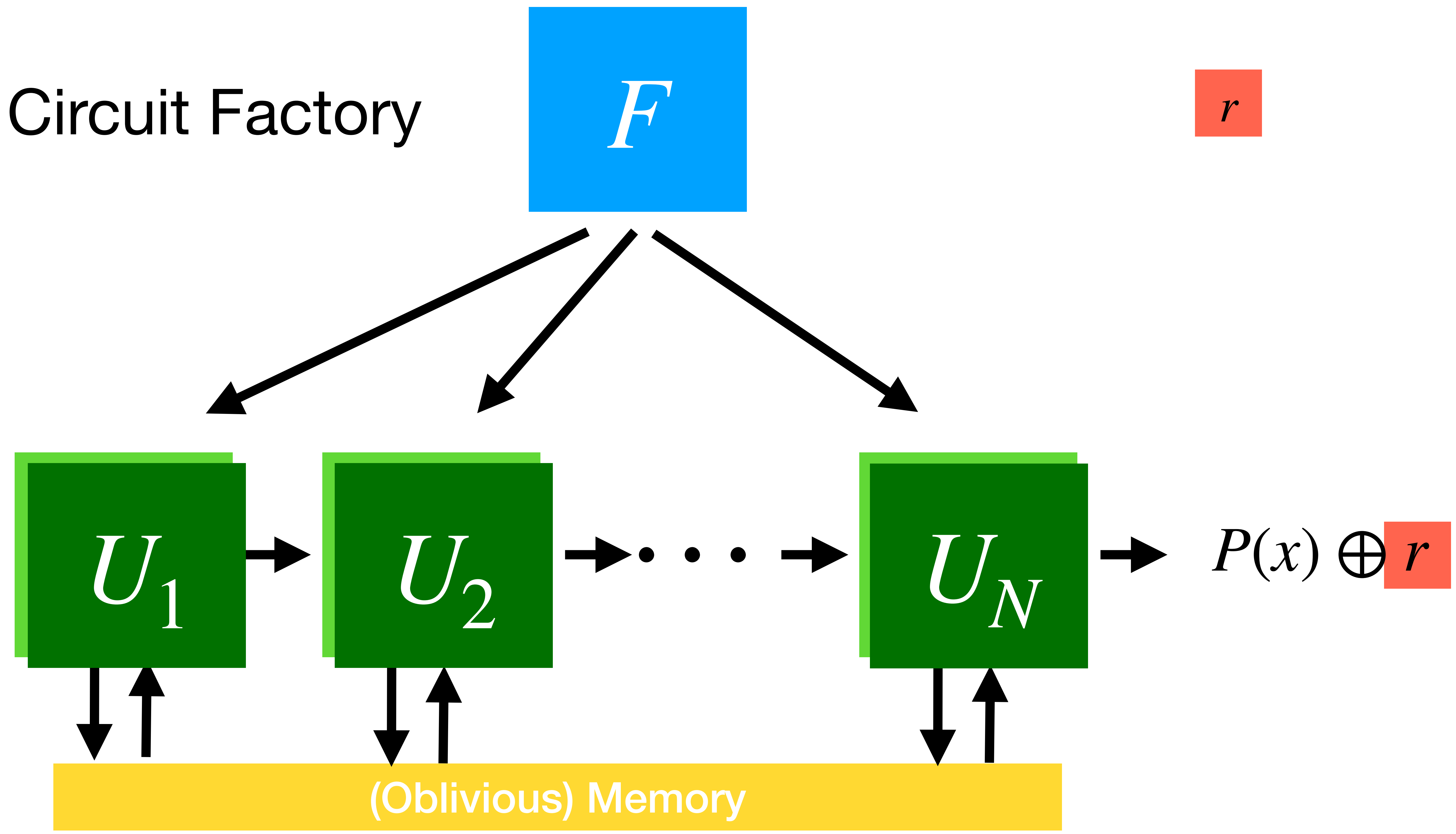
Succinct Garbling

- TM/RAM Computation via Step Circuits
- Oblivious memory access pattern

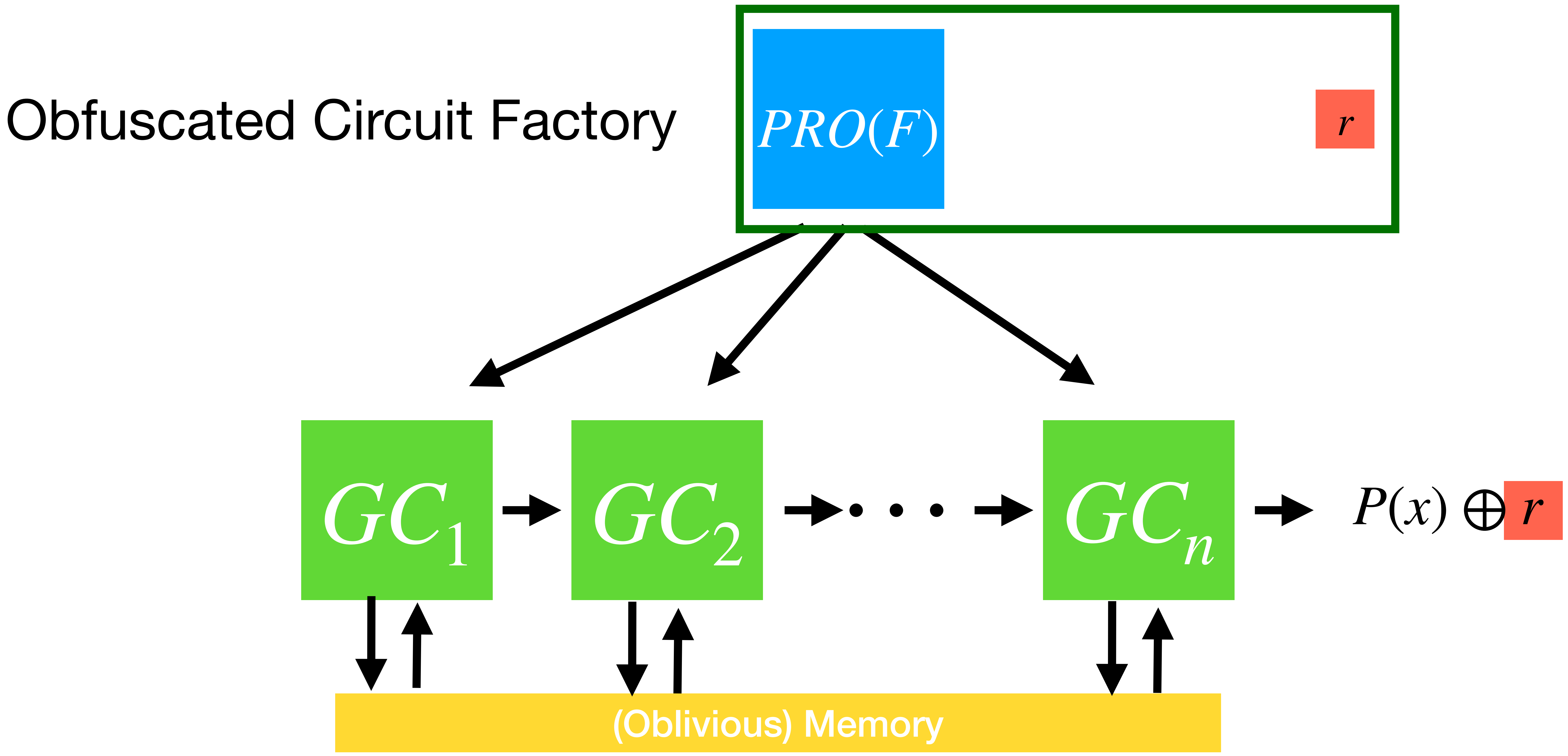


Succinct Garbling

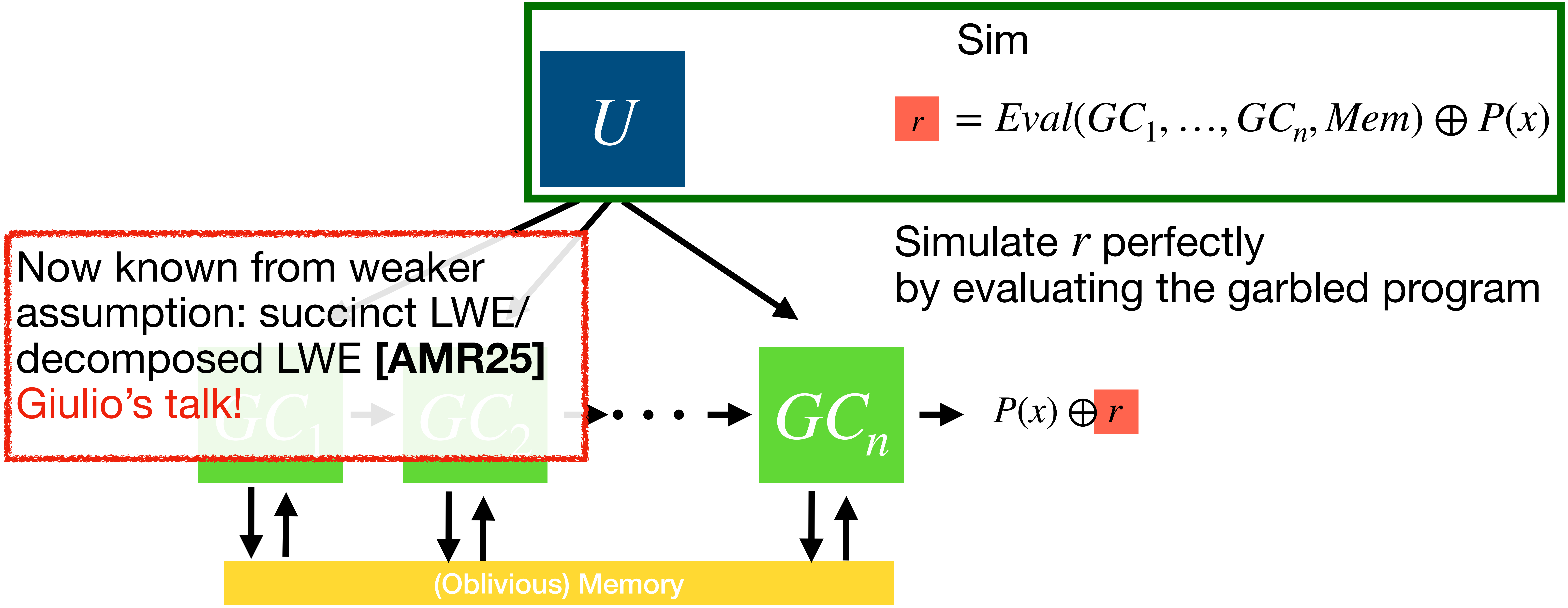
- Garble step circuits
- Last GC outputs a share of the result
- Other share given in plain
- [BLSV'18]: Point&Permute GC are pseudorandom ("Blind GC")
- Produce GCs via circuit factory



Succinct Garbling



Succinct Garbling



Simulate r perfectly
by evaluating the garbled program

Pseudorandom Witness Encryption

[GGHRSW'13] recipe

- Fix NP-language $\mathscr{L}$

C[m,x,K](w): Output

m if $Verify_{\mathscr{L}}(x, w) = 1$

$PRF_K(w)$ otherwise

prWE.Enc(x,m) = PRO(C[m,x,K])

