Using chaos to characterize a programmable analog quantum simulator

Adam Shaw
Endres Lab

Mostly following:
The entanglement challenge

Start with some simple initial state...
The entanglement challenge

Start with some simple initial state...

... create a useful entangled state (as a resource for computation, simulation, metrology, etc...)
The entanglement challenge

Start with some simple initial state…

... create a useful entangled state (as a resource for computation, simulation, metrology, etc…)

But we are always prone to errors...
And we can’t even detect when they happen!
The entanglement challenge

Start with some simple initial state...

But we are always prone to errors
And we can't even detect when they happen!

... create a useful entangled state
(as a resource for computation, simulation, metrology, etc...)

So how do we scale up quantum systems while maintaining control, minimizing errors, and verifying we've done the correct evolution?
Outlook

**Benchmarking**
For a large scale analog quantum simulator

**Applications**
Entanglement estimation, noise learning, etc
Rydberg atom arrays

Optical tweezers:
focused laser beams which can trap single atoms

Can create large arrays in multiple dimensions with full positional control ("rearrangement")
Rydberg atom arrays

In our experiment we work with arrays of strontium atoms.

Lots of interesting atomic physics to discuss...

Can create large arrays in multiple dimensions with full positional control (“rearrangement”)
Rydberg atom arrays

In our experiment we work with arrays of strontium atoms.
Lots of interesting atomic physics to discuss...
But not for this talk!

Can create large arrays in multiple dimensions with full positional control ("rearrangement")
Making atoms interact

\[ |g\rangle = \text{atom in the ground state} \]

Making atoms interact

Atom in the ground state

$|g\rangle = \text{electron} \rightarrow \text{nucleus}$

$|r\rangle =$

Electron wavefunction
$n \sim 100$
(radius can be hundreds of nanometers)

Rydberg atom

Making atoms interact

\[ |g\rangle = \text{nucleus} \quad \text{atom in the ground state} \]

\[ |r\rangle = \text{electron} \]

Try exciting two atoms...

Electron wavefunction
\[ n \sim 100 \]
(radius can be hundreds of nanometers)

Rydberg atom

Making atoms interact

\[ |g\rangle = \includegraphics[width=0.2\textwidth]{nucleus} \quad \text{electron} \quad |r\rangle = \includegraphics[width=0.2\textwidth]{lectron} \]

Small separations*: Double excitation is **blocked**

*Visual is exaggerated

Try exciting two atoms...

Electron wavefunction \( n \sim 100 \)
(radius can be hundreds of nanometers)

Rydberg atom

Energy

\[ |rr\rangle, |gr\rangle, |rg\rangle, |gg\rangle \]

Large separations: Both atoms can be excited

Atomic separation

Making atoms interact

Large separations:
Both atoms can be excited

Atom in the ground state

Electron wavefunction
\( n \sim 100 \)
(radius can be hundreds of nanometers)

Small separations*:
Double excitation is **blocked**

Try exciting two atoms...

One excitation becomes **shared** across both atoms...

**Entanglement!**

Physics with atom arrays, a small selection (pre 2024)

New phases of matter
Semeghini, Science (2021)  
de Leseleuc, Science (2019)

Young, Science (2022)

Young, Science (2022)

Yan*, Spar*, PRL (2022)

Optimization problems
Norcia, Science (2019)  
Madjarov, PRX (2019)

Shaw*, Finkelstein*, Nat Phys (2023)  
Eckner, Nature (2023)

Hubbard physics
Yan*, Spar*, PRL (2022)

Quantum magnetism
Ebad, Nature (2022)

Scholl, Nature (2021)  
Chen, Nature (2023)

Quantum phase transitions

de Leseleuc, Science (2019)

Shaw*, Finkelstein*, Nat Phys (2023)

Hubbard physics
Yan*, Spar*, PRL (2022)

Quantum randomness
Choi*, Shaw*, Nature (2023)

Quantum algorithms
Bluvstein, Nature (2022)  
Graham, Nature (2022)

High-fidelity two-qubit entanglement
Evered*, Bluvstein*, Kalinowski*, Nature (2023)

Scholl*, Shaw*, Nature (2023)
Both quantum simulation and quantum computation applications!

