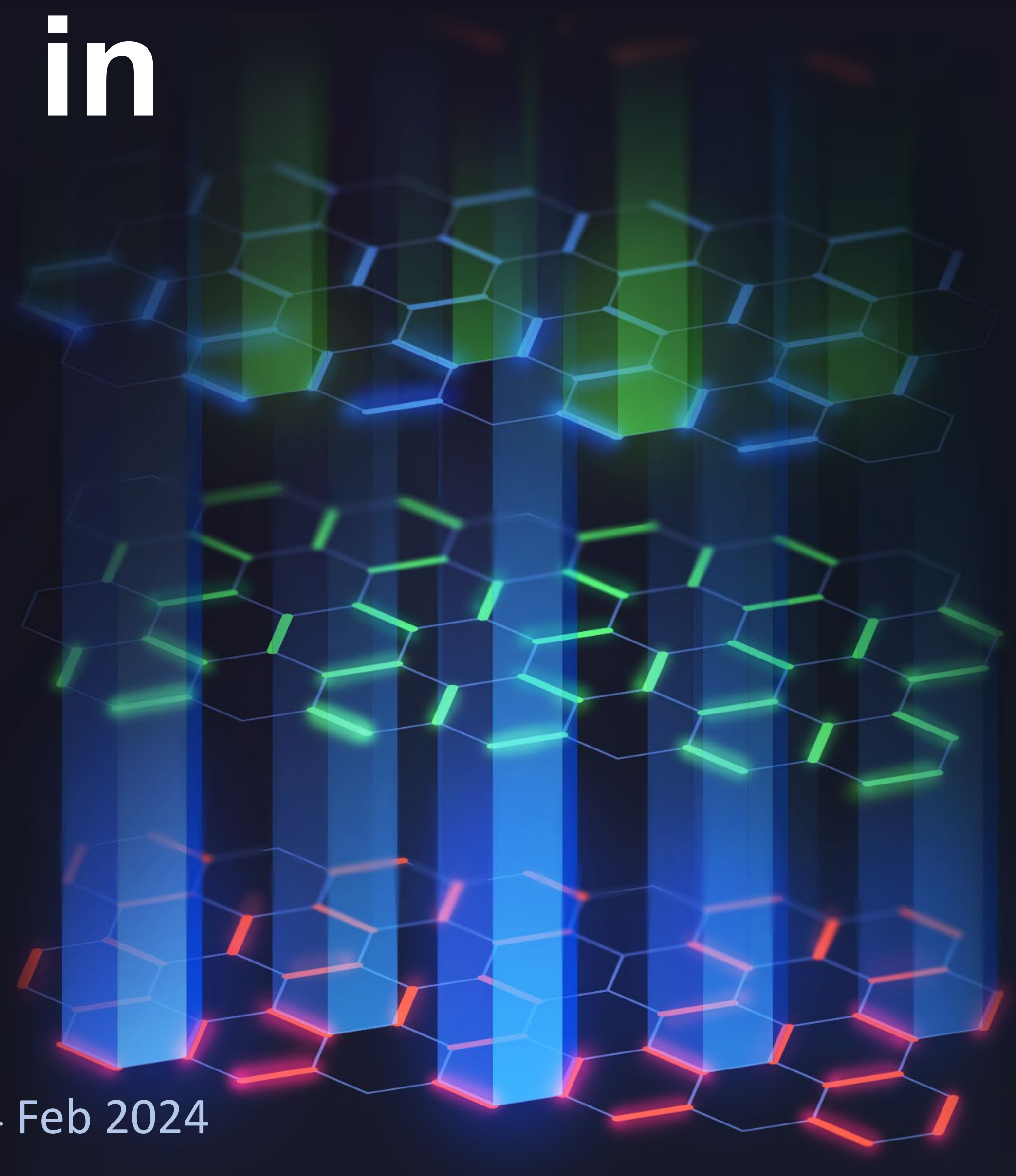
Logical operations in Floquet codes



Margarita Davydova (MIT)

Advances in Quantum Coding Theory, Simons Institute, 14 Feb 2024

arXiv:2307.10353

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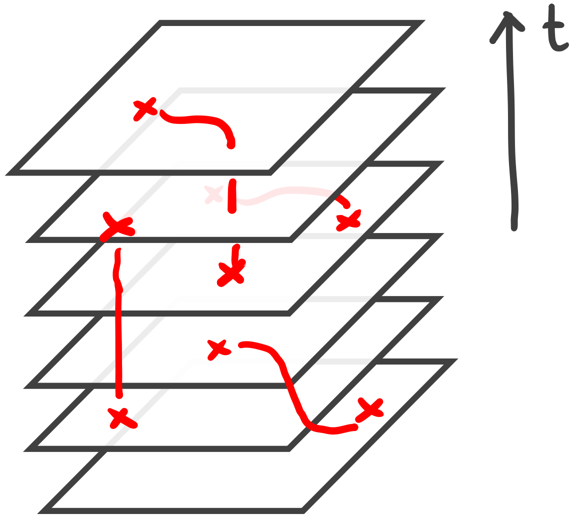
Kavli Institute for
Theoretical Physics

I. Introduction

Stabilizer vs Floquet codes

Toric/surface code:

Repeat measurement of stabilizers (parity checks) for time $\sim O(d)$:



to guarantee $\Pr(\text{failure}) \leq \left(\frac{p}{p_{th}}\right)^{O(d)}$

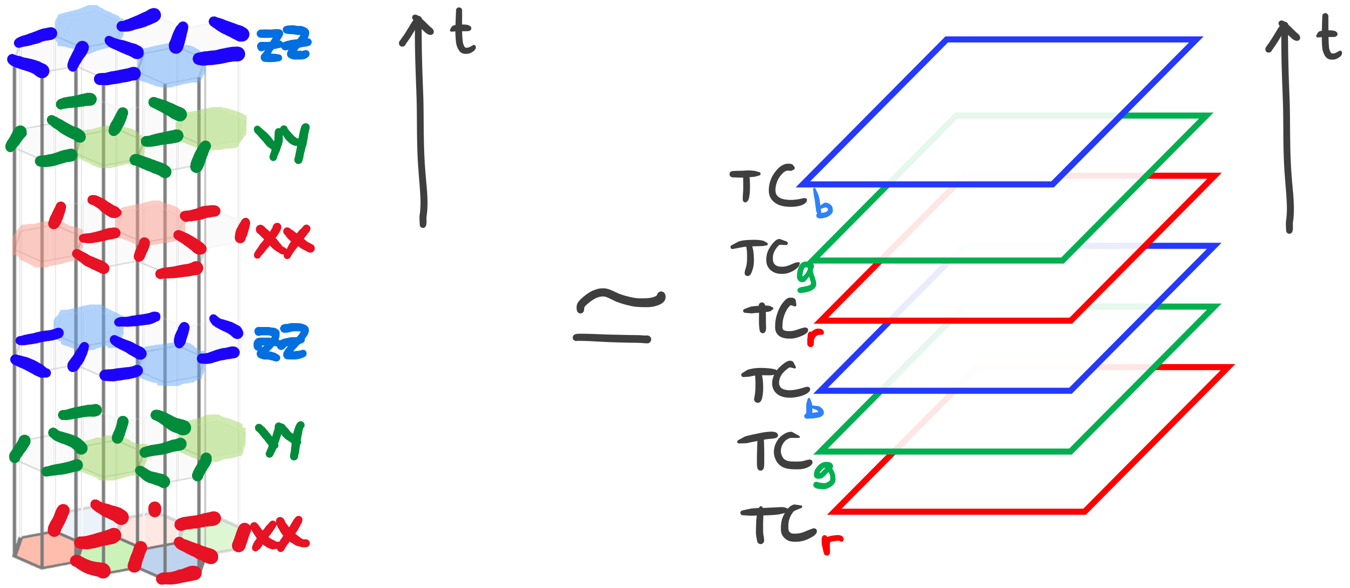
- Geometrically local stabilizer codes (in $d \leq 3$) always require some sort of repetition.

Stabilizer vs Floquet codes

Compare:

Hastings-Haah honeycomb code, [Quantum 5, 564 (2021)]:

repeat measurements of low-weight checks (that anticommute between different rounds) for time $\sim O(d)$,



- Instantaneous stabilizer group (ISG) of the toric code at each step
- Detects errors ('spacetime detectors'
  Kesselring et al '22, Delfosse Paetznick '23)
- e-m automorphism (logical $\bar{X} \leftrightarrow \bar{Z}$) each period

Stabilizer vs Floquet codes

Floquet code:

Periodic sequence of low-weight measurements (different rounds do not commute) that:

- Preserves logical information from round to round
- Detects errors = extracts syndromes (‘spacetime detectors’; Delfosse Paetznick ‘23)

Measurement sequence can be thought of as incorporating syndrome extraction and evolving the code, simultaneously.

Stabilizer vs Floquet codes:
logical operations

Want to achieve fault-tolerant universal quantum computation on encoded information.