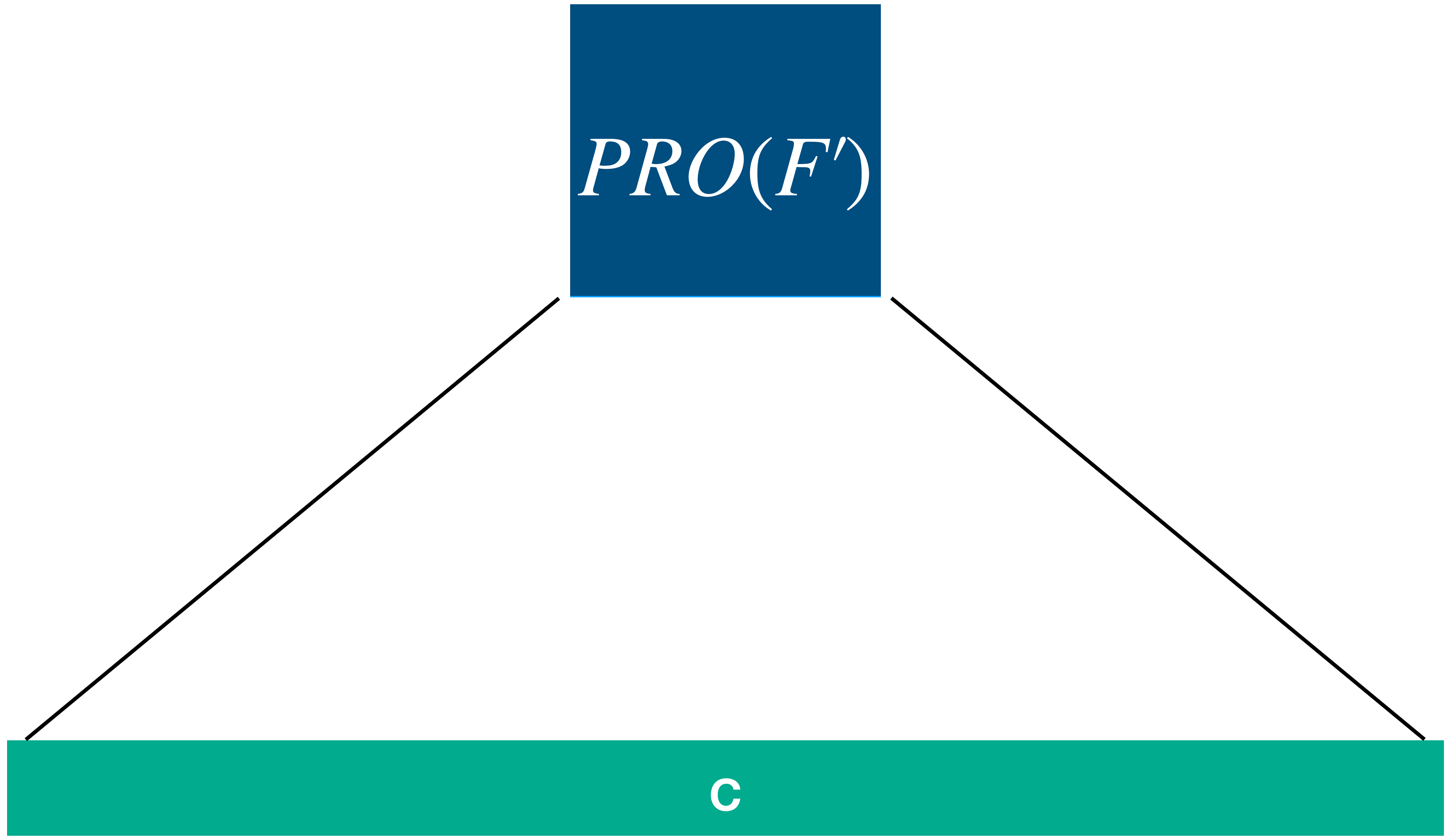
prWE.Dec(c,w) = c(w)

- If $x \notin \mathscr{L}$ then $C[m, x, K](\cdot) \equiv PRF_K(\cdot)$
- By PRO-security prWE.Enc(x,m) is pseudorandom if $x \notin \mathscr{L}$

Succinct Pseudorandom Witness Encryption

Compress further via PRO

- $|WE(x, m)| = poly(|C[x, K]|) = poly(|w|)$
- However, if $x \notin \mathscr{L}$, WE(x,m) is pseudorandom
- Hence $F'_{x,m}(i) = (prWE.Enc(x, m))_i$ is a pseudorandom function with domain-size $log(|w|)$
- Hence define $prWE'.Enc(x, m) = PRO(F'_{x,m})$
- Decrypting $c = prWE'.Enc(x, m)$ given witness w:
  - Set $c' = (c(i))_{i \in [|w|]}$
  - $m = c'(w)$



Issues with PRO

Counterexample to PRO

- Just seen: PRO ⇒ <u>pr</u>WE
  - Hides prWE hides false statements!
- The notion of PRO is “self-defeating”
- Idea: aux is prWE for following statement:
  - x s.t. exists small circuit C st. x = TT(c)
  - i.e. x has small Kolmogorov complexity

Counterexample to PRO

Precondition

	aux
$x = TT(PRF_K)$	prWE(“x is TT of small C”,0^λ)
$\approx$	
u	prWE(“u is ~~TT of~~ small C”,0^λ)
$\approx$	
u	prWE(“u’ is TT of small C”,0^λ)
$\approx$	
u	prWE(“x is TT of small C”,0^λ)

Counterexample to PRO

Postcondition

$x = TT(PRF_K) = TT(PRO(PRF_K))$

aux

$w =$ $PRO(PRF_K)$

prWE(“x is TT of small C”,0^λ)

