Emergent Universal Randomness

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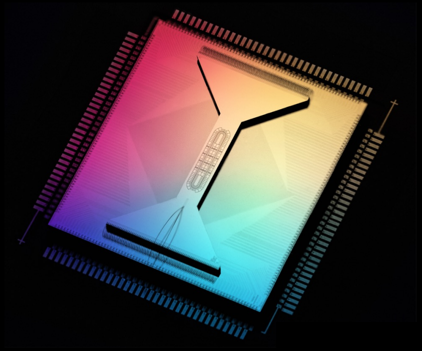
April 23rd, 2024

Exciting Time for Quantum Information Science

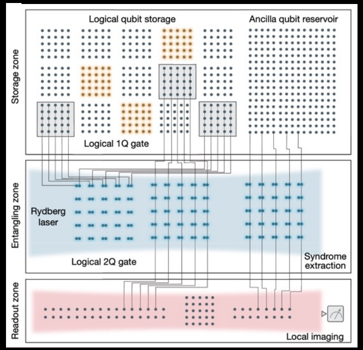
High fidelity operations



Quantinuum

✓ Quality

Logical quantum circuits

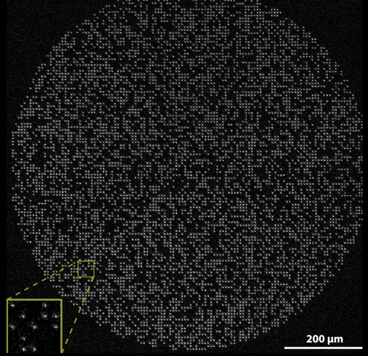


Lukin group

✓ Control

Large qubit arrays

6100 atoms



Endres group

✓ Size

What should we do with quantum devices?

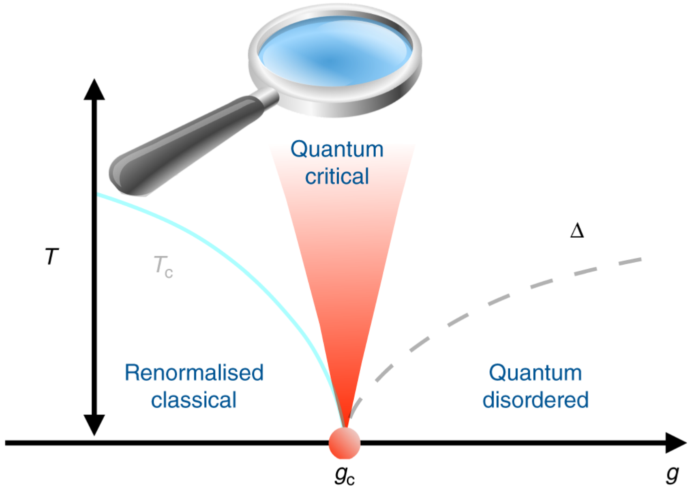
- Further **improve** them
- Realize quantum **algorithms** or **applications**
- Solve **open questions** in science
- Today → **Explore new science** and (maybe) discover interesting stuff

Universal Randomness Emerges from Natural Quantum Many-Body Systems

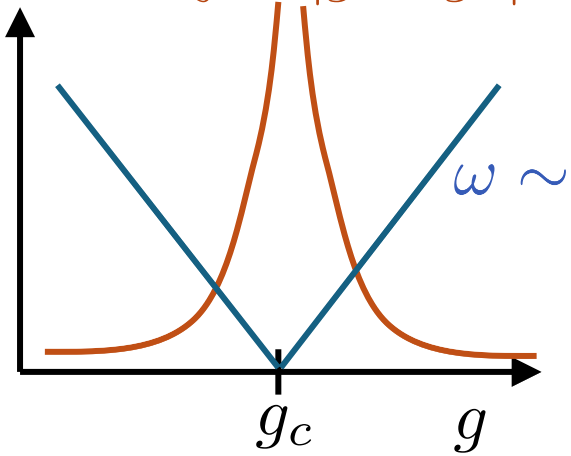
Universality

the emergence of important features independent of microscopic details

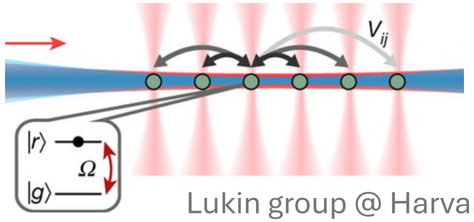
Universal scaling at criticality



Nat. Comm. 10, 577 (2019)

