Tensor network toy models of AdS/CFT: locality & causality

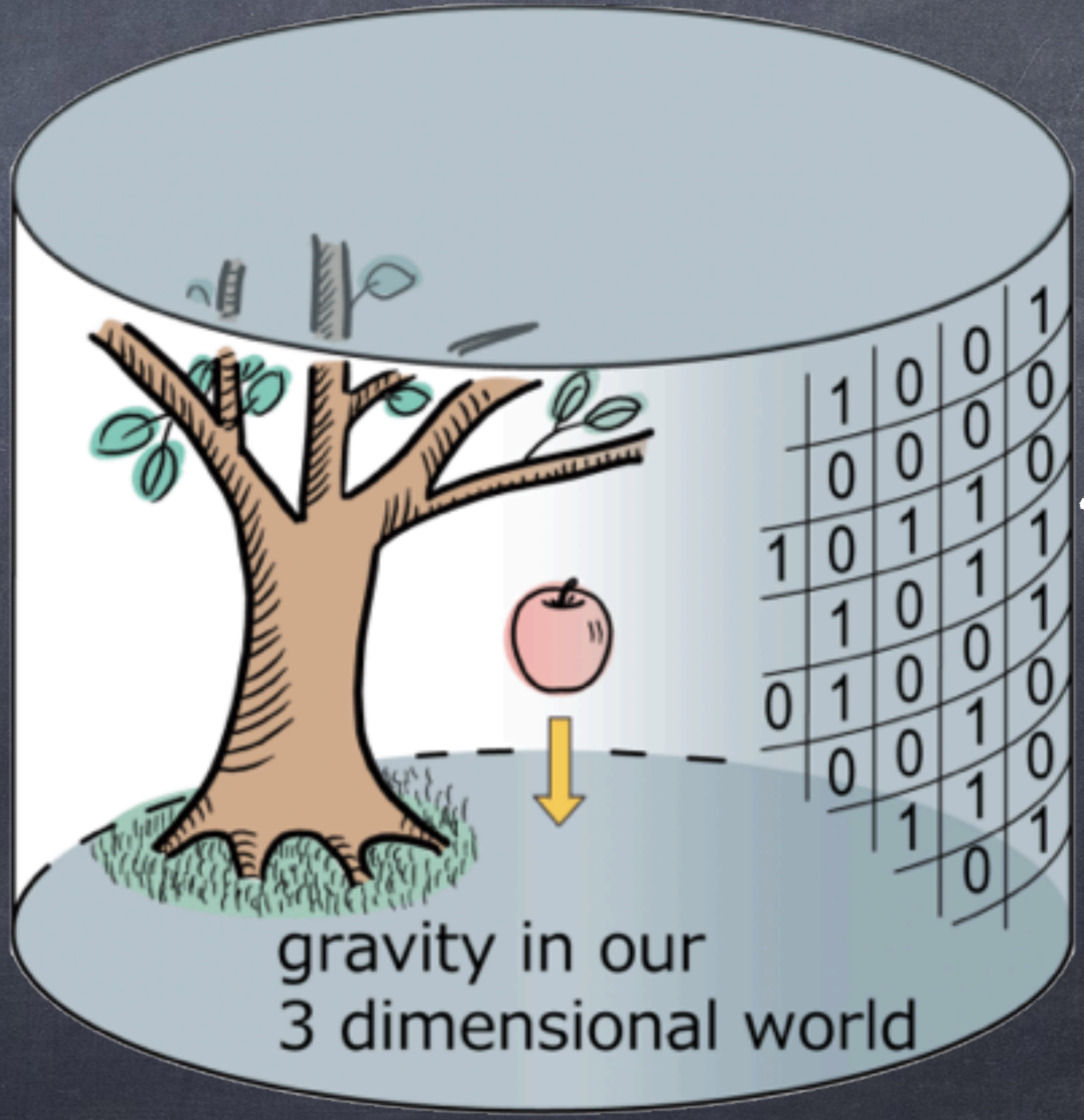
Joint work with Harriet Apel (UCL), Toby Cubitt (UCL), David Perez-Garcia (Universidad Complutense de Madrid) and Patrick Hayden (Stanford)

arxiv:1810.08992, arxiv:2003.13753, arxiv:2105.12067, arxiv:2401.09058

What is AdS/CFT?

The holographic principle: quantum gravity in (d+1)-dimensional spacetime is equivalent to a many body system defined on its boundary

- AdS/CFT is the key example of the holographic principle
- No complete mathematical description of the correspondence
- Exactly solvable toy models using quantum info techniques

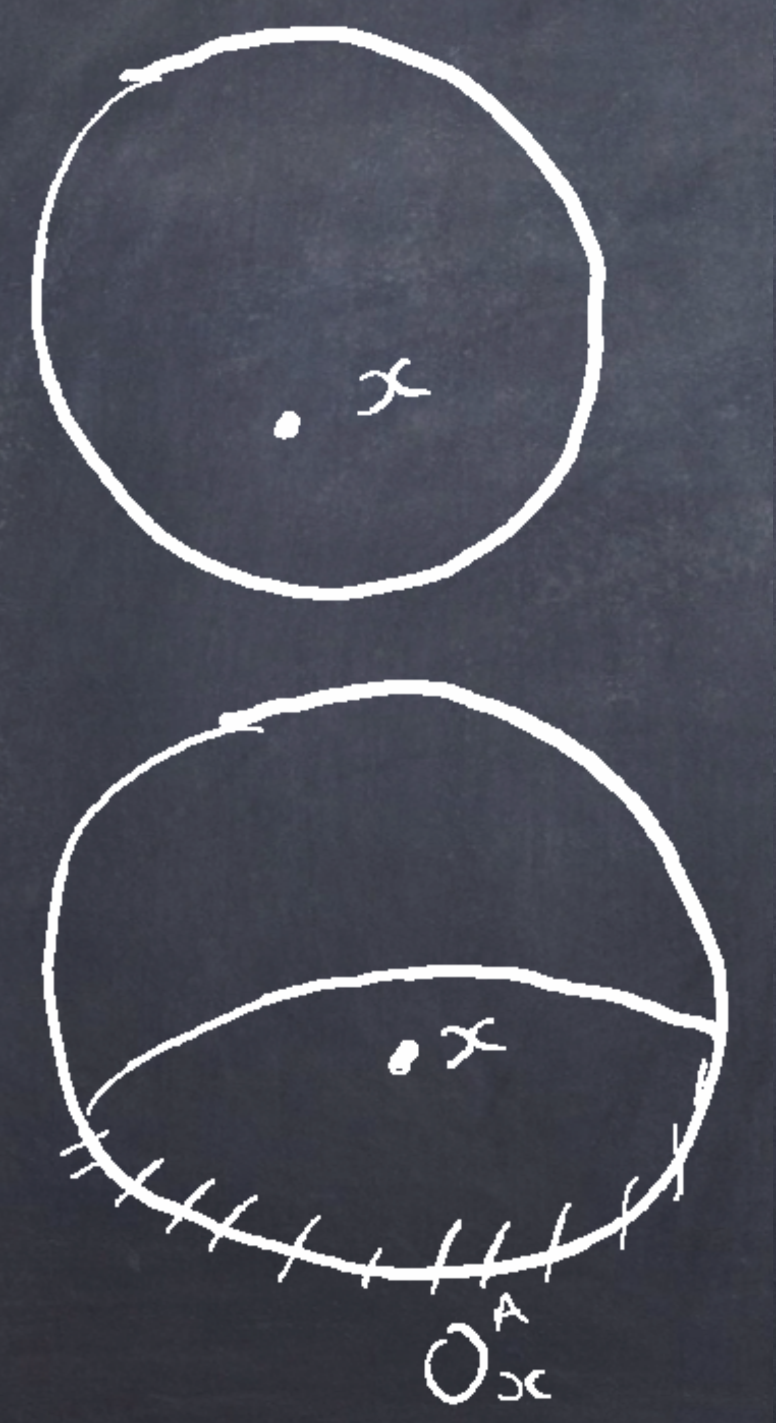


data projected onto boundary

What makes a good toy model?