Pauli stabilizer codes

- Transversal gates:
  - no universal transversal gateset in the same code
  [Eastin-Knill '09]
  - 2D: at most Clifford gates
  - 3D: 3rd level of Clifford hierarchy
  [Bravyi Koenig '12]

Code switching
2D ↔ 3D

- Lattice surgery; twist defects (Clifford gates)

- Magic state injection (non-Clifford gates)

Stabilizer vs Floquet codes: logical operations

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Floquet codes

- Instantaneous stabilizer → can adapt methods from stabilizer codes
  - lattice surgery; Haah Hastings arXiv:2110.09545
  - defect braiding; Ellison et al. arXiv:2306.08027
  - transversal gates

- **Can absorb gates into code’s “evolution” /syndrome extraction circuit**
  - dynamic automorphism codes

II. Automorphisms

Topological quantum error-correcting codes

This talk: Floquet codes whose instantaneous stabilizer group $S(t) = \{s_i\}$ is that of a **topological quantum code.**

Information encoded in the ground states of a 'topologically ordered' Hamiltonian

$$H = -\sum_i s_i$$

local

- States locally indistinguishable → protection
- Excitations = anyons (endpoints of strings in 2D)
- (Pauli) logical operators are generated by wrapping anyons along nontrivial cycles
- Examples: toric code (TC), color code, Kitaev quantum doubles

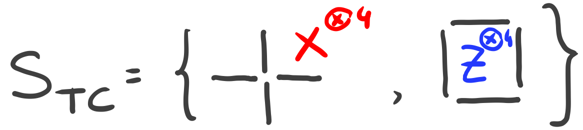
Automorphisms of the topological order

(also: *topological symmetries; anyon permutation symmetries*)

- Permutations of anyons that preserve topological order (fusion, braiding).
- Consequently, they permute respective logical operators, i.e. **quantum gates.**

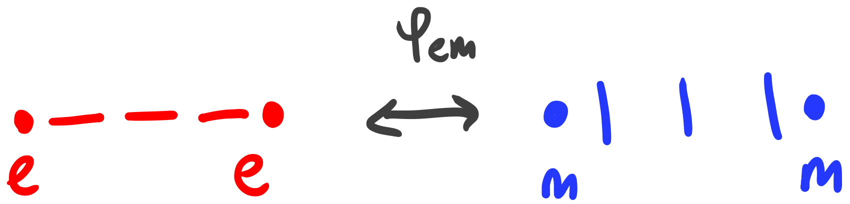
e-m automorphism of the toric code (TC)

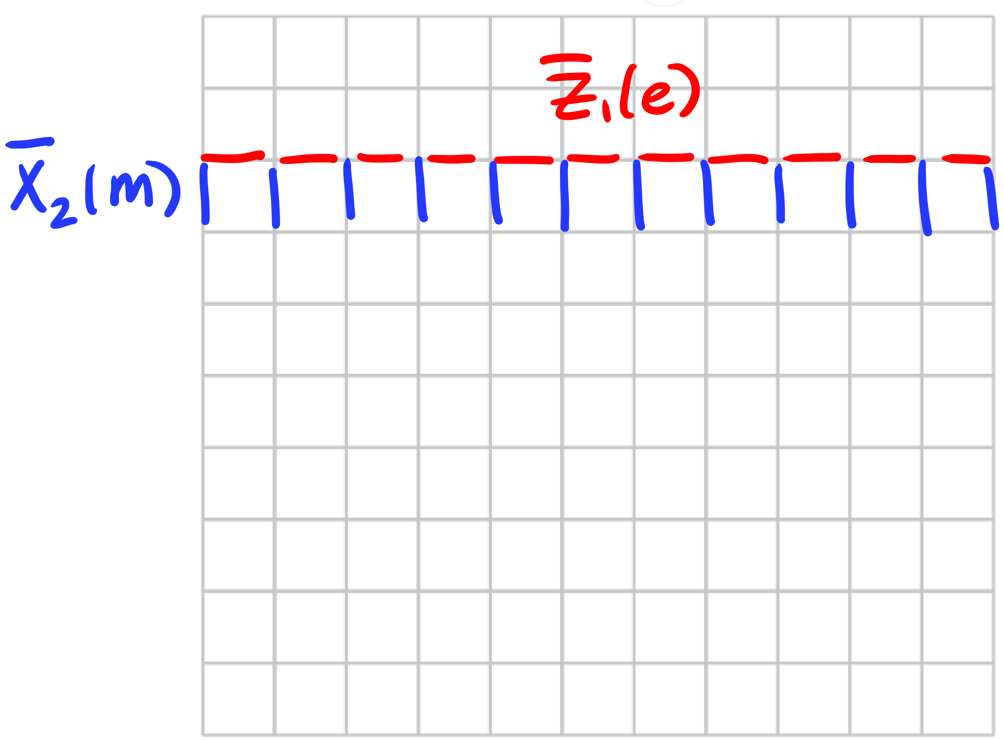
consider square lattice toric code



e-m automorphism $\varphi_{e\text{-}m}$:

preserves the stabilizer group but swaps excitations and logical operators:



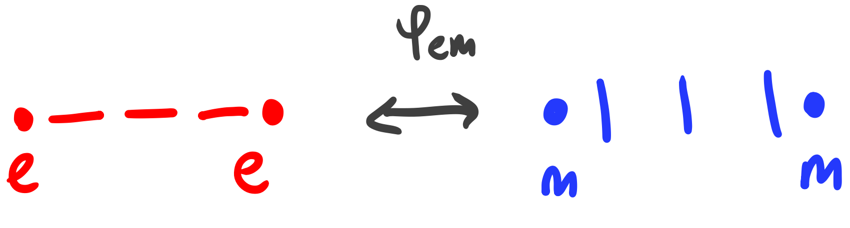


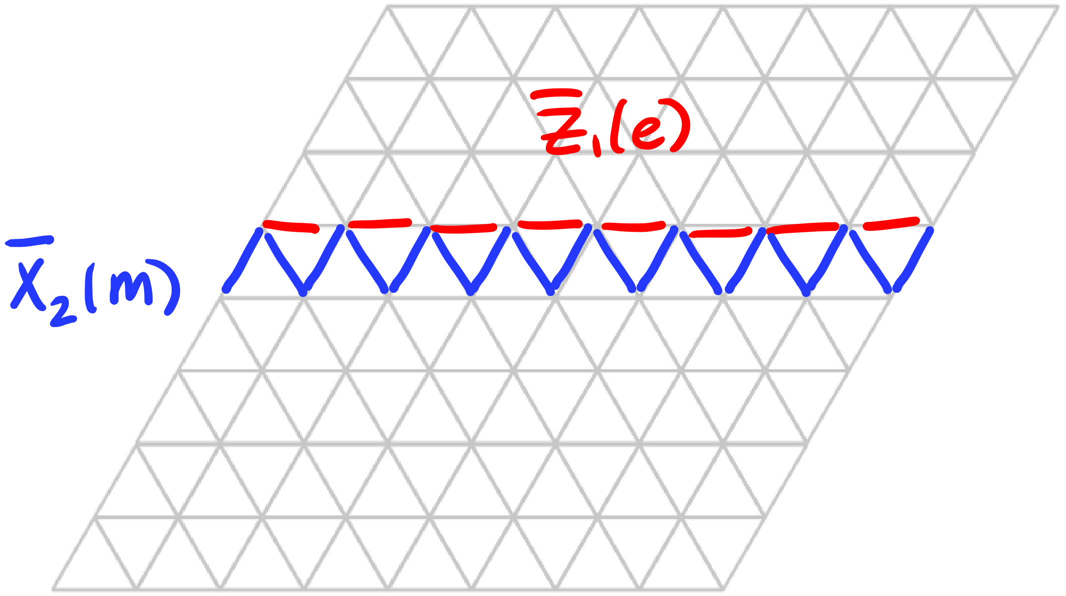
Lattice details don't matter! *("topological")*

~~square lattice toric code~~

triangular lattice toric code

$S_{TC} = \{ \ast X^{\otimes 6}, \triangle Z^{\otimes 3}, \nabla Z^{\otimes 3} \}$





Automorphisms of the topological order

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- Permutations of anyons that preserve topological order (fusion, braiding).
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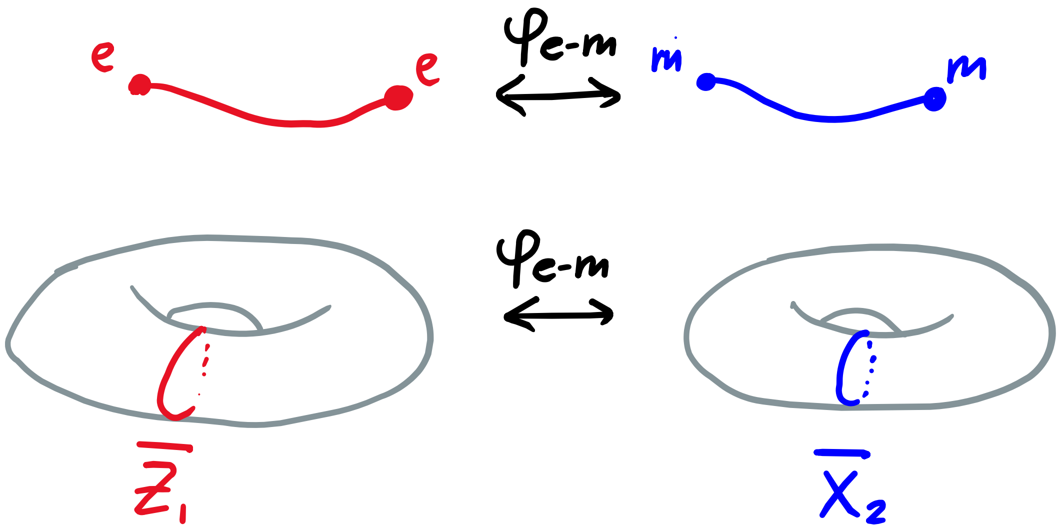
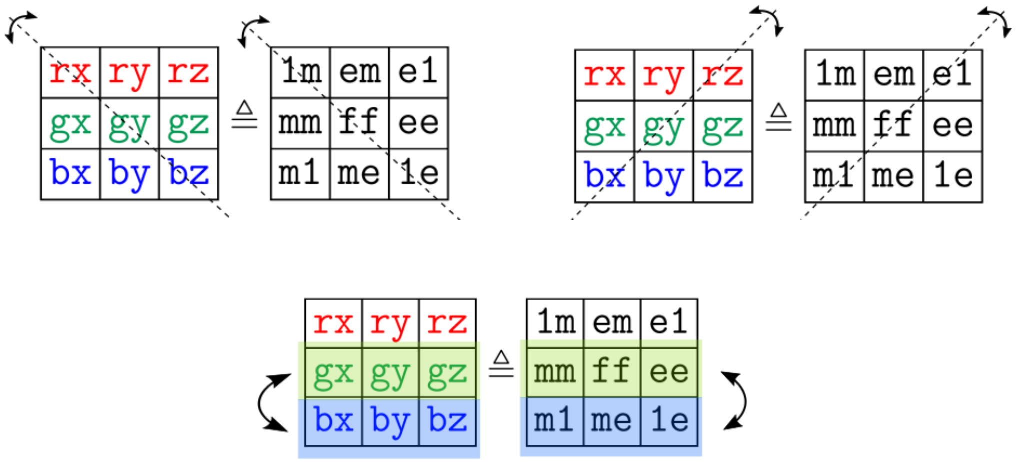
Examples:

- e-m automorphism of the toric code $\varphi_{e\text{-}m}$



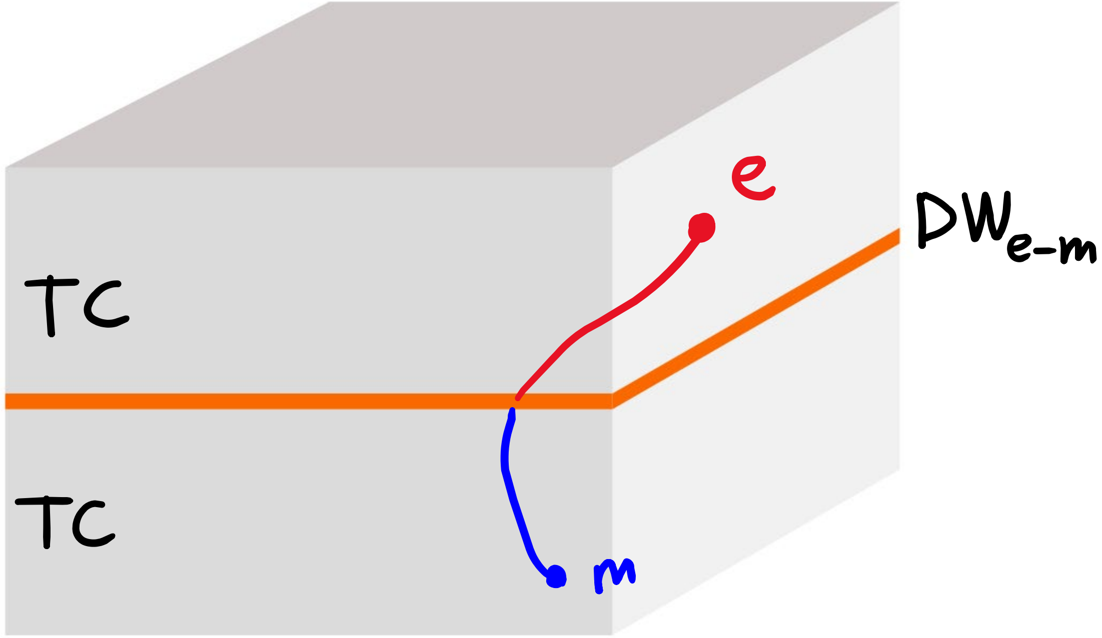
- symmetries of two toric codes=color code

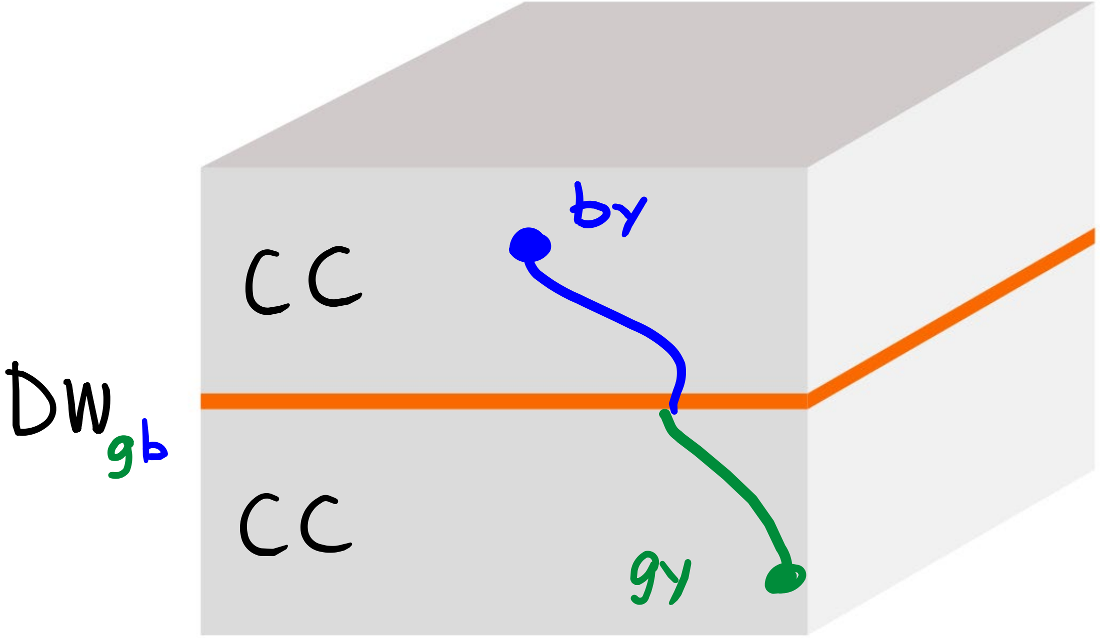


Domain walls

Automorphism can occur across a domain wall in spacetime (“symmetry defect”).

If the domain wall is temporal ≡ we applied a logical gate.

