# Intro to Quantum Gibbs Sampling: Algorithmic Ingredients.

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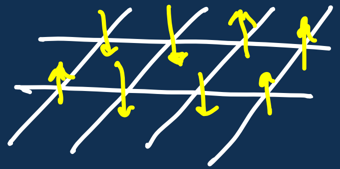
# Classical Gibbs Distribution

Ex:



Classical local Hamiltonian

Ising $-\sum_{\langle i,j \rangle} Z_i \cdot Z_j$

Eigenstates : $S_n = \{0,1\}^n$

Goal is to sample from $p(x) = \dfrac{e^{-\beta E_x}}{Z}$, $Z = \sum_x e^{-\beta E_x}$

## Discrete Time Markov Chain

Random walk on $S_n$

$P(X_t = j \mid X_{t-1} = i) = P_{ij}$

$X_t \in S_n$ location at time $t$.

$P$ : stochastic matrix.

# Classical Metropolis Algorithm

Set of jump operators $A^a$

$A^a$ selected with probability $p(a)$.

Ex: flip a randomly chosen spin.

Current state $= x$.

- Pick $A^a$ w.p. $p(a)$.
- Apply $A^a$ to $x \to y$.
- Accept move w.p. $\underbrace{\min\{1, \exp(-\beta(E_y - E_x))\}}_{\gamma_\beta(E_x, E_y)}$
- If reject, go back to $x$.

$\beta = O(\mathrm{poly}(n))$

# Classical Metropolis Algorithm

Gibbs distribution $\pi_\beta$ is a fixed point of Metropolis

Detailed balance: $e^{-\beta E_x} \gamma_\beta(E_x - E_y) = e^{-\beta E_y} \gamma_\beta(E_y - E_x)$

If jump operators mix enough then the fixed point is unique.

# Quantum Gibbs Sampling

Quantum Hamiltonian $H$

Eigenstates $\{|\psi_j\rangle\}_j$ Eigenvalues $\{E_j\}_j$

Prepare distribution: $|\psi_j\rangle$ w.p. $e^{-\beta E_j}/Z$.

Mixed State:

$$\rho_\beta = \sum \frac{e^{-\beta E_j}}{Z} |\psi_j\rangle\langle\psi_j| = \frac{\exp(-\beta H)}{\mathrm{Tr}[\exp(-\beta H)]}$$

# Quantum Gibbs Sampling

Challenges:

1. Given eigenstate $|\psi_j\rangle$ cannot calculate $E_j$ exactly

2. Rejection procedure requires backing up after a quantum measurement.

# Classical Continuous-time Markov Chains

State $X(t)$ is a function of $t \in \mathbb{R}^{\geq 0}$

Markov:

$$\Pr(X(t+s)=j \mid X(s)=i)$$

$$= \Pr(X(t+s)=j \mid X(s)=i, \text{ events before } t-s)$$

Time-Homogeneous

$$\Pr(X(t+s)=j \mid X(s)=i) = P_{ij}(t)$$

independent of start time s.

# Classical Continuous-time Markov Chains

One way to specify cont-time M.c. $P_{ij}(t)$ for all $t$.

Instead use generator matrix $Q$:

$$Q \triangleq \lim_{\Delta t \to 0} \frac{P(\Delta t) - I}{\Delta t}$$

Properties of $Q$:

- $P_{ij} \geq 0$ if $i \neq j$.
  $P_{ii} \leq 0$
- Rows sum to 0.

# Classical Continuous-time Markov Chains

$$\frac{dP}{dt} = P \cdot Q \qquad \frac{d\pi}{dt} = \pi Q$$

$\pi$: distribution over states.

$$\Rightarrow \quad P(t) = e^{Qt} = \sum_{n=0}^{\infty} \frac{1}{n!} Q^n t^n$$

From

Lecture Notes by Miranda Holmes-Cerfon (UBC)

$\pi_0$ is a fixed point if

$$\pi_0 = \pi_0 P(t) \quad \forall t \iff \pi_0 Q = 0.$$

# Classical Continuous-time Markov Chains

$$Q = \begin{bmatrix} \ddots & & \geq 0 \\ & \ddots & \\ \geq 0 & & \ddots \end{bmatrix}$$

# Classical Continuous-time Markov Chains

$$Q = \begin{bmatrix} & & & & & \\ q_{i1} & q_{i2} & \cdots & \boxed{\phantom{q}} & \cdots & q_{id} \\ & & & & & \end{bmatrix} \qquad q_{ii} = -\sum_{j \neq i} q_{ij}$$

For any $\pi$

$$\sum_i \frac{d\pi_i}{dt} = 0 = \pi Q \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

# Generator for Classical Continuous Metropolis:

Jump operators $A^a$ non-negative matrices (not nec. stoch.)

$\sum_a p(a) A^a$ is symmetric

$$Q^\beta_{ij} = \sum_a p(a) \Big[ \underbrace{\gamma_\beta(E_j - E_i)}_{\text{accept}} A^a_{ij} - \delta_{ij} \underbrace{\sum_{k \neq i} \gamma_\beta(E_k - E_i)}_{\text{reject.}} A^a_{ki} \Big]$$

$$\pi_\beta \cdot Q^\beta_{ij} = 0$$

# Evolution of Closed Quantum Systems.

Hamiltonian $H$

Schrödinger's Equation $\quad \frac{d|\psi\rangle}{dt} = -iH|\psi\rangle$

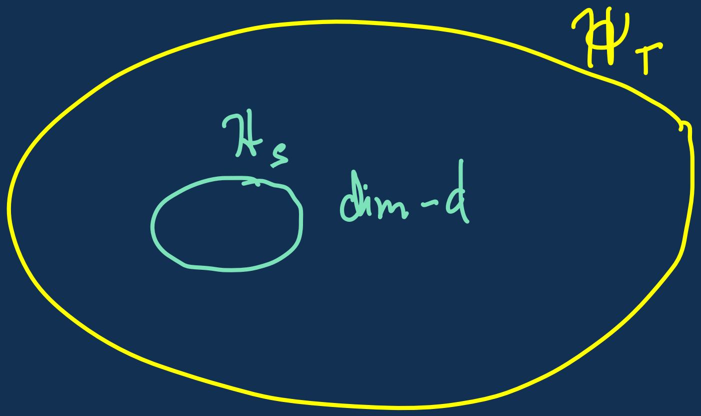
$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

Simulation in QC
[Abrams + Lloyd '97]

Generalize: Classical → Quantum
Quantum Closed → Quantum Open

$\Big\}$ Lindbladian.

# ~~Closed~~ Open Quantum Systems




$\dim \mathcal{H}_T = D.$

Pure state $|\psi_T\rangle$

$D$-dim vector $\quad \langle\psi_T|\psi_T\rangle = 1$

$\Downarrow$ trace out $T-S$

Ensemble of states

$\{|\psi_j\rangle, p_j\} \qquad |\psi_j\rangle \in \mathcal{H}_S.$

$$\rho_S = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

density matrix: positive operator

Trace = 1.

# A Bit About Mixed States (density matrices)

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Unitary evolution $\rho \xrightarrow{U} U\rho U^\dagger$

Schrödinger's Equation $\frac{d|\psi\rangle}{dt} = -iH|\psi\rangle$

$$\frac{d\rho}{dt} = -i(H\rho - \rho H) = -i[H,\rho]$$

$$[A,B] \triangleq AB - BA$$

Fock-Liouville

$\underset{d\times d}{\rho} \to \underset{d^2\times 1.}{|\rho\rangle}$

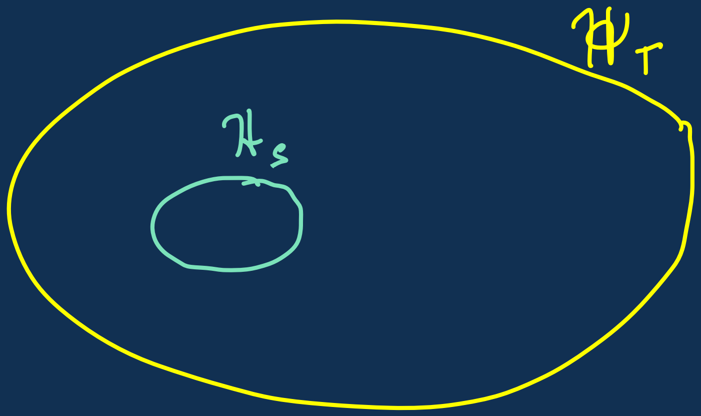
$$\mathrm{Tr}[\phi^* \rho] = \langle\phi|\rho\rangle$$

Linear op $\mathcal{L}$ on $\rho$

$$\tilde{\mathcal{L}}|\rho\rangle$$

$d^2 \times d^2$

# Open Quantum Systems




$\mathcal{H}_T$ evolves according to $H_T$.

$$\begin{tikzcd}
\lvert\psi(0)\rangle \arrow[r, "e^{-iH_T t}"] \arrow[d, "\text{trace out } T-S"] & \lvert\psi(t)\rangle \arrow[d] \\
\rho_0 \arrow[r, "V(t)"] & \rho_T
\end{tikzcd}$$

Completely positive trace-preserving map. (CPTP)

Is there a linear map $\mathcal{L}$

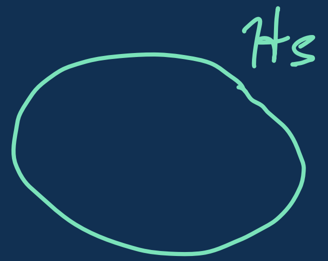
$$\frac{d\rho}{dt} = \mathcal{L}\rho$$

$V(t) = e^{\mathcal{L}t}$ for all $t$ ? (Not Always)

Ref: A Short Intro to the Lindblad Master Equation, D. Manzano.