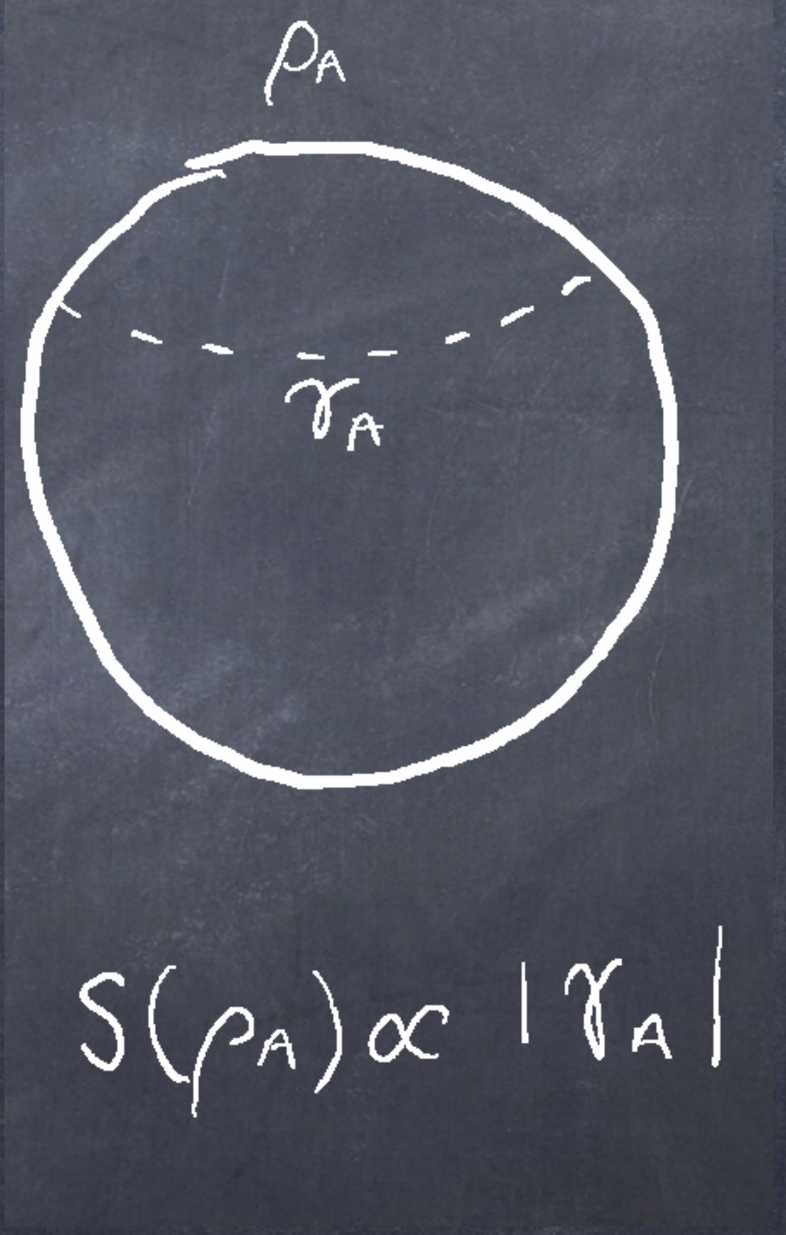
1. Error correction

AdS/CFT acts as an erasure code



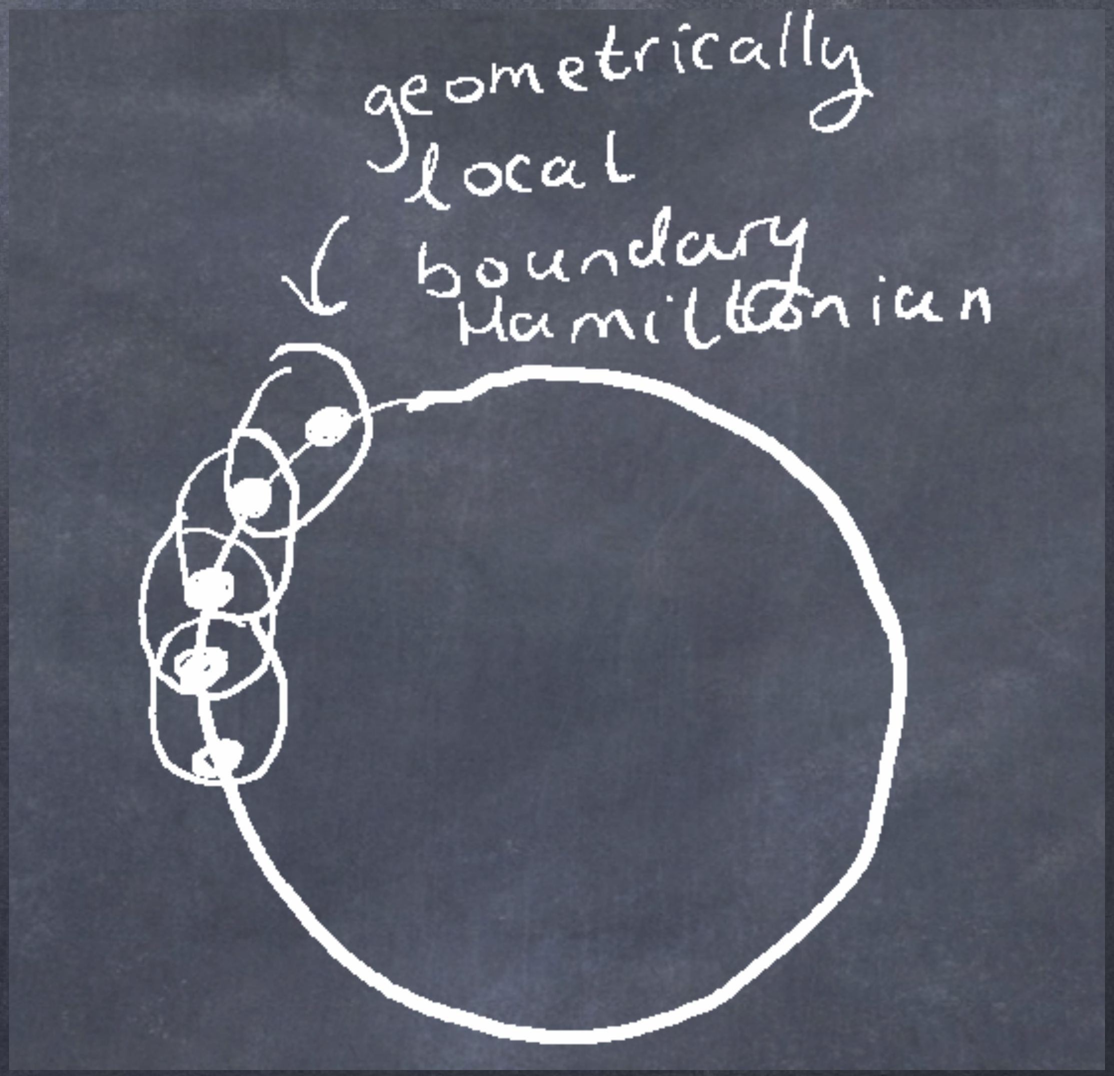
2. Entanglement

Boundary entanglement is related to bulk geometry



3. Local Hamiltonians

The boundary Hamiltonian should be geometrically local

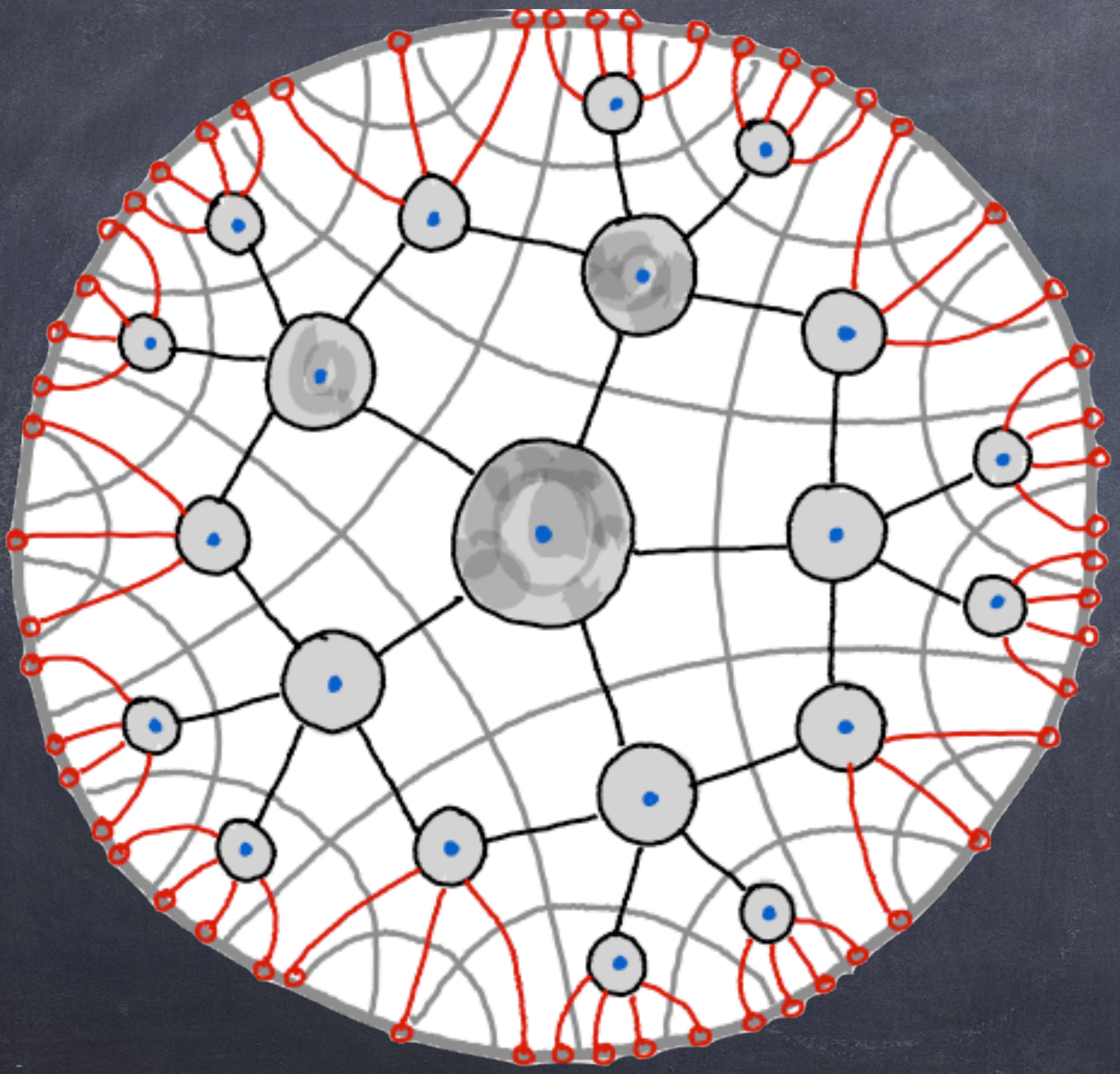


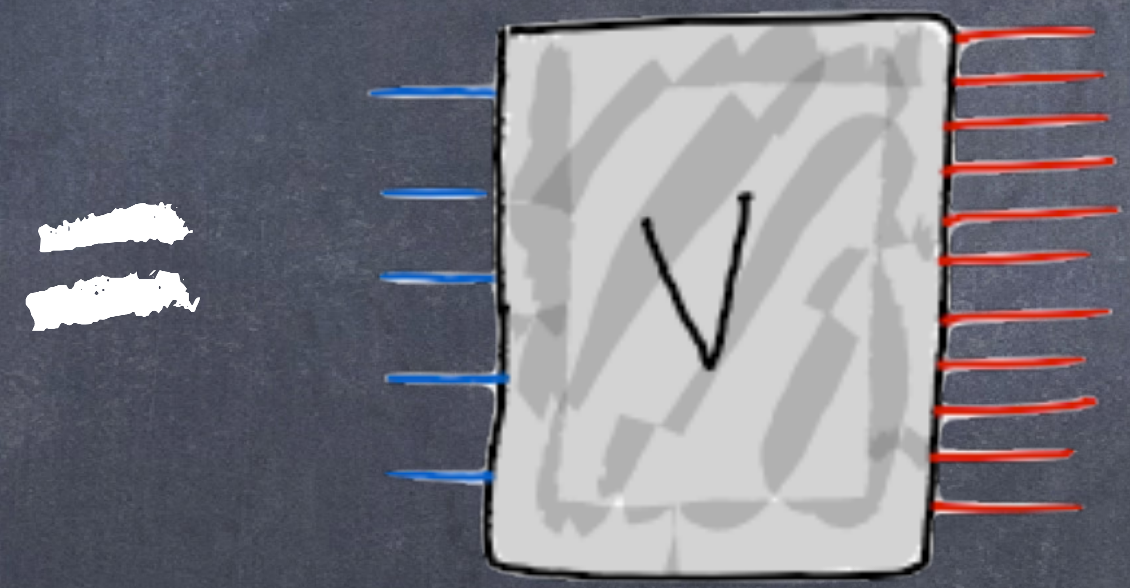
Outline of talk

- Holographic quantum error correcting codes
- Hamiltonian simulation techniques
- Our construction
- Applications: position based cryptography
- Open questions

Holographic quantum error correcting codes

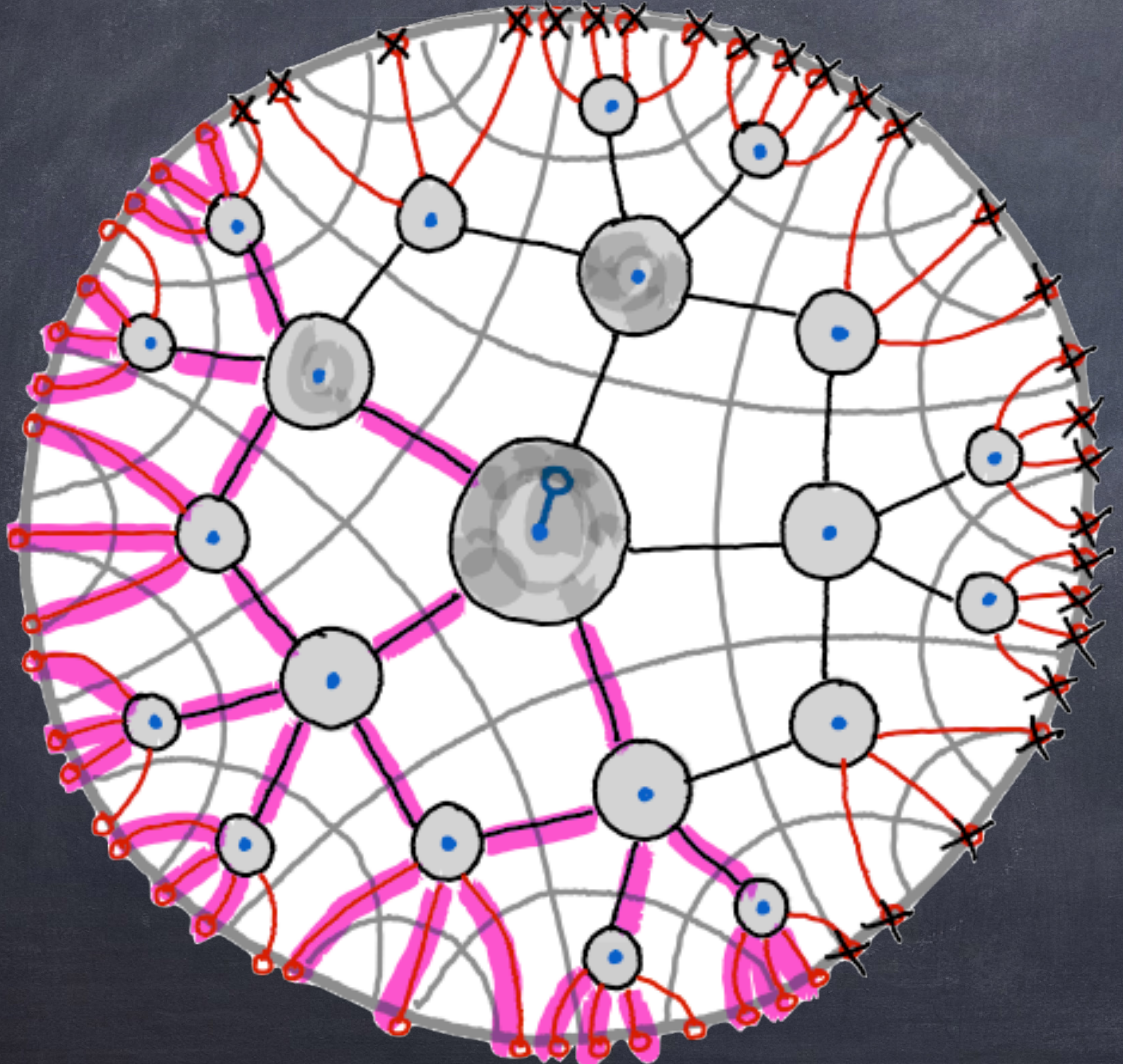
Holographic quantum error correcting codes from perfect tensors [HaPPY 2015]





Perfect tensors: isometries across any bipartition, good QECC

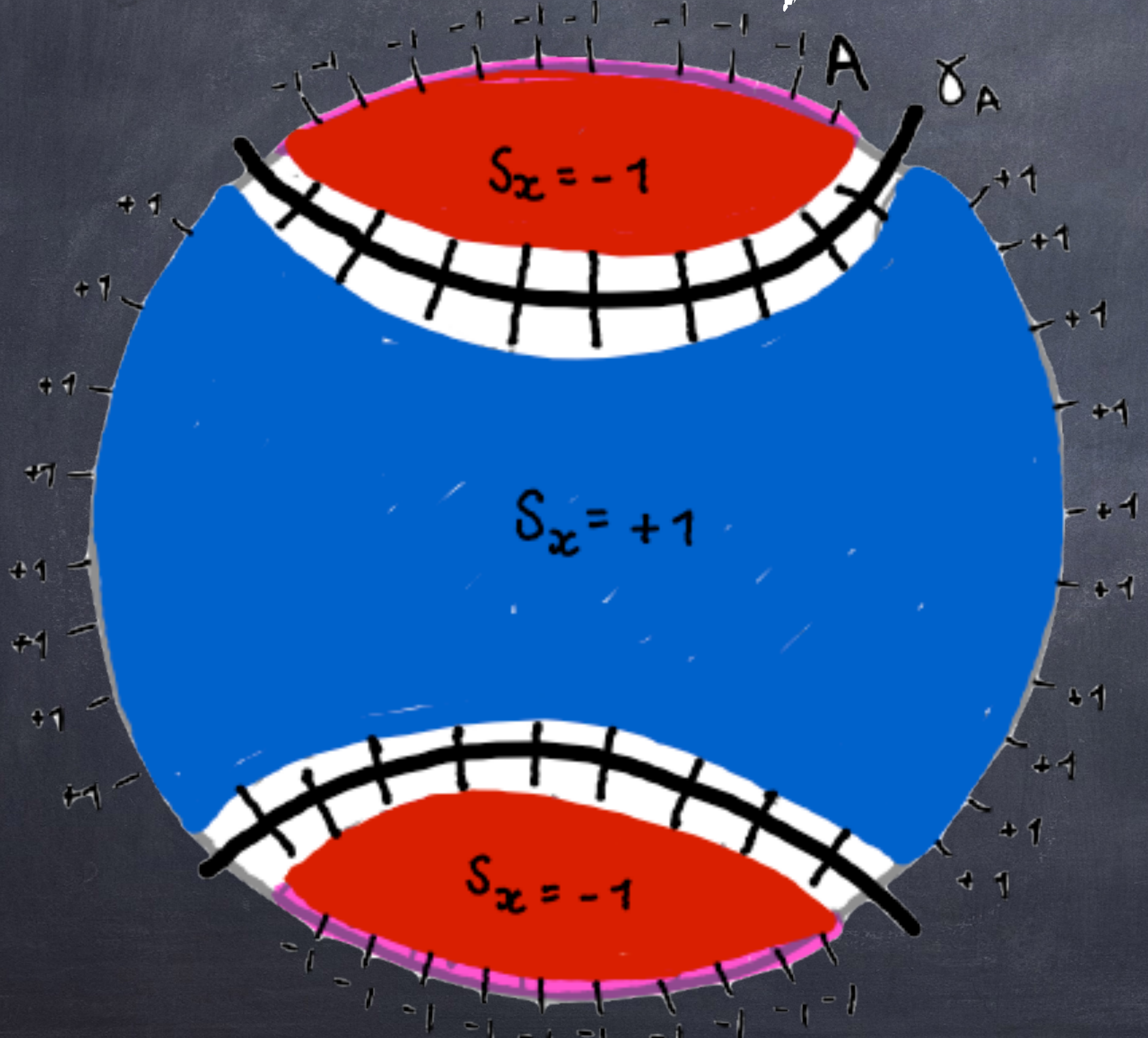
Holographic quantum error correcting codes from perfect tensors [HaPPY 2015]