**Analog:**
Simulating complex quantum dynamics by mapping to the natural system evolution

**Digital:**
Solving computational problems using a discrete and universal set of quantum operations
A tale of two qubits

Two different choices of qubit...

Rydberg state $|r\rangle$

metastable state $|e\rangle$

ground state $|g\rangle$

Approximate energy levels of strontium, the atom we use
A tale of two qubits

Two different choices of qubit...

Approximate energy levels of strontium, the atom we use

Digital:
Long-lived qubit, amenable to gate-based operation, Rydberg state is only excited transiently
A tale of two qubits

Two different choices of qubit...

Digital:
Long-lived qubit, amenable to gate-based operation, Rydberg state is only excited transiently

Analog:
Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior

Rydberg state
metastable state
ground state

Approximate energy levels of strontium, the atom we use
A tale of two qubits

Digital qubit: CZ gate fidelity (measured with RB) of 0.9973(4)*

An error model for the experiment shows the probability of transitioning between states. The published value is 0.9935.

Digital:
Long-lived qubit, amenable to gate-based operation, Rydberg state is only excited transiently.

Analog:
Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior.

Finkelstein*, Tsai*, Shaw, Endres, arXiv:2402.16220 (2024)

Other gate results from: Saffman, Lukin, Thompson, Kaufman, Bernien, Zhan, and more.
A tale of two qubits

**Digital**

Long-lived qubit, amenable to gate-based operation, Rydberg state is only excited transiently

**Analog**

Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior

**Analog qubit: Bell state fidelity of ~0.99992** (with erasure conversion*)

Detailed error modeling shows that ~0.9999 is experimentally realistic

Scholl*, Shaw*, et al., Nature 622 (2023)

*for erasure conversion, see also theory: Wu,..., Thompson, Nat Comm (2022)
experiment: Ma,...,Thompson, Nature 622 (2023)
A tale of two qubits

Digital qubit: Long-lived qubit, amenable to gate-based operation

Two different ways of defining a qubit on this three level system...

- for erasure conversion, see also theory: Wu,..., Thompson, Nat Comm (2022)
- experiment: Ma,..., Thompson, Nature 622 (2023)

Analog qubit: Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior

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experiment: Ma,..., Thompson, Nature 622 (2023)

But what about larger systems?

Analog: Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior
Fidelity benchmarking

One-dimensional array of up to 60 atoms
Fidelity benchmarking

One-dimensional array of up to 60 atoms

\[ \hat{H}/\hbar = \Omega \sum_i \hat{S}_i^x - \Delta \sum_i \hat{n}_i + \frac{C_6}{\alpha^6} \sum_{i>j} \frac{\hat{n}_i \hat{n}_j}{|i-j|^6} \]

- Evolve under Ising-like Hamiltonian
- No errors
- Half-chain entanglement entropy grows linearly with a system-size independent rate, but saturates at system size dependent time/level
Fidelity benchmarking

One-dimensional array of up to 60 atoms

\[
\hat{H}/\hbar = \Omega \sum_i \hat{S}_i^x - \Delta \sum_i \hat{n}_i + \frac{C_6}{\alpha^6} \sum_{i>j} \frac{\hat{n}_i \hat{n}_j}{|i-j|^6}
\]
Fidelity benchmarking

One-dimensional array of up to 60 atoms

\[ |0\rangle \quad |0\rangle \quad |0\rangle \quad \vdots \quad |0\rangle \quad |0\rangle \]

Evolve under Ising-like Hamiltonian

No errors

\[ |\Psi\rangle \]

Errors during evolution

\[ \hat{\rho}_{\text{exp}} \]

Fidelity: \[ F = \langle \Psi | \hat{\rho}_{\text{exp}} | \Psi \rangle \]

Fidelity is the probability that we don’t make an error

Exponentially difficult to measure for large systems!
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Exponentially difficult to measure for large systems!

Fidelity benchmarking

One-dimensional array of up to 60 atoms

If the dynamics are explicitly randomized, you can estimate fidelity by measuring $q(z)$, the experimental bitstring probability distribution in a fixed basis.