What is the most general form for linear map $\mathcal{L}$

$$\frac{d}{dt}\rho = \mathcal{L}\rho$$




Such that $V(t) = e^{\mathcal{L}t}$ is CPTP?

If $V$ is a linear map $B(\mathcal{H}) \longrightarrow B(\mathcal{H})$

Bounded lin ops on $\mathcal{H}$.

(1) $V$ is completely positive iff $V(\rho) = \sum_i L_i \rho L_i^\dagger$

(2) $V$ is CPTP iff also $\sum_i L_i L_i^\dagger = I$

Start with $V(t)$ is C.P.

basis for ops.

$\frac{1}{\sqrt{d}} I = K_1, \dots K_{d^2}$

$Tr(K_i^\dagger K_j) = 0 \quad i \neq j.$

$\Rightarrow Tr(K_j) = 0 \quad j \geq 2.$

$$\mathcal{L} = \lim_{\Delta t \to 0} \frac{V(\Delta t) - I}{\Delta t}$$

$$V(\Delta t)\rho = c(\Delta t)_{ij} K_i \rho K_j^\dagger$$

$$\mathcal{L}\rho = \sum_{i,j=2}^{d^2} g_{ij} K_i \rho K_j^\dagger + \underbrace{\sum_{i=2}^{d^2} g_{i,1} K_i \rho + \sum_{j=2}^{d^2} g_{1,j} \rho K_j^\dagger + g_{1,1} I}_{G\rho + \rho G^\dagger}$$

$$\mathcal{L}\rho = \sum_{i,j=2}^{d^2} g_{ij} K_i \rho K_j^\dagger + \underbrace{\sum_{i=2}^{d^2} g_{i,1} K_i \rho + \sum_{j=2}^{d^2} g_{1,j} \rho K_j^\dagger + g_{11} I}_{G\rho + \rho G^\dagger}$$

$$G = \underset{\text{anti-herm}}{F} - i\underset{\text{Herm.}}{H}$$

$$= \underbrace{\sum_{i,j=2}^{d^2} g_{ij} K_i \rho K_j^\dagger + F\rho + \rho F}_{\text{Trace} = 0.} \underbrace{- iH\rho + i\rho H.}_{-i[H,\rho].}$$

Apply trace-preserving condition

$$0 = \mathrm{Tr}\left[\frac{d\rho}{dt}\right] = \mathrm{Tr}[\mathcal{L}\rho] \qquad F = -\frac{1}{2}\sum_{i,j=2}^{d^2} g_{ij} K_i K_j^\dagger$$

# Lindbladian Master Equation

$\{A, B\} = AB + BA$
$[A, B] = AB - BA$

$$\mathcal{L}\rho = \underbrace{\sum_k L_k \rho L_k^\dagger}_{\text{"Accept"}} - \underbrace{\frac{1}{2}\{L_k L_k^\dagger, \rho\}}_{\text{"Reject"}} - \underbrace{i[H, \rho]}_{\text{coherent}}$$

to preserve trace.

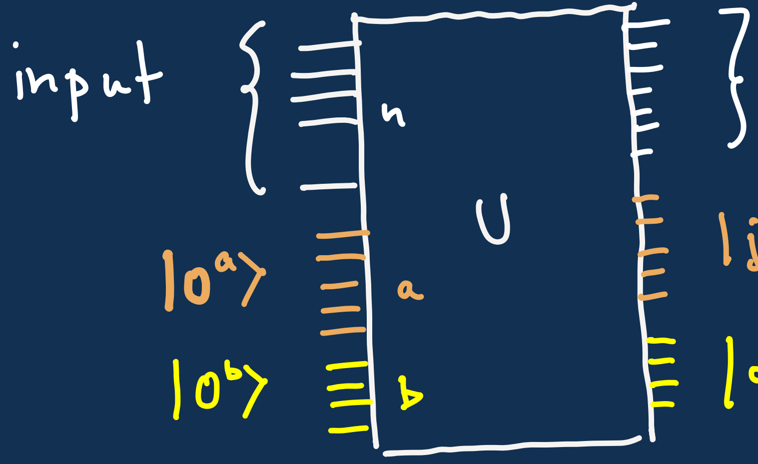
w.l.o.g $\mathrm{Tr}[L_k] = 0$

Goal: Simulate $e^{\mathcal{L}t}$

How to specify $\{L_k\}_k$ ?

# Block Encoding

$\sum_j L_j \otimes |j\rangle\langle j|$




output of $L_j$

$|j\rangle\langle j|$ index for $L_j$

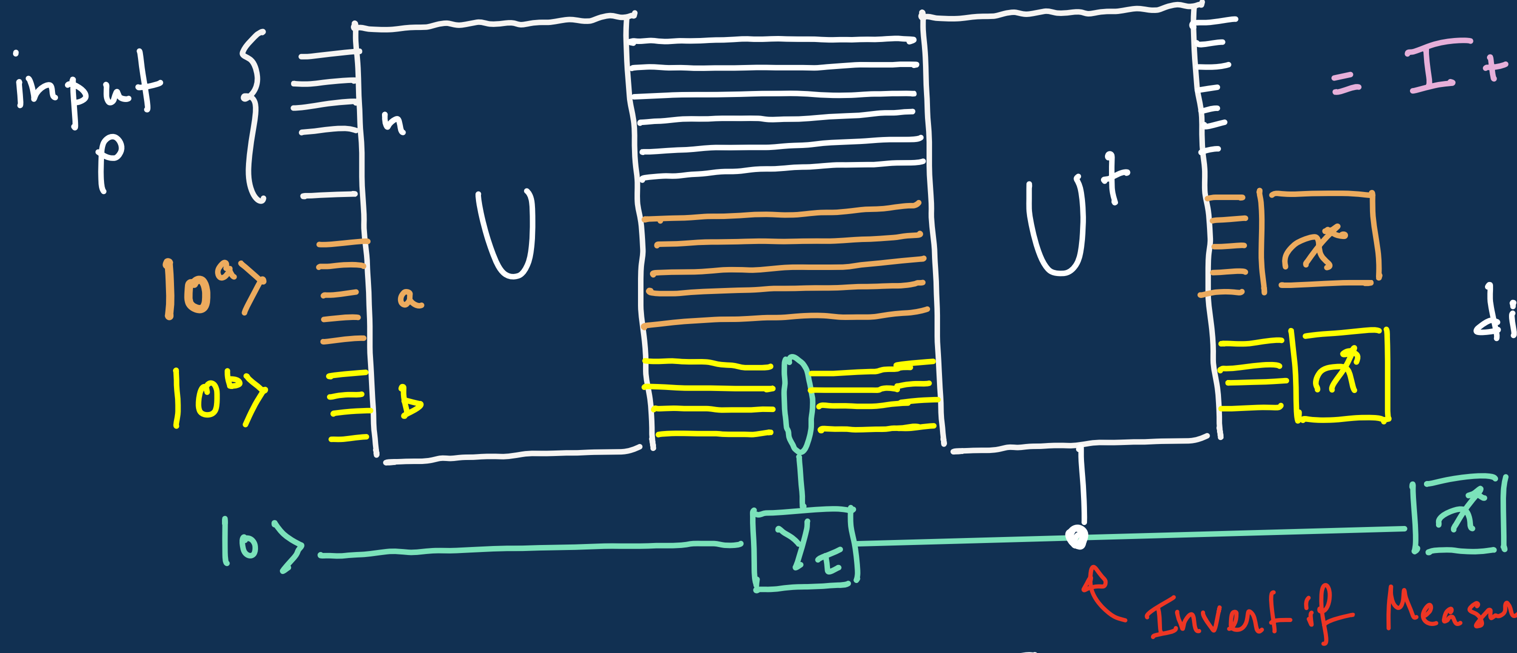
$|0\rangle\langle 0|$ condition on last reg = 0.