The error correction properties of AdS/CFT are well captured by this toy model

Some of the expected entanglement structure

Holographic quantum error correcting codes from random tensors

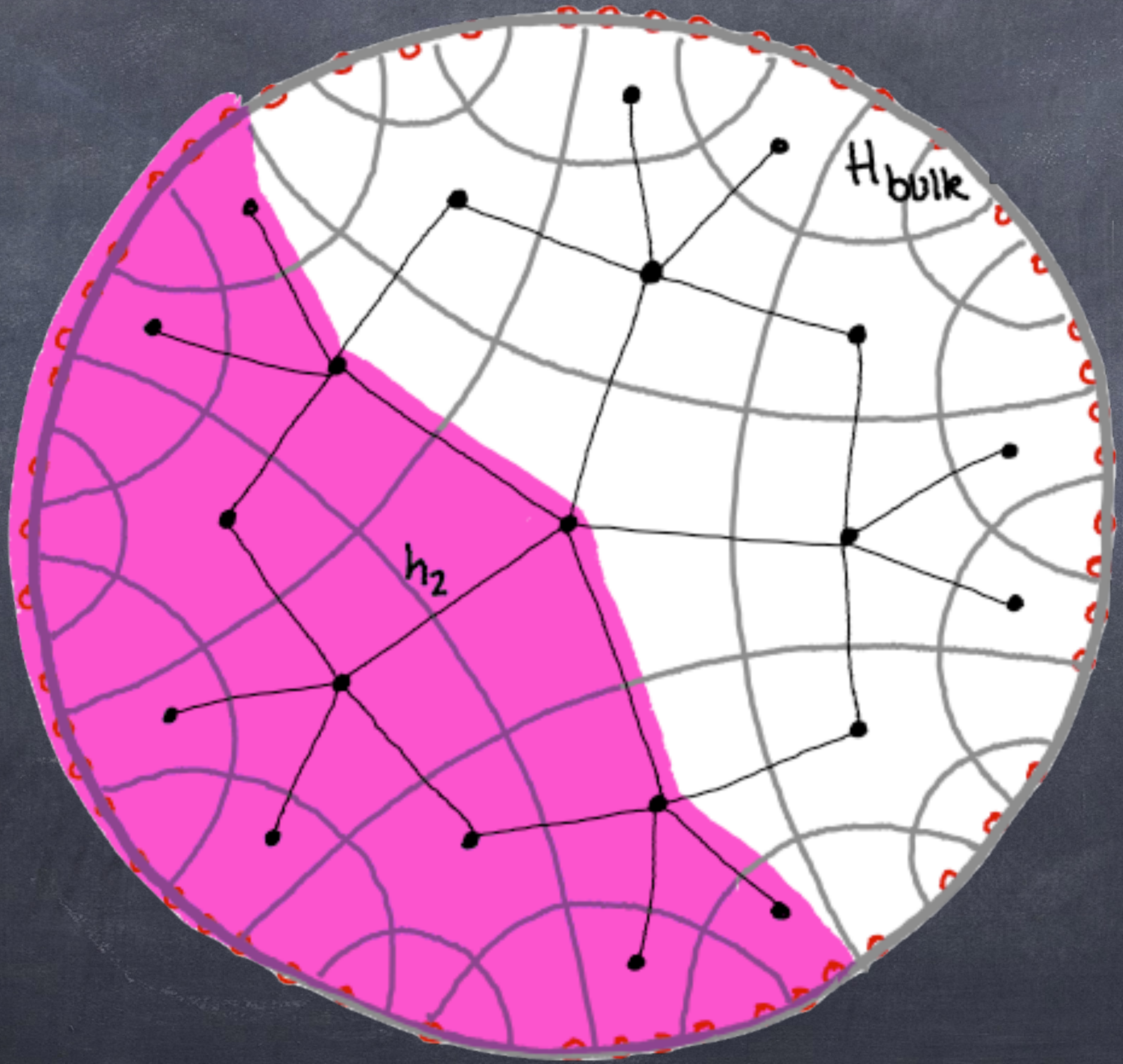


Replace perfect tensors with random tensors:

- better error correction properties
- captures the entanglement properties of AdS/CFT
- requires large local dimension

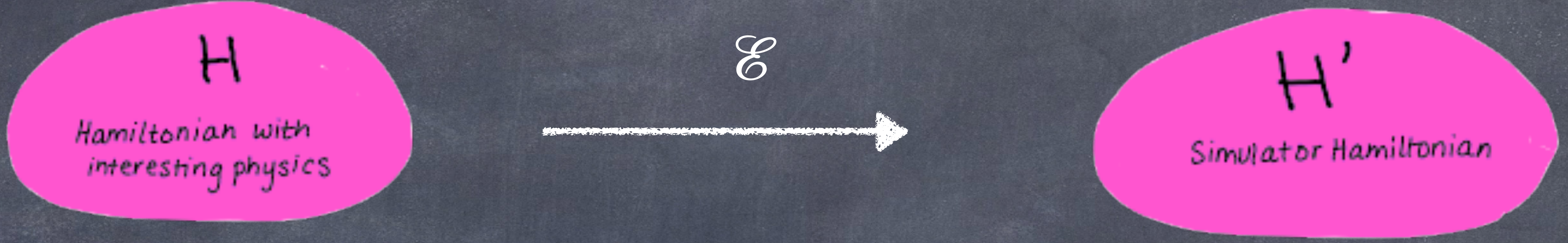
[Hayden et al 2016]

What do boundary Hamiltonians look like in this model?



Hamiltonian simulation

Perfect Hamiltonian Simulation



- Perfect simulation below Δ if $||\mathscr{E}(H) - H'|_{\Delta}|| = 0$
- Operator norm: whole spectrum, all measurement outcomes, thermal properties preserved
- But H' and H can have different interaction graphs

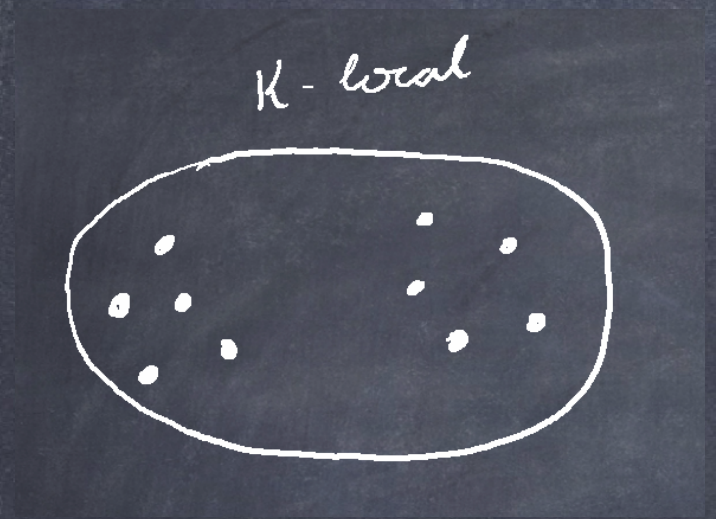
Approximate Hamiltonian simulation

- We say H′ is a (Δ, ϵ, η)-simulation of H if below Δ H′ approximately simulates H
- ϵ controls the error in the eigenvalues of H
- η controls the error in the eigenvectors of H
- Errors in the simulation grow as $2\epsilon t + \eta$

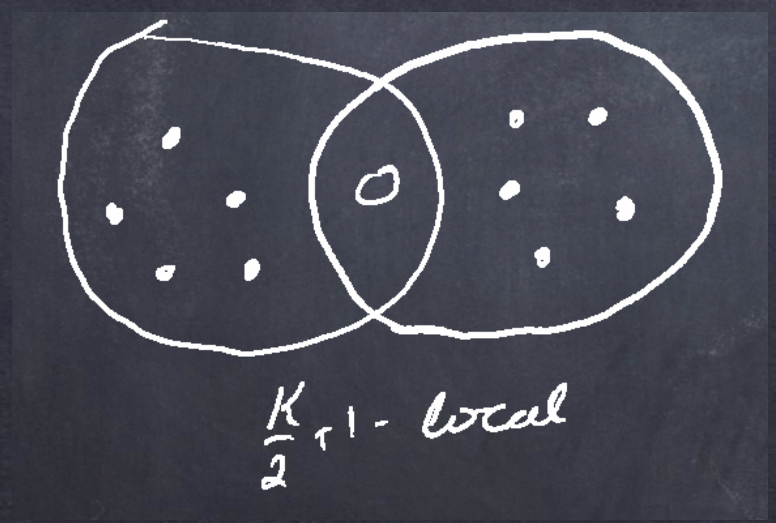
Perturbation gadgets

Subdivision

Reduce the weight of interactions to make the Hamiltonian local

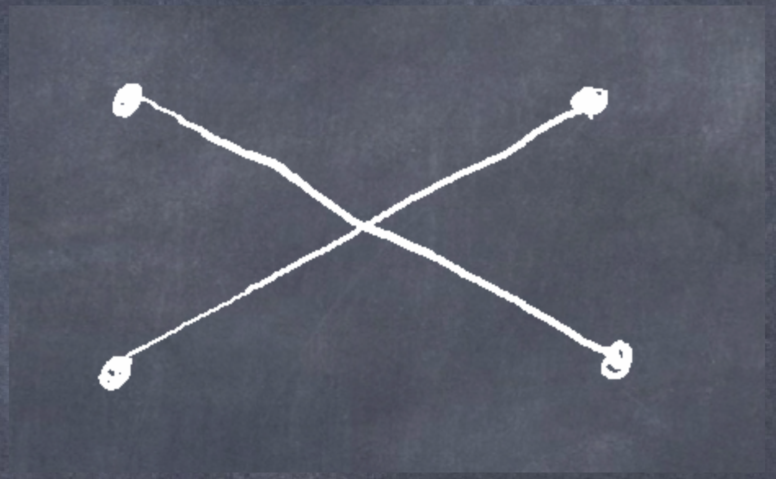


is simulated by:

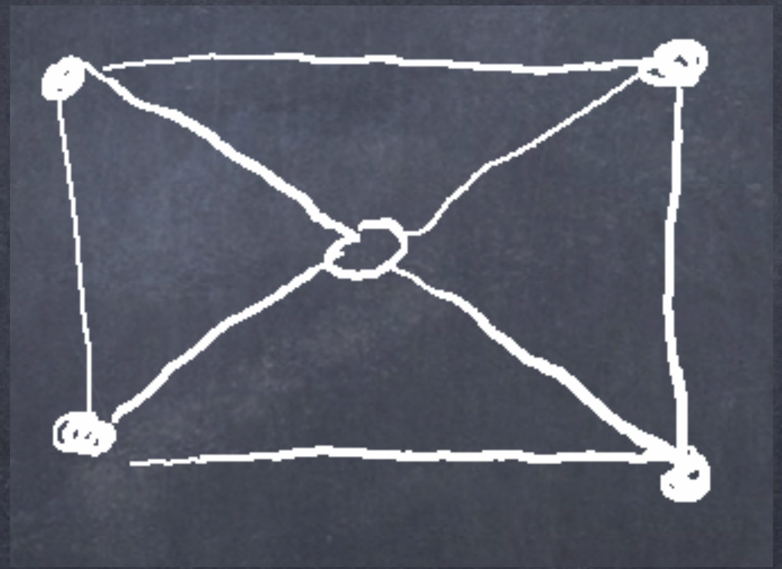


Crossing

Remove crossings so the Hamiltonian is geometrically local

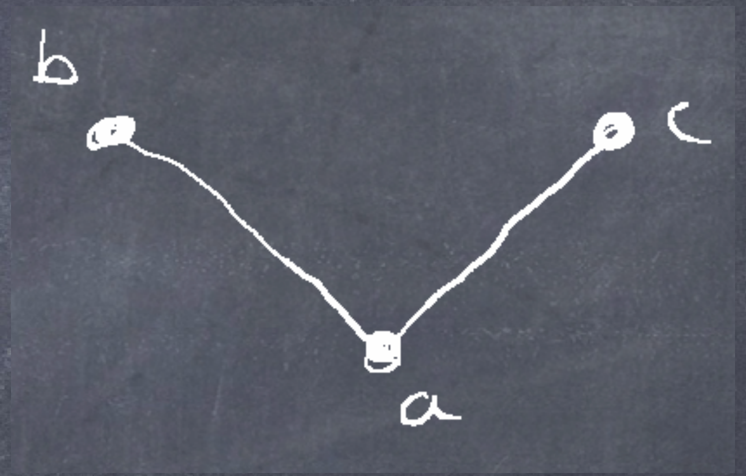


is simulated by:

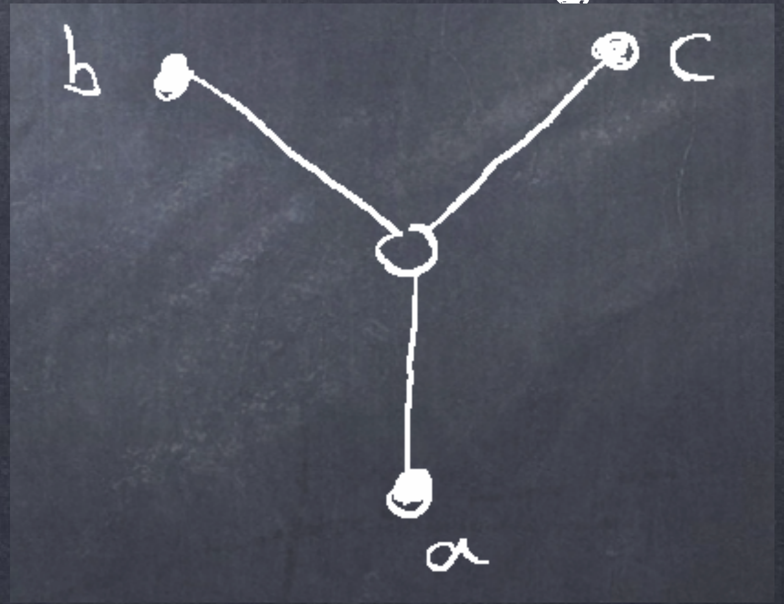


Fork

Reduce degree of interaction graph to place Hamiltonian on a lattice



is simulated by:



Perturbation gadgets

The mediator qubit is acted on by a heavily weighted projector:

$$H_0 = J|0\rangle\langle 0|$$

for $J \gg 1$.

Each perturbation gadget acts on a single Pauli term - handle general Hamiltonians via linearity.

