Constructions and performance of hyperbolic and semi-hyperbolic Floquet codes

Oscar Higgott (Google Quantum AI)
Work completed while at UCL with Niko Breuckmann (Bristol)
arXiv: 2308.03750

Motivation: reducing QEC overhead

- Fault-tolerant applications require millions of qubits
- Surface codes and honeycomb codes incur an enormous overhead

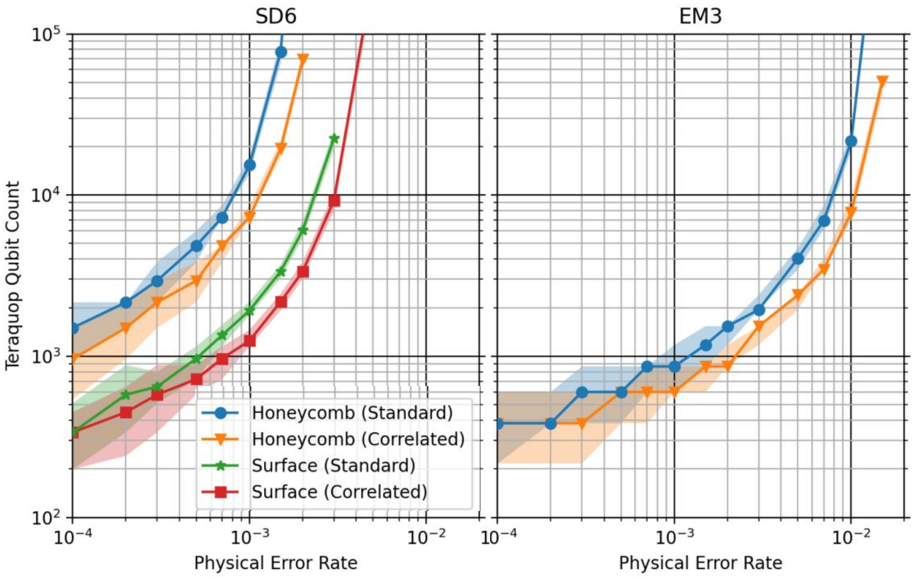
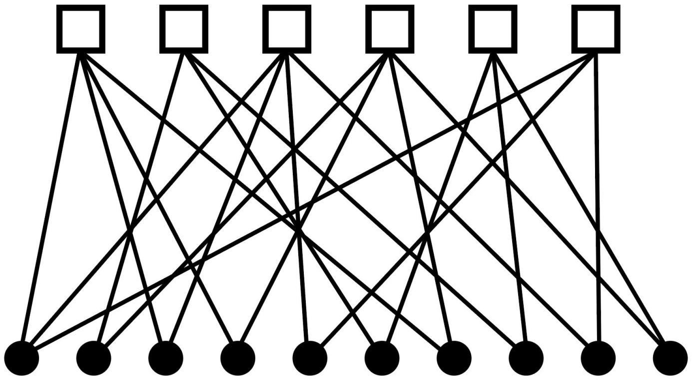


Figure from [1]

[1] Gidney, Craig, et al. "A fault-tolerant honeycomb memory." Quantum 5 (2021): 605.

Potential solution: LDPC codes

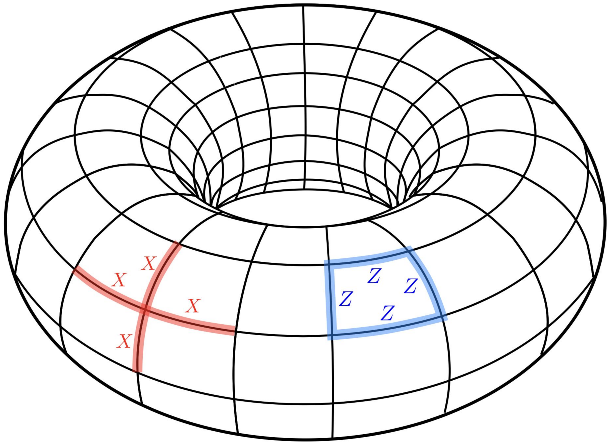


Code	Dimension *k*	Distance *d*
Toric	2	$\Theta(\sqrt{n})$
Hypergraph product	$\Theta(n)$	$\Theta(\sqrt{n})$
Good qLDPC	$\Theta(n)$	$\Theta(n)$
Hyperbolic	$\Theta(n)$	$\Theta(\log n)$

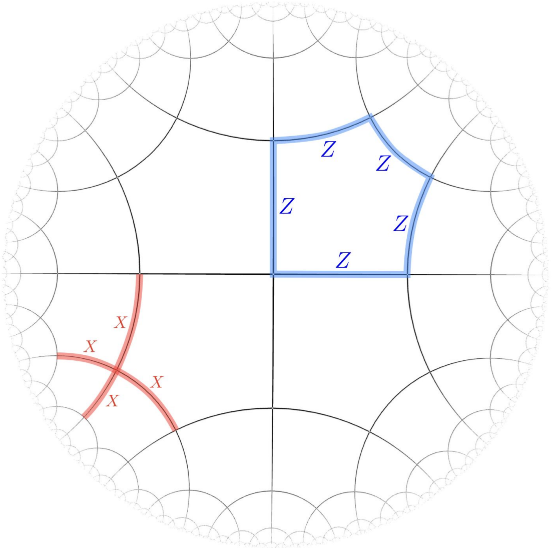
For practical purposes, we care about finite system size performance, not asymptotics

Background: hyperbolic surface codes

Toric code



Hyperbolic surface code



Genus = 1 ⟶ $k = 2$

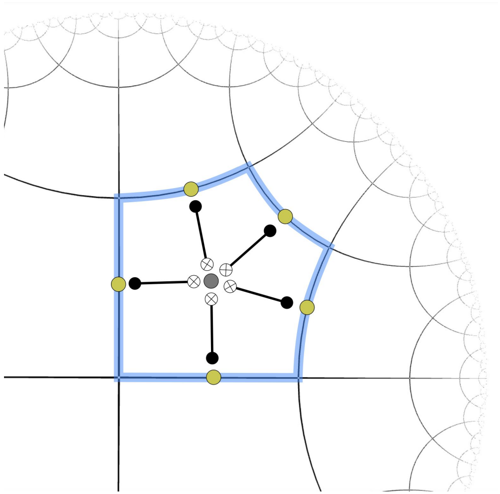
$d = \sqrt{n}$

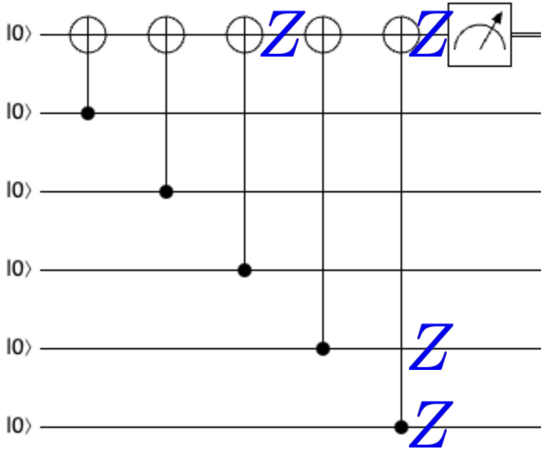
Genus scales with area ⟶ $k = O(n)$

$d = O(\log n)$

[1] Breuckmann, Nikolas P., and Barbara M. Terhal. "Constructions and noise threshold of hyperbolic surface codes." IEEE transactions on Information Theory 62.6 (2016): 3731-3744.