Given a block-encoding of $\{L_j\}$ can simulate $e^{t\mathcal{L}}$

$$\mathcal{L}[\rho] = \sum_j L_j \rho L_j^\dagger - \frac{1}{2} L_j L_j^\dagger \rho - \frac{1}{2} \rho L_j L_j^\dagger$$

[Cleve-Wang '17, Li-Wang '22, CGKB '23]

# Weak-Measurement Lindblad Implementation




$$= I + \tau\mathcal{L} + O(\tau^2)$$
$$\approx e^{\tau\mathcal{L}}$$

$$Y_\tau = |0^b\rangle\langle 0^b| \otimes \begin{bmatrix} \sqrt{1-\tau} & -\sqrt{\tau} \\ \sqrt{\tau} & \sqrt{1-\tau} \end{bmatrix} + \left(I - |0^b\rangle\langle 0^b|\right) \otimes I.$$

# Weak-Measurement Lindblad Implementation

Mixing time for $\mathcal{L}$: $T_{mix}$

\# steps for weak-meas. implementation:

$$T_{mix}/\tau$$

Total error: $\dfrac{T_{mix}}{\tau} \cdot \tau^2 = T_{mix}\,\tau$

Need $\tau = O(1/T_{mix})$ # steps $\sim (T_{mix})^2$

# Quantum Phase Estimation

$$V|\Phi\rangle = e^{i\varphi 2\pi}|\Phi\rangle$$

Input: $\underbrace{|0^r\rangle}_{\sim \log n}\underbrace{|\Phi\rangle}_{n}$ $\quad 2^r = R \sim \mathrm{poly}(n).$

$$\downarrow H^{\otimes n} \otimes I$$

$$\frac{1}{\sqrt{R}}\sum_t |t\rangle|\Phi\rangle \quad t \in \{0,..,R-1\}$$

$$\downarrow |t\rangle\langle t| \otimes V^t$$

$$\frac{1}{\sqrt{R}}\sum_t e^{i\varphi 2\pi t}|t\rangle|\Phi\rangle$$

$$\downarrow QFT^\dagger$$

$$|\tilde{\varphi}\cdot R\rangle|\Phi\rangle$$

$$V = e^{iH/k}.$$

$$k \geq \frac{\|H\|}{2\pi}.$$

$$H|\psi_j\rangle = E_j|\psi_j\rangle$$

$$V|\psi_j\rangle = e^{i\varphi_j 2\pi}|\psi_j\rangle$$

$$E_j = \varphi_j \cdot 2\pi \cdot k.$$

# Quantum Phase Estimation

$$V|\Phi\rangle = e^{i\varphi 2\pi}|\Phi\rangle$$

Input: $|0^r\rangle|\Phi\rangle$ $\qquad 2^r = R$

$$\downarrow\ H^{\otimes n}\otimes I$$

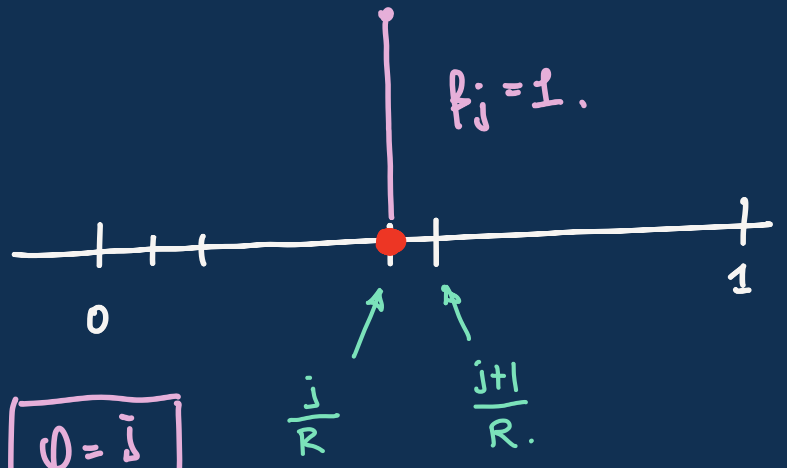
$$\frac{1}{\sqrt{R}}\sum_t |t\rangle|\Phi\rangle \qquad t\in\{0,..,R-1\}$$

$$\downarrow\ |t\rangle\langle t|\otimes V^t$$

$$\frac{1}{\sqrt{R}}\sum_t e^{i\varphi 2\pi t}|t\rangle|\Phi\rangle$$

$$\downarrow\ QFT^\dagger$$

$$|\tilde{\varphi}\cdot R\rangle|\Phi\rangle = \sum_j \beta_j |j\rangle|\Phi\rangle$$




$$\boxed{\varphi = \frac{j}{R}}$$

# Quantum Phase Estimation

$$V|\Phi\rangle = e^{i\varphi 2\pi}|\Phi\rangle$$

Input: $|0^r\rangle|\Phi\rangle$  $2^r = R$

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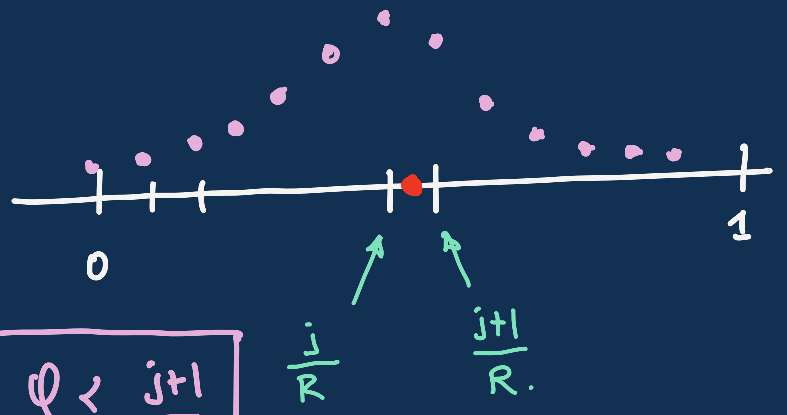
$$|\tilde{\varphi}\cdot R\rangle|\Phi\rangle = \sum_j \beta_j |j\rangle|\Phi\rangle$$

$$\boxed{\frac{j}{R} < \varphi < \frac{j+1}{R}}$$




① Precision $R \sim$ poly

② Non-deterministic.

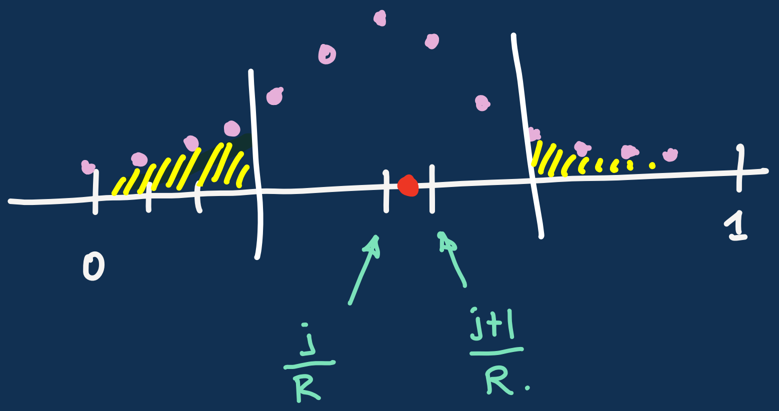
# Quantum Phase Estimation

$$V|\Phi\rangle = e^{i\varphi 2\pi}|\Phi\rangle$$

Input: $|0^r\rangle|\Phi\rangle \qquad 2^r = R$

$\Downarrow$ QPE.

$$|\tilde{\varphi}\cdot R\rangle\,|\Phi\rangle = \sum_j \beta_j\,|j\rangle|\Phi\rangle$$