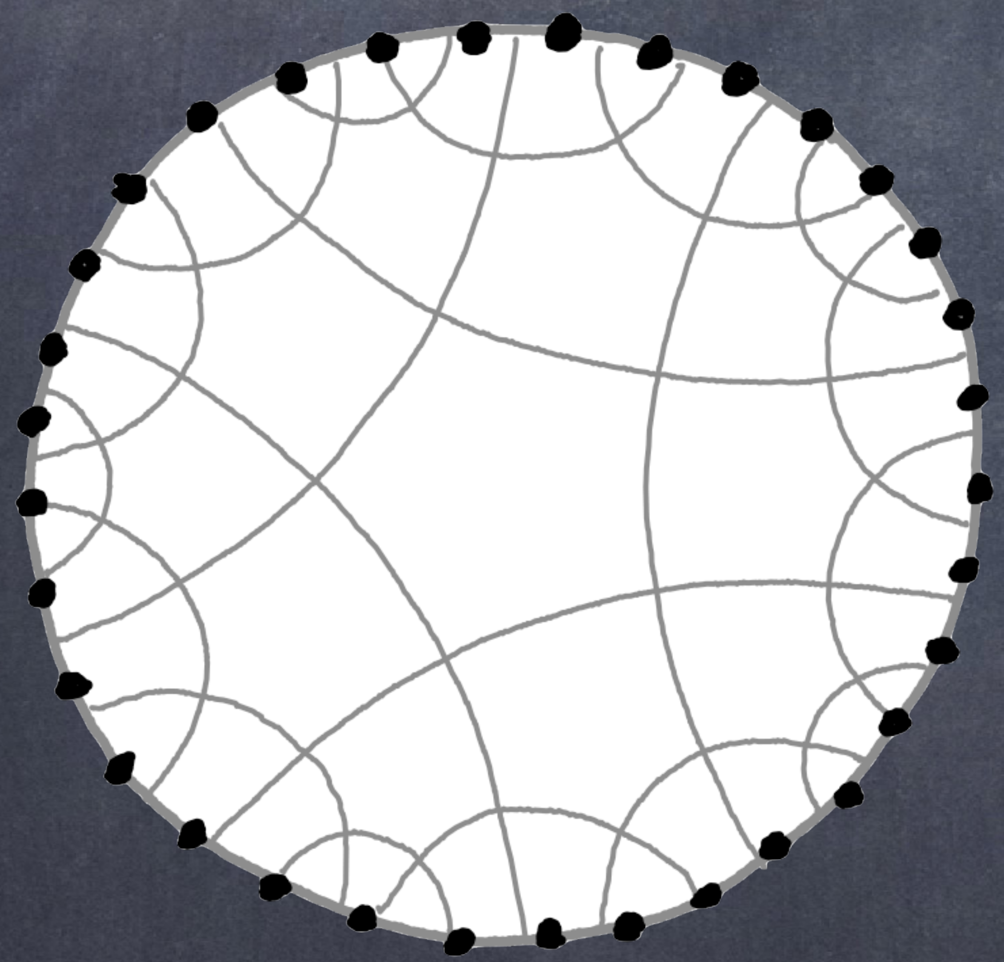
Costs of simulation:

- increase in norm of Hamiltonian
- additional degrees of freedom

Our construction

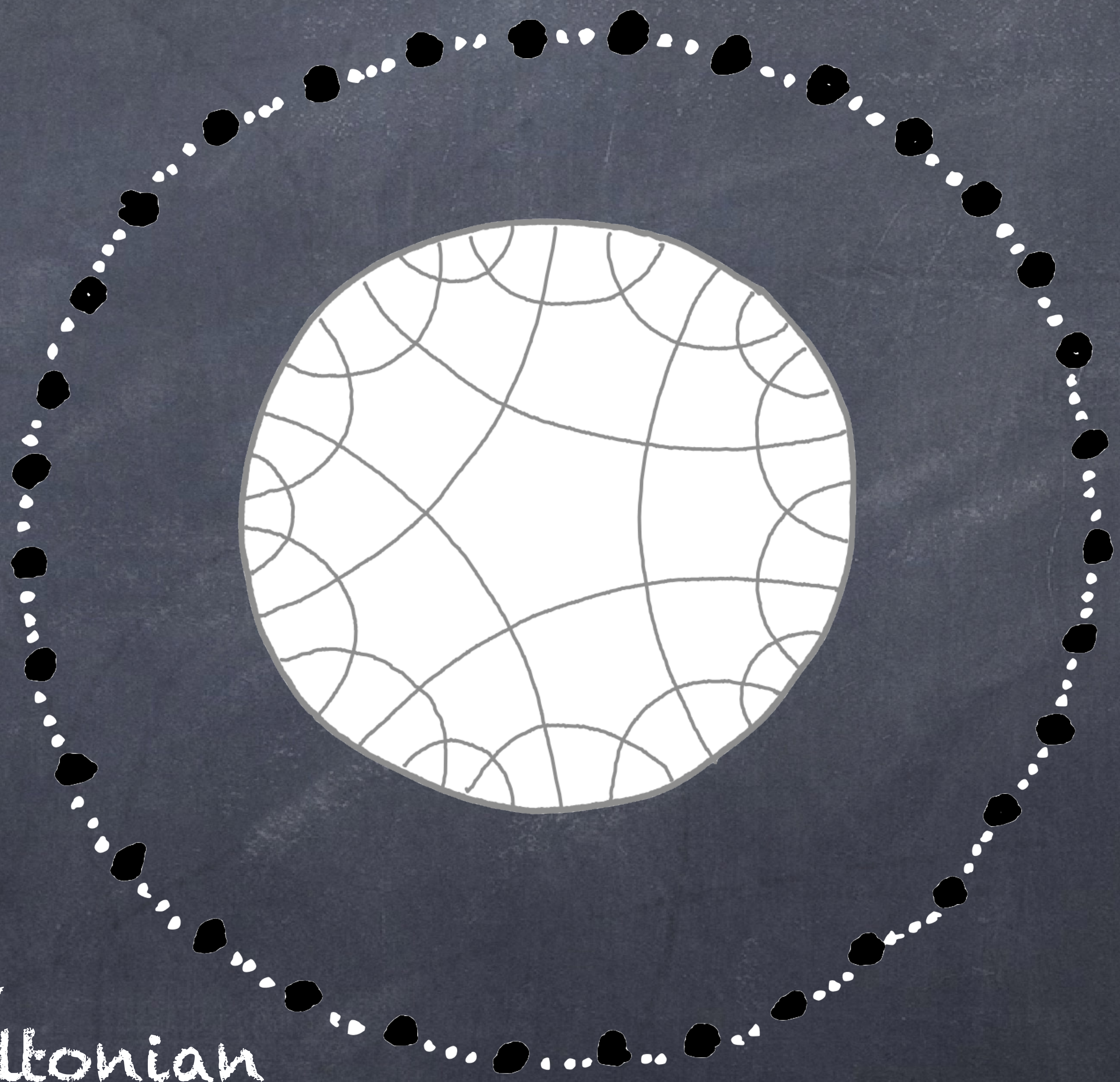
Can we localise the boundary of a random tensor HQECC?

HQECC constructed from Haar random tensors



non-local Hamiltonian on n qudits

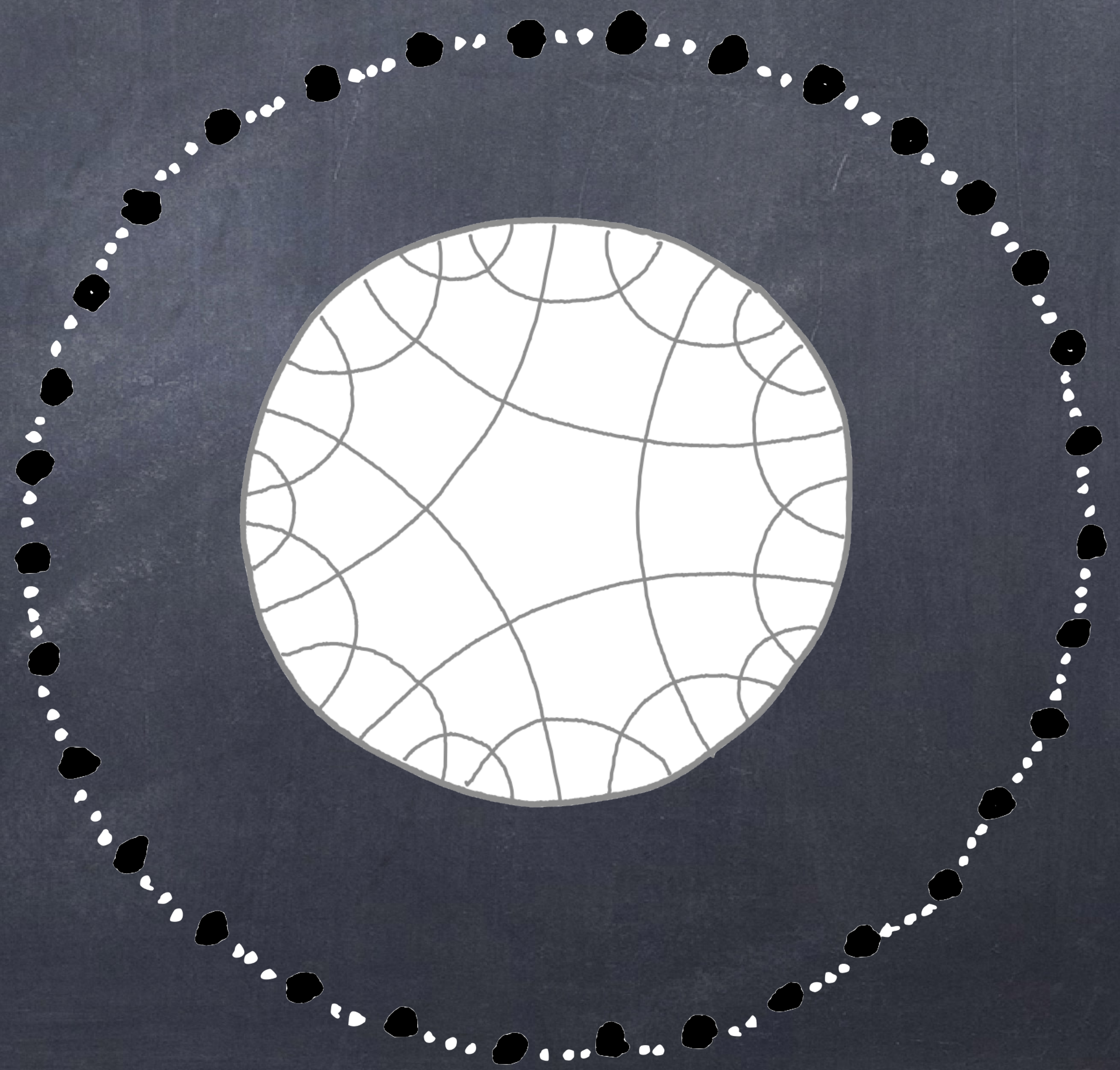
Hamiltonian simulation techniques →



local Hamiltonian on exp(n) qudits

Why has the boundary size increased by so much?

- k-local terms deep in the bulk map to O(n) terms on the boundary
- these terms have Pauli rank exp(n)
- making each term local only requires log(n) rounds of perturbation theory, but exp(n) ancillas

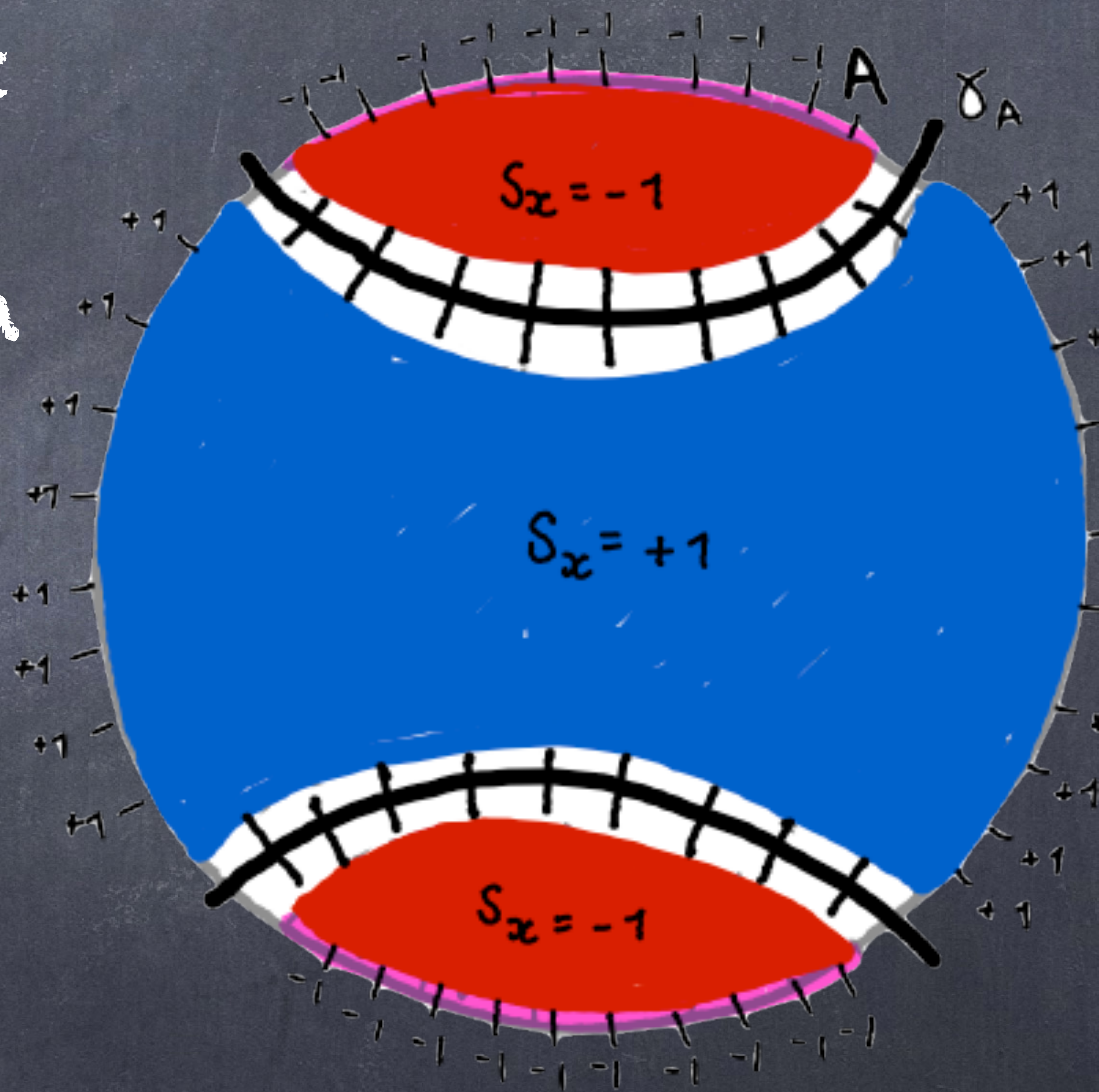


What about using random stabiliser tensors?