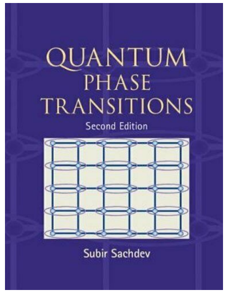


Quantum Simulator



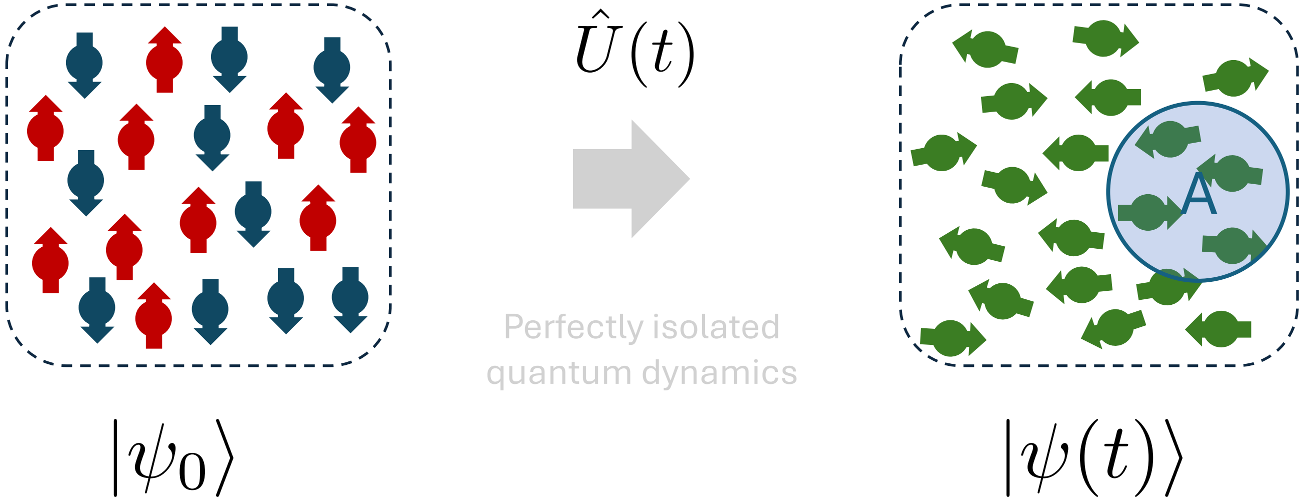
Lukin group @ Harvard

$\approx H_{\text{Ising}}$



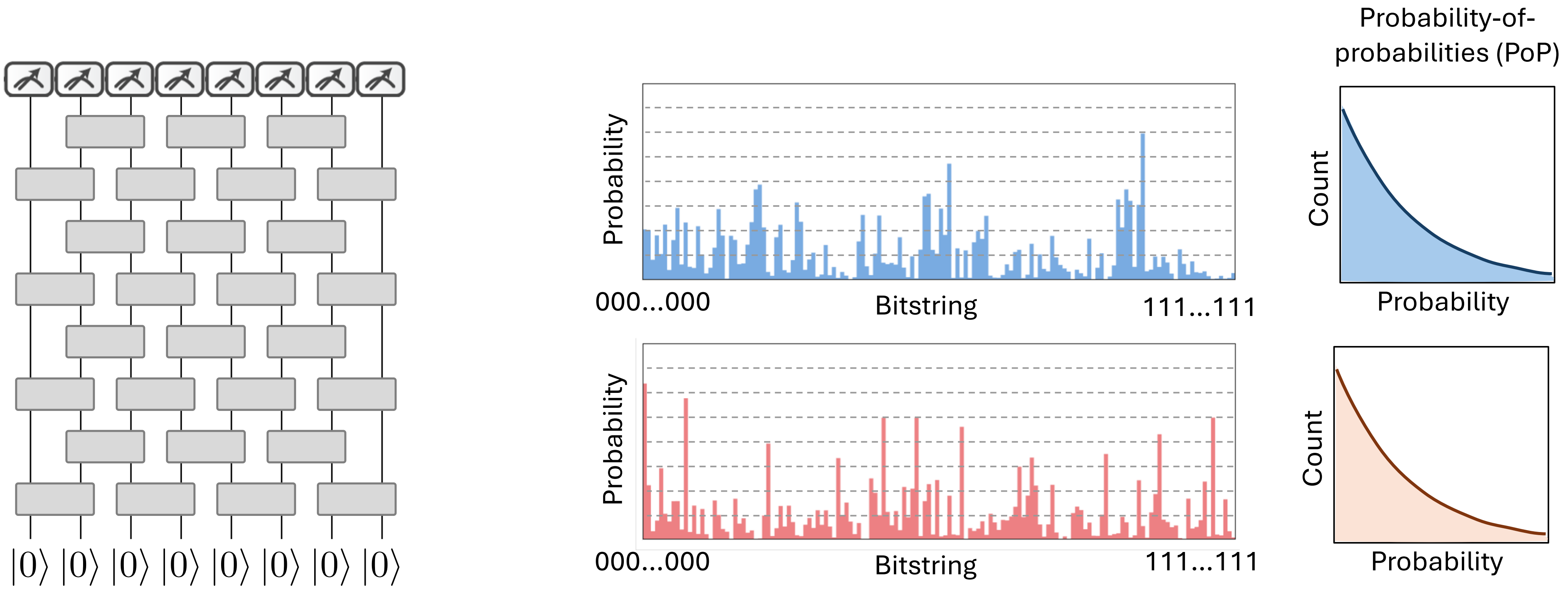
Quantum thermalization

$$\rho_A \sim e^{-\beta H_A}$$



Independent of the initial states and the choice of subsystems!

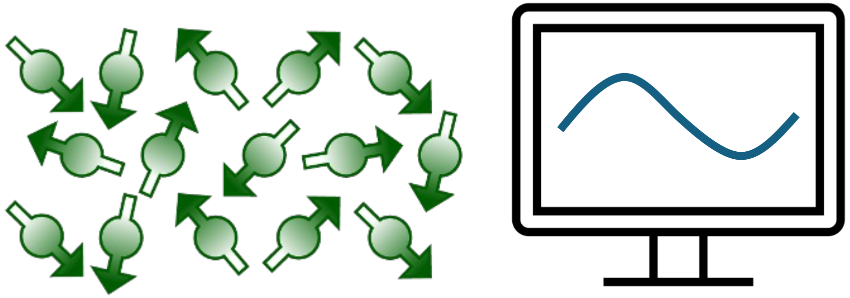
Random Unitary Circuits



Universal PoPs!

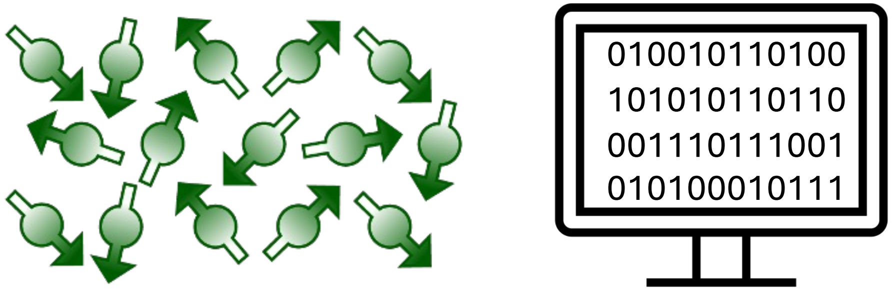
Quantum Experiments

Traditional



- Local observables
- Spatially averaged
- Temporally averaged
- Open systems

Modern

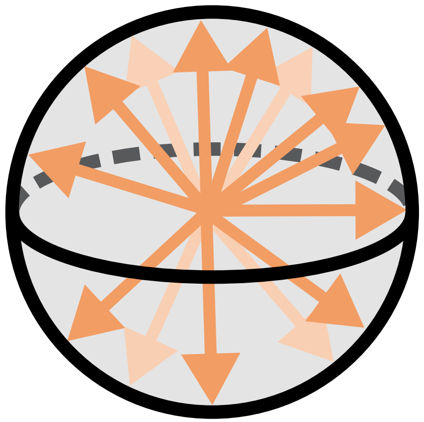


- Nonlocal, microscopic observables
- Spatially resolved
- Temporally resolved
- Well isolated

Modern experiments enable probing new universal phenomena

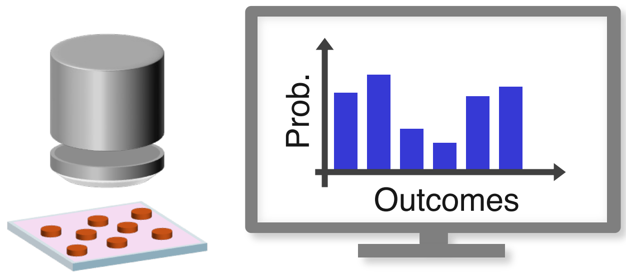
Emergent Randomness in Natural Quantum Systems

Projected Ensemble



Temporal Ensemble

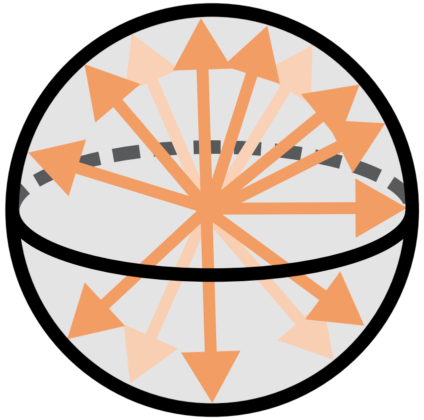
Applications



$$F \approx F_d$$

Emergent Randomness in Natural Quantum Systems

Projected Ensemble



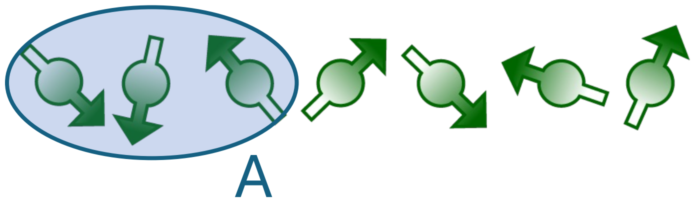
Temporal Ensemble

Applications

$F \approx F_d$

Projected Ensemble

Conventional

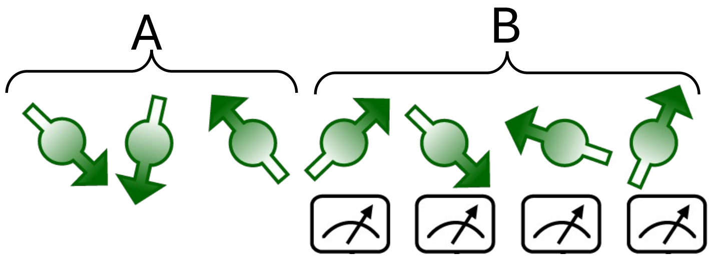


$|\Psi(t)\rangle$

$$\rho_A$$

- ✓ All local observables
- ✓ Average

New settings

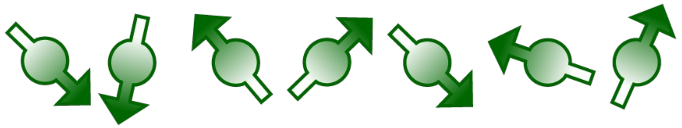


$$|\psi_{000}\rangle, |\psi_{001}\rangle, |\psi_{010}\rangle, \ldots$$

$$\mathcal{E}_{\text{proj}} = \{p_z, |\psi_z\rangle\}$$

- ✓ Correlations between A and B
- ✓ All statistical moments
- ✓ Strictly more information

