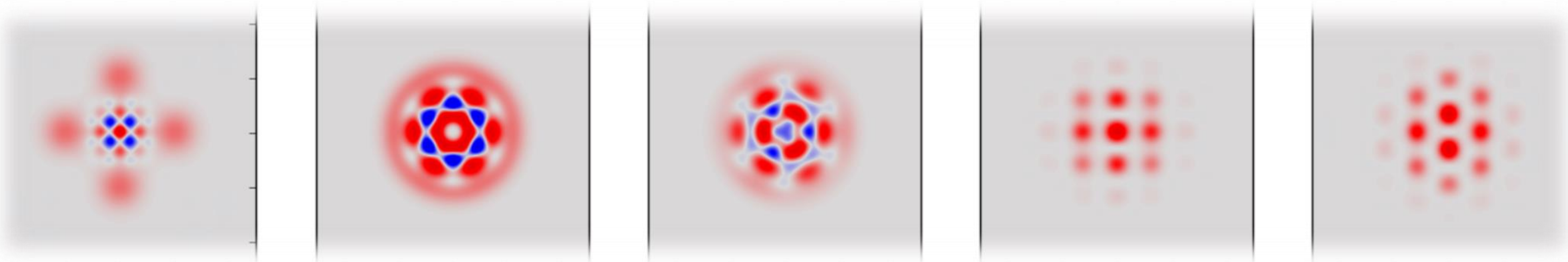
Tutorial on Bosonic Codes



For details and references, see:

- arXiv:2211.05714
- errorcorrectionzoo.org/c/oscillators

Advances in Quantum Coding Theory Workshop

SIMONS INSTITUTE for the Theory of Computing

Presented by Victor V. Albert

Joint Center for Quantum Information and Computer Science

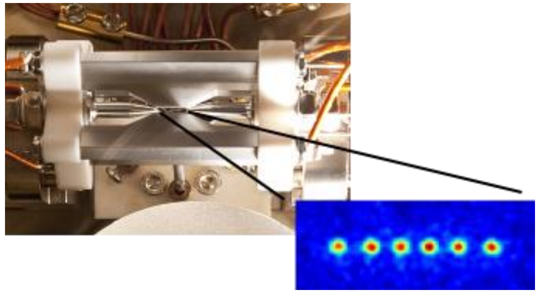
NIST

University of Maryland 1856

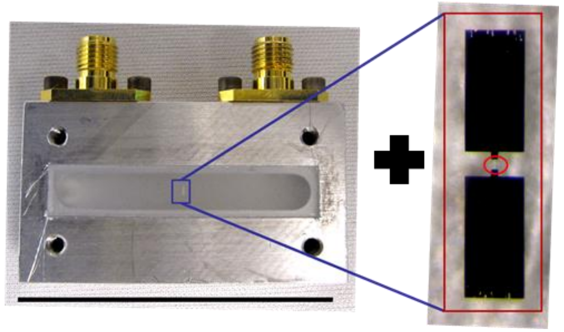
CONTINUOUS-VARIABLE (CV or "BOSONIC") SYSTEMS

- Characterized by continuous degrees of freedom, such as **position** and **momentum** of a mechanical oscillator or **quadrature components** for an electromagnetic field mode.
- We **already use** them:
  - Communication (A)
  - Excitations of current systems (C)
- We **should study** them:
  - Long natural lifetimes (C, E)
  - Extra degrees of freedom (B, C)
  - Different geometry, states, operations (binary vs. lattices)

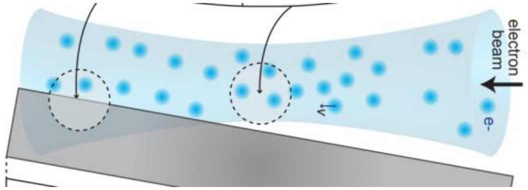
A. Optical fibers, free space



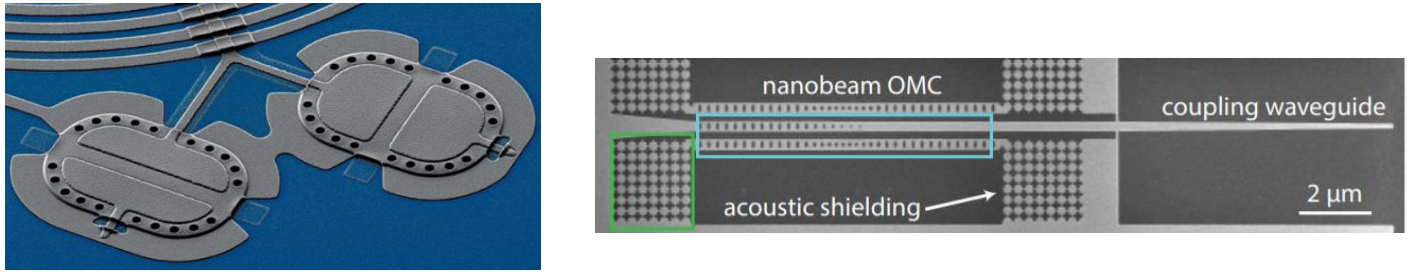
B. Vibrations of atoms/molecules



C. Microwave cavities + superconducting circuits



D. Free-electron–light interactions



E. Mechanical/acoustic resonators

O. Painter, M. Mirhosseini, A. Faraon, J. Teufel, M. Spiropulu, R. Schoelkopf, M. Devoret, J. Home, I. Kaminer groups, many more

∞ DIMENSIONS circumvents NO-GO THMS

1. GKP states are powerful **non-Gaussian resources**

   GKP QEC + Gaussian states = universal computation (no magic-state distill.) arXiv:1903.00012

   Protecting against small shifts in a logical mode (no-go thm in DV) arXiv:1903.12615

2. Continuous-parameter families of **transversal gates** (Eastin-Knill no-go thm in DV).

   arXiv:1902.07714 and many others

3. **Hamiltonian-based bias-preserving gates** (no-go thm in DV).

   arXiv:1905.00450 (cat code CNOT gate)

4. Q-simulation; commuting-projector models for **chiral topological phases** (no-go thm in DV).

   arXiv:1902.06756, 1906.08270, 2107.02817 (e.g., fractional quantum Hall on the lattice)

5. **Hardware efficiency:** small CV codes $\geq$ small DV codes; cat codes $>$ 1D Ising repetition code

   arXiv:2205.09767 (cat codes + 2D classical Ising model)

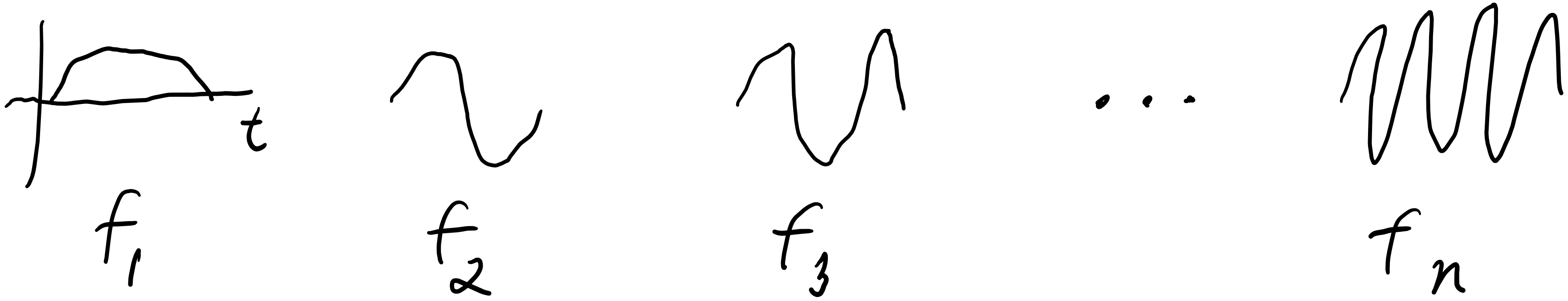
6. Single-mode codes do not have thresholds, but concatenation can **improve qubit thresholds**.

   arXiv:2107.13589 and refs. therein

WHAT IS A qu-MODE?

QUANTIZING SIGNALS

- Pick some basis of **normal modes** for your signal (either in space or time).



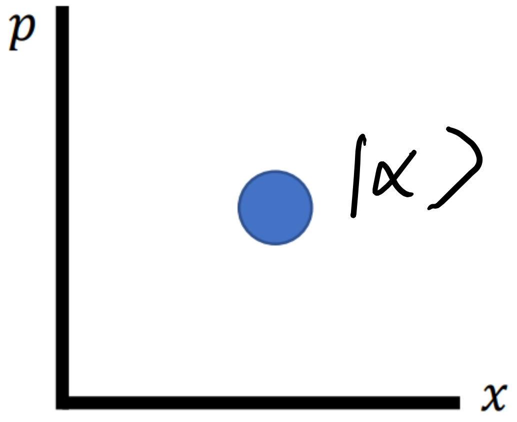
- Classical signal is continuous! Can be expanded as **linear combination** of normal modes.

$$\psi(t) = \sum_{j=1}^{n} \alpha_j f_j(t) \ , \quad \alpha_j \in \mathbb{C}$$

Shannon 1947

- In quantum state space, signal corresponds to tensor product of **coherent states**:

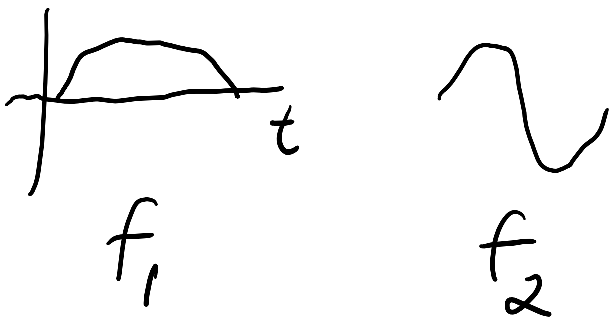
$$|\psi\rangle = \bigotimes_{j=1}^{n} |\alpha_j\rangle \ \in \ \ell^2(\mathbb{R}^n)$$