In practice we need a circuit, not just a code



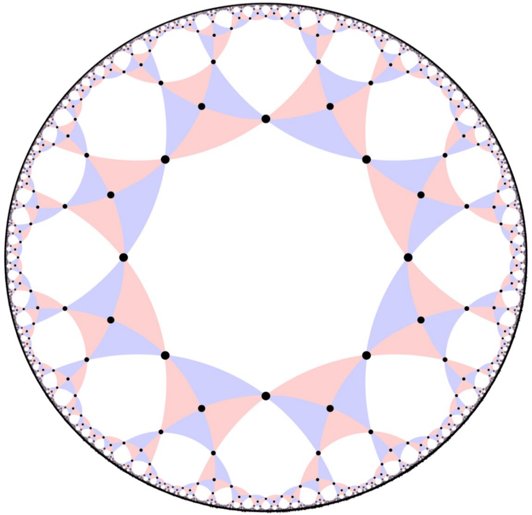


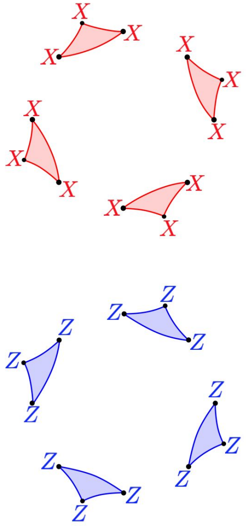
Quantum circuit constrained by:

- Idling, gate and measurement noise
- Native physical operations
- Qubit connectivity

Previous work: subsystem hyperbolic codes

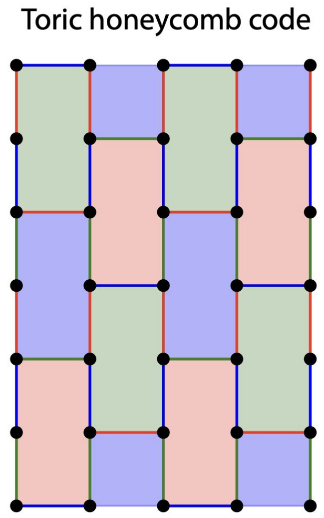
- Subsystem hyperbolic codes [2]:
  - Weight-3 checks
  - 4.3x saving for circuit noise
  - Pseudo-thresholds >0.5%
  - Flexibility arises from allowing ISG to vary
  - Can we do better?

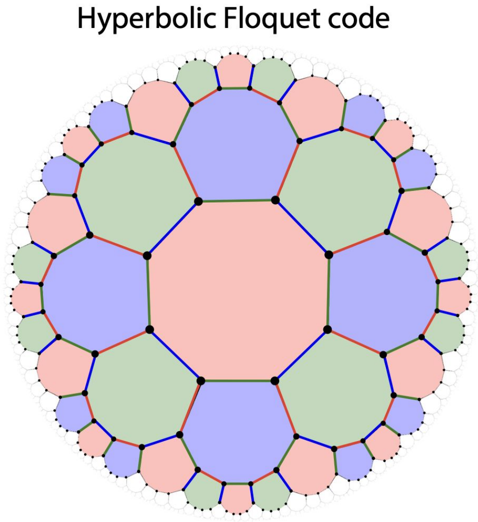




[1] Breuckmann, Nikolas P., and Barbara M. Terhal. "Constructions and noise threshold of hyperbolic surface codes." IEEE transactions on Information Theory 62.6 (2016): 3731-3744.
[2] Higgott, Oscar, and Nikolas P. Breuckmann. "Subsystem codes with high thresholds by gauge fixing and reduced qubit overhead." Physical Review X 11.3 (2021): 031039.

This talk



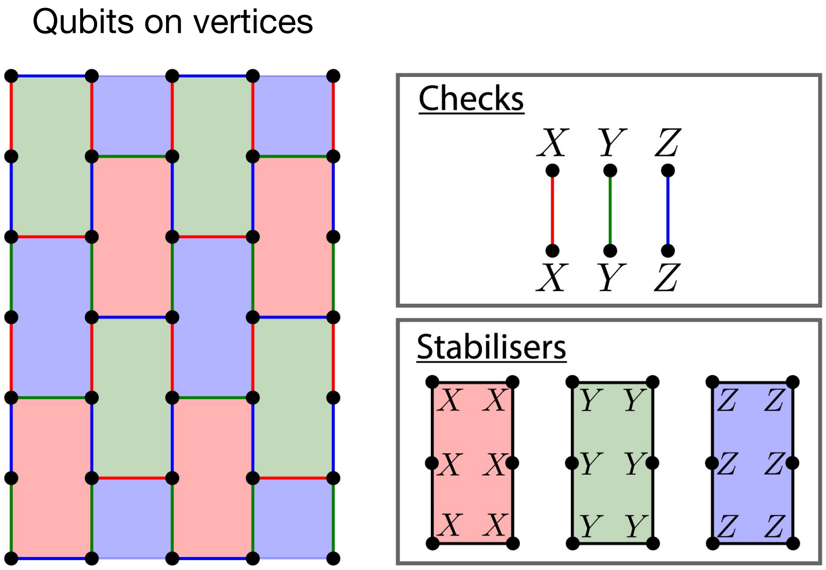


Properties:

- Weight-two checks
- Degree-three connectivity
- Biplanar & modular
- Efficient decoder (MWPM/UF)
- High threshold (pair measurements)

Floquet codes

- Defined on three-colourable lattice
  - Each edge is the colour of the faces it connects
- Measure weight-two anti-commuting checks
  - XX on red, YY on green, ZZ on blue

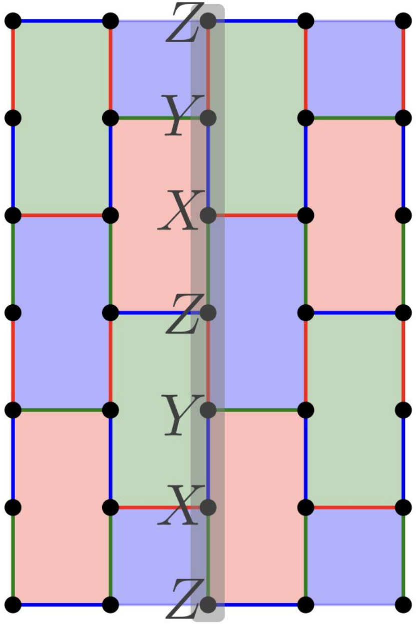


[1] Hastings, Matthew B., and Jeongwan Haah. "Dynamically generated logical qubits." Quantum 5 (2021): 564.

Empty subsystem code

- Treat each edge as a gauge generator
- Stabilisers are cycles of edge operators
  - Including homologically non-trivial loops!
- With all stabilisers fixed, no logical degrees of freedom

“Inner logical” commutes with checks



Floquet code schedule

- Measure:
  - XX on red edges
  - YY on green edges
  - ZZ on blue edges
- Can show this only measures *bi-coloured cycles*
- But bi-coloured cycles are homologically trivial
- Homologically non-trivial cycles preserved (“inner logical operators”)

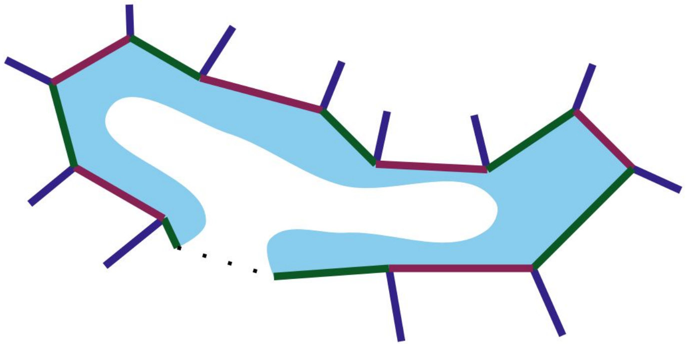
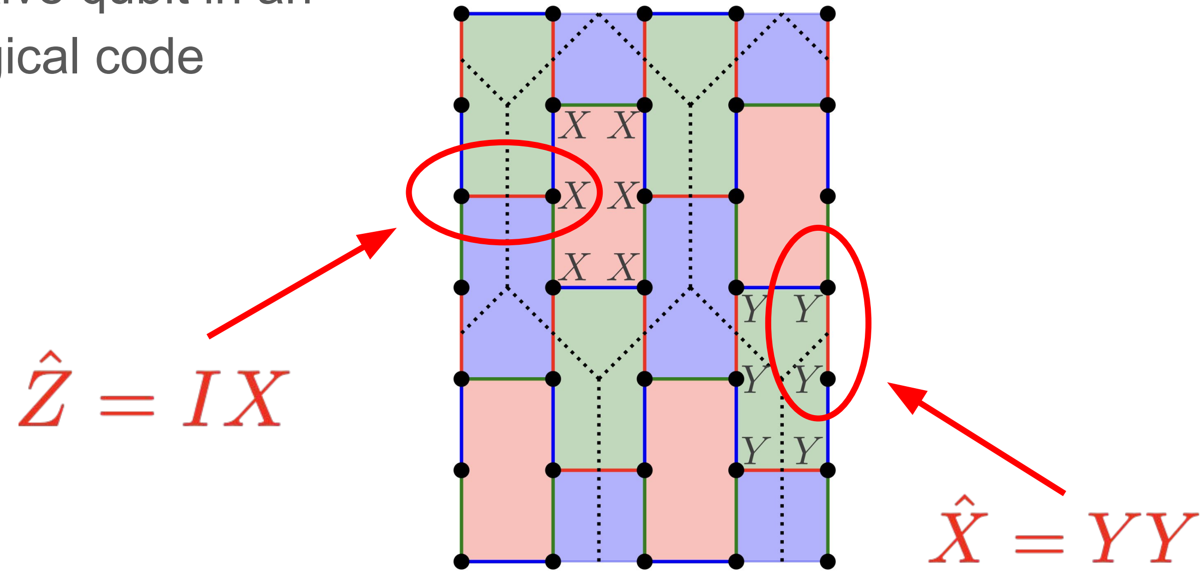