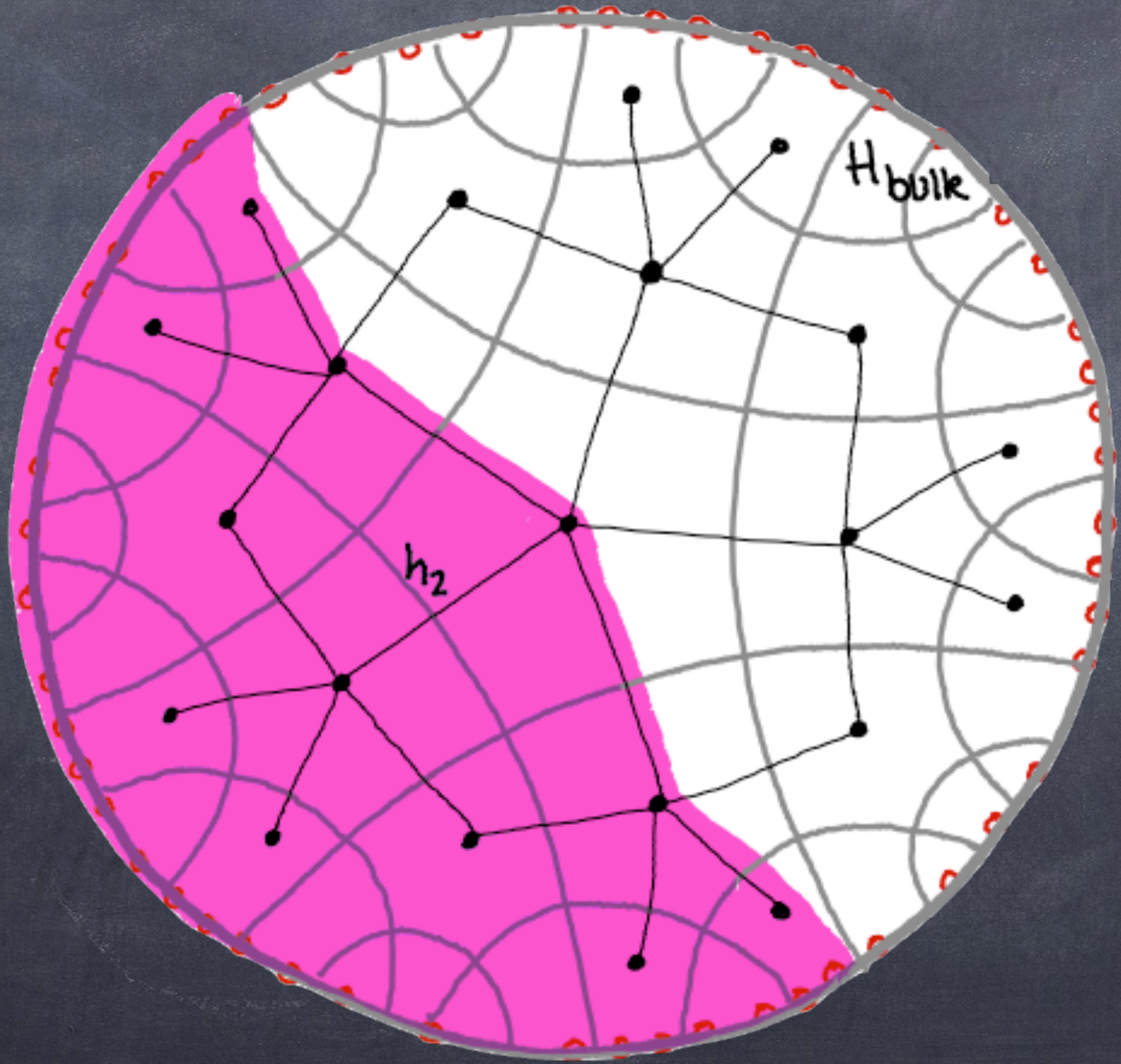
- Our construction uses random stabiliser tensors instead of Haar random tensors
- Random stabiliser tensors are obtained by applying a random Clifford to a reference state
- Random Cliffords form a unitary 2-design

What about using random stabiliser tensors?

- Random stabiliser tensors are exactly perfect with high probability - retain all the error correction properties of the HaPPY code with high probability
- Using the mapping to the Ising model for random tensors plus quantisation of entropy in stabiliser states can show that the Ryu-Takayanagi entropy formula is obeyed exactly



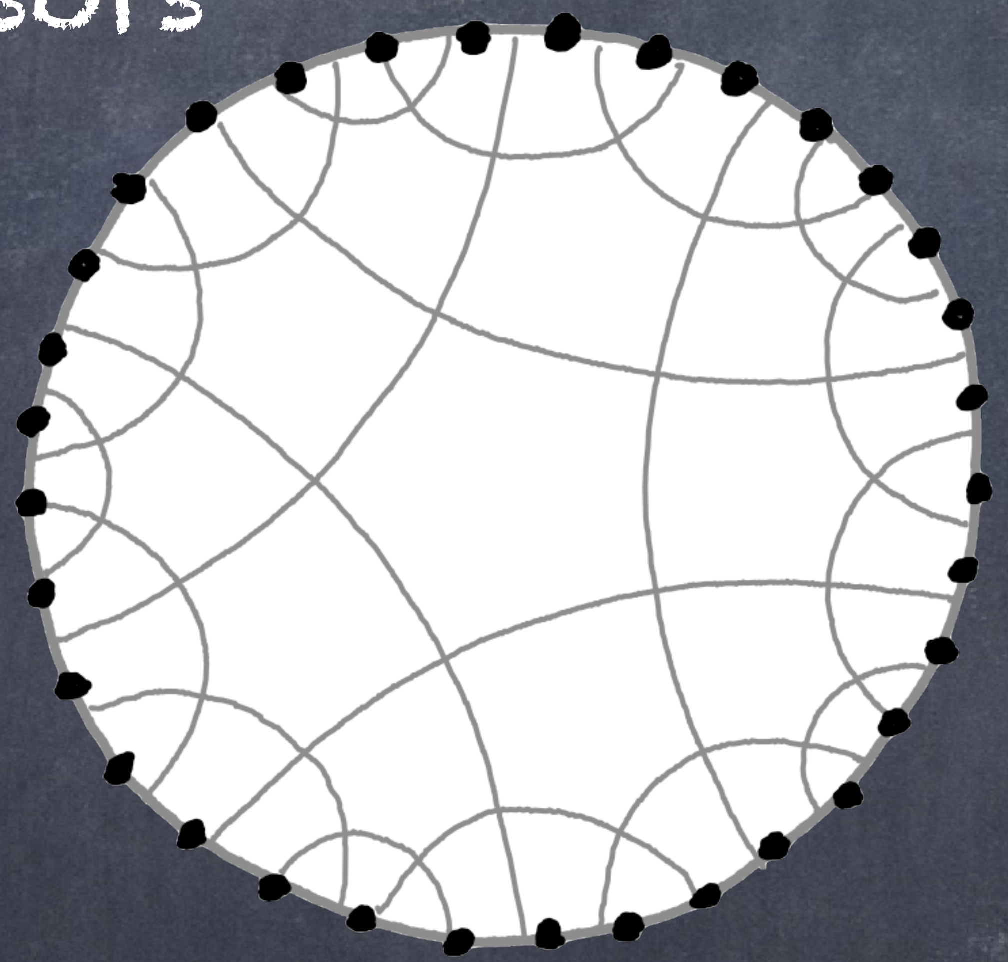
What about using random stabiliser tensors?



Stabiliser tensors preserve the Pauli rank of operators

What about using random stabiliser tensors?

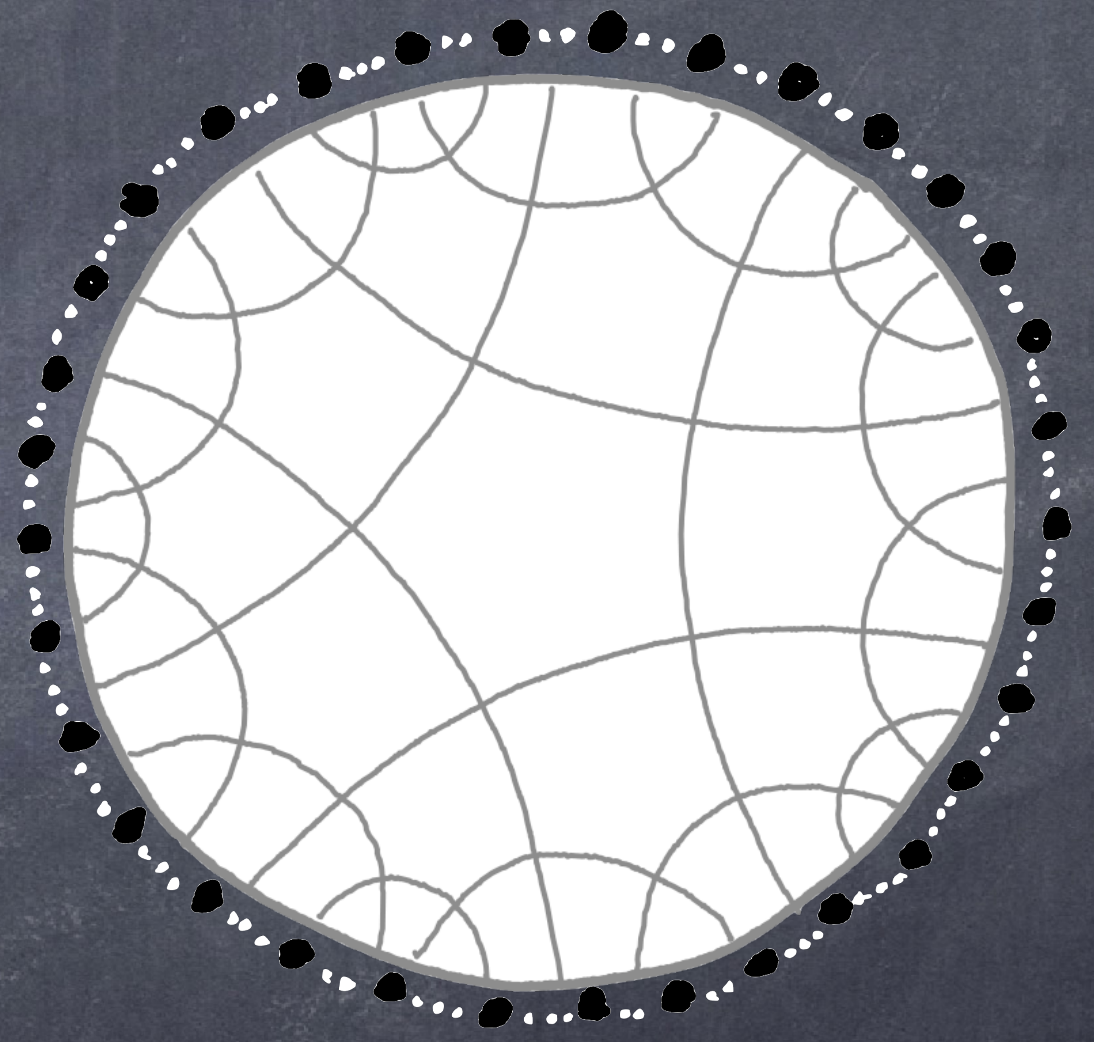
HQECC constructed from stabiliser random tensors



non-local Hamiltonian on n qudits

Hamiltonian simulation techniques

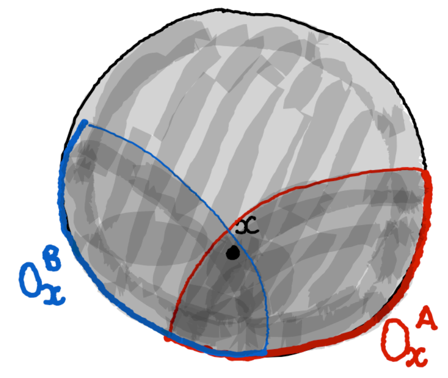
⟶



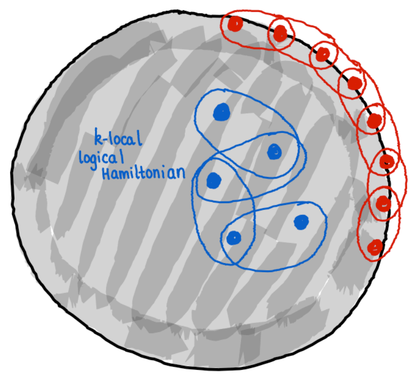
local Hamiltonian on n(poly(log(n))) qudits

What about using random stabiliser tensors?

① ERROR CORRECTION



② LOCAL HAMILTONIANS



③ ENTANGLEMENT

$$S(\rho_A) \propto |\gamma_A|$$

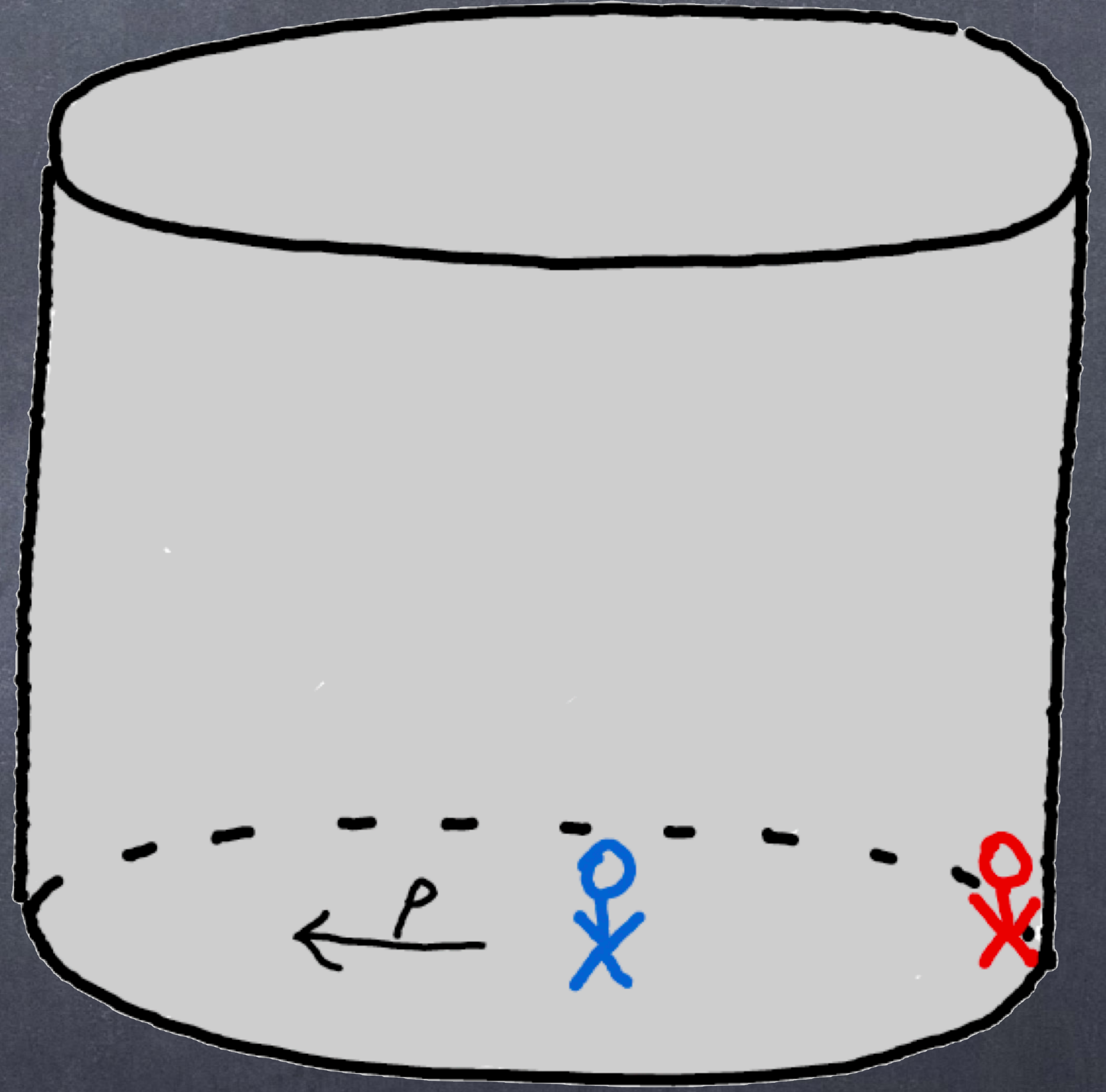
The boundary Hamiltonian has large interaction strengths to keep errors small...