$WE.Dec$

0^λ

Distinguisher!

u'

prWE(“x is TT of small C”,0^λ)

$WE.Dec$

$\perp$

A Weaker Notion

Indistinguishability PRO (iPRO)

$TT(F_K)$ $\approx$ U

$\Downarrow$

$iPRO(F_{K_0})$ $\approx$ $iPRO(F_{K_1})$

- iO for Pseudorandom Functions
- Implied by iO $\Rightarrow$ no counterexamples!

xiPRO: $|iPRO(C)| = |TT(C)|^{1-\epsilon}$

From xiPRO to xiO

$xiPRO(PRF_K(\cdot) + C(\cdot))$ $\cancel{xiPRO(PRF}_K(x))$

$xiPRO((i,j) \mapsto e(g_1^{x_i}, g_2^{x_2}) \cdot g_T^{C(i,j)})$ $e(g_1^{x_i}, g_2^{x_j})$ ~~via~~ $\cancel{(g_1^{x_i})_i, (g_2^{y_j})_j}$

via $QFE.Enc((x_i)_i, (y_j)_j)$
and $\{sk_{i,j}\}_{i,j}$

$\approx Sim(\{e(g_1^{x_i}, g_2^{x_j})\}_{i,j})$

- Hide the $g_1^{x_i}$, $g_2^{x_j}$ using wrapper of quadratic FE **[Wee'20]**
- Layer of amortisation of quadratic FE keys
- Does not go through local PRGs, LPN: "Coding Hardness"