Figure from [2]

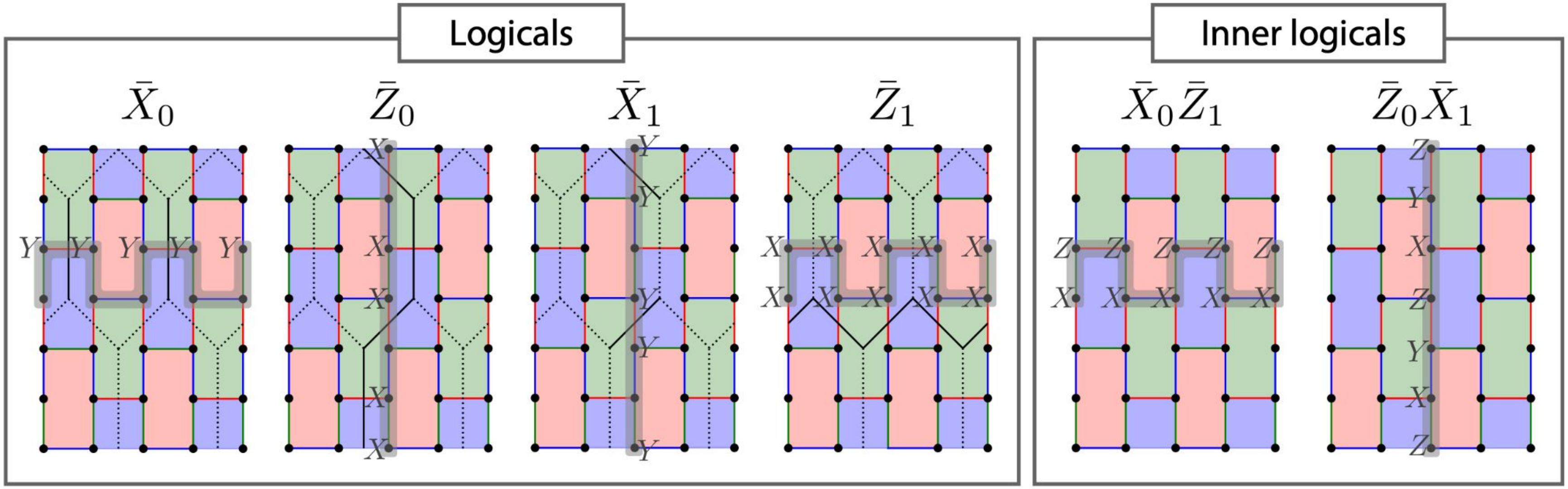
[1] Hastings, Matthew B., and Jeongwan Haah. "Dynamically generated logical qubits." Quantum 5 (2021): 564.
[2] Vuillot, Christophe. "Planar floquet codes." arXiv preprint arXiv:2110.05348 (2021).

The embedded homological code

- Each edge projected into a one-qubit subspace after measurement
- This forms an effective qubit in an embedded homological code



Logical operators for the toric honeycomb code



Decoding floquet codes

- Each “detector” compares a stabiliser in consecutive rounds
- Detectors formed from the parity of 12 edge measurements spanning 5 sub-rounds
- Can decode with MWPM or UF

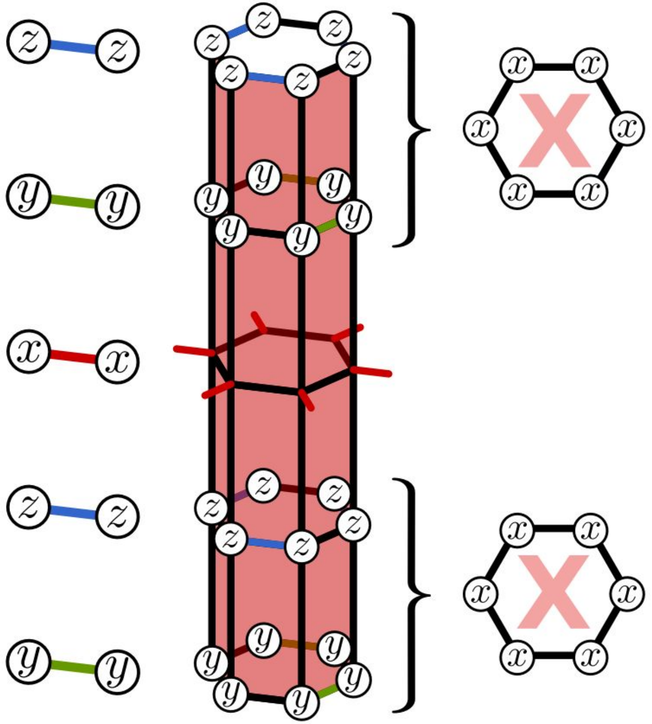
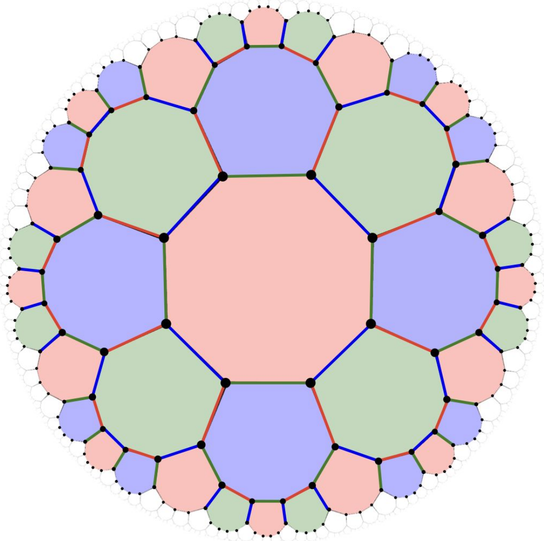


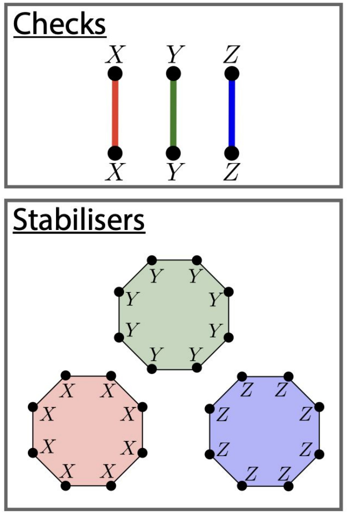
Figure from [1]

[1] Kesselring, Markus S., et al. "Anyon condensation and the color code." arXiv preprint arXiv:2212.00042 (2022).

Floquet codes from hyperbolic tilings

- Can construct a floquet code from *any* colour code tiling [1,2]
- Vuillot [2] proposed using hyperbolic colour code tilings
- Asymptotic parameter scaling:
  - Finite encoding rate k/n
  - Distance: log(n)
  - $kd^2/n=\log^2(n)$
- Explicit examples not previously studied





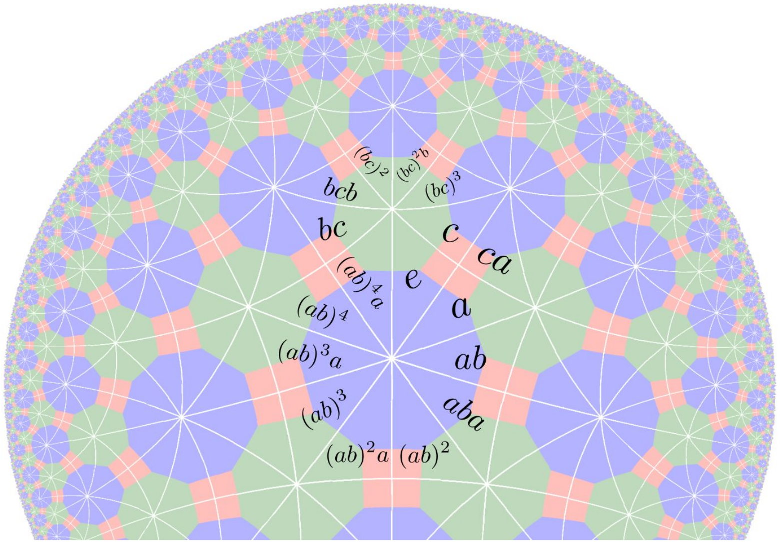
[1] Hastings, Matthew B., and Jeongwan Haah. "Dynamically generated logical qubits." Quantum 5 (2021): 564.
[2] Vuillot, Christophe. "Planar floquet codes." arXiv preprint arXiv:2110.05348 (2021).