Projected Ensemble is Pseudo-random

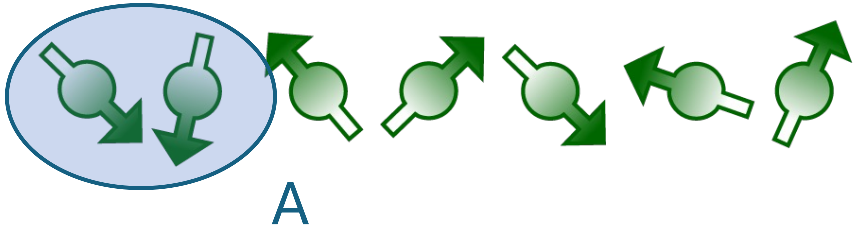
$|\Psi(t)\rangle$ 

$$\mathcal{E}_{\text{proj}} = \{p_z, |\psi_z\rangle\}$$

Claim. For a wide class of quantum states $|\Psi(t)\rangle$ obtained from natural quantum dynamics*, $\mathcal{E}_{\text{proj}}$ is universal. Furthermore, it is statistically indistinguishable from Haar random at infinite effective T.

Projected Ensemble is Pseudo-random

$|\Psi(t)\rangle$

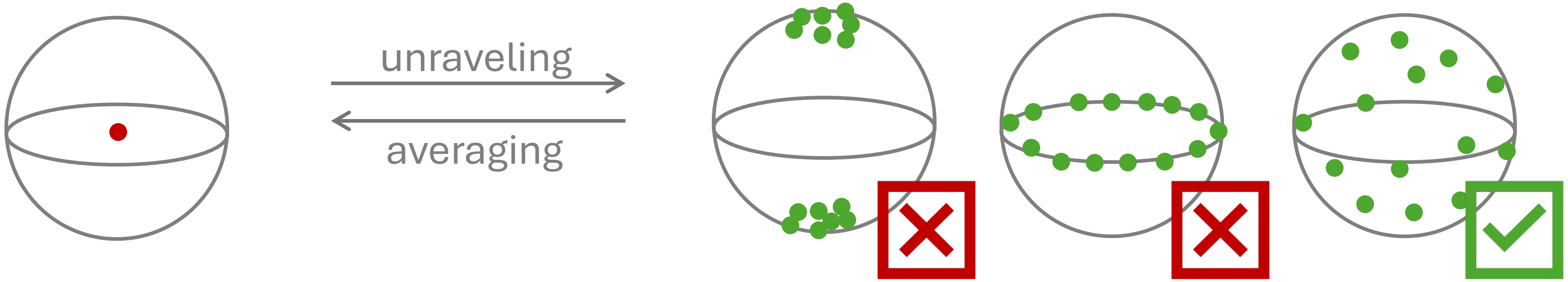


Quantum thermalization

@ $T = \infty$

$\rho_A \propto \mathbb{I}$

$\mathcal{E}_{\text{proj}} = ?$



- ✓ Same density matrix.
- ✓ Different statistical properties.
- ✓ Different information content

Many examples and evidence!

Cotler *et al.*, PRXQ (2023)

Projected Ensemble is Pseudo-random

For **almost all states** $|\Psi\rangle$, $\mathcal{E}_{\mathrm{proj}}$ forms approximate state designs.

Theorem 1. *Let $|\Psi\rangle$ be chosen uniformly at random from Hilbert space $\mathcal{H}$. The ensemble $\mathcal{E}_{A,\Psi}$ forms an ε-approximate k-design with probability at least 1-δ if* $N_B = \Omega\left(k\ N_A + log\left(\frac{1}{\varepsilon}\right) + log\ log\left(\frac{1}{\delta}\right)\right)$.

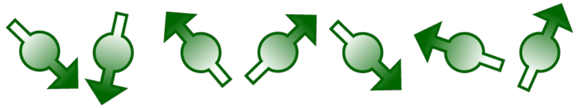
Low-complexity states $|\Psi\rangle$, $\mathcal{E}_{\mathrm{proj}}$ forms approximate state designs.

Theorem 2. *Let $|\Psi\rangle$ be chosen uniformly at random from an ε'-approximate k'-design. Then, the ensemble $\mathcal{E}_{A,\Psi}$ forms an ε-approximate k-design with probability at least 1-δ if …*

Ho and Choi, PRL (2022)

Solvable model I: **Kicked Ising model**

1D spin-1/2 chain



Floquet dynamics

$$U_F = R_y \, e^{-iH_{\text{Ising}}\tau}$$

$$R_y = e^{-i(\pi/2)\sum_j S_j^y}$$

$$H_{\text{Ising}} = \sum_j J S_j^z S_{j+1}^z + g S_j^z$$

$$J\tau = \pi$$

$$|\Psi(t)\rangle = (U_F)^t |+\rangle^N$$

Theorem. *For a subsystem of size $N_A \le t$, the projected ensemble forms exact quantum state k-designs for all k in the thermodynamic limit $N \to \infty$.*

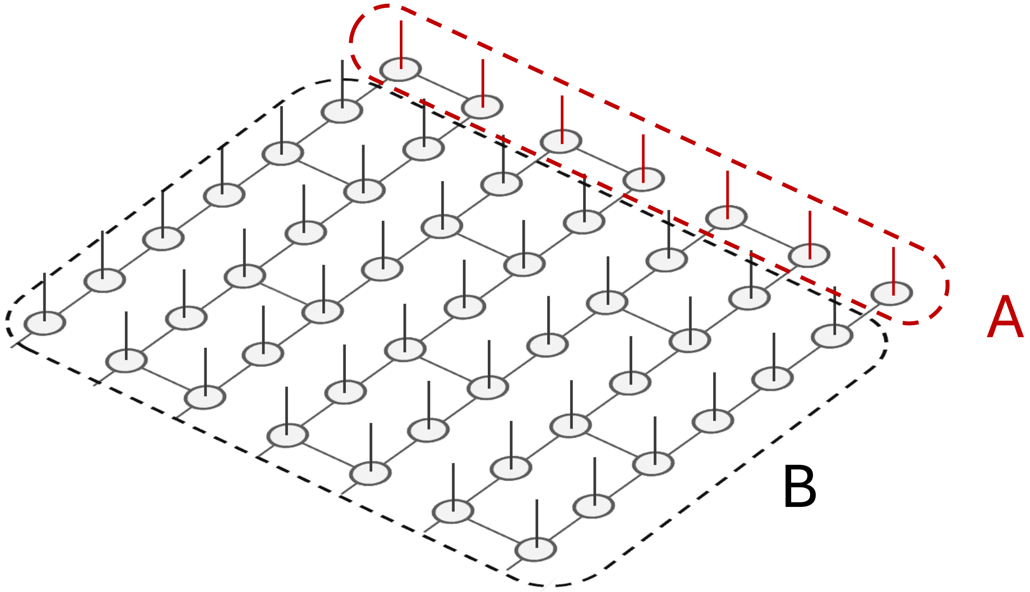
Main ingredients

- ✓ Tensor-network rep.
- ✓ Self dual unitary
- ✓ Constructing Universal gate set

Cotler *et al.*, PRXQ (2023)

Solvable model II: **Time-independent Hamiltonian evolution**

2D square lattice



$$H_{\text{Ising}} = \sum_i h_i \sigma_i^x + \sum_{(i,j)\in E} J_0 \sigma_i^x \sigma_j^x$$

$$|\Psi\rangle = e^{-iH_{\text{Ising}}\tau} |0\rangle^N$$

✓ Cluster state (meas. based QC)

✓ Arbitrary k for arbitrary N_A

Cotler *et al.*, PRXQ (2023)

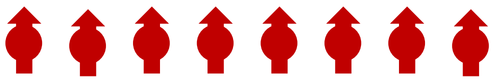
Numerical evidence: **Unitary time evolution**