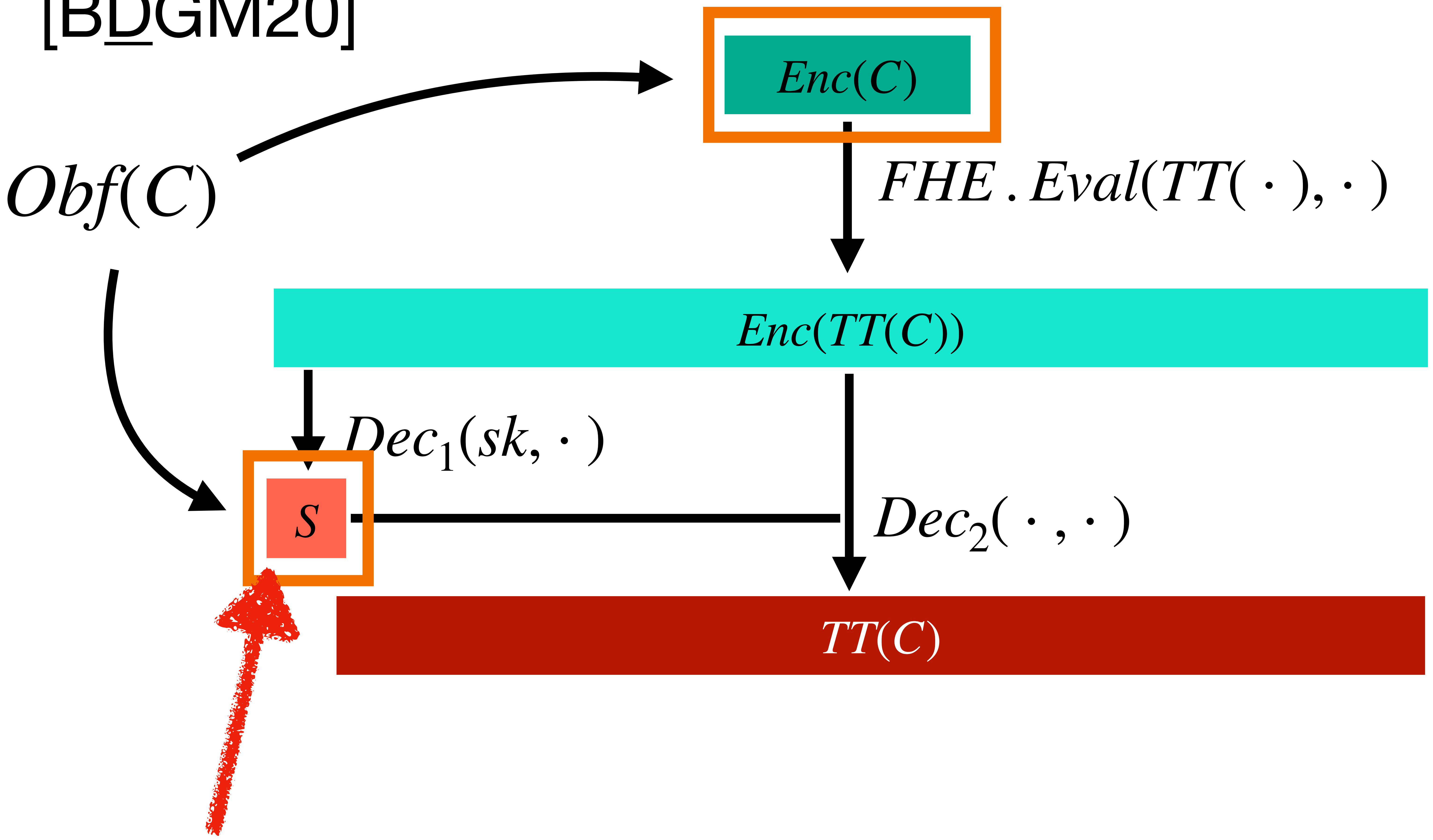
PRO from private-coin evasive LWE

xPRO: PRO with polynomial compression

- Suffices to construct exponentially efficient version xPRO
- Bootstrapping similar to **[BV15,AJ15]**
- Additional ingredients: Blind Garbled Circuits **[BLSV18]** and blind LFE (new)

BDGM Approach to xiO

[BDGM20]