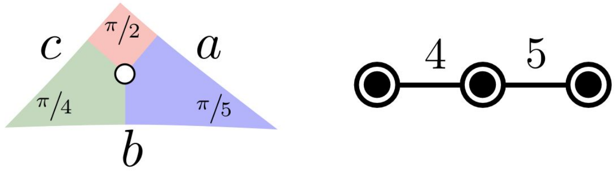
Hyperbolic colour code tilings

Wythoff's kaleidoscopic construction:

- Tiling generated through reflections across the sides of a triangle
- Can generate r.g.b uniform tiling from triangle group Δ(r/2,g/2,b/2)
- Finite tilings generated from quotients of Δ(r/2,g/2,b/2)
- Hyperbolic if 1/r + 1/g + 1/b < 1/2

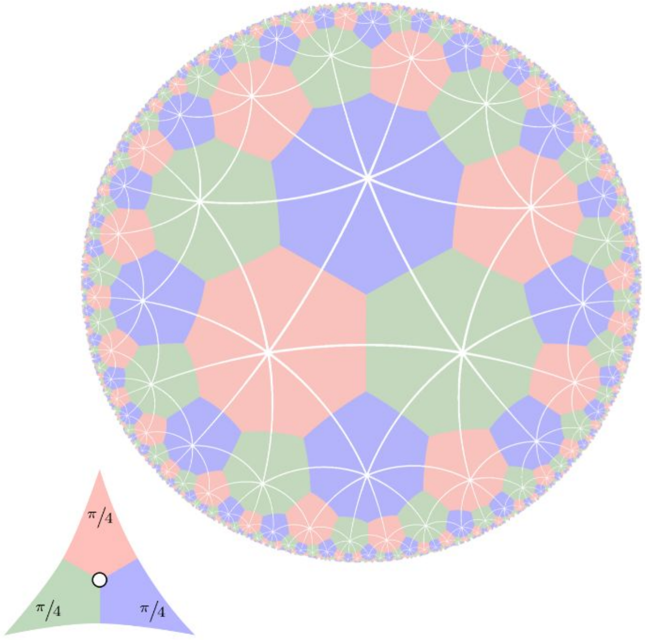
4.8.10 uniform tiling



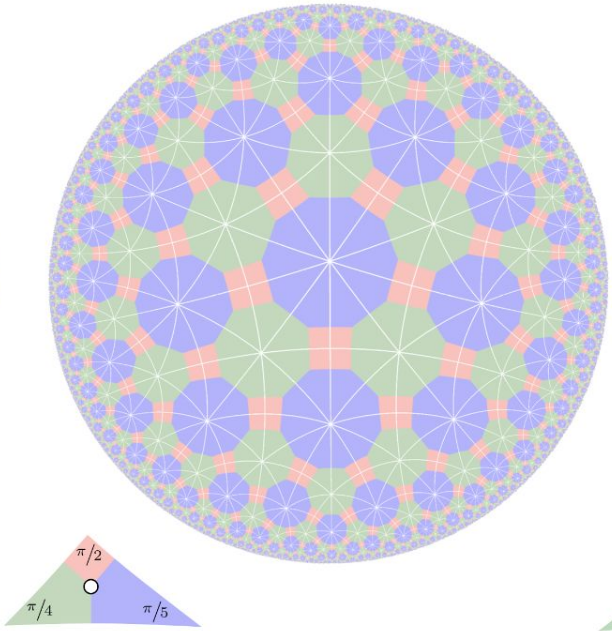


Families of uniform tilings we construct

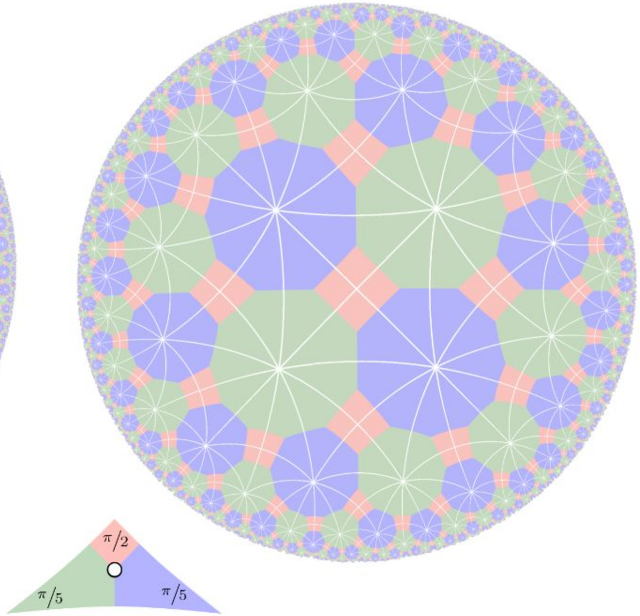
8.8.8



4.8.10

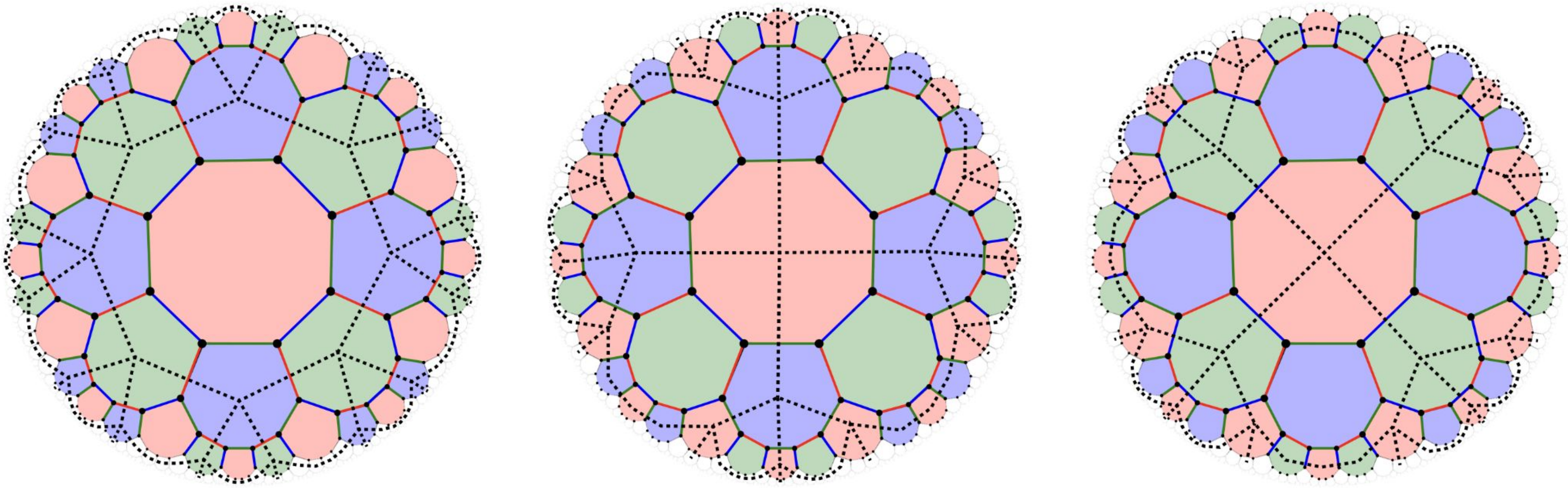


4.10.10



Embedded homological codes of 8.8.8 Floquet codes

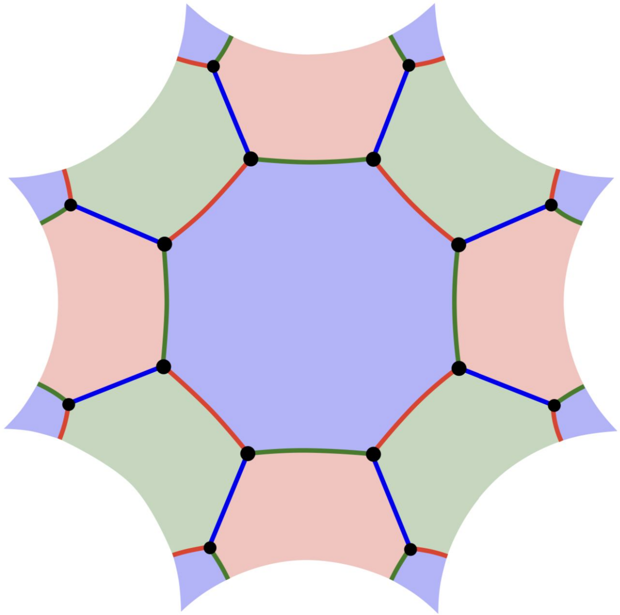
Red restricted lattice — Green restricted lattice — Blue restricted lattice