Time dilation in HQECC

The AdS metric is given by:

$$ds^2 = \alpha^2[-cosh^2\rho dt^2 + d\rho^2 + sinh^2\rho d\Omega^2]$$

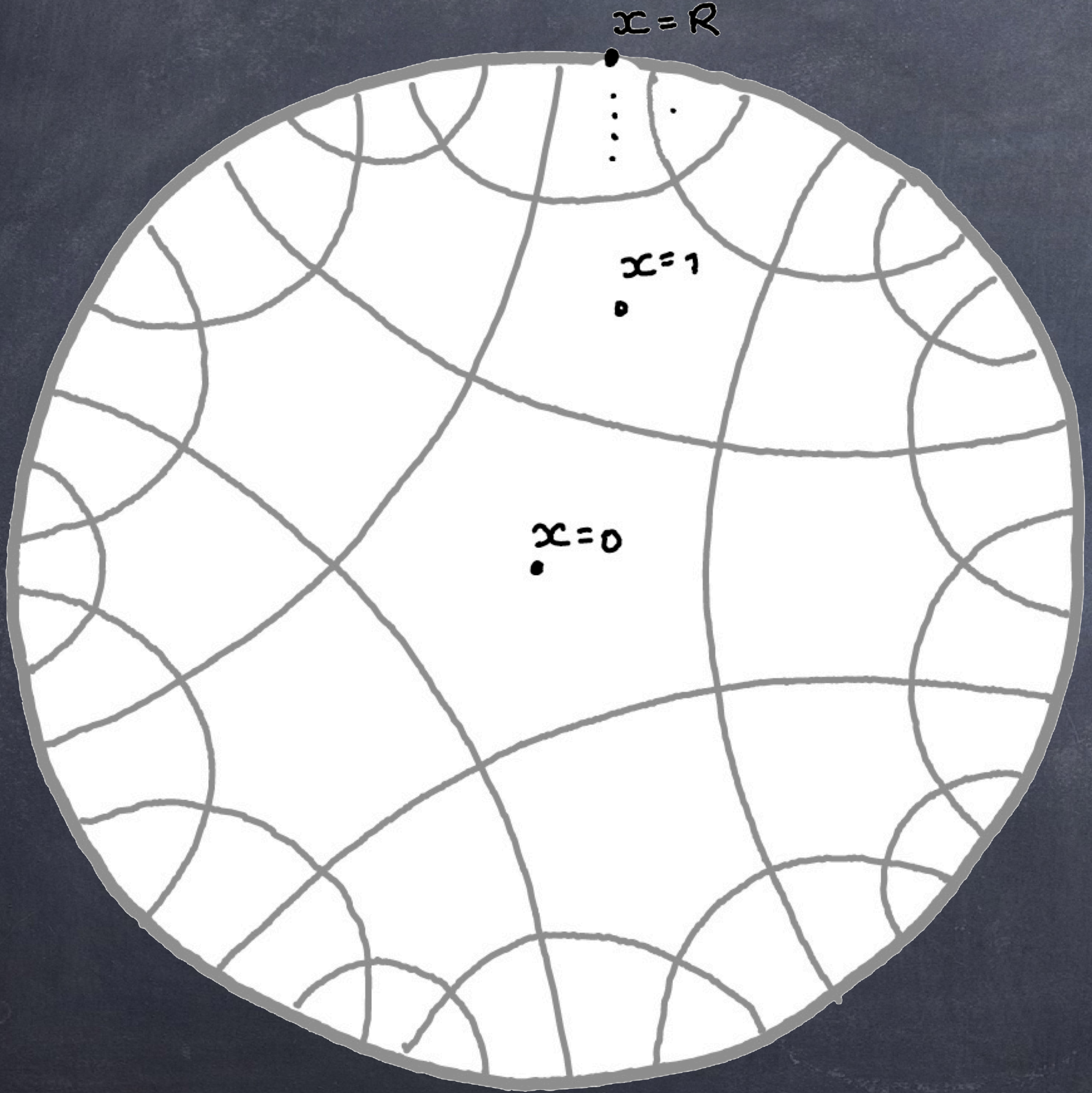
-> time is dilated in the centre of the bulk



For stationary observers at different points:

$$dt_0 = \frac{cosh(\rho_1)}{cosh(\rho_0)}dt_1 \approx e^{\rho_1 - \rho_0}dt_1$$

Time dilation in HQECC



Translating to model parameters, coordinate time at boundary is related to coordinate time in layer x by:

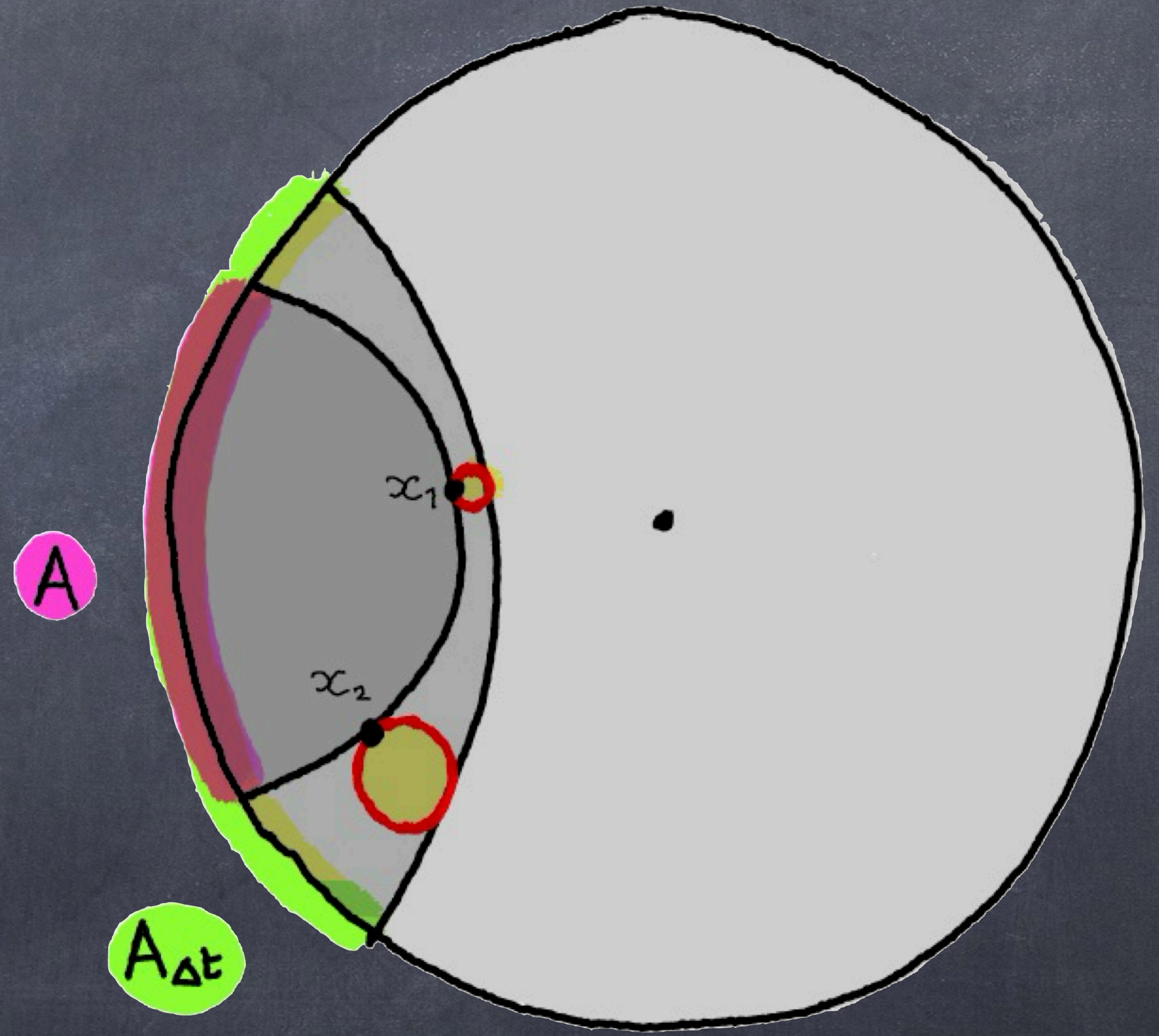
$$t_x = \tau^{R-x} t_R$$

Insert time dilation by hand via scaling Hamiltonian interaction strengths:

$$||h_x|| = O(\tau^{x-R})$$

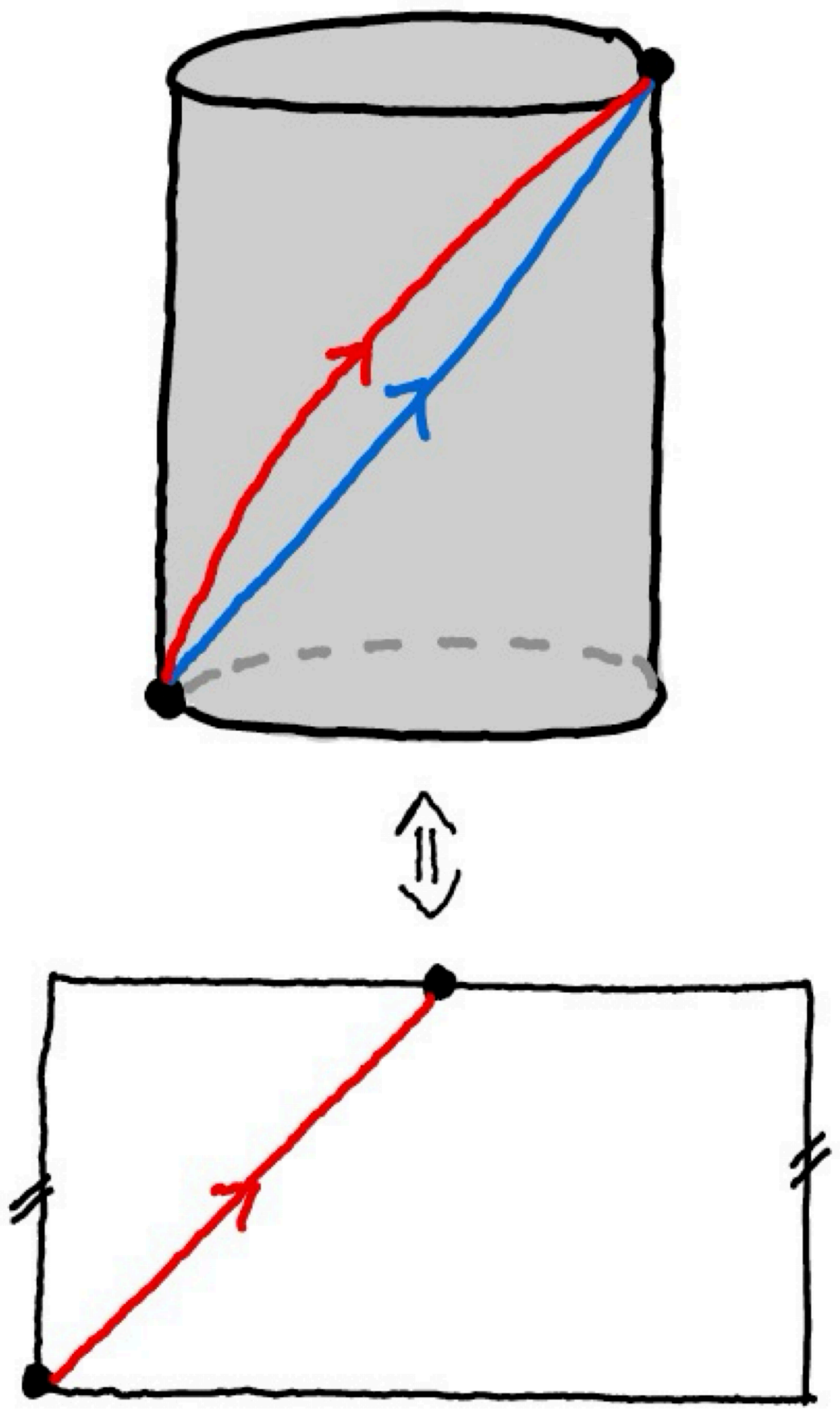
Butterfly velocities

Butterfly velocities: capture how fast information propagates on the boundary in the code subspace