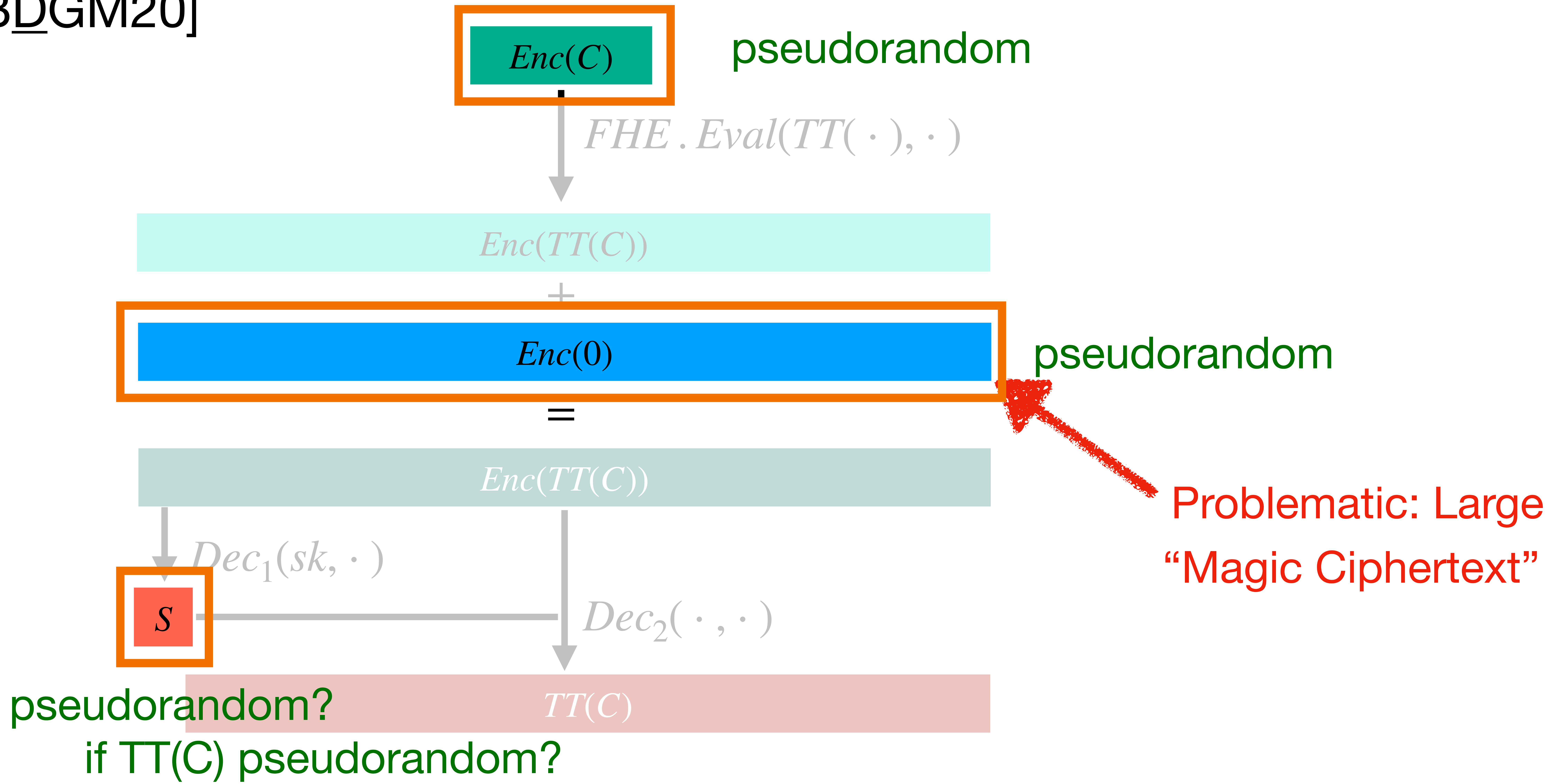


Dual-Regev Enc
(think hybrid encryption)

Leaks information about $FHE.sk$, C