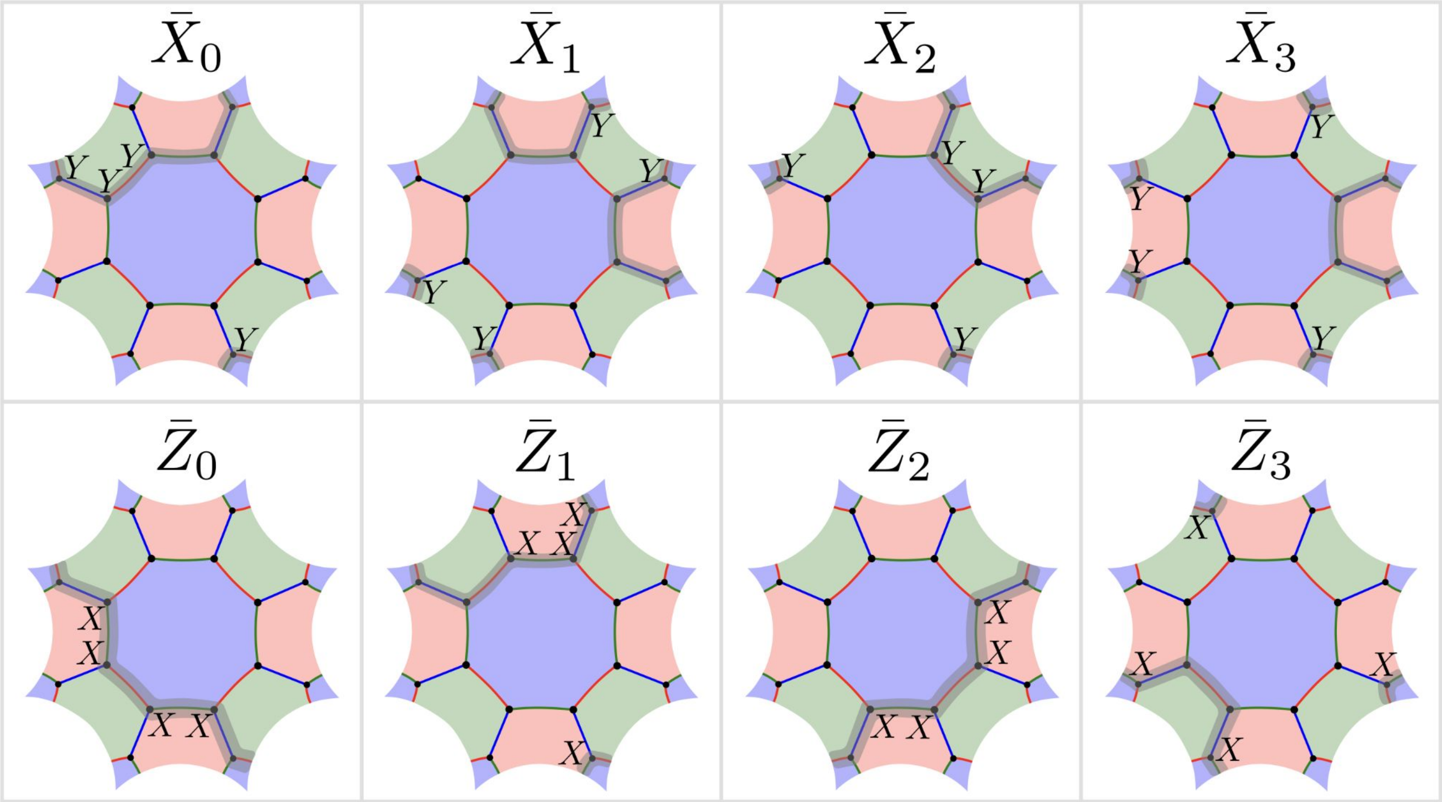
Definition: The *embedded* distance is the minimum distance of any of the three embedded homological codes

Small example: Bolza Floquet code

- Floquet code from 8.8.8 tiling of the genus-2 Bolza surface
- Opposite sides are identified
- Encodes four logical qubits into 16 physical qubits

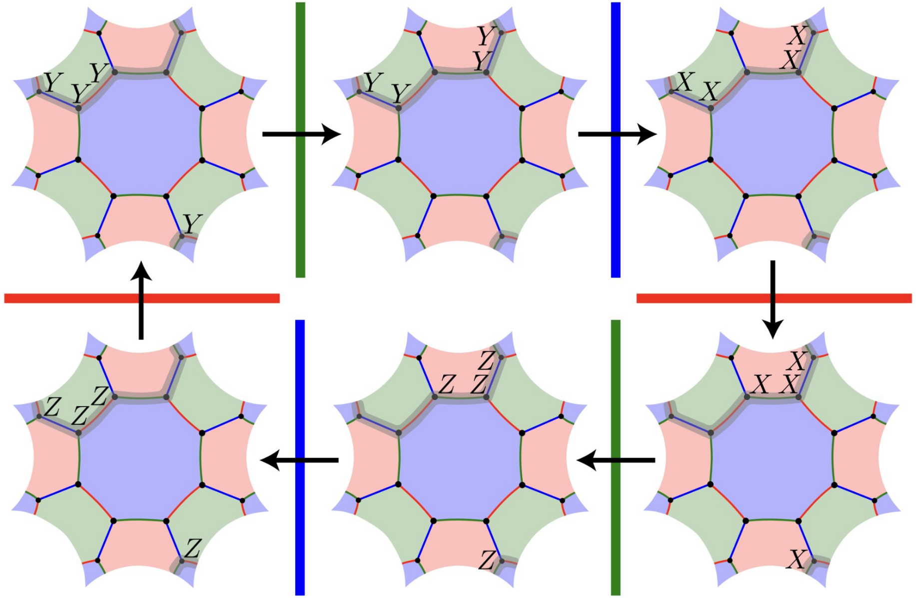


Logical operators of the Bolza code



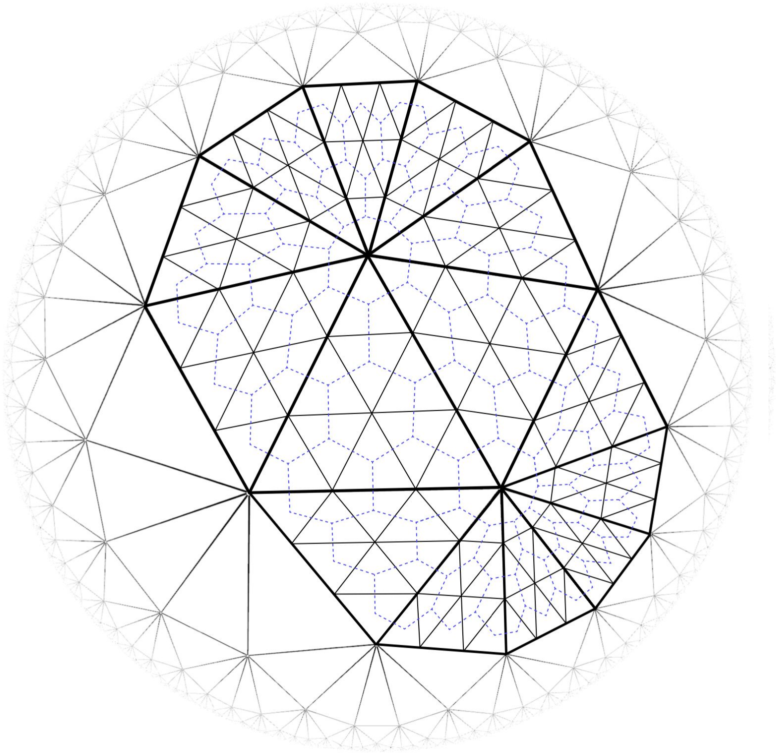
Movement of the logical operators

- No static generating set of logical operators for Floquet codes
- After C sub-round, for C in {R,G,B}:
  - Multiply C-checks into logical operator if they lie within homologically non-trivial logical path