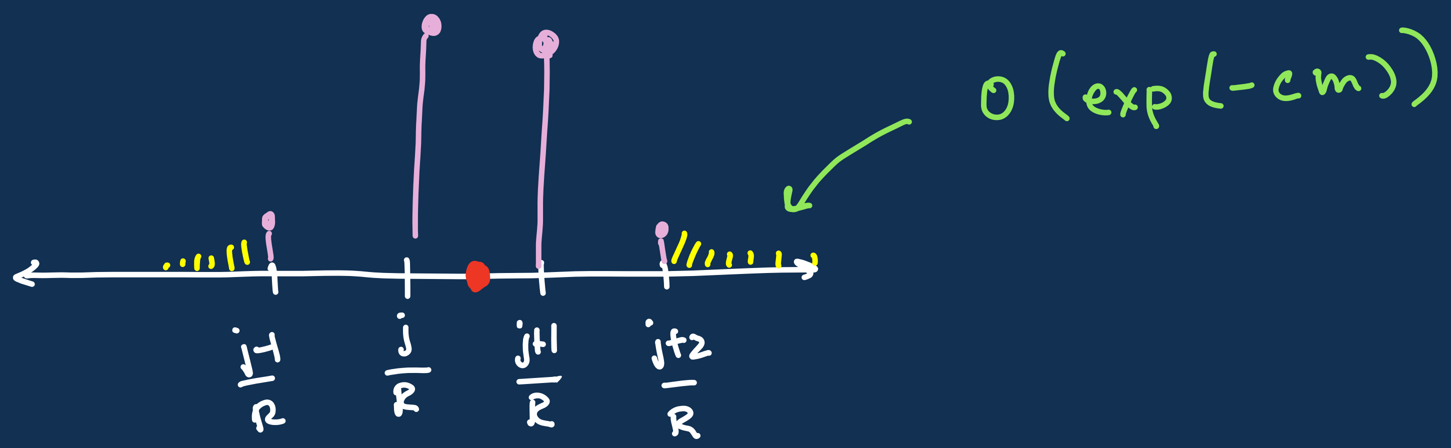



$$\sum_{\substack{j: \\ |j-\varphi R| > R^\alpha}} |\beta_j|^2 \sim \frac{1}{R^{1-\alpha}}$$

# Boosted QPE: Median of multiple runs. [NC]

$$\underbrace{|0^r\rangle|0^r\rangle\cdots|0^r\rangle}_{\otimes m}|0^r\rangle|\Phi\rangle \xrightarrow{QPE^{\otimes m}} |j_1\rangle\cdots|j_m\rangle|0^r\rangle|\Phi\rangle$$

$$\xrightarrow{\text{median}} |j_1\rangle\cdots|j_m\rangle|\bar{j}\rangle|\Phi\rangle.$$



# Boosting QPE by Filters

Input: $|0^r\rangle|\Phi\rangle$ $\quad 2^r = R$

$\downarrow H^{\otimes n} \otimes I$

$\frac{1}{\sqrt{R}} \sum_t |t\rangle|\Phi\rangle \quad t \in \{0,..,R-1\}$

prepare filter.

$\downarrow |t\rangle\langle t| \otimes V^t$

$\frac{1}{\sqrt{R}} \sum_t e^{i\varphi 2\pi t}|t\rangle|\Phi\rangle$

$\downarrow QFT^\dagger$

$|\tilde{\varphi}\cdot R\rangle|\Phi\rangle$

- Kaiser Filter for QPE
  arXiv 2209.13581 '22
- Patel, Tan, Subasi, Sornborger
  2024

## CGKB: Gaussian Filter

$$|g_\sigma\rangle = \frac{1}{\sqrt{Norm}} \sum_t e^{-\frac{t^2}{4\sigma^2}}|t\rangle$$

$$QFT^\dagger|g_\sigma\rangle \approx |g_{N/\sigma}\rangle$$

# Boosting QPE by Filters

$$|g_\sigma\rangle = \frac{1}{\sqrt{Norm}} \sum_t e^{-\frac{t^2}{4\sigma^2}} |t\rangle$$

Input: $|0^r\rangle|\Phi\rangle$ $\quad 2^r = R$

$$\downarrow$$

$$|g_\sigma\rangle|\Phi\rangle$$

$$\Big\downarrow \; |t\rangle\langle t| \otimes U^t$$

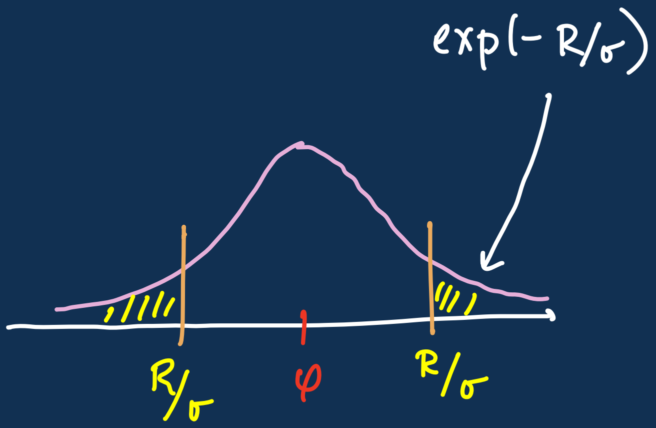
$$\frac{1}{\sqrt{Norm}} \sum_t e^{-\frac{t^2}{4\sigma^2}} e^{i\varphi 2\pi t} |t\rangle |\Phi\rangle$$

$$\Big\downarrow \; QFT^\dagger$$

$$|\tilde{\varphi} \cdot R\rangle |\Phi\rangle$$

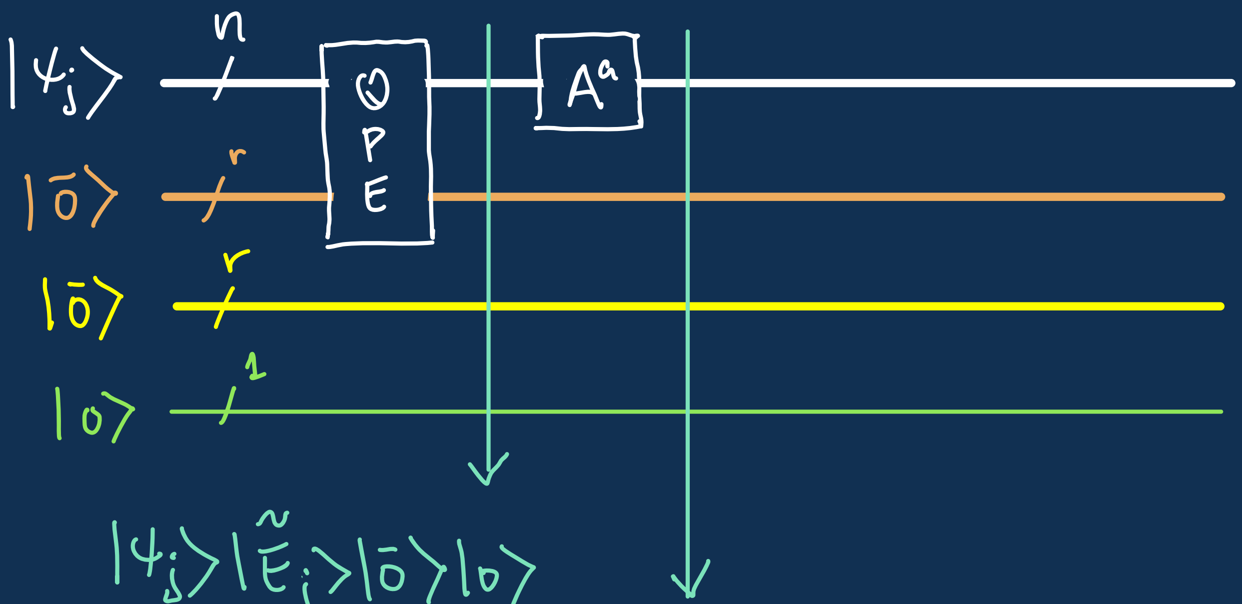
$|\tilde{\varphi} R\rangle = |g_{R/\sigma}\rangle$

shifted by $\varphi \cdot R$.