BDGM Approach to xiO

[BDGM20]



Learning with Errors

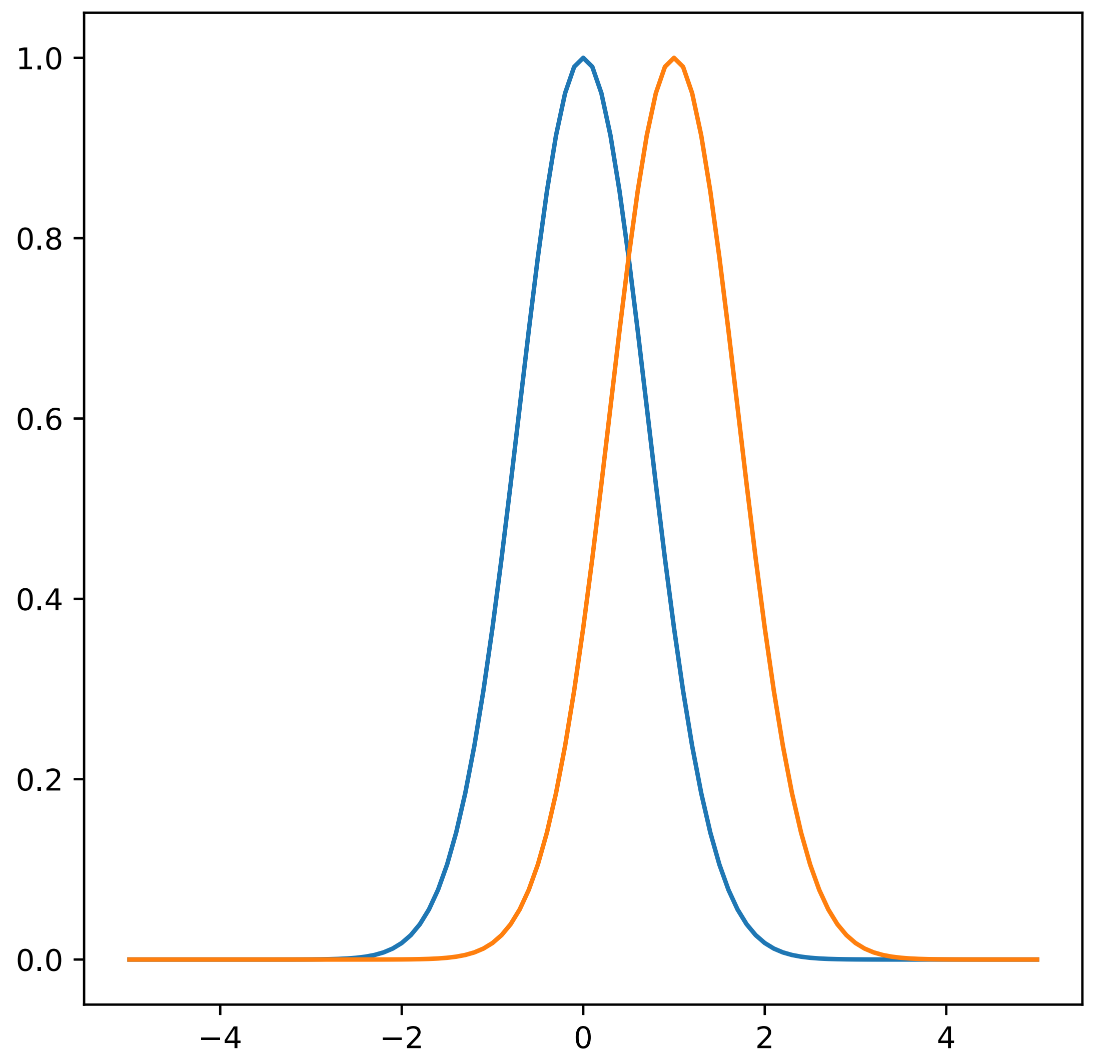
[Regev05]