Fidelity: $F = \langle \Psi | \hat{\rho}_{\text{exp}} | \Psi \rangle$

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Fidelity is the probability that we don’t make an error

If the dynamics are explicitly randomized, you can estimate fidelity by measuring \( q(z) \), the experimental bitstring probability distribution in a fixed basis


We showed this also holds for time-independent Hamiltonian systems!

Choi*, Shaw* et al, Nature (2023)

Mark, Choi, Shaw et al, PRL (2023)

Cotler, Shaw et al, PRXQ (2023)
Fidelities from bitstrings

Choi*, Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, Phys Rev Lett (2023)

Also see: Arute et al, Nature (2019)
Fidelities from bitstrings

Quantum dynamics maps to near-random distribution...
Like a quantum fingerprint

Very sensitive to the exact initial state and dynamics!

Choi*, Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, Phys Rev Lett (2023)

Also see: Arute et al, Nature (2019)
Fidelities from bitstrings

Impose a phase flip error on a single qubit: initially not visible! (because error is orthogonal to measurement basis)

Quantum dynamics maps to near-random distribution... Like a quantum fingerprint

Choi*, Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, Phys Rev Lett (2023)

Also see: Arute et al, Nature (2019)
Fidelities from bitstrings

Quantum dynamics maps to near-random distribution...
Like a quantum fingerprint

But error soon scrambles, changing the distribution!
"Butterfly effect"

Choi*, Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, Phys Rev Lett (2023)

Also see: Arute et al, Nature (2019)
Fidelities from bitstrings

Fidelity estimator proportional to Theory-Experiment correlation!

Choi*, Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, Phys Rev Lett (2023)

Also see: Arute et al, Nature (2019)

Quantum dynamics maps to near-random distribution...
Like a quantum fingerprint

But error soon scrambles, changing the distribution!
“Butterfly effect”
Demonstration of benchmarking with experiment

First, use noisy simulation to verify fidelity estimator is accurate to the model fidelity.

Creating maximum entanglement entropy states

Choi*, Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, PRL (2023)
Demonstration of benchmarking with experiment

Creating maximum entanglement entropy states

First, use noisy simulation to verify fidelity estimator is accurate to the model fidelity

Can accurately benchmark the experiment!

Choi*, Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, PRL (2023)
Creating maximum entanglement entropy states

First, use noisy simulation to verify fidelity estimator is accurate to the model fidelity. Can accurately benchmark the experiment!

Needs an exact classical simulation...

What about LARGER systems?

Can accurately benchmark the experiment!

Choi*; Shaw* et al, Nature (2023)
Mark, Choi, Shaw et al, PRL (2023)
What makes classical simulation hard?

- System size
- Evolution time

Exactly simulatable
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- System size
- Evolution time

Exactly simulatable

At early times, the system hasn’t built up much entanglement.
What makes classical simulation hard?

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We can simulate by using Matrix product states (MPS)!
What makes classical simulation hard?

System size

Exactly simulatable

Exactly simulatable

Evolution time

MPS perfectly simulates dynamics while entanglement is below the “bond dimension”

At early times the system hasn’t built up much entanglement

We can simulate by using Matrix product states (MPS)!
What makes classical simulation hard?

- **System size**
  - Exactly simulatable
  - Not exactly simulatable
  - Exactly simulatable

- **Evolution time**
  - MPS perfectly simulates dynamics while entanglement is below the "bond dimension".

At early times, the system hasn't built up much entanglement.

We can simulate by using Matrix product states (MPS)!
Fidelity benchmarking breakdown

- **I** Classically exact
- **II** Classically approximate

The diagram illustrates the relationship between system size and evolution time.
For large enough bond dimension, we exactly benchmark the system.
Fidelity benchmarking breakdown

For large enough bond dimension, we exactly benchmark the system.

When bond dimension is too small, classical accuracy drops.

And so does the benchmarked quantum fidelity!

Fidelity estimate \( \approx \) Experimental true fidelity \( \times \) Simulation fidelity

(Not exactly, but qualitatively)
Fidelity benchmarking breakdown

For large systems, can’t go to high enough bond dimension to benchmark regime II!

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Fidelity benchmarking breakdown

For large systems, can’t go to high enough bond dimension to benchmark regime II!

