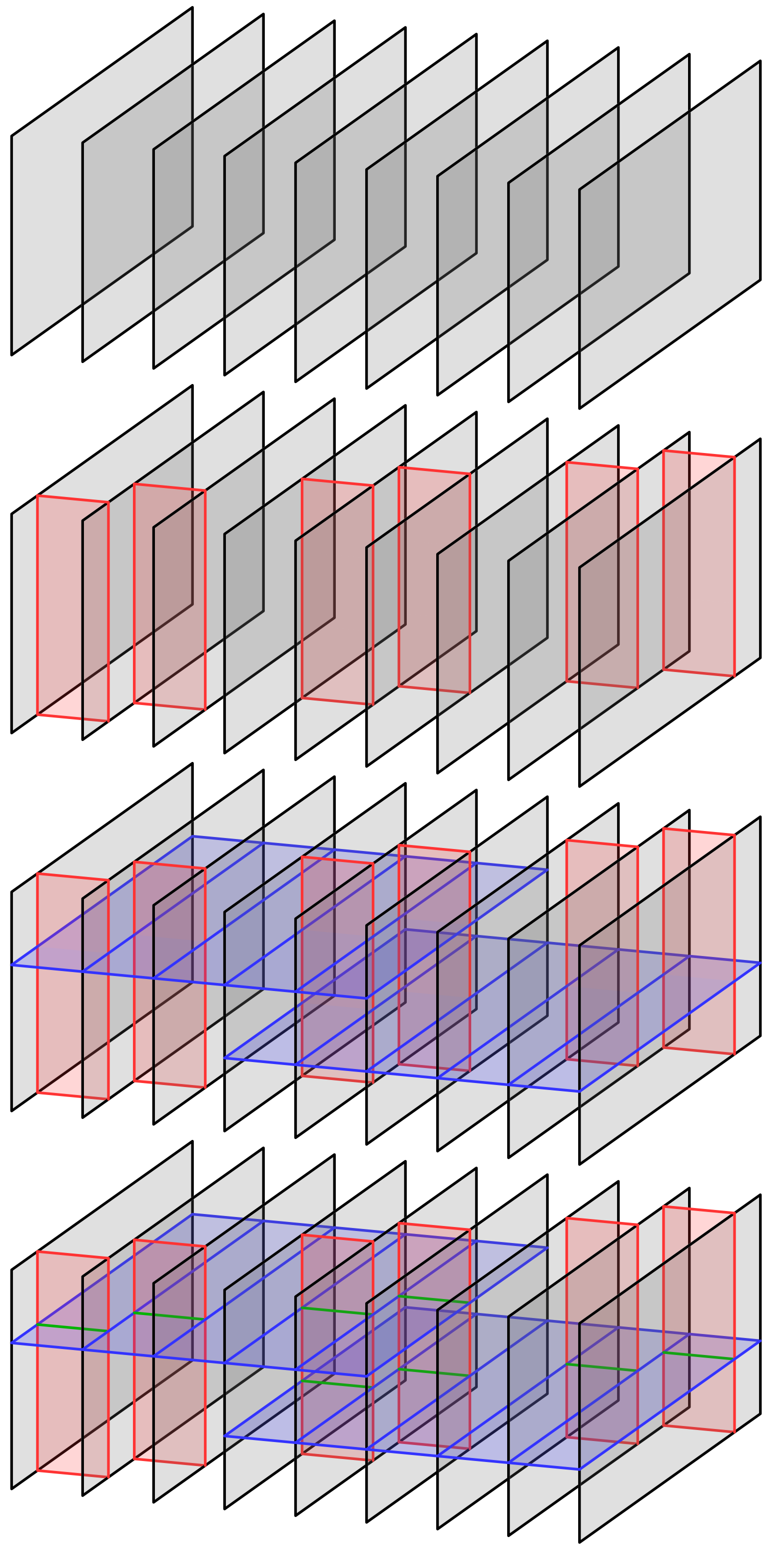
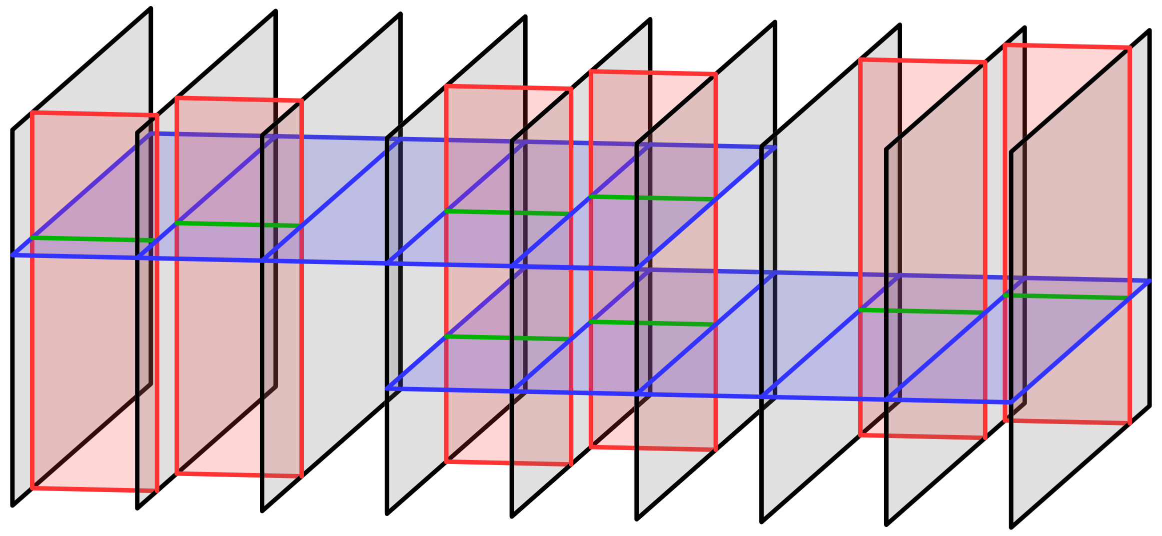


Layer Codes

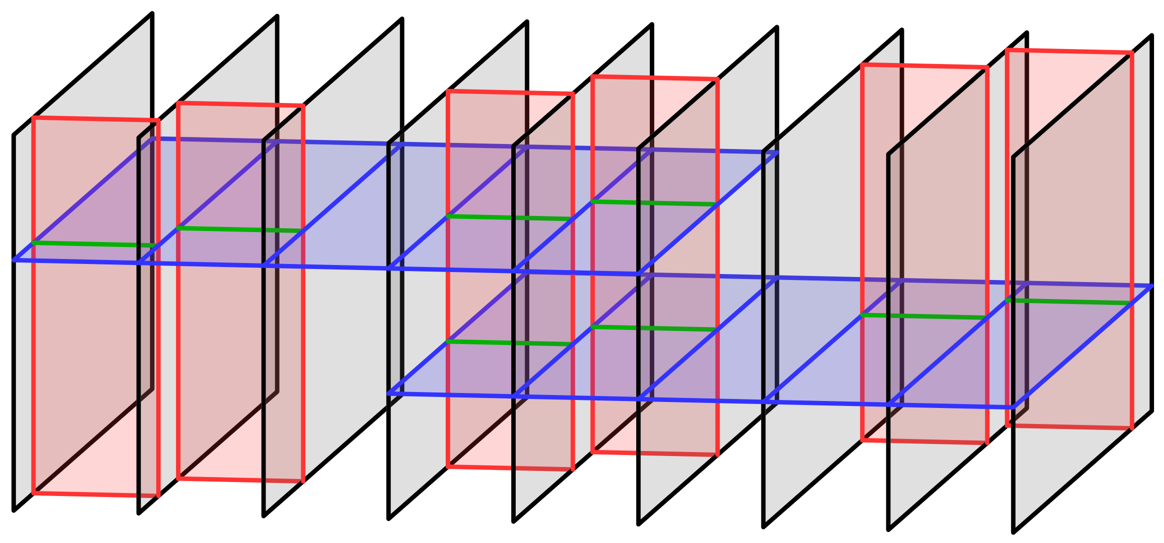


Dom Williamson
The University of Sydney → IBM

arxiv:2309.16503 Joint w/ Nouédyn Baspin
related work: Portnoy 2023
Lin, Wills, Hsieh 2023

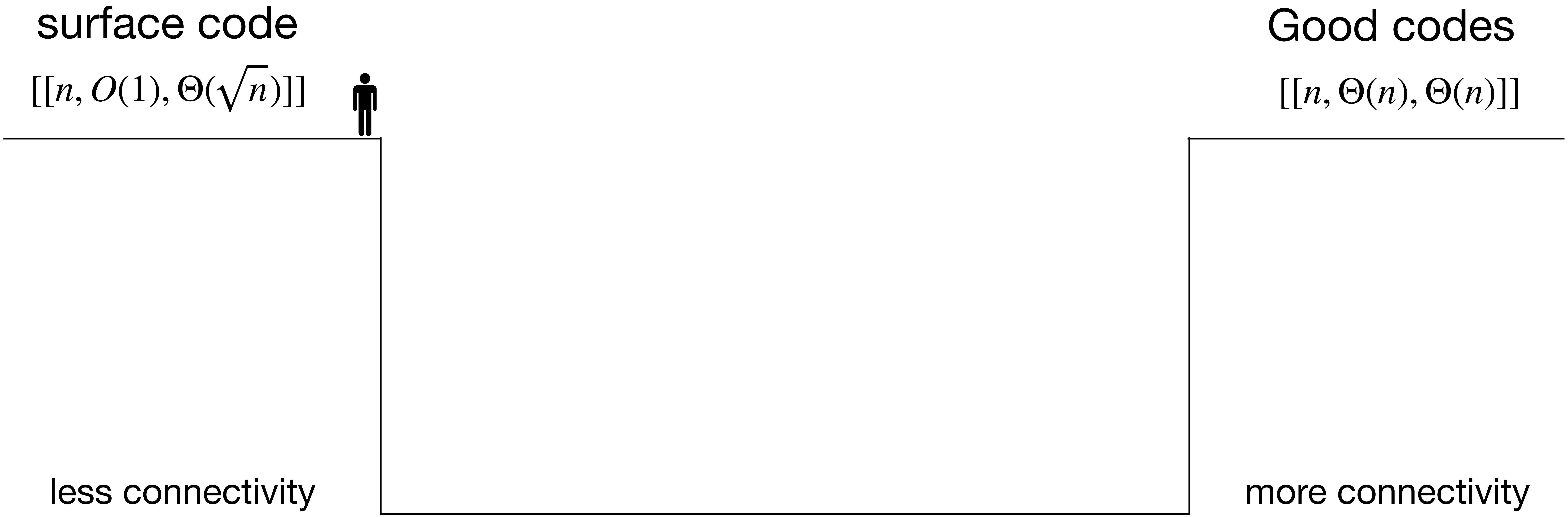
Overview

- Best possible codes in 3D from coupled layers of surface code



layer code

Motivation


surface code
$[[n, O(1), \Theta(\sqrt{n})]]$
Good codes
$[[n, \Theta(n), \Theta(n)]]$
less connectivity
more connectivity

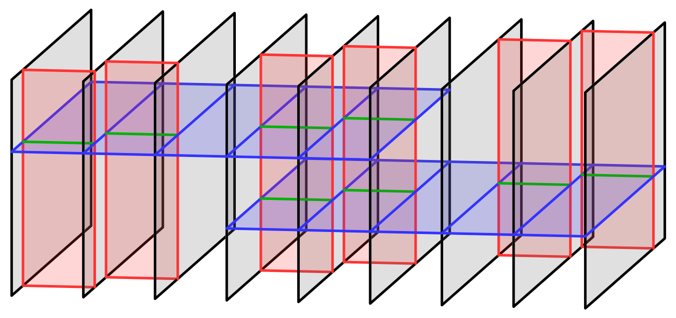
Motivation

surface code

$[[n, O(1), \Theta(\sqrt{n})]]$

layer codes

$[[n, \Theta(n^{1/3}), \Theta(n^{2/3})]]$



Good codes

$[[n, \Theta(n), \Theta(n)]]$

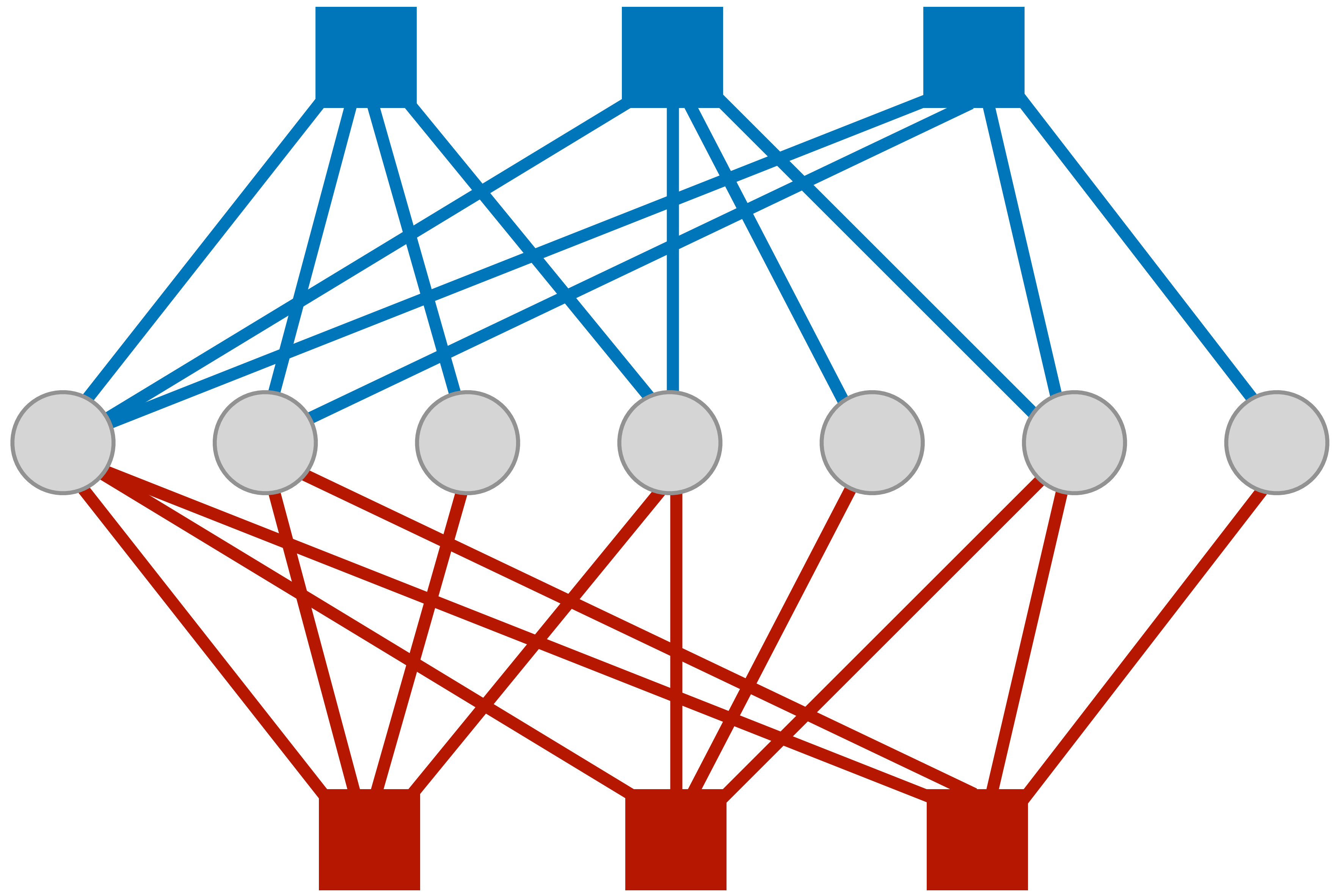
less connectivity

more connectivity

Outline

- Good CSS LDPC codes exist!
- LDPC codes embedded in dimension D are restricted
- The surface code is optimal in 2D
- We construct *Layer Codes* by combining surface codes with Good LDPC codes to find optimal codes in 3D

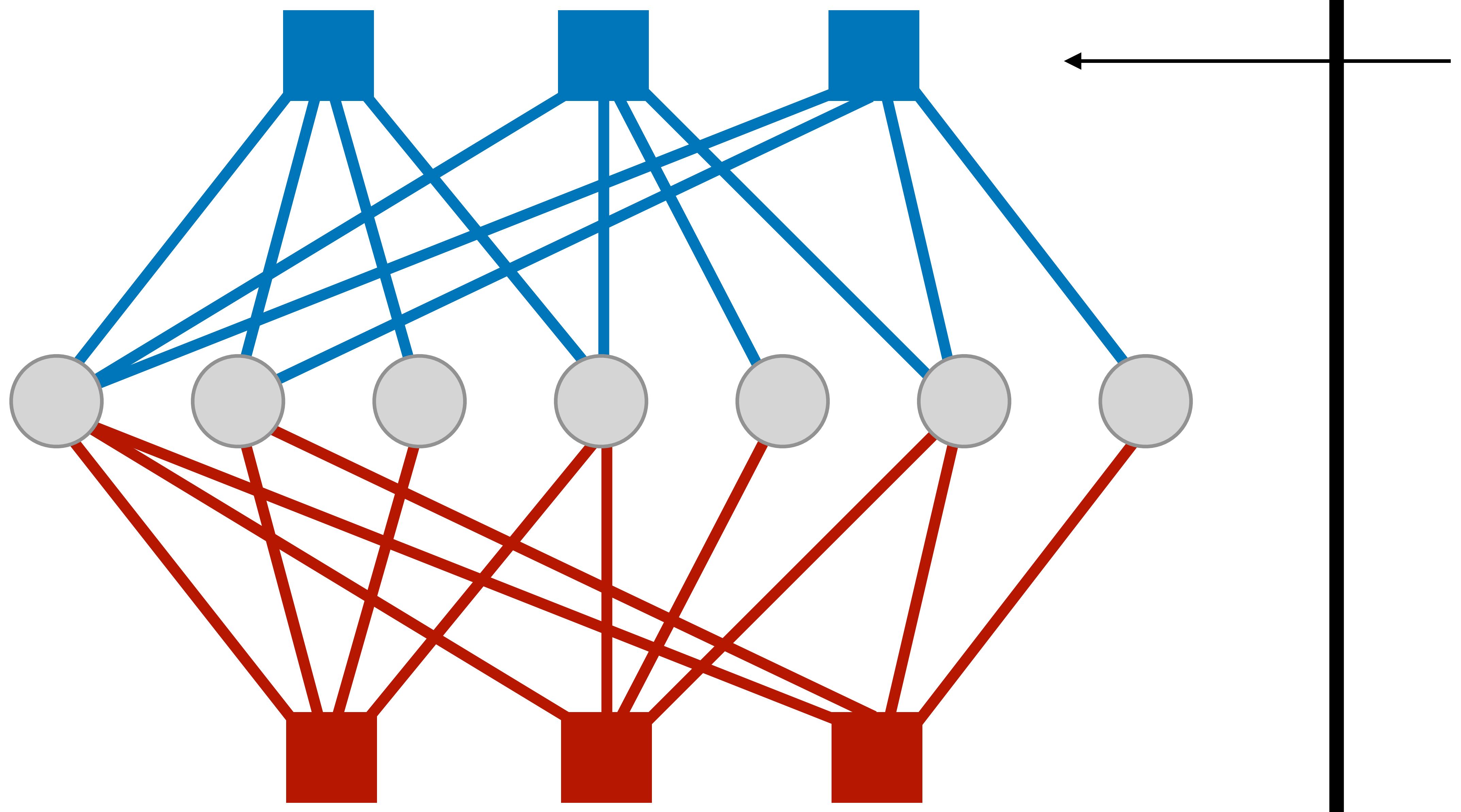
CSS codes



Tanner graph

Calderbank Shor 96
Steane 96

CSS codes



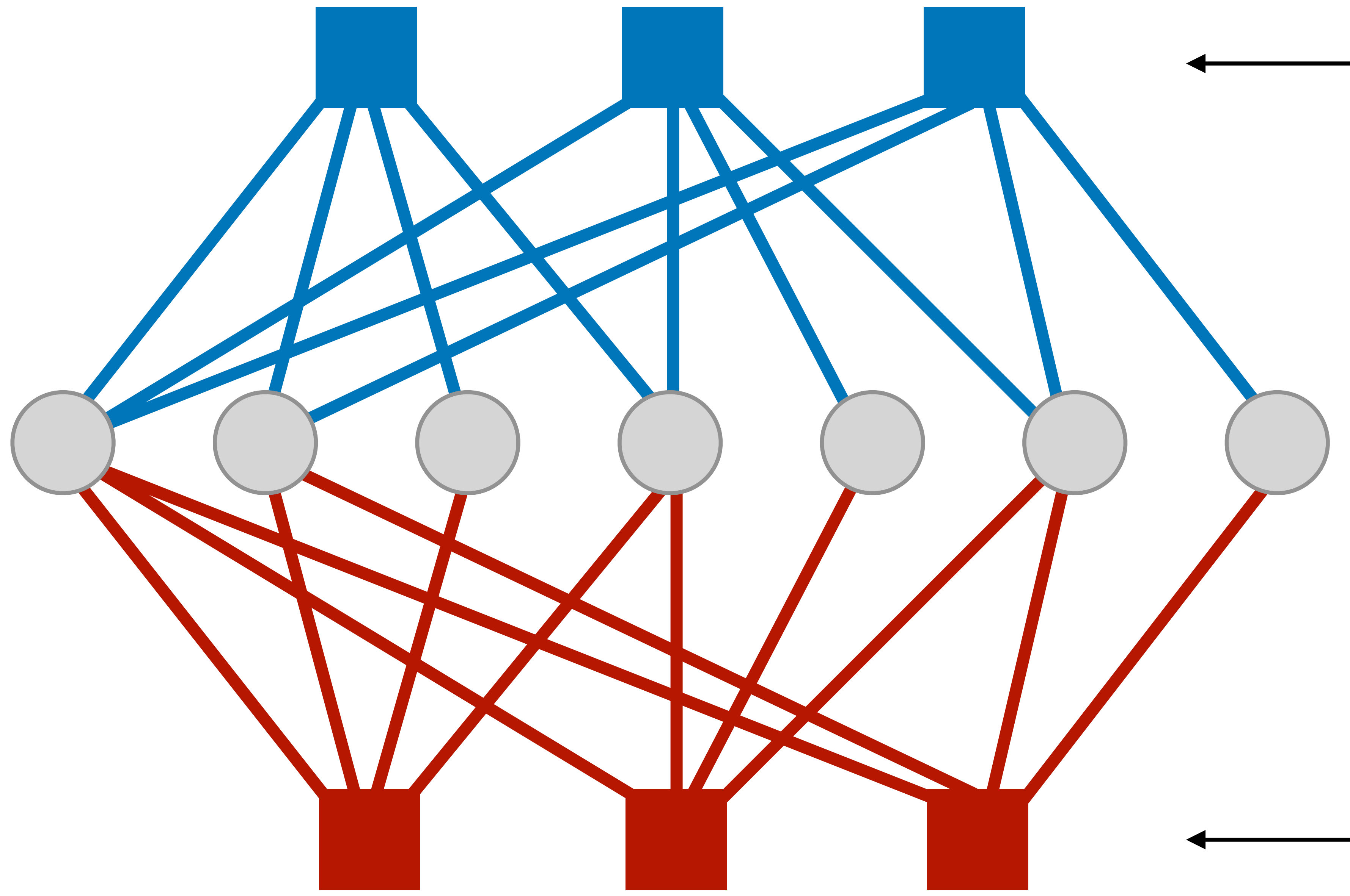
Tanner graph

X checks

$$A_i = \prod_{j \in a_i} X_j$$

Calderbank Shor 96
Steane 96

CSS codes



Tanner graph

X checks

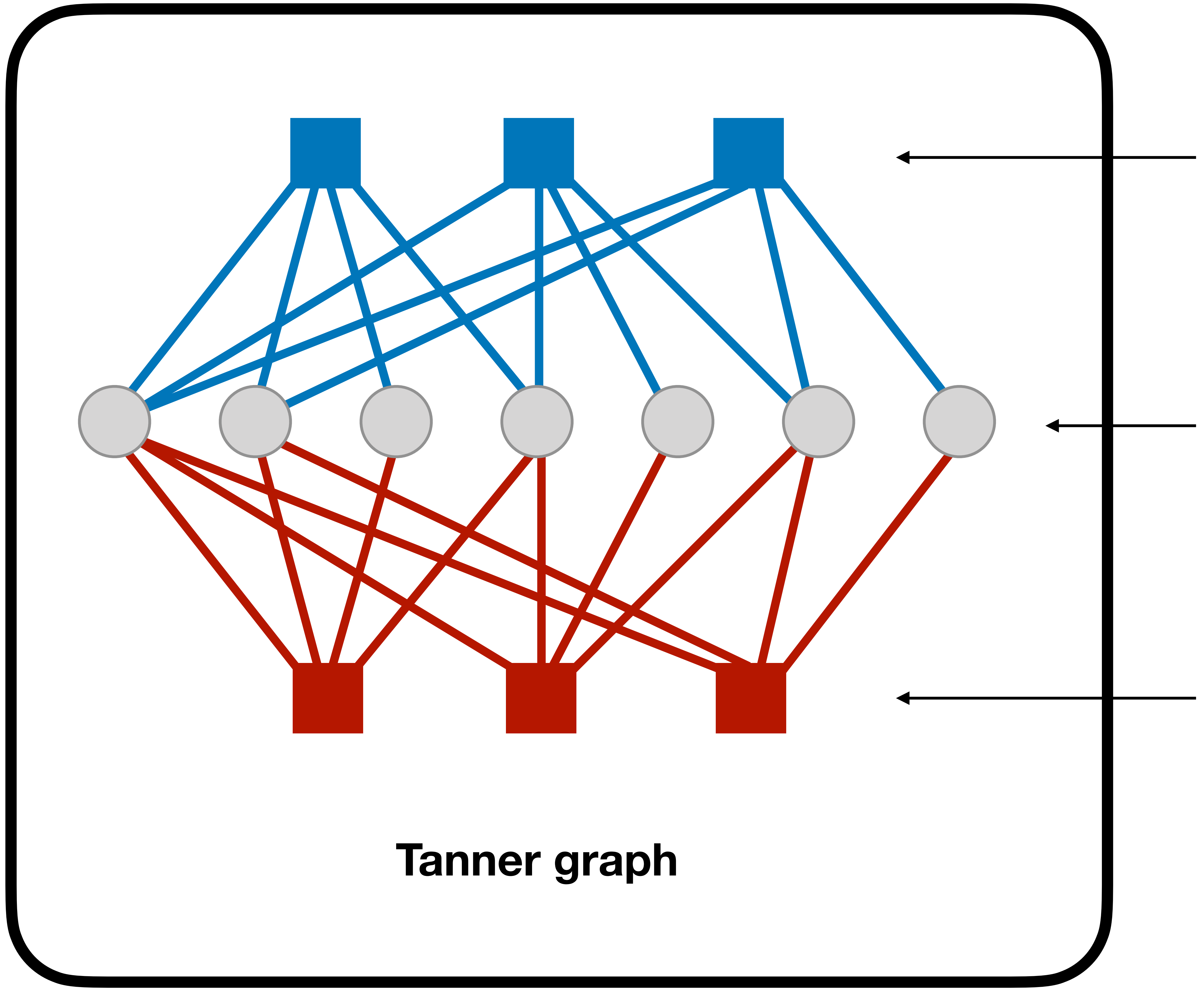
$$A_i = \prod_{j \in a_i} X_j$$

Z checks

$$B_i = \prod_{j \in b_i} Z_j$$

Calderbank Shor 96
Steane 96

CSS codes



X checks $A_i = \prod_{j \in a_i} X_j$

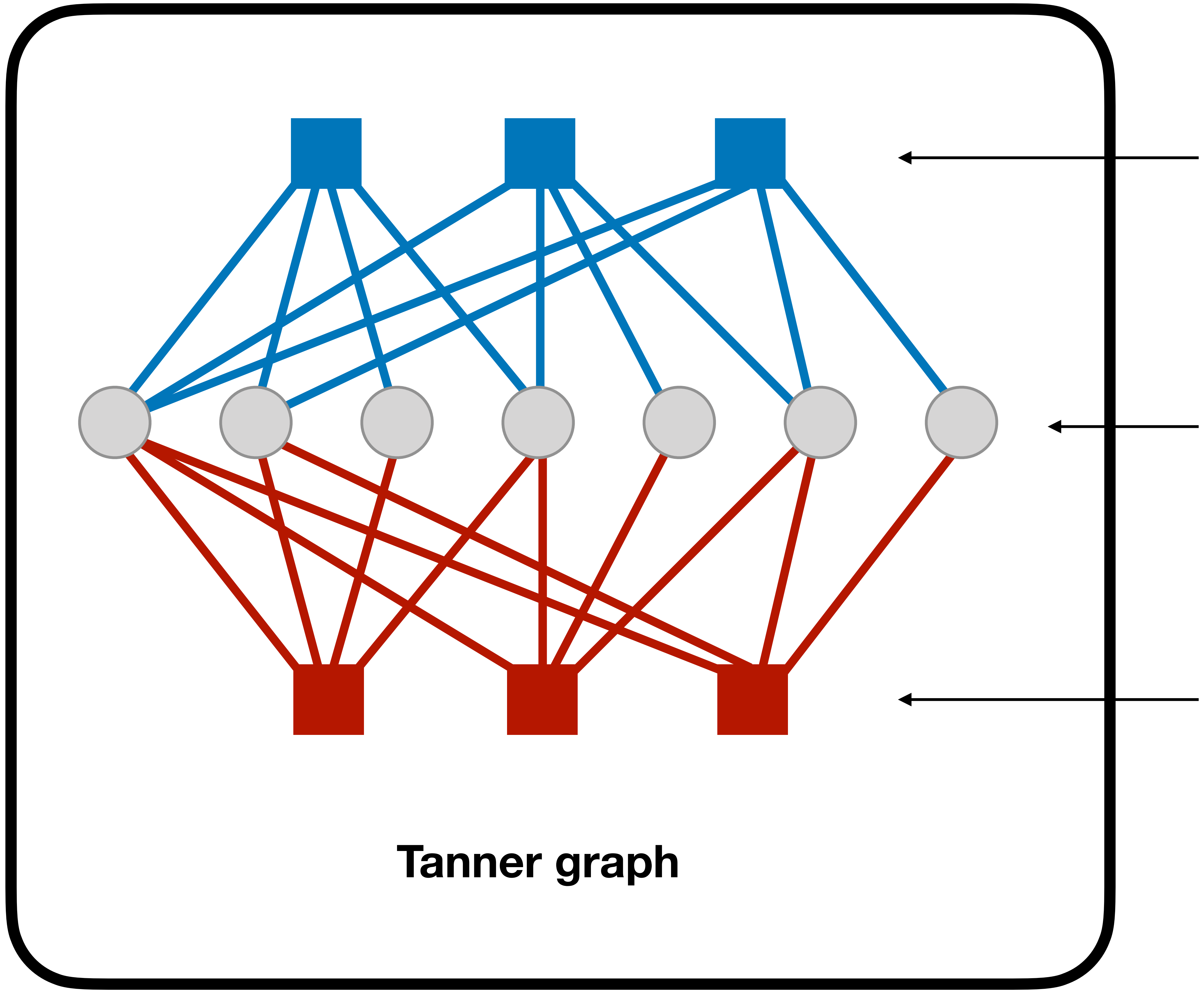
qubits $\{ |\Psi\rangle : A_i |\Psi\rangle = B_i |\Psi\rangle = |\Psi\rangle \}$

code space

Z checks $B_i = \prod_{j \in b_i} Z_j$

Calderbank Shor 96
Steane 96

CSS codes



X checks $A_i = \prod_{j \in a_i} X_j$

qubits $\{\, |\Psi\rangle : A_i|\Psi\rangle = B_i|\Psi\rangle = |\Psi\rangle \} \cong (\mathbb{C}^2)^{\otimes k} \subseteq (\mathbb{C}^2)^{\otimes n}$

code space

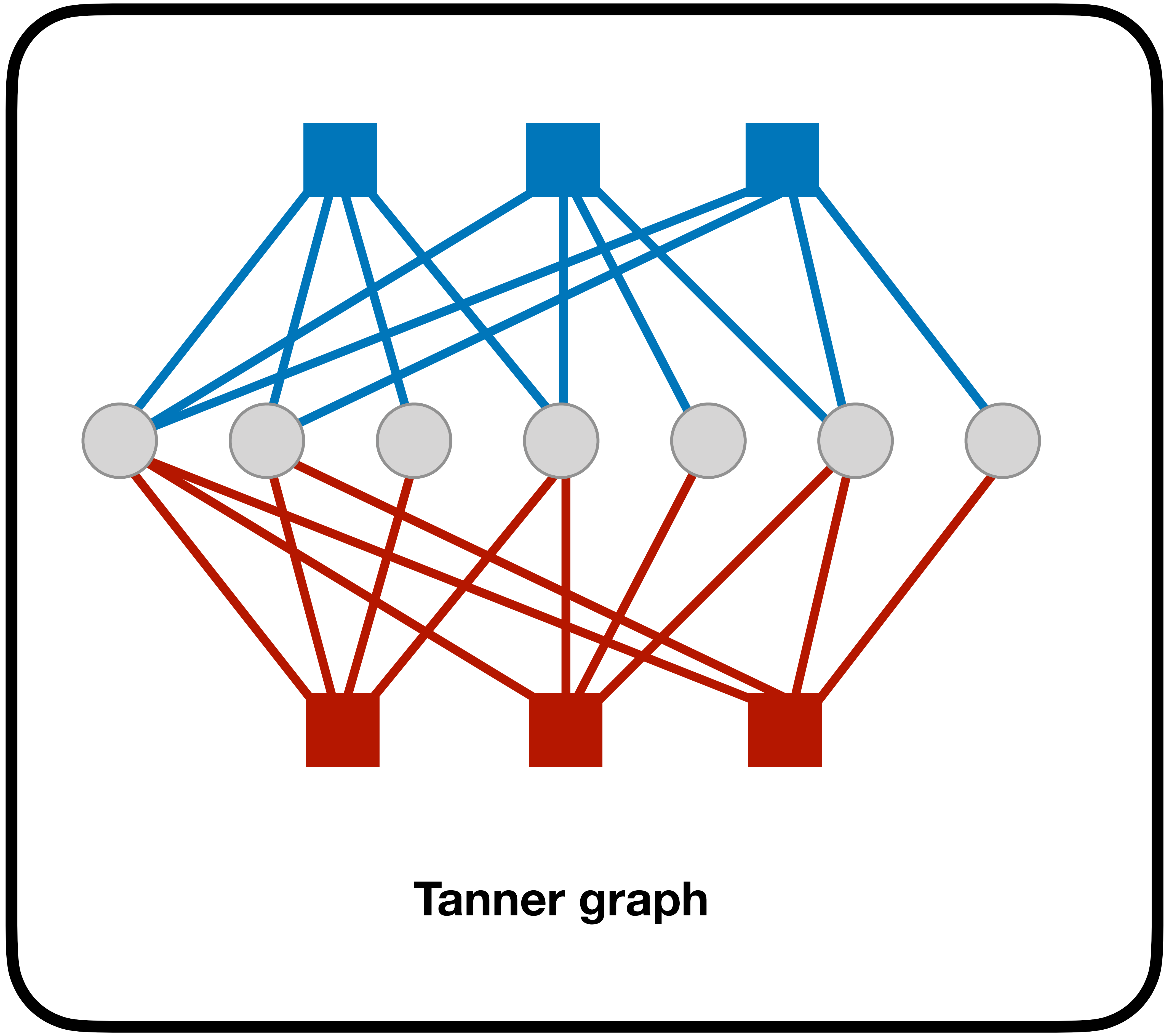
logical qubits

physical qubits

Z checks $B_i = \prod_{j \in b_i} Z_j$

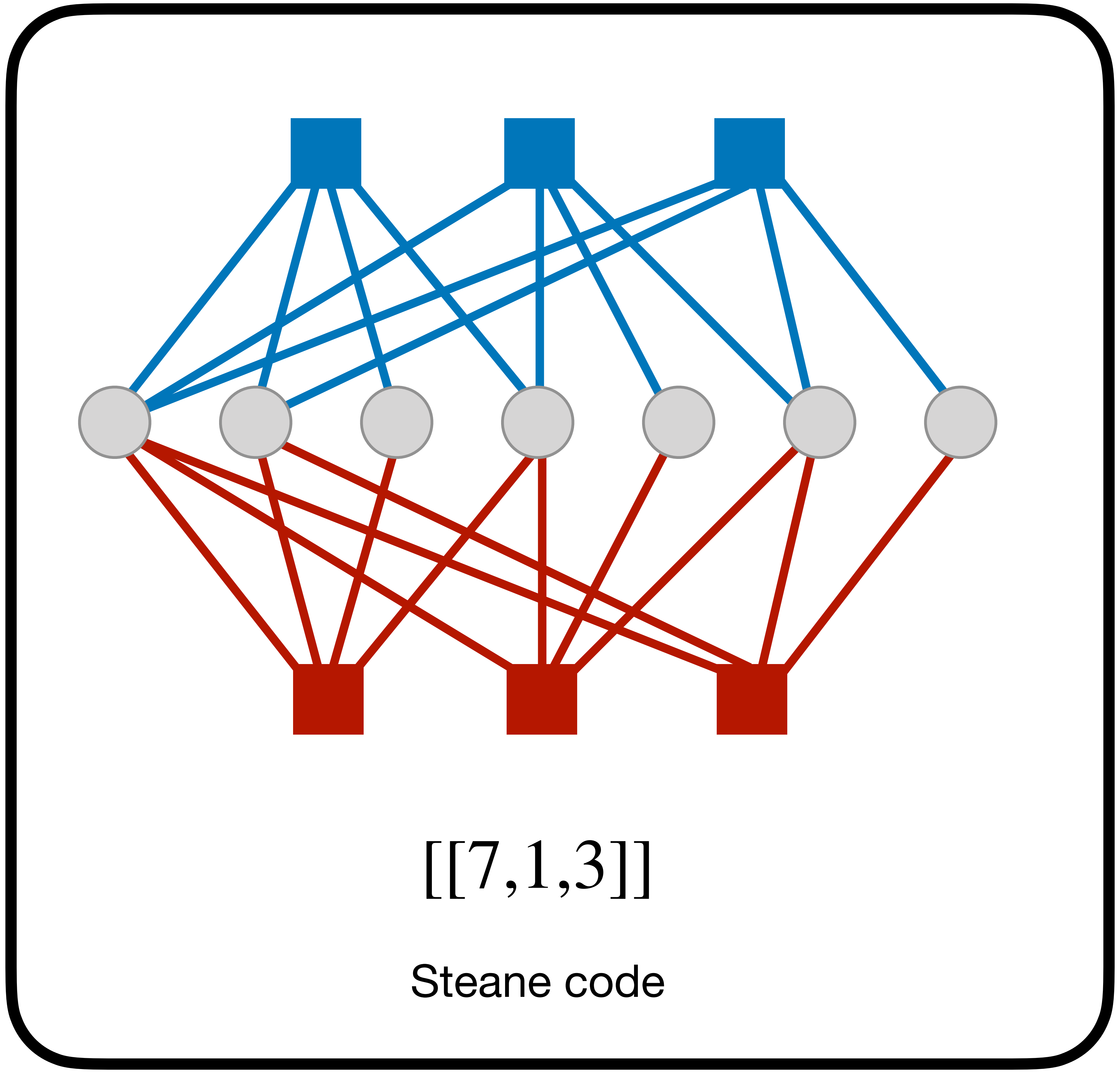
Calderbank Shor 96
Steane 96

QLDPC codes



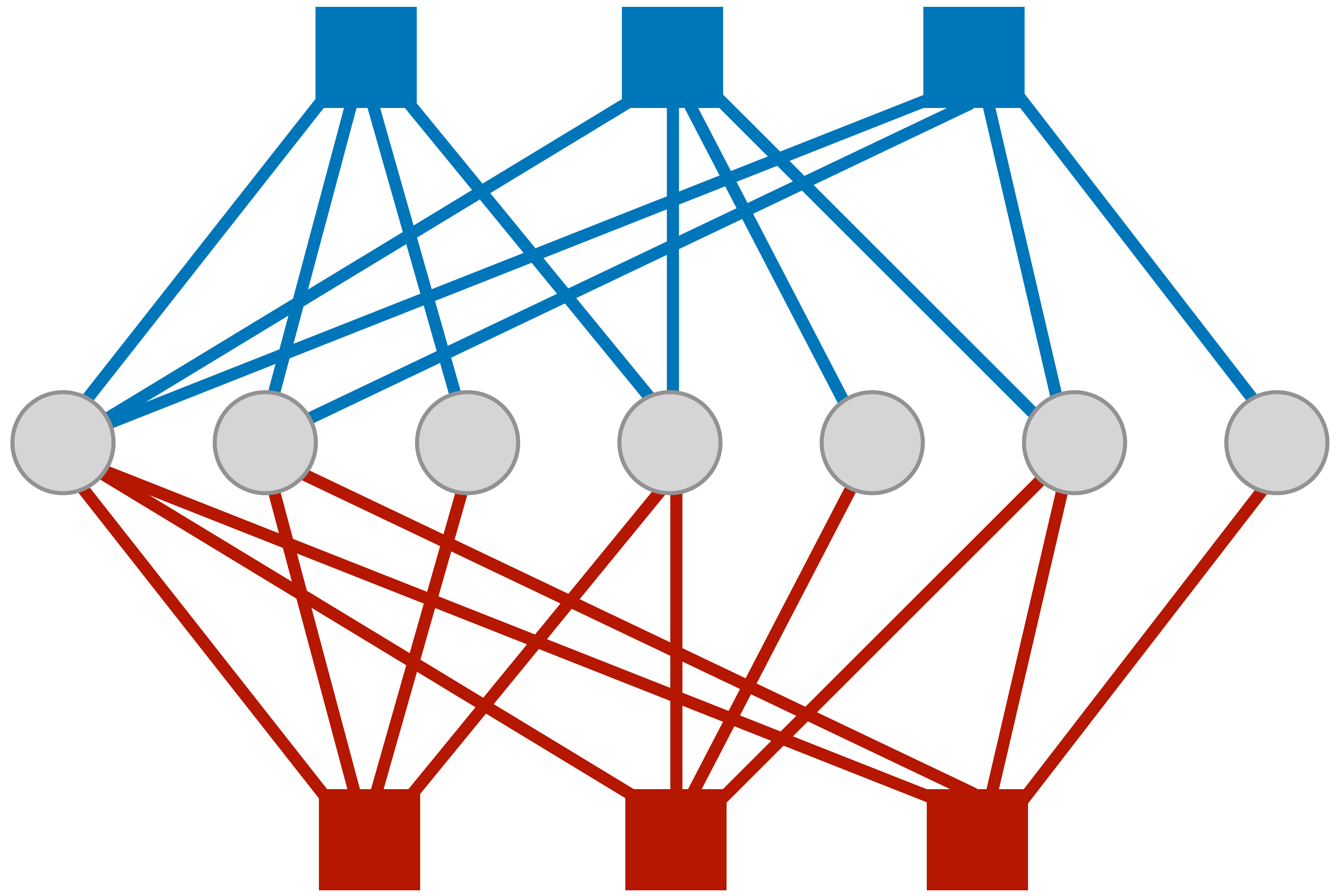
- LDPC $\leftrightarrow$ sparse Tanner graph