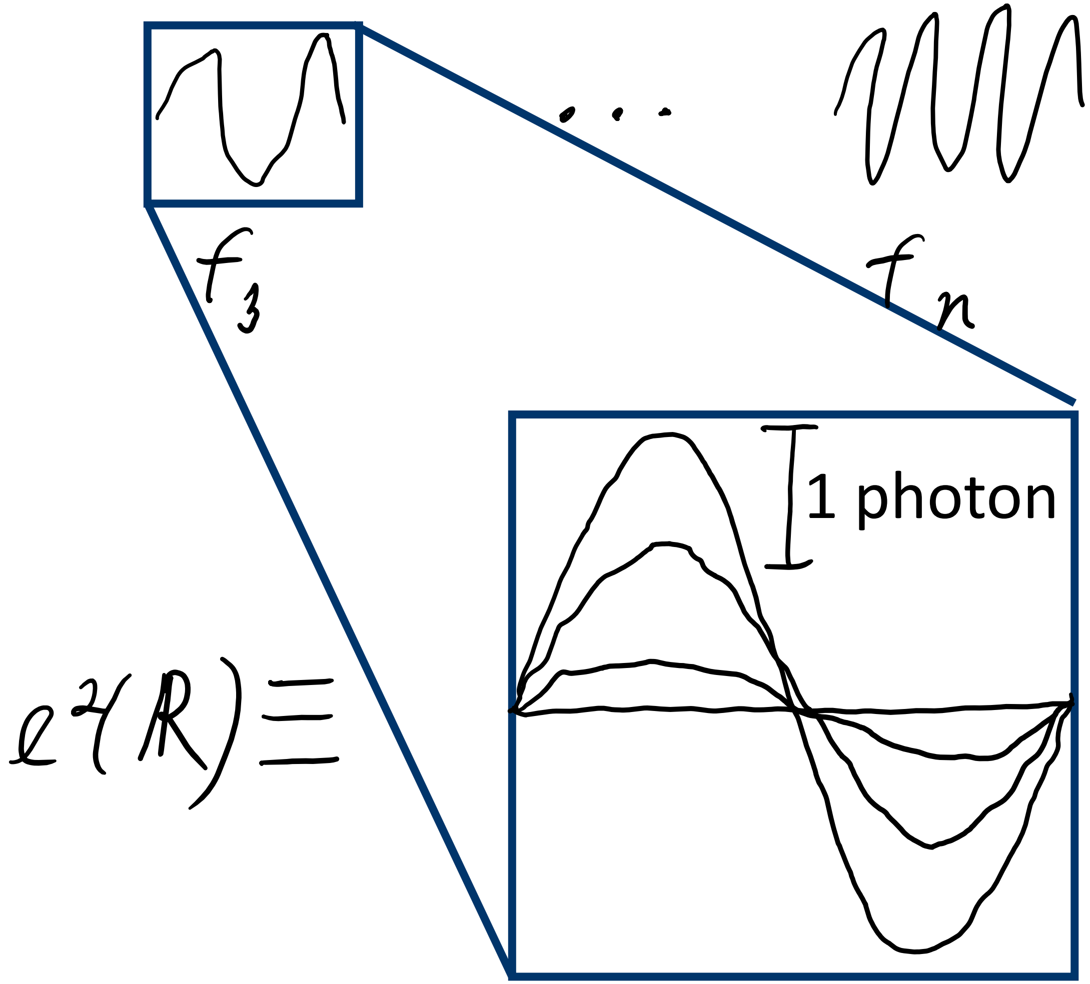


Blow Loudon Phoenix Shapherd 1992, Banaszek Kunz Jachura Jarzyna 2020, Thesis – N. Fabre 2021

WHAT IS A qu-MODE???

- Find **normal modes** of Laplace's equation for a box (either in space or time).





- Quantum mode, or qu-mode, arises from quantizing the **amplitude** of **each** (classical) normal mode.

- This allows us to consider **superpositions** of coherent states...

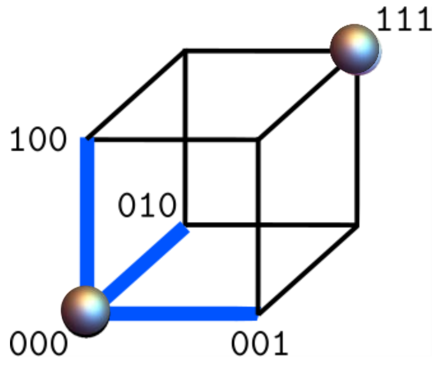
Blow Loudon Phoenix Shapherd 1992, Banaszek Kunz Jachura Jarzyna 2020, Thesis – N. Fabre 2021

CLASSICAL → QUANTUM

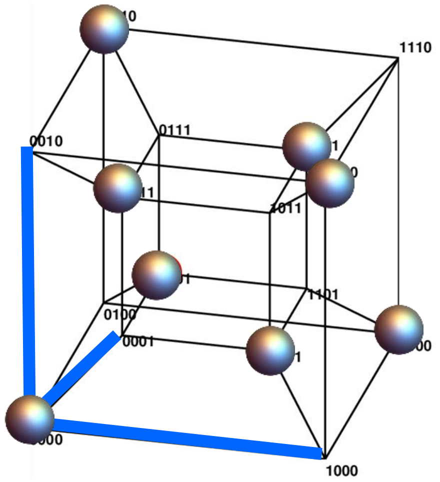
$$|\bar{0}\rangle = \sum_{\alpha \in \bullet} |\alpha\rangle \qquad |\bar{1}\rangle = \sum_{\alpha \in \circ} |\alpha\rangle$$

Classical codewords pack space in well-separated way, protecting from **bit-flip errors**.

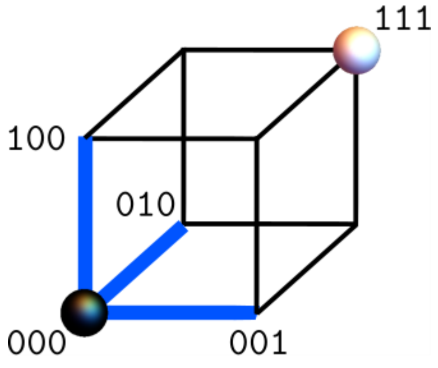
Quantum codewords are **superposed** classical codewords.*



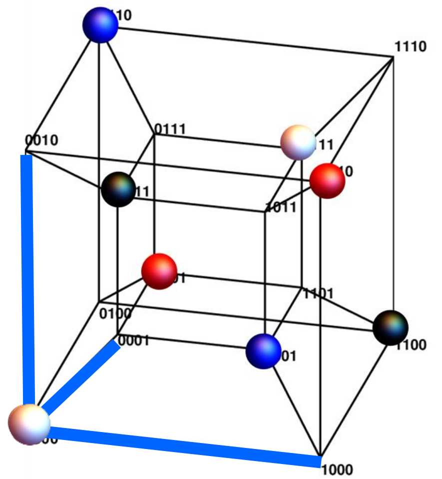
[3,1,2] repetition



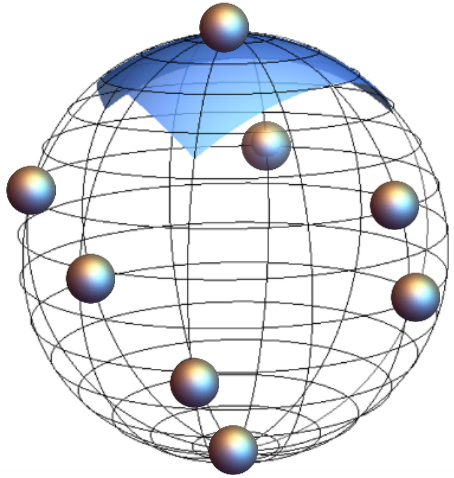
[4,3,2] single parity check



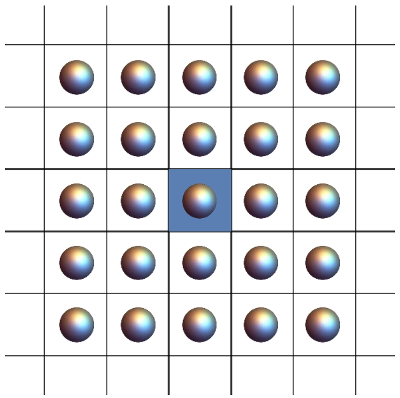
[[3,1]] quantum repetition



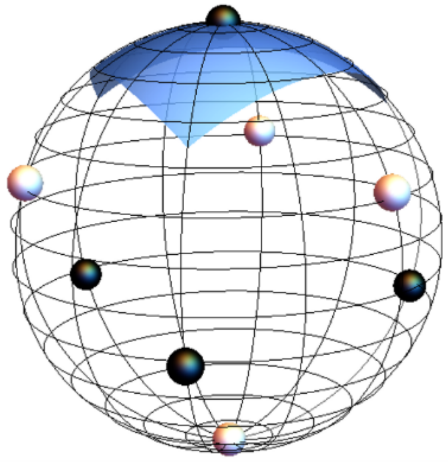
[[4,2,2]]



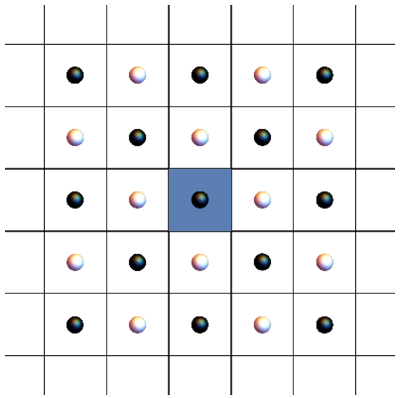
spherical code



lattice



quantum spherical (QSC)