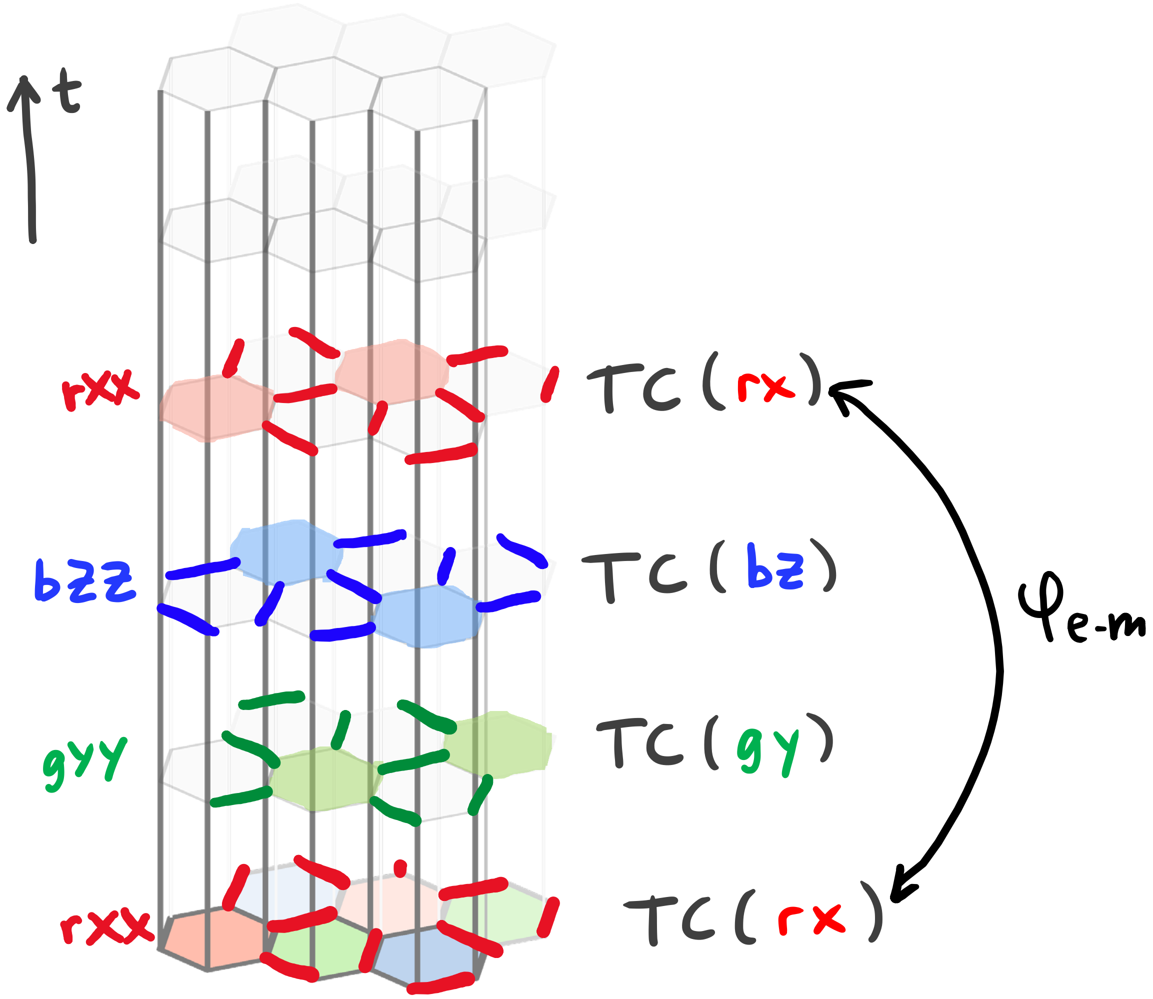


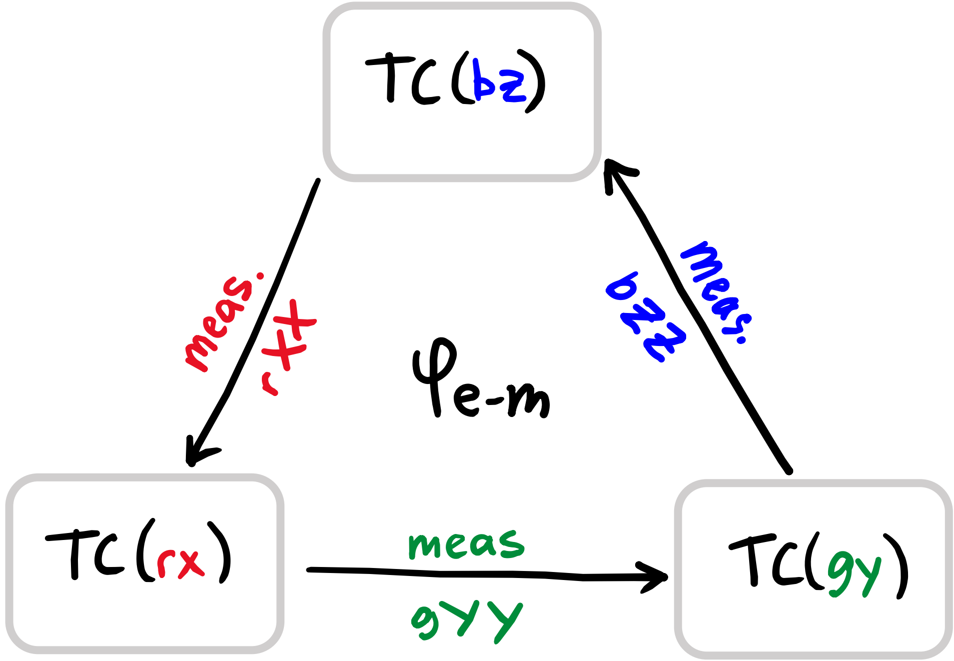


III. Automorhipsms in Floquet codes

e-m automorphism in the honeycomb code

[Hastings, Haah, Quantum 5, 564 (2021)]





Anyon condensation

Parent code:

- S_{PAR} contains all plaquette stabilizers from every round
- L_{PAR} contains logical operators from every round

Instantaneous/child codes:

- obtained by anyon condensation (measuring checks) from the parent model.

Anyon condensation

Kesselring et al '22

$CC \simeq TC \times TC$

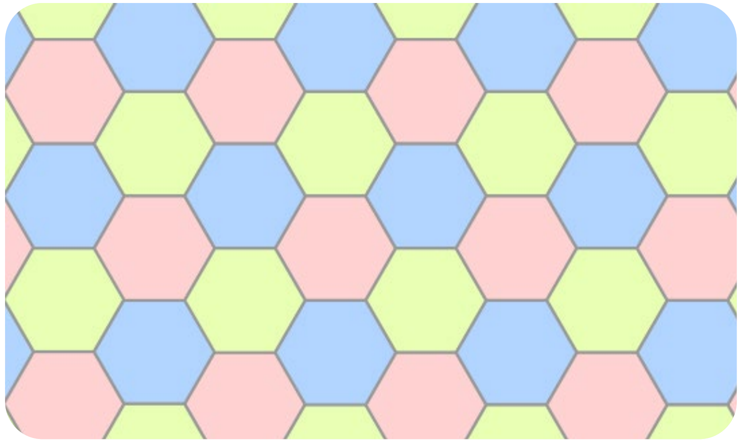
[Kubica, et al NJP 17.8 (2015): 083026.]

Honeycomb code example:

Parent model: color code CC

qubits @ vertices

stabilizer group S = {plaq$_i$(X), plaq$_i$(Z)}



anyon table

CC

rx	ry	rz
gx	gy	gz
bx	by	bz

$\triangleq$

TC × TC

1m	em	e1
mm	ff	ee
m1	me	1e

rows: rx × ry × rz = 1

columns: rx × gy × bz = 1

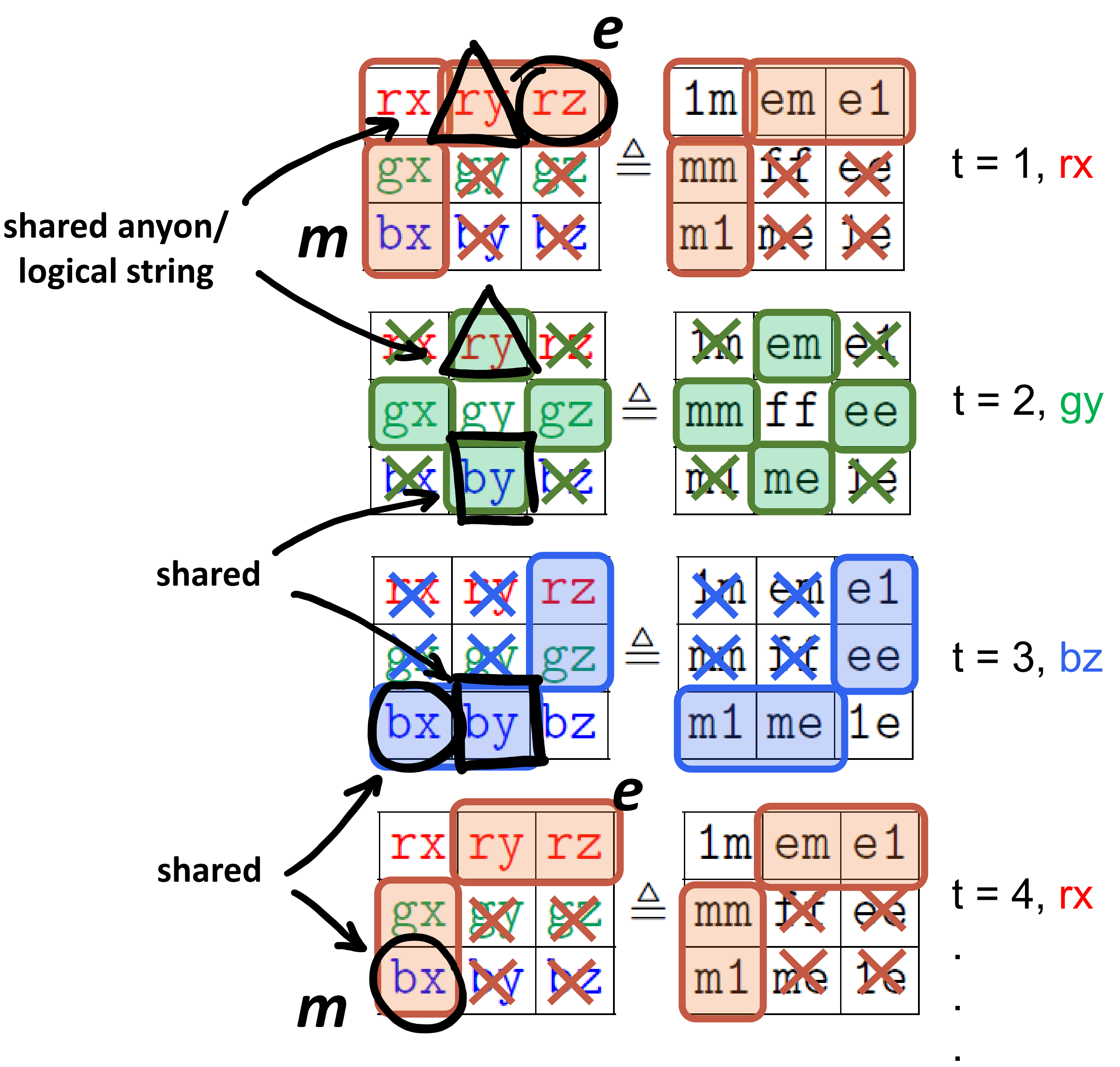
1m × em × e1 = 1

1m × mm × m1 = 1

e × e = 1, m × m = 1,
e × m = f

e-m automorphism from anyon condensation

honeycomb code:



***e*-*m* automorphism:**

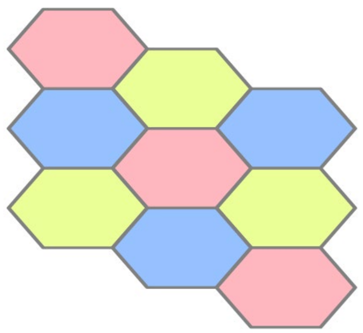
rz ↔ bx

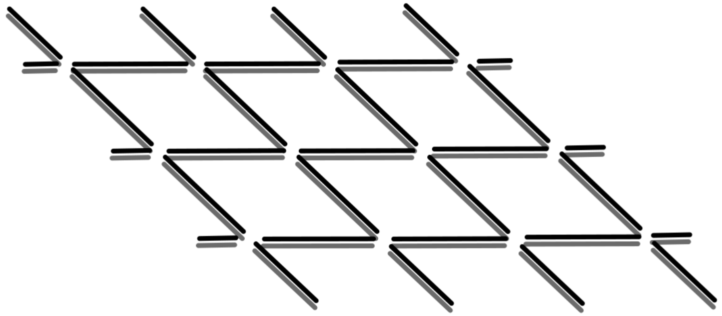
$\overline{Z_i} \leftrightarrow \overline{X_{i'}}$

Anyon condensation

Lattice doesn't matter (again).