QLDPC codes



- LDPC ↔ sparse Tanner graph
- Code properties $[[n, k, d]]$

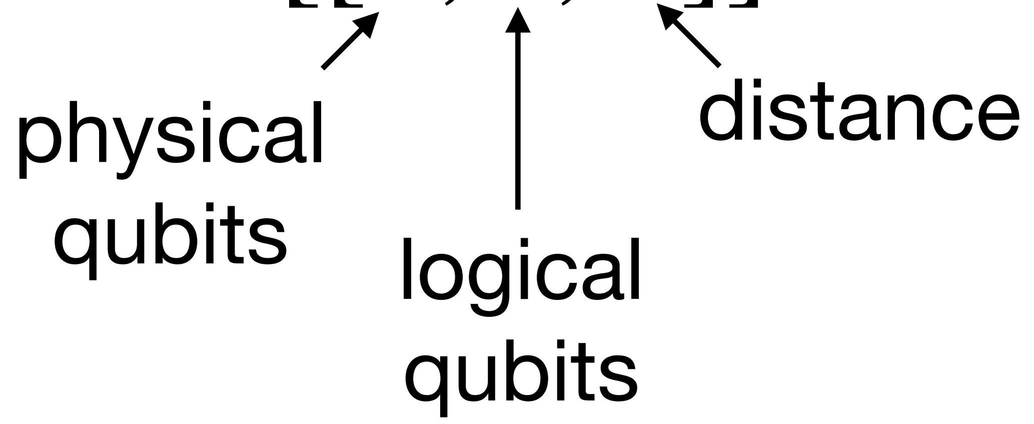
QLDPC codes



$[[7,1,3]]$

Steane code

- LDPC ↔ sparse Tanner graph
- Code properties $[[n, k, d]]$

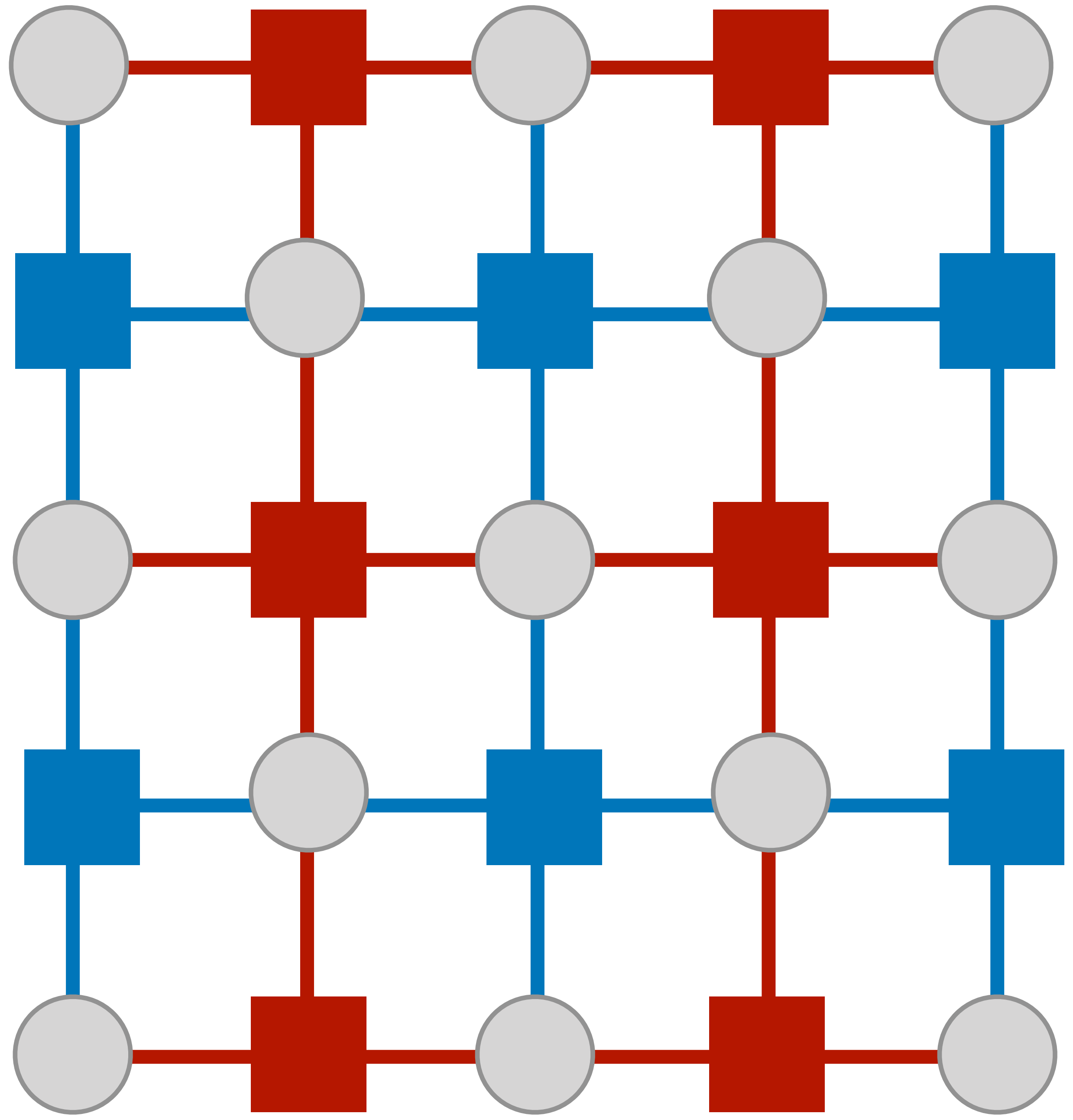


- Good codes exist! $[[n, \Theta(n), \Theta(n)]]$

…, Pantaleev Kalachev 21, …

Topological codes

- Local checks on a lattice
- Correct all local errors
- Ground space of local Hamiltonian

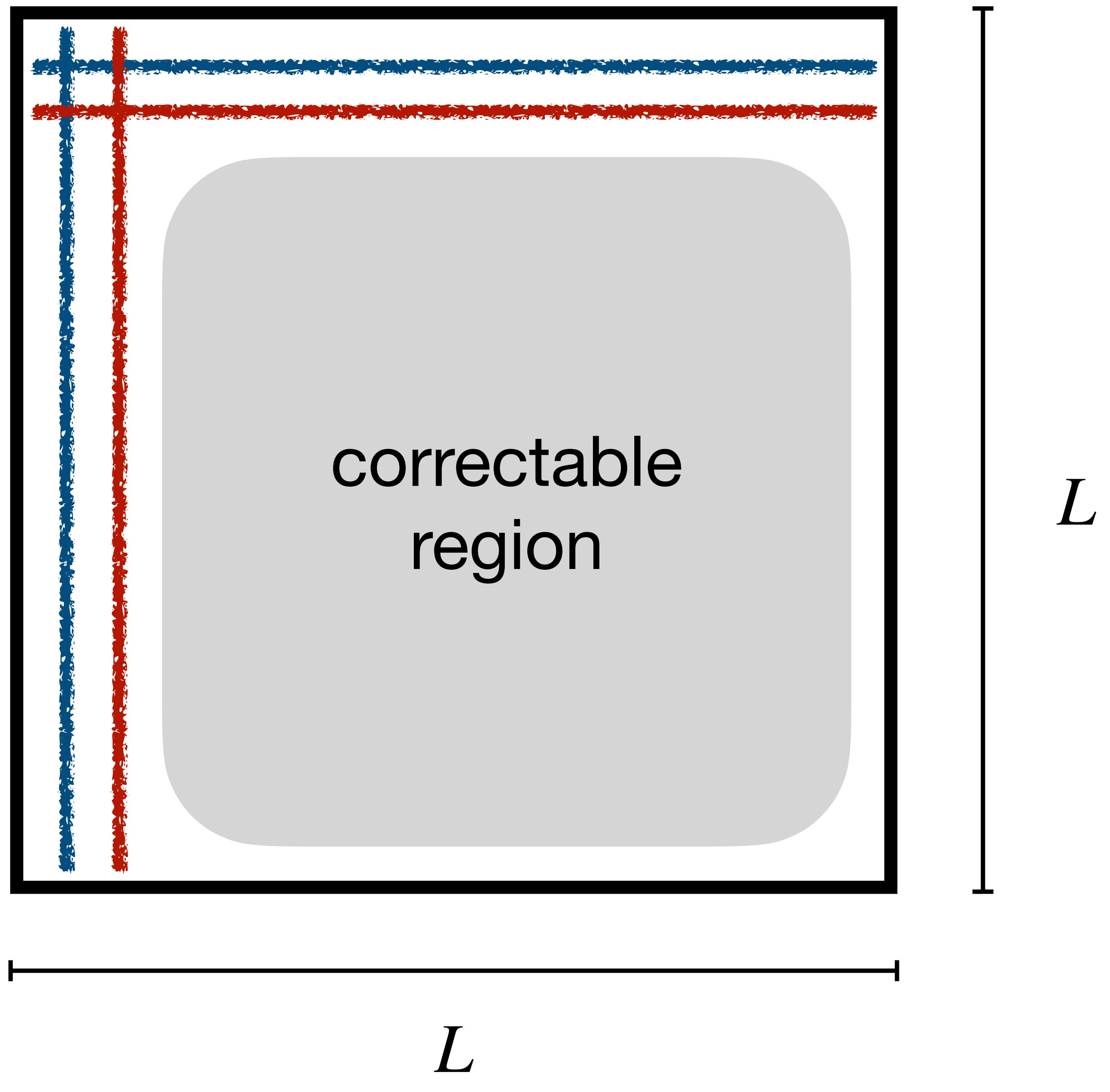


BPT bound

For topological code in D dimensions:

- BPT $kd^{\frac{2}{D-1)}} = O(L^D)$
- BT $d = O(L^{D-1})$
- Haah* $k = O(L^{D-2})$

*homogeneity assumption



Bravyi Terhal 08
Bravyi Poulin Terhal 09
Haah 2020

BPT bound

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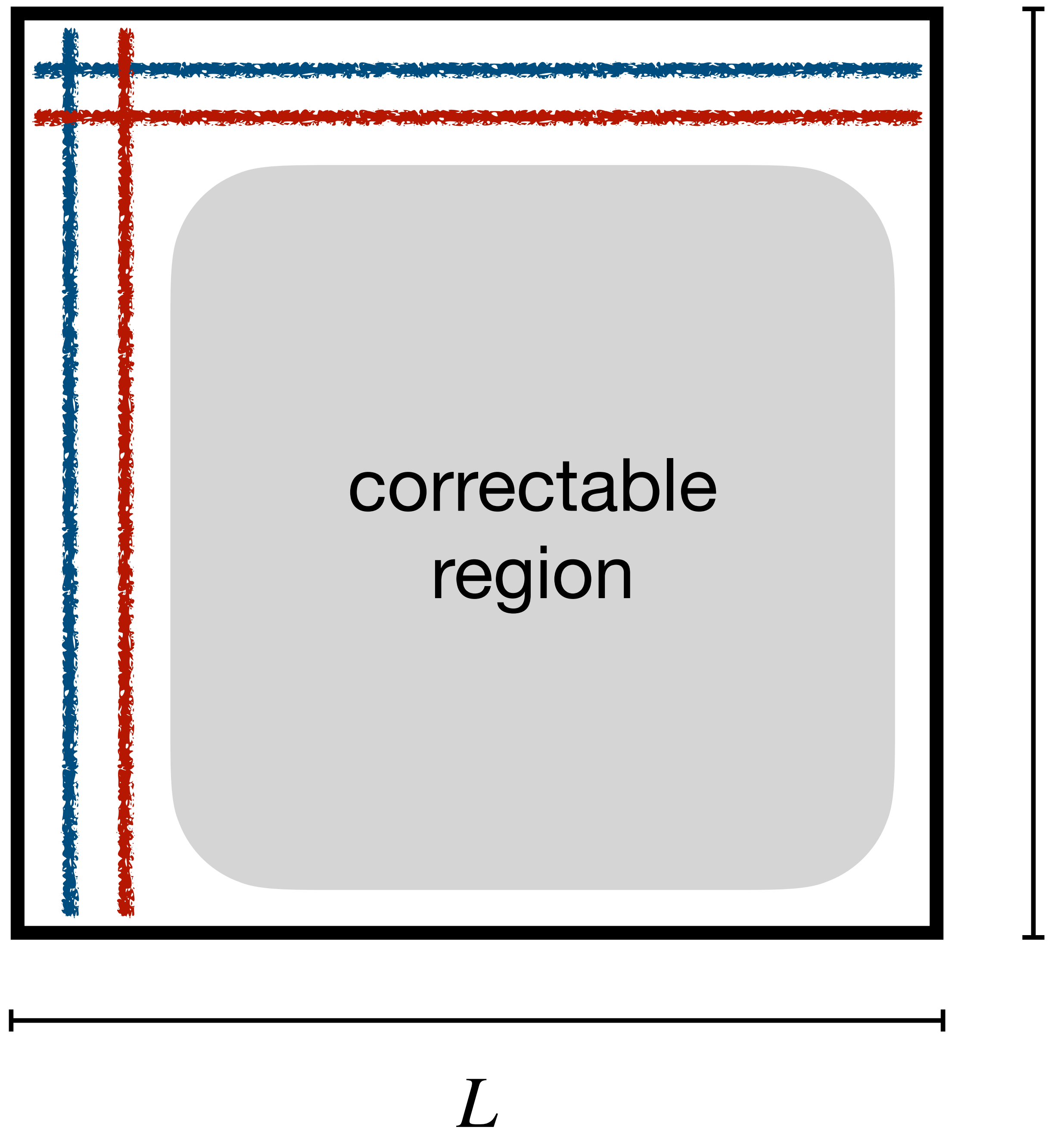
- Best you can do in 2D: $[[L^2, 1, L]]$



Bravyi Terhal 08
Bravyi Poulin Terhal 09
Haah 2020

BPT bound

For topological code in D dimensions:

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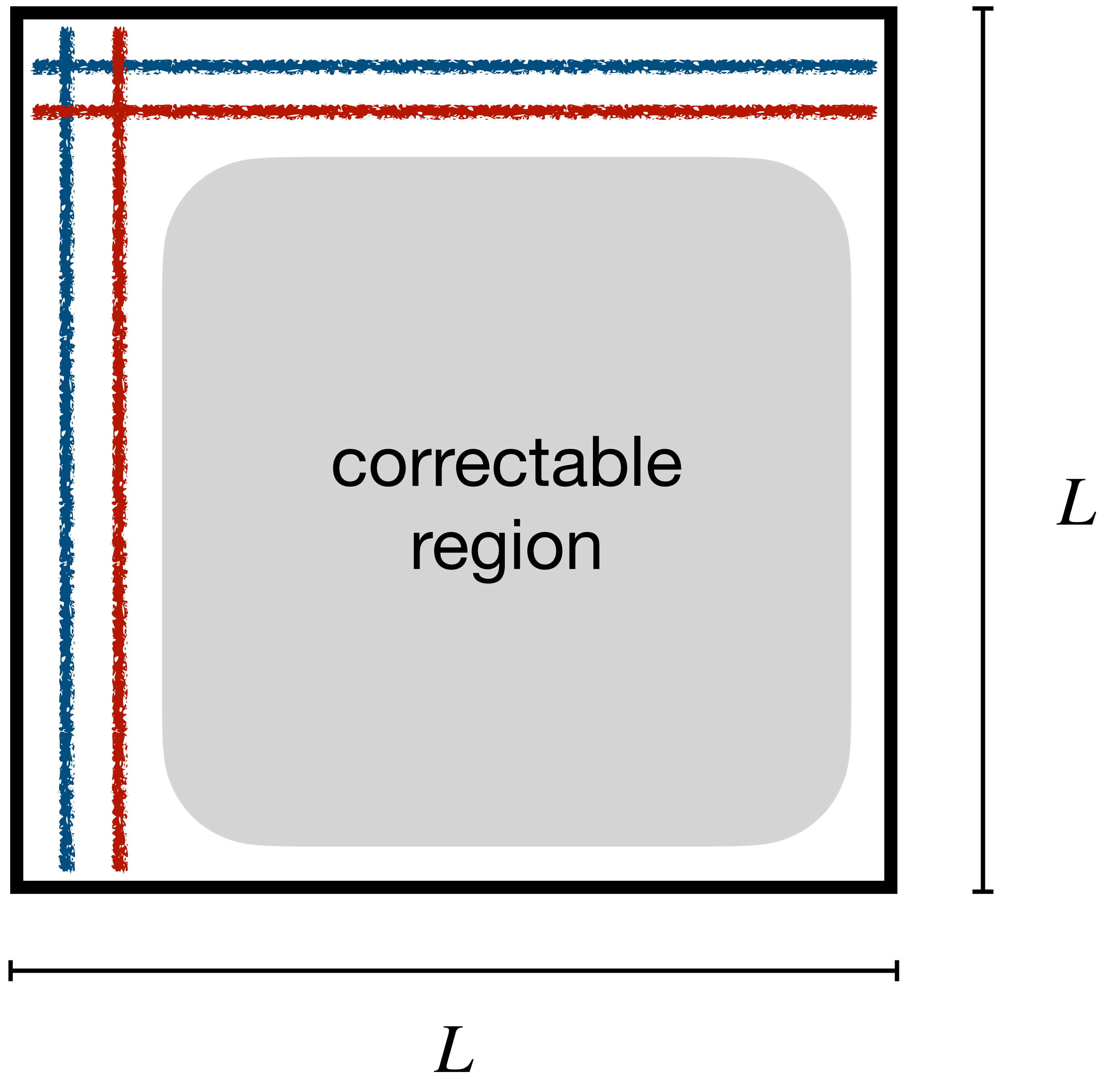
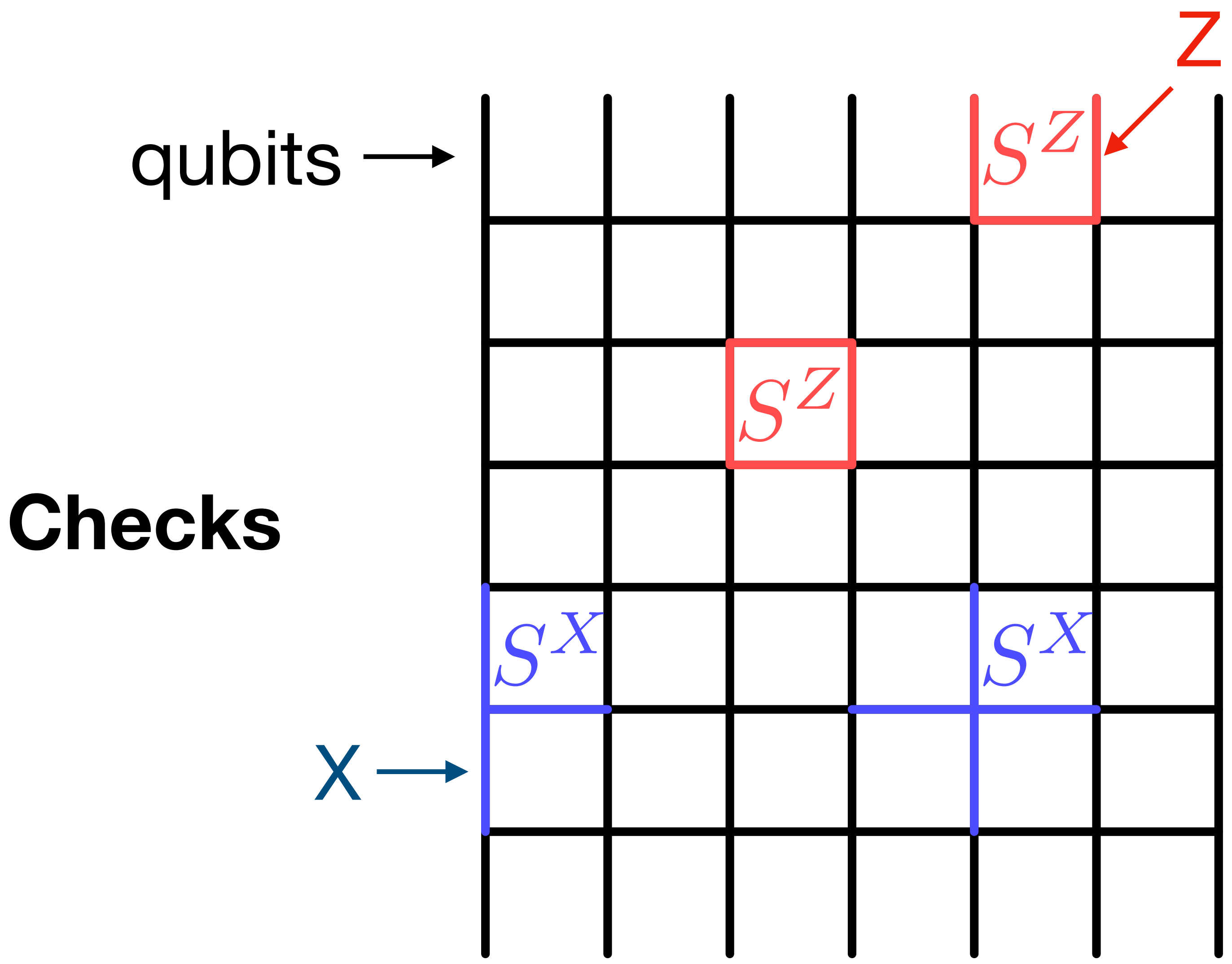
- Best you can do in 2D: $[[L^2, 1, L]]$
- Best you can do in 3D: $[[L^3, L, L^2]]$



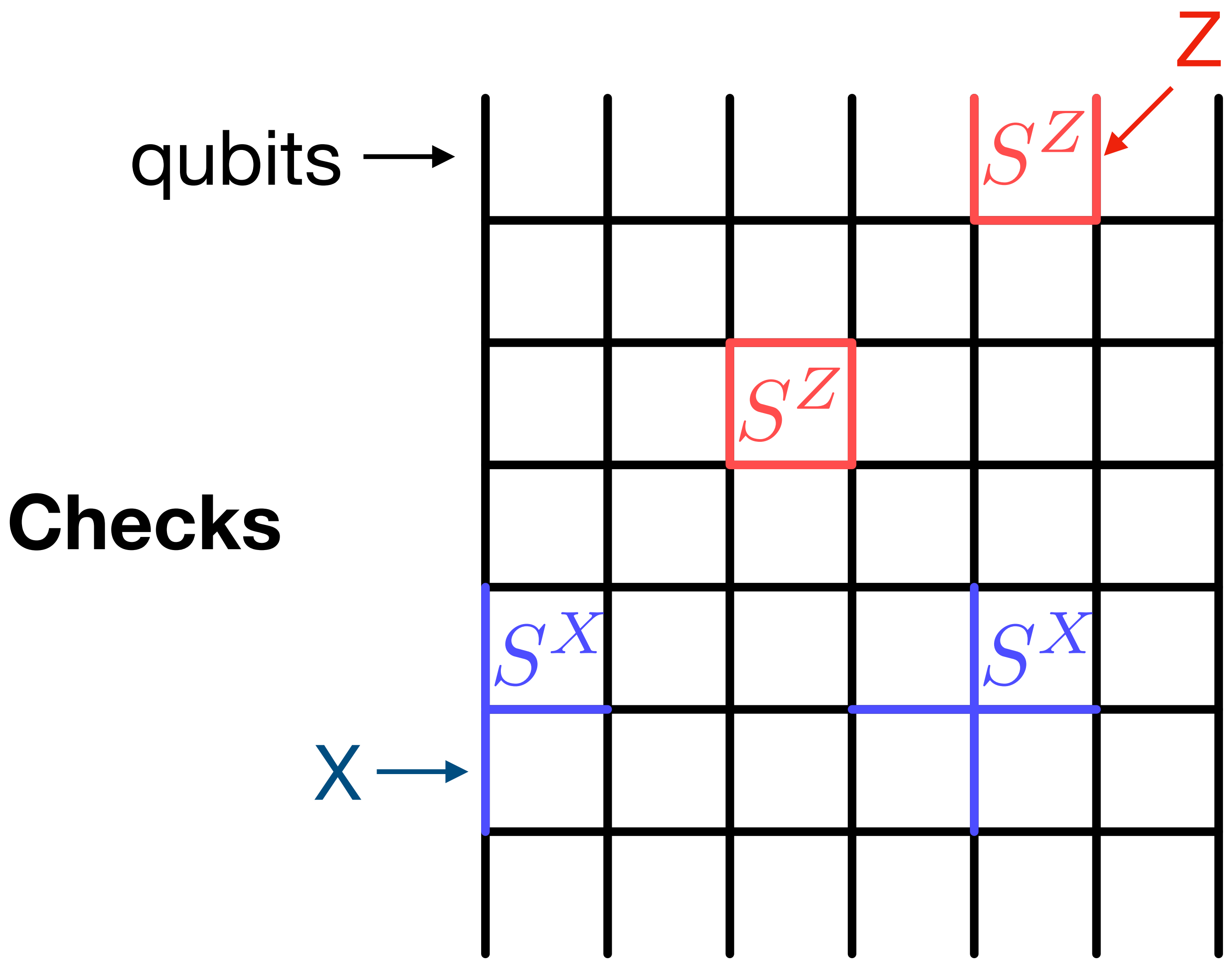
Bravyi Terhal 08
Bravyi Poulin Terhal 09
Haah 2020