Semi-hyperbolic Floquet codes

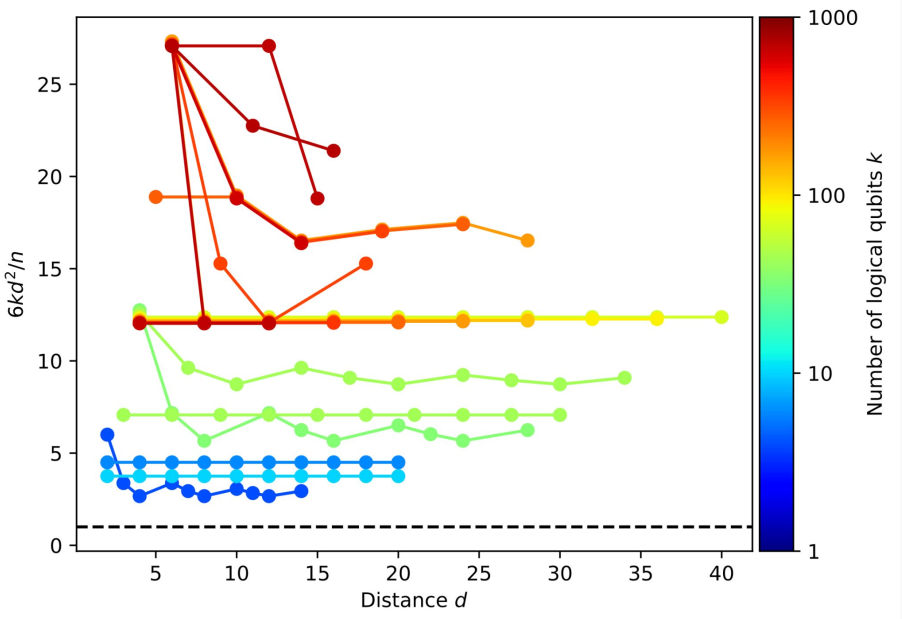
- So far: logarithmic distance
- Need 10^{-12} logical error rates
- Can weaken curvature using semi-hyperbolic lattices
- Distance scales as $\sqrt{n}$
- Can achieve $n/k = cd^2$ for smaller c



[1] Breuckmann, Nikolas P., et al. "Hyperbolic and semi-hyperbolic surface codes for quantum storage." Quantum Science and Technology 2.3 (2017): 035007.

Parameter improvement for semi-hyperbolic families

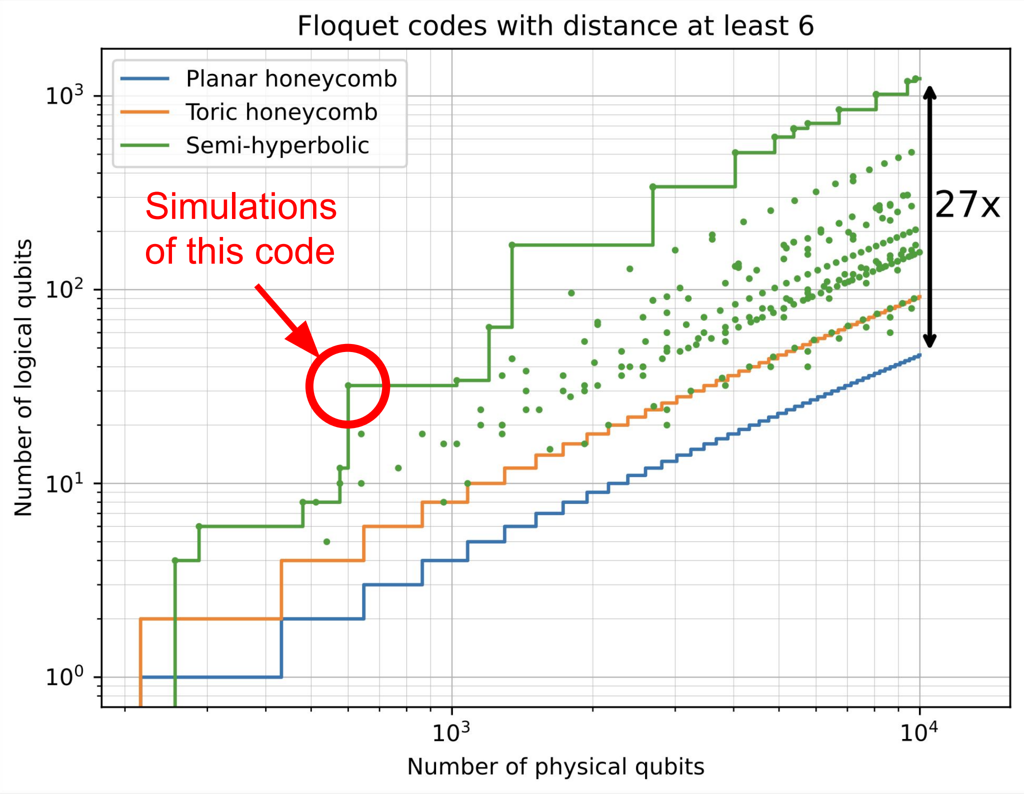
- Does fine-graining reduce advantage relative to honeycomb codes?
- Each line is a semi-hyperbolic code family
- Small (or no) reduction as distance scaled up with fine-graining



Logical qubits protected with embedded distance at least 6

Parameter improvement over planar honeycomb codes is:

- 6x using 300 qubits
- 10x-15x using 600 qubits
- 27x using 1400 qubits



Circuits and noise models

EM3 [1,2]

- Direct XX, YY and ZZ measurements
- Two-qubit depolarising noise correlated with measurement error
- Majorana-inspired [1]

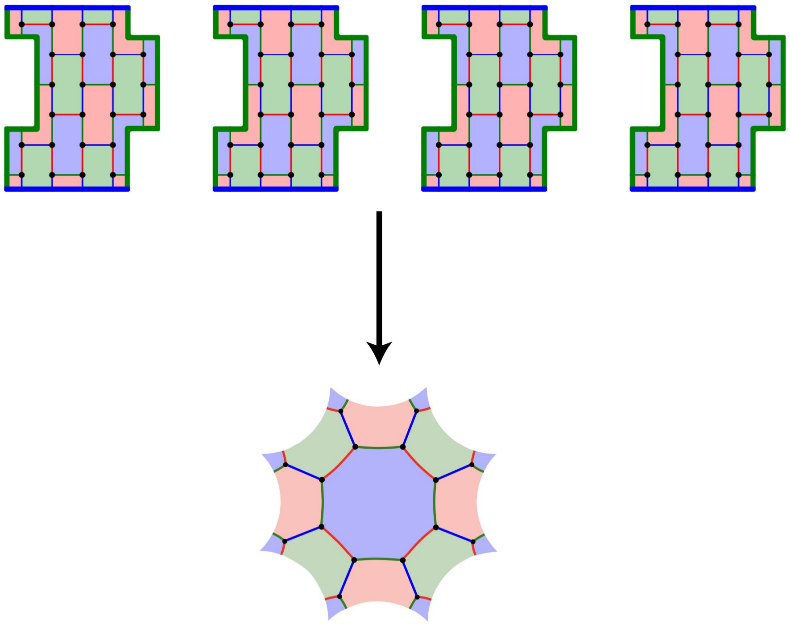
SD6 [2]

- Standard circuit-level depolarising noise
- Use an ancilla and CNOTs for each edge measurement

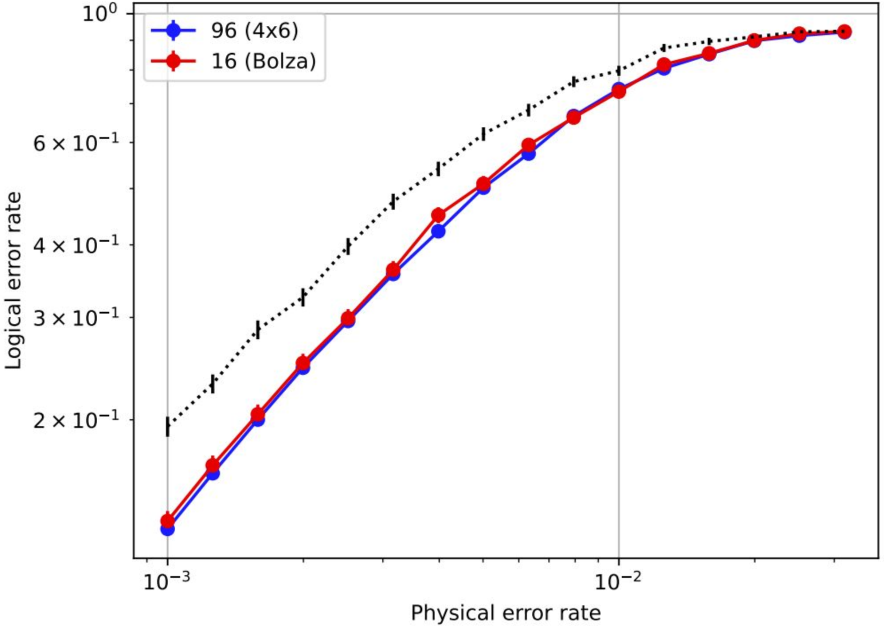
[1] Chao, Rui, et al. "Optimization of the surface code design for Majorana-based qubits." Quantum 4 (2020): 352.
[2] Gidney, Craig, et al. "A fault-tolerant honeycomb memory." Quantum 5 (2021): 605.