For example,





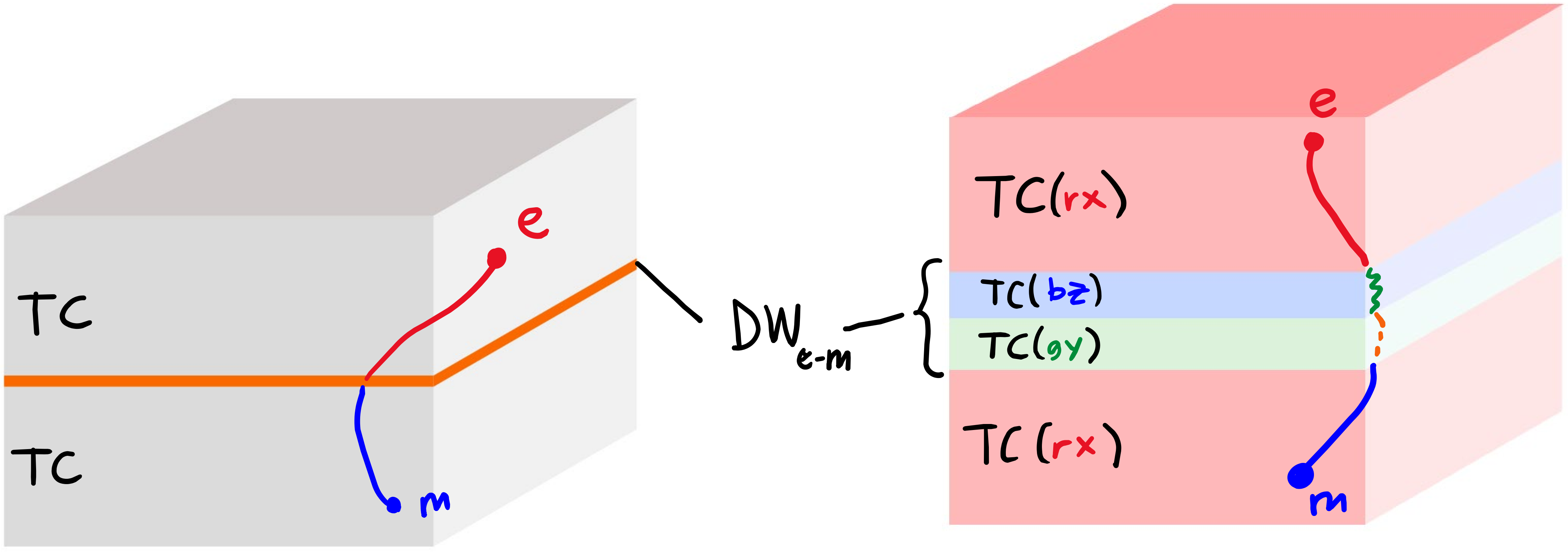
Can both be parent models for the `honeycomb code'

(the stabilizer and check weights might vary)

also see Bauer '23

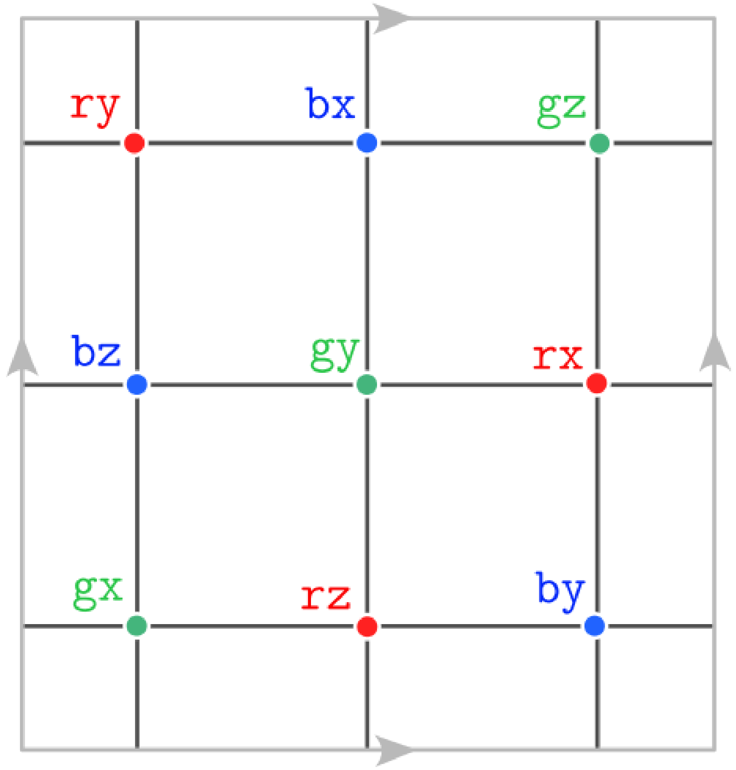
Transversal gates and domain walls

Symmetry defect in spacetime = logical operation



Putting all together: condensation graph

The family of honeycomb codes:

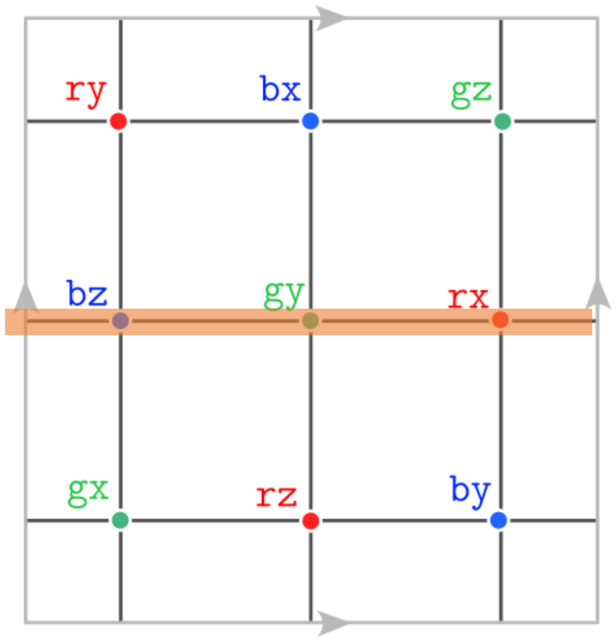


- **Vertices are instantaneous codes;** labeled by the anyon condensed from the parent model
- **Edges are reversible transitions** (conserve logical information)
- Honeycomb codes: **torus**!
- Loops are labeled by **automorphisms**

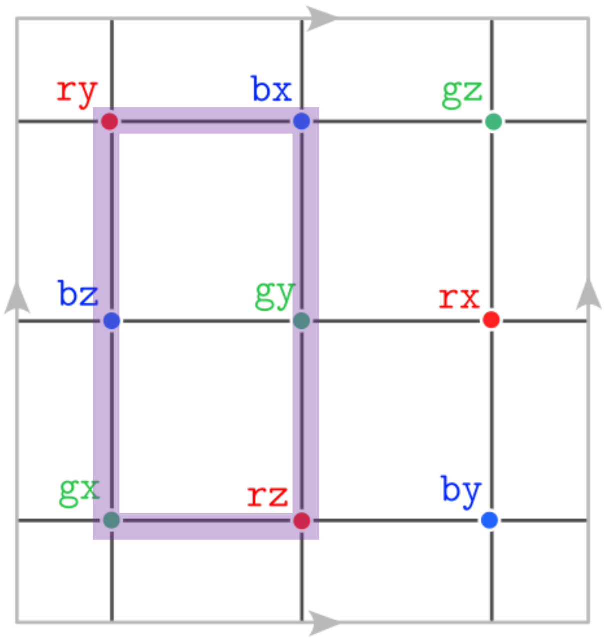
Putting all together: condensation graph

The family of honeycomb codes:

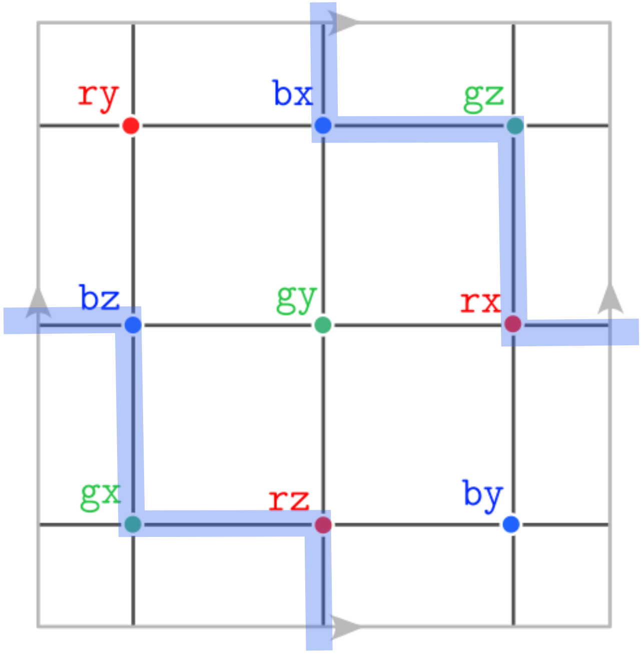
e-m automorphism
Hastings-Haah



trivial automorphism
any contractible cycle

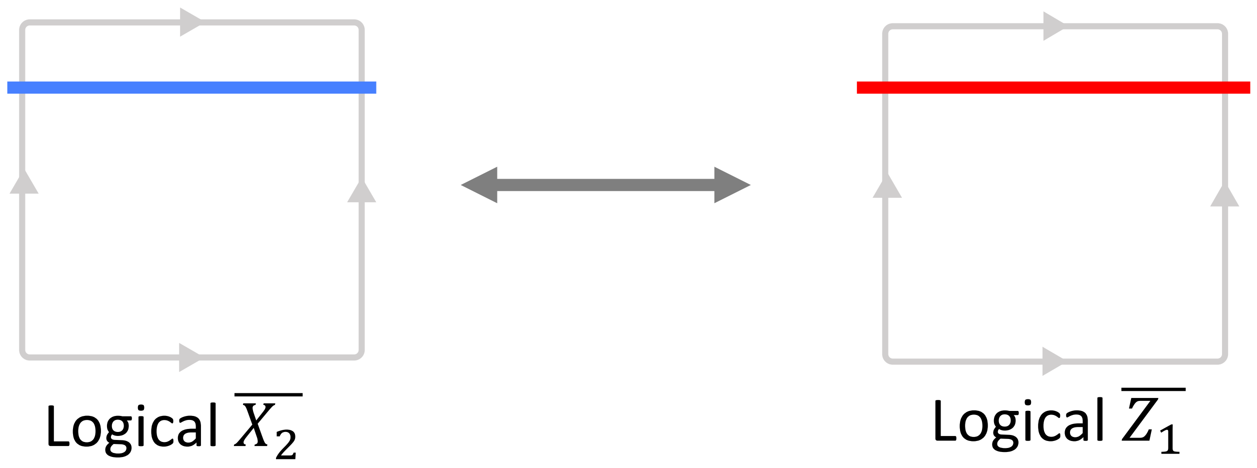


trivial automorphism
of the CSS honeycomb code (both cycles)