Mixed field Ising model*

$$H = \sum_j h_x X_j + h_y Y_j + J X_j X_{j+1}$$

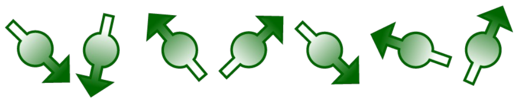
Quenched time evolution

$|\Psi(0)\rangle$



e^{-iHt}

$|\Psi(t)\rangle$



Projected ensemble

$$\mathcal{E} = \{|\Psi_A(z_B)\rangle, p(z_B)\}$$

Evaluate the k-th moment

$$\rho_A^{(k)} = \sum_z p_z \left(|\Psi_A(z)\rangle\langle\Psi_A(z)|\right)^{\otimes k}$$

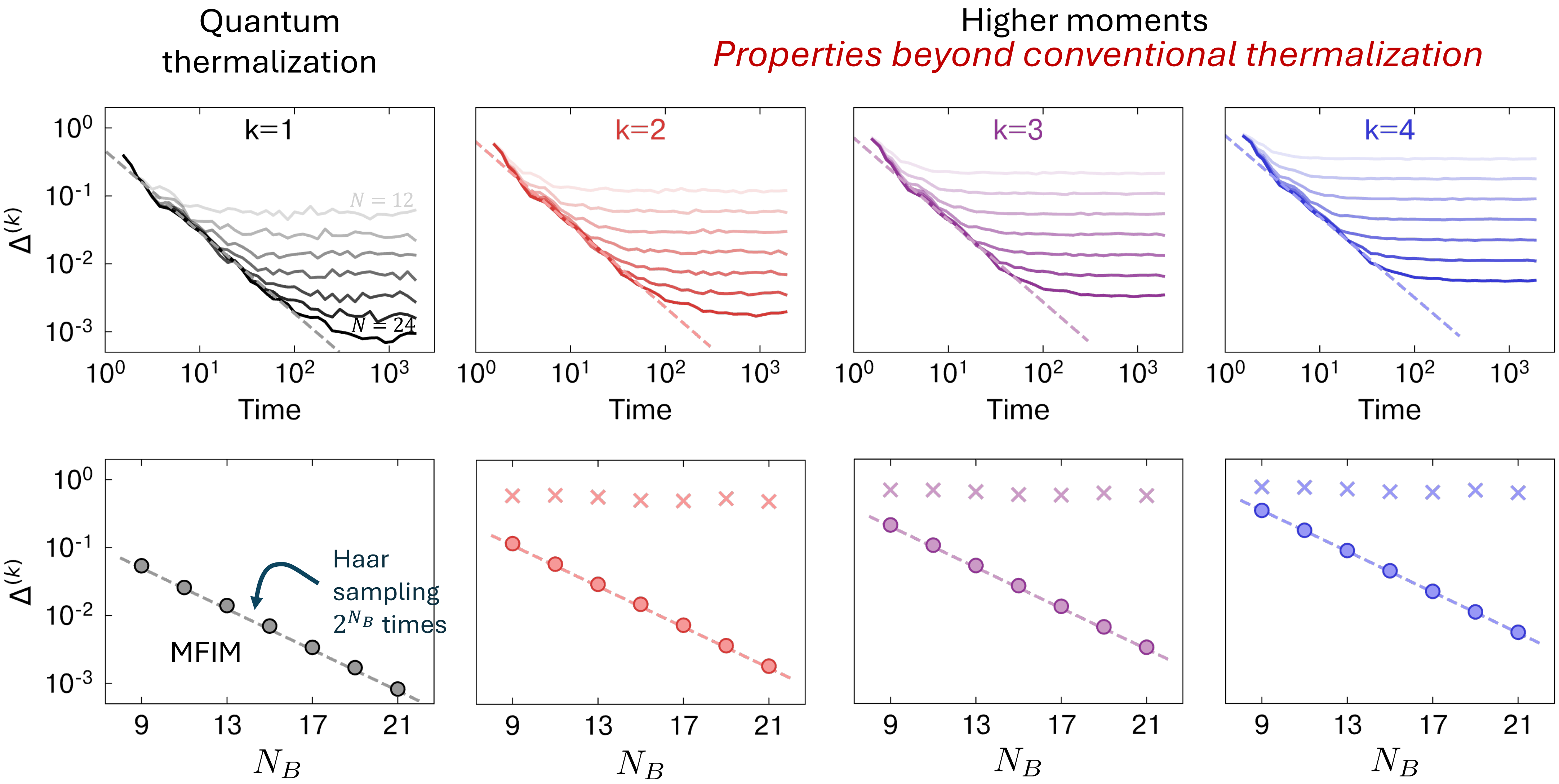
Check the convergence to k-design

$$\Delta^{(k)} = \frac{1}{2}\left|\left|\rho_A^k - \rho_{\text{Haar}}^{(k)}\right|\right|_1$$

We use the parameters $h_x = 0.8090, h_y = 0.9045, J = 1$ adapted from [Kim *et al*, PRE 90, 052105 (2014)].

Emergent k-designs from quench dynamics

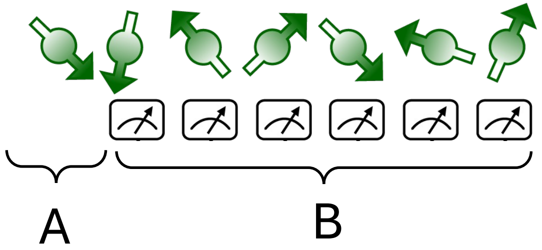
$$\Delta^{(k)} = \frac{1}{2}\left\|\rho_A^{(k)} - \rho_{\mathrm{Haar}}^{(k)}\right\|_1$$

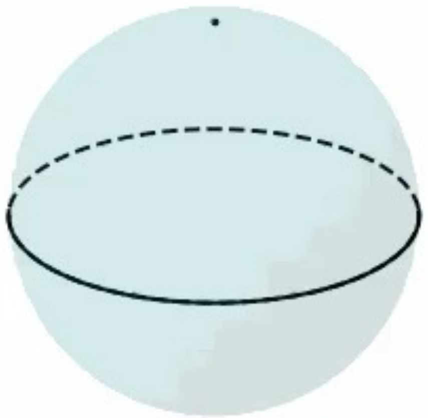


Convergence as good as 2^{N_B} Haar random samples!

Emergent k-designs from quench dynamics

$|\Psi(t)\rangle$





Projected Ensemble forms Pseudo-random states

Ho, Choi, PRL (2022)
Ippoliti, Ho, PRXQ (2023)
Ippoliti, Ho, Quantum (2022)
Claeys *et al.*, Quantum (2022)
Choi *et al.*, Nature (2023)
Cotler *et al.*, PRXQ (2023)
Lucas *et al.*, PRA (2023)
Shrotryia, Ho, arXiv:2305.08437
Chan, Luca, arXiv:2402.16939
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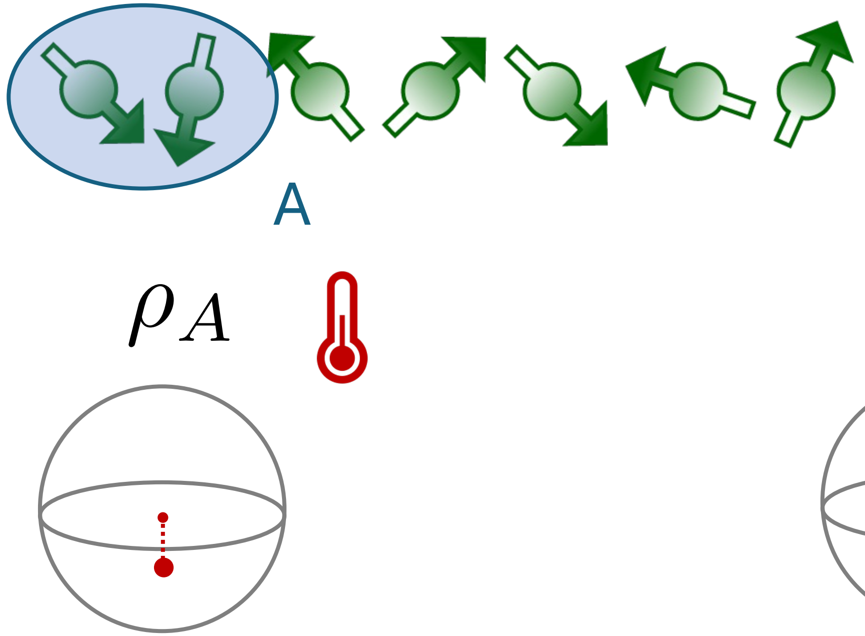
“Deep thermalization”