Small example: Bolza code, four logicals (EM3)

96 qubits

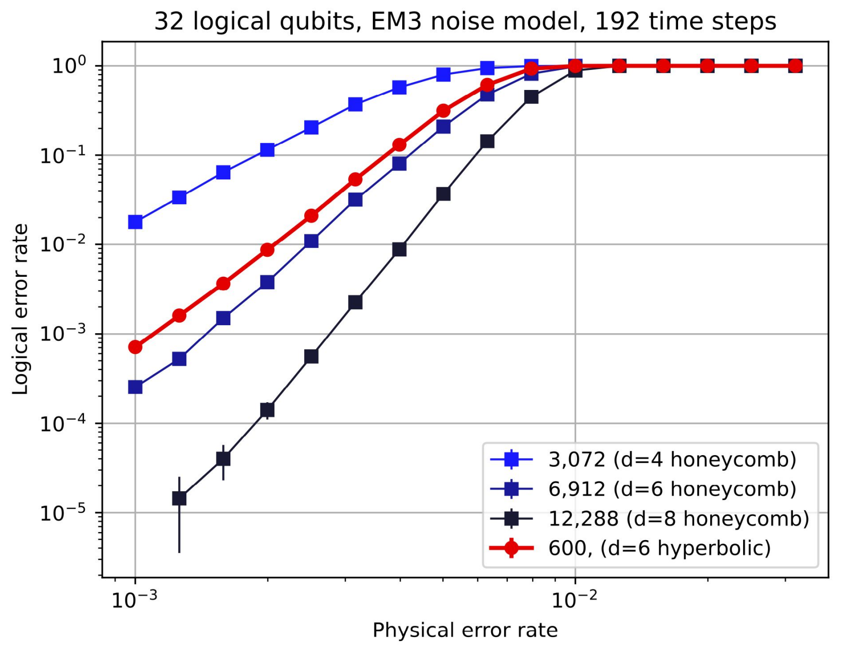


16 qubits



Medium-sized example: encoding 32 logicals (EM3)

- 4.10.10 tiling
- Hyperbolic: 600 qubits, d=6
- Honeycomb: 6,912 qubits
- ~12x improvement over honeycomb
- >20x improvement over surface



Large example: 674 logical qubits

EM3

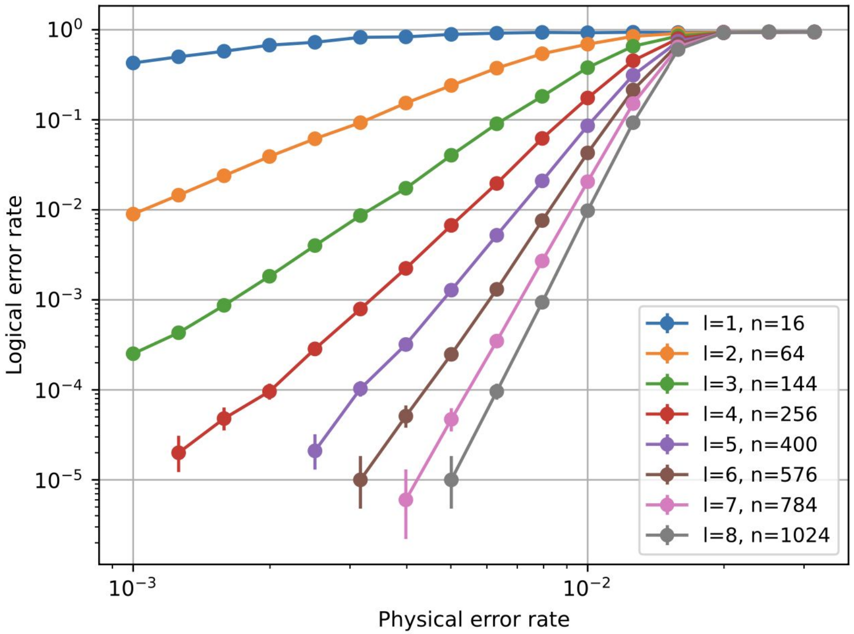
- 48x fewer qubits than honeycomb codes
- 21,504 physical qubits
- Teraquop footprint of 32 physical per logical
- Distance 12

SD6

- 30x fewer qubits than honeycomb codes
- 53,760 physical qubits
- 5.6x fewer qubits than surface codes
- Distance ~21

Semi-hyperbolic threshold: Bolza surface

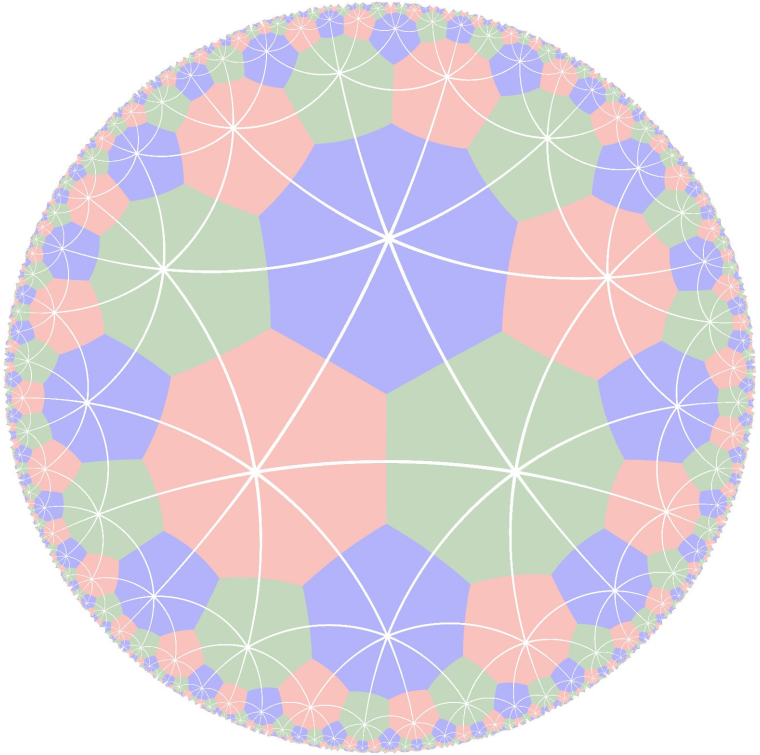
- 1.5%-2% threshold with EM3 noise
- Consistent with honeycomb code



How to implement hyperbolic connectivity?

Two-qubit gates local on hyperbolic surface. We propose two architectures:

- Biplanar
- Modular



Background: thin planar architecture [1]

- Use few layers of couplers
- Couplers within each layer do not cross, but may be long-range
- Number of layers given by *thickness* of qubit connectivity graph

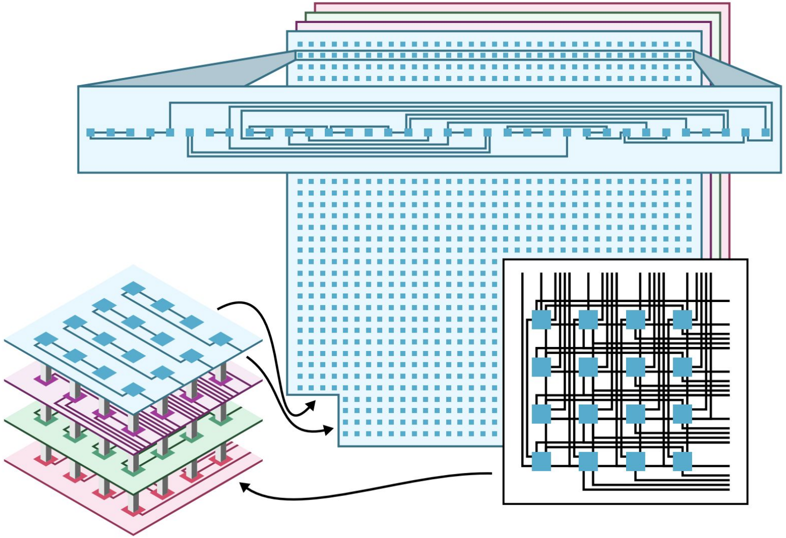


Figure from [1]

[1] Tremblay, Maxime A., Nicolas Delfosse, and Michael E. Beverland. "Constant-overhead quantum error correction with thin planar connectivity." Physical Review Letters 129.5 (2022): 050504.
[2] Halton, John H. "On the thickness of graphs of given degree." Information Sciences 54.3 (1991): 219-238.