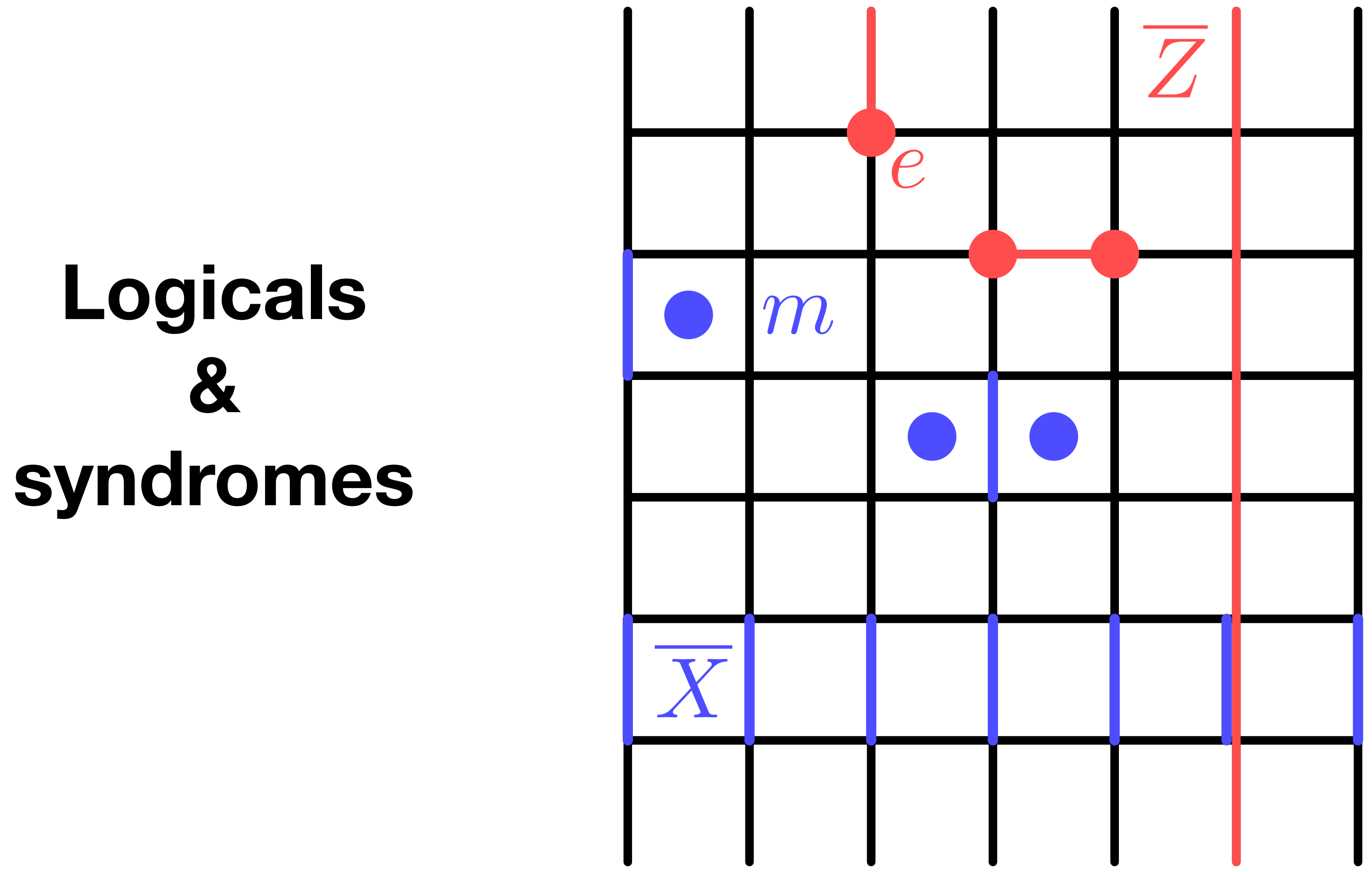
Surface code



Kitaev 97
Bravyi Kitaev 97

Surface code

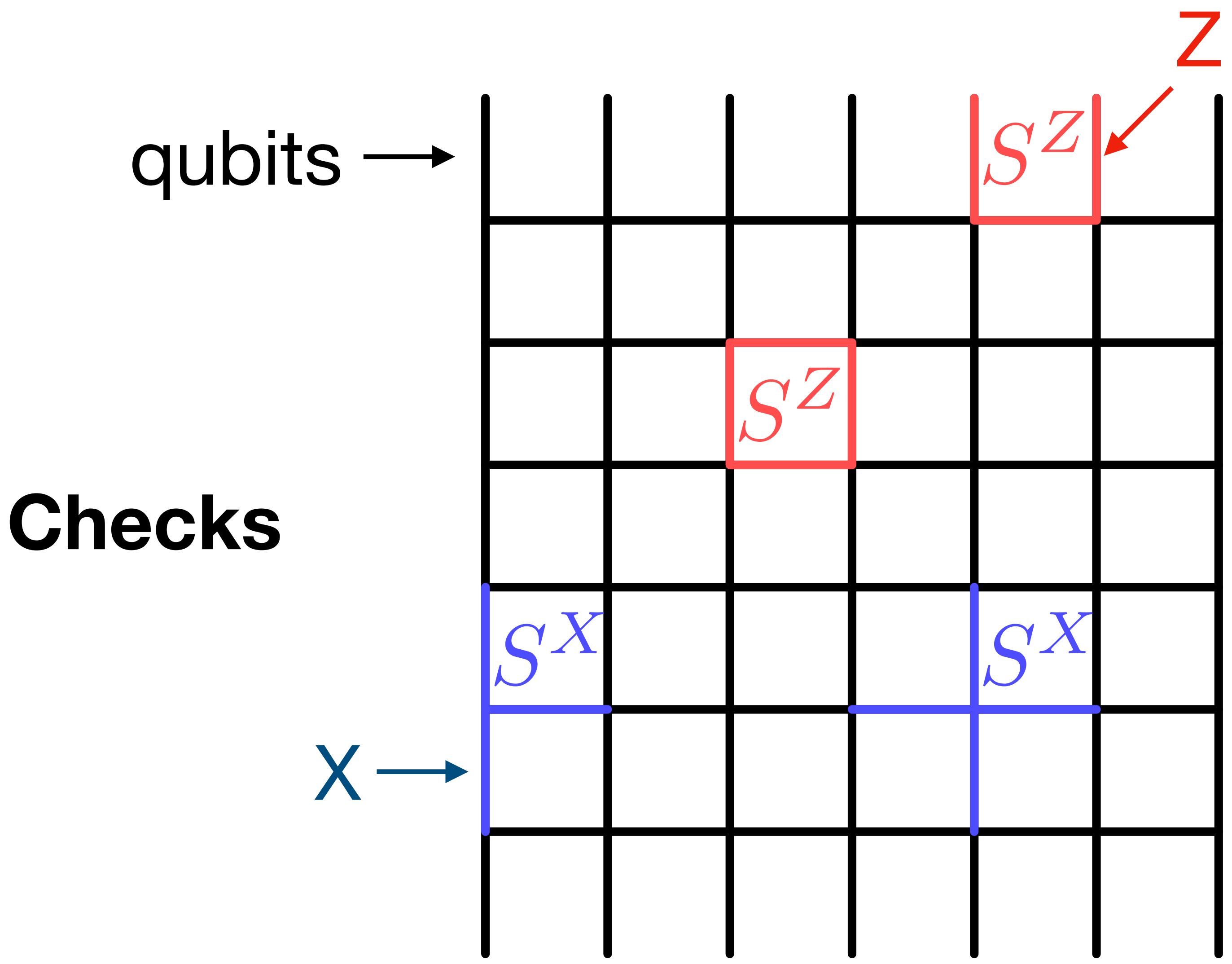


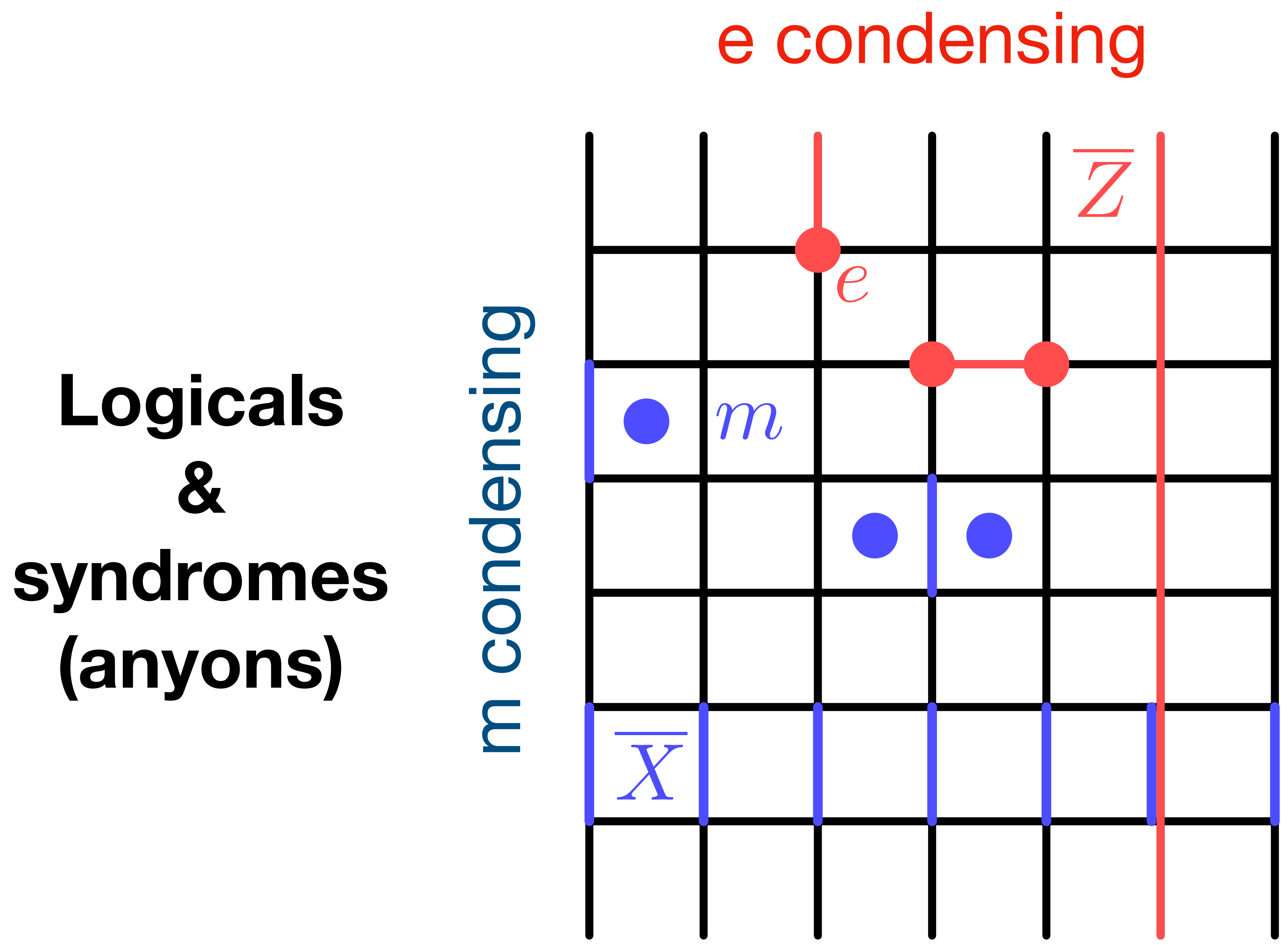


- Optimal parameter scaling in 2D: $[[L^2, 1, L]]$

Kitaev 97
Bravyi Kitaev 97

Surface code

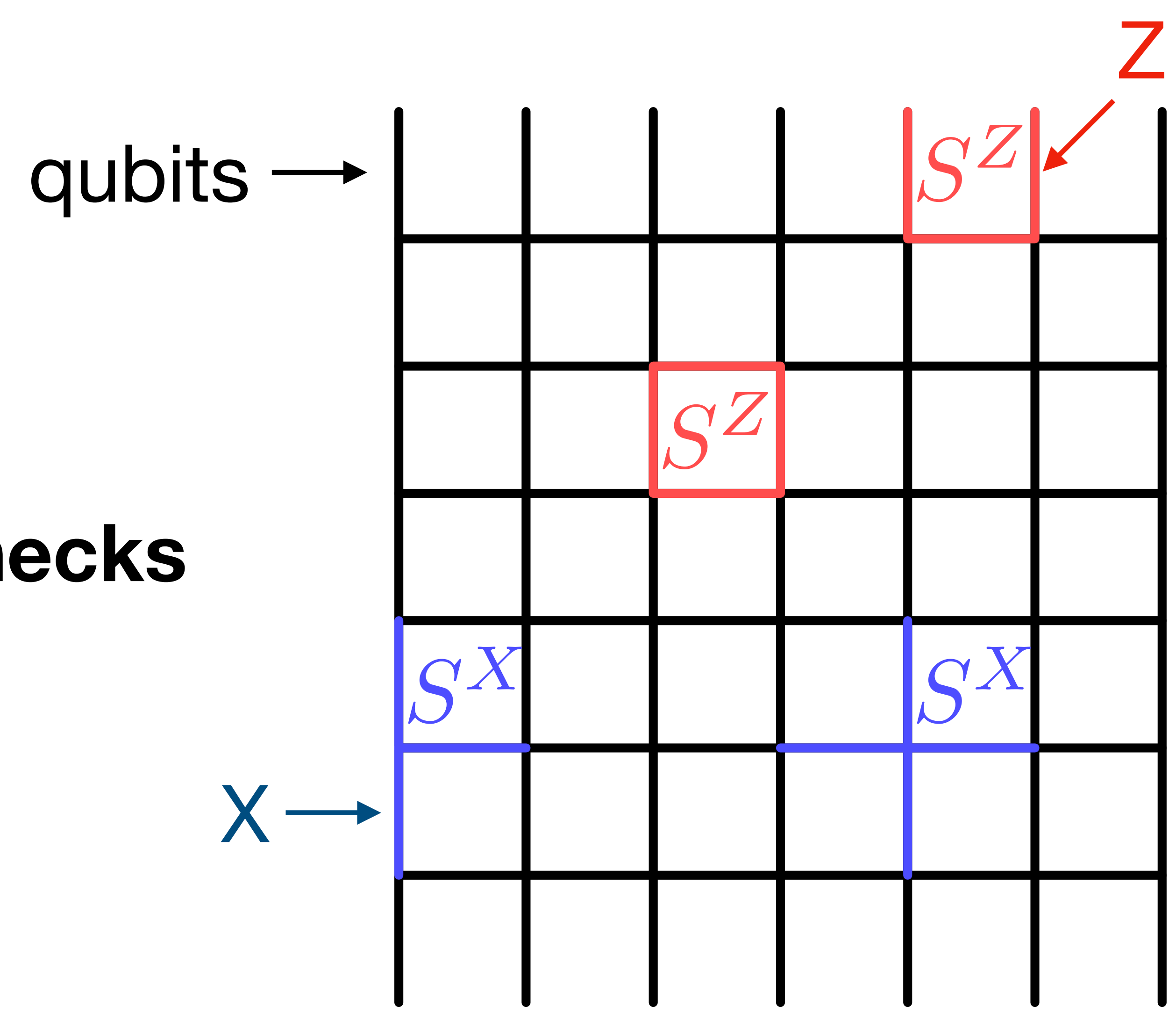




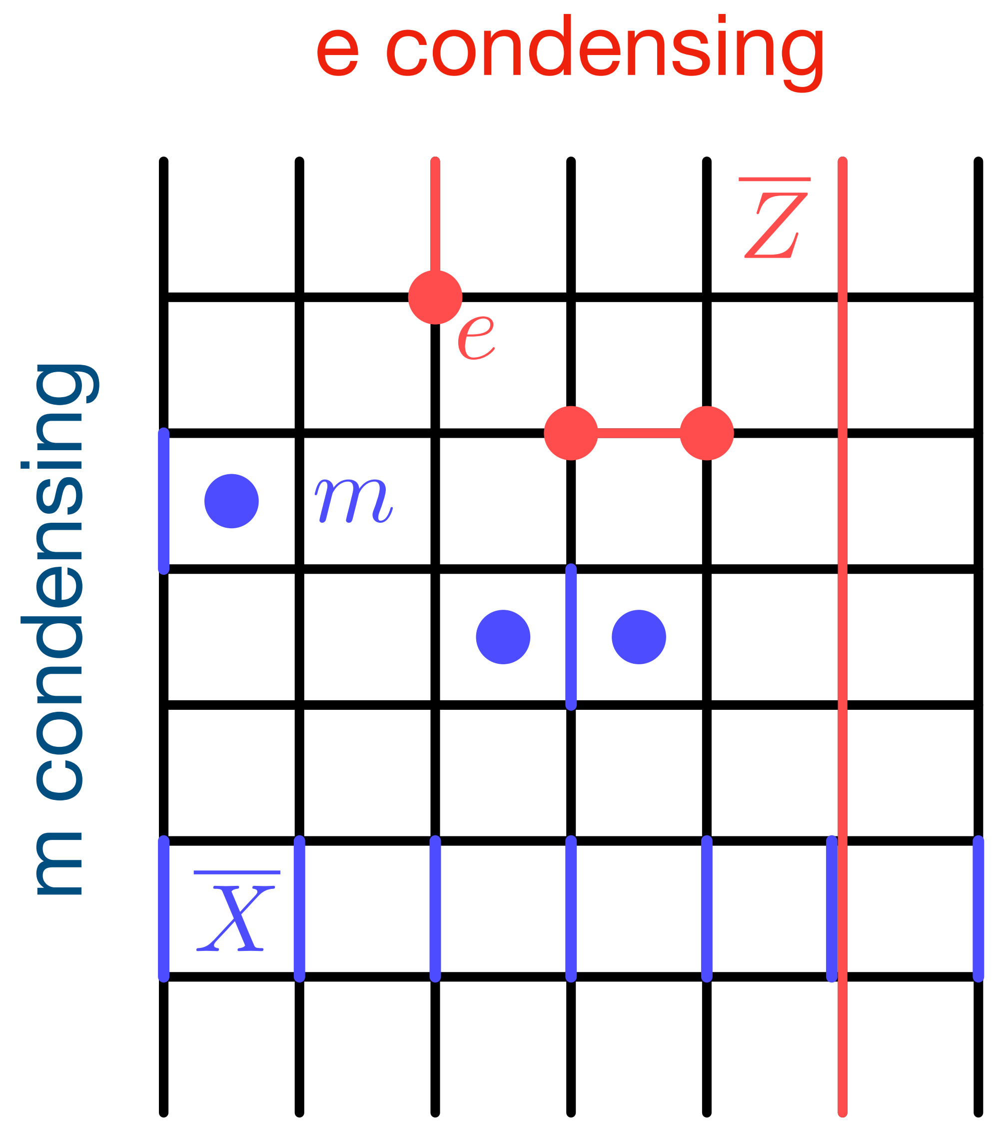
Kitaev 97
Bravyi Kitaev 97

Surface code

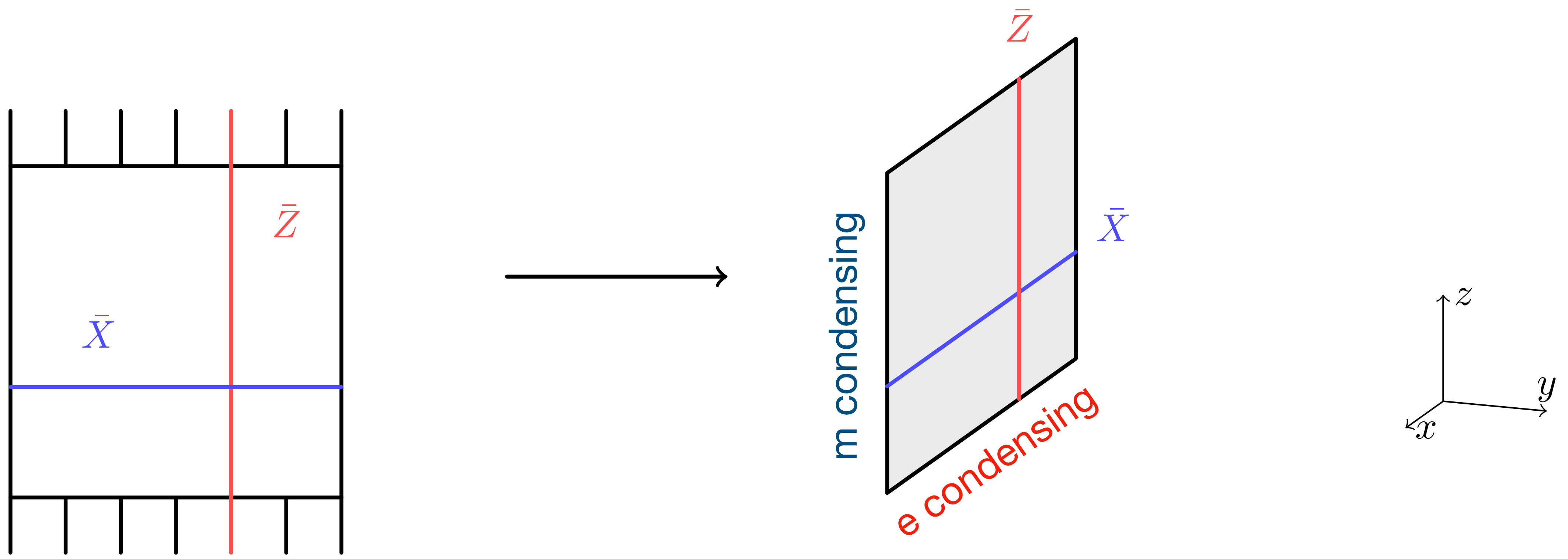
Checks



Logicals & syndromes (anyons)

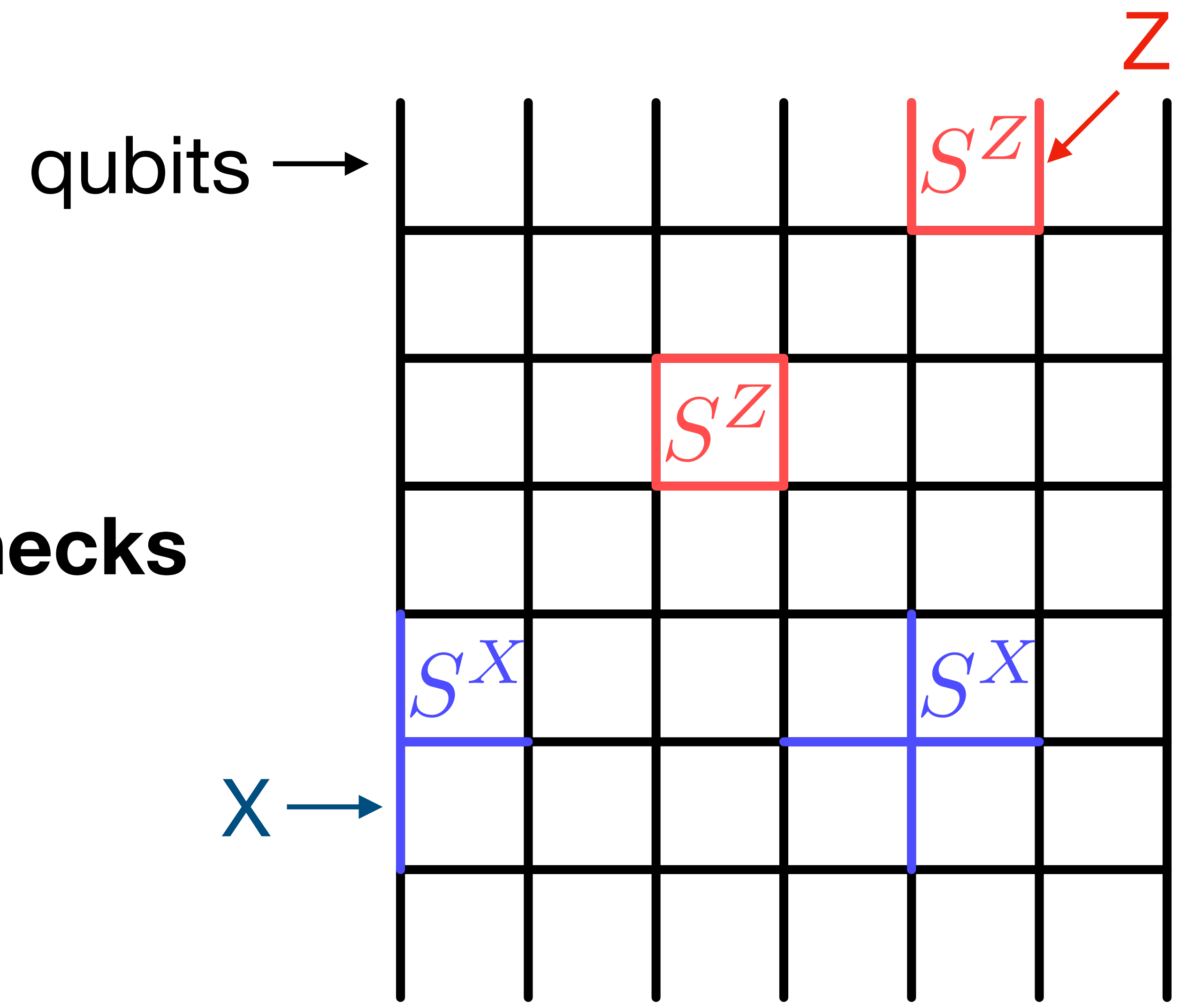


Abstract picture

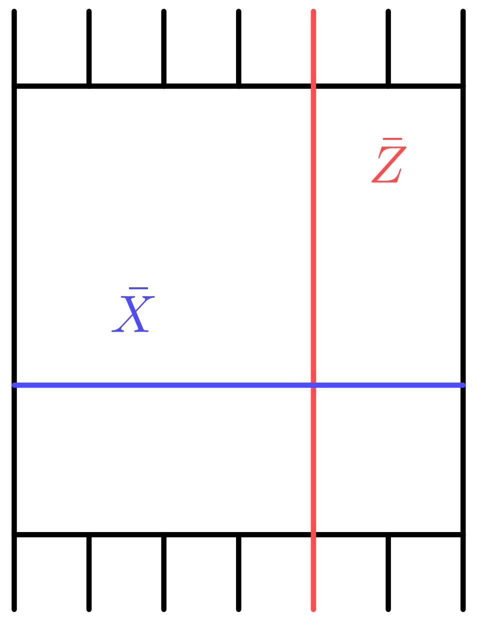


Kitaev 97
Bravyi Kitaev 97

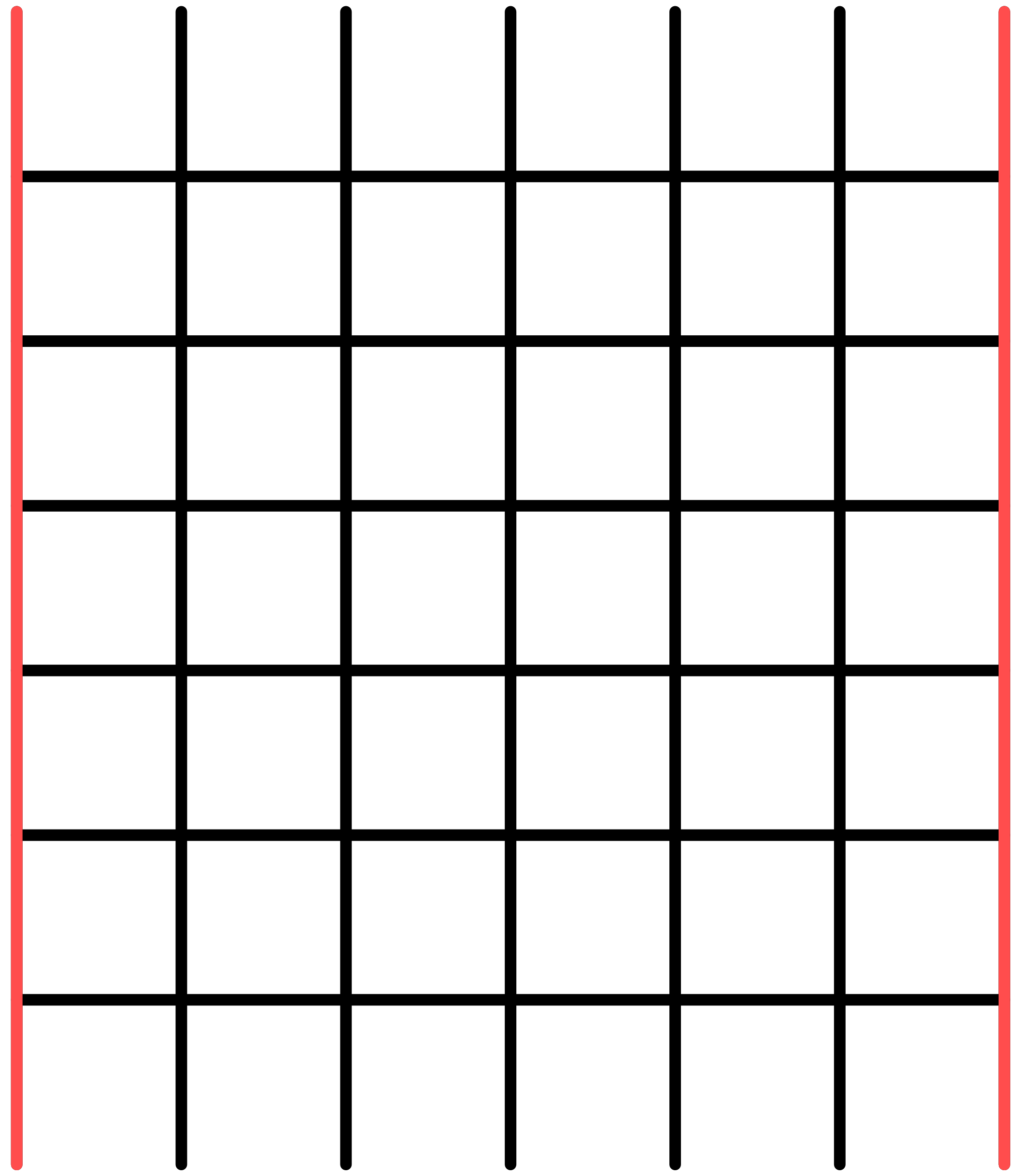
Surface code



Abstract picture



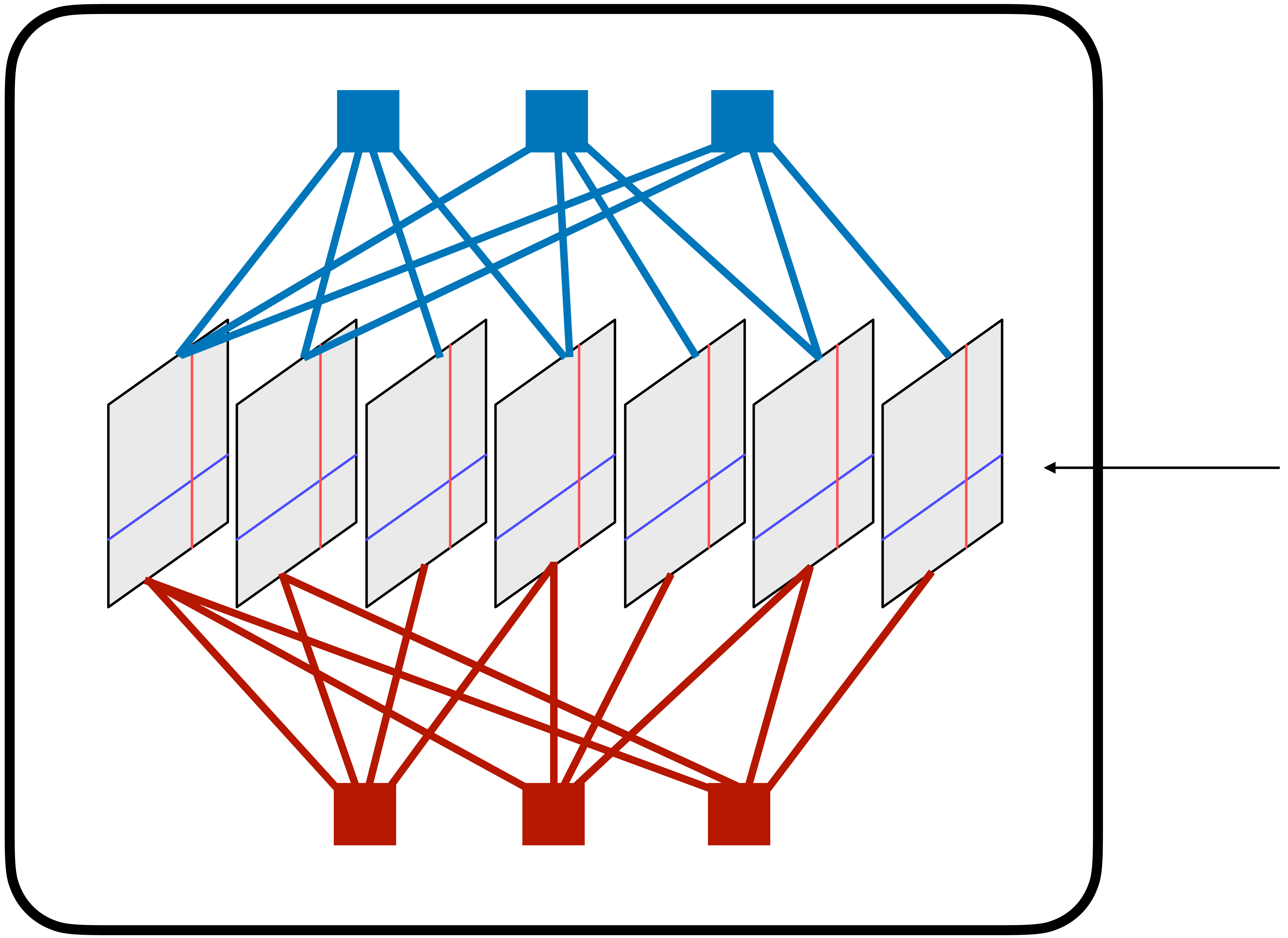
A very useful property!



$$\prod S^Z = \overline{Z}_l \overline{Z}_r$$

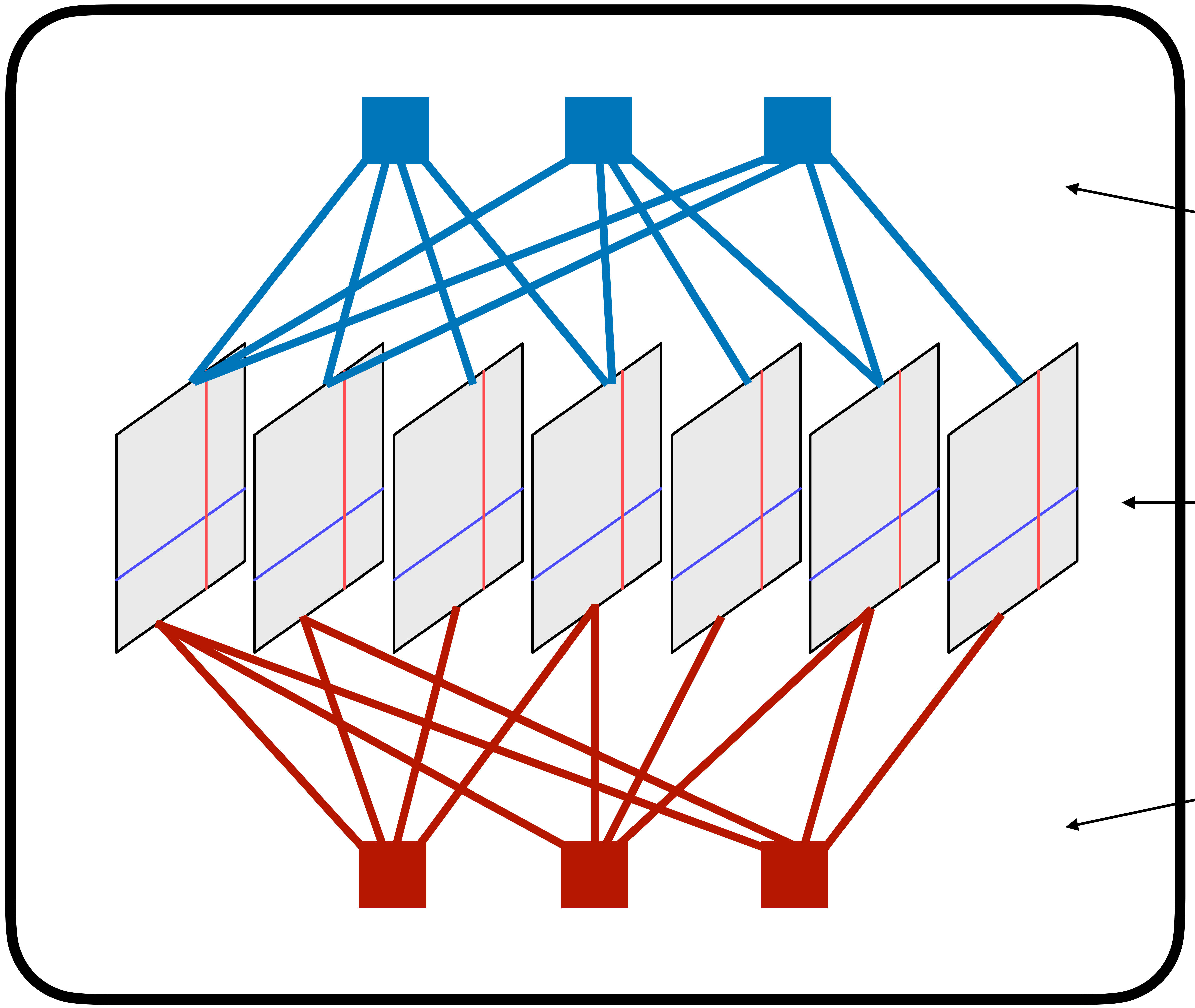
Kitaev 97
Bravyi Kitaev 97

Concatenated codes



Surface codes

Concatenated codes

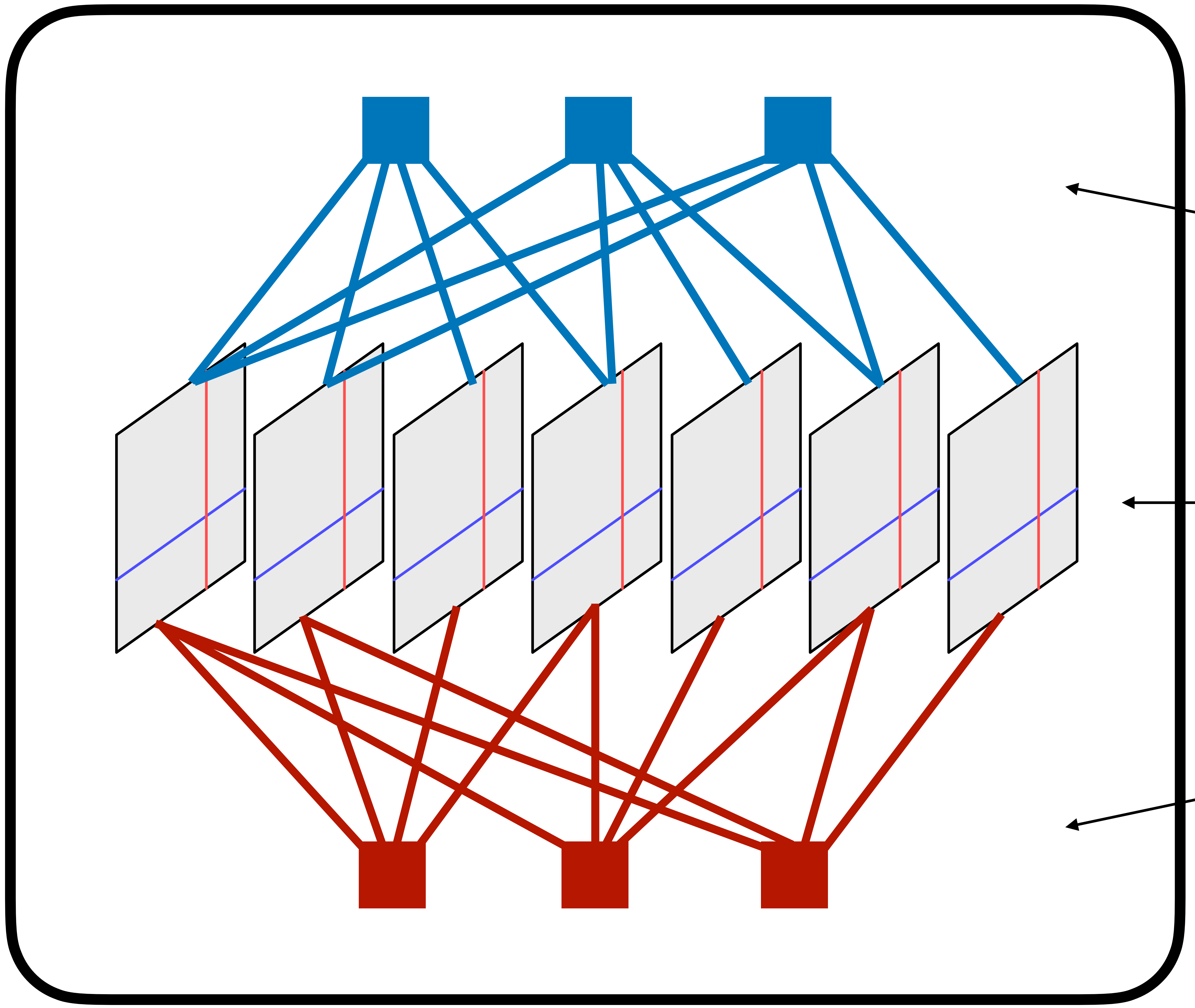


Logical X checks $A_i = \prod_{j \in a_i} \overline{X}_j$

Surface codes

Logical Z checks $B_i = \prod_{j \in b_i} \overline{Z}_j$

Concatenated codes



Logical X checks $A_i = \prod_{j \in a_i} \overline{X}_j$

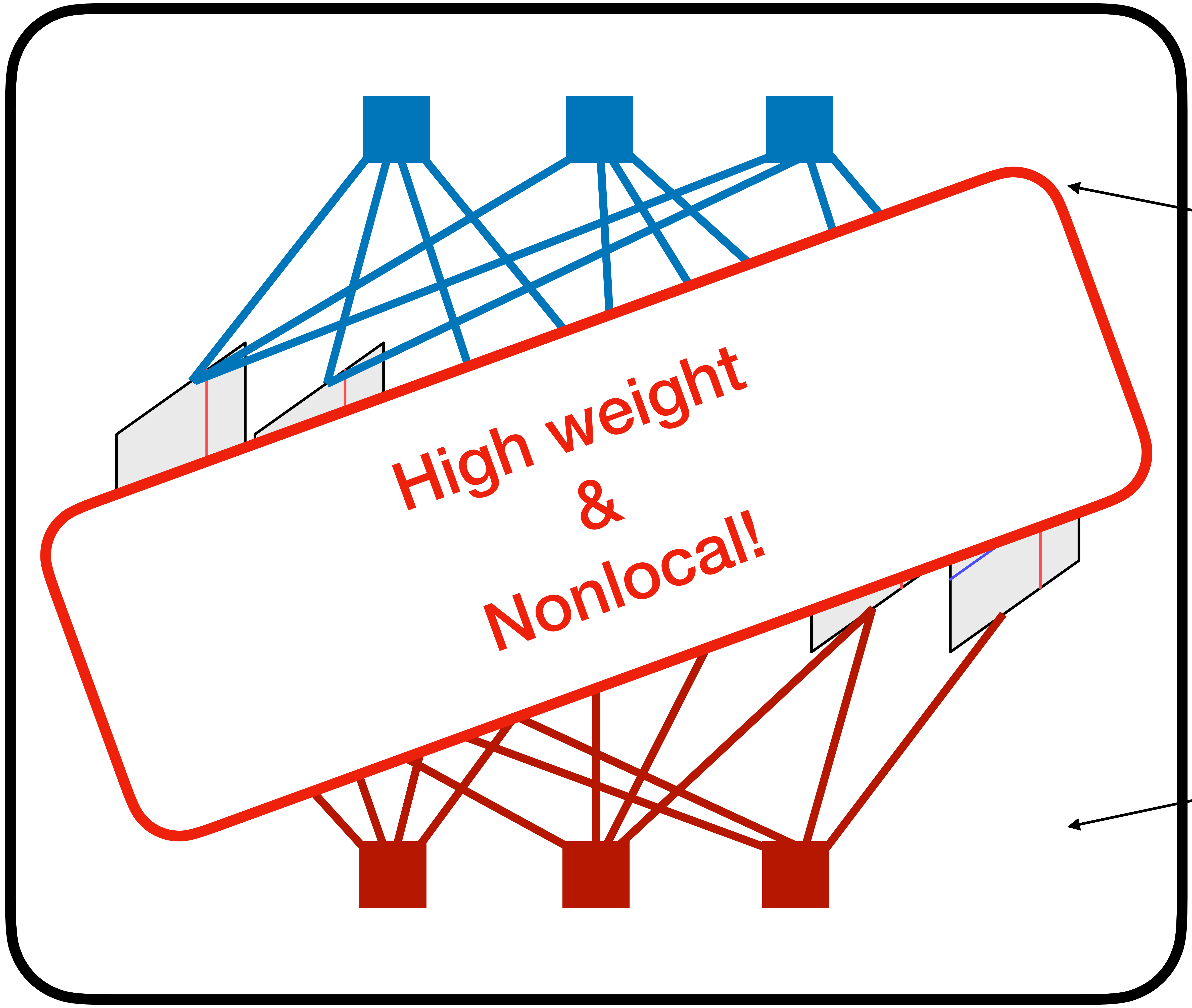
Surface codes

Logical Z checks $B_i = \prod_{j \in b_i} \overline{Z}_j$

- Achieves $[[L^3, L, L^2]]$

☺

Concatenated codes



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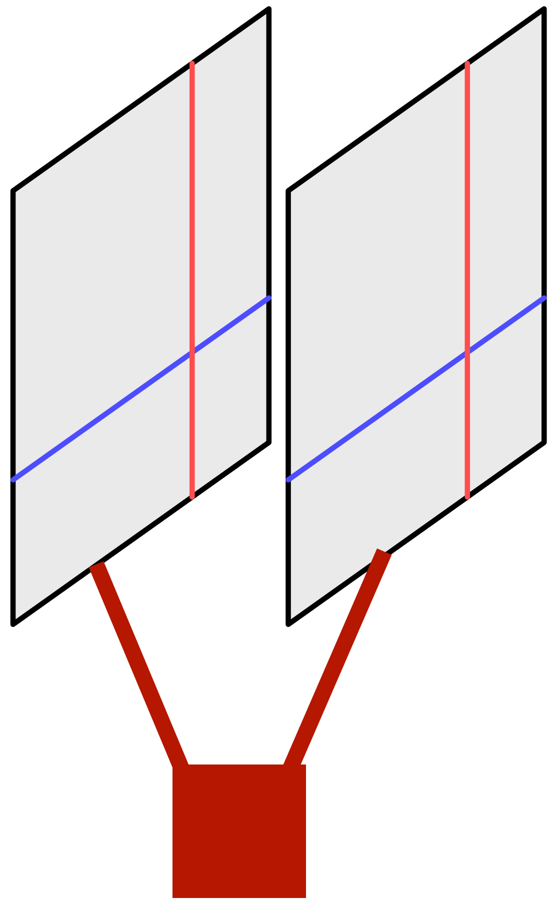
Surface codes

Logical Z checks $B_i = \prod_{j \in b_i} \overline{Z}_j$

- Achieves $[[L^3, L, L^2]]$

Topological defects (lattice surgery)

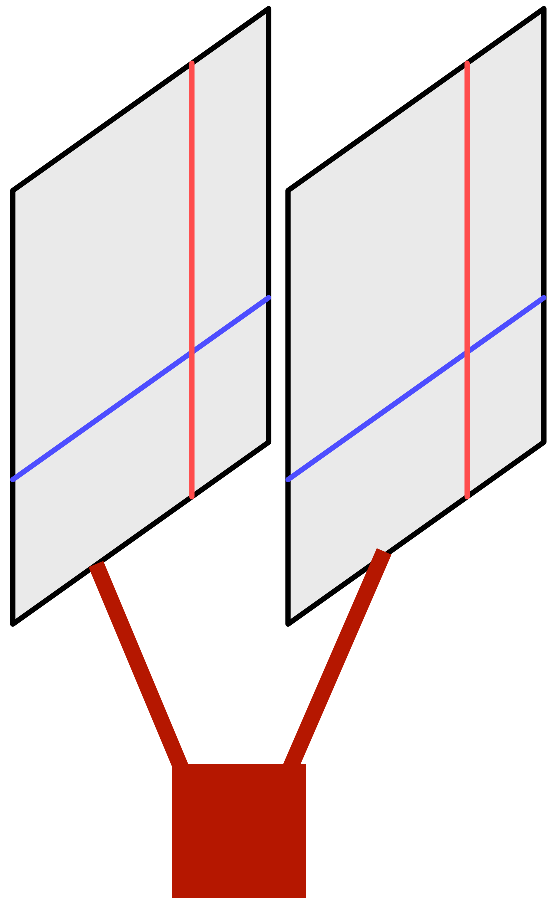


Concatenated check

High weight
&
Nonlocal!