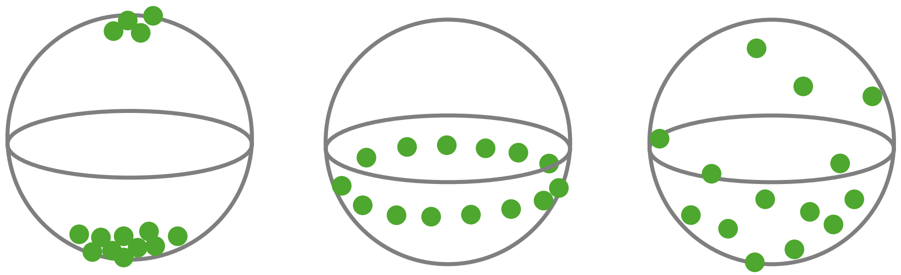
Projected ensemble at **finite temperature**

$|\Psi(t)\rangle$



Quantum thermalization
@ $T < \infty$ $(\beta \neq 0)$

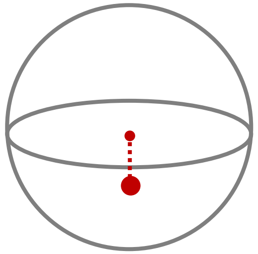
$\mathcal{E}_{\mathrm{proj}} = ?$



Claim. $\mathcal{E}_{\mathrm{proj}}$ approaches the (generalized) **Scrooge ensemble**

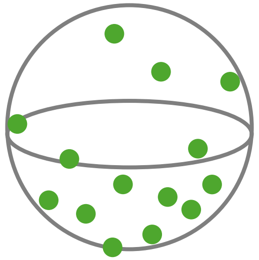
Scrooge Ensemble

ρ_A



Thermally deformed
Haar ensemble

$\mathcal{E}_{\text{Scrooge}}$



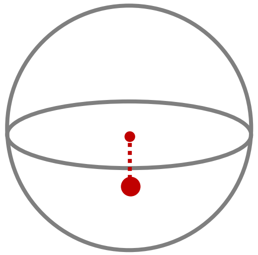
Sampling from $\mathcal{E}_{Scrooge}$

1. Pick a Haar random state $|\phi\rangle$
2. Obtain $|\tilde{\Psi}\rangle = \sqrt{\rho_A}\,|\phi\rangle$ and normalize it
3. Accept it with Pr ~ $\langle\phi|\rho_A|\phi\rangle$.

Jozsa, Robb, Wootters, PRA 49, 668 (1994)

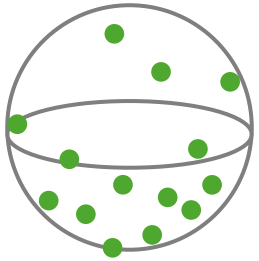
Scrooge Ensemble

ρ_A



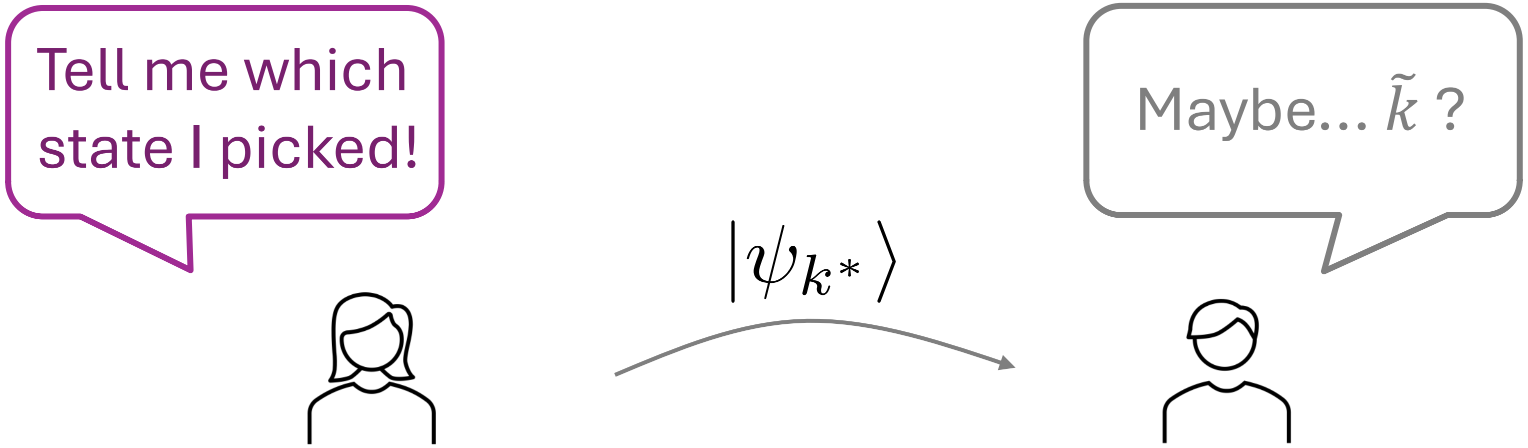
The most *stingy unraveling* of ρ_A.

$\mathcal{E}_{\text{Scrooge}}$



Jozsa, Robb, Wootters, PRA 49, 668 (1994)

Scrooge Ensemble



Pick a random k^*

$|\psi_{k^*}\rangle \in \mathcal{E}_{\text{Scrooge}}$

Perform the *optimum* POVM($\mathcal{E}_A$).

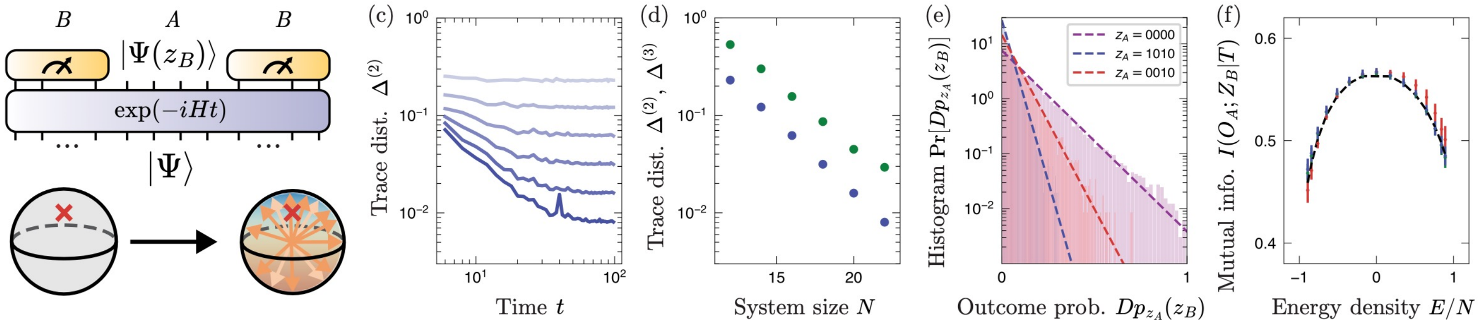
- $\mathcal{E}_{Scrooge}$ *minimizes* the accessible information.
- $\mathcal{E}_{Scrooge}$ is maximally difficult to compress, distinguish, or use for information transmission,

Jozsa, Robb, Wootters, PRA 49, 668 (1994)

Projected Ensemble approximates (generalized) Scrooge Ensemble

See Mark *et al*, arXiv:2403:11970

✓ Numerical confirmation: "stringy behavior"



✓ Analytic arguments / "Derivation"

Nature is stingy, hiding/scrambling information as thoroughly as possible.