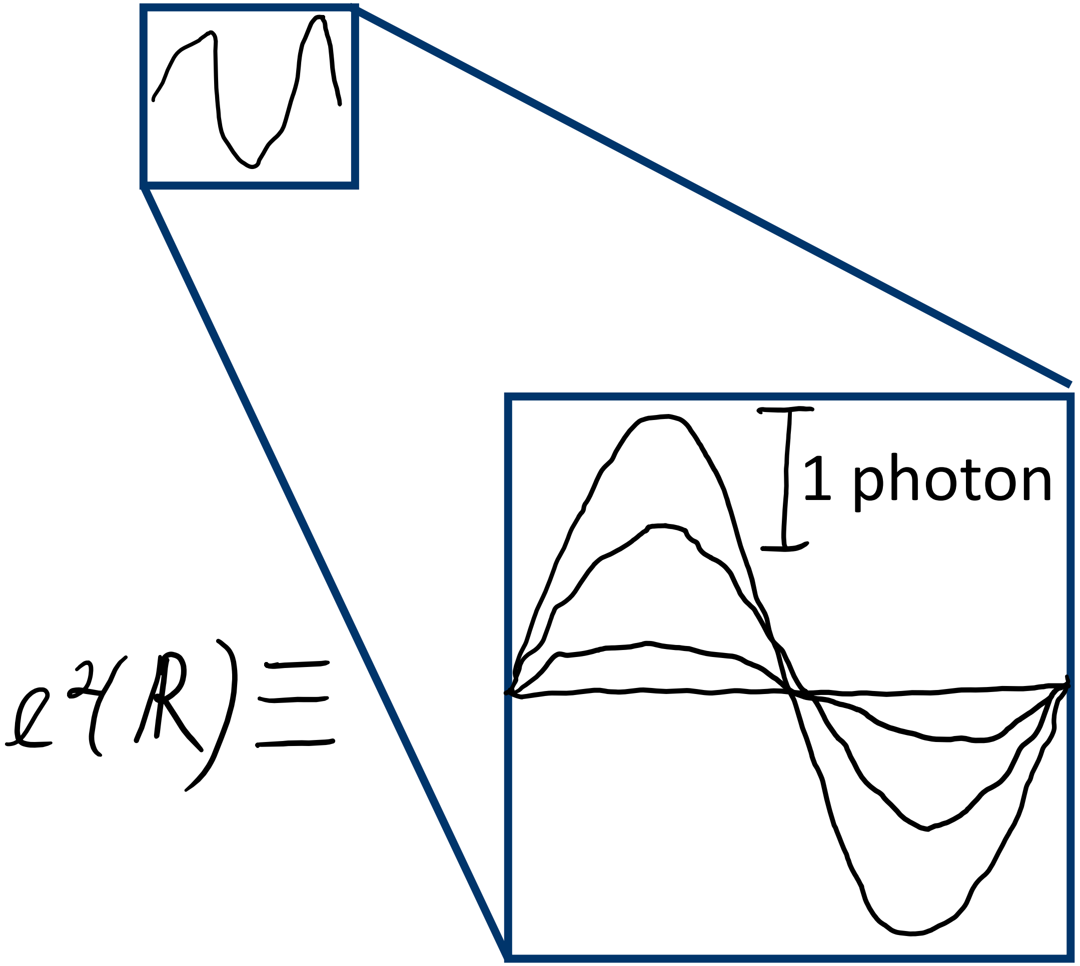
quantum lattice (GKP)

*nonuniform superpositions yield “non-CSS” codes

GENERAL STATES

- More generally, a (single-mode) **quantum state** $|\psi\rangle \in L^2(\mathbb{R})$ is a vector with finite occupation-number moments.
  - ✓ $\langle\psi|\hat{n}^\ell|\psi\rangle < \infty$ for all $\ell \geq 0$.
  - ✓ Such states form **Schwartz space** $S(\mathbb{R})$.

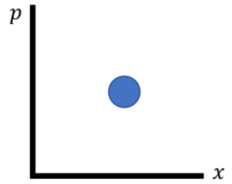
- Ideal CV codewords are often not in $L^2(\mathbb{R})$, but **"regularized"** versions in $S(\mathbb{R})$ yield the same basic protection.



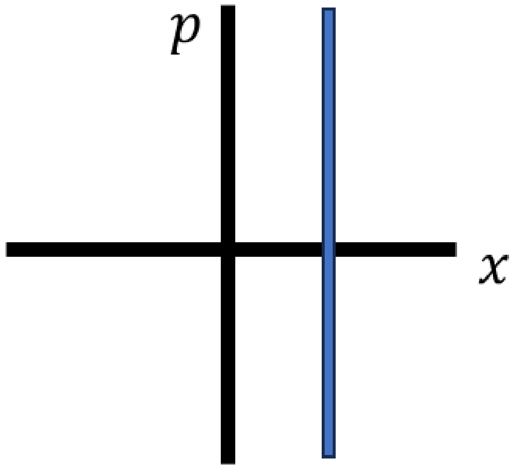
arXiv:2211.05714

OTHER CV "BASES"

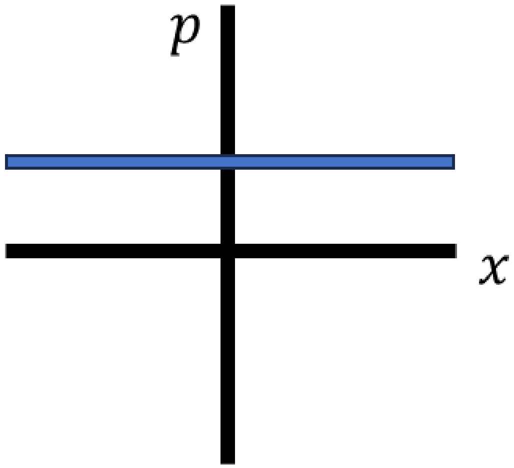
Coherent states $|\alpha\rangle$



Cartesian conjugate bases

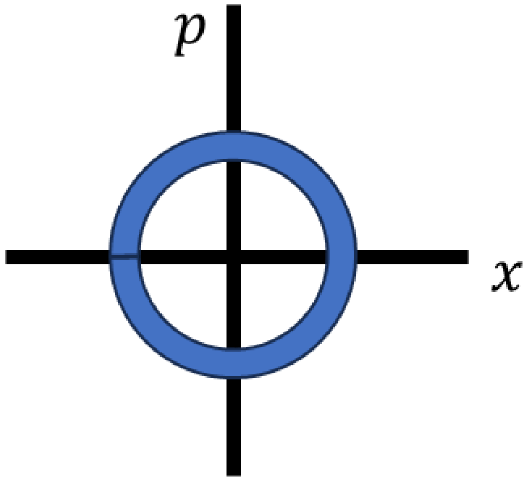


Position states
$|x\rangle$

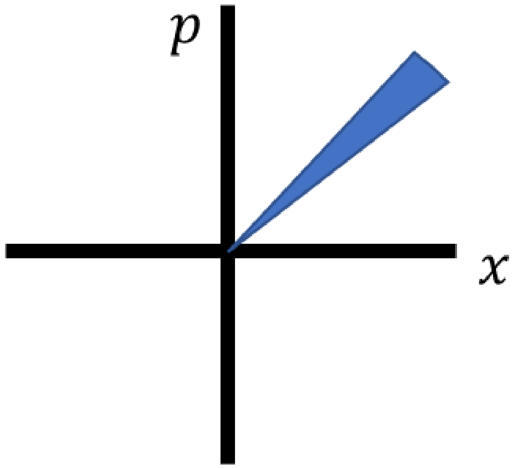


Momentum states
$|p\rangle$

Angular "conjugate" bases



Fock states
$|n\rangle$

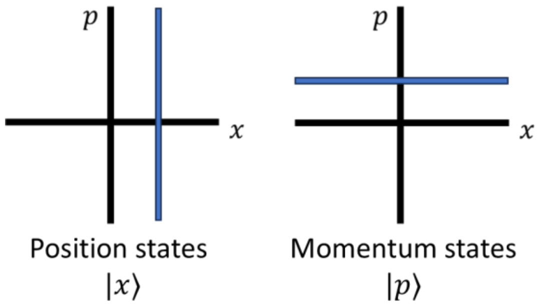


Phase states
$|\phi\rangle$

- Each is a "basis" for **states**.
- Each offers ways to **construct** new codes and **embed** old ones.
- Each conjugate pair comes with an **error basis** for operators.

CARTESIAN CODES

CARTESIAN ERROR BASIS



Position/momentum displacement operators $D(\boldsymbol{\gamma})$ are the closest analogues of **Pauli strings**.

$$D(\gamma) \equiv \exp\left(\gamma a - \gamma^* a^\dagger\right)$$

They form a complete and "orthonormal" **basis**.

$$\mathrm{tr}\left(D(\gamma)D(\eta)^\dagger\right) = \pi\delta^2(\gamma - \eta)$$

Any physical (bounded) error operator can be **expanded**:

$$U(\theta) = \frac{1}{\pi}\int d^2\gamma\ u_\theta(\gamma)D(\gamma)$$

Two notions of distance:

1. Hamming weight, $\Delta(\boldsymbol{\alpha})$

   → analogous to Pauli weight

ANALOG STABILIZER CODES

$\mathbb{Z}_2$ [[4,1,2]] qubit		$\mathbb{R}$ $[[4,1,2]]_{\mathbb{R}}$ CV
$\lvert x \rangle = \left(\sum_{y \in \mathbb{Z}_2} (-1)^{xy} \lvert y \rangle^{\otimes 2} \right)^{\otimes 2}$	codewords	$\lvert x \rangle = \left(\int_{\mathbb{R}} dy\, e^{ixy} \lvert y \rangle^{\otimes 2} \right)^{\otimes 2}$
$ZZ11$		$e^{i\hat{x}} e^{-i\hat{x}}\, 1\, 1$
$11ZZ$	Stabilizer "generators"	$1\, 1\, e^{i\hat{x}} e^{-i\hat{x}}$
$XXXX$		$e^{i\hat{p}} e^{i\hat{p}} e^{-i\hat{p}} e^{-i\hat{p}}$

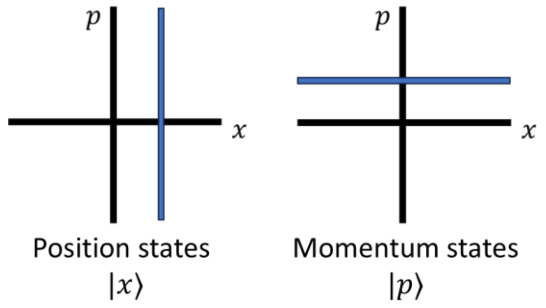
Stabilizer group is **continuous**, can look at Lie algebra (a.k.a. **nullifiers**):

$$\hat{x}_1 - \hat{x}_2,\ \hat{x}_3 - \hat{x}_4,\ \text{and}\ \hat{p}_1 + \hat{p}_2 - \hat{p}_3 - \hat{p}_4$$

$$(\hat{x}_1 - \hat{x}_2)\lvert x_{\text{four-mode}}\rangle = \int_{\mathbb{R}} \mathrm{d}y \int_{\mathbb{R}} \mathrm{d}z\, e^{ix(y+z)} (y - y)\lvert y, y, z, z\rangle = 0$$