## A Metropolis-like Lindbladian

Select jump op $A^a$ with prob. $p(a)$.




$$|\psi_j\rangle|\tilde{E}_j\rangle|\bar{0}\rangle|0\rangle$$

$$\sum_k \alpha_{jk} |\psi_k\rangle|\tilde{E}_j\rangle|\bar{0}\rangle|0\rangle$$

# A Metropolis-like Lindbladian

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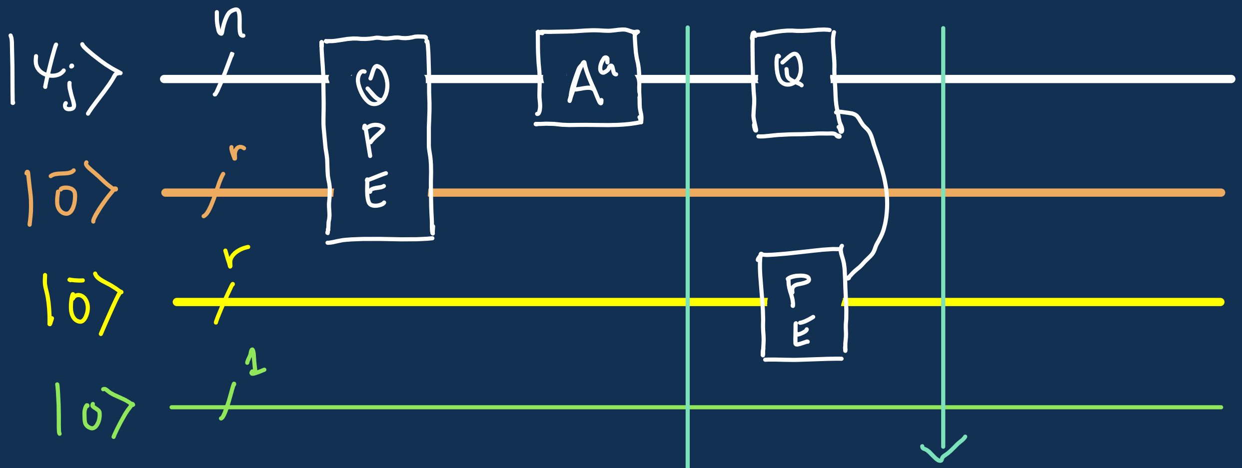
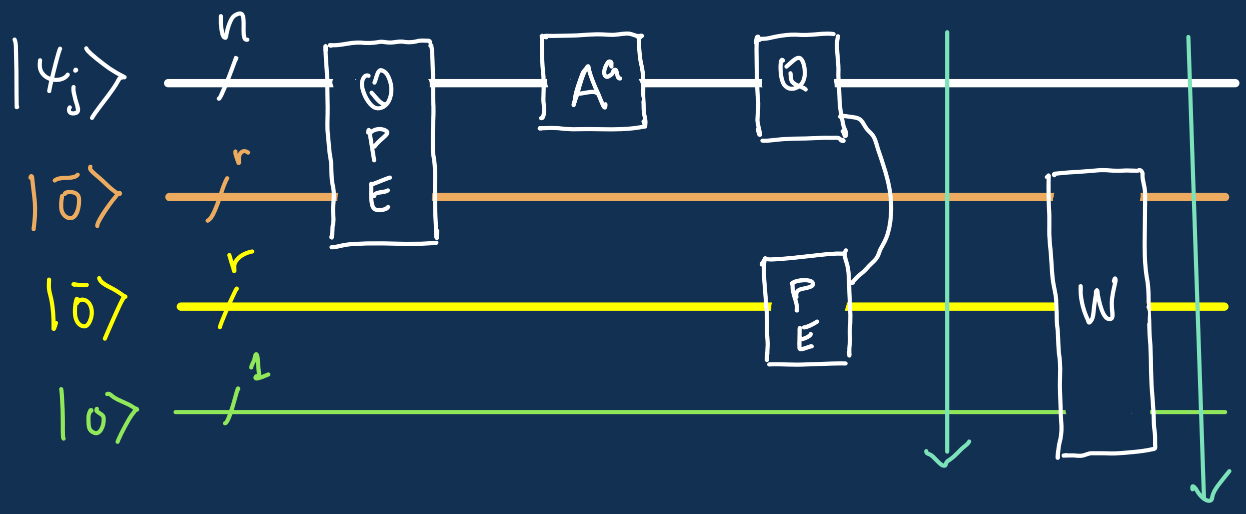



$$\sum_k \alpha_{jk} |\psi_k\rangle |\tilde{E}_j\rangle |\tilde{E}_k\rangle |0\rangle$$

$$\sum_k \alpha_{jk} |\psi_k\rangle |\tilde{E}_j\rangle |\bar{0}\rangle |0\rangle$$




## A Metropolis-like Lindbladian

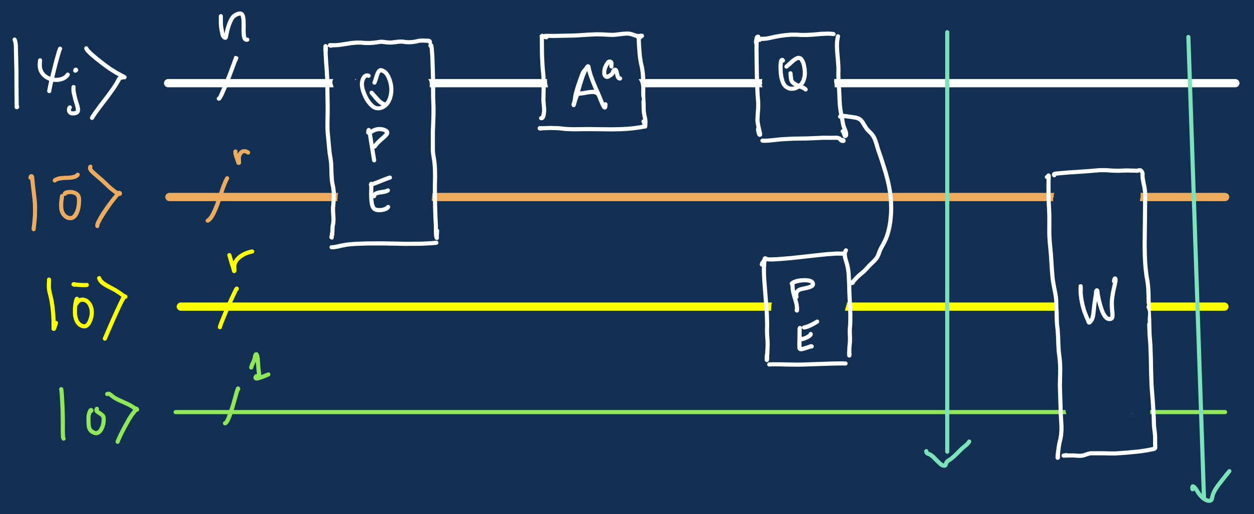
Select jump op $A^a$ with prob. $p(a)$.

$$\gamma = \min\{1, \exp(-\beta(E_k - E_j))\}$$

$$\sum_k \alpha_{jk} |\psi_k\rangle |\tilde{E}_j\rangle |\tilde{E}_k\rangle |0\rangle$$

$$\sum_k \alpha_{jk} |\psi_k\rangle |\hat{E}_j\rangle |\hat{E}_k\rangle \otimes \big(\underbrace{\sqrt{1-\gamma}|0\rangle}_{\text{Rej}} + \underbrace{\sqrt{\gamma}|1\rangle}_{\text{Acc}}\big)$$

# A Metropolis-like Lindbladian




Select jump op $A^a$ with prob. $p(a)$.

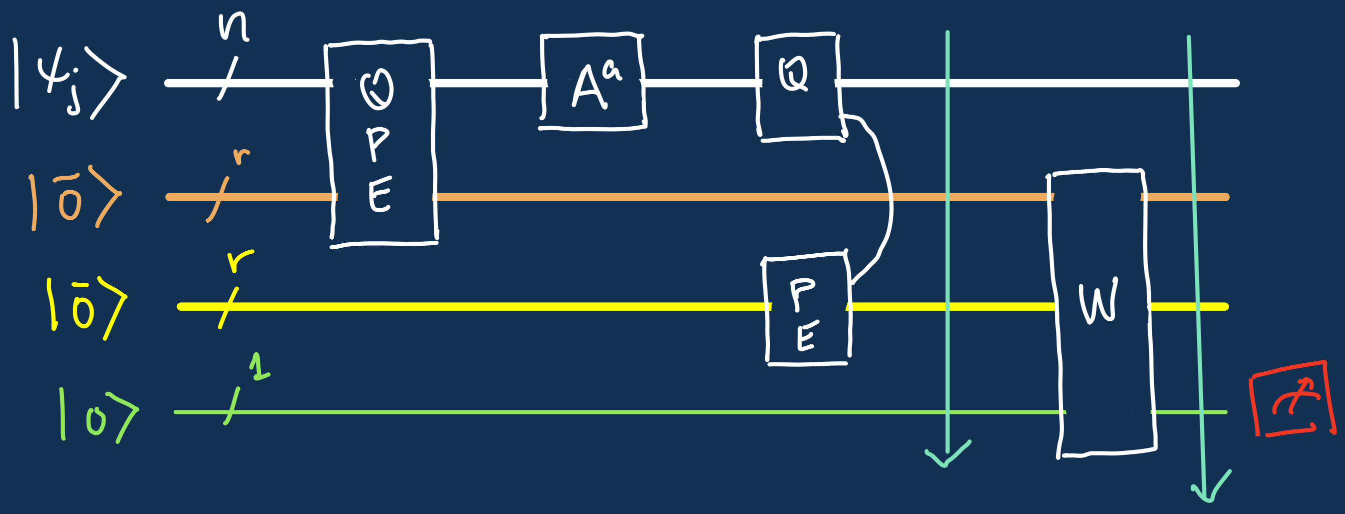
$$\gamma = \min\{1, \exp(-\beta(E_k - E_j))\}$$

$$\sum_k \alpha_{jk} |\psi_k\rangle |\underline{\tilde{E}_j}\rangle |\underline{\tilde{E}_k}\rangle |0\rangle$$

$$\sum_k \alpha_{jk} |\psi_k\rangle |\hat{E}_j\rangle |\hat{E}_k\rangle \otimes \big(\underbrace{\sqrt{1-\gamma}\,|0\rangle}_{\text{Rej}} + \underbrace{\sqrt{\gamma}\,|1\rangle}_{\text{Acc}}\big)$$

$$W = \sum_{EE'} |EE'\rangle\langle EE'| \otimes \begin{bmatrix} \sqrt{1-\gamma_{EE'}} & -\sqrt{\gamma_{EE'}} \\ \sqrt{\gamma_{EE'}} & \sqrt{1-\gamma_{EE'}} \end{bmatrix}$$

TOVPV 2013




Meas 1: Accept.
Meas 0: Reject.