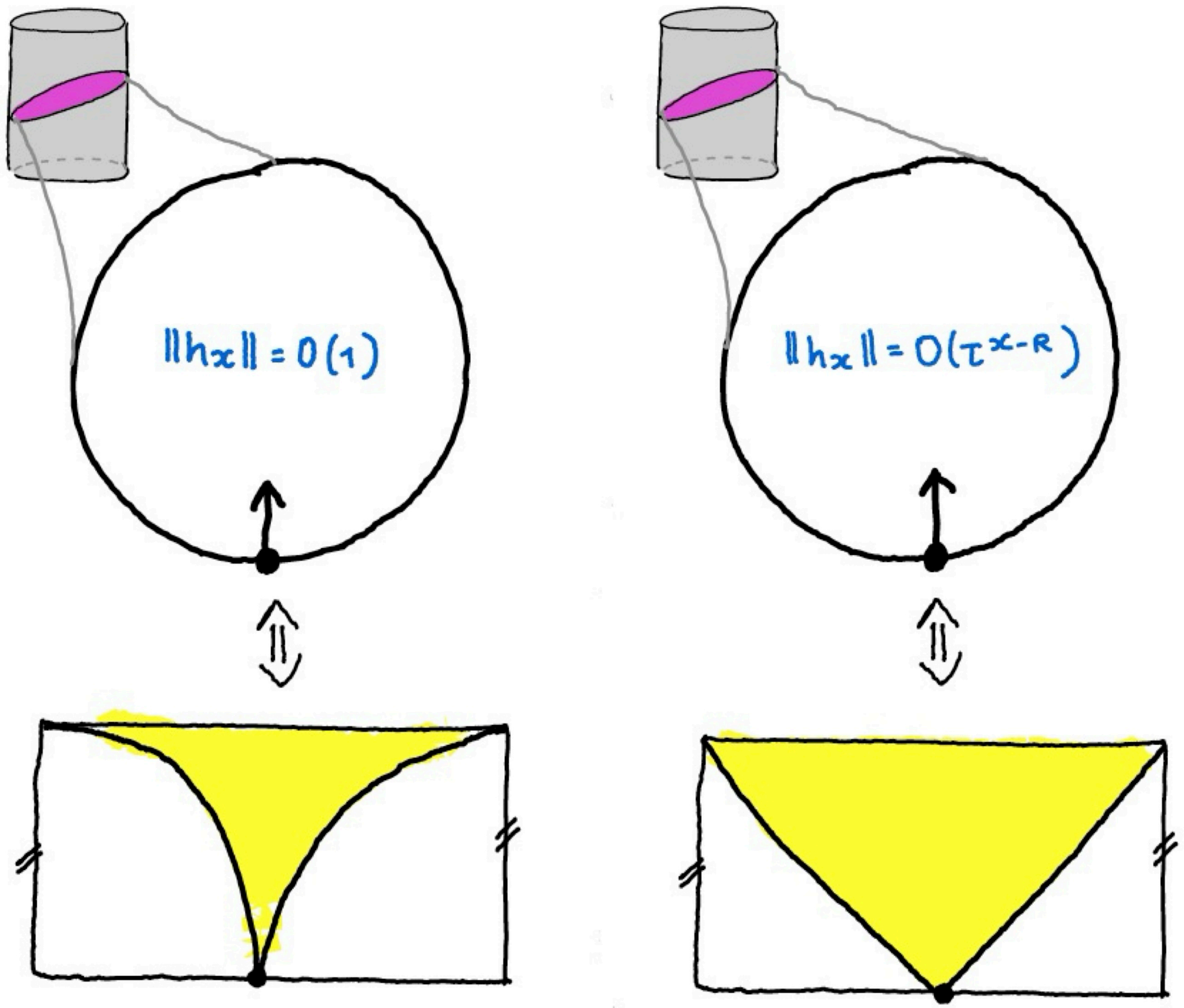


Boundary signalling in toy models

① Particle travelling across bulk/boundary



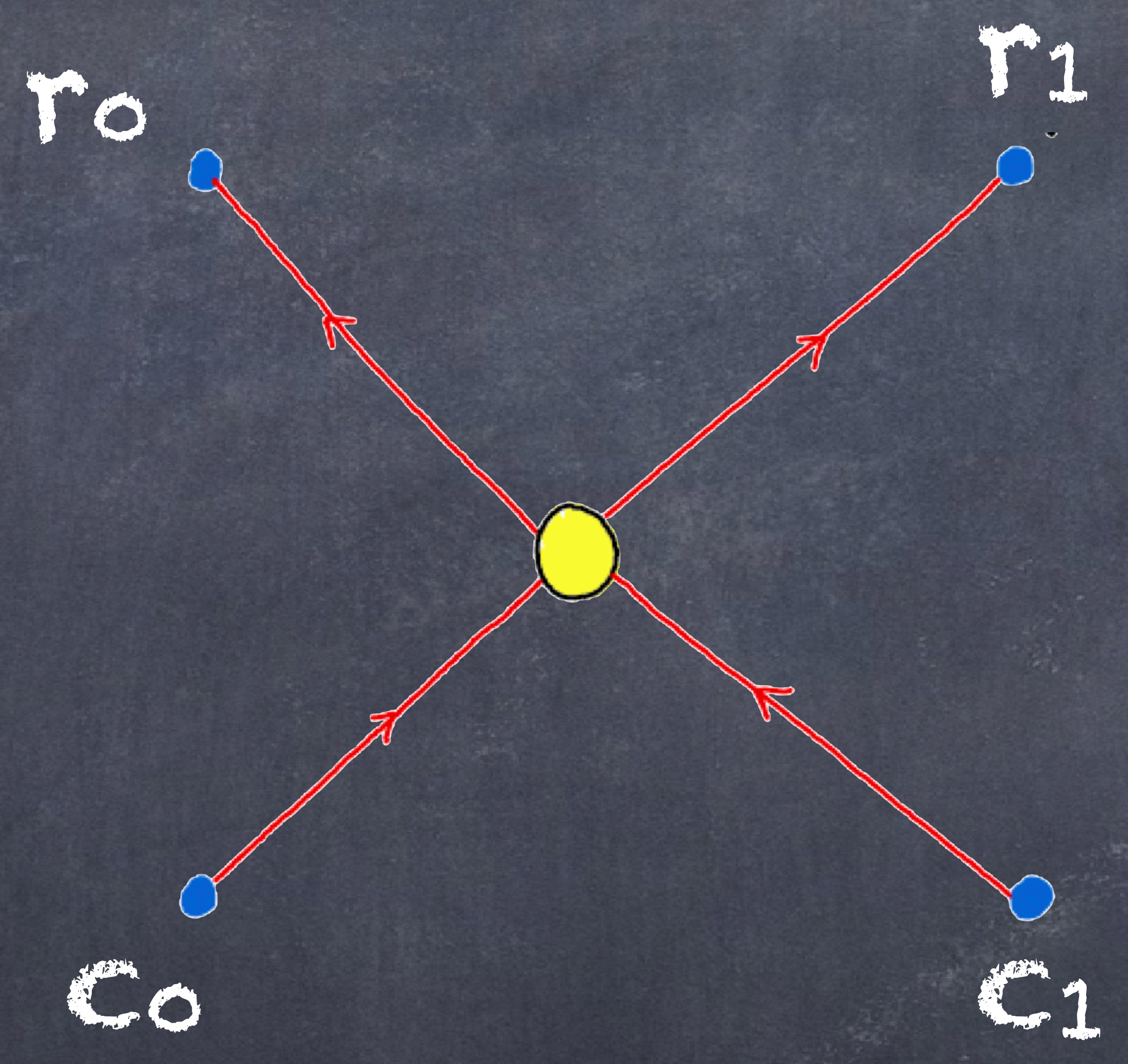
② Straight light cones for butterfly velocities



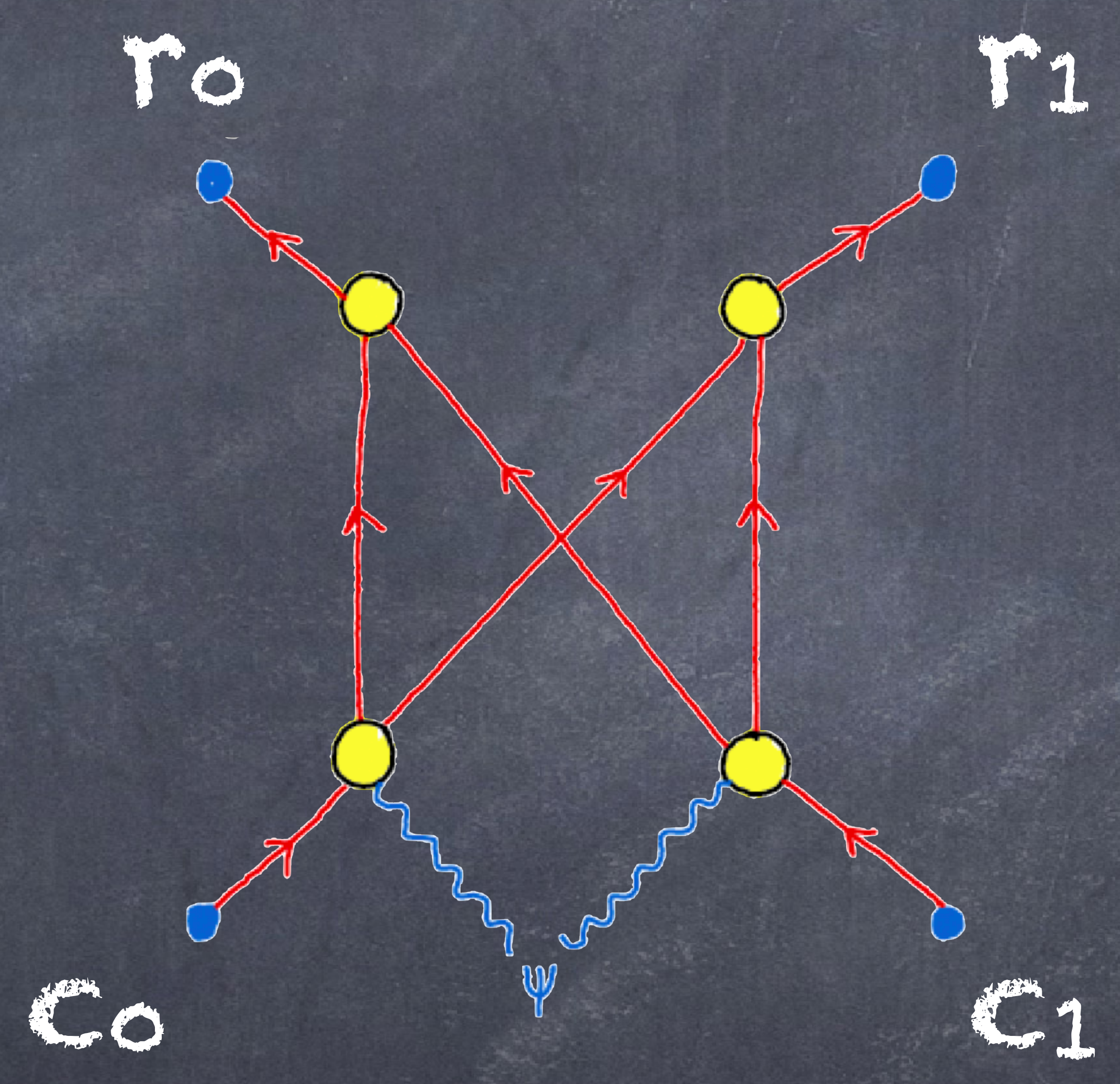
Applications: position based cryptography

Position based quantum cryptography

PBQC uses the provers position in spacetime as its credential

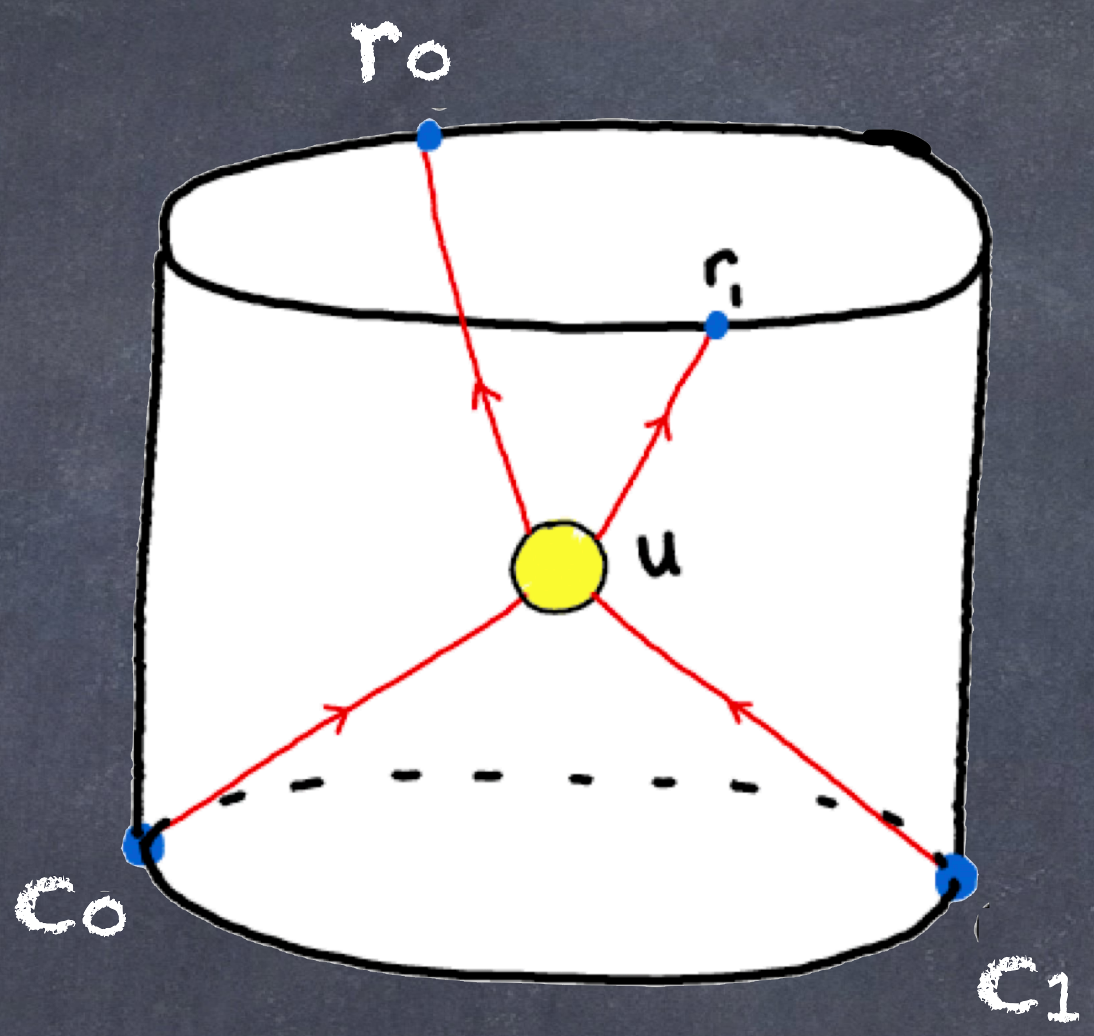


Honest protocol: unitary is applied locally at the required position

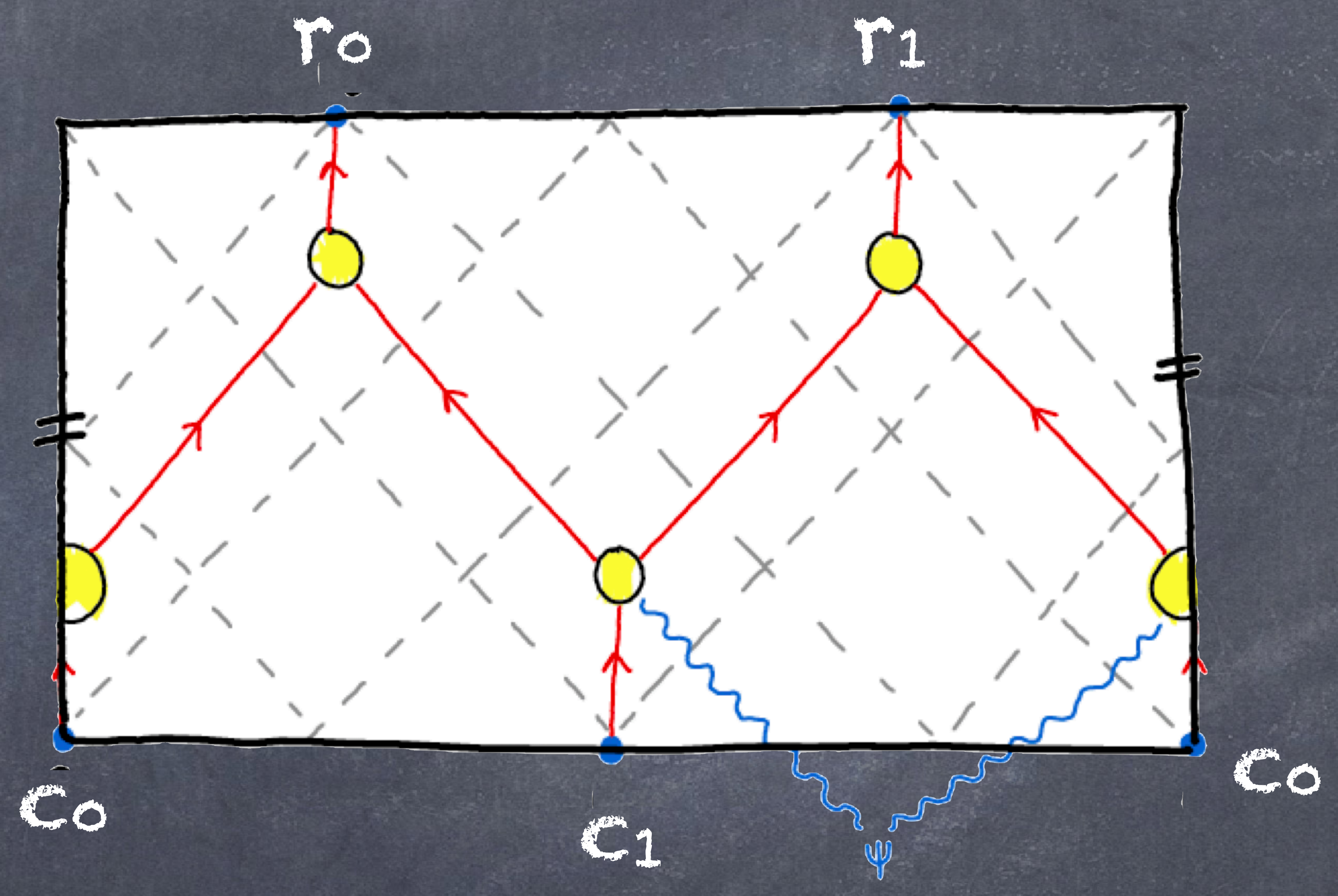


Non-local attack: using shared entanglement attackers 'spoof' the required position