Lloyd, Slotine, Braunstein late 1990s; see errorcorrectionzoo.org/c/analog_stabilizer

CARTESIAN ERROR BASIS



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Two notions of distance:

1. Hamming weight, $\Delta(\boldsymbol{\alpha})$
→ analogous to Pauli weight

2. Displacement length, $||\boldsymbol{\alpha}||_2$
→ relevant to GKP codes

GKP CODES: POSITION STATE FORMULATION

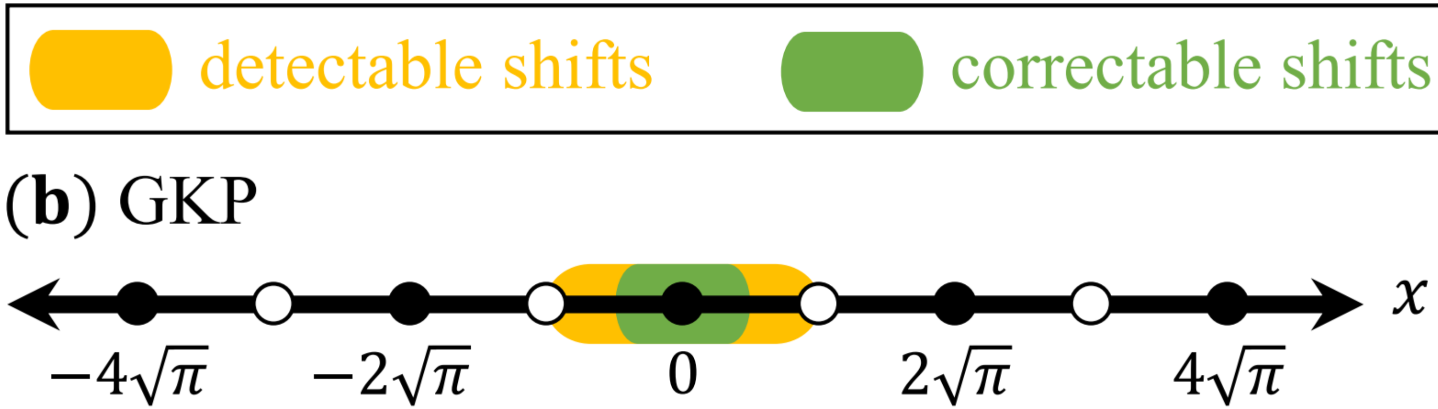
- Stabilizer group is infinite, but **discrete.**
- **Finite**-dimensional (qudit) codespace
- Protect against **small** position and momentum displacements.

$$\mathsf{S}_{\mathrm{GKP}} = \langle S_1, S_2 \rangle = \langle e^{i\sqrt{2\pi N}\hat{x}}, e^{-i\sqrt{2\pi N}\hat{p}} \rangle = \{ S_1^a S_2^b \,|\, a, b \in \mathbb{Z} \}$$

- Codewords form 1D lattices w/ position states (but form 2D lattice w/ coherent states!).

$$|0_{\mathrm{GKP}}\rangle = \sum_{\ell \in \mathbb{Z}} |\, (2\ell)\sqrt{\pi} \rangle \quad \text{and} \quad |1_{\mathrm{GKP}}\rangle = \sum_{\ell \in \mathbb{Z}} |\, (2\ell + 1)\sqrt{\pi} \rangle$$

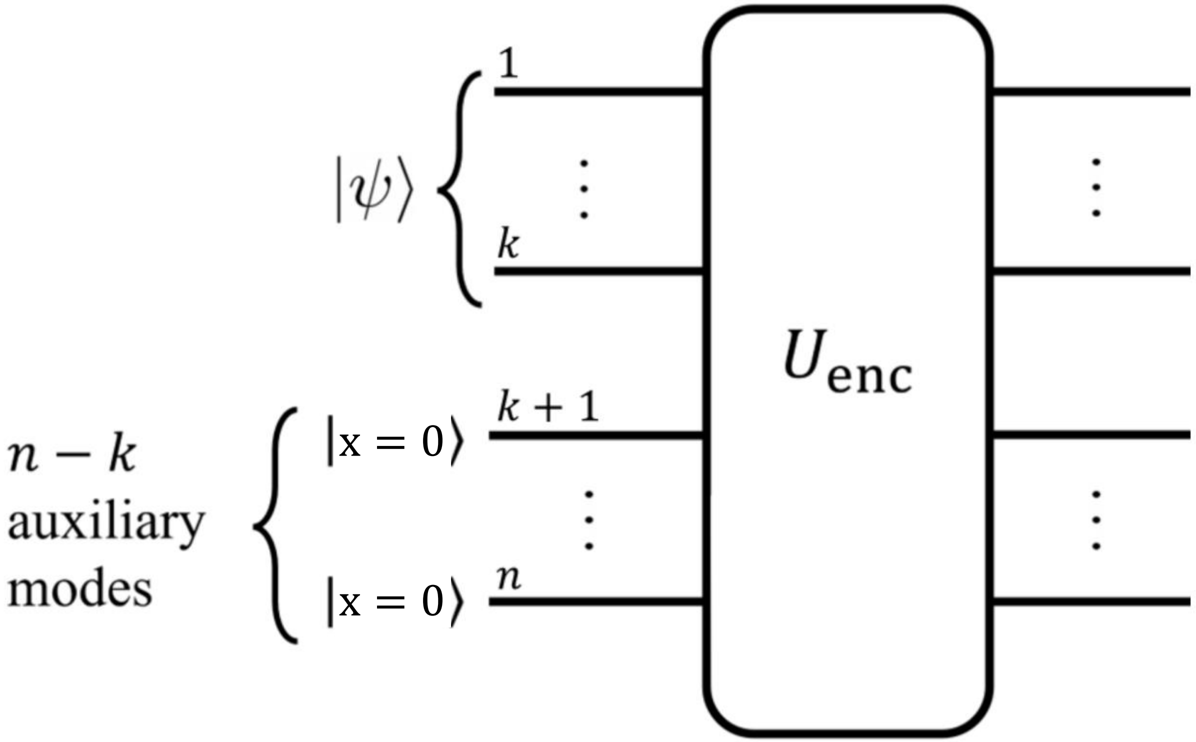
- Same picture holds in momentum space.

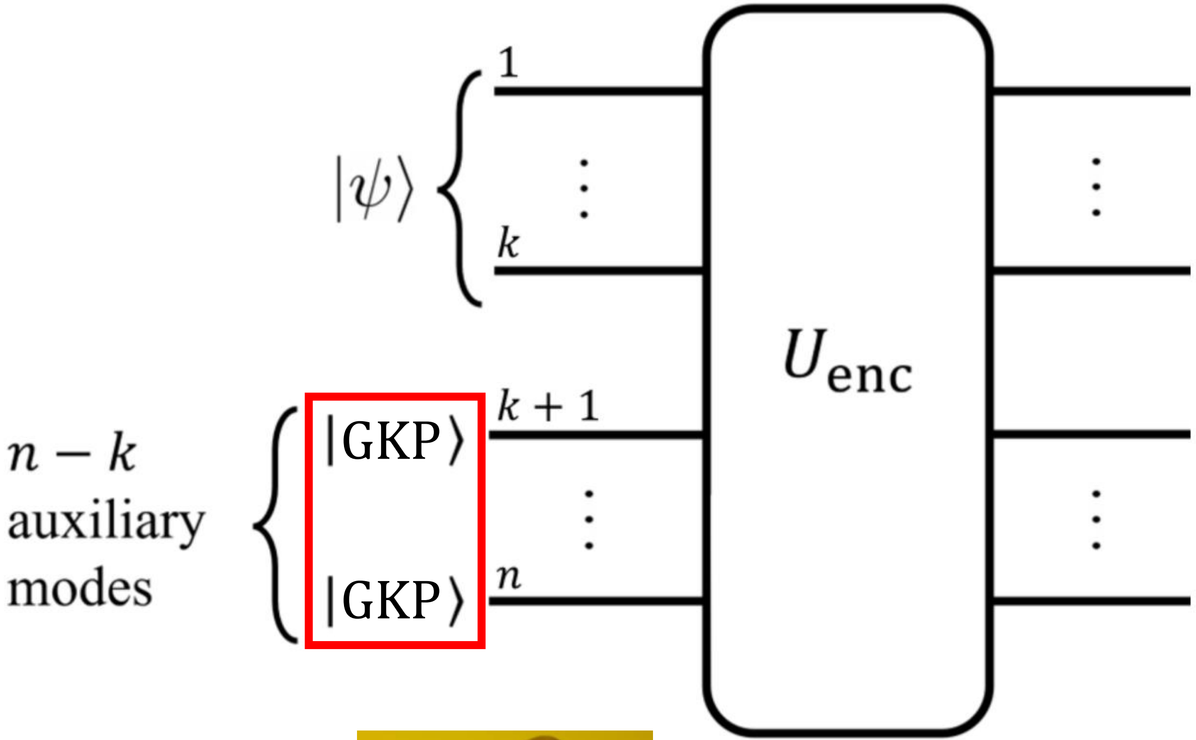


Gottesman, Preskill, Kitaev; see errorcorrectionzoo.org/c/gkp

GKP-STABILIZER CODES

- Consider a GKP code and **remove** some of the stabilizers -> degenerate lattice
- GKP-stabilizer codes encode one **logical mode** into **physical modes**
- Protect infinite-dim state space against **small displacements**.