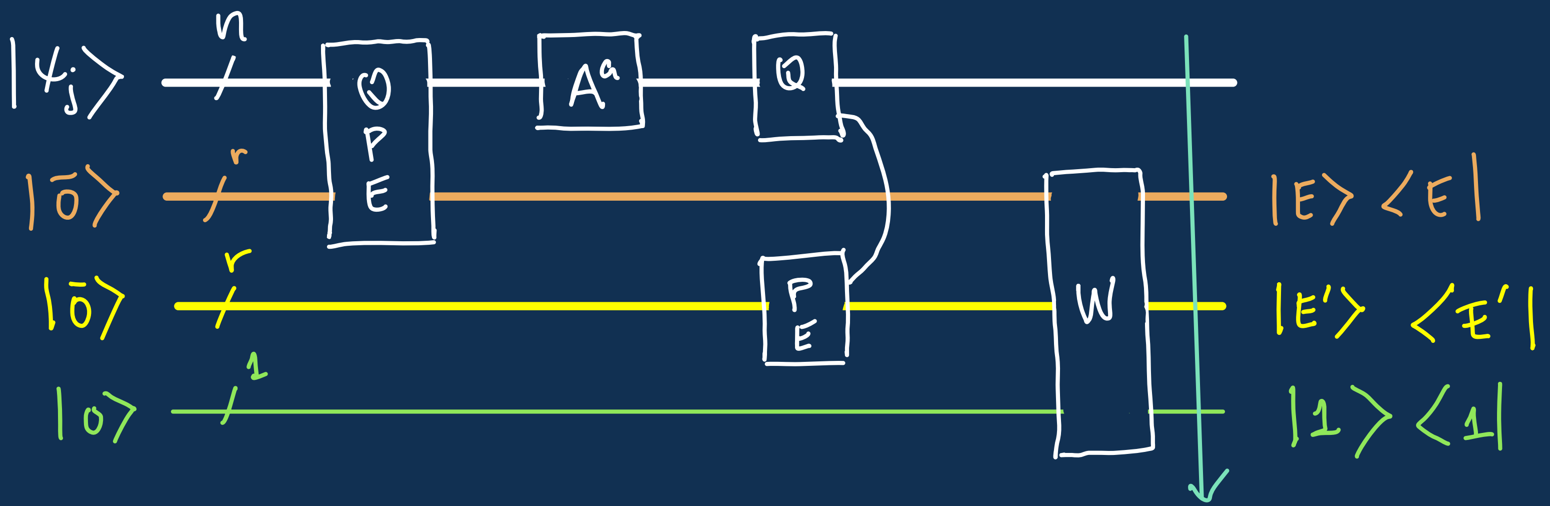
Back up via Mariott-Watrous.

$$\sum_k \alpha_{jk} |\psi_k\rangle |\tilde{E}_j\rangle |\tilde{E}_k\rangle |0\rangle$$

$$\sum_k \alpha_{jk} |\psi_k\rangle |\hat{E}_j\rangle |\hat{E}_k\rangle \otimes (\underbrace{\sqrt{1-\gamma}|0\rangle}_{\text{Rej}} + \underbrace{\sqrt{\gamma}|1\rangle}_{\text{Acc}})$$

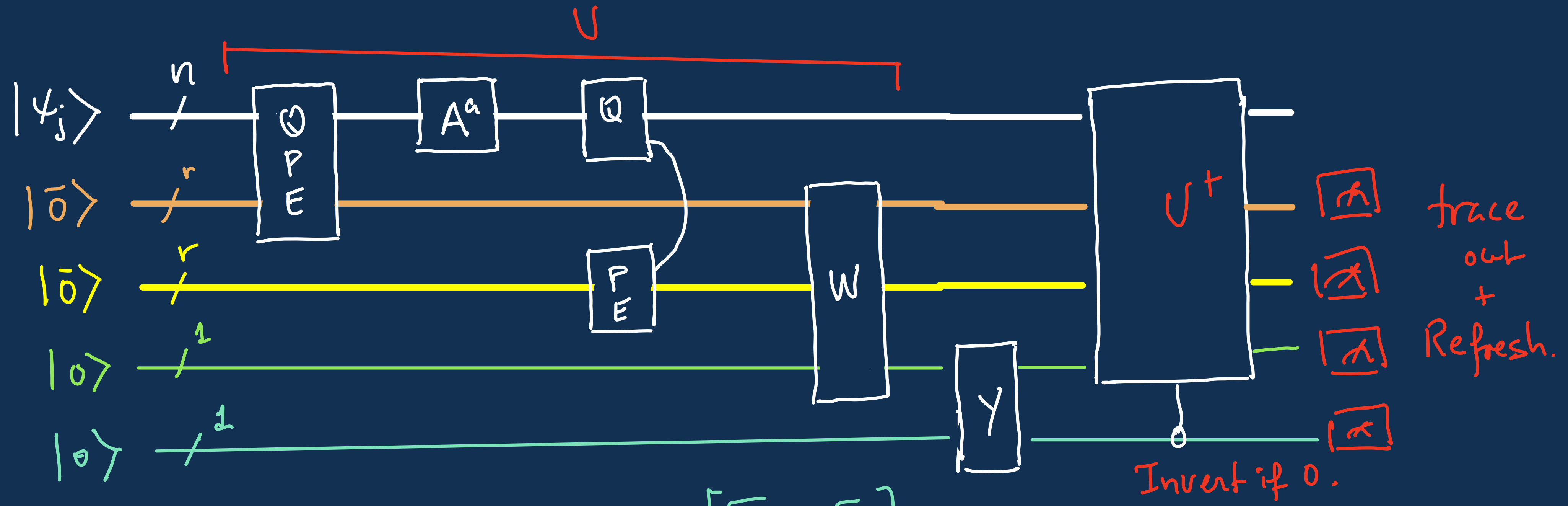
$$W = \sum_{EE'} |EE'\rangle\langle EE'| \otimes \begin{bmatrix} \sqrt{1-\gamma_{EE'}} & -\sqrt{\gamma_{EE'}} \\ \sqrt{\gamma_{EE'}} & \sqrt{1-\gamma_{EE'}} \end{bmatrix}$$

# Define Lindbladian Operators $\{L_{EE'}\}_{EE'}$




$$\sum_k \alpha_{jk}|\psi_k\rangle|\hat{E}_j\rangle|\hat{E}_k\rangle \otimes \left(\underbrace{\sqrt{1-\gamma}\,|0\rangle}_{Rej} + \underbrace{\sqrt{\gamma}\,|1\rangle}_{Acc}\right)$$

Define Lindbladian Operators $\{L_{EE'}\}_{EE'} \Rightarrow \mathcal{L}$




$$Y = \underbrace{|1\rangle\langle 1|}_{A} \otimes \begin{bmatrix} \sqrt{1-\tau} & \sqrt{\tau} \\ \sqrt{\tau} & \sqrt{1-\tau} \end{bmatrix}$$

$$+ \underbrace{|0\rangle\langle 0|}_{R.} \otimes I$$

Implement $e^{\mathcal{L}t}$ via weak measurement.

# Lindbladian Operators from CGKB '23

Select $A^a$ w.p. $p(a)$.