An attack on PBQC from holography



In the bulk there exists a position where the unitary can be applied locally



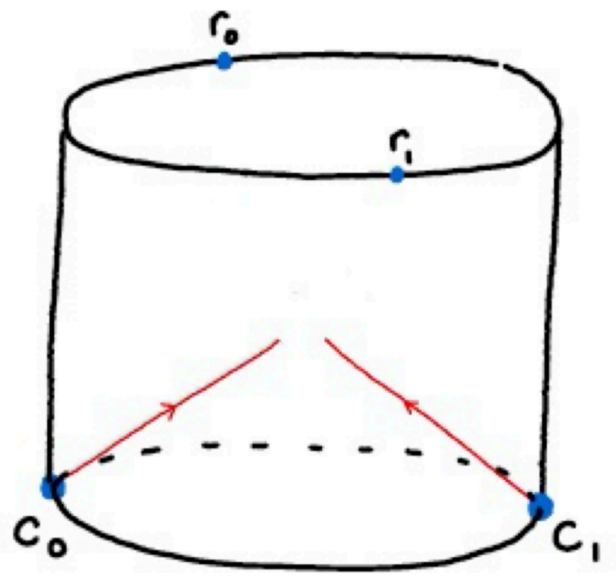
In the boundary there's no intersection of the lightcones: the unitary must be applied non-locally, using only linear entanglement

[May 2019]

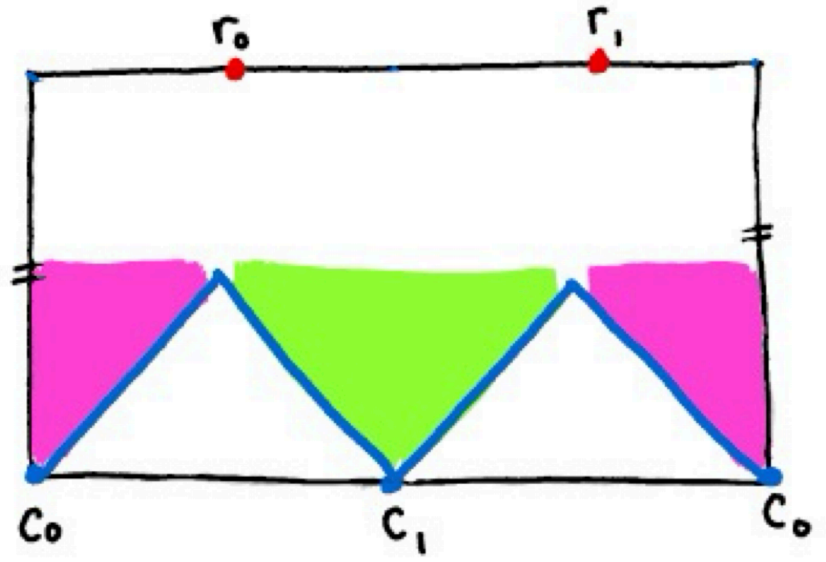
Can we replicate this attack in toy models?

① PROPAGATE IN

Local SWAPS with exp decaying weight translate inputs to the centre of the bulk.

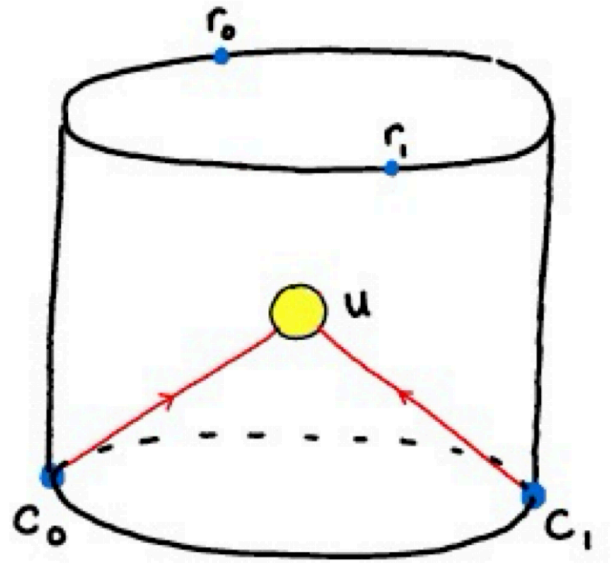


Boundary butterfly velocity obey straight light cones.

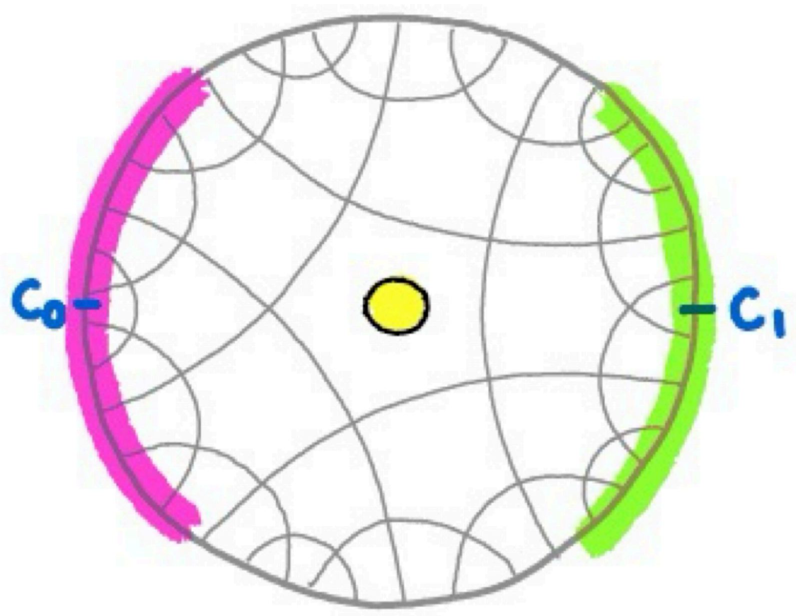


② SIMULATE UNITARY

General local unitary U generated by n-local bulk Hamiltonian H.



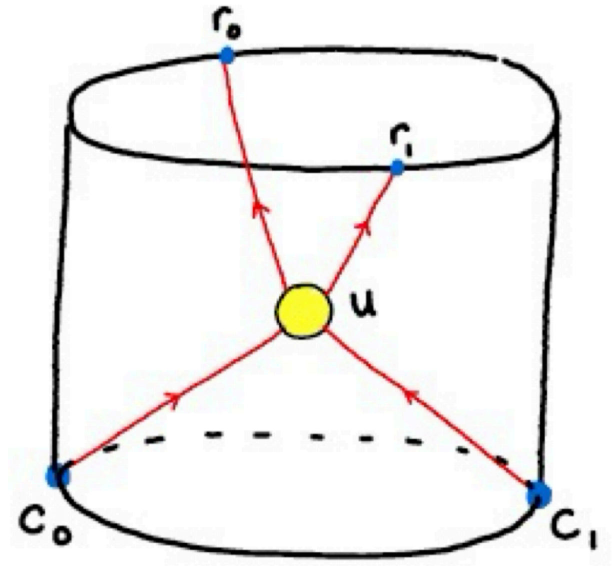
Tensor network map → tensor product operator over two regions



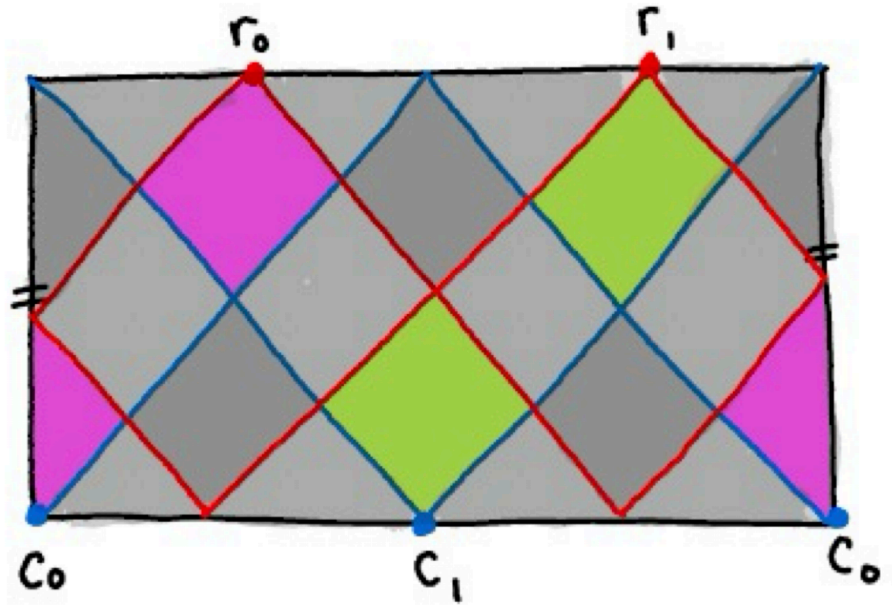
Need a GOOD simulator to keep boundary causal.

③ PROPAGATE OUT

Local SWAPS again to deliver outputs



Causal structure on boundary maintained during protocol.



Tensor network obeys RT so can bound entanglement of non-local attack.

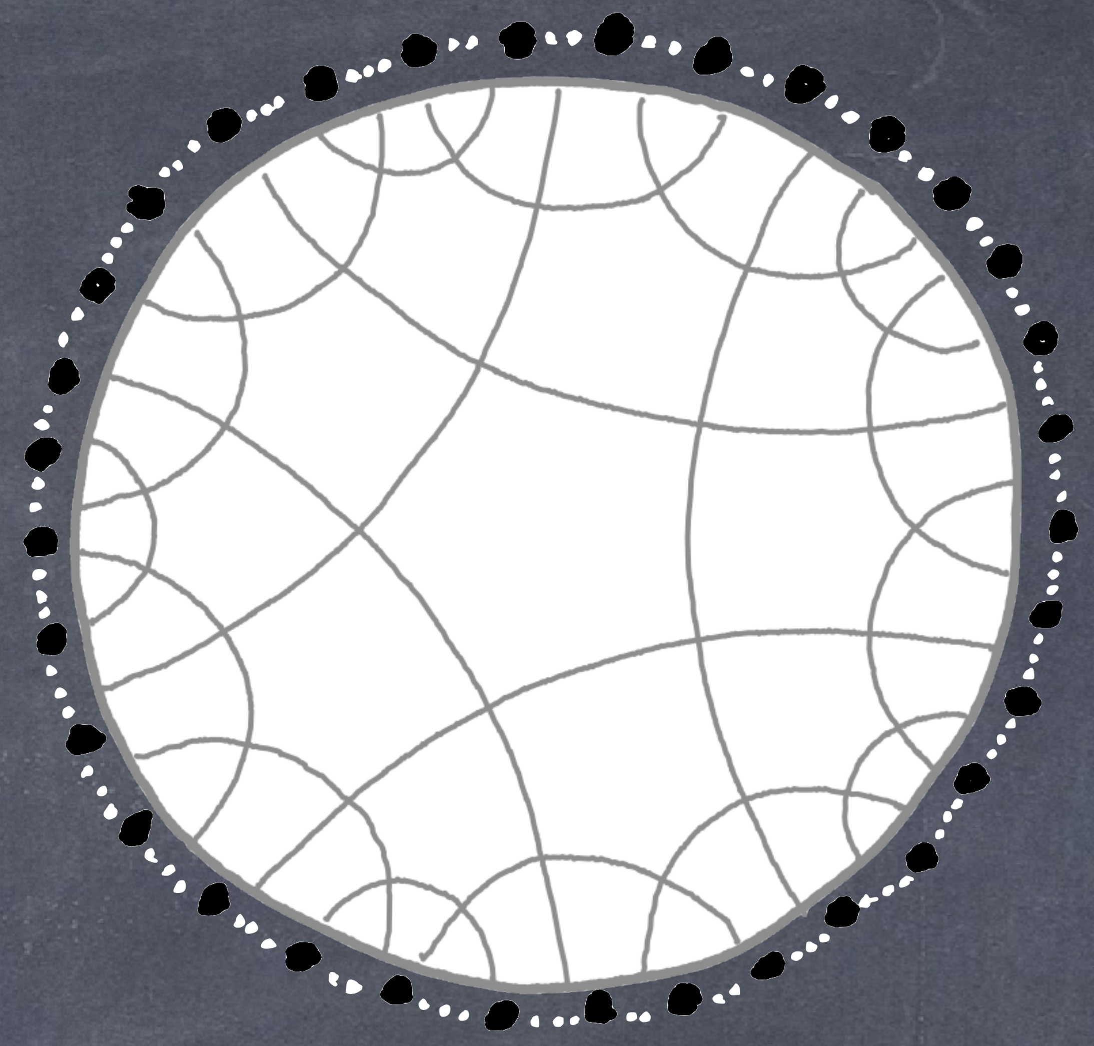
What counts as a `good simulator'?

- To maintain causal structure during boundary implementation of U, while maintaining small enough errors to break PBQC we need a simulator Hamiltonian with interaction strengths scaling as poly($\frac{n^a}{\epsilon^b}||H_{target}||$) with a+b < 1
- If we could construct such a simulator then we could break PBQC using only linear entanglement
- If PBQC can be proven to be secure by other means this implies a limit on how good simulations can be

How tight is this bound?

- For general Hamiltonians the bound isn't tight
- We can apply this bound to bulk unitaries generated by k-local Hamiltonians - this gives a bound on simulating sparse boundary Hamiltonians
- Using history state simulation methods we can construct new simulation methods optimised for simulating sparse Hamiltonians - this gives simulators with interactions strengths scaling as poly($\frac{n^a}{\epsilon^b}||H_{target}||$) with a+b $\geq$ 1.5

Open questions



- Where's the gravity??
- Toy models with conformal symmetries in the boundary & time dynamics?
- Toy models that capture the full entanglement spectrum correctly?
- Other applications of these toy models?