Mark *et al.*, arXiv:2403:11970
Shaw *et al.* arXiv:2403.11971

Emergent Randomness in Natural Quantum Systems

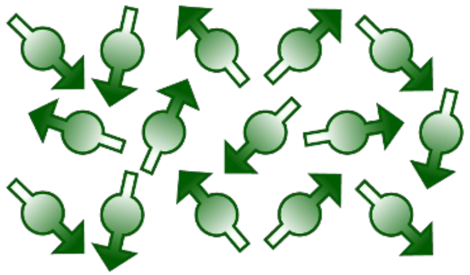
Projected Ensemble

Temporal Ensemble

Applications

$$F \approx F_d$$

Temporal Ensemble



$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$$

Conventional

$$\rho_d \equiv \mathbb{E}_t\big[|\Psi(t)\rangle\langle\Psi(t)|\big]$$

New

$$\mathcal{E}_{\mathrm{temp}} = \{|\Psi(t)\rangle\}$$

Claim. $\mathcal{E}_{\mathrm{temp}}$ is as random as possible up to the energy conservation.

Temporal ensemble is the random phase ensemble

$$\mathcal{E}_{\text{temp}} = \mathcal{E}_{\text{rand phase}} = \left\{ \sum_{j=1}^{D} |c_j| e^{i\phi_j} |j\rangle \middle| \phi_j \sim \text{Unif}([0, 2\pi)) \right\}$$

Theorem. *The infinite-time temporal ensemble is equal to a random phase ensemble if and only if the Hamiltonian H satisfies all k-th no-resonance conditions*

Statistical moments

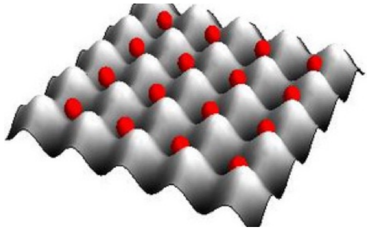
Haar random states

$$\mathbb{E}_{\text{temp}}\left[|\Psi\rangle\langle\Psi|^{\otimes k}\right] = \rho_d^{\otimes k} \sum_{\sigma \in S_k} \text{Perm}(\sigma) + O(\text{tr}(\rho_d^2))$$

Relation between ergodic quantum dynamics and pseudo-randomness.

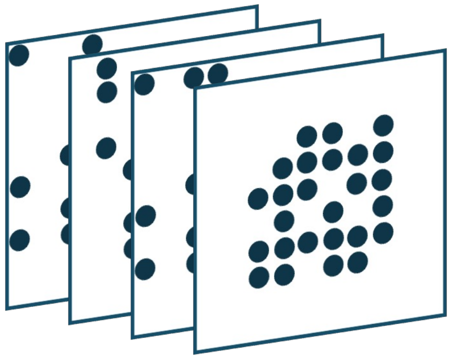
Universal fluctuation from ergodic quantum dynamics

Natural Hamiltonian evolution

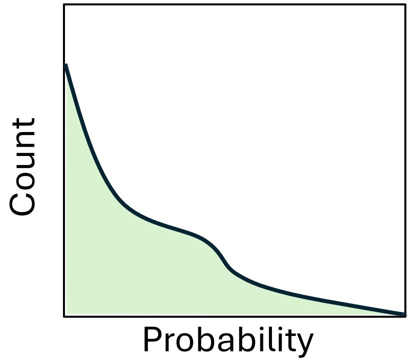


$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$$

Projective measurement in a complete basis $\{|z\rangle\}$



Non-universal PoP

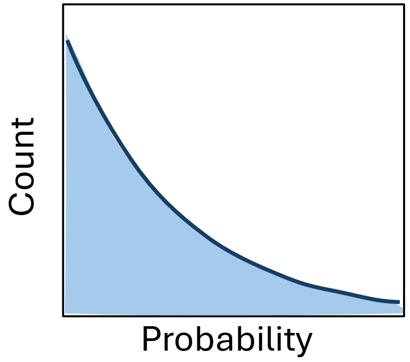


$$p(z,t) = \underbrace{p_{\text{avg}}(z)}_{\text{Systematic}} \times \underbrace{\tilde{p}(z,t)}_{\text{Fluctuation}}$$

Systematic $= \langle z|\rho_d|z\rangle$

Fluctuation

Universal PoP



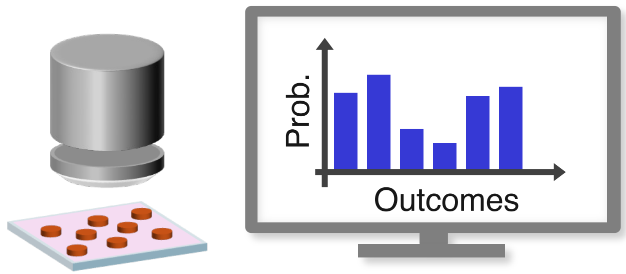
Mark, Choi, Shaw, Endres, SC, *PRL* (2023)
See also Choi, ... SC, Endres, *Nature* 613, 468 (2023)

Emergent Randomness in Natural Quantum Systems

Projected Ensemble

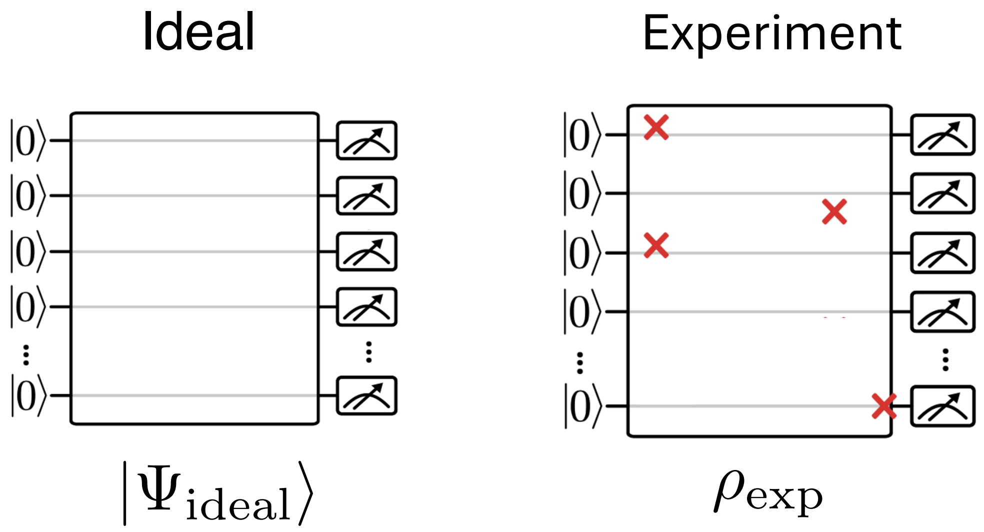
Temporal Ensemble

Applications



$$F \approx F_d$$

Benchmarking Analog Quantum Devices



- Verify the Hamiltonian $H(\vec{E}, \vec{B}, \Omega, \omega)$?
- Decoherence or noise level?
- Calibration?