Horsman et al 2012

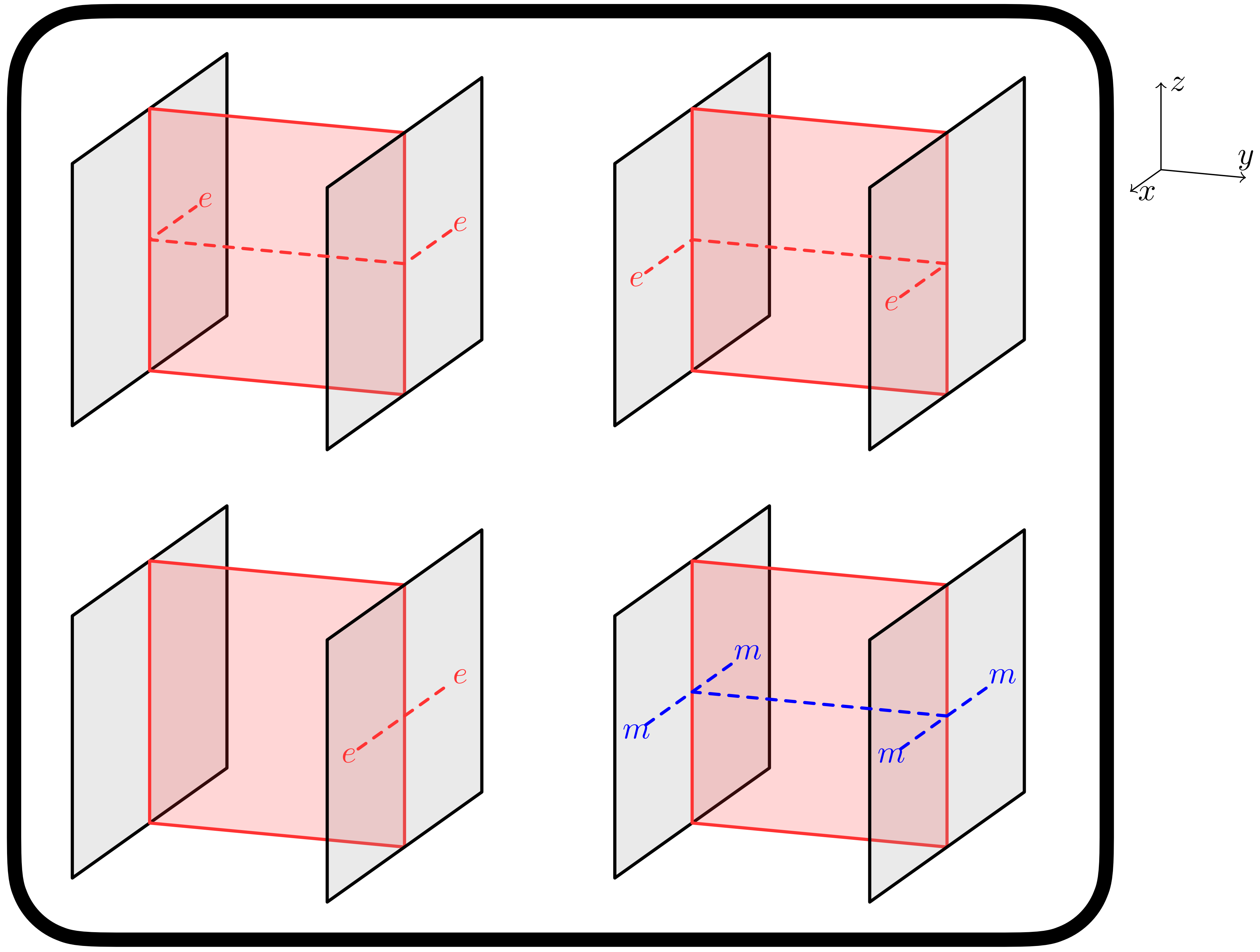
Topological defects (lattice surgery)



Concatenated check

High weight
&
Nonlocal!