More logical operations?

- e-m automorphism of the honeycomb code is a Hadamard-like **logical gate***



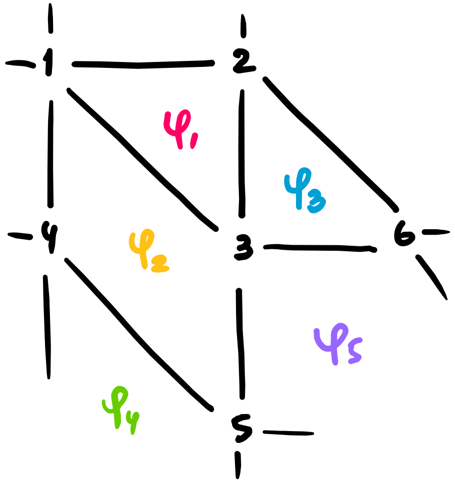
- How do we get more gates? Find a Floquet code where instantaneous code has more **automorphisms = gates**

* technically, it is $(H \otimes H)\ SWAP$

Dynamic automorphism codes

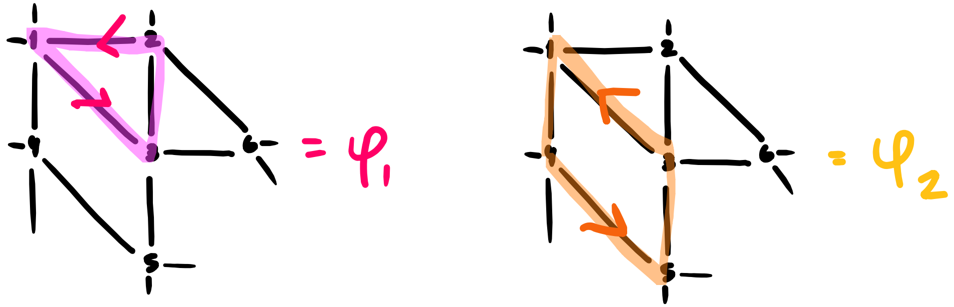
(non-periodic) **generalization of Floquet codes that can do logical operations**

General condensation graph:

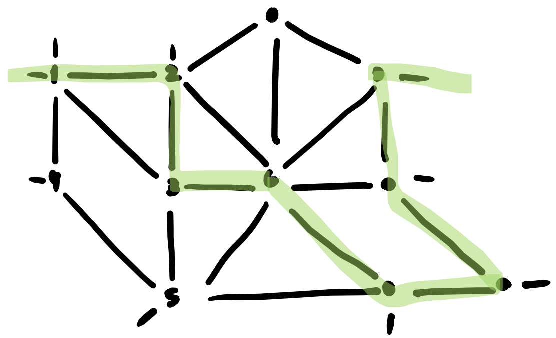


- **Some general parent code**
- **Vertices** are instantaneous codes;
- **Edges** are information-preserving transitions
- **Closed loops labeled by automorphisms/gates**

For a desired gate, choose one loop.



For computation, combine them:



Dynamic code that stores information, but also **computes all by short measurement sequences.**

IV. Dynamic automorphism codes

Logical operations in dynamic codes

What are the possibilities?

arXiv:2307.10353

(1) Child models = Pauli stabilizer codes

- **2D:** anyon strings → logical Pauli operators X_i, Y_j, Z_k
- Anyon permutations = (at most) Clifford gates

dynamic automorphism color code
- 72 auts. of the color code
- Clifford group on a stack of triangles

✓

arXiv:2307.10353

- **3D:** anyon strings → logical Pauli Z_k

  flux membranes → Pauli X_i

  "Cheshire excitations" → Clifford S_j / CZ_{ij}

3D dynamic automorphism color code
- Non-Pauli measurements → non-Clifford (CCZ) gate

✓

(2) Beyond Pauli stabilizer codes?

Yes!

- non-Abelian 2D models (weird algebra of anyon strings)
- FT non-Clifford gates in 2D → **universal fault-tolerant QC**

* [upcoming work w/ B. Brown, A.Bauer, J.C. Magdalena de la Fuente, D. Williamson…]

* see also H. Bombin [1810.09571], B. Brown [sciadv.aay4929]

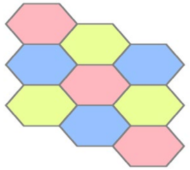
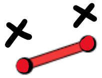
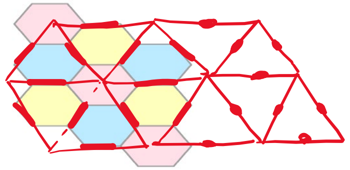
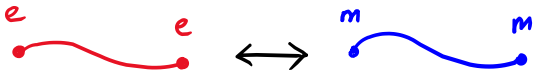
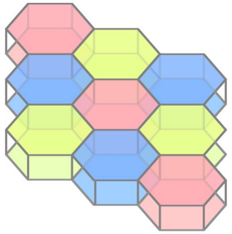
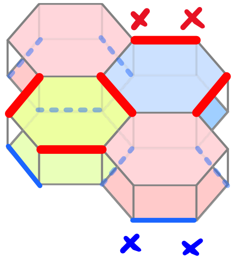
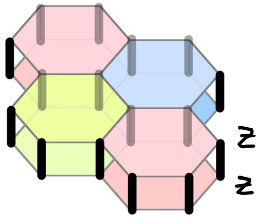
** earlier works:

Cong, Cheng, Wang '17

Laubscher, Loss, Wootton' 18

V. Dynamic automorphism color codes

Dynamic automorphism codes, arXiv:2307.10353

	parent model	measurements types and child codes	Possible Auts.	Gates from Auts.
(2D) honeycomb codes	CC 	TC e.g. TC(rx)  	$\{1, \varphi_{e-m}\}$ 	One gate $(H \otimes H)\, SWAP$
2D dynamic automorphism color code	CC × CC 	TC × TC ≃ CC  	72 auts of the CC rx ry rz gx gy gz bx by bz	

Automorphisms of the CC $\simeq$TC $\times$ TC

Symmetries of the anyon table

rx	ry	rz
gx	gy	gz
bx	by	bz

72 automorphisms :

$(S_3 \times S_3) \rtimes Z_2$

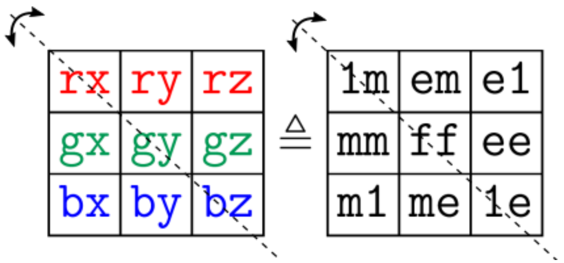
group structure: real Clifford on 2 qubits

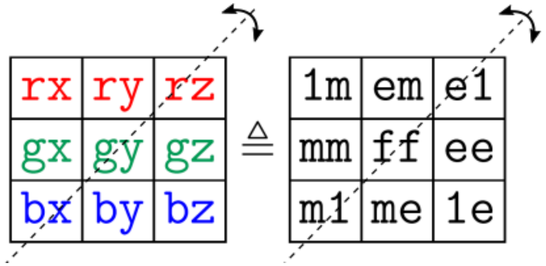
Automorphisms of the CC ≃TC × TC

Symmetries of the anyon table

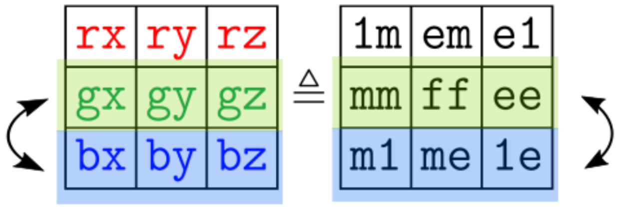
Generators:

- e-m in each copy of the TC (∼ H_1 and H_2-like)