ANALOG STABILIZER

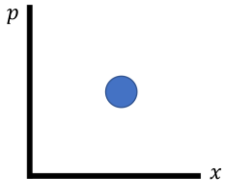
GKP-STABILIZER

Noh, Girvin, Jiang; see errorcorrectionzoo.org/c/gkp-stabilizer

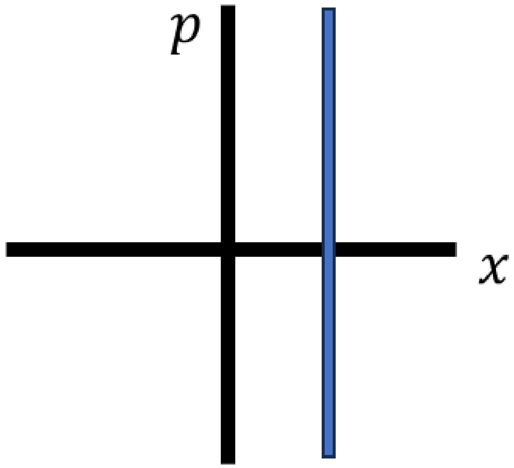
ANGULAR CODES

OTHER CV "BASES"

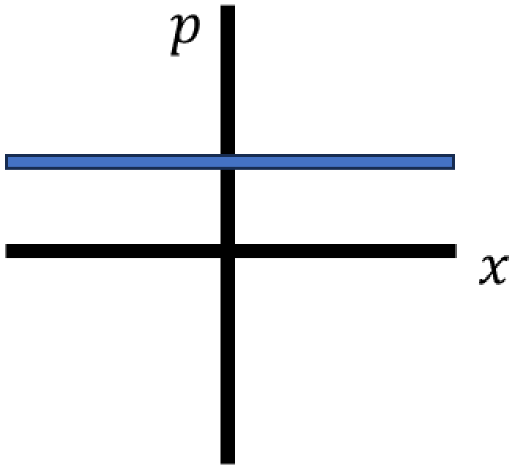
Coherent states $|\alpha\rangle$



Cartesian conjugate bases

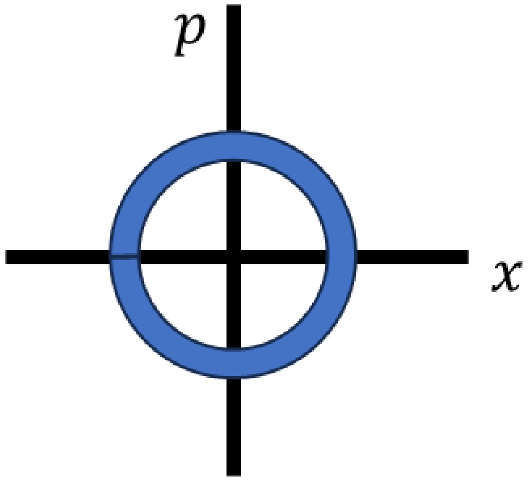


Position states $|x\rangle$

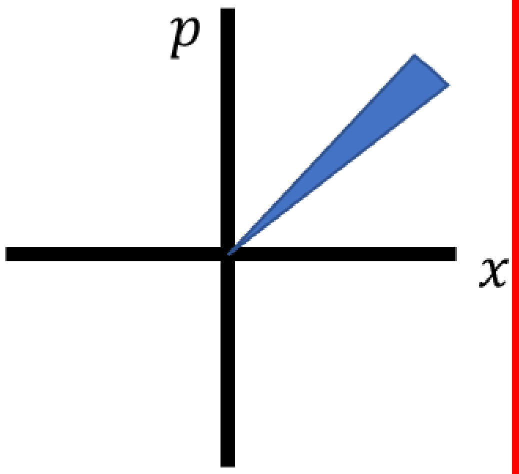


Momentum states $|p\rangle$

Angular "conjugate" bases



Fock states $|n\rangle$



Phase states $|\phi\rangle$

- Each is a "basis" for **states**.
- Each offers ways to **construct** new codes and **embed** old ones.
- Each conjugate pair comes with an "operator basis" for **errors**.

THE BIG THREE STATE SPACES

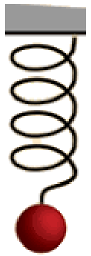
Qubit

Discrete position states $|0\rangle, |1\rangle$

Discrete momentum states $|\pm\rangle$

Pauli/Clifford groups

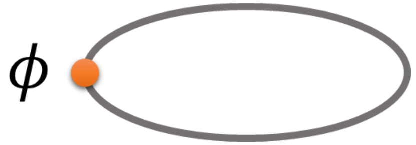
CV



Real position states $|x\rangle$

Real momentum states $|p\rangle$

Displacements & Gaussian ops.



Planar rotor

Angle position states $|\phi\rangle$

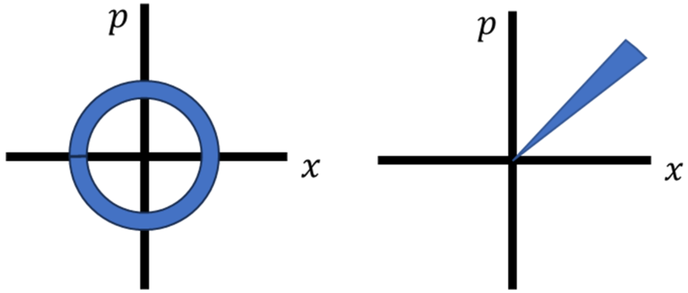
Integer momentum states $|\ell\rangle$

rotor Pauli/Clifford groups

arxiv:2311.07679

NUMBER-PHASE "ROTOR"

- **Phase** and **Fock states** are position and momentum states of a "rotor" with **no negative momentum**.
- "Momentum" kicks and Fock-space rotations mimc the momentum kicks and position shifts of a rotor.



"Momentum" kicks (photon loss/gain)

$$\hat{E}_\ell = \begin{cases} \sum_{n\geq 0} |n+\ell\rangle\langle n| & \ell \geq 0 \\ \sum_{n\geq 0} |n\rangle\langle n+|\ell|| = \hat{E}^\dagger_{|\ell|} & \ell < 0 \end{cases} \quad \text{for} \quad \ell \in \mathbb{Z}$$

Position shifts (rotations)

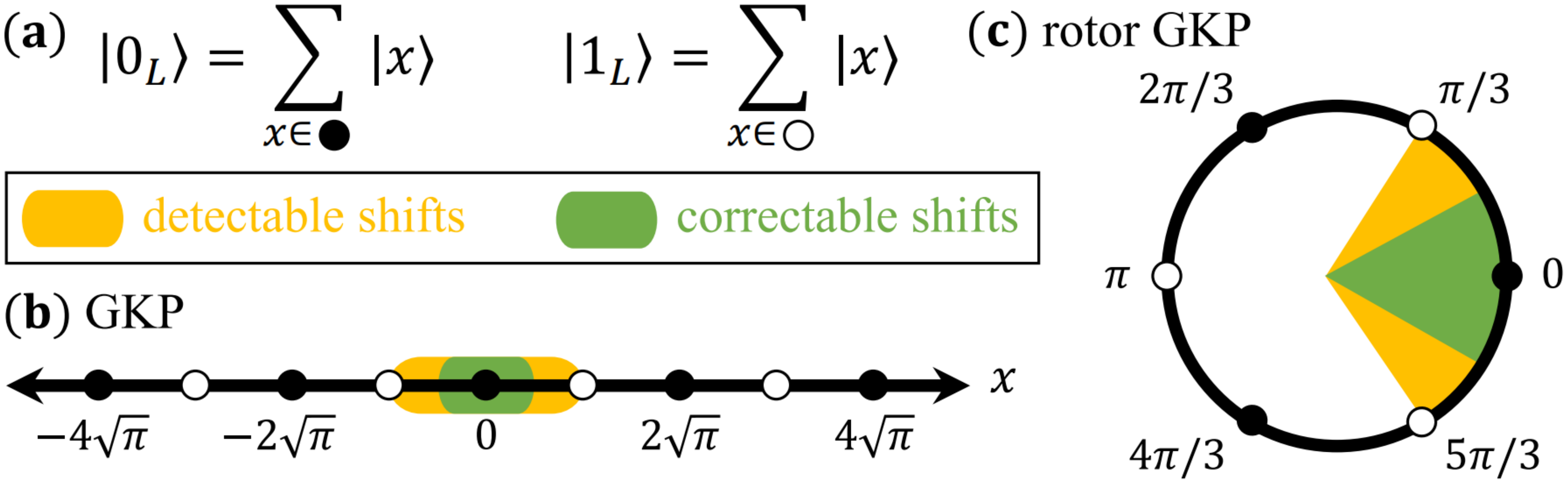
$$e^{i\phi\hat{n}} = \sum_{n\geq 0} e^{i\phi n}|n\rangle\langle n|$$

- Their products form a complete **basis.**
- Any physical (bounded) error operator can be **expanded**.

Pegg, Barnett, Susskind, Glogouwer, etc.; see arXiv:2211.05714, see also arXiv:2311.13670

ROTOR GKP CODES

- GKP codes can be similarly constructed for the rotor:
  line → circle
- Correct against
  - small **shifts** in the angular position
  - small **kicks** in angular momentum.



see errorcorrectionzoo.org/c/rotor_gkp

NUMBER-PHASE CODES

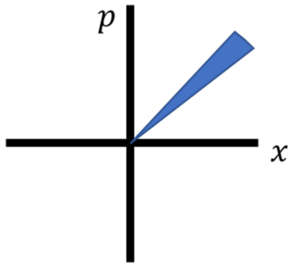
- **CV analogues** of bona-fide rotor GKP codes
- Spaced apart in **both** phase-state and Fock-state basis.
- Protect against Fock-space rotations and “momentum” kicks (photon losses/gains).

$$|0_{\text{num-ph}}\rangle = \left(|\phi = 0\rangle + \left|\phi = \tfrac{2\pi}{3}\right\rangle + \left|\phi = \tfrac{4\pi}{3}\right\rangle\right)/\sqrt{3}$$

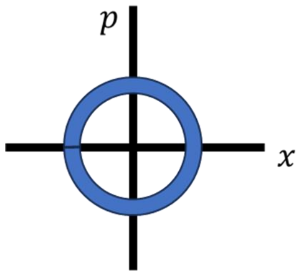
$$|1_{\text{num-ph}}\rangle = \left(\left|\phi = \tfrac{\pi}{3}\right\rangle + |\phi = \pi\rangle + \left|\phi = \tfrac{5\pi}{3}\right\rangle\right)/\sqrt{3}$$

$$|+_{\text{num-ph}}\rangle = \left(|0_{\text{num-ph}}\rangle + |1_{\text{num-ph}}\rangle\right)/\sqrt{2} \propto \sum_{n\geq 0} |6n\rangle$$

$$|-_{\text{num-ph}}\rangle = \left(|0_{\text{num-ph}}\rangle - |1_{\text{num-ph}}\rangle\right)/\sqrt{2} \propto \sum_{n\geq 0} |6n+3\rangle$$