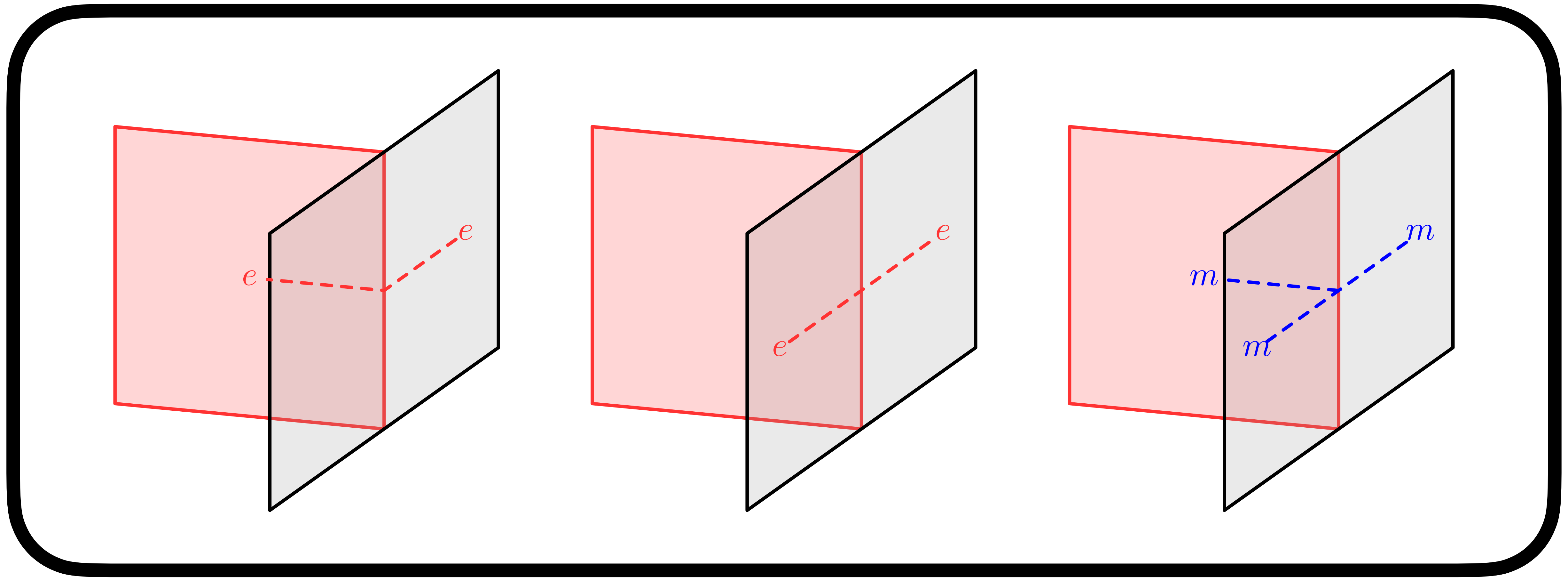
Resolve via
local defects



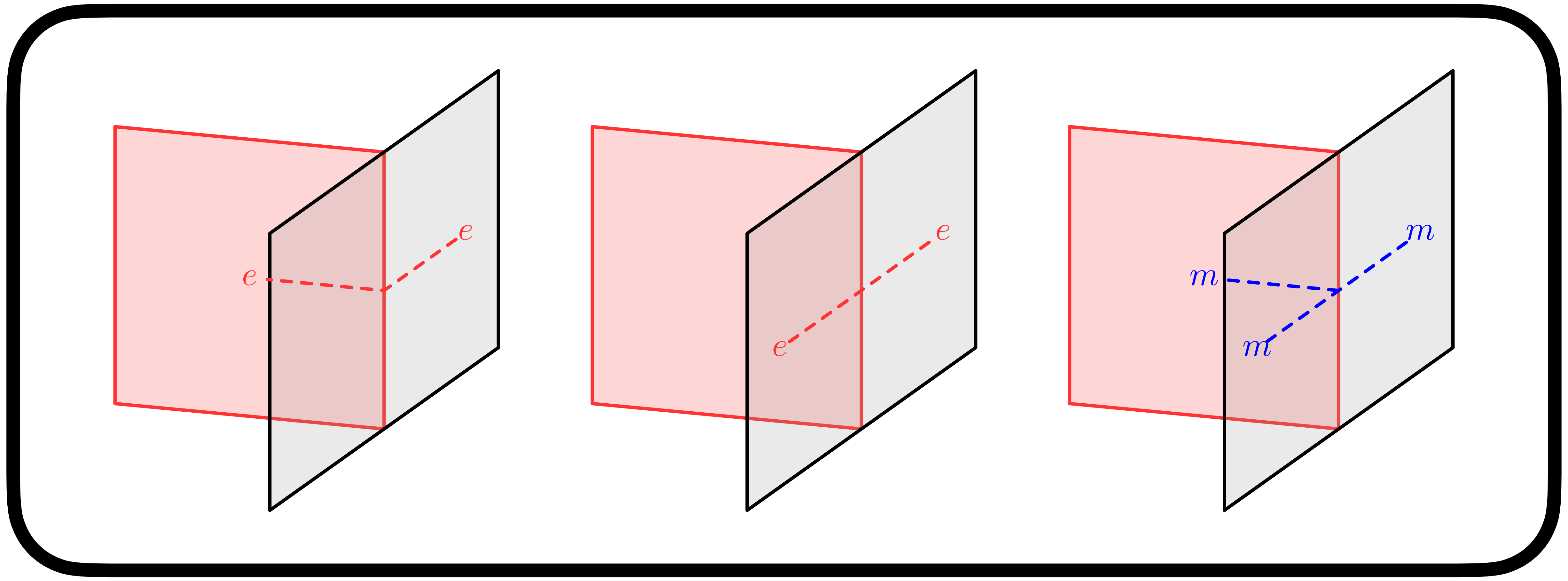
low weight
&
local

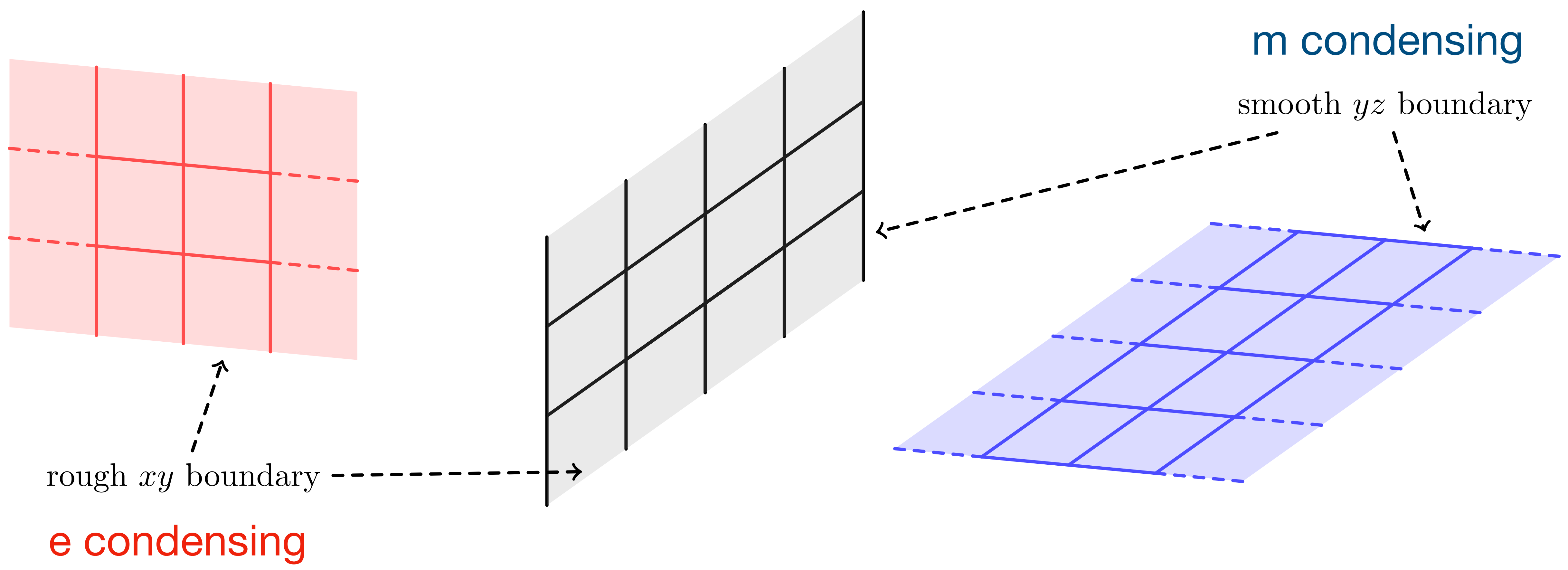
Horsman et al 2012

Topological defects

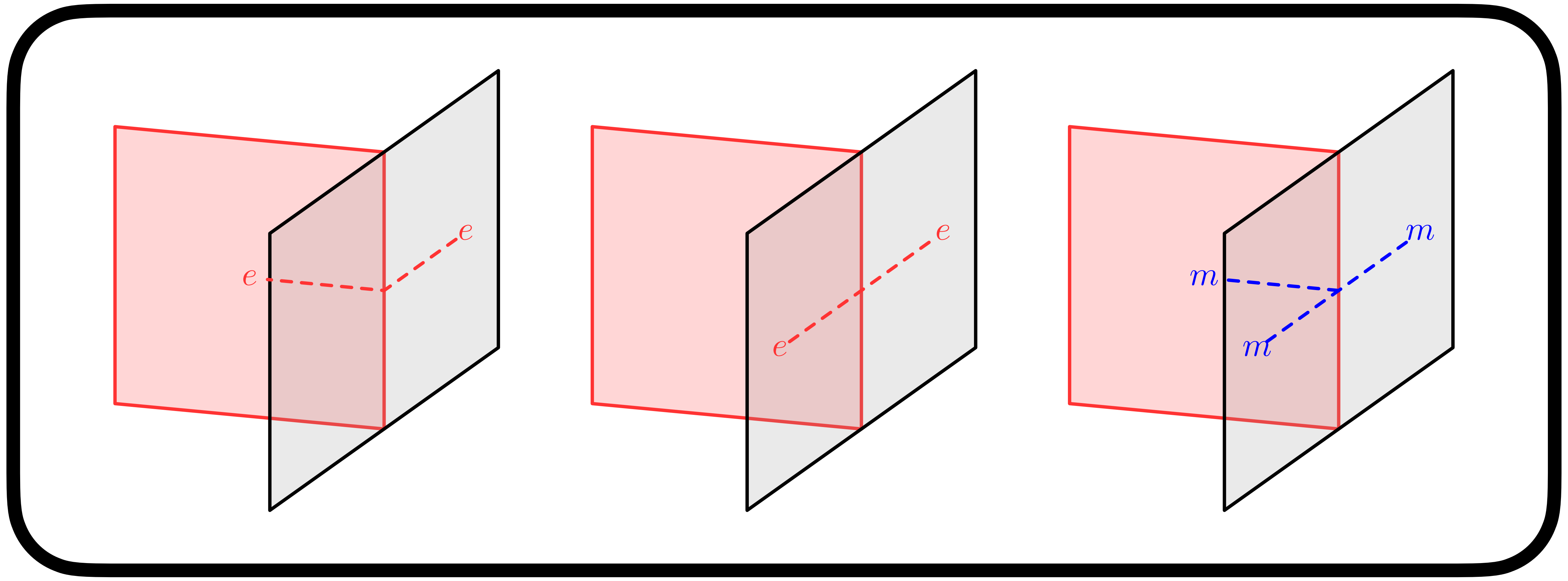


Topological defects

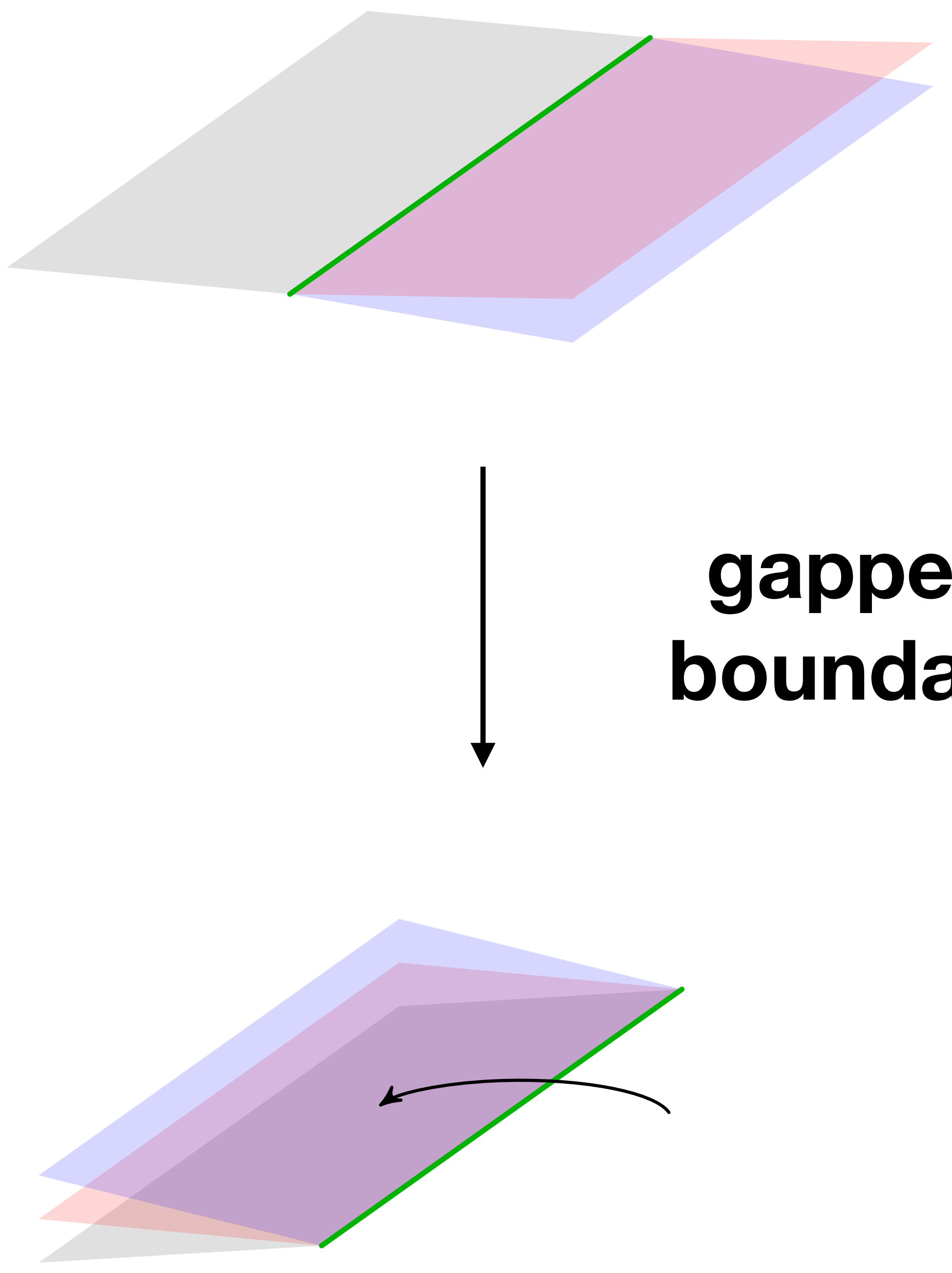




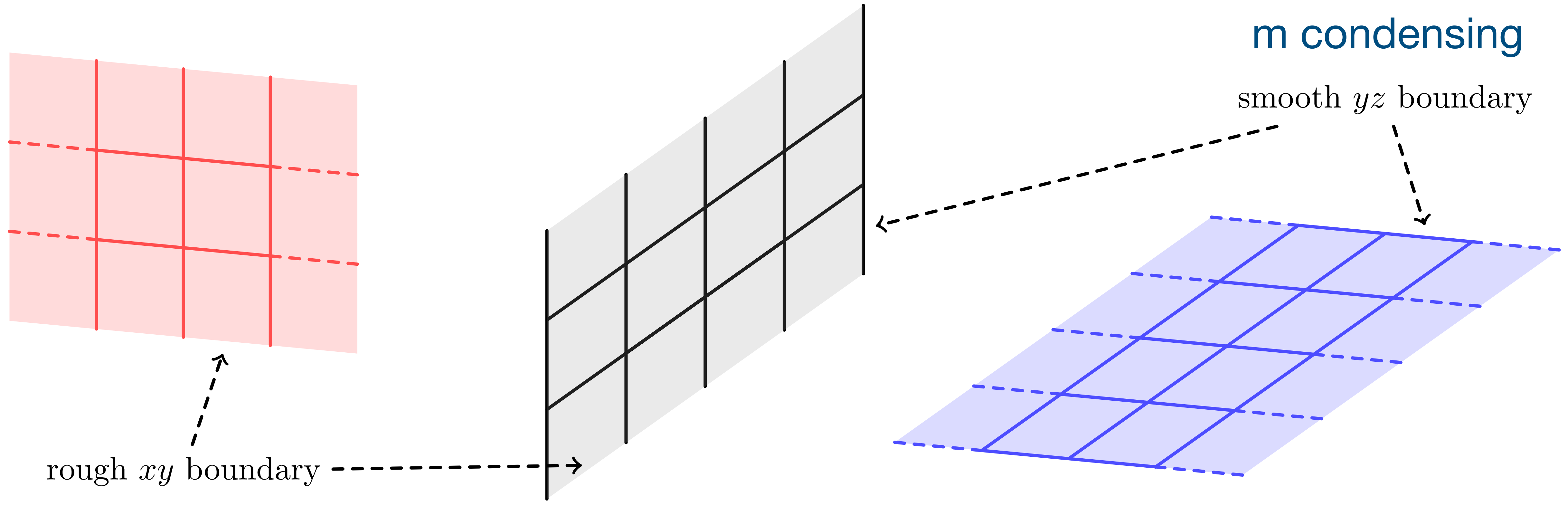
Topological defects



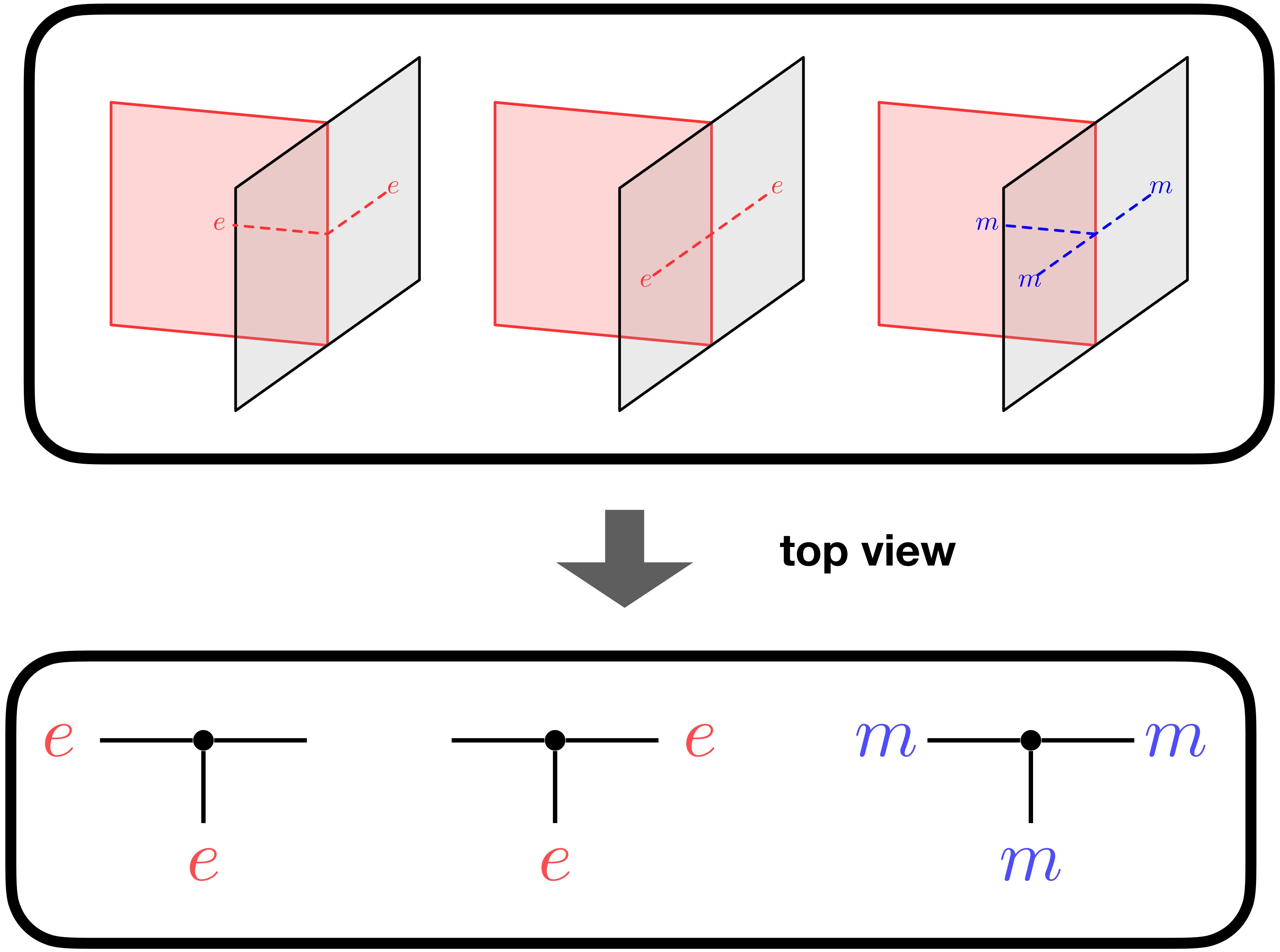
line defect

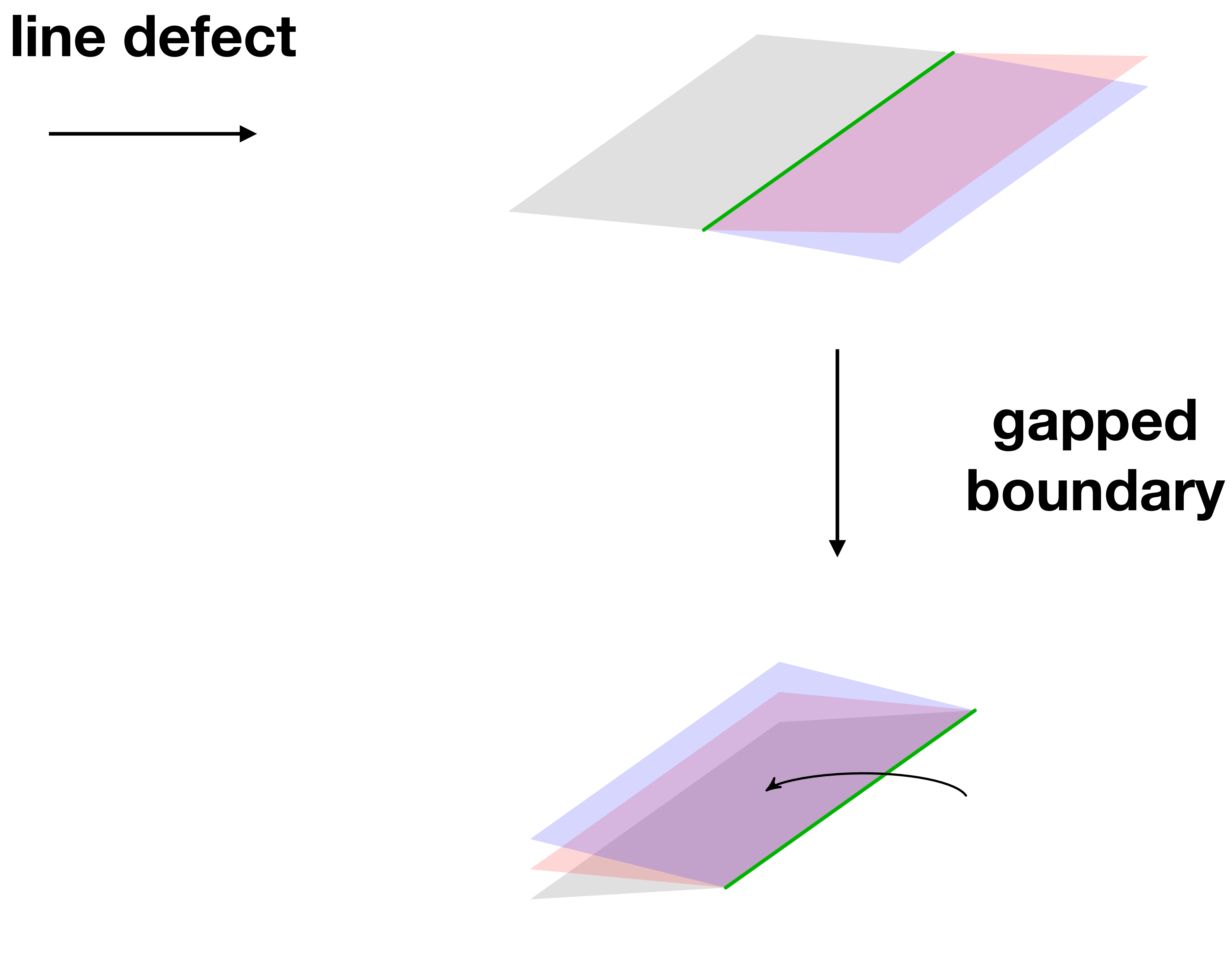


gapped boundary



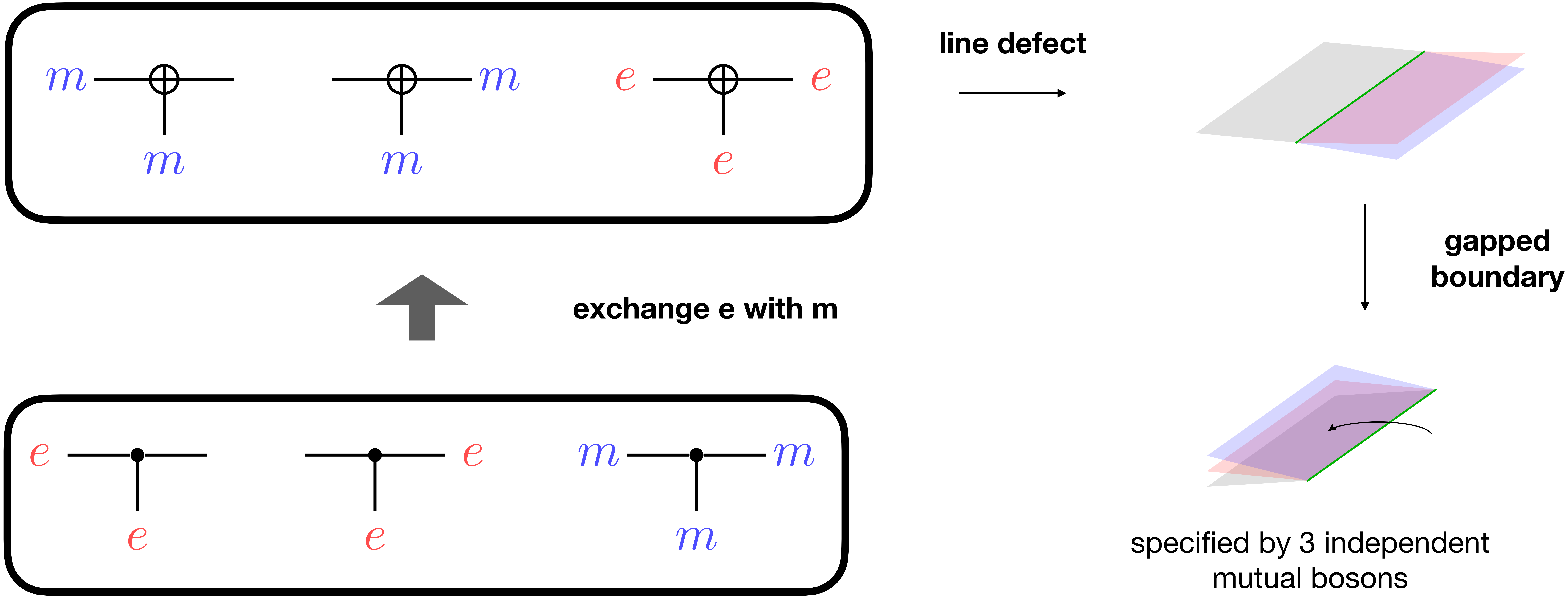
Topological defects



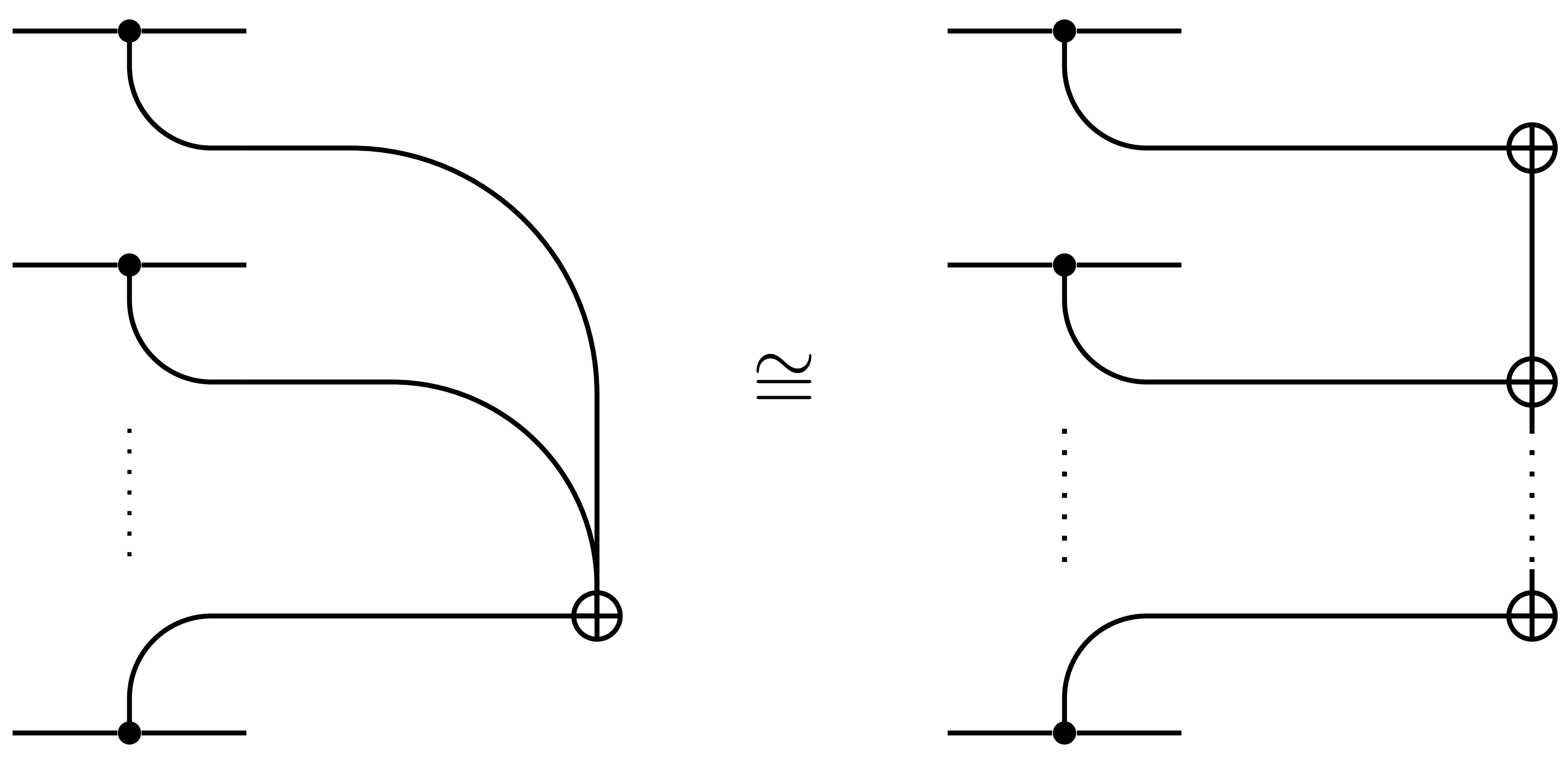


specified by 3 independent
mutual bosons

Topological defects

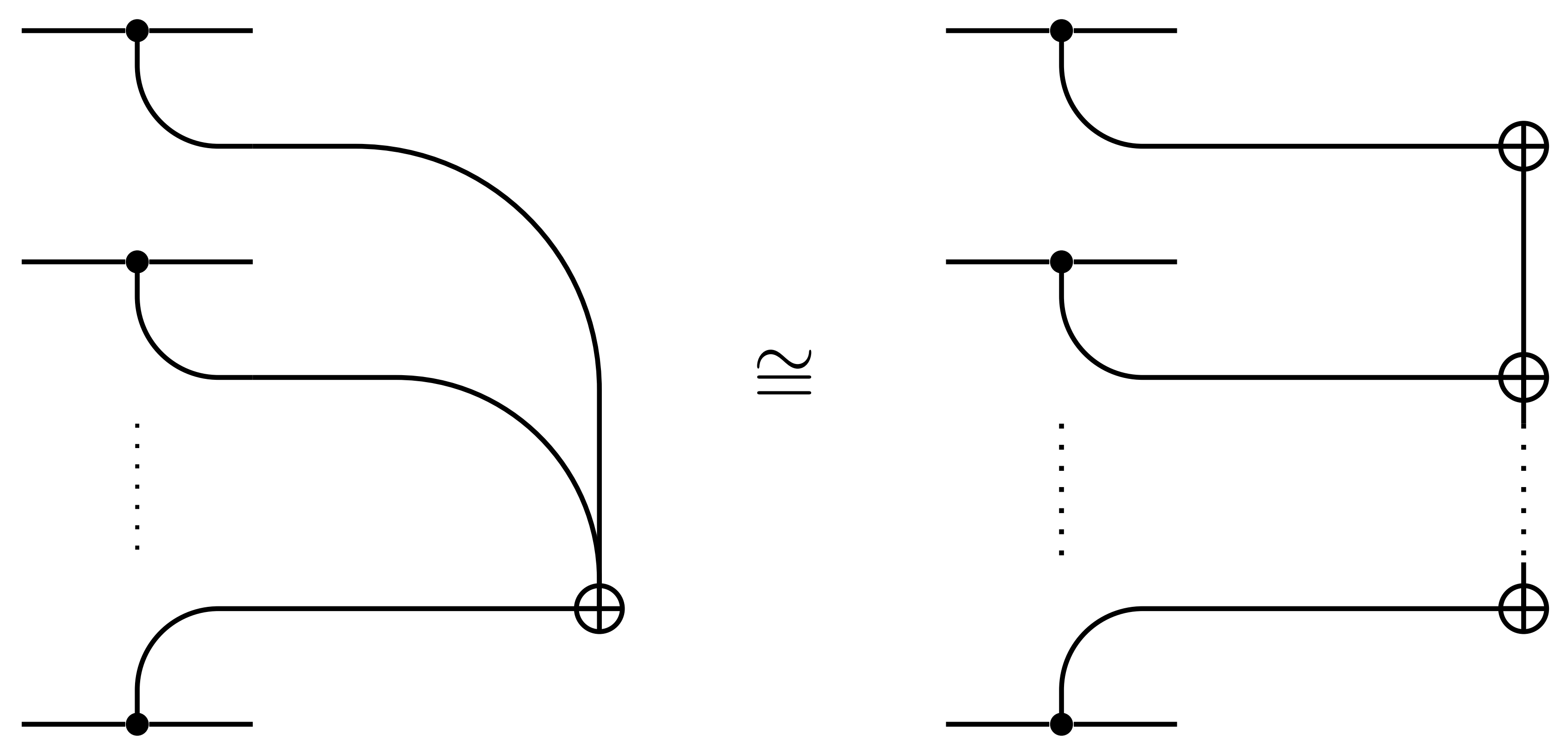


Topological defects



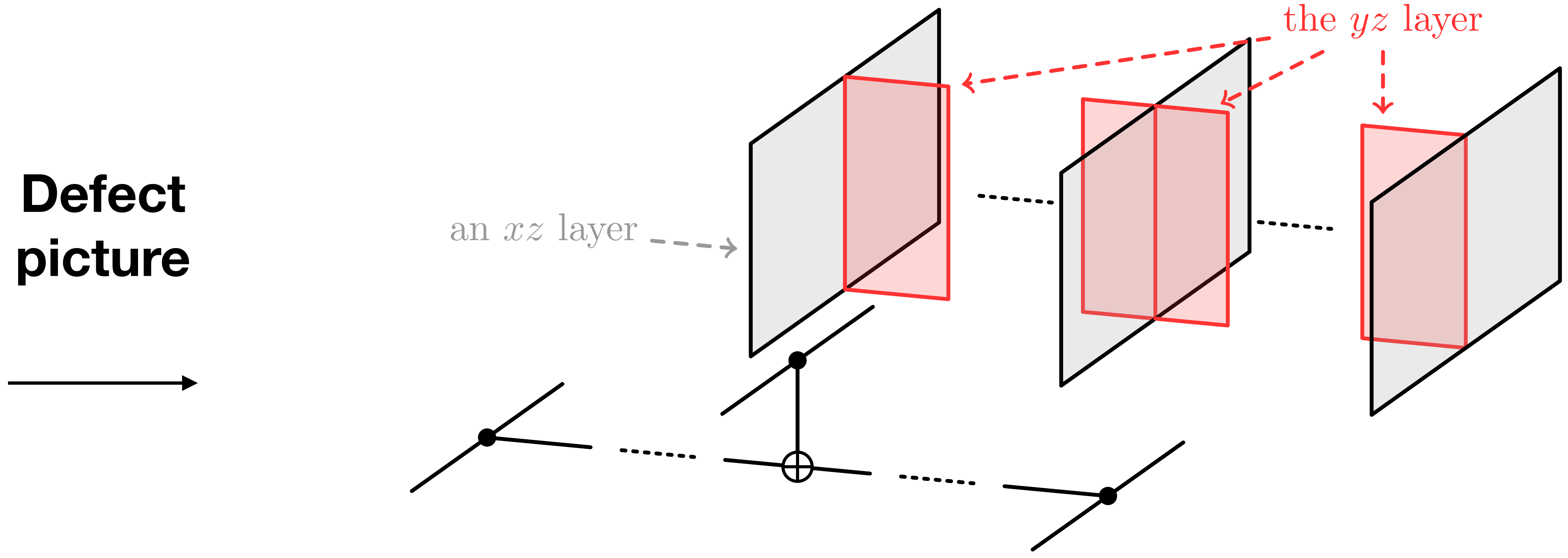
Z-check decomposition

Topological defects

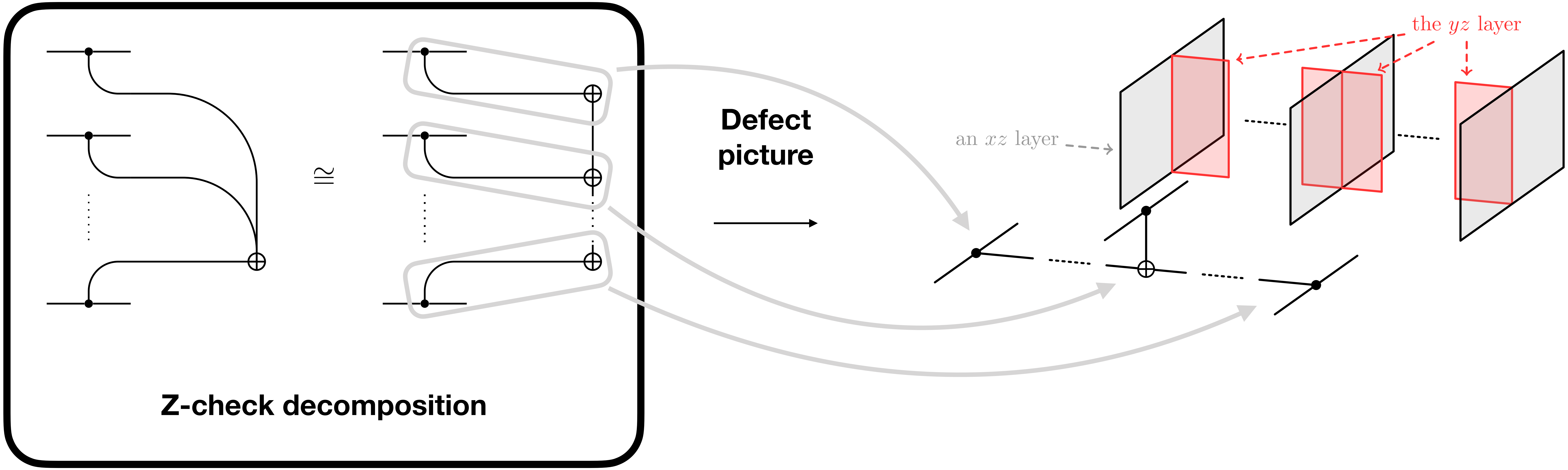


Z-check decomposition

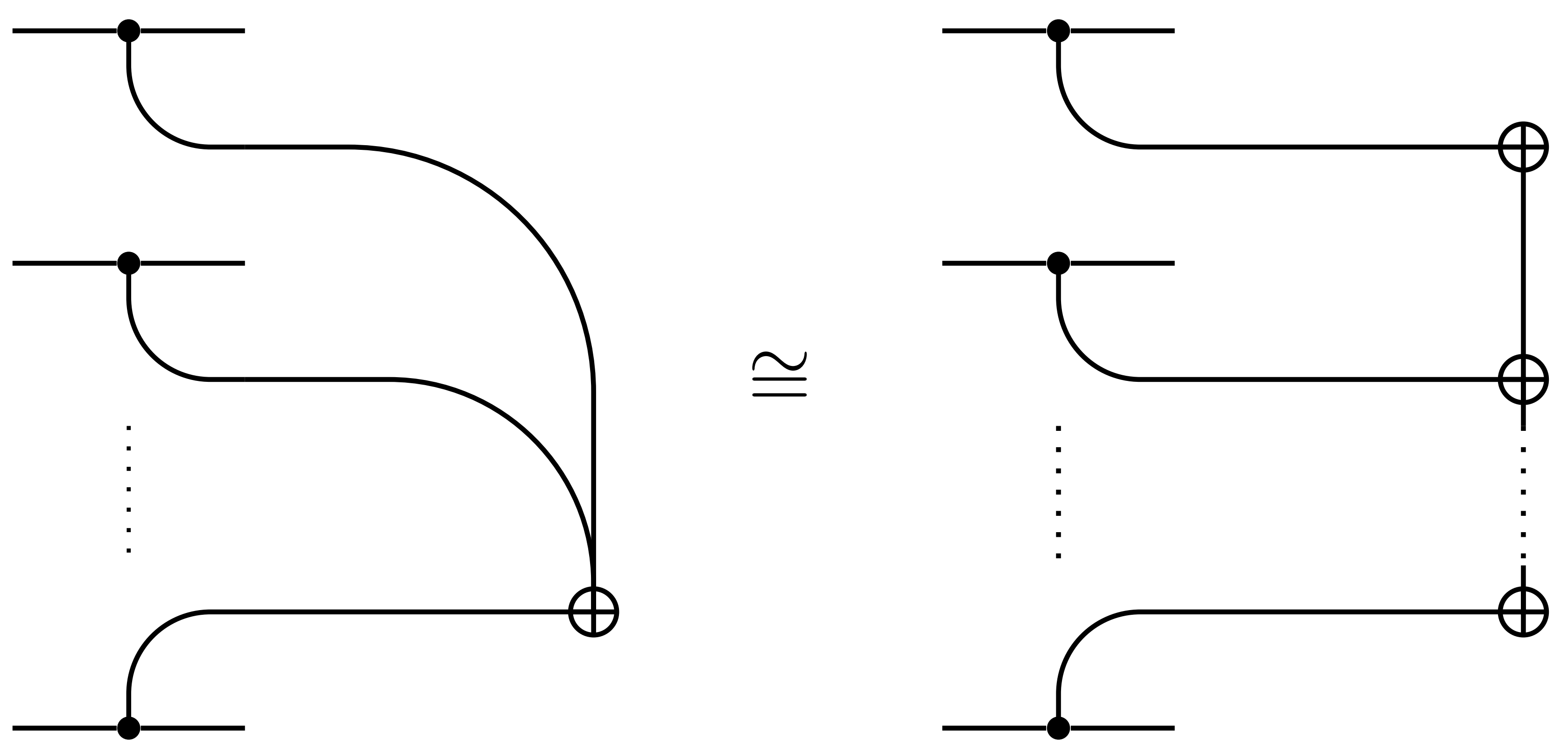
Defect picture



Topological defects

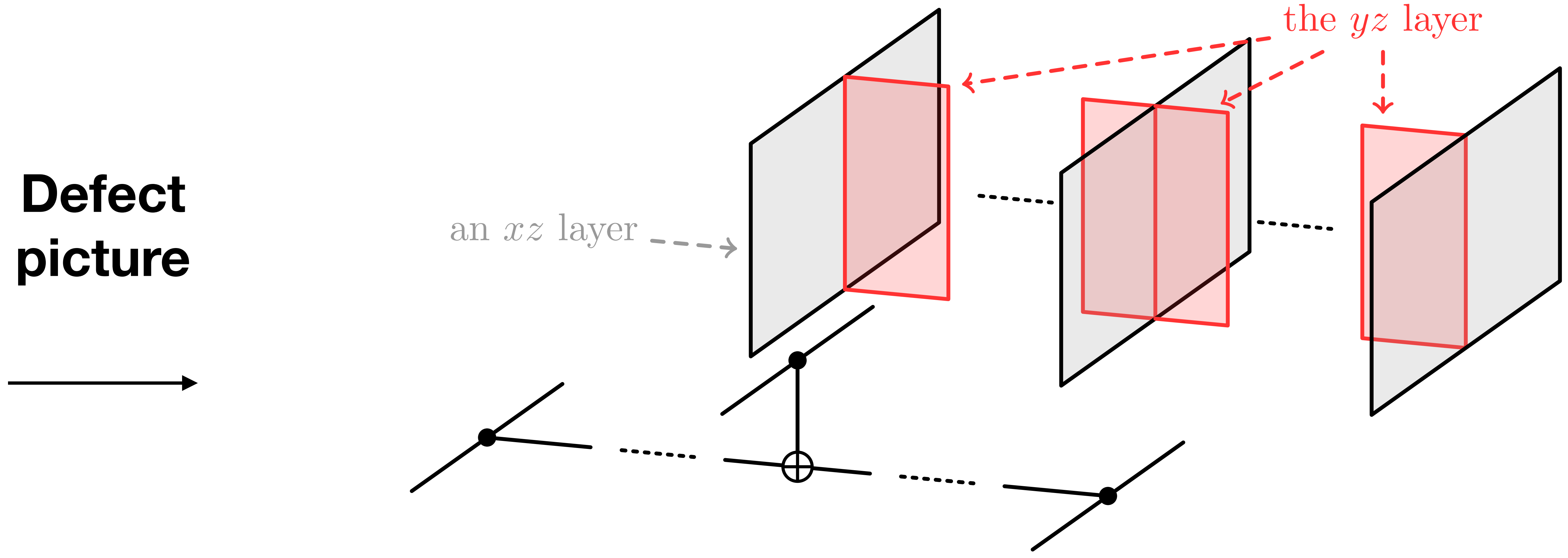


Topological defects

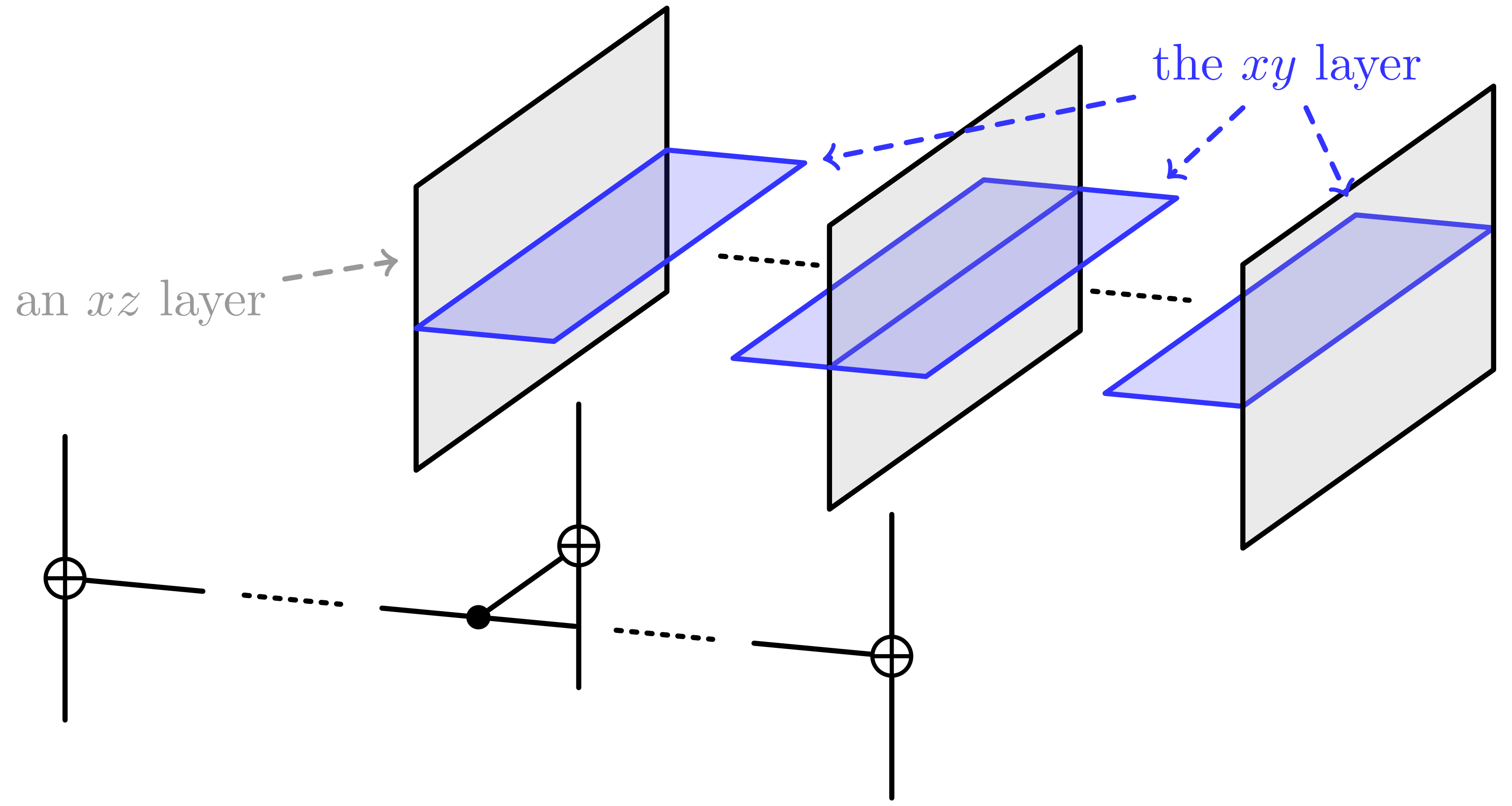


Z-check decomposition

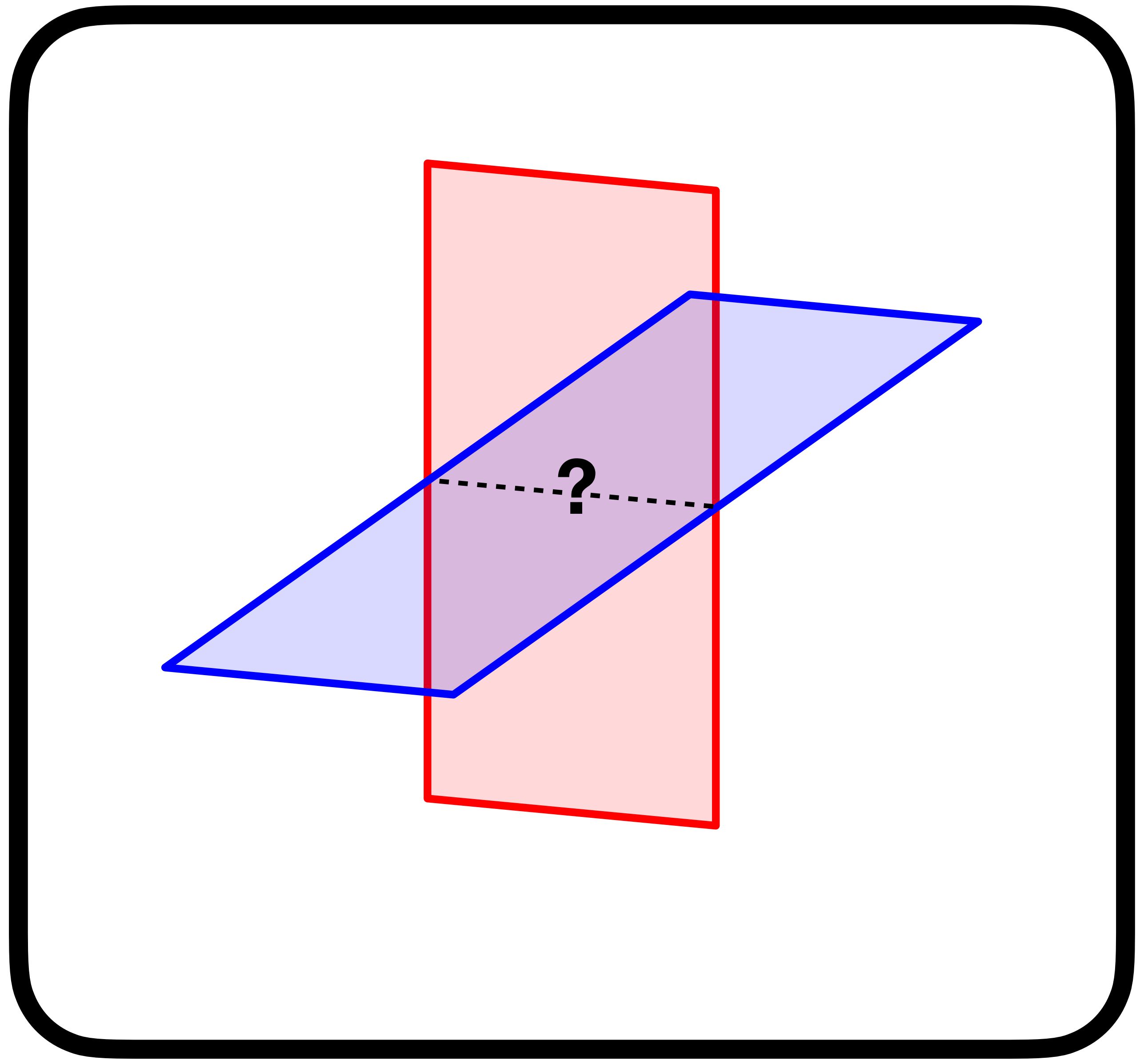
Defect picture



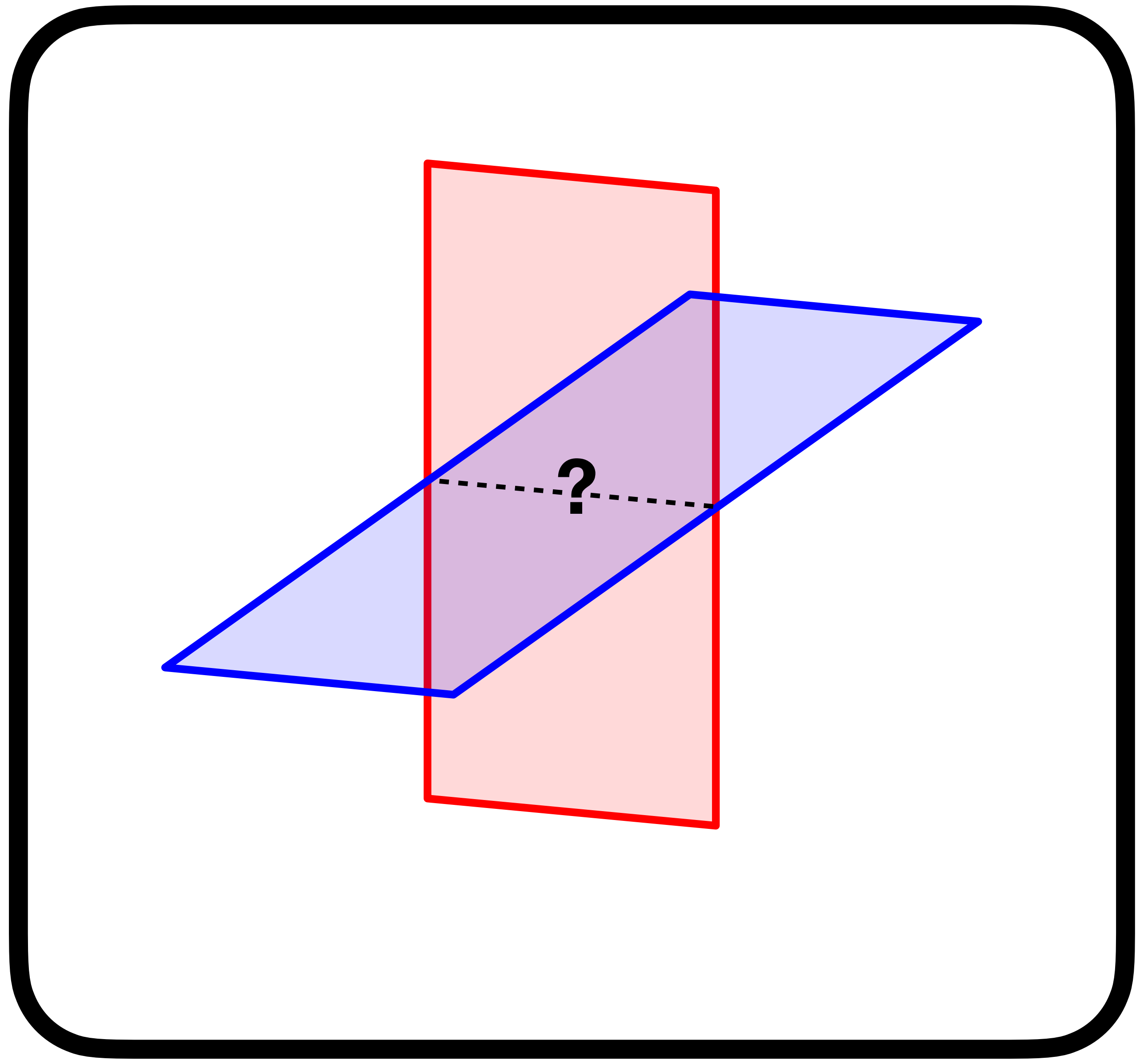
Similar for X-check defect

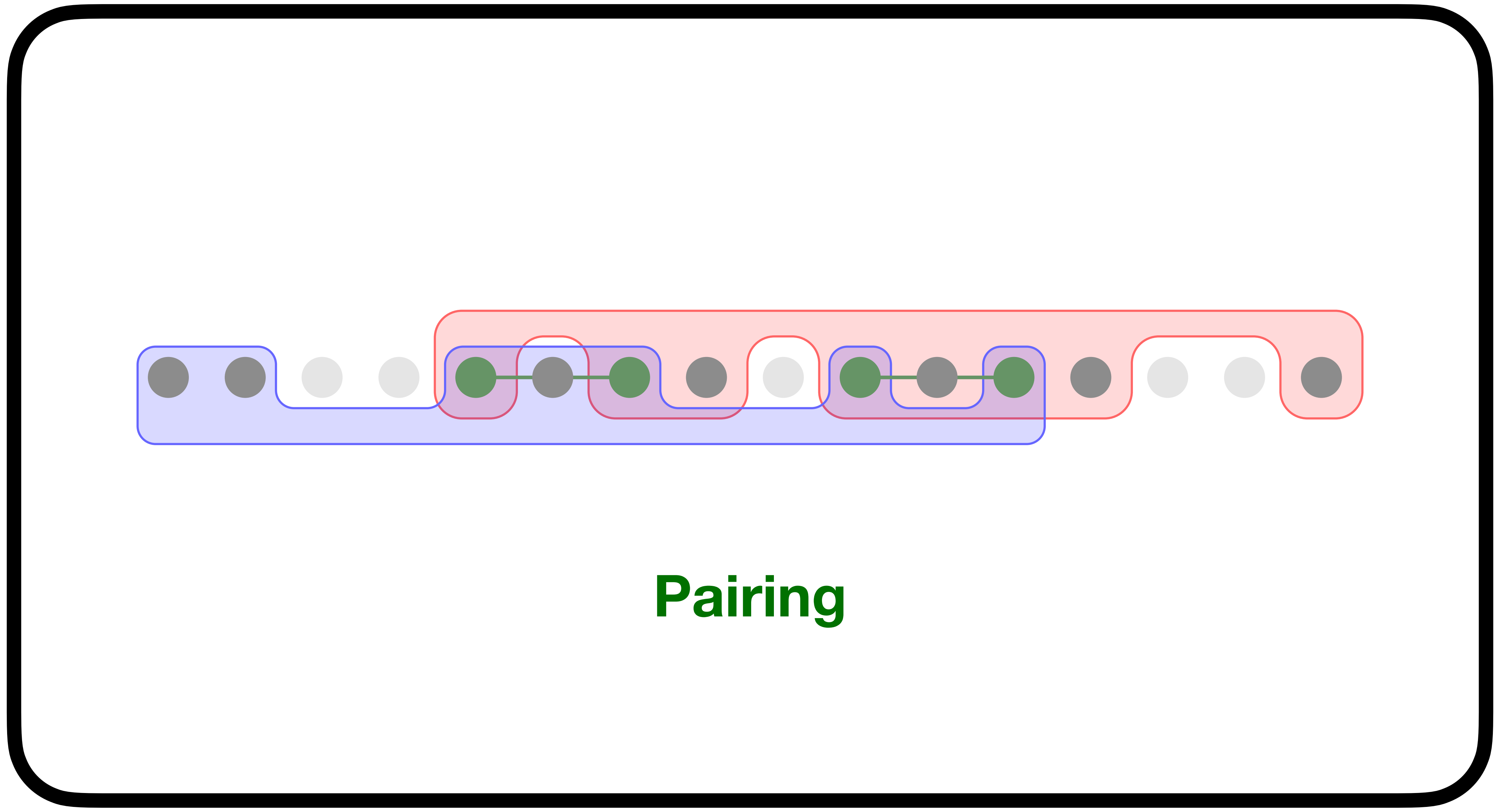


Intersection line defects

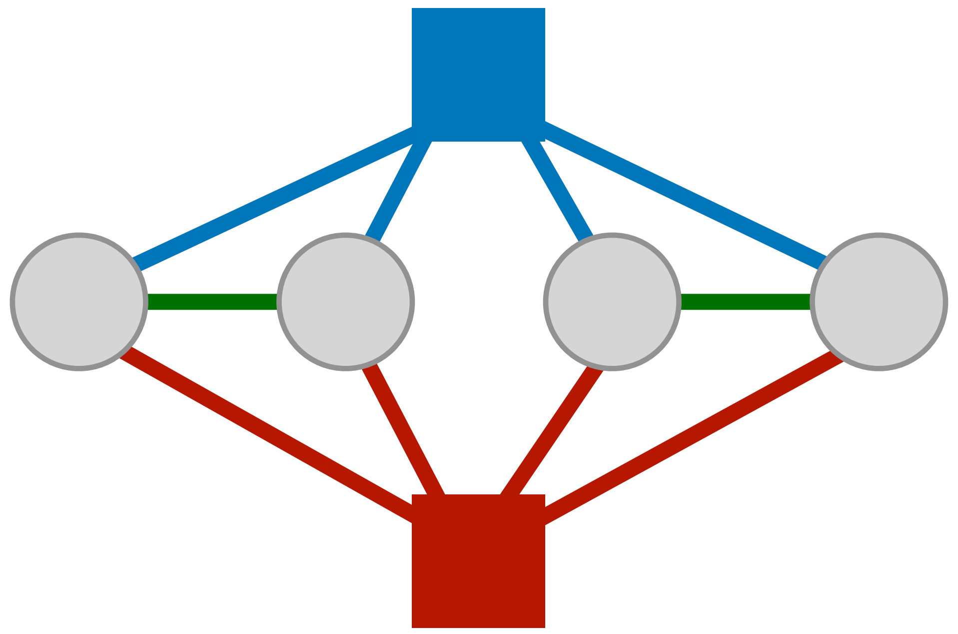


Intersection line defects



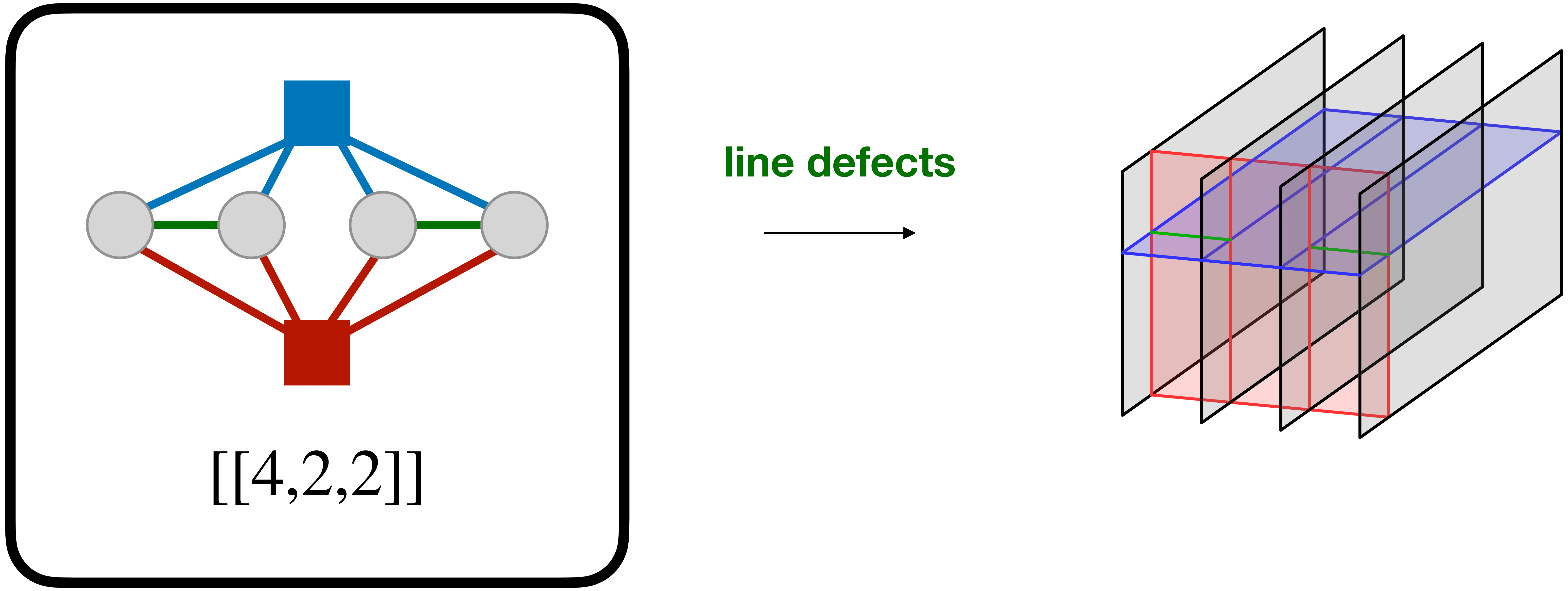


Intersection line defects

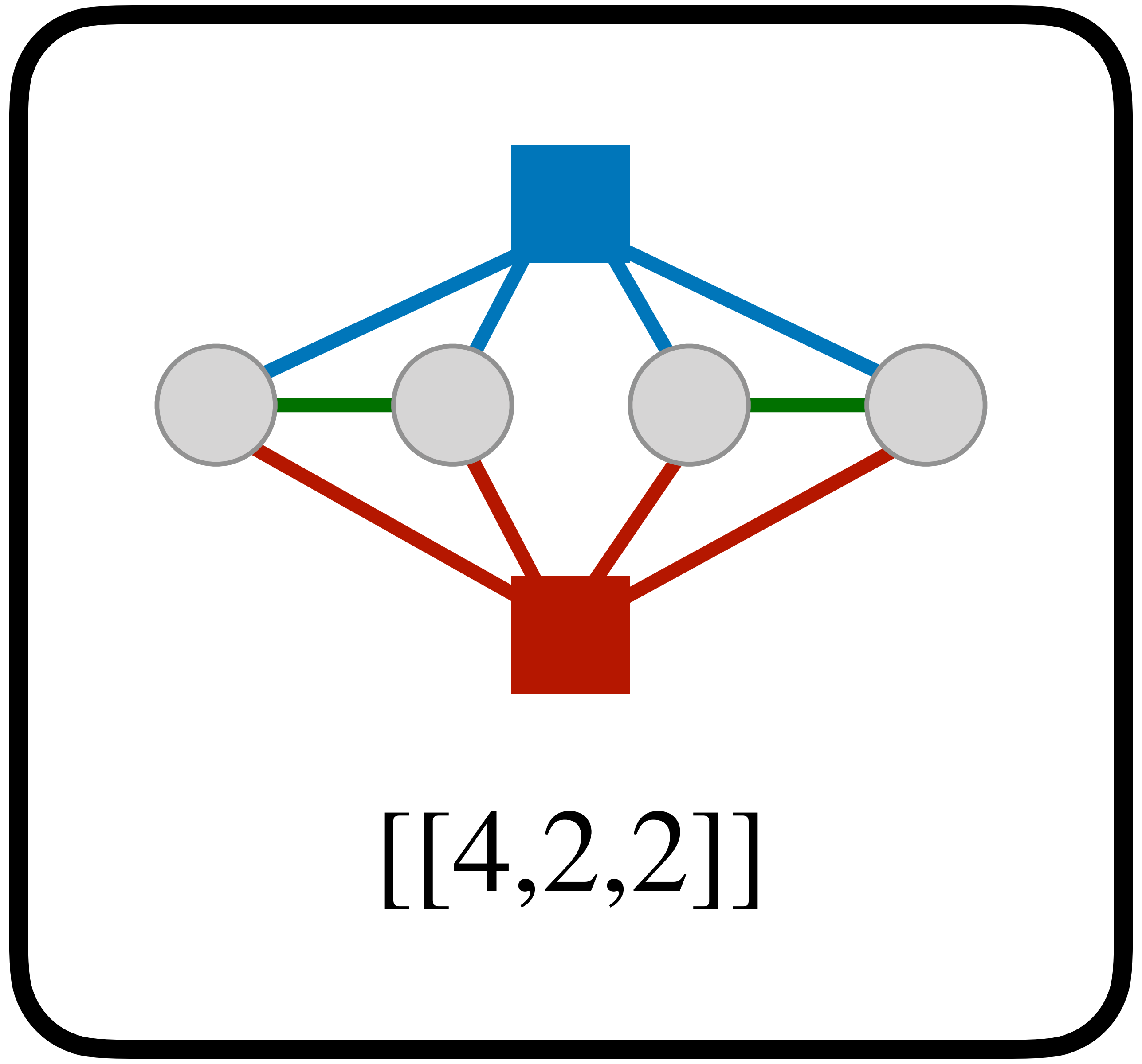


[[4,2,2]]