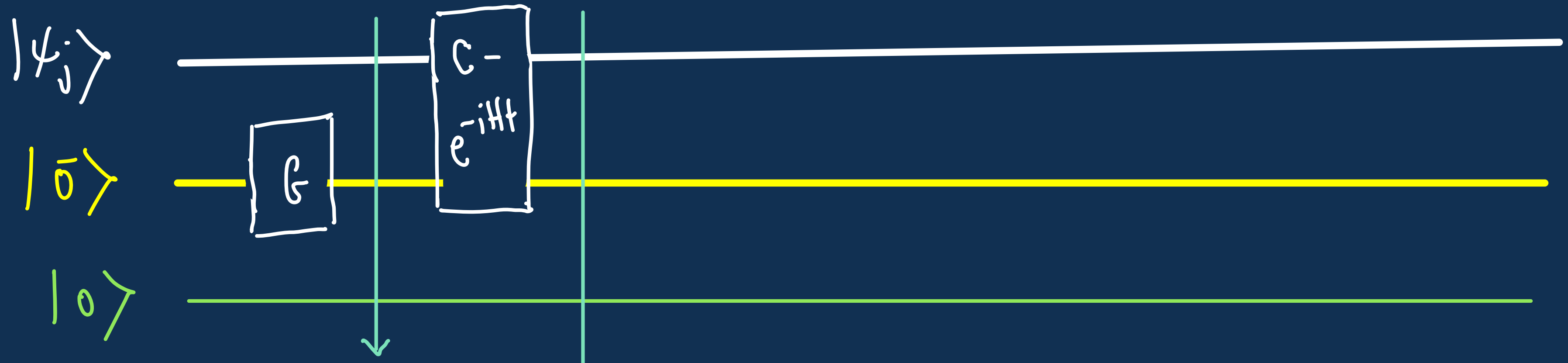

$$|\psi_j\rangle \otimes \sum_t g_t |t\rangle \otimes |0\rangle$$

Apply controlled time-evolved jump operator

$$\left[ e^{iHt} A^a e^{-iHt} \right] \otimes |t\rangle\langle t|$$

# Lindbladian Operators from CGKB '23

Select $A^a$ w.p. $p(a)$.

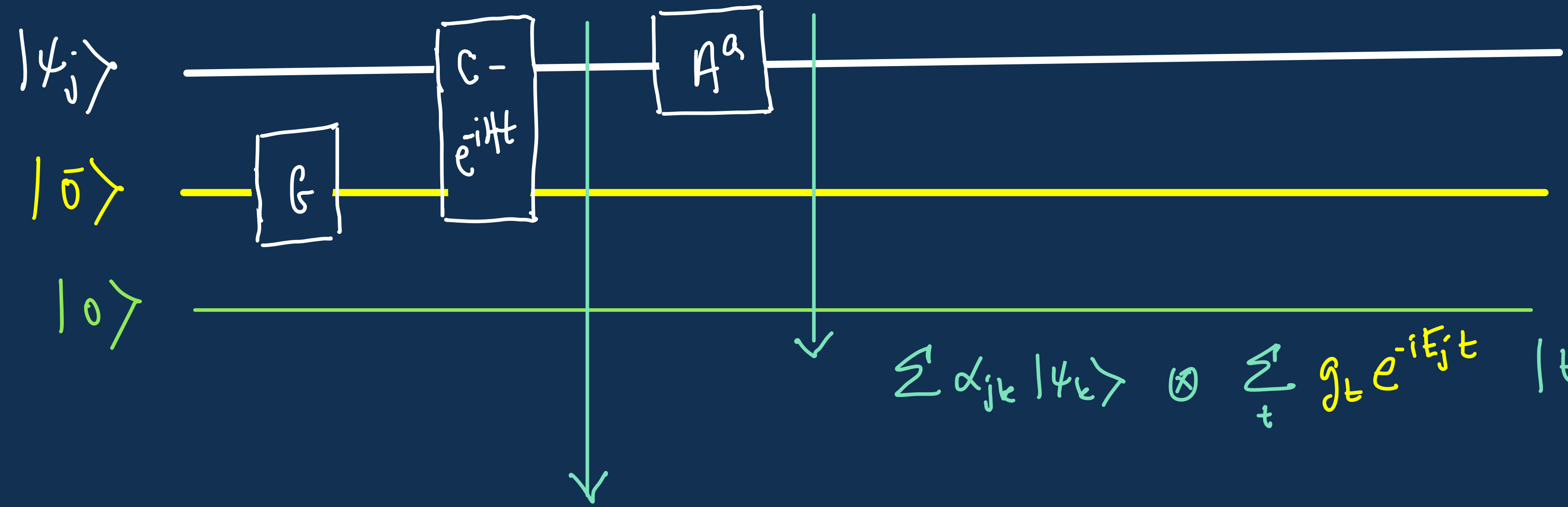



$$\underbrace{|\psi_j\rangle} \otimes \underbrace{\sum_t g_t |t\rangle} \otimes |0\rangle$$

$$\underbrace{|\psi_j\rangle} \otimes \underbrace{\sum_t g_t e^{-iE_j t} |t\rangle} \otimes |0\rangle$$

# Lindbladian Operators from CGKB '23

Select $A^a$ w.p. $p(a)$.

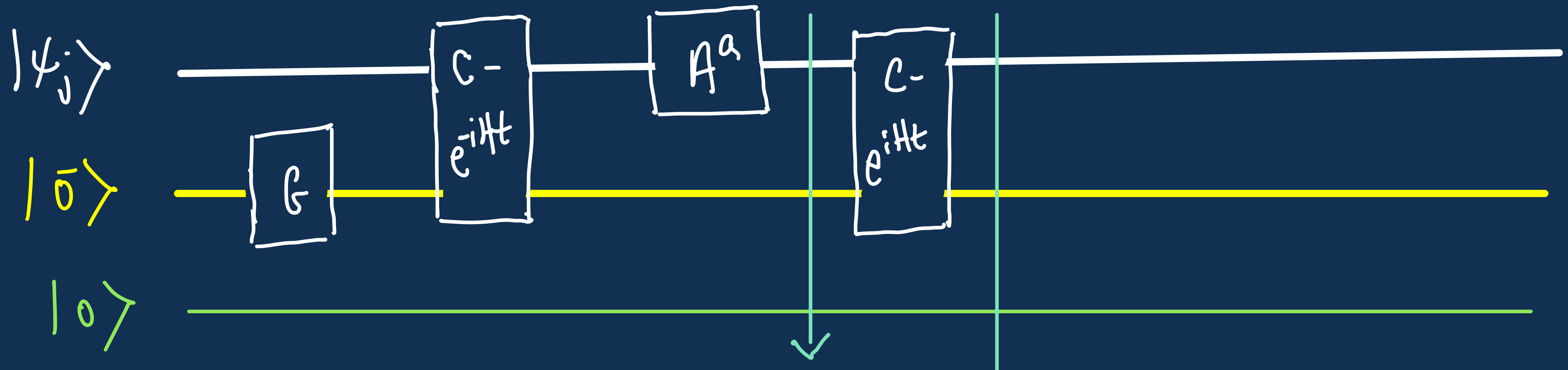



$$\sum \alpha_{jk} |\psi_k\rangle \otimes \sum_t g_t e^{-iE_j t} |t\rangle \otimes |0\rangle$$

$$|\psi_j\rangle \otimes \sum_t g_t e^{-iE_j t} |t\rangle \otimes |0\rangle$$

# Lindbladian Operators from CGKB '23

Select $A^a$ w.p. $p(a)$.

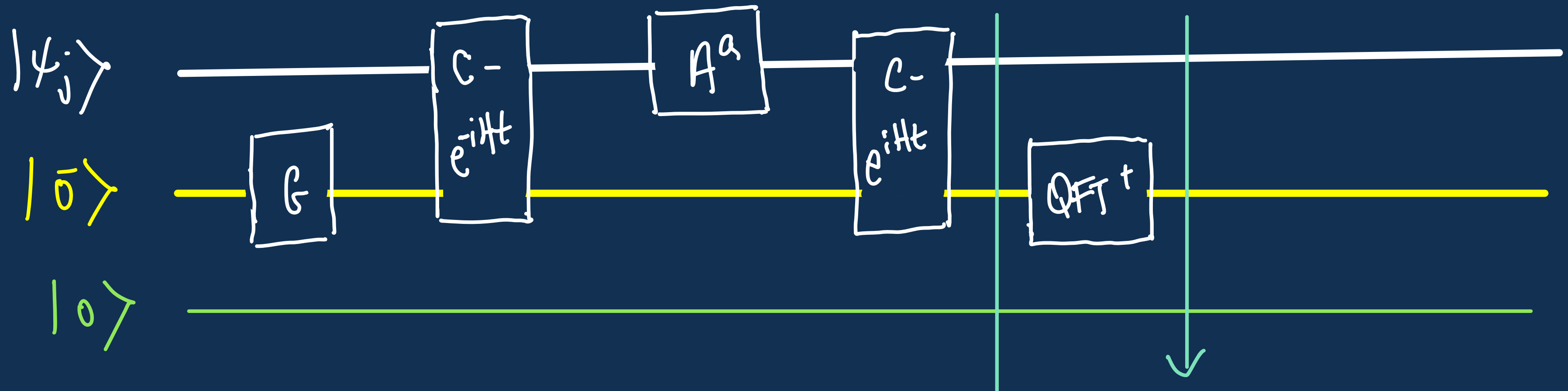



$$\sum \alpha_{jk} |\psi_k\rangle \otimes \sum_t g_t e^{-iE_j t} |t\rangle \otimes |0\rangle$$

$$\sum \alpha_{jk} |\psi_k\rangle \otimes \sum_t g_t e^{i(E_k - E_j)t} |t\rangle \otimes |0\rangle$$

# Lindbladian Operators from CGKB '23

Select $A^a$ w.p. $p(a)$.