Phase states
$|\phi\rangle$



Fock states
$|n\rangle$

- Number-phase codes come with a set of “stabilizers”, but it is **no longer a group**:

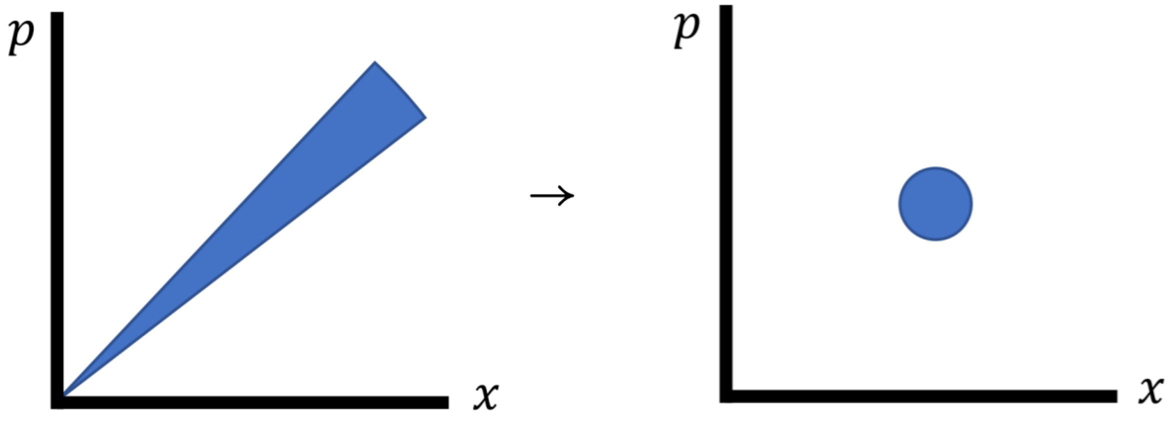
$$\mathsf{S}_{\text{num-phase}} = \{e^{i\frac{2\pi}{N}\hat{n}a}\,\hat{E}^{\dagger}_{2Nb} \mid a \in \mathbb{Z}_N,\, b \in \mathbb{N}_0\}$$

Combes, Baragiola, Grimsmo; see errorcorrectionzoo.org/c/number_phase

CAT CODES are regularized NUMBER-PHASE CODES

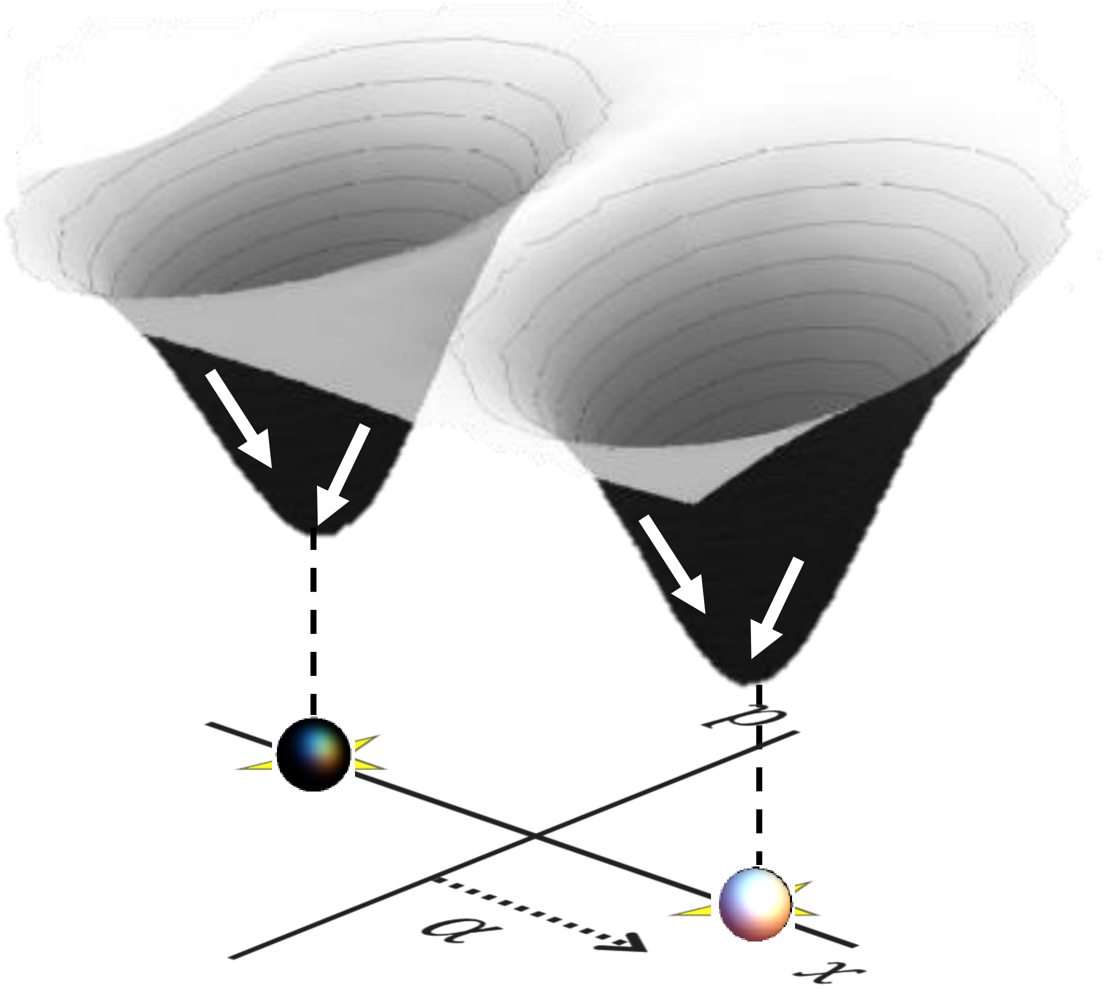
- Cat codes ~ "regularized" number-phase codes.

$$|\phi\rangle \propto \sum_{n\geq 0} e^{i\phi n}|n\rangle \quad \rightarrow \quad e^{-\frac{1}{2}\alpha^2}\sum_{n\geq 0}\frac{\alpha^n}{\sqrt{n!}}e^{i\phi n}|n\rangle$$



(a) phase state **(b)** coherent state

- Two-component cat code, $|\overline{0}\rangle = |\alpha\rangle$ and $|\overline{1}\rangle = |-\alpha\rangle$, "stabilized" by **quantum double well** Lindbladian master eqn.



$$\frac{\partial\rho}{\partial t} = \mathcal{L}(\rho) = 2F\rho F^\dagger - F^\dagger F\rho - \rho F^\dagger F$$

$$F = a^2 - \alpha^2$$

Cochrane, Milburn, Munro 1999, Leghtas et al 2013; see errorcorrectionzoo.org/c/cat

SINGLE-MODE CAT CODES

- Two-component cat codes: errorcorrectionzoo.org/c/two-legged-cat EC zoo
  1. **Hardware-efficient** CV version of repetition code.
  2. **Self-correcting** classical memory (vs. dephasing). arXiv:2008.02816, 2205.09767
  3. Several stabilization schemes (realized). arXiv:1312.2017, 1412.4633, 1605.09408, 1907.12131
  4. Practical X-type gates (realized). arXiv:1312.2017, 1705.02401
  5. **Bias-preserving** gates. arXiv:1904.09474, 1905.00450

- Four-component cat codes: errorcorrectionzoo.org/c/cat EC zoo
  1. **Protection vs. loss** in concatenated schemes. arXiv:1207.0679
  2. **Fault-tolerant** syndrome msmnt (realized). arXiv:1207.0679, 1311.2534, 1803.00102
  3. Track errors without correcting (realized)

1st break-even QEC arXiv:1311.2534

FOCK-STATE CODES

FOUR-QUBIT CODE → BINOMIAL CODE

- Add up individual binary labels of the [[4,1,2]] code of each basis state for each codeword:

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle) \longrightarrow |0_{\text{bin}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |4\rangle)$$

$$|1_L\rangle = \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle) \longrightarrow |1_{\text{bin}}\rangle = |2\rangle$$

- Finite-support version of the cat code.
- Maintains the same spacing in Fock space as before.

$$|0/1_{\text{bin}}\rangle = \frac{1}{2^{D/2}} \sum_{m\sim\text{even/odd}} \sqrt{\binom{D+1}{m}} |Nm\rangle$$

Michael et al 2015; see errorcorrectionzoo.org/c/binomial

FOCK-STATE CODES

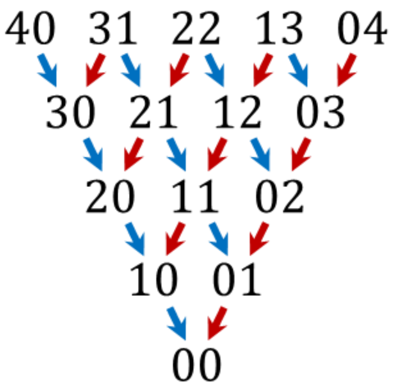
- **Finite-support** codes
- Codewords expressed conveniently w/ **Fock states**.

Binomial

$$|0/1_{\text{bin}}\rangle = \frac{1}{2^{D/2}} \sum_{m\sim\text{even/odd}} \sqrt{\binom{D+1}{m}} |Nm\rangle$$

Chuang-Leung-Yamamoto

$$|\bar{0}\rangle = \frac{1}{\sqrt{2}}(|40\rangle + |04\rangle)$$
$$|\bar{1}\rangle = |22\rangle \ .$$



Wasilewski-Banaszek + Ouyang-Chao

$$|\bar{0}\rangle = \frac{1}{\sqrt{3}}(|003\rangle + |030\rangle + |300\rangle)$$
$$|\bar{1}\rangle = |111\rangle$$

GNU code

$$|\pm\rangle = \sum_{m=0}^{D} \frac{(\pm 1)^m}{\sqrt{2^D}} \sqrt{\binom{D}{m}} |\text{Dicke}^n_{Nm}\rangle$$

Dual-rail: $|10\rangle$ and $|01\rangle$

Even more exotic codes:
- Matrix-model (Swingle et al)
- RG Cat
- Penrose tilings

see errorcorrectionzoo.org/c/fock_state

SYNDROMES

ERROR SYNDROMES

- Measuring linear or angular basis is TMI → syndromes are **modular**.
- Codes with continuous syndromes require **continuous** ancilla msmnts, large ancillas.
  - **In theory!** Yale expts went beyond break even with **first bit** of syndrome.