Intersection line defects

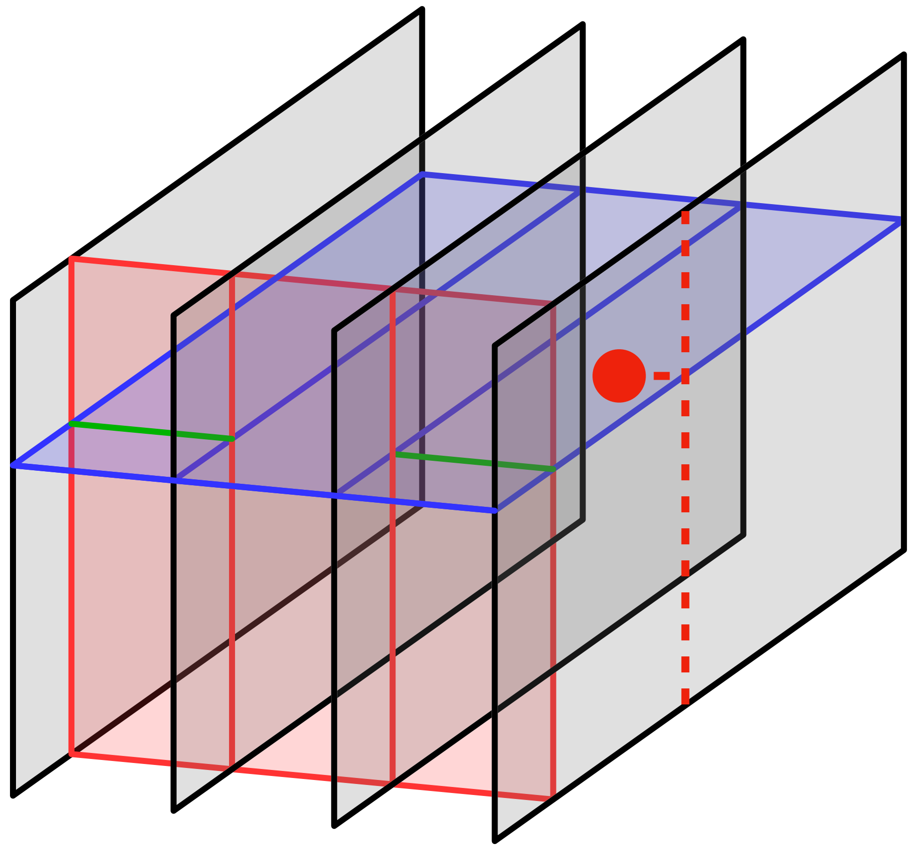


Intersection line defects

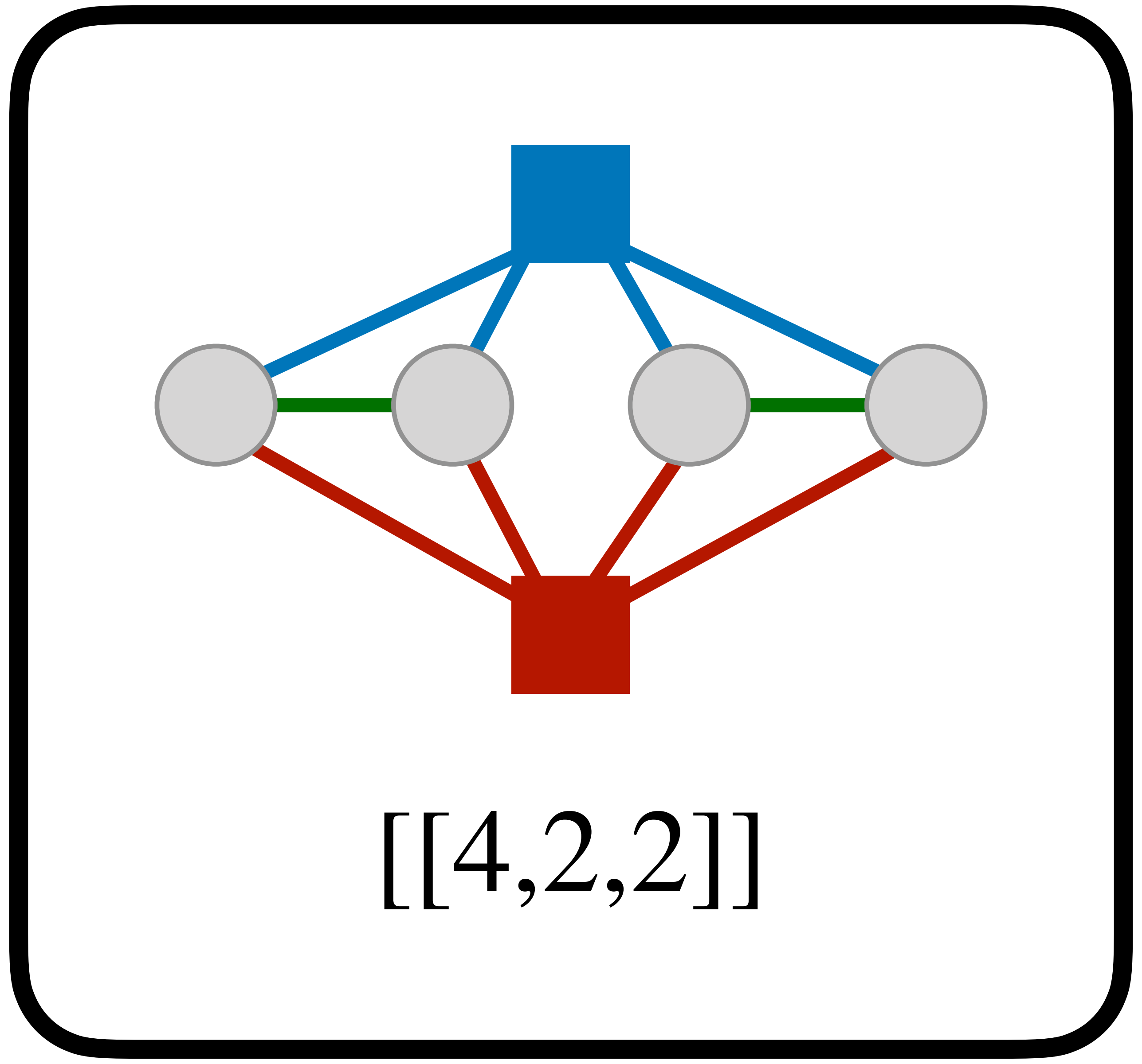


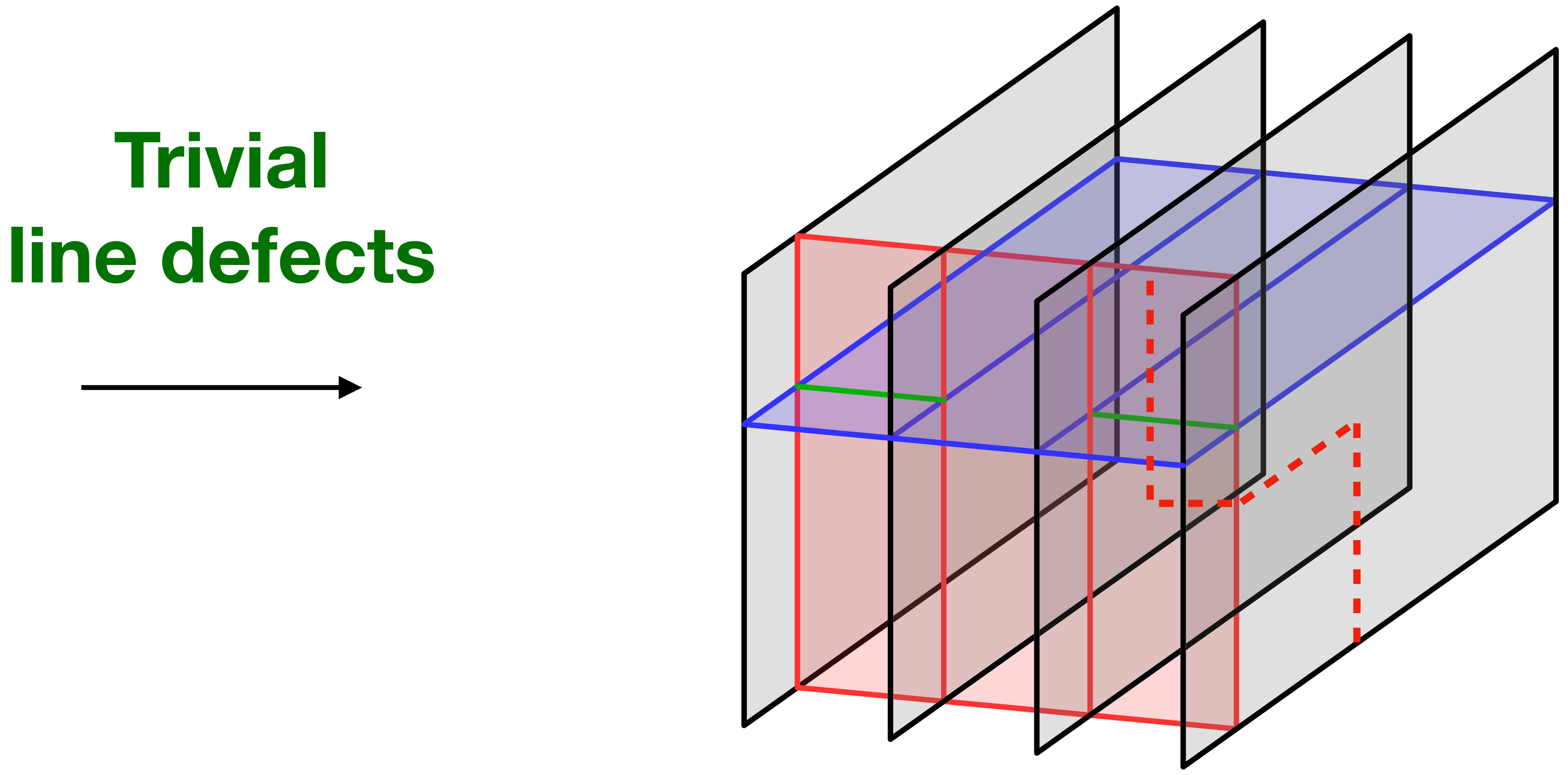
Trivial line defects

→

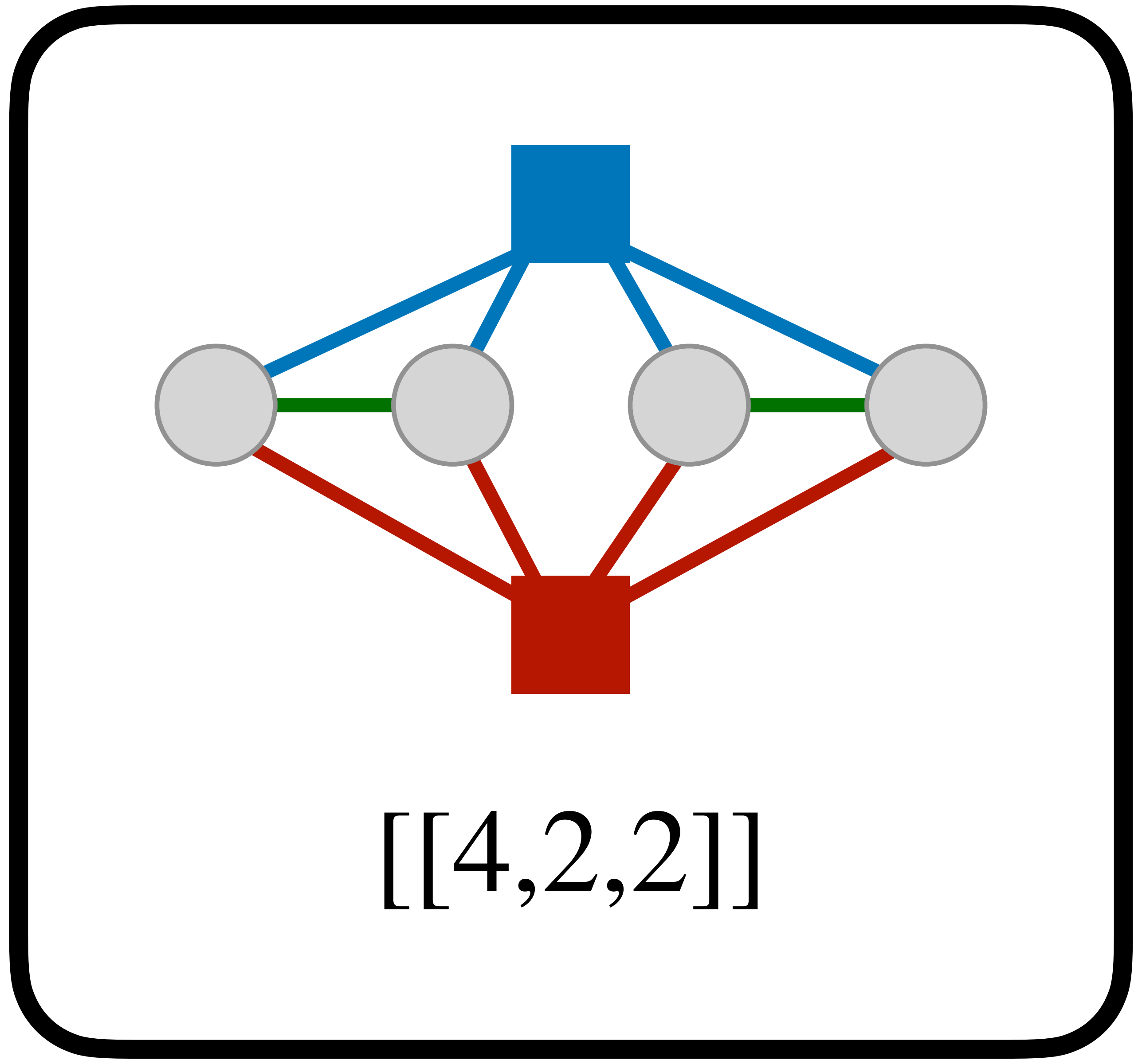


Intersection line defects

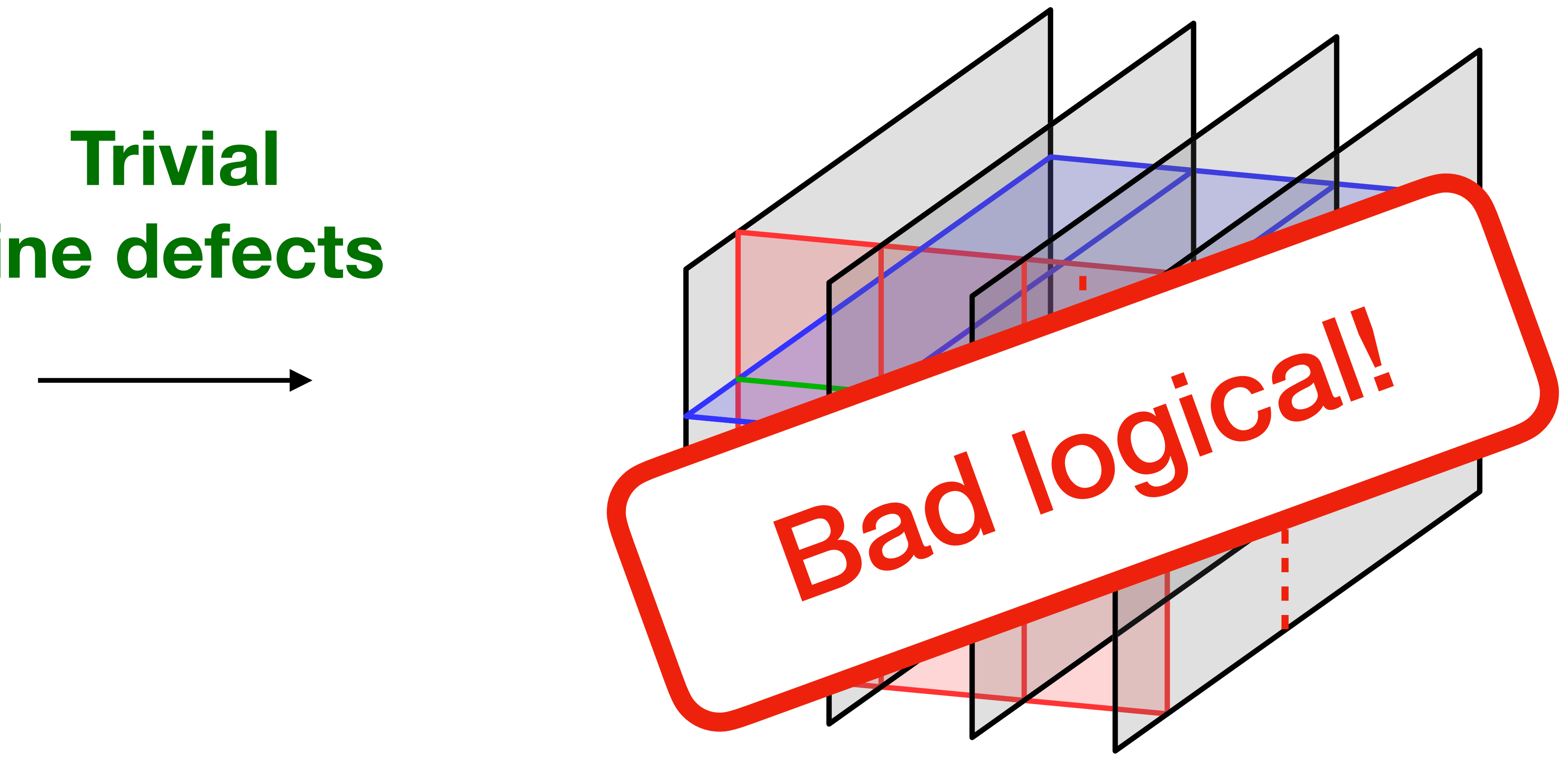




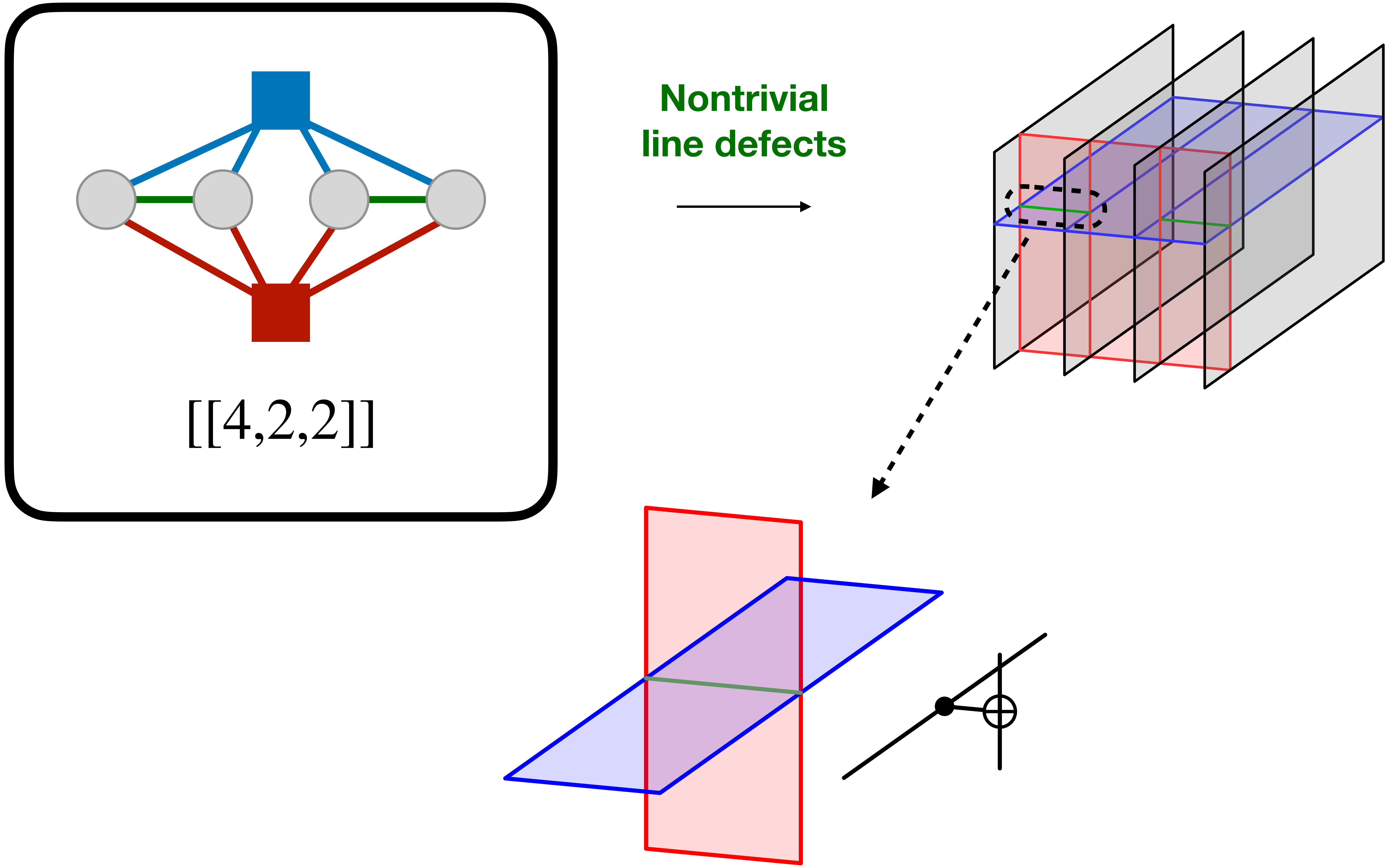
Intersection line defects



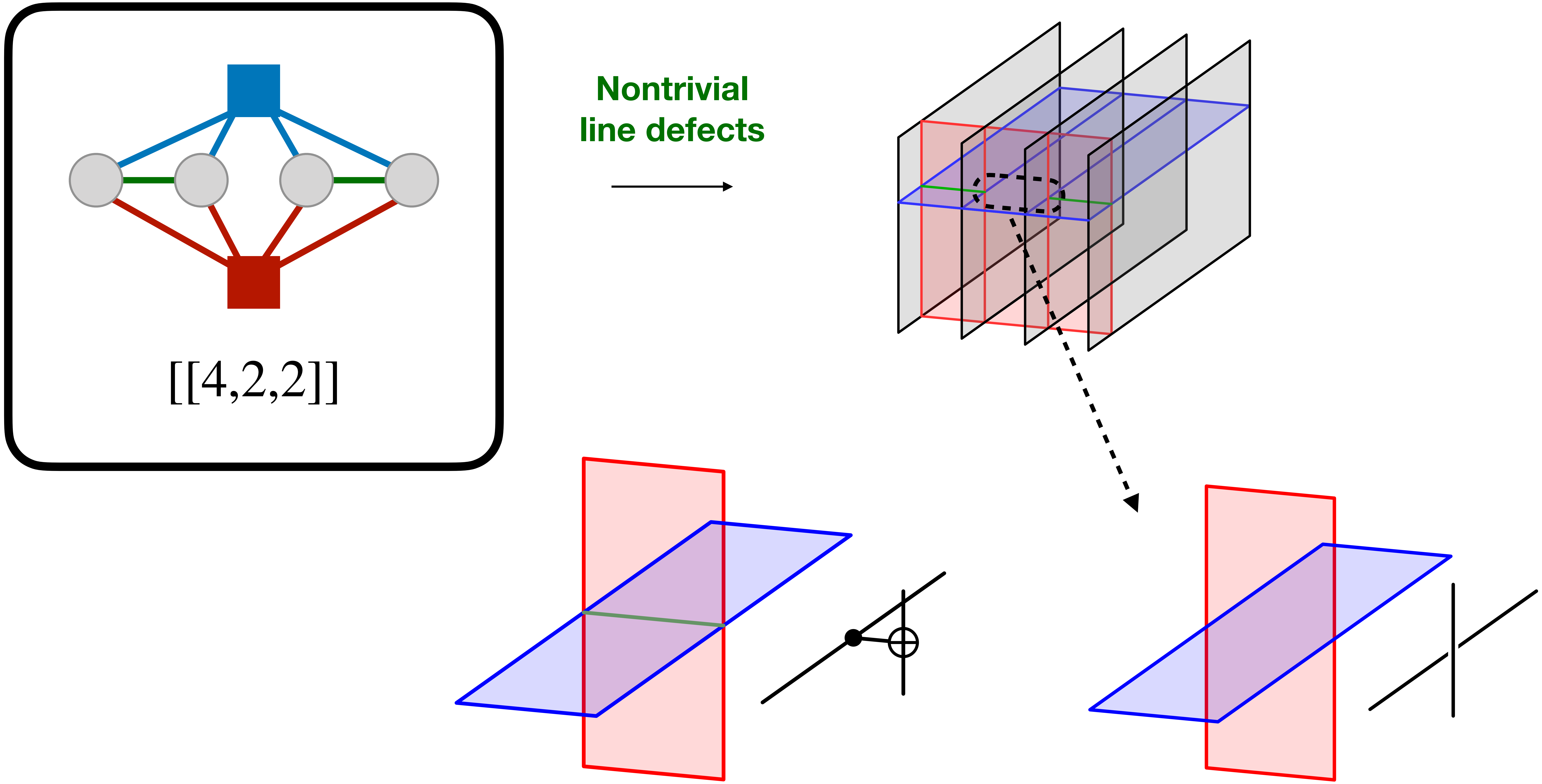
Trivial
line defects



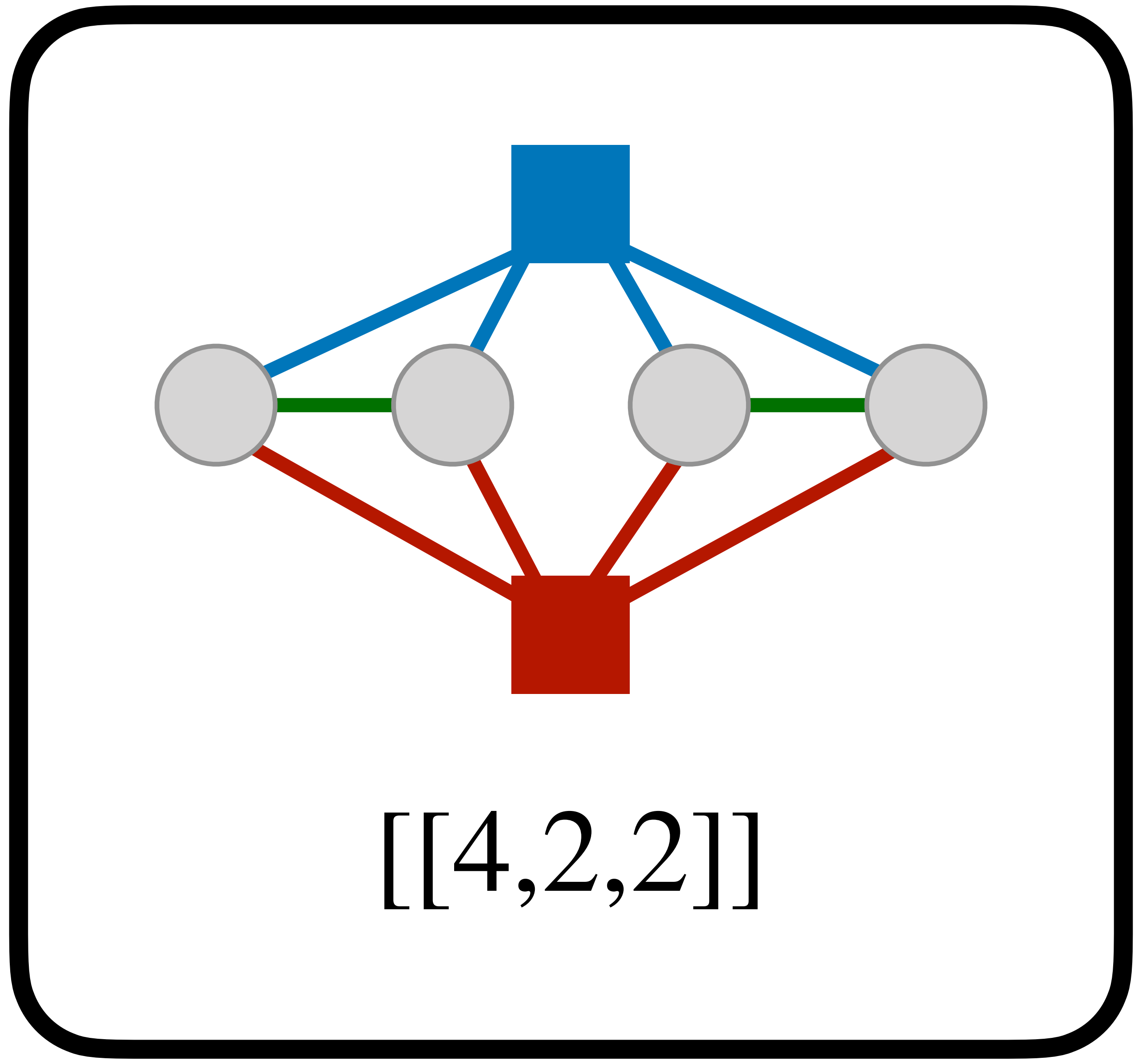
Intersection line defects



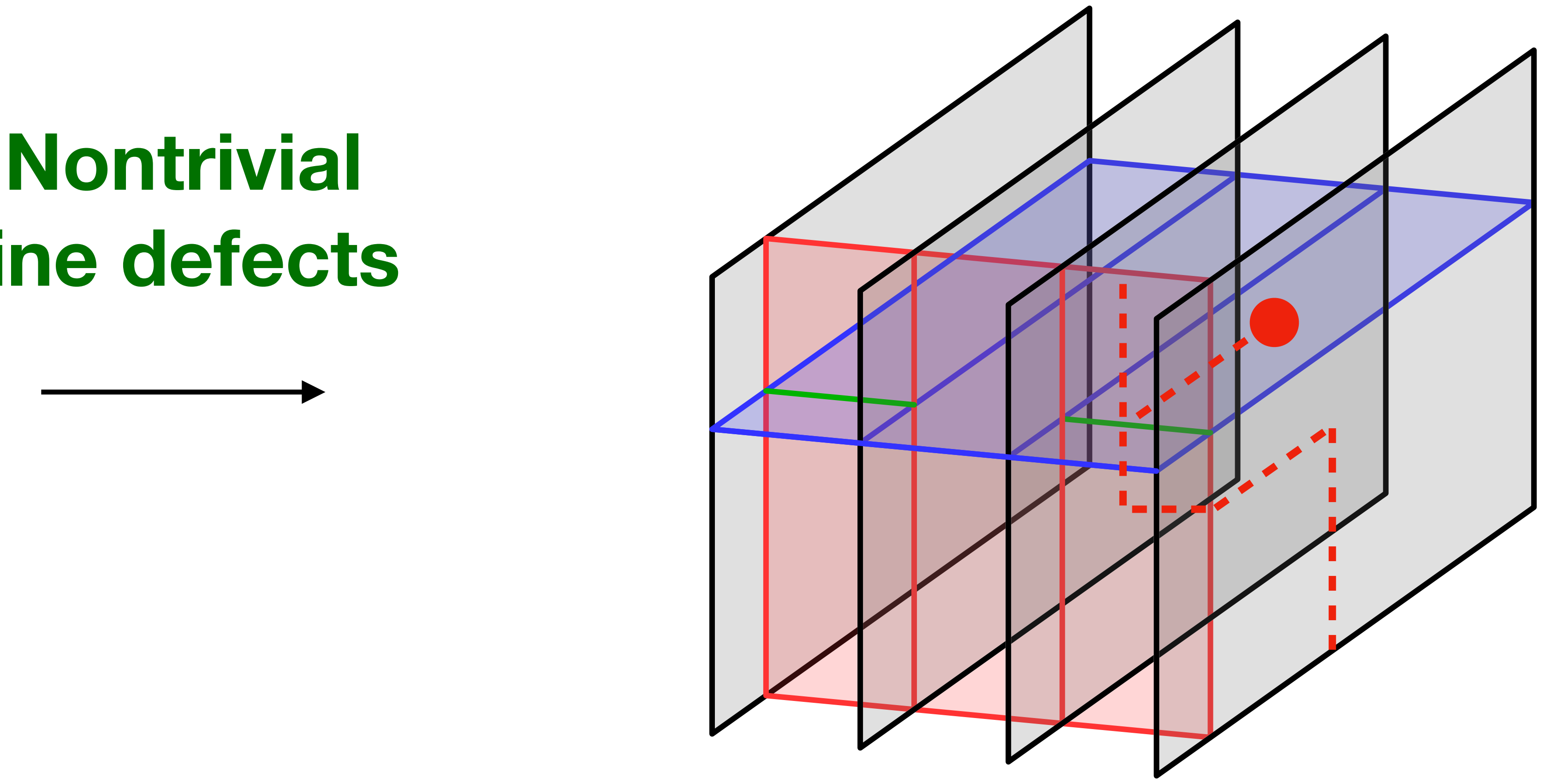
Intersection line defects



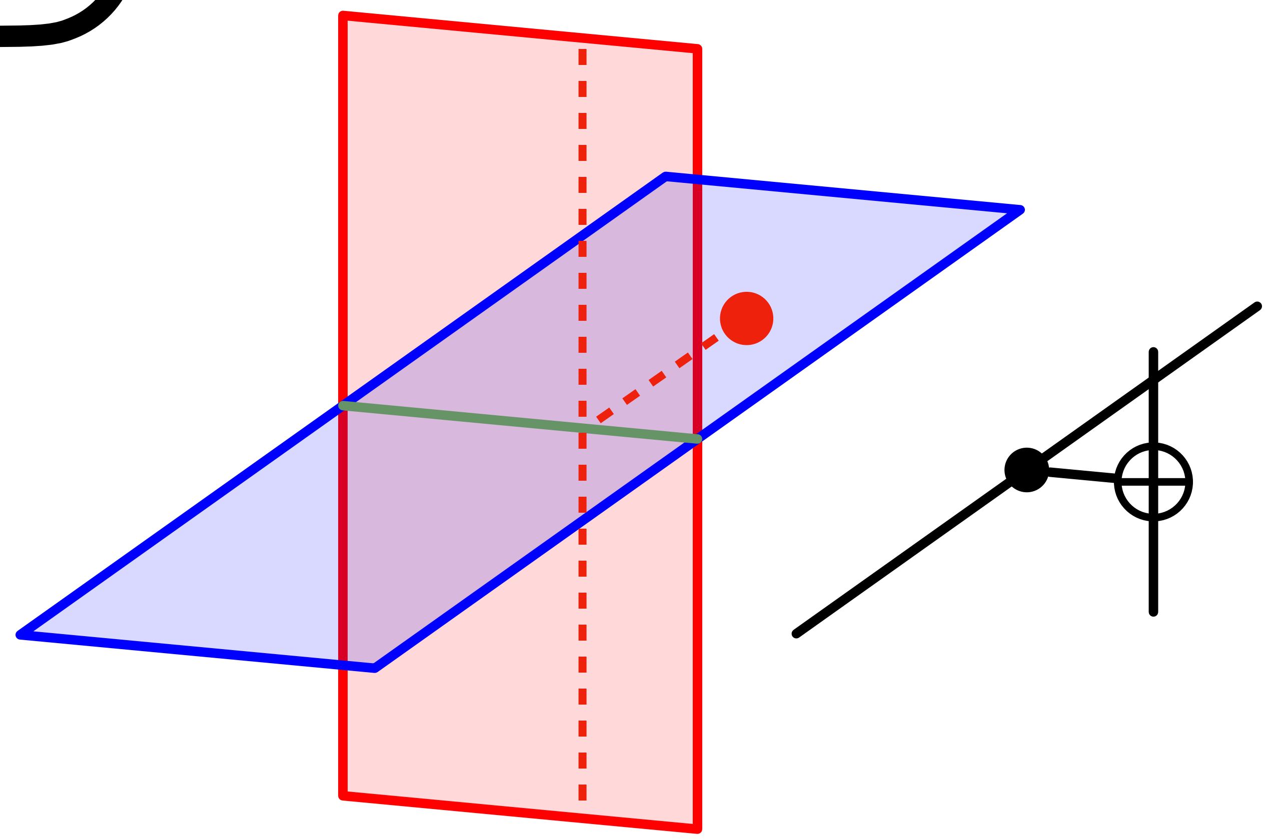
Intersection line defects



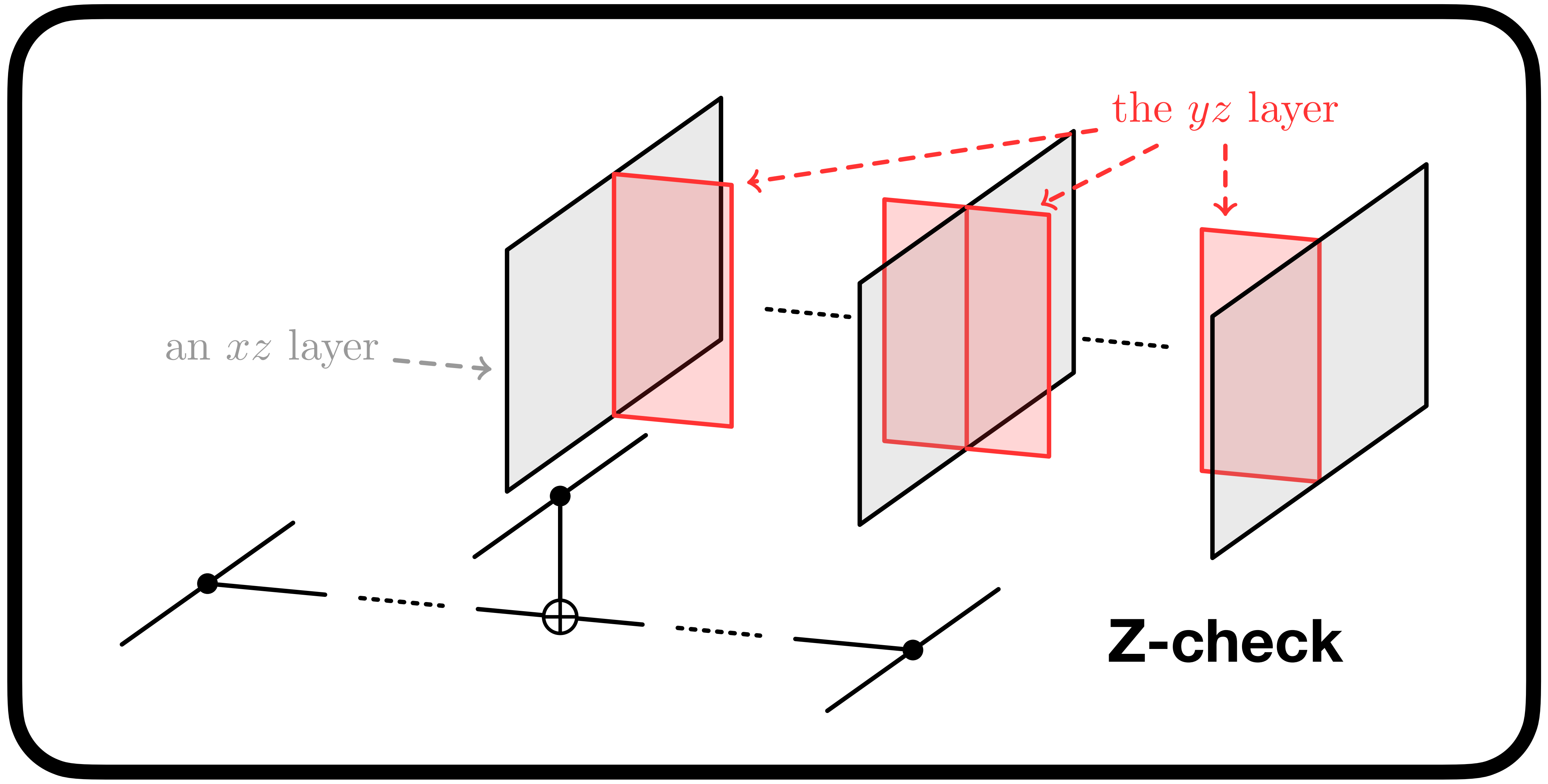
Nontrivial
line defects

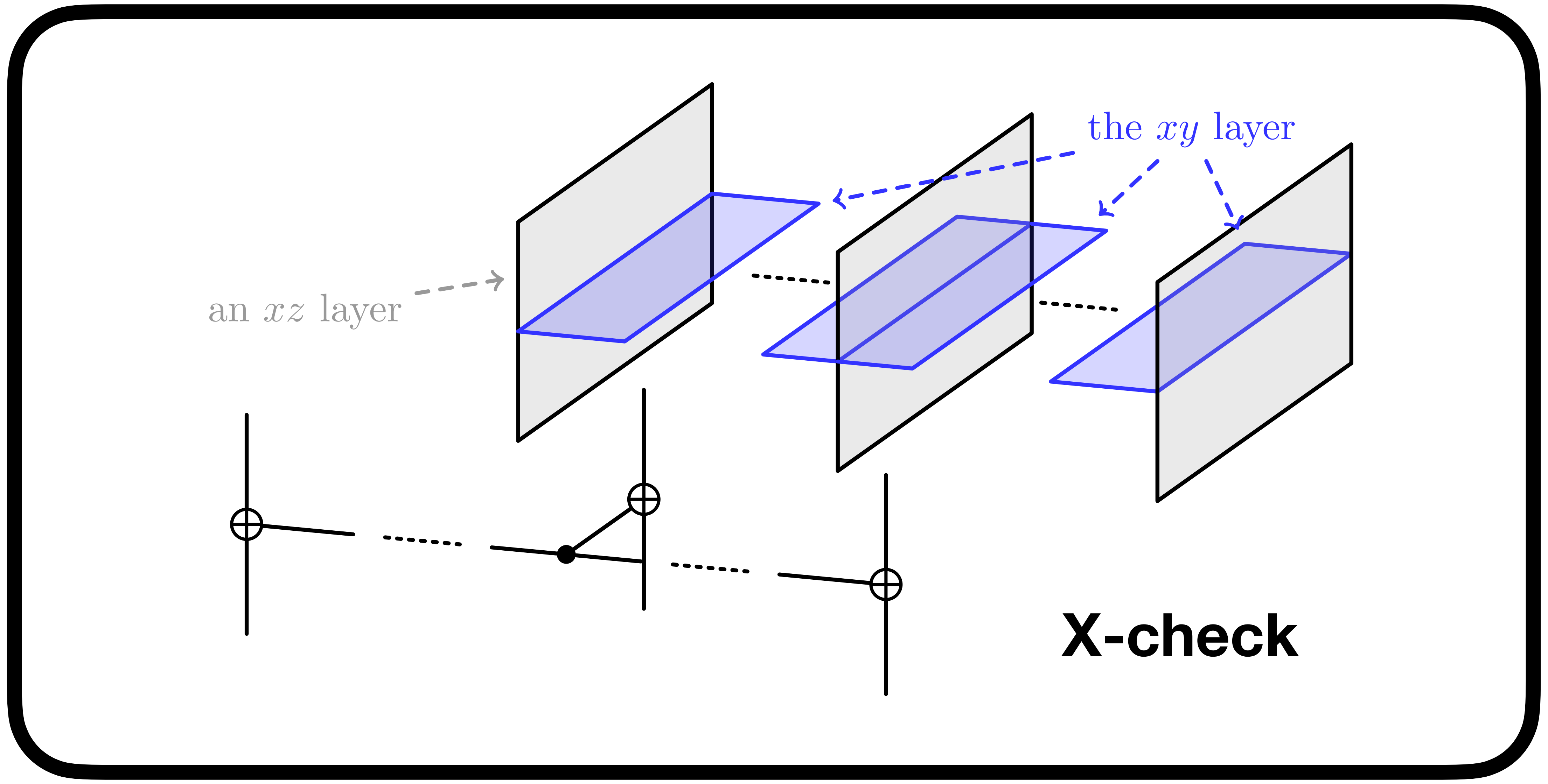


Removes bad logicals!

