What is your logical qubit?

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Advances in Quantum Coding Theory
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Wanted: Logical Qubits

• Long Q computation needs good qubits and good operations.
• Physical qubits do not meet the bar,
  • though there are interesting states and dynamics
    • Interesting sampling problem
    • Time crystal
    • Topologically ordered states (phase?)

• With logical qubits
  • Factorize integers
  • Estimate ground state energies
  • Probe Q dynamics
  • Simulate materials
  • Try models of AI
In logical qubit demos...

- logical error rate < physical error rate
- instances of a scalable family of codes.
- error-correction, not detect-and-postselect.
- a universal set of logical components
- Algorithms on small instances

Do you believe you have a Turing machine?
Why?
Probably because you’ve seen your laptop
- Doing calculations with increasing complexity
- Results are crosschecked,
- Building up to overwhelming certainty about correctness
  (you only worry of programming bugs and occasional CPU burn-out)
Apples to Oranges

• With physical qubits
  • maybe never: initialize a qubit in T state, measure two-qubit Paulis.
  • maybe always: unitary CZ, single-qubit arbitrary rotation. Z measurements.

• In surface code architecture in 2d, with code patches
  • Unlikely: CNOT (but usually do with atoms)
  • Always: measure XX, ZZ, and perhaps measure XZ or do H. Embed T into a code patch.

• With a code of large number of encoded qubits (high rate LDPC)
  • maybe never: any Clifford unitary
  • maybe always: measure P1*P2, and inject T.

• If I needed two code blocks to realize single-qubit Clifford unitaries, and made two blocks, did I make one logical qubit or two?

• Despite the differences, a ton of oranges are more useful than a few apples.

? RISC 1GHz vs CISC 800MHz ?
Small code demos: does QEC ever work?

• [Duke & Maryland, 2009.11482]
  • Logical Pauli eigenstates on \([[[9,1,3]]]\)
    → syndrome extraction, deferred to the end
    → logical eigenvalue measurement.
  • Some initialized states are more fiducial when encoded than unencoded. (~1%)
  • Single logical qubit rotation about one axis. S gate.

• [Abobeih et al. 2108.01646]
  • Nitrogen-vacancy in diamond
  • \([[[5,1,3]]]\) with 2 ancillas, full Clifford on one logical qubit.
  • Two versions of state prep, FT / nFT
  • FT version is better: 95% vs 81%

• [Postler et al. 2111.12654]
  • Ion trap
  • 2x \([[[7,1,3]]]\), T injection,
  • Encoded T prepared by Chamberland-Cross’s flagged H-measurement.
  • Not comparison. Demo of a universal gate set.
Small code demos: break-even?

- [Quantinuum, 2107.07505]
  - Logical Pauli eigenstates on $[[7,1,3]]$
    → syndrome extraction cycles
    → logical eigenvalue measurement.
  - Logical SPAM error rate $1.7e^{-3} < $ Physical SPAM $2.4e^{-3}$.
  - “We note that a complete understanding of the overhead associated QEC will have to wait until logical qubit entangling operations are characterized.”

- [Quantinuum, 2208.01863]
  - $2 \times [[7,1,3]]$ with logical CNOT, creating a Bell pair
  - Physical fidelity $[0.985, 0.990] < $ Logical fidelity $[0.9957, 0.9963]$

Lessons (for me)

- The noise physics does not deviate much from a local model.
- Ion manipulation technology is pretty good.
Bosonic codes

(Photon) number

- Studied codes take superposition of either number eigenstates or coherent states
- Cat codes, GKP codes, Binomial codes

Coherent state: \( \hat{a} | \alpha \rangle = \alpha | \alpha \rangle \)
- \( \hat{a} | n \rangle = \sqrt{n} | n - 1 \rangle \)
- an eigenstate of \( \hat{a} \).
- Nonhermitian operator.
- Eigenvalue = any complex number.
- So, a coherent state = a point in the complex plane

\( \hat{a} : \) annihilation operator

- Pauli stabilizer qubit codes: Many qubits provide a large Hilbert space
- Bosons live in an infinite Hilbert space. Select two-dimensional subspace.
One qubit from many photons in cavity