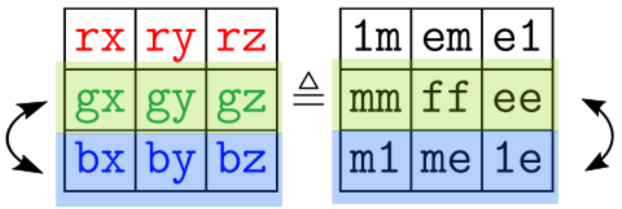


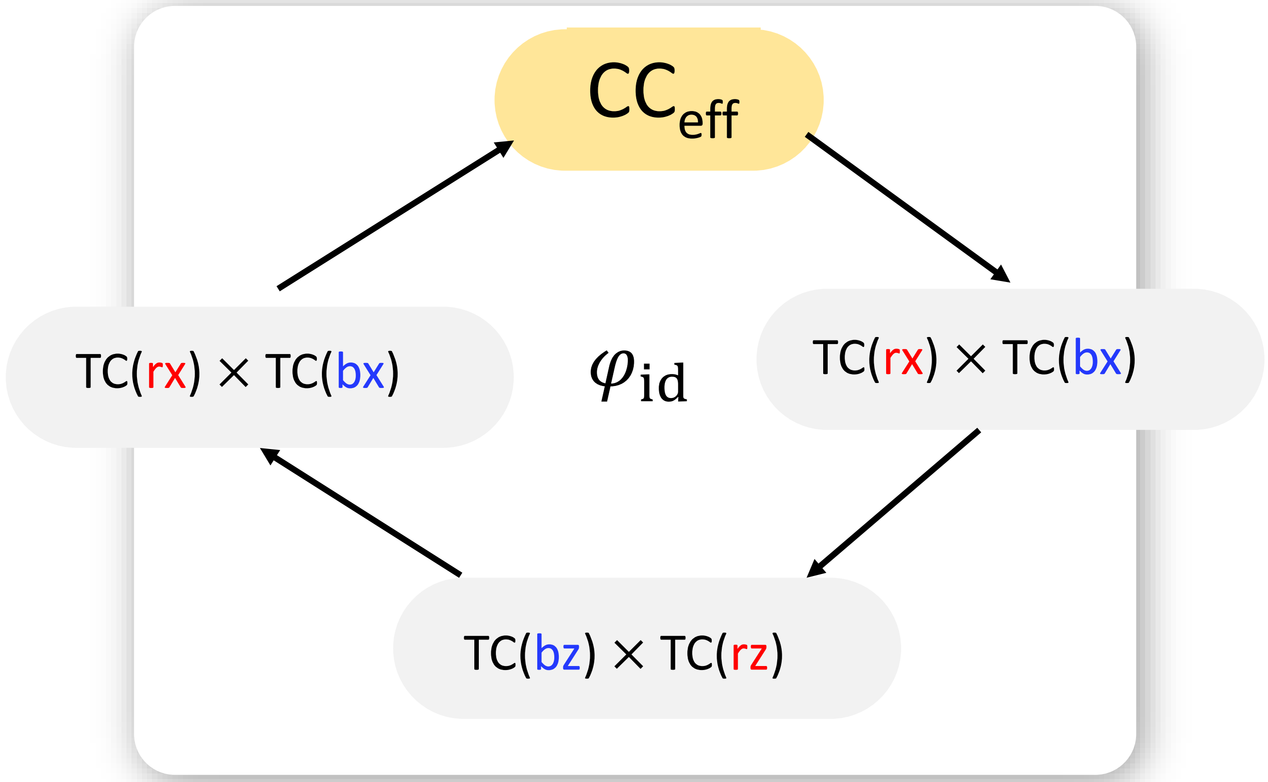
- row permutation/color swap (∼ CNOT gate-like)



row permutation / g-b swap / CNOT gate*

- row permutation/color swap (~ CNOT gate-like)





*technically, it is $CNOT_{12}$ $CNOT_{34}$ for 4 qubits on the torus

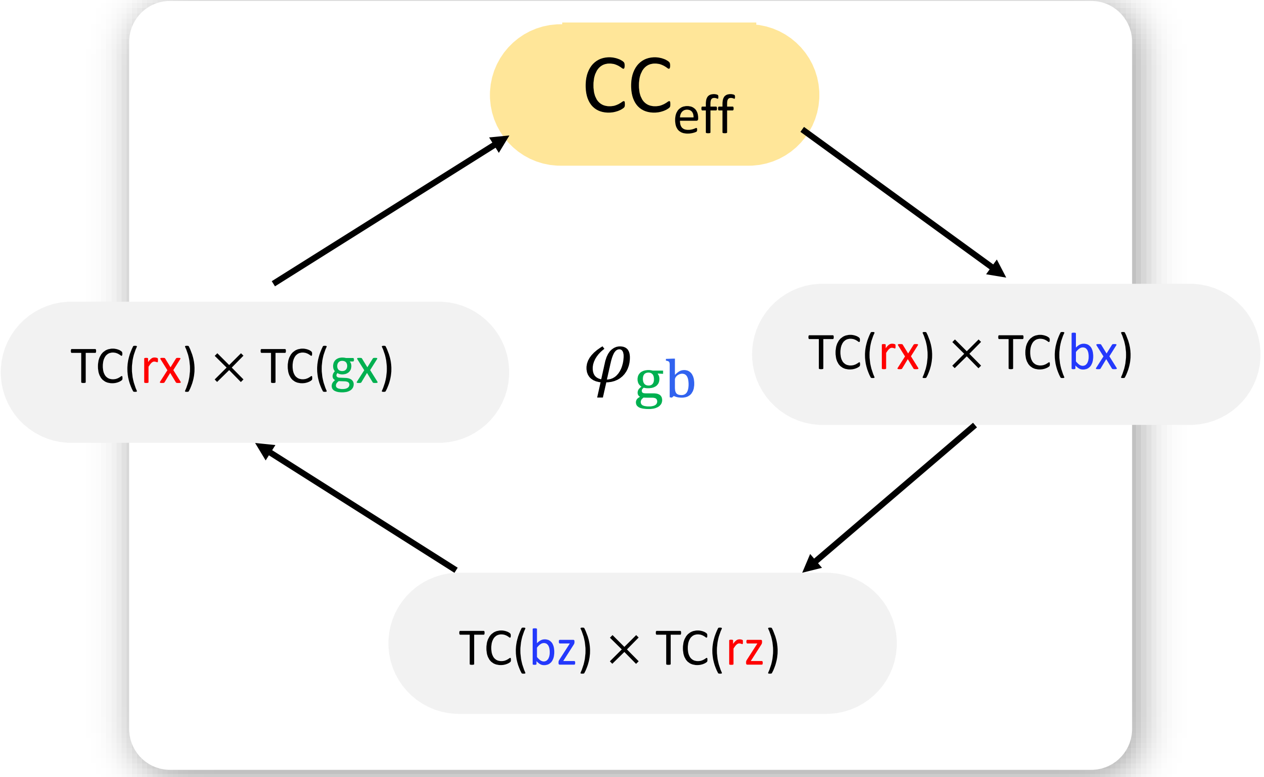
row permutation / g-b swap / CNOT gate*

- row permutation/color swap (~ CNOT gate-like)

rx	ry	rz
gx	gy	gz
bx	by	bz

≜

1m	em	e1
mm	ff	ee
m1	me	1e



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Dynamic automorphism codes, arXiv:2307.10353