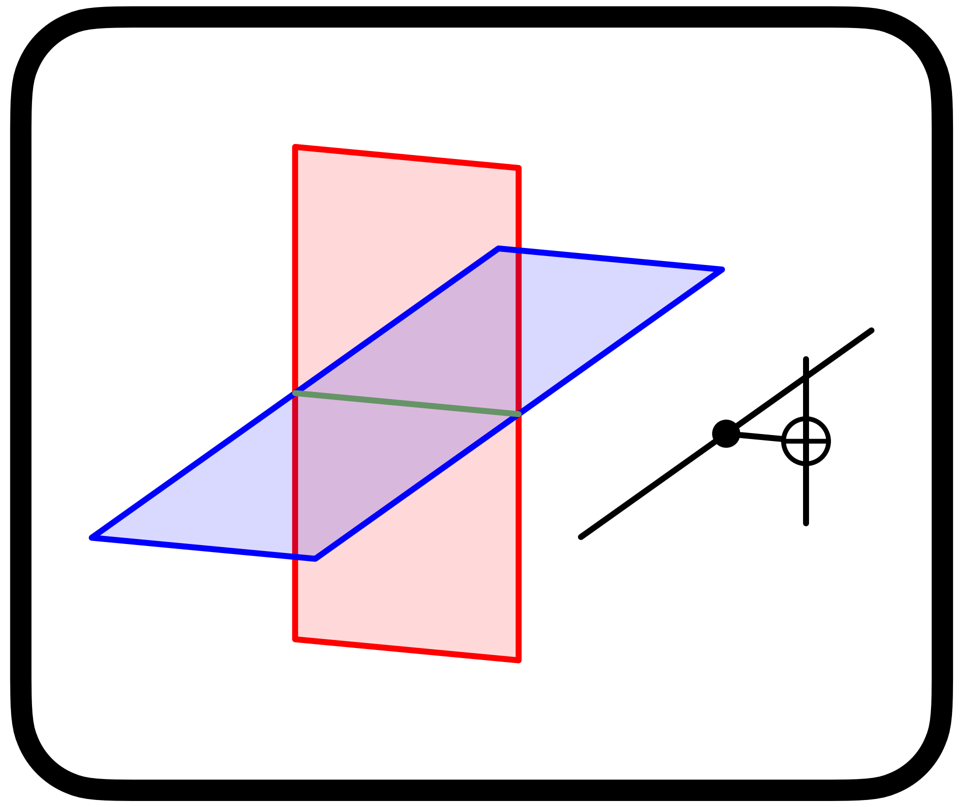


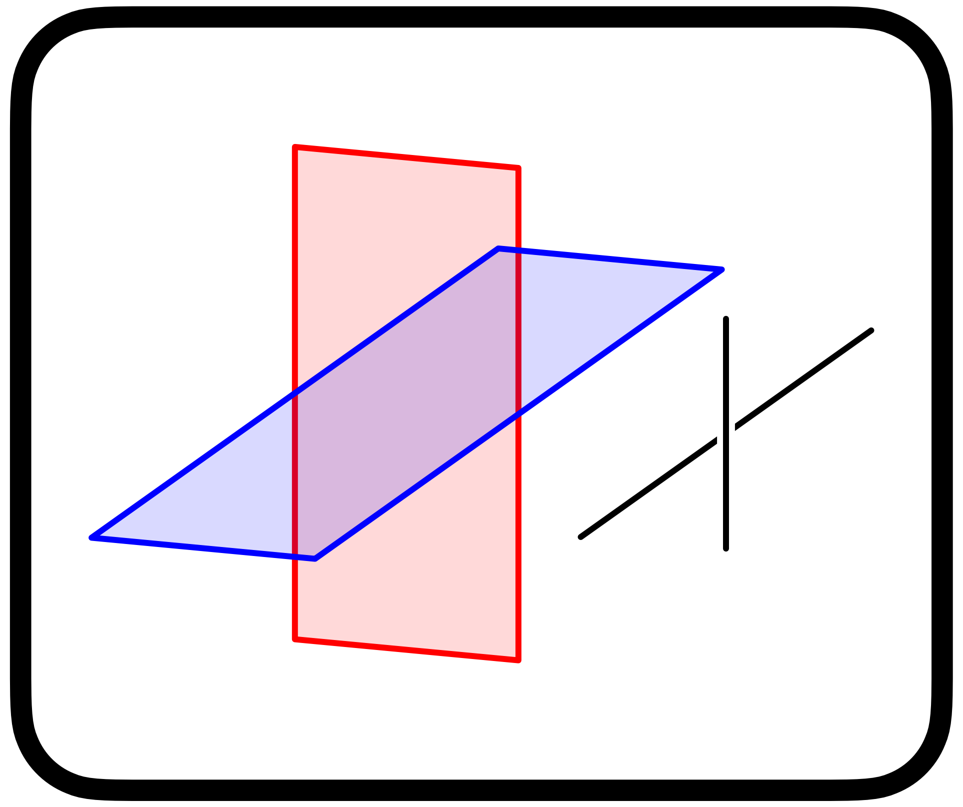
Summary: Layer code line defects



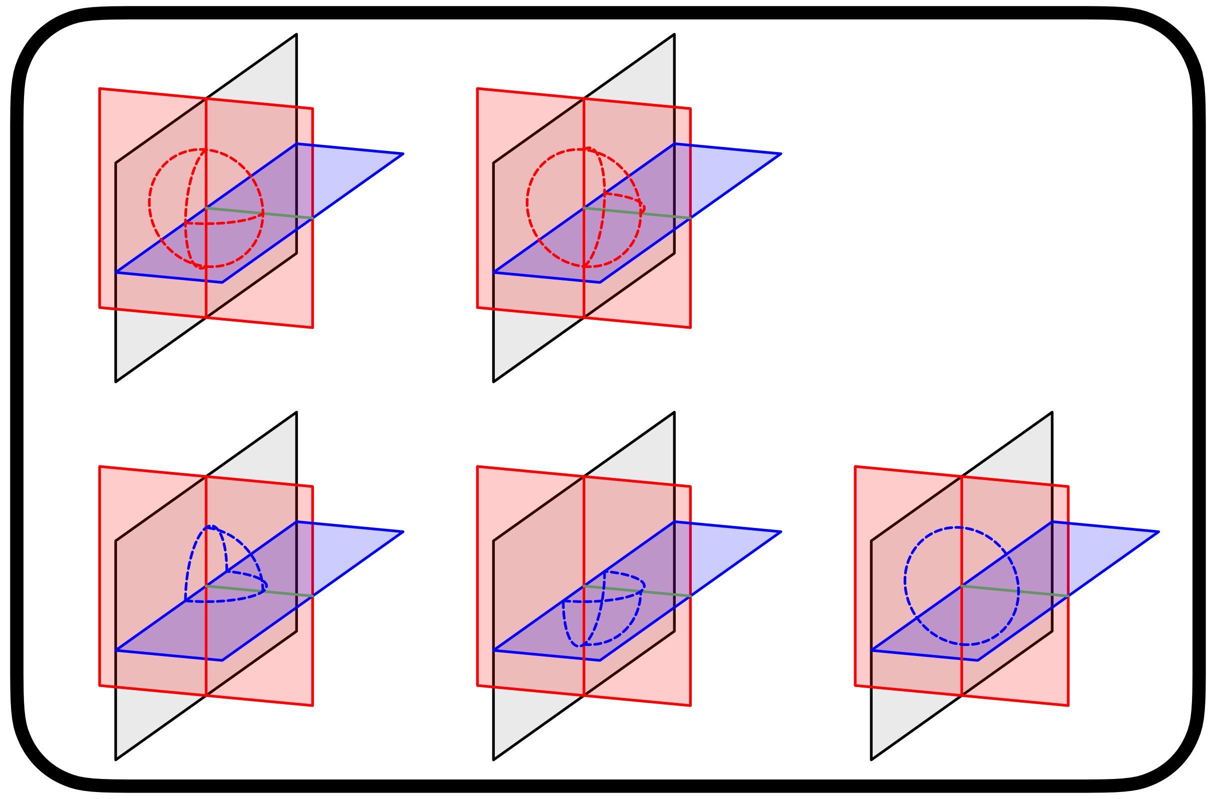


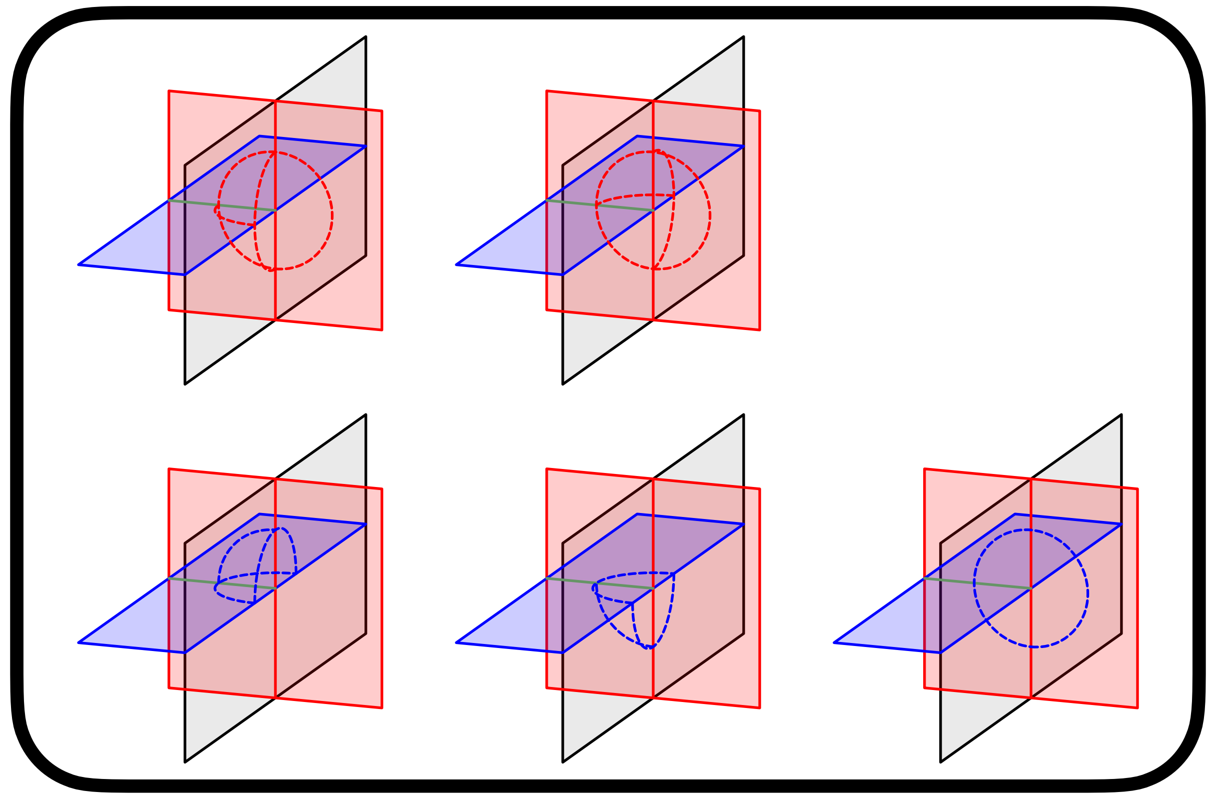
Intersections:

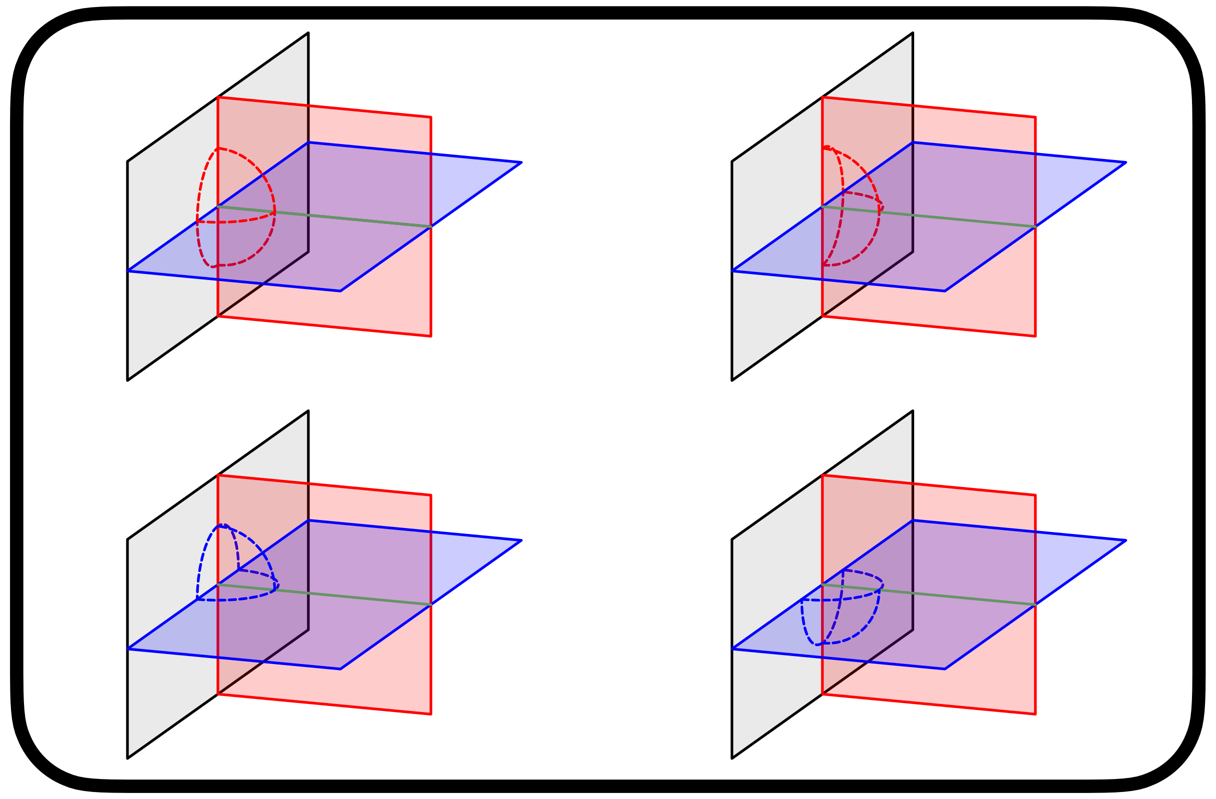


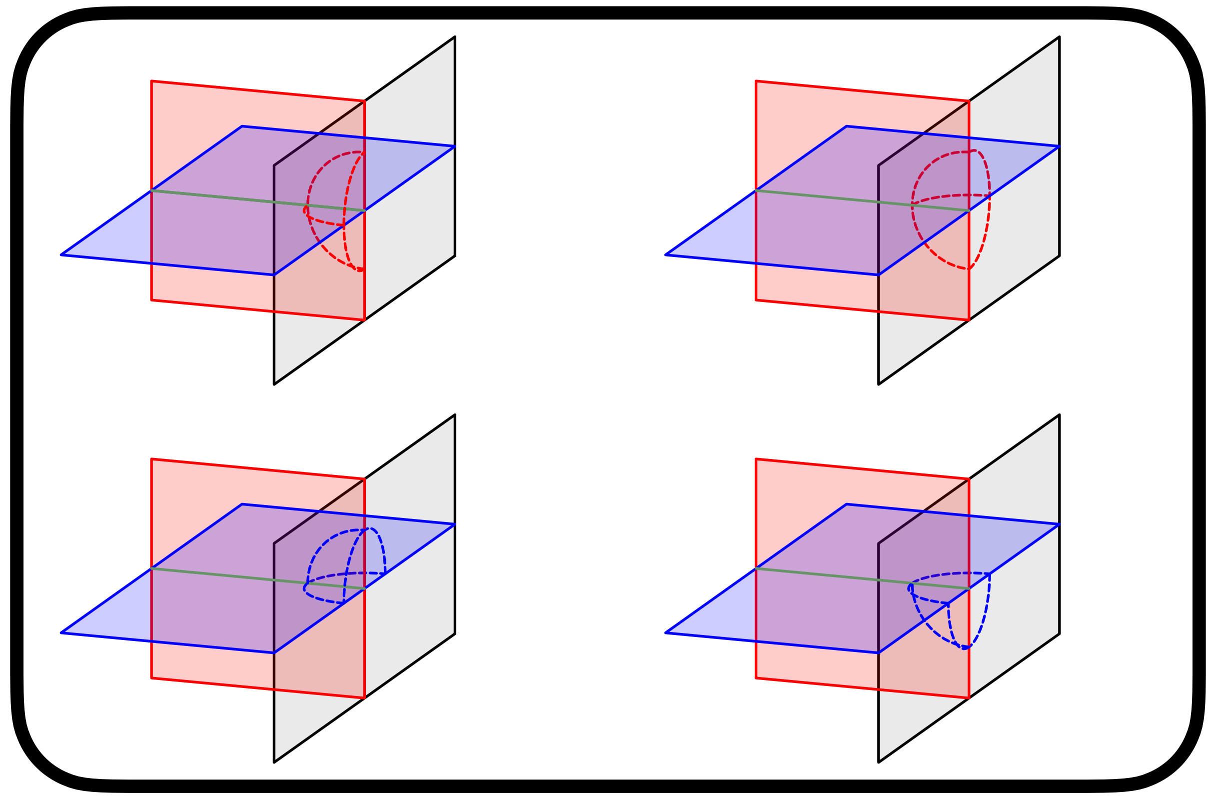


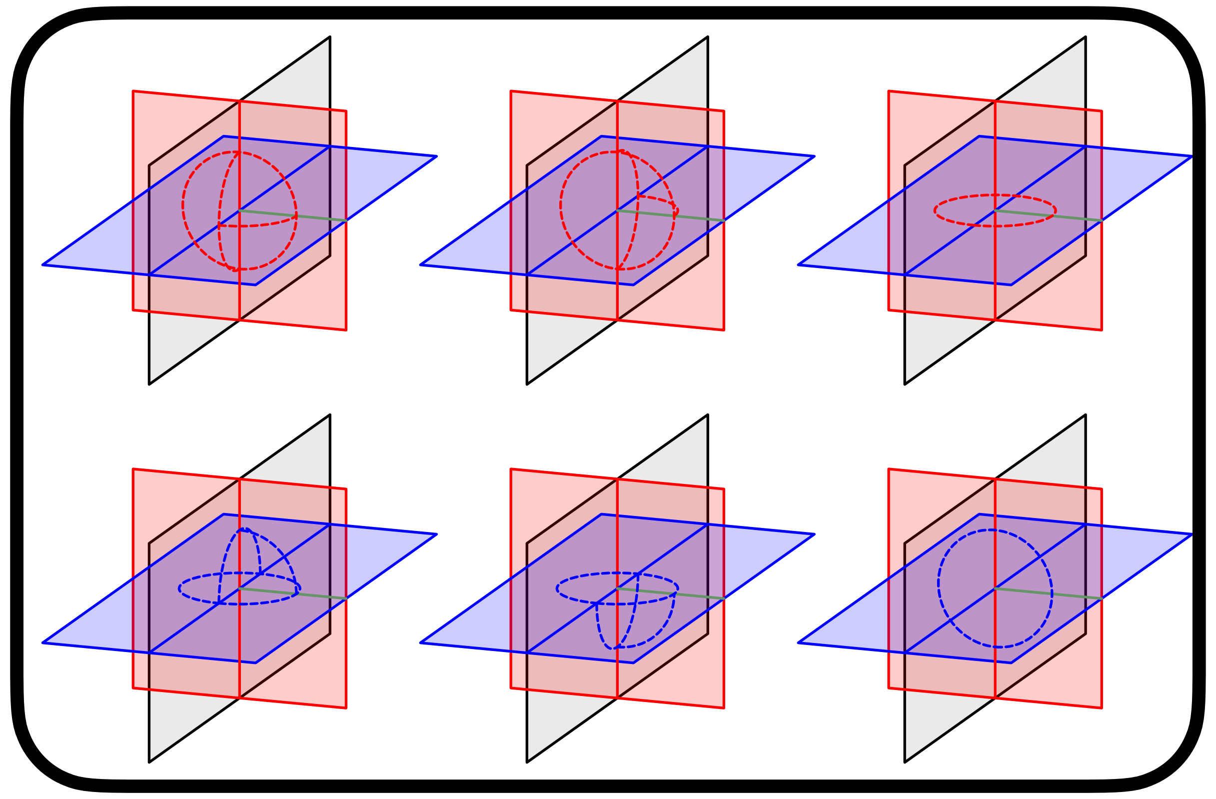
Point defects

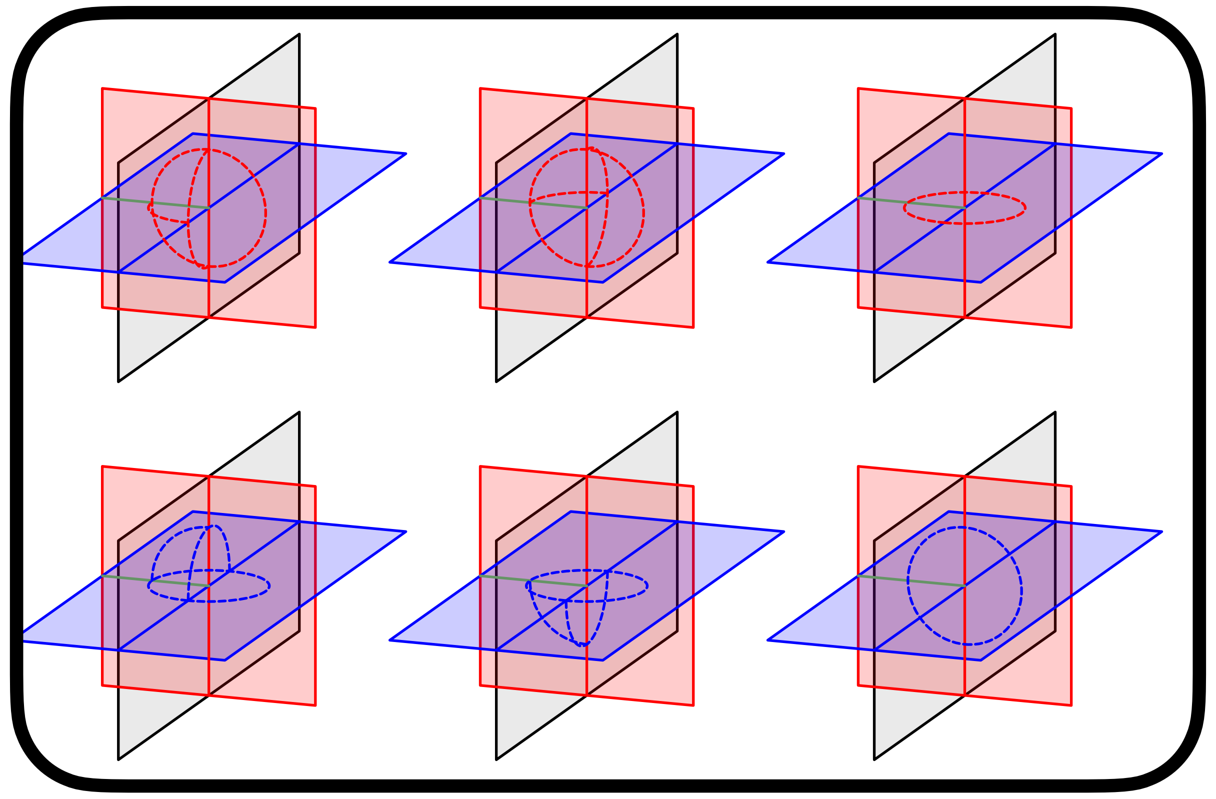


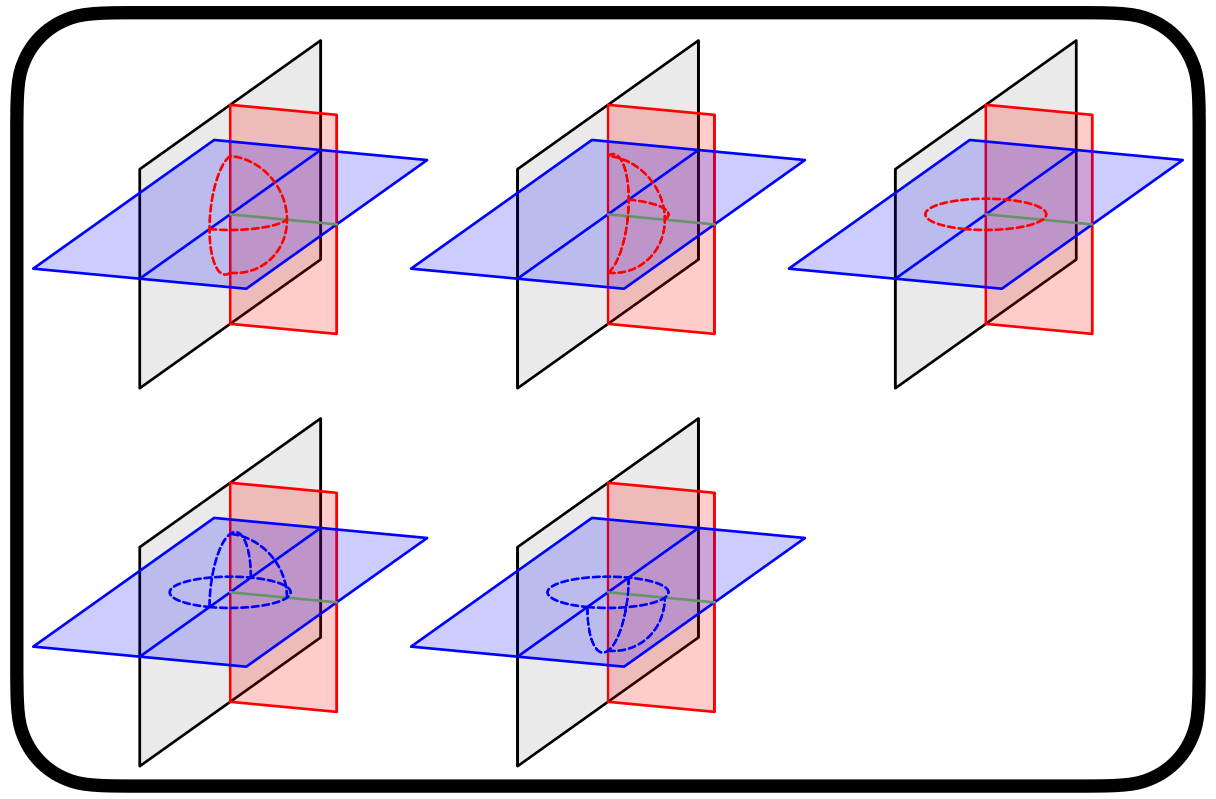


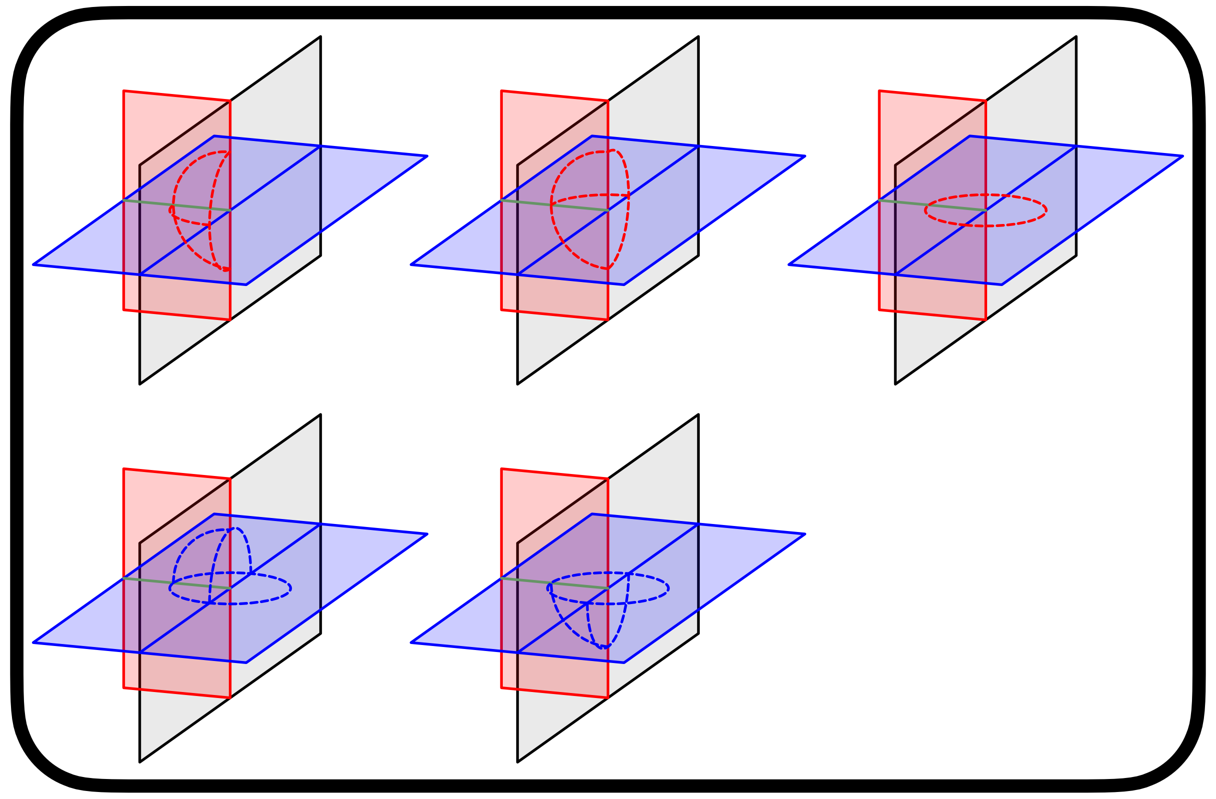




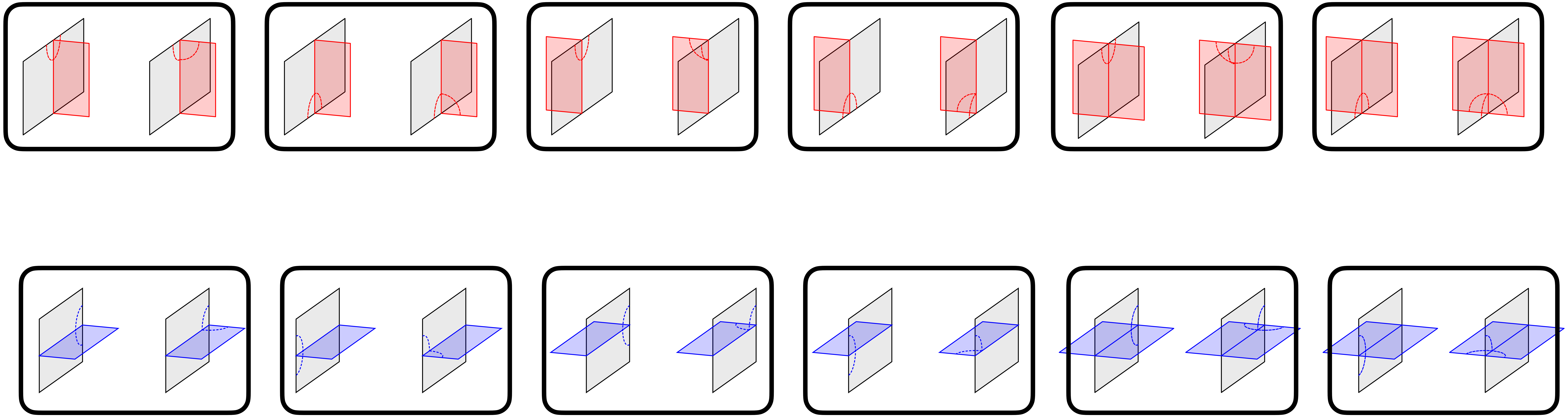




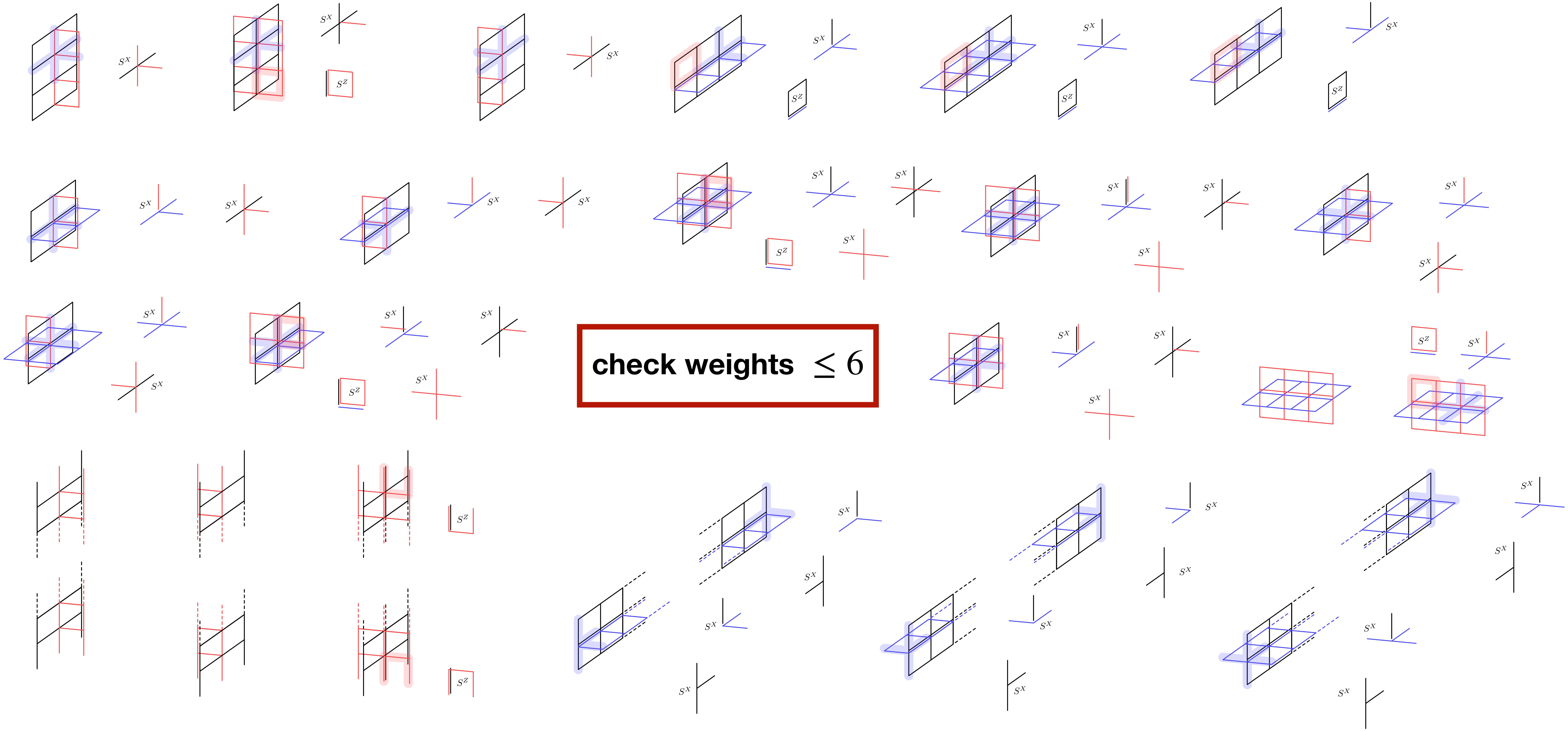




Boundary point defects

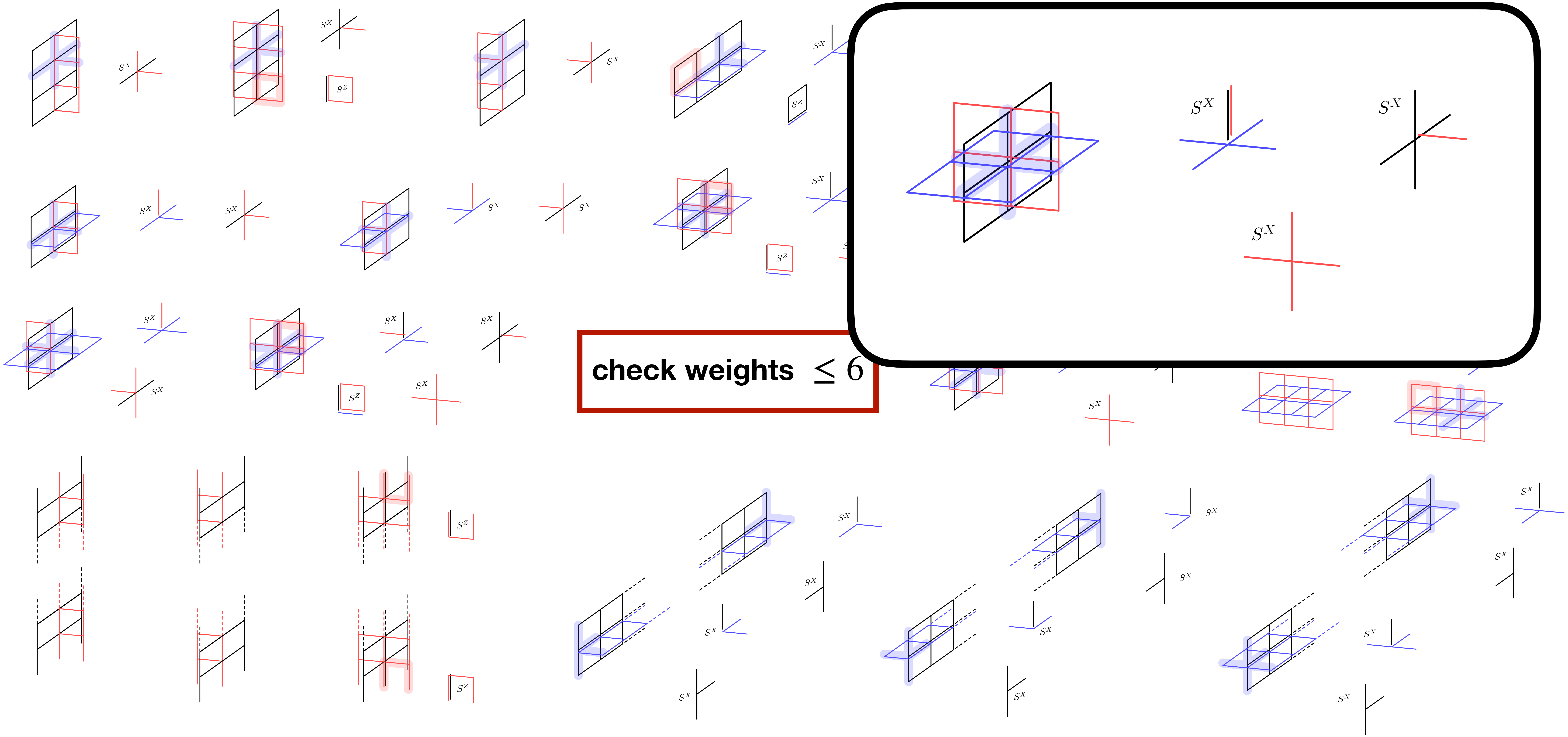


Lattice model



check weights ≤ 6

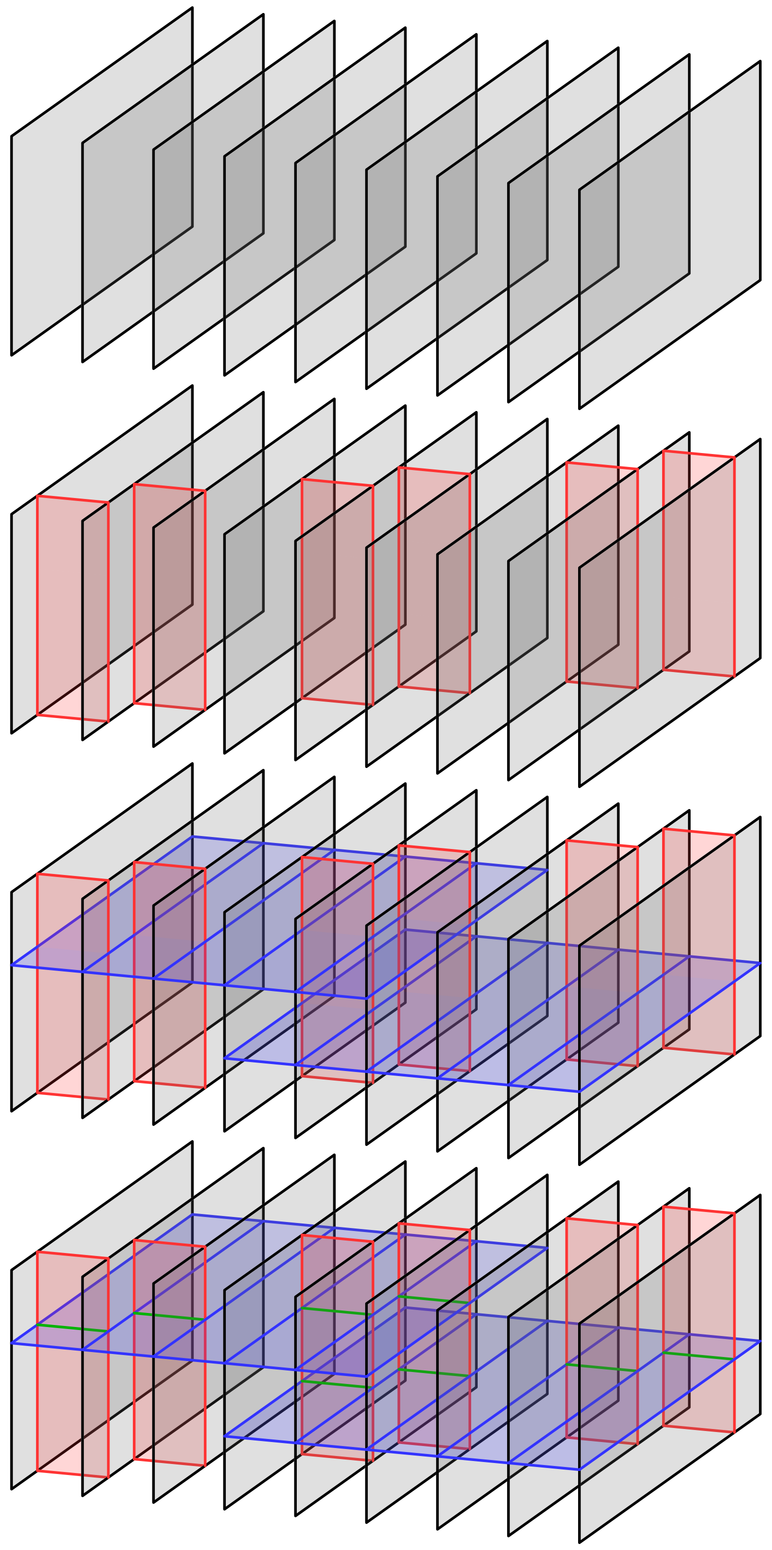
Lattice model

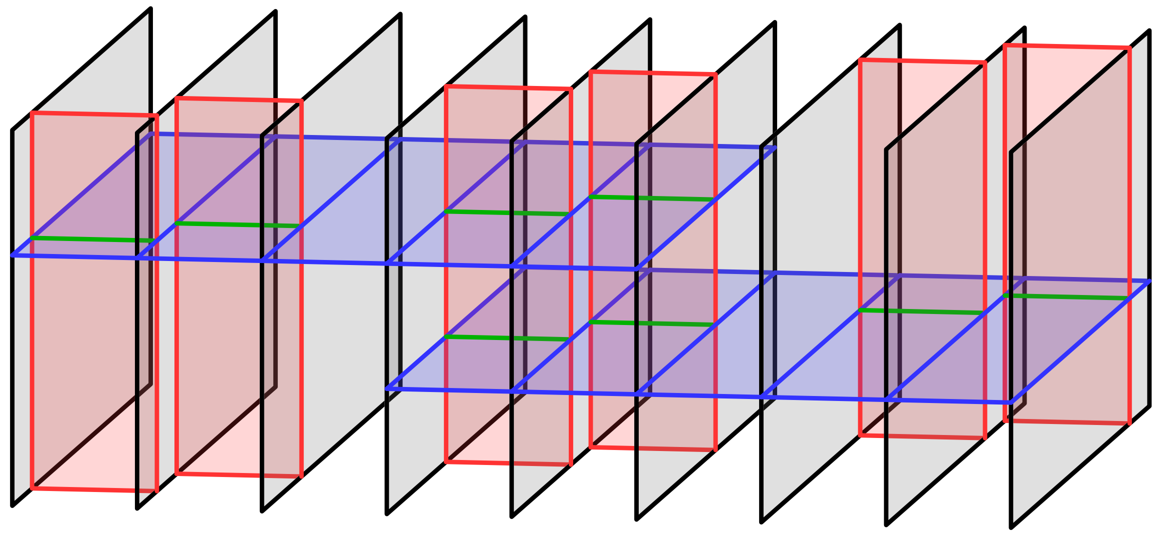


check weights ≤ 6

Layer code recipe

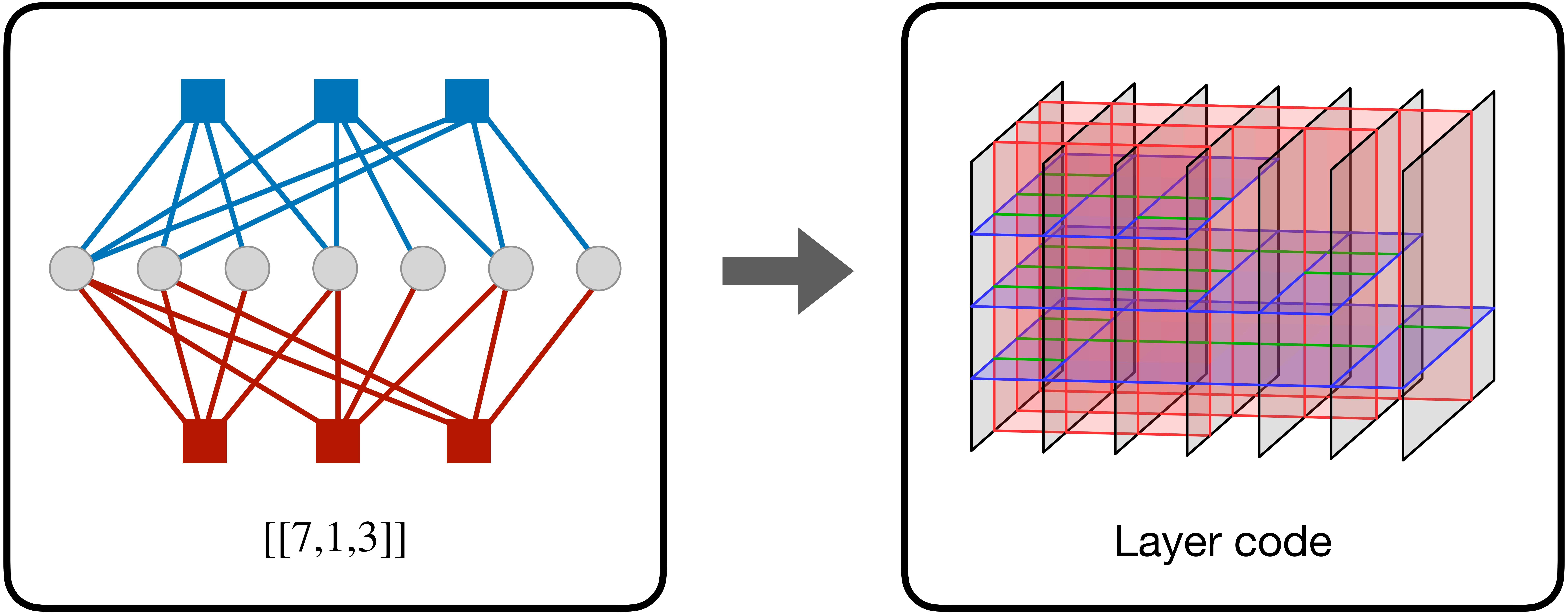
- Data layers
- Z-check layers
- X-check layers
- y-defects





- Layer code

Steane code example



Layer code parameters

For topological code in 3D:

- BPT $kd = O(L^3)$
- BT $d = O(L^2)$
- Haah* $k = O(L)$

*homogeneity assumption

Good LDPC code

$$[[L, \theta(L), \theta(L)]]$$

↓

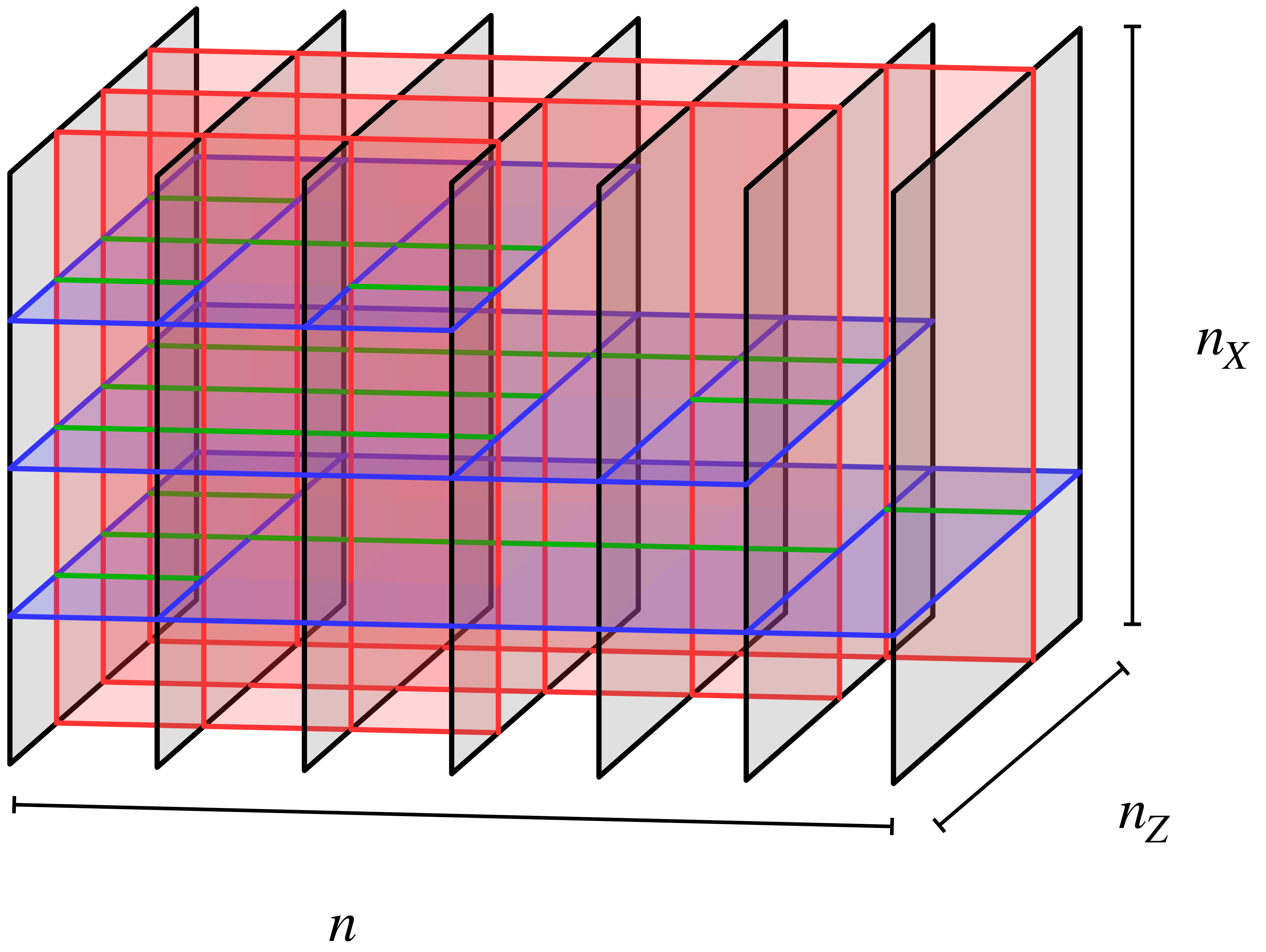
$$[[\theta(L^3), \theta(L), \theta(L^2)]]$$

Optimal 3D code

N

$$N = \Theta(n n_x n_z)$$

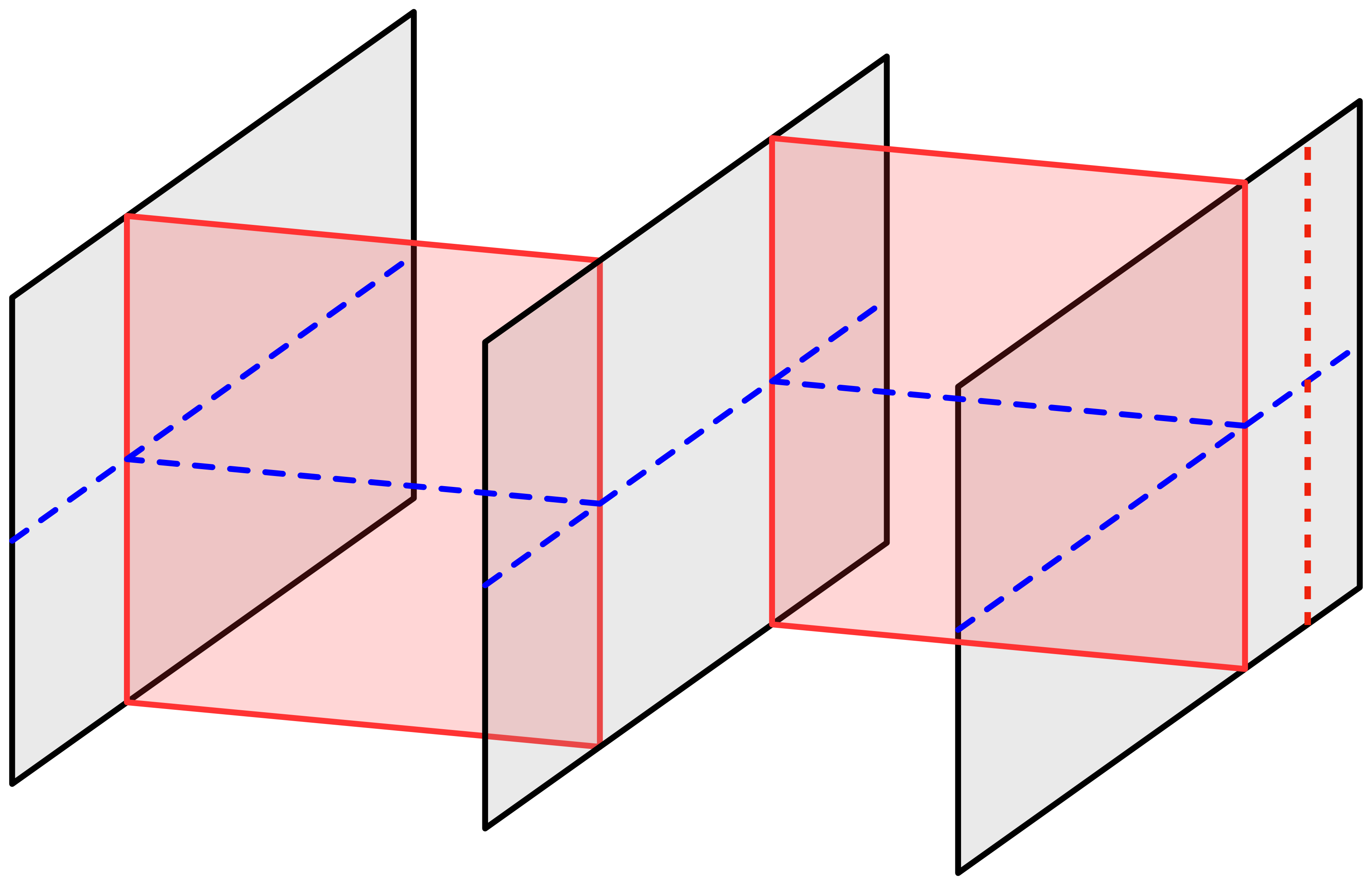
Good LDPC code $\rightarrow \Theta(L^3)$



K

$$K = \Theta(k)$$

Good LDPC code $\longrightarrow \Theta(L)$

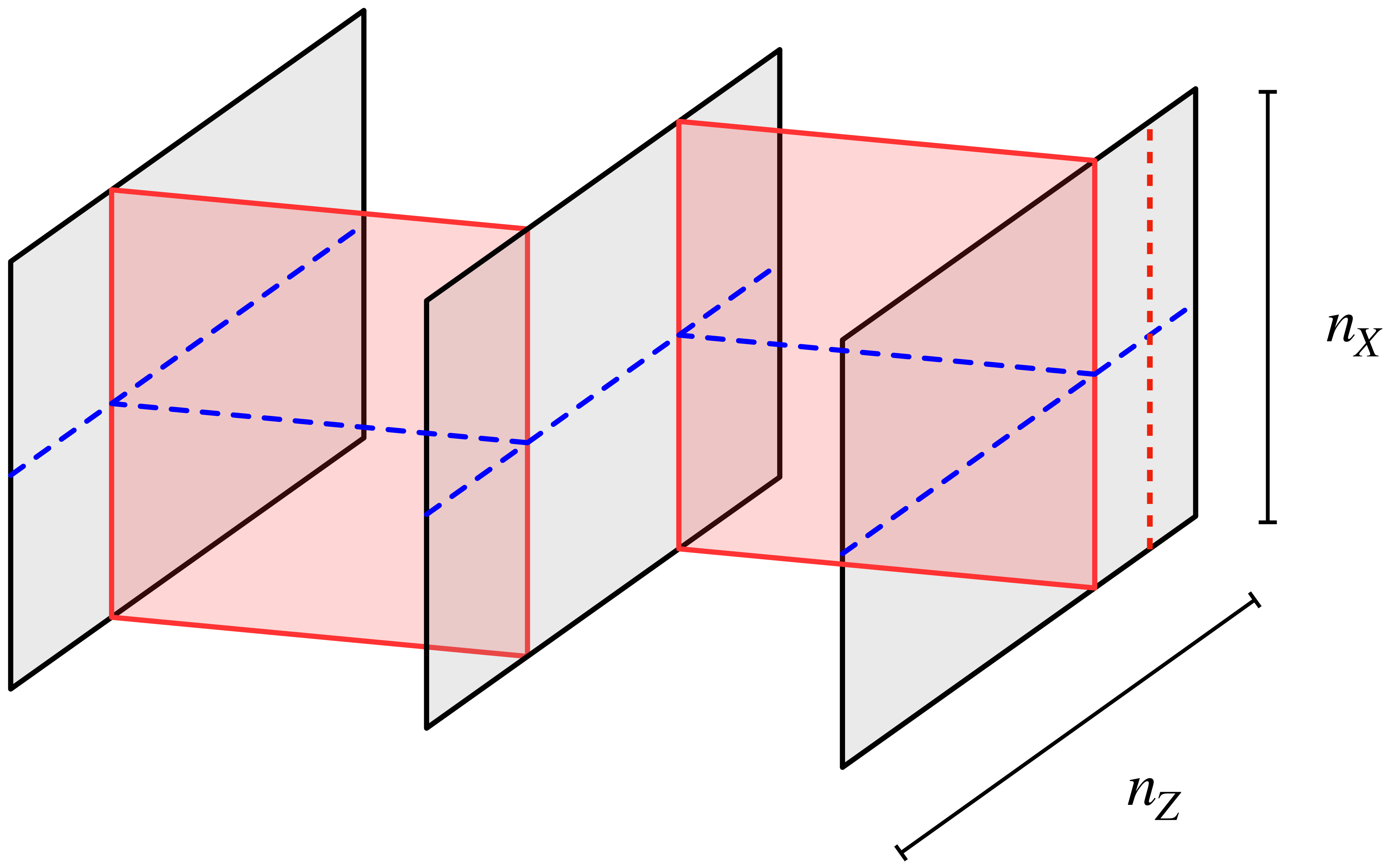


quasi-concatenated logical

D

$$D = \Theta(\frac{1}{w} d \min(n_x, n_z))$$

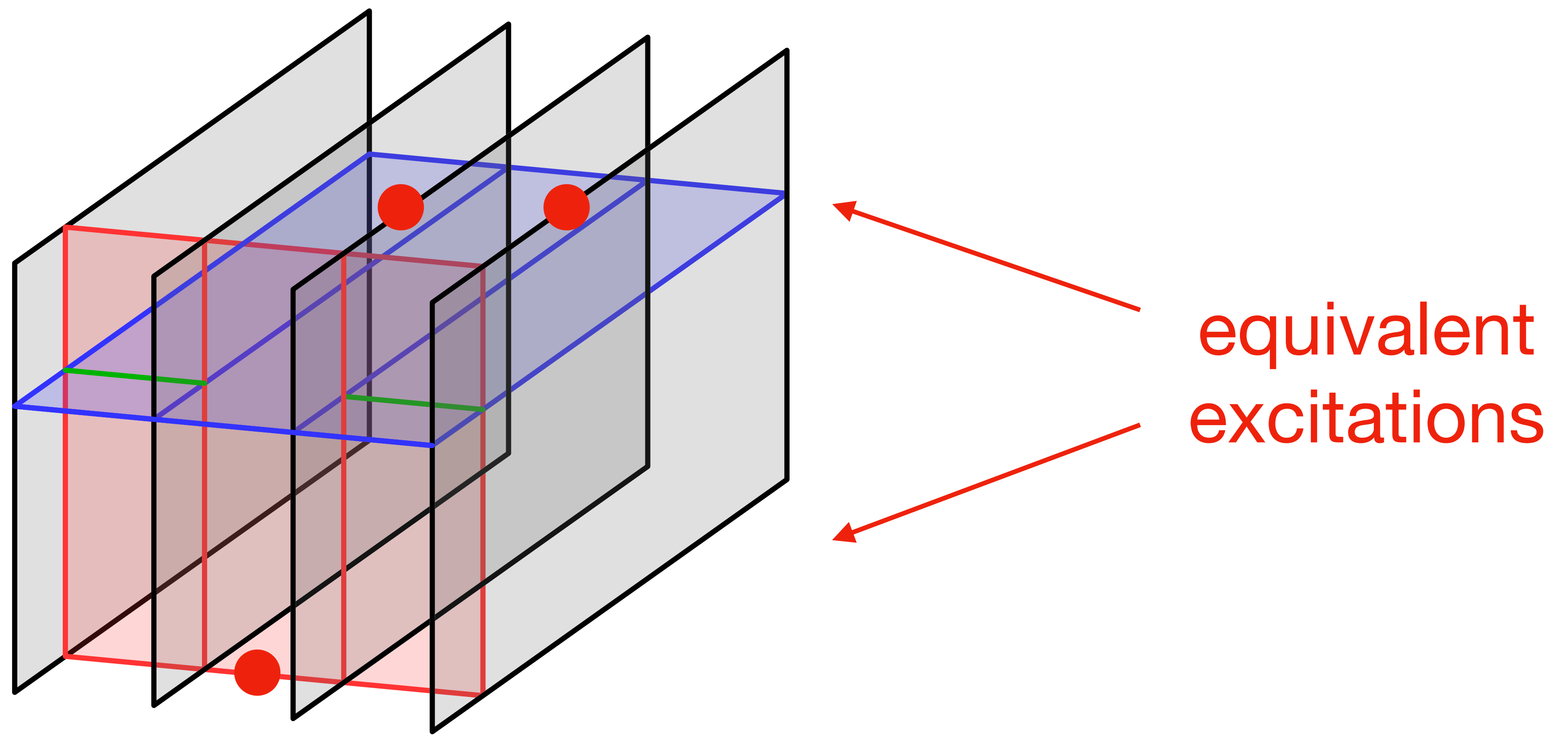
Good LDPC code $\rightarrow \Theta(L^2)$



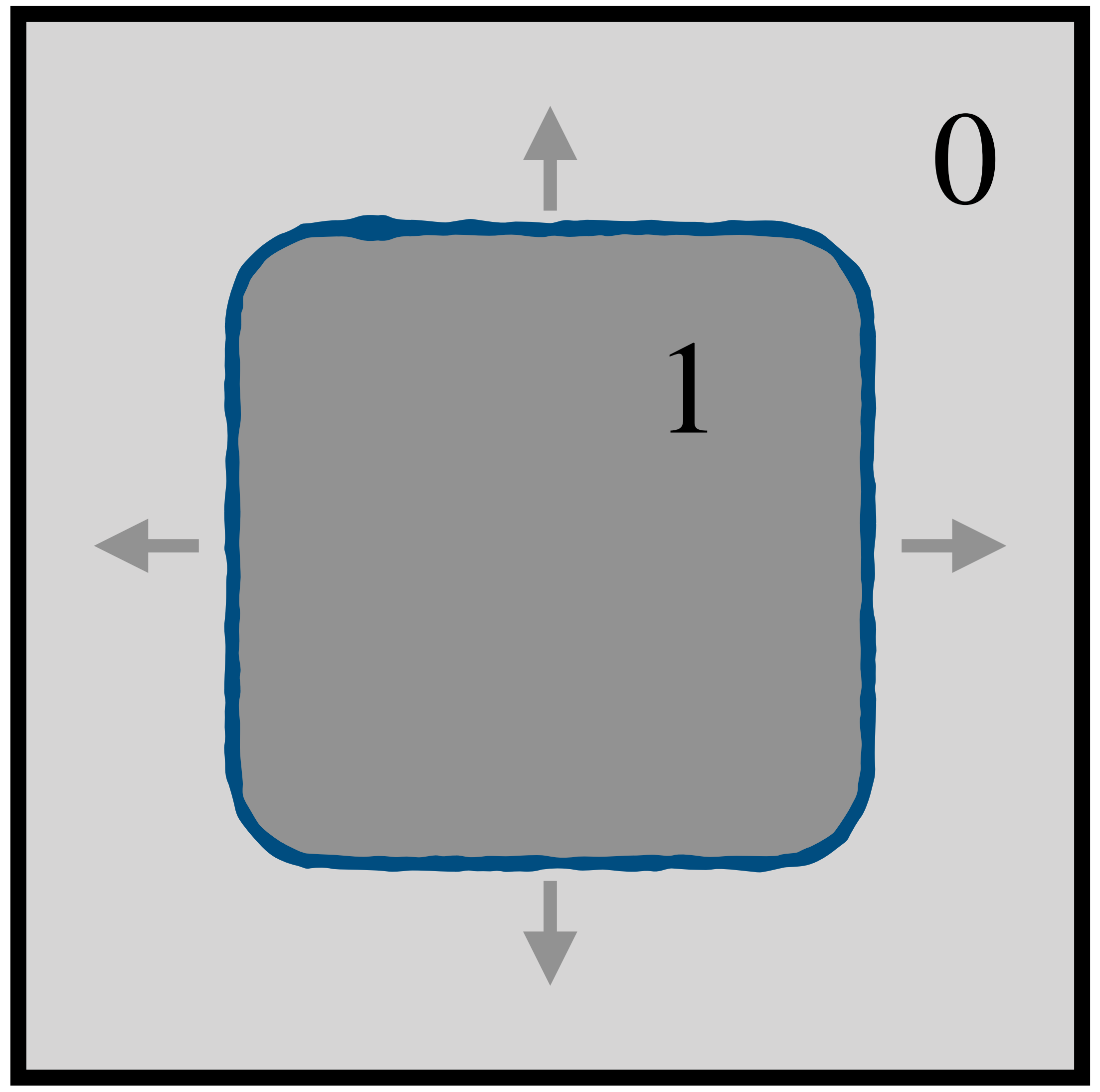
D

$$D = \Theta(\frac{1}{w} d \min(n_x, n_z))$$

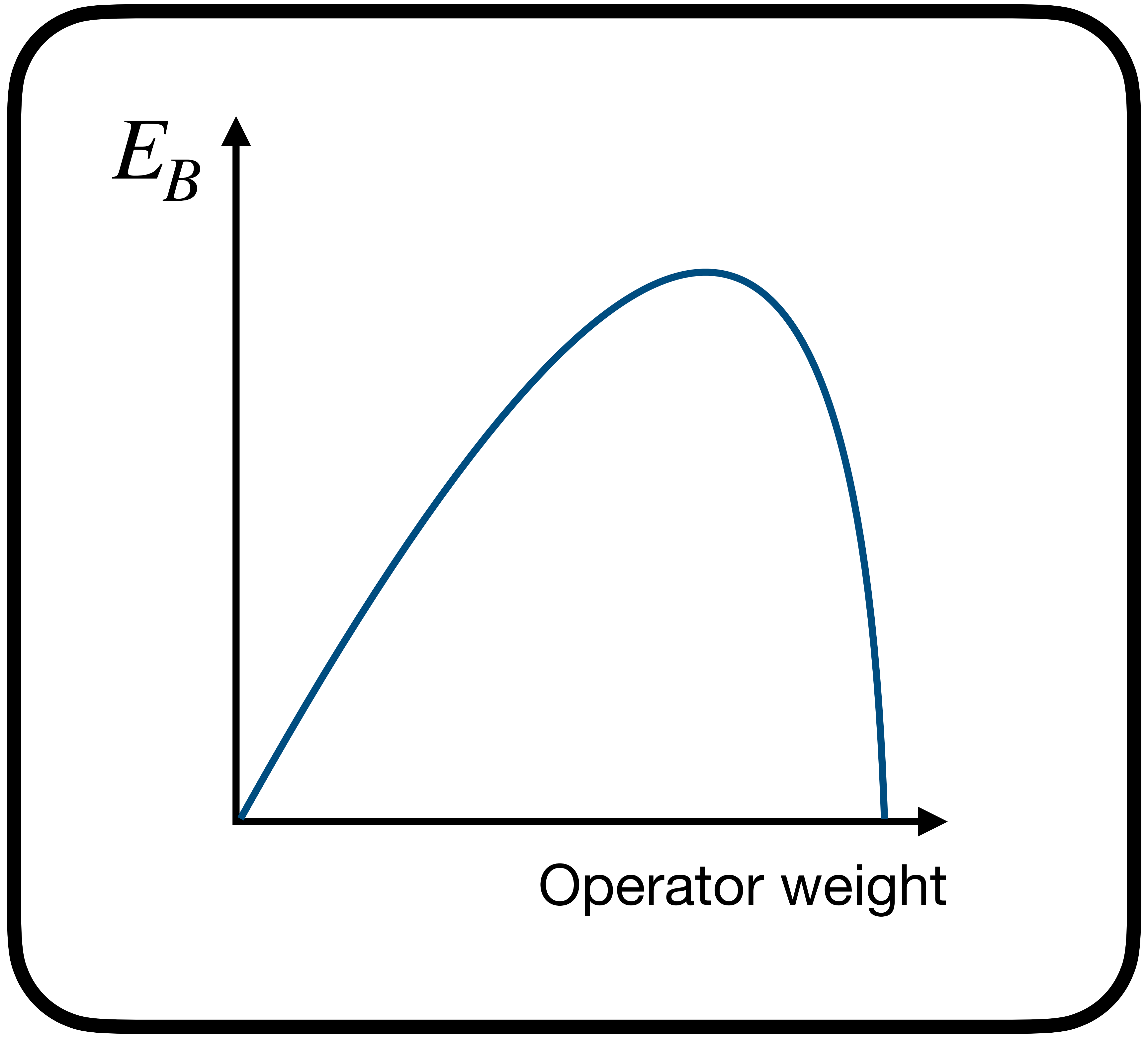
Good LDPC code $\rightarrow \Theta(L^2)$



Energy barrier



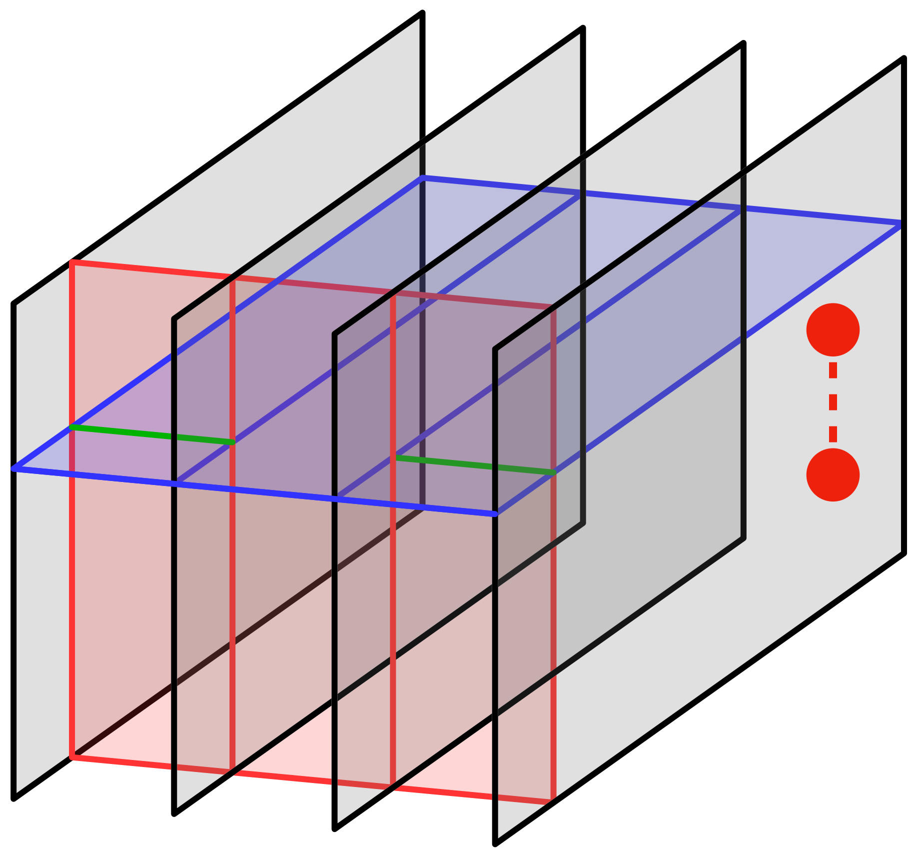
Ising Ferromagnet

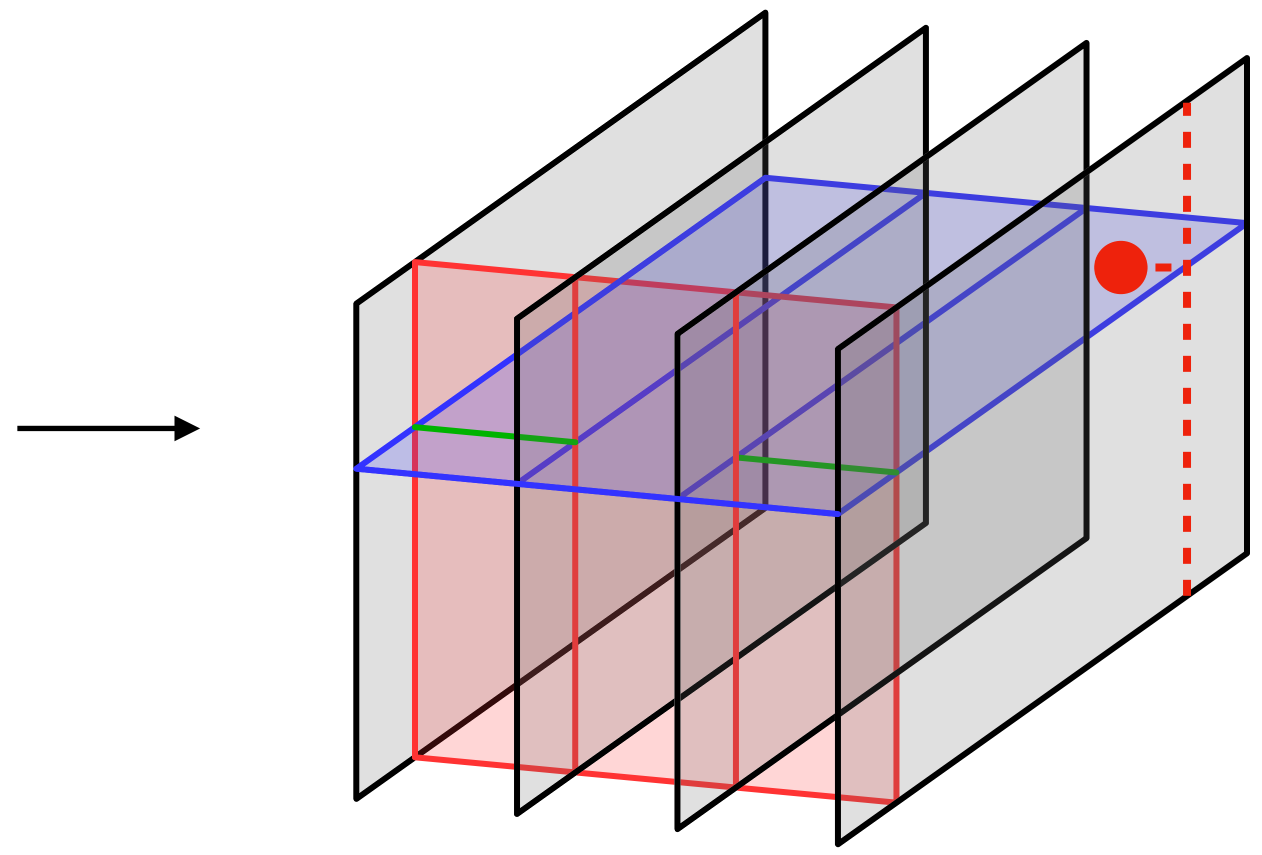


Energy barrier

$$E_B = \Theta(\frac{1}{ww'}L)$$

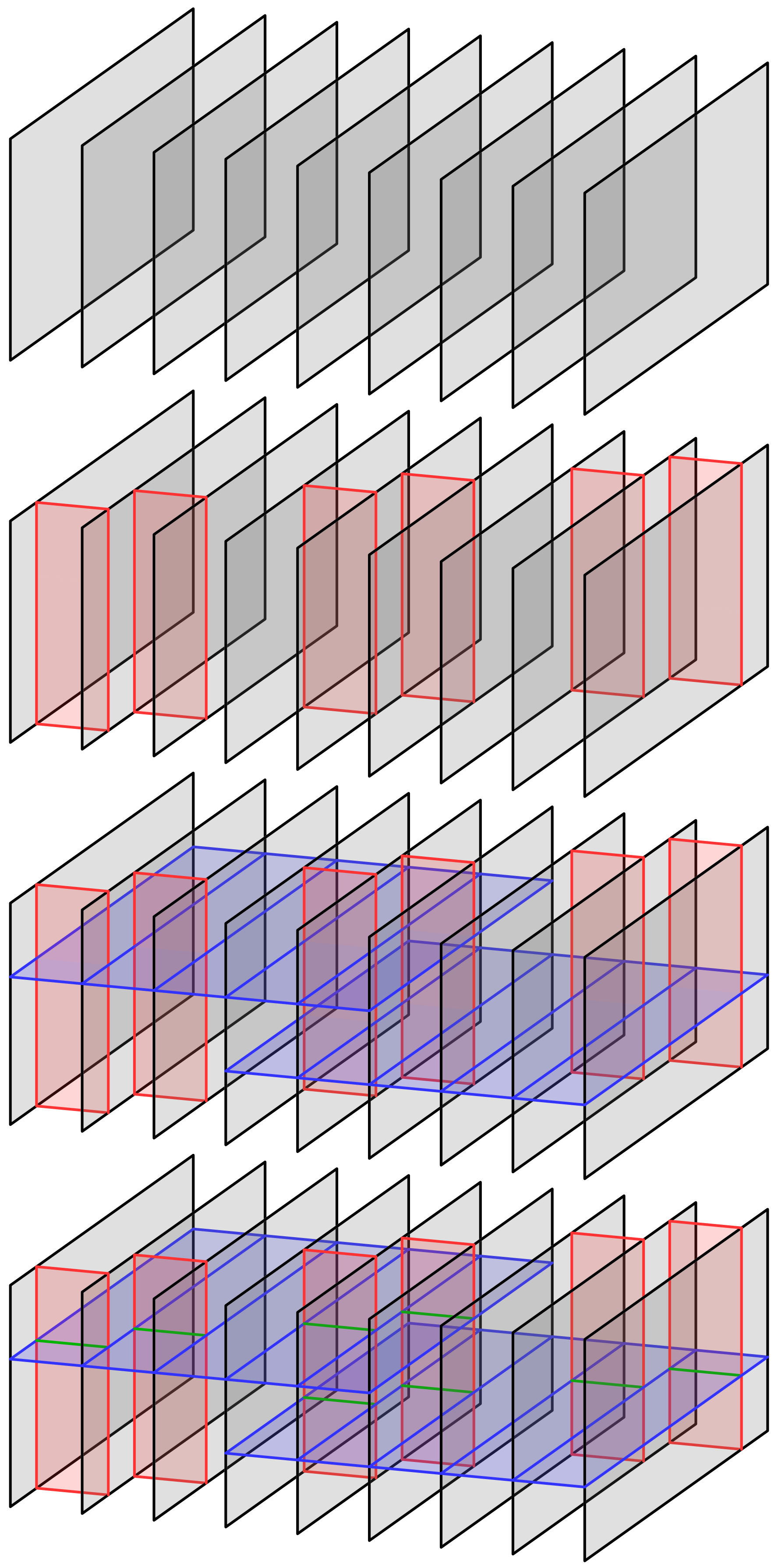
(For good LDPC code)





quasi-concatenated error

Questions



- Self-correction
- Decoders
- Fault-tolerant gates
- Equivalences
- Weight reduction
- Optimal codes for different architectures