Bosonic code	Encoding	Check operators
analog stabilizer	logical modes	nullifiers
GKP	logical qudits	modular position & modular momentum
GKP-stabilizer	logical modes	modular position & modular momentum
number-phase	logical qudits	modular number & modular phase
cat	logical qudits	modular number
binomial & Chebyshev	logical qudits	modular number
dual-rail	logical qubit	number (error-detecting only)
CLY	logical qudit	number

BOSONIC EXPERIMENTS

modes	Name	Platforms
4	Niset-Andersen-Cerf	Photonics (Anderson group)
9	Lloyd-Slotine analog	Photonics (Furusawa group)
1	Binomial	Microwave cavities (Devoret, Sun groups)
1	Two-legged cat	Microwave cavities (Devoret, Leghtas, Wang groups, Alice & Bob)
1	Four-legged cat	Microwave cavities (Schoelkopf group; break-even QEC)
1	GKP	Trapped ions (Home group), Microwave cavities (Devoret group; beyond break-even), Photonics (Furusawa group)

For references, see:
https://errorcorrectionzoo.org/list/realizations

COMMERCIAL PROPOSALS

microwave

Yale

microwave

arXiv:1907.11729
arXiv:2204.09128

Cat + repetition

ALICE & BOB

Inria

phononic

arXiv:2012.04108, 2103.06994
https://youtu.be/fN5-UO2fy0c

Cat/GKP + rep-n/surface

aws Caltech

optical

arXiv:2010.02905
https://youtu.be/vzc53S3ACWw

GKP + cluster states

XANADU

optical

FBQC arXiv:2101.09310,
Interleaving arXiv:2103.08612,
https://youtu.be/E_dD1XUTaq4

Dual-rail + fusion-based QC

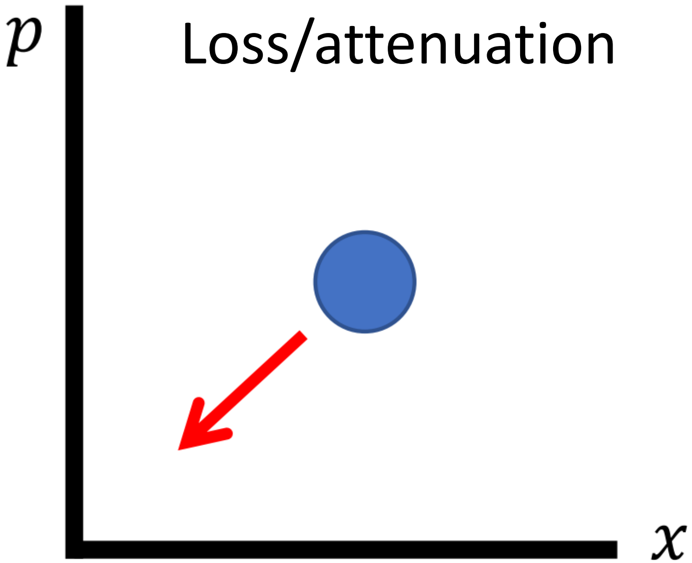
Ψ PsiQuantum

NOISE

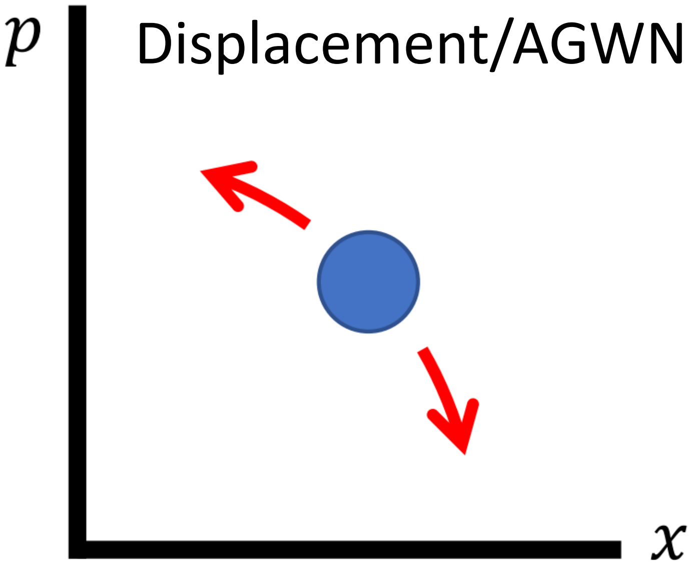
NOISE MODELS

Kraus form

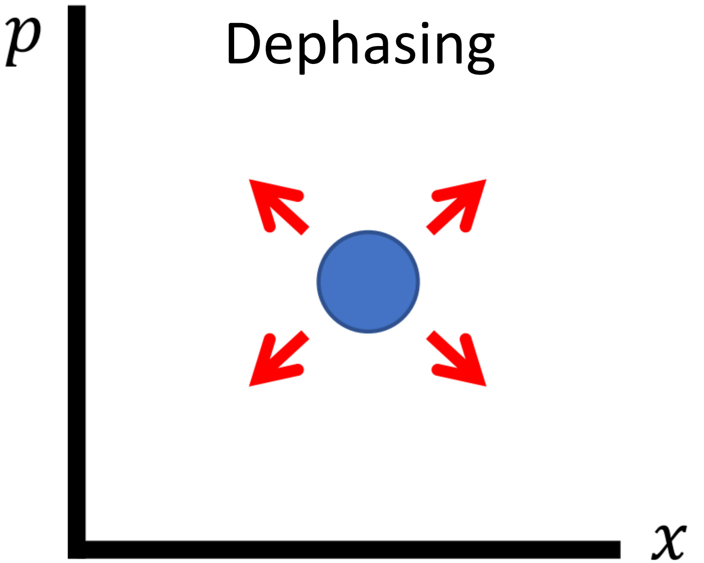
$$\mathcal{N}(\rho) = \sum_\ell N_\ell \rho N_\ell^\dagger$$



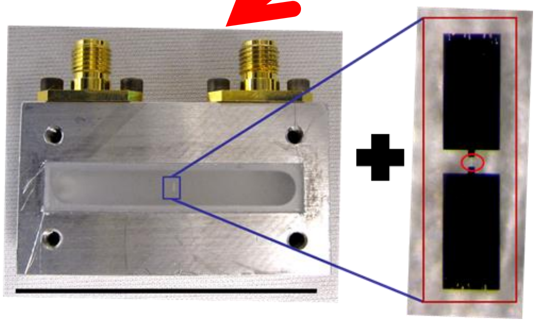
$$N_\ell^{\text{loss}} = \frac{(1-e^{-\chi})^{\ell/2}}{\sqrt{\ell!}} e^{-\frac{1}{2}\chi\hat{n}}\hat{a}^\ell$$



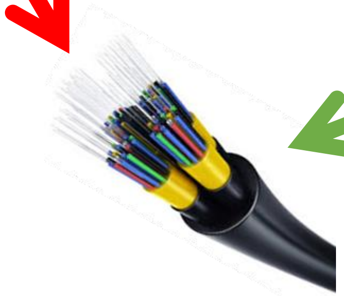
$$N_{\mathbf{q}}^{\text{disp}} = \frac{e^{-\frac{|\mathbf{q}|^2}{2}}}{\sqrt{\pi}} e^{i\chi(q_1\hat{x}-q_2\hat{p})}$$



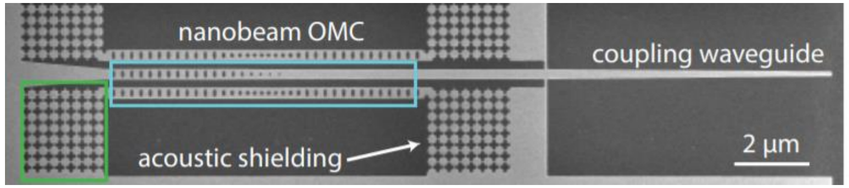
$$N_\ell^{\text{deph}} = \frac{\chi^{\ell/2}}{\sqrt{\ell!}} e^{-\frac{1}{2}\chi\hat{n}^2}\hat{n}^\ell$$



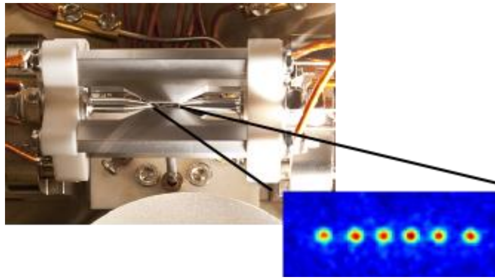
Microwave cavities + superconducting circuits