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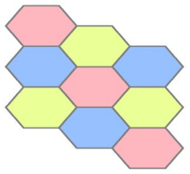
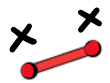
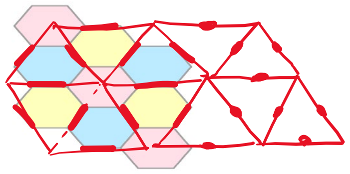
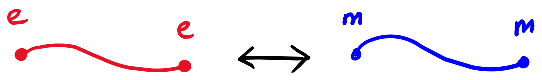
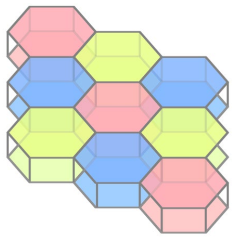
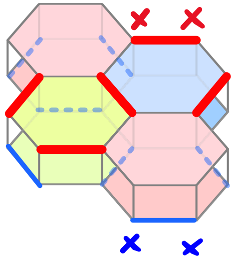
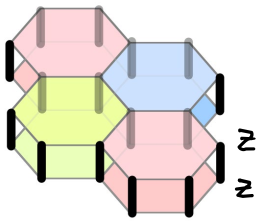
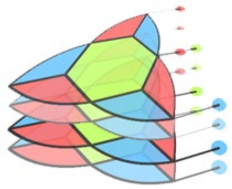
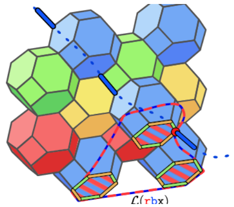
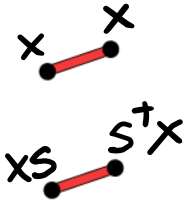
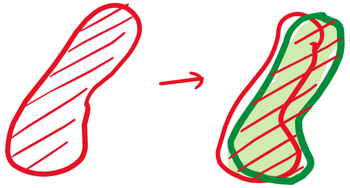
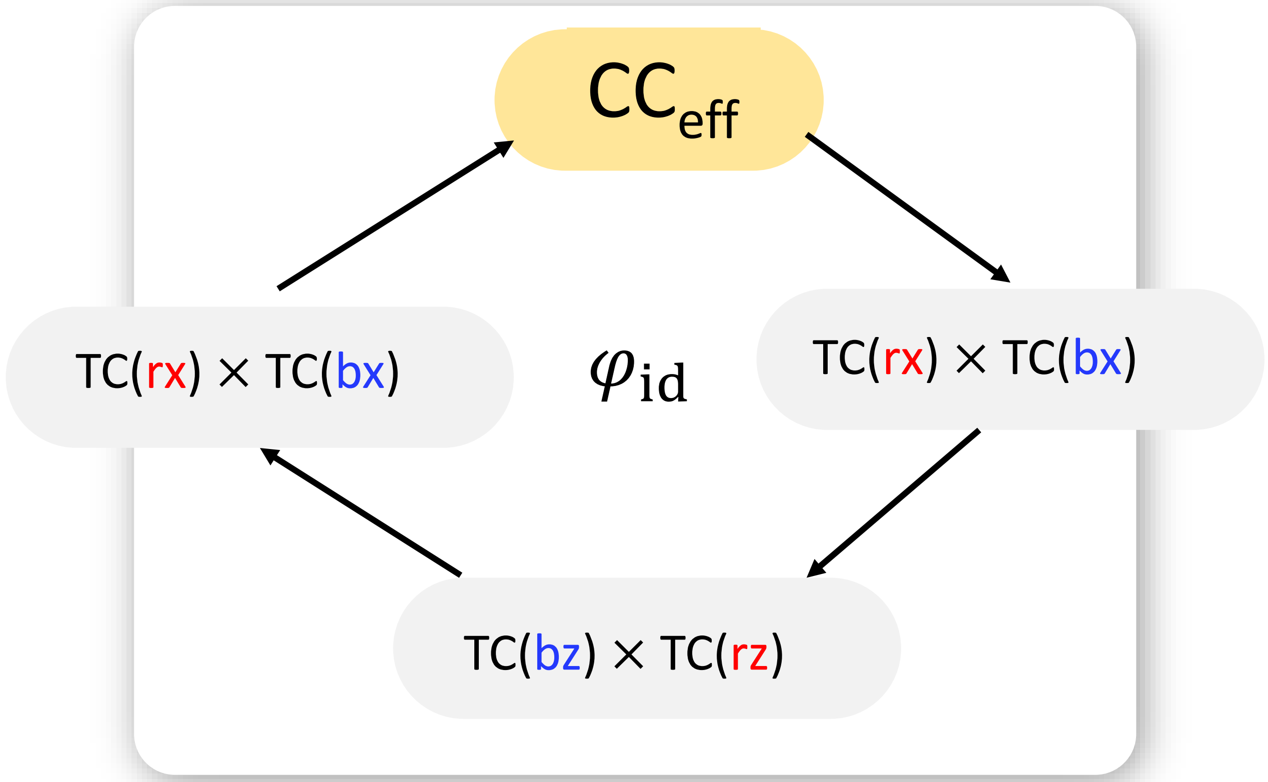
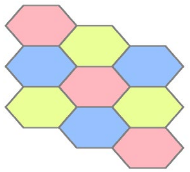
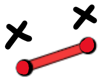
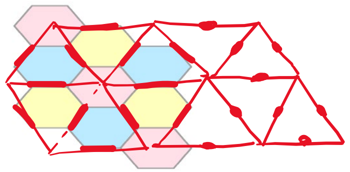
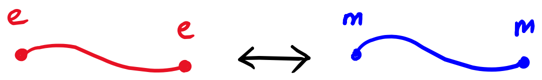
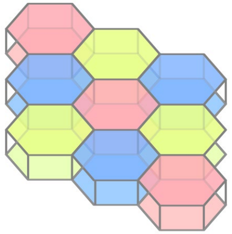
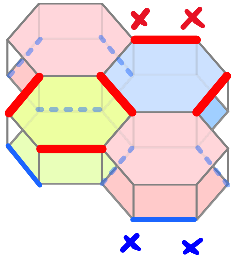
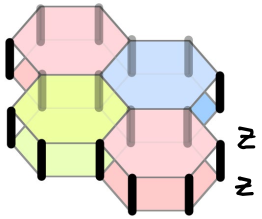
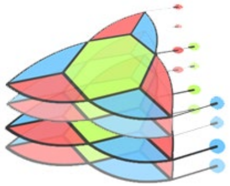
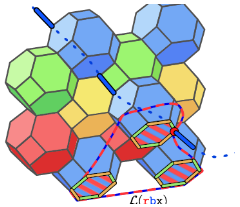
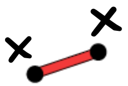
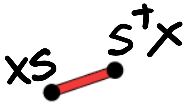
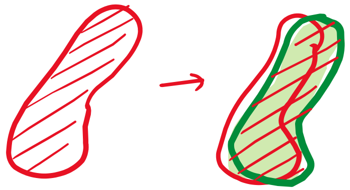
(2D) honeycomb codes	CC 	TC e.g. TC(rx)  	$\{1, \varphi_{e-m}\}$ 	One gate $(H \otimes H)\,SWAP$
2D dynamic automorphism color code	CC × CC 	TC × TC ≃ CC  	72 auts of the CC rx ry rz gx gy gz bx by bz	w/boundaries: full Clifford group 
3D dynamic automorphism color code	3DCC ×3DCC × 3DCC 	TC × TC × TC ≃ 3DCC measurements: 2-body Pauli 2-body Clifford 	X → XS automorphism (involving 'cluster state' excitation ~ S) 	non-Clifford gate ~ CCZ

Non-Clifford gate in 3D: analogous recipe to 2D

Caveat: need to insert a round of Pauli feedback to fix Z-stabilizers to +1



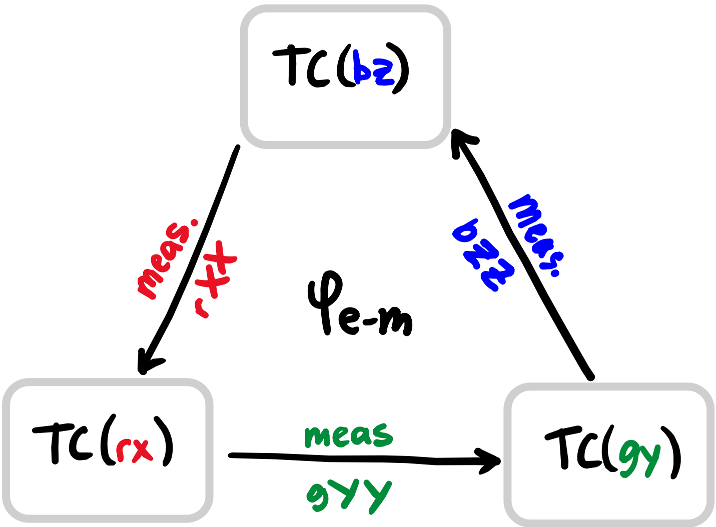
Dynamic automorphism codes, arXiv:2307.10353

	parent model	measurements types and child codes	Possible Auts.	Gates from Auts.
(2D) honeycomb codes	CC 	TC e.g. TC(rx)  	$\{1, \varphi_{e-m}\}$ 	One gate $(H \otimes H)\, SWAP$
2D dynamic automorphism color code	CC × CC 	TC × TC ≃ CC  	72 auts of the CC rx ry rz gx gy gz bx by bz	w/boundaries: full Clifford group 
3D dynamic automorphism color code	3DCC ×3DCC × 3DCC 	TC × TC × TC ≃ 3DCC measurements: 2-body Pauli 2-body Clifford  	X → XS automorphism (involving 'cluster state' excitation ~ S) 	non-Clifford gate ~ CCZ

VI. Outlook & conclusions

New view on Floquet/dynamic codes?

Break down syndrome extraction circuit to low-weight measurements (anticommuting between different rounds), such that **the syndrome extraction itself can implement logical operations.**



Logical operations in dynamic codes

(1) **Child models = Pauli stabilizer codes** arXiv:2307.10353

- **2D:** anyon strings → logical Pauli operators X_i, Y_j, Z_k
- Anyon permutations = (at most) Clifford gates

dynamic automorphism color code
- 72 auts. of the color code
- Clifford group on a stack of triangles

✓

arXiv:2307.10353

- **3D:** anyon strings → logical Pauli Z_k

  flux membranes → Pauli X_i, Clifford S_j / CZ_{ij}

3D dynamic automorphism color code
- non-Clifford (CCZ) gate

✓

(2) **Beyond stabilizer codes?**

Yes!

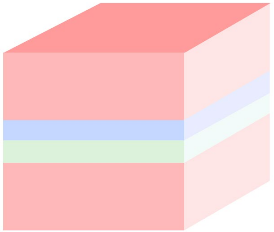
- 2D: non-Abelian models (weird algebra of anyon strings)
- Transitions between Abelian and non-Abelian models
- FT non-Clifford gates in 2D → **universal FT QC**

[upcoming work w/ B. Brown, A.Bauer, J.C. Magdalena de la Fuente, D. Williamson…]

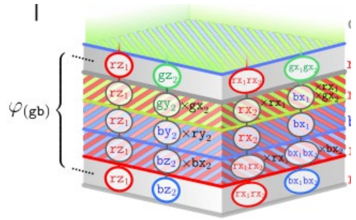
see also H. Bombin [1810.09571], B. Brown [sciadv.aay4929]

What's next?

Universal quantum computation in dynamic codes framework

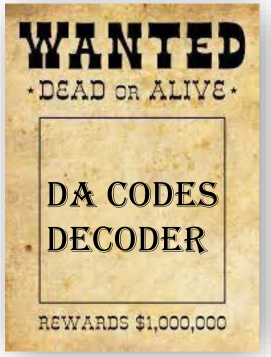


Non-Abelian DA codes; new generalizations

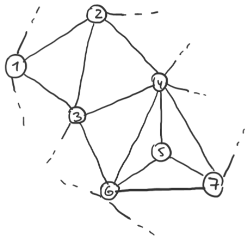


General theory of fault-tolerance; spacetime overheads

Decoders for DA codes



Dynamic codes beyond manifolds?
LDPC dynamic codes?
Gates?



Thank you for your attention!