Fidelity estimate $\approx$ Experimental true fidelity $\times$ Simulation fidelity

(Not exactly, but qualitatively)

Existing benchmarking methods (e.g. system-size extrapolation via patch/elided circuits like Google) do not work for our system because of analog noise sources.
For large systems, can’t go to high enough bond dimension to benchmark regime II!

Fidelity

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Experimental true fidelity</th>
<th>Simulation fidelity</th>
</tr>
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<tr>
<td>≈</td>
<td></td>
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</tbody>
</table>

(Not exactly, but qualitatively)

But we can extrapolate as a function of bond dimension!

Classical exact

Classically approximate

Late time quantum fidelity

Classical resources (MPS bond dimension)

N=6

N=60
Large scale fidelity

Large scale fidelity

This led us to discover certain non-Markovian noise leads to power law fidelity decay!

Large scale fidelity

Once we knew what to look for, clearly visible in small system size error model simulations.

This led us to discover certain **non-Markovian** noise leads to power law fidelity decay!

*Shaw*, *Chen*, *Choi*, *Mark*, et al, *Nature*, 2024
Large scale fidelity

This led us to discover certain non-Markovian noise leads to power law fidelity decay!

**Fidelities**

Our system - 0.095 @ N=60 in 1D*

*10% vs. 0.000000000000000001%
17 orders-of-magnitude better than random chance
Large scale fidelity

This led us to discover certain non-Markovian noise leads to power law fidelity decay! Fidelities

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These results are consistent with a two-qubit fidelity of ~0.999

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Large scale fidelity

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Fidelities

* Our system - 0.095 @ N=60 in 1D
** Google - 0.003 @ N=53 in 2D

These results are consistent with a two-qubit fidelity of ~0.999

*10% vs. 0.0000000000000000001%
17 orders-of-magnitude better than random chance

**Not a fair comparison because of different level of control, but gives a general sense of scale. Higher values in more recent papers (Morvan et al, 2023)
What is the actual classical cost?

Which better represents the quantum world?
Quantum experiment or classical computer?

Physical error vs approximation error
What is the actual classical cost?

Which better represents the quantum world?
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Physical error vs approximation error

Find minimum classical resources for classical computer to have higher fidelity than experiment
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180 core-days on the Caltech supercomputer (using a highly optimized algorithm)
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Quantum is hard

**Sycamore supremacy circuit**

Complex 2D random unitary circuit
Quantum is hard

**Sycamore supremacy circuit**
Complex 2D random unitary circuit

**Our “circuit”**
Time-independent, global, 1D evolution

\[ e^{-i\hat{H}t/\hbar} \]
Quantum is hard

Sycamore supremacy circuit
Complex 2D random unitary circuit

Our “circuit”
Time-independent, global, 1D evolution

Even for the simplest quantum evolution, classical computers struggle to keep up!
Applications!

**Hamiltonian estimation**

- Normalized $P_c$ vs $\Omega/2\pi$ (MHz)
- Normalized $P_c$ vs $\Delta/2\pi$ (MHz)
- Normalized $P_c$ vs $C_6$ (GHz μm$^6$)

**Noise learning**

- Distribution of Gaussian coherent errors
- Distribution of Global depolarizing errors
- Distribution of Local amplitude damping errors

**Unusual thermalization**

- Correlator vs Correlator distance
- Time vs Convolutor distance

**Entanglement estimation**

- Mixed state entanglement proxy vs Time (cycles)
- Mixed state entanglement proxy vs Effective system size
Applications!

Hamiltonian estimation

- Normalized $F_c$ vs $\Omega/2\pi$ (MHz)
- $\Delta/2\pi$ (MHz) vs $C_6 (\text{GHz } \mu\text{m}^6)$

Noise learning

- Gaussian coherent errors
- Global depolarizing errors
- Local amplitude damping errors

Unusual thermalization

- Correlator vs correlator distance
- Standard deviations from theory

Entanglement estimation

- Mixed state entanglement proxy vs time (cycles)
- Effective system size vs time (cycles)
How entangled are we?

Whenever we’ve talked about entanglement, we’ve meant pure state entanglement.
How entangled are we?

Whenever we’ve talked about entanglement, we’ve meant pure state entanglement.