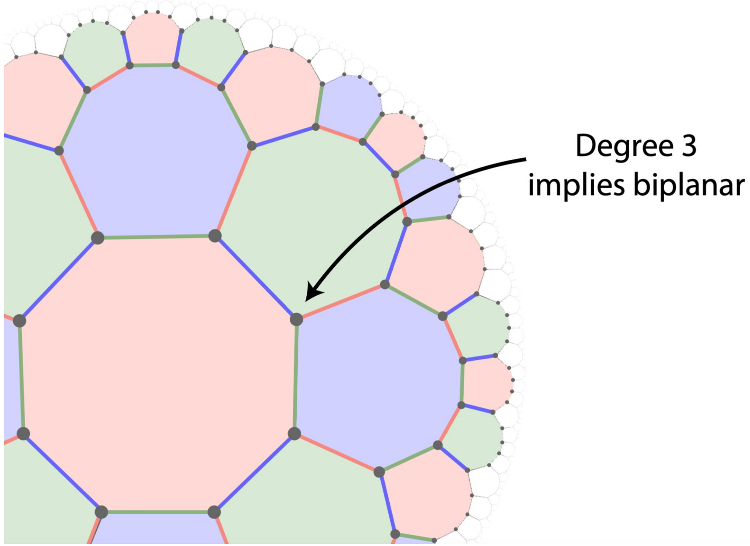
Background: graph thickness results

- The **thickness** of a graph is the minimum number of planar subgraphs into which the graph can be decomposed
- Any graph of **degree *d*** has thickness at most **ceil(d/2)** [1, Corollary 5]
- Any planar graph always has a planar representation in which the nodes are placed in arbitrary positions [1, Theorem 8]

[1] Halton, John H. "On the thickness of graphs of given degree." Information Sciences 54.3 (1991): 219-238.

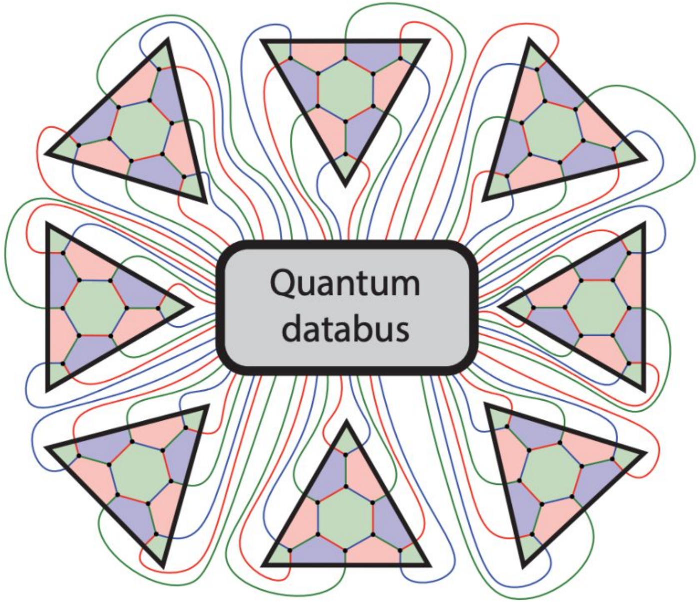
Biplanar architecture

- Floquet code circuits are degree 3
- Can use *two* layers of couplers between qubits
- Couplers within each layer do not cross (planar graph) but may be long-range



Modular architecture

- Small modules with Euclidean connectivity
- Long-range connections between modules
- Tolerance to noisy links between modules? [1,2]



[1] Fowler, Austin G., et al. "Surface code quantum communication." Physical review letters 104.18 (2010): 180503.
[2] Ramette, Joshua, et al. "Fault-Tolerant Connection of Error-Corrected Qubits with Noisy Links." arXiv preprint arXiv:2302.01296 (2023).

Comparison with other recent work on LDPC codes

- Practical implementations of hypergraph and lifted product codes in [1,2]
- [1,2] perform better for standard depolarising noise (larger savings for smaller system sizes)
- Hyperbolic Floquet codes perform better for pair measurements
- [3] also constructs hyperbolic Floquet codes, uses a different noise model and contains new examples

[1] Bravyi, Sergey, et al. "High-threshold and low-overhead fault-tolerant quantum memory." arXiv preprint arXiv:2308.07915 (2023).
[2] Xu, Qian, et al. "Constant-overhead fault-tolerant quantum computation with reconfigurable atom arrays." arXiv preprint arXiv:2308.08648 (2023)
[3] Fahimniya, Ali, et al. "Fault-tolerant hyperbolic Floquet quantum error correcting codes." arXiv preprint arXiv:2309.10033 (2023).

Comparison with IBM’s Bivariate Bicycle codes

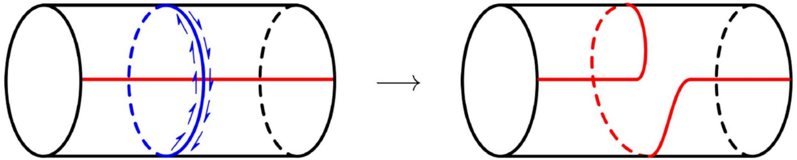
	Bivariate Bicycle [1]	Hyperbolic Floquet
Overhead reduction:	~13x	~12x
System size:	288 qubits	600 qubits
Noise model:	Standard circuit-level	Pair measurements
Connectivity degree:	6	3
Decoder:	BP+OSD (cubic runtime)	MWPM (linear runtime)

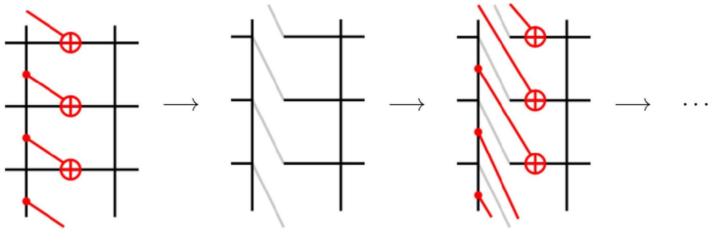
[1] Bravyi, Sergey, et al. "High-threshold and low-overhead fault-tolerant quantum memory." arXiv preprint arXiv:2308.07915 (2023).

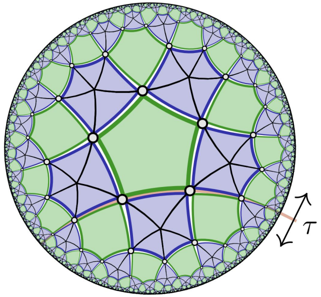
Future work: logical gates

Adapt techniques for hyperbolic codes:

- Dehn twists & lattice surgery [1]
  - Constant cumulative degree?
  - Preserve biplanarity?
- Fold-transversal gates [2]







[1] Breuckmann, Nikolas P., et al. "Hyperbolic and semi-hyperbolic surface codes for quantum storage." Quantum Science and Technology 2.3 (2017): 035007.
[2] Breuckmann, Nikolas P., and Simon Burton. "Fold-transversal Clifford gates for quantum codes." arXiv preprint arXiv:2202.06647 (2022).

Conclusions

- Constructed Floquet codes from hyperbolic and semi-hyperbolic tilings
- For pair measurement architectures (EM3):
  - >48x more efficient than honeycomb and surface codes
  - Reach teraquop regime with 32 physical qubits per logical qubit
- Small examples with as few as 16 qubits, experimentally feasible
- All constructions implementable in biplanar or modular architectures
- Efficient to decode with MWPM or UF

Thank you