Optical fibers & cavities

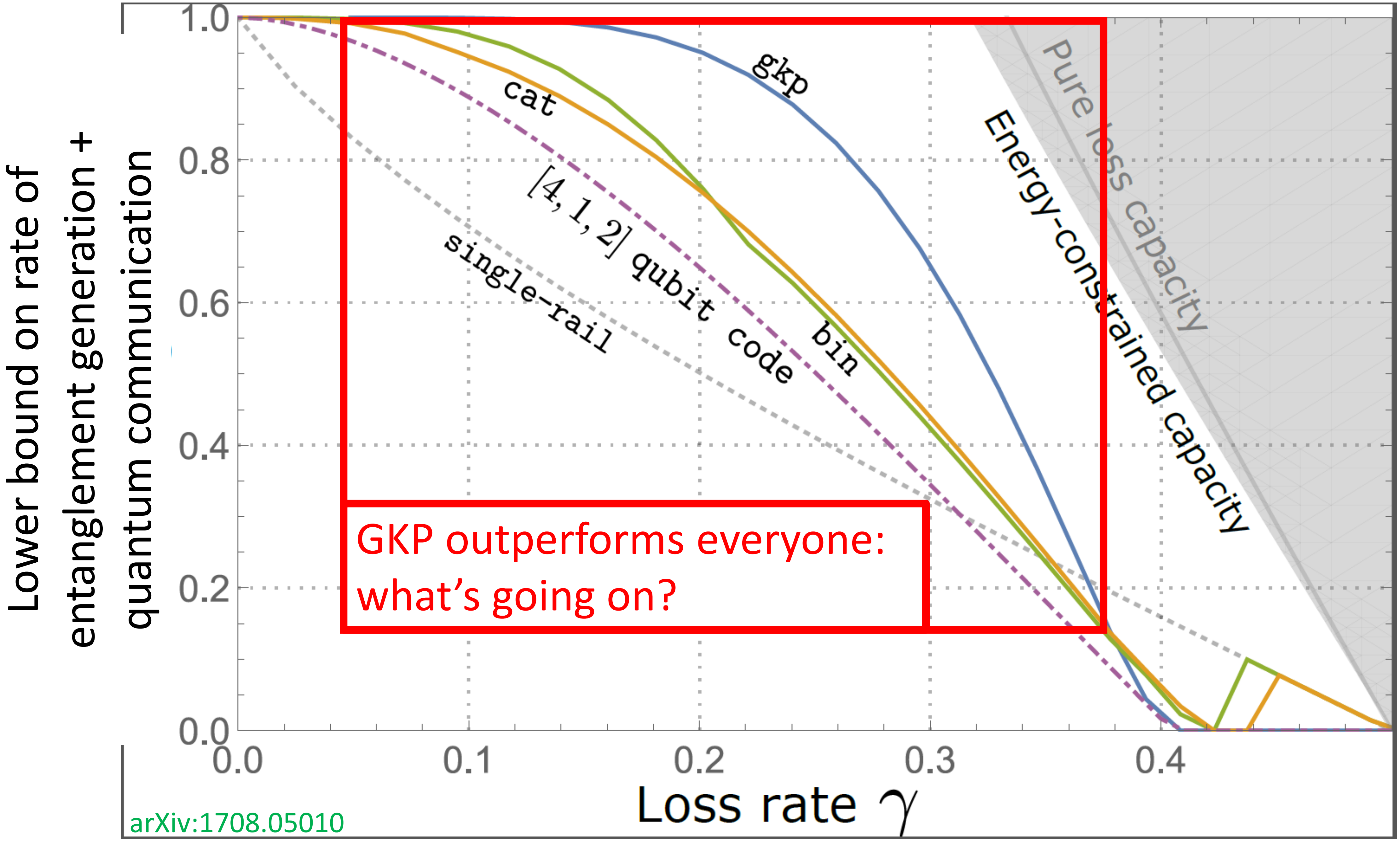


Nano-acoustic resonators



Motion of trapped ions

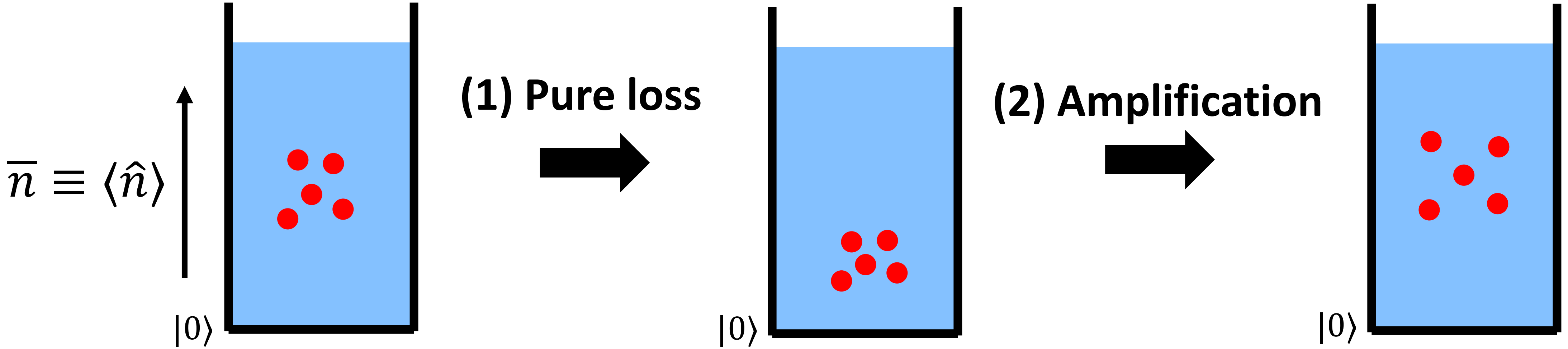
Hashing bound vs. loss rate γ for codes with energy constraint $\overline{n} \leq 10$



arXiv:1708.05010

GKP vs. PHOTON LOSS

- Consider a **particular (non-optimal) recovery operation** --- (A) amplification + (B) conventional GKP recovery.



(1) Pure loss + (2) Amplification = (3) Displacement noise

- But GKP codes can handle displacement!
- They **achieve capacity up to constant*** against loss channel
- Achieve hashing bound of displacement (AGWN) channel.

VVA, Noh, ... Jiang; arXiv:1708.05010v2; see also Cerf, Leuchs, Polzik book and Caruso Giovanetti Holevo, arXiv:quant-ph/0609013
*arXiv:quant-ph/0105058,arXiv:1708.07257,arXiv:1801.04731,arXiv:1801.07271; see L. Jiang's BBQ2024 talk

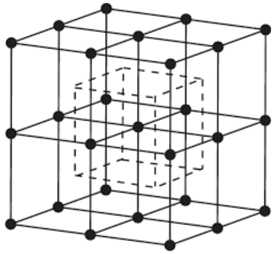
WRAPPING UP

MANY THINGS FIT INTO CV SYSTEMS!

Cartesian

Analog

GKP

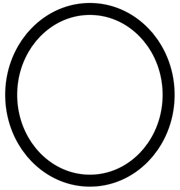


GKP-stabilizer

(degenerate lattice)

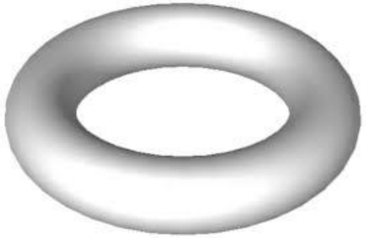
Angular

Number-phase (1 mode)



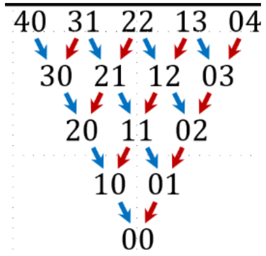
Quantum spherical (n-mode)

Homological number-phase (n-mode)



Fock-state

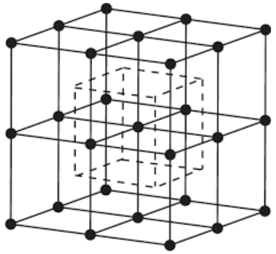
Chuang-Leung-Yamamoto, binomial, chi-squared, Ouyang-Chao permutation-invariant, matrix-model, pair-cat



MANY OF THEM ARE GROUPS...

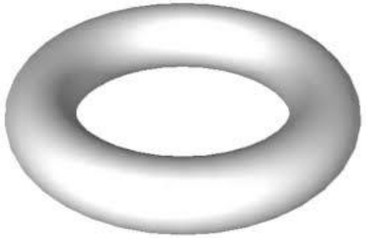
Analog

GKP

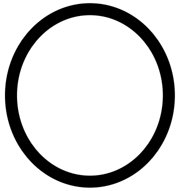


GKP-
stabilizer

Homological
number-phase
(n-mode)



Number-phase
(1 mode)



arXiv:1911.00099
errorcorrectionzoo.org/c/group_gkp

Space	G	H	Related code
n qubits	$\mathbb{Z}_2^n$	$\mathbb{Z}_2^m$	qubit CSS
n modular qudits	$\mathbb{Z}_q^n$	$\mathbb{Z}_q^m$	modular-qudit CSS
n modes	$\mathbb{R}^n$	$\mathbb{R}^m$	analog stabilizer
n modes	$\mathbb{R}^n$	$\mathbb{Z}^n$	multimode GKP
n modes	$\mathbb{R}^n$	$\mathbb{Z}^{m<n}$	GKP-stabilizer
planar rotor	$U(1)$	$\mathbb{Z}_n$	rotor GKP
rigid body	$SO(3)$	point group	molecular

Table I: Special cases of group GKP codes

...BUT SOME ARE NOT!

	Classical codewords are **elements** of an alphabet.	Quantum codewords are **functions** of an alphabet.
$\mathbb{Z}_2^n = \mathbb{F}_2^n$	bits	qubits
$\mathbb{F}_q^n$	q-ary strings	Galois qudits
$\mathbb{Z}_q^n$	q-ary strings over $\mathbb{Z}_q$	modular qudits
$\mathbb{R}^n$	reals	oscillators
G	finite group	group-valued qudit

ℓ^2 ()

Spheres?

Two-pt homogeneous spaces?

More general?

⋮

⋮

FUTURE DIRECTIONS

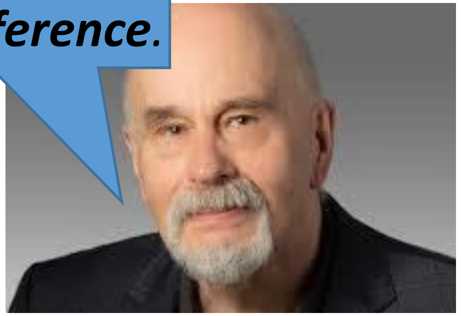
(emphasis mine)

1. Encodings utilizing the entire **multi**mode space.
2. **“Non-CSS”** encodings → non-uniform superpositions.
   - Non-symplectic lattice codes?
   - Non-CSS spherical codes? See next talk!
3. Code **bounds**, can they encompass all codes?
4. Understanding soft (**average-energy**) cutoffs.
5. Mapping **qubit primitives**:
   - Chain-complex industry
   - Qubit-inspired Fock-state codes
   - Designs [arXiv:2211.05127]
   - Schur-Weyl-Howe duality
   - Clifford hierarchy (for rotors?)
   - Spacetime circuits
6. There are more codes and **code classes** to explore.

*A bosonic code encoding one qubit in one mode will only have a **limited amount** of error-correction capability … . One worthwhile approach … is to find bosonic codes that use multiple modes **without requiring concatenation.***

arXiv:2210.15844

*I’m interested in **small codes** that make a **difference**.*



*A drink that makes you live **300 years**, but makes you function **10 times** slower.*

This week’s talks (paraphrasing)