Evolve under Ising-like Hamiltonian:

- **No errors**
  - $|\Psi\rangle$
- **Errors during evolution**
  - $\hat{\rho}_{exp}$

But what about the actual experimental mixed state entanglement?

Notoriously hard to measure, even theoretically!
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\[ |\Psi\rangle \]

Errors during evolution

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But what about the actual experimental mixed state entanglement?

Notoriously hard to measure, even theoretically!

We developed a new mixed state entanglement proxy:

\[ E_{\text{mixed}} = E_{\text{pure}} + \log(F) \]

*For experts, this is a proxy for the negativity.
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Mixed state entanglement entropy is notoriously hard to measure, even theoretically!

Comparing analog quantum simulators and digital quantum processors on the same plot!

General purpose, cross-platform, evaluation metric including both qubit quantity and quality!
How entangled are we?

Mixed state entanglement entropy is notoriously hard to measure, even theoretically!

General purpose, cross-platform, evaluation metric including both qubit quantity and quality!

Closely related to questions of:
- How many Bell pairs could we possibly extract?
- What is the classical simulation complexity?

Comparing analog quantum simulators and digital quantum processors on the same plot!
Applications!

**Hamiltonian estimation**

- Normalized $P_c$ versus $\Omega/2\pi$ (MHz)
- $\Delta/2\pi$ (MHz) versus $C_0$ (GHz $\mu$m$^6$)

**Unusual thermalization**

- Correlator distance versus time (cycles)
- Standard deviations from theory

**Noise learning**

- Distribution of Gaussian coherent errors, Global depolarizing errors, Local amplitude damping errors

**Entanglement estimation**

- Mixed state entanglement proxy versus time (cycles)
- Effective system size versus Effective system proxy

- Data points from different sources: This work (Caltech), Sycamore (Google), Zuchongzhi (USTC), H2 (Quantum)
Unusual thermalization

Entanglement estimation

Hamiltonian estimation

Noise learning

Applications!

- Gaussian coherent errors
- Global depolarizing errors
- Local amplitude damping errors

theory by Daniel Mark
Consider random unitary circuit (RUC) evolution
Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

We will be studying the probability-of-probabilities (PoP) distribution... For RUCs, this is well known to be an **exponential distribution**

*the PoP counts how many probabilities fall into each bin (gray lines)

* a Porter-Thomas distribution
Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

If one error occurs, the PoP distribution will still be an independent exponential distribution!*

*Assuming dynamics is sufficiently scrambling

The PoP counts how many probabilities fall into each bin (gray lines)

*~a Porter-Thomas distribution
Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

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** Assuming dynamics is sufficiently scrambling

Consider random unitary circuit (RUC) evolution

\[ K = N \times D \] possible error locations
(spacetime volume of circuit)

If many independent errors occur, the probability-of-probabilities (PoP) distributions will be many independent exponential distributions.*

* a Porter-Thomas distribution

** Assuming dynamics is sufficiently scrambling

Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

K = NxD possible error locations (spacetime volume of circuit)

No error Probability: F

Errors Probability: (1-F)/K

The probability of observing the different distributions is given by the fidelity, F

K possibilities

We can't keep track of all microscopic errors, so the aggregate PoP is an incoherent sum over all of them!

* Porter-Thomas distribution
** Assuming dynamics is sufficiently scrambling

If many independent errors occur, the probability-of-probabilities (PoP) distributions will be many independent exponential distributions. Assuming dynamics is sufficiently scrambling, the probability of observing the different distributions is given by the fidelity, $F$. The probability of each PoP is

- One distribution with weight $F$
- $K$ distributions with weight $(1-F)/K$

Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

\[ P_{\text{Hypo}}(x) = \bigotimes_i P_{\text{Exp}}(x/\omega_i) = \sum_i \omega_i \exp(-x/\omega_i) \prod_{j \neq i} \frac{\omega_i}{\omega_i - \omega_j} \]

The probability of observing the different distributions is given by the fidelity, \( F \)

The probability-of-probabilities (PoP) distributions will be many independent** exponential distributions*

*Assuming dynamics is sufficiently scrambling

\*Shaw*, Mark*, et al., arXiv:2403.11971
Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

- Incoherent sum over
- One distribution with weight \( F \)
- \( K \) distributions with weight \((1-F)/K\)