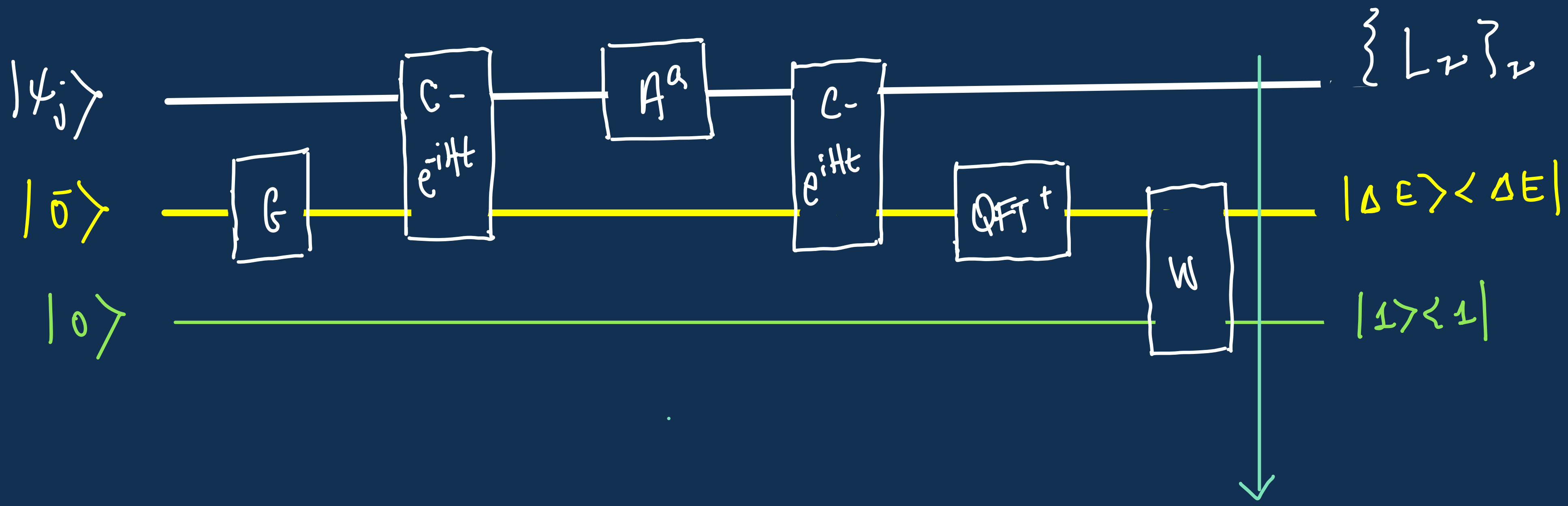



$$\sum_{\alpha_{jk}} \alpha_{jk} |\psi_k\rangle \, |\widetilde{(E_k - E_j)}\rangle \, |0\rangle$$

$$\sum \alpha_{jk} |\psi_k\rangle \otimes \sum_t g_t \, e^{i(E_k - E_j)t} |t\rangle \otimes |0\rangle$$

# Lindbladian Operators from CGKB '23

Select $A^a$ w.p. $p(a)$.

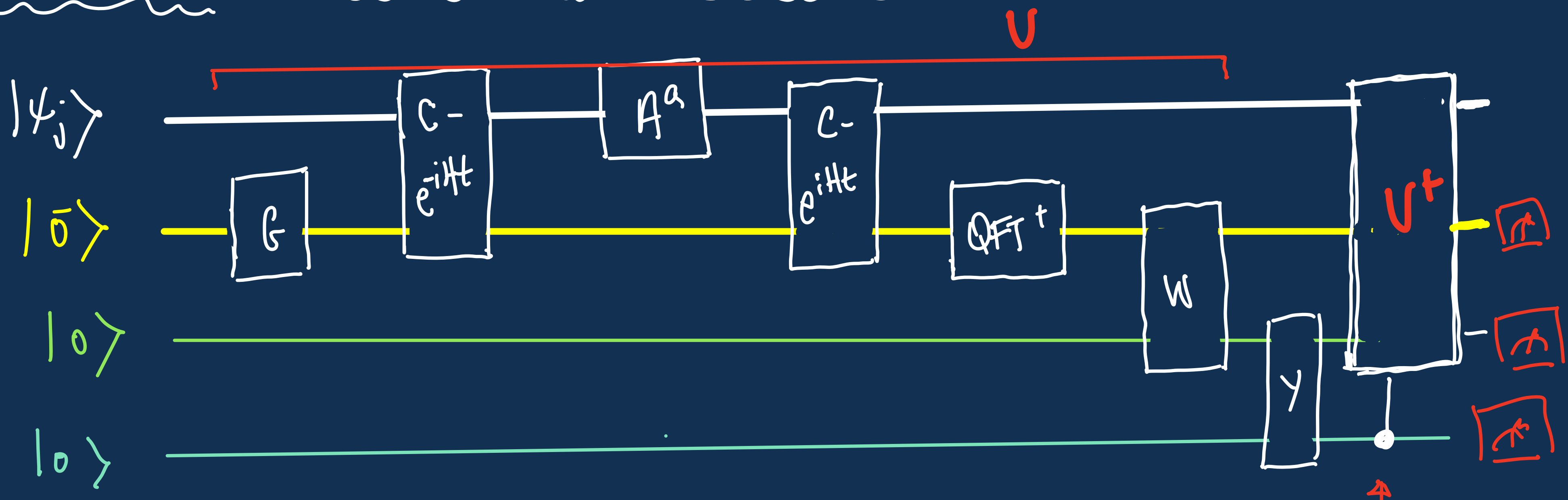



$$\sum_{\alpha_{jk}} \alpha_{jk} |\psi_k\rangle \, |(\widetilde{E_k - E_j})\rangle \left[\sqrt{1-\gamma}\,|0\rangle + \sqrt{\gamma}\,|1\rangle\right]$$

$$\gamma = \min\{1, \exp(-\beta(\widetilde{E_k - E_j}))\}$$

# Lindbladian Operators from CGKB '23

Select $A^a$ w.p. $p(a)$.




$$Y = |1\rangle\langle 1| \otimes \begin{bmatrix} \sqrt{1-\tau} & -\sqrt{\tau} \\ \sqrt{\tau} & \sqrt{1-\tau} \end{bmatrix}$$

$$+\ |0\rangle\langle 0| \otimes I$$

Invert if 0.

One iteration of CGKB using weak-measurement Lindbladian implementation.

# Metropolis-Style Algorithm [Jiang, I.]

$$|\psi_j\rangle|\bar{0}\rangle|\bar{0}\rangle|0\rangle$$

$$W|EE'\rangle|0\rangle \to |EE'\rangle\left[\sqrt{1-T\gamma}\,|0\rangle + \sqrt{T\gamma}\,|1\rangle\right]$$

$$U\left\{\boxed{\begin{matrix} QPE_{1,2} \\ A^0 \\ QPE_{1,3} \end{matrix}}\right.$$

$$\sum_k \alpha_{jk}|\psi_k\rangle|E\rangle|E'\rangle|0\rangle$$

$U \to \boxed{W \text{ Meas.}}$

$\boxed{W \text{ Meas.}} \xrightarrow{1} \text{Acc}$

$\boxed{W \text{ Meas.}} \xrightarrow{0} \boxed{W^\dagger \text{ meas}}$

$\boxed{W^\dagger \text{ meas}} \xrightarrow{1} \text{Alternate-Acc}$

$\boxed{W^\dagger \text{ meas}} \xrightarrow{0} \boxed{U^\dagger} \to \text{Reject.}$

$\text{Acc} \overset{\approx}{\longleftrightarrow} \text{Alternate-Acc}$

$$\approx |\psi_j\rangle|\bar{0}\rangle|\bar{0}\rangle|0\rangle$$

Thank You!