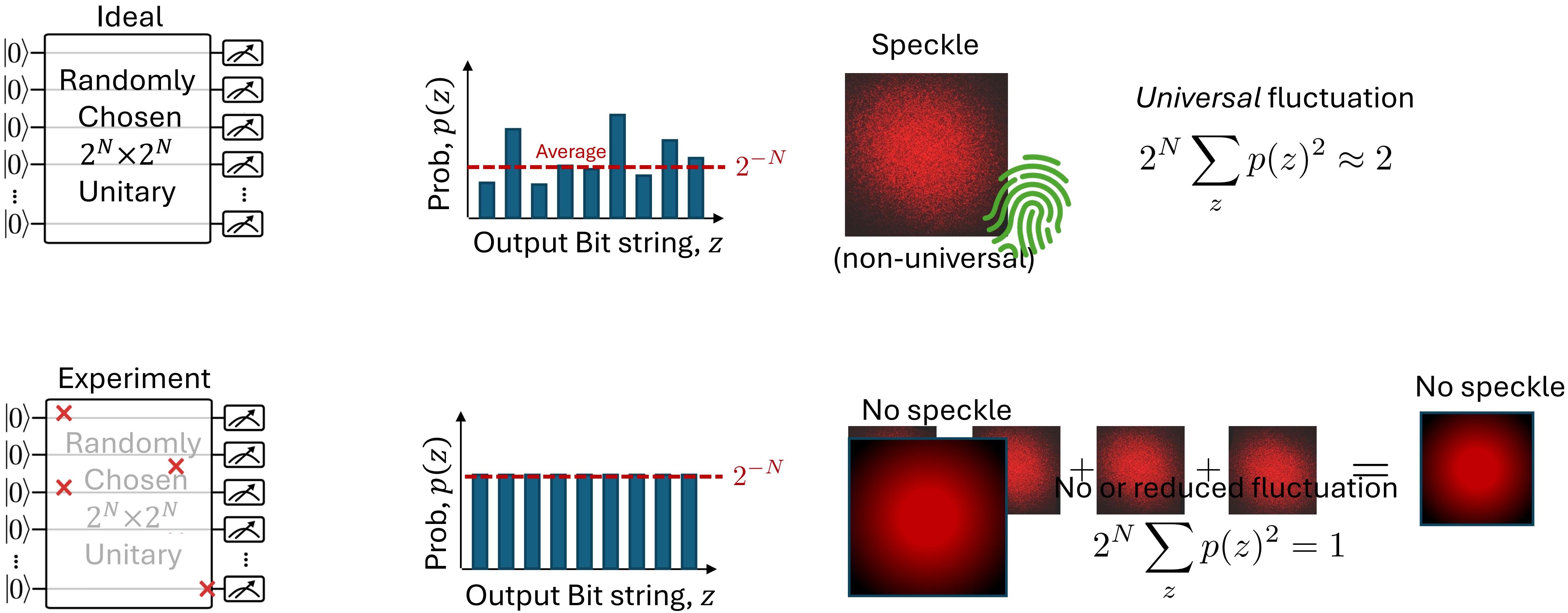
Fidelity estimation

$$\mathcal{F} = \langle \Psi_{\rm ideal} | \rho_{\rm exp} | \Psi_{\rm ideal} \rangle$$

Challenges

- Efficiently measurable?
- Limited controllability

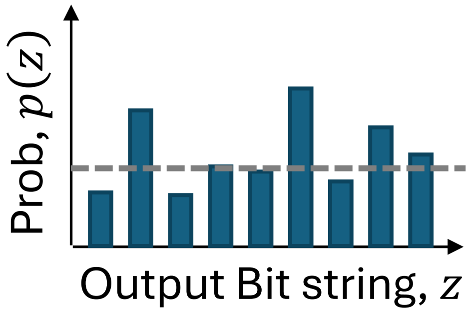
Benchmarking *Digital* Quantum Devices via Speckle patterns



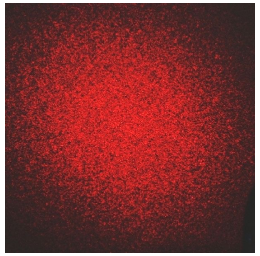
See Arute *et al*, Nature 574 (2019) , Elben *et al*, PRL 124 (2020), Huang *et al*, Nat. Phys. 16 (2020)

Comparing Speckle Patterns: XEB

Ideal theory

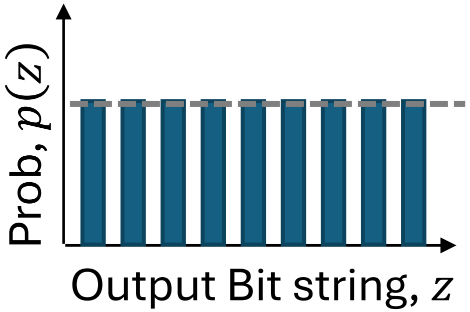


Speckle

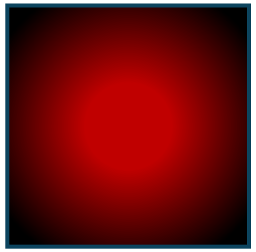


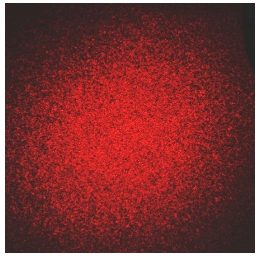
VS

Experiments



No or incorrect speckle





Linear Cross-Entropy Benchmark (XEB)

Sample efficient!

Experiment **Ideal state**

$$F_{\mathrm{XEB}} = 2^N \sum_z p_1(z) p_0(z) - 1$$

$$\approx 2^N \langle p_0(z) \rangle_{\mathrm{sample}} - 1 \approx F$$

See Arute *et al*, Nature 574 (2019) , Elben *et al*, PRL 124 (2020), Huang *et al*, Nat. Phys. 16 (2020)

Comparing Speckle Patterns: XEB

- Need to apply random unitary
- Require long coherence time

Challenging for analog quantum simulators

Improved method using *emergent randomness*

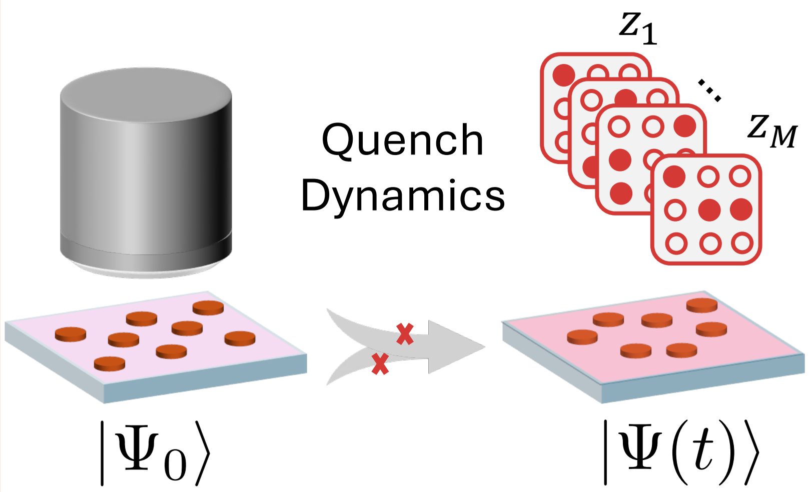
$$p(z,t) = p_{\mathrm{avg}}(z) \times \tilde{p}(z,t)$$

See Arute *et al*, Nature 574 (2019) , Elben *et al*, PRL 124 (2020), Huang *et al*, Nat. Phys. 16 (2020)

Mark, Choi, Shaw, Endres, SC, *PRL* (2023)
Choi, … SC, Endres, *Nature* 613, 468 (2023)

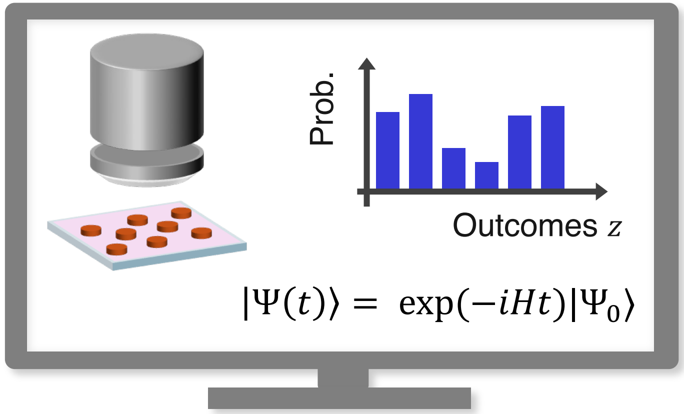
Our protocol

Experiments



1. Prepare $|\Psi_0\rangle$ of interest
2. Evolve under H
3. Measure snapshots $\{z_i\}$
   - Bitstrings or particle configs.