$S \; A + E \approx U$ given A

gaussian

Noise Drowing/Flooding/Smudging

$E + Z \approx_s E$



Non-compact Magic Ciphertext

$\equiv$ xPRO Precondition

- Magic ciphertext is dual-Regev encryption of 0
- Idea: Magic ciphertext can be simulated given S and $TT(C)$
- E drowns noise artefacts
- Replace $Enc(C)$ with $Enc(0)$
- Pseudorandomness of TT(C) makes (simulated) magic ciphertext pseudorandom
- **Establishes PRO precondition**

$$Enc(0)$$

$$=$$

$$S' \quad P + E'$$

$$\approx$$

$$SP - Enc(TT(C)) + TT(C) + L(sk) + E$$

$$\approx$$

$$SP - Enc(TT(C)) + TT(C) + E$$

$$\approx$$

$$SP - Enc(TT(0)) + TT(C) + E$$

$$\approx$$

$$U$$

Compressing the Magic Ciphertext

Pseudodrowning

- Magic ciphertext $SP + E$ is as large as truth table $TT(C)$
- Need to compress $SP + E$
- **Evasive LWE Recipe/Heuristic:** Actual scheme contains a term $SB + E$ and a short matrix $B^{-1}(P)$ with $B \cdot B^{-1}(P) = P$

Compressing the Magic Ciphertext

Rationale

$$\left(S \; B + E \right) \; B^{-1}(P)$$

$$= S \; P \; + \; E \; B^{-1}(P)$$

In low rank subspace

Heuristic: behaves like

$$S \; P \; + \; E'$$

full rank

Private Coin Evasive LWE

[Tsabary'22,Wee'22,VWW'22]

If

Precondition

$$S B + E \;,\; S P + E' \;\approx\; U \;,\; U' \quad \text{given} \quad \text{aux}$$

Then

Postcondition

$$S B + E \;\approx\; U \quad \text{given} \quad B^{-1}(P) \quad \text{aux}$$

Compressing the Magic Ciphertext

Pseudodrowning

- **Evasive LWE Heuristic**: Only thing you can do with $B^{-1}(P)$ is expand samples and then ignore it
- Noise term in subspace behaves like a fresh gaussian noise term, use it to drown artefacts
- Evasive LWE is (essentially) always used for “pseudodrowning”

Direct Counterexamples to evasive LWE

- Evasive LWE counterexample from PRO is *contrived/pathologic*
- Direct/natural counterexample questioning the “pseudo-drowning” heuristic

Direct Counterexamples to evasive LWE

Precondition

aux

$$sB + e \quad sP + e' \quad sP - t \qquad t + e' \to e' \mod 2$$

$$\approx$$

$$u \quad u' \quad sP - t$$

Mask t reveals exactly LSBs of error e'

Direct Counterexamples to evasive LWE

aux

$B^{-1}(P)$ $\quad sP + eB^{-1}(P)$ $\quad sP - t$ $\qquad eB^{-1}(P) \mod 2$

$$\Cap$$

$Span(B^{-1}(P) \mod 2)$

Distinguisher!

$B^{-1}(P)$ $\quad u'$ $\quad sP - t$ $\qquad \tilde{u} \mod 2$

$$\not\Cap$$

$Span(B^{-1}(P) \mod 2)$

Takeaways

- PRO powerful tool to augment iO-like constructions with pseudorandomness properties
- In general too good to be true
- Notion of PRO suffers from counterexamples
- Concept of PRO shed light on issues with evasive LWE
- Fallback: iO for pseudorandom functions (iPRO), implies iO using standard pairing assumptions
- Looking ahead: Stronger Notions of PRO that do not suffer from counterexamples?

Thanks!