The probability of observing the different distributions is given by the fidelity, \( F \)

\[
P_{\text{Hypo}}(x) = \bigotimes_i P_{\text{Exp}}(x/\omega_i) = \sum_i \omega_i^{-1} \exp(-x/\omega_i) \prod_{j \neq i} \frac{\omega_i}{\omega_i - \omega_j}
\]

Parameterization is defined by the noise channel

The probability of many independent exponential distributions

\*Assuming dynamics is sufficiently scrambling
Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution

*Assuming dynamics is sufficiently scrambling

Errors Probability: \( \frac{(1-F)}{K} \)

If many errors occur, the probability-of-probabilities (PoP) distributions will still be exponential distributions

\( K = N^D \) possible error locations (spacetime volume of circuit)

The probability of observing the different distributions is given by the fidelity, \( F \)

Technical details aside, the take-home message is:

Given a noise channel (and a measured fidelity) we can always write the corresponding hypoexponential weights and analytically predict the PoP distribution

Parameterization is defined by the noise channel
What to notice: results from **numerical simulations (bars)** agree very well with corresponding analytical predictions (**same color lines**), while being clearly distinct from analytical predictions for other noise channels (faint lines).
Learning noise from bitstring measurements

What to notice: results from **numerical simulations (bars)** agree very well with corresponding analytical predictions (**same color lines**), while being clearly distinct from analytical predictions for other noise channels (**faint lines**).
Learning noise from bitstring measurements

What to notice:
results from numerical simulations (bars) agree very well with corresponding analytical predictions (same color lines), while being clearly distinct from analytical predictions for other noise channels (faint lines).

But why does this work for Hamiltonian systems?
Learning noise from bitstring measurements

See Soonwon’s talk just before mine about new theoretical discoveries (and experimental confirmations) that **ergodic Hamiltonian systems universally behave like random unitary circuits**...

![Graph showing universal behavior between RUC and ergodic Hamiltonian systems](image)

Mark, Elben, Surace, **Shaw** et al, arXiv:2403.11970
Learning noise from bitstring measurements

Can apply to experiment* using measured fidelity!

*Because of finite-sampling costs, should actually compare low-order moments of the PoP, which are sample-efficient and still predictive

Learning noise from bitstring measurements

Can apply to experiment* using measured fidelity!

Can define a distance from experimental to predicted PoP

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Learning noise from bitstring measurements

Can apply to experiment* using measured fidelity!

Can define a distance from experimental to predicted PoP
And learn experimentally consistent error models!

*Because of finite-sampling costs, should actually compare low-order moments of the PoP, which are sample-efficient and still predictive
Learning noise from bitstring measurements

Can apply to experiment* using measured fidelity!

RUC simulation shows learning noise models in this way is accurate!

Can define a distance from experimental to predicted PoP
And learn experimentally consistent error models!

*Because of finite-sampling costs, should actually compare low-order moments of the PoP, which are sample-efficient and still predictive

Quantitative benchmarking enables both **improving quantum science**, and realizing **new science applications**

Moments analysis

Can more efficiently learn noises just from their effects on the **moments** of the PoP

<table>
<thead>
<tr>
<th>Noise channel</th>
<th>$\kappa_2$</th>
<th>$\kappa_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global depolarization</td>
<td>$F^2$</td>
<td>$2F^3$</td>
</tr>
<tr>
<td>Local incoherent</td>
<td>$F^2 + (1 - F)^2/k$</td>
<td>$2\left(F^3 + (1 - F)^3/k^2\right)$</td>
</tr>
<tr>
<td>Global Gaussian coherent</td>
<td>$F/\sqrt{2}$</td>
<td>$2F^2/\sqrt{3}$</td>
</tr>
</tbody>
</table>
Local noise -> Global depolarizing
Identifying scaling behavior

As the bond dimension is increased, fidelity estimate rises before saturating...
Identifying scaling behavior

As the bond dimension is increased, fidelity estimate rises before saturating...

For large systems, we can’t reach the "saturation bond dimension"
Identifying scaling behavior

As the bond dimension is increased, fidelity estimate rises before saturating...

For large systems, we can’t reach the "saturation bond dimension"

But we can extrapolate as a function of the bond dimension!