Simulation



4. Simulate dynamics
5. Obtain outcome probabilities

$$p(z) = |\langle z|\Psi(t)\rangle|^2,$$

$$p_{\text{avg}}(z) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \text{d}t\, p(z,t)$$

Data Processing

6. Evaluate our *computationally assisted statistic:*

$$F_d(z_1, z_2, \ldots, z_M)$$

$$\approx 2\frac{\frac{1}{M}\sum_{i=1}^M \tilde{p}(z_i)}{\sum_z p_{\text{avg}}(z)\tilde{p}(z)^2} - 1 \qquad \tilde{p}(z) \equiv p(z)/p_{\text{avg}}(z) \quad \tilde{q}(z) \equiv q(z)/p_{\text{avg}}(z)$$

Claim:

$$F_d \approx F$$

- State prep $|\Psi_0\rangle$
- Quench Dyn. H
- Measurements

✓ Generic ergodic dynamics
✓ Weak, independent noise
✓ Sample efficient
✓ **Easy to implement**

✓ **State overlap benchmarking**
✓ **Evolution benchmarking**

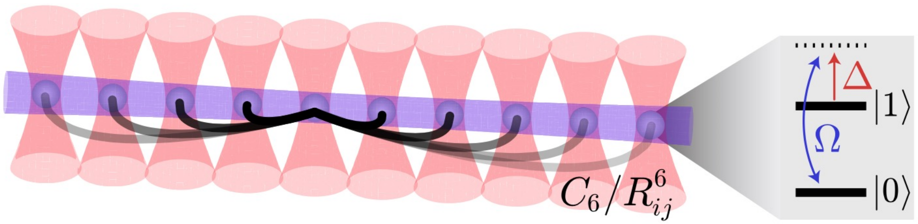
✗ Not suitable for large systems

Benchmarking Analog Quantum Simulators

Choi Shaw Endres

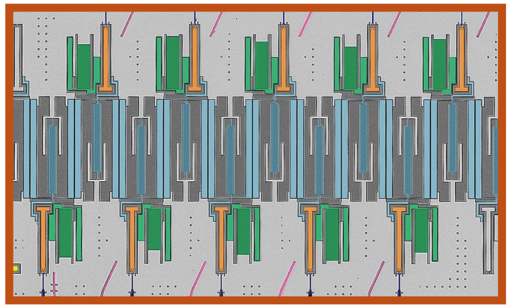
Zhang Kim Painter

60-atom Rydberg Quantum Simulator



Nature 613, 468 (2023)
Nature 628, 71 (2024)

10 Superconducting Qubits
42 metamaterial resonators

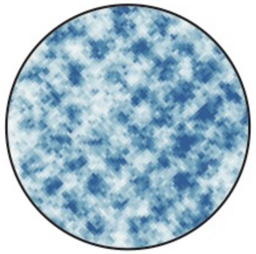


Science 379, 278 (2023)

Mark, Choi, Shaw, Endres, SC, *PRL(2023)*

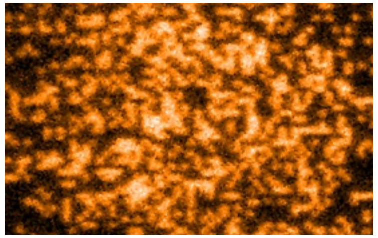
Benchmarking Analog Quantum Simulators

Bose-Hubbard



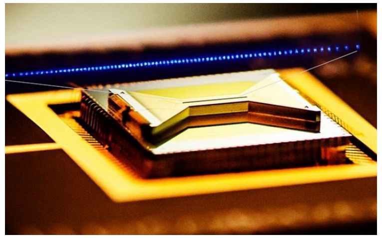
Greiner group @ Harvard

Fermi-Hubbard



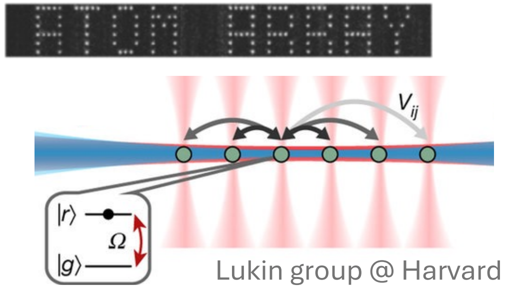
Zwierlein group @ MIT

Trapped Ions

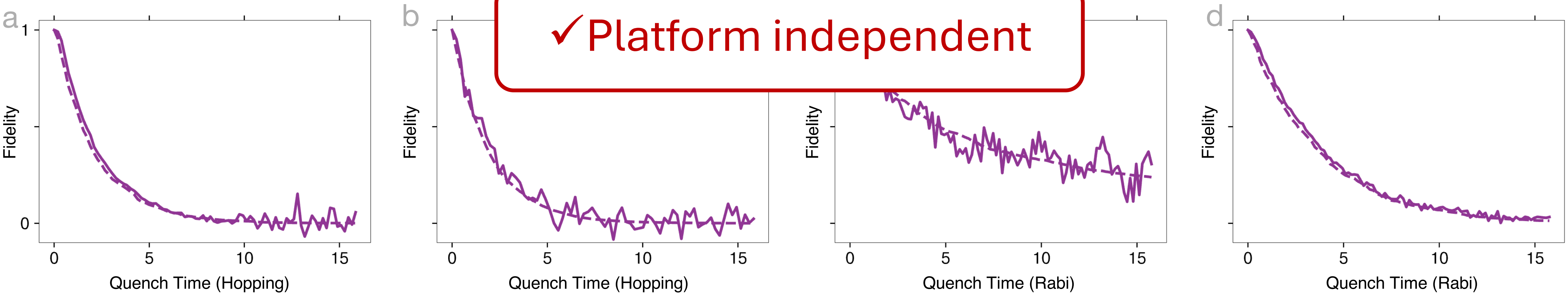


Monroe group @ UMD

2D Rydberg square array



Lukin group @ Harvard



✓Platform independent

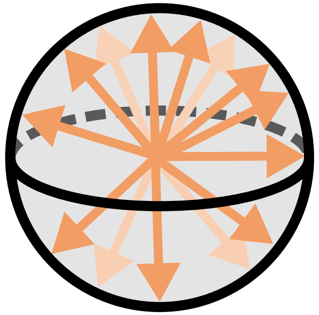
— — — True fidelity

——— Our formula

a. 10 particles on 10 sites in 1D, particle dephasing; b. Spin-1/2 half-filling 10 particles 10 sites (integrable), particle dephasing; c. 1D Long-range TFIM 14 spins, depolarization noise; d. 5x5 square lattice PXP model + detuning, dephasing.

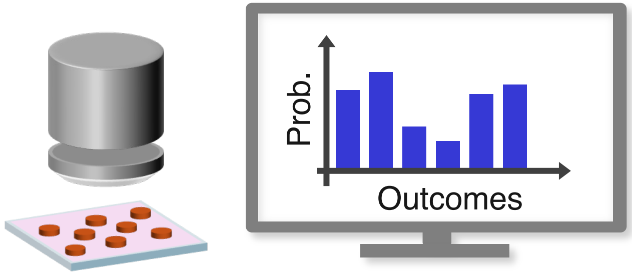
Emergent Randomness: Take Home Message

Projected Ensemble



Temporal Ensemble

Applications



$$F \approx F_d$$

✓ Natural quantum systems display a **universal** form of randomness.

✓ Nature is **stingy**, hiding/scrambling information

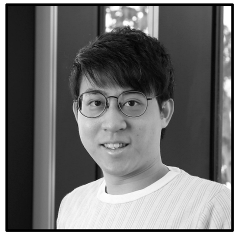
✓ Ergodic quantum dynamics as **resource** for application

✓ Near-term quantum devices produce new science

Collaborators


Daniel Mark


Jordan Cotler


Robert Huang


Hannes Pichler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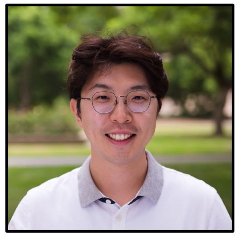
Gil Refael


Manuel Endres


Adam Shaw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Thank you