  - Cat code: coherent state superpositions $|\alpha\rangle + |\alpha\rangle$, and $|i\alpha\rangle + |i\alpha\rangle$
  - Errors push code states to vacuum, the origin of the complex plane
  - Code basis states are not exactly orthogonal.
  - Coherence time gain: 1.1 over best physical qubit (two lowest photon-number eigenstates)

- Sivak et al. Nature 616, 50; Ni et al. ibid, 56 (2023)
  - GKP code: lattice-superposition of coherent states (Yale)
  - Binomial code: superpositions of photon number eigenstates (SUST)
  - Many QEC cycles with sophisticated feedback mechanism.
  - Coherence time gain: 2.27 in Sivak et al., and 1.16 in Ni et al.

- Clifford operations are possible by Gaussian operations
  - Performance not reported on those encoded qubits.

- How do logical operations improve?
- Is the longevity of encoded qubits necessary or sufficient?
Relativity

“A drink that makes you live 300 years.”

• Fine print: All humans must take it.
• All your activity will last/take longer by a factor of 10.
• Your laughing.
• Your blinking.
• Your walking.
• Your writing.
• Your thinking.

A question of a system admin

• This server burns out every year. It is too expensive.
  • Turn down the CPU clock rate by 2x. This will make the machine last 18 mos.
• Excuse me? Our revenue is the number of floating-point arithmetic operations.
  • Then, turn up the CPU clock rate by 2x. And replace the machine every 9 mos.
• That’s great!
Computational tasks

• [Wang et al. 2309.09893] using Quantinuum H1-1
  • One-bit adder: $1.1 \times 10^{-3} < 9.5 \times 10^{-3}$
  • Used $[[8,3,2]]$. Error detection.

• [Menendez et al. 2309.08663] on IBM and IonQ
  • CCZ gate performance improves if encoded on $[[8,3,2]]$.

• [Bluvstein et al. 2312.03982]
  • $2^4 \times [[8,3,2]]$ (1 of 4 different experiments. Will discuss later)
  • Fast scrambling circuit. Sampling task. (cf. 2402.03211 faster classical)
    ($\{\text{CCZ, CZ, Z}\}$ within a block, and CNOT between blocks.) $\times$ (many rounds)
  • Linear XEB scored better in postselected encoded qubits than in unencoded counterpart.
“Larger codes work better”
Happens to be all about surface codes.
Threshold (& Pseudo...)

- There exists a local noise model, parametrized by $\epsilon$, and a positive number $\epsilon_0$ such that if $\epsilon < \epsilon_0$, then any quantum circuit of size $M$ can be mapped efficiently to a noisy circuit of size $M \ polylog M$, controlled by a perfect classical computer with runtime $M \ polylog(M)$, such that the output distributions have TV-distance $\leq 0.1$.

- “Pseudothreshold” is the error rate $p$ of the physical qubits/operations such that the logical error rate is also $p$.
  - Prerequisite:
    - All noise needs to be parametrized by one real number $p$.
    - The logical error rate must be defined!
    - Logical Error Rate = $f(p)$
Thresholds for Id, CNOT...

• There exists a local noise model, parametrized by $\epsilon$, and a positive number $\epsilon_0$ such that if $\epsilon < \epsilon_0$, then any quantum circuit of size $M$ can be mapped efficiently to a noisy circuit of size $M \ polylog M$, controlled by a perfect classical computer with runtime $M \ polylog(M)$, such that the output distributions have TV-distance $\leq 0.1$.

• Any Id/CNOT circuit of size $M$ can be mapped efficiently to another circuit of size $M \ polylog M$, controlled by a perfect classical computer with runtime $M \ polylog(M)$, such that the output distributions have TV-distance $\leq 0.1$.

• Therefore, the threshold for id and CNOT is universally $\infty$. 
Nonetheless, we say

• “The threshold of the toric/surface code as a quantum memory is around 1%”
  • Detailed examination of the implementation makes clear what would happen during logical operations.
  • Any logical operation is a minor modification in the limit of large code distances.

• “A quantum code’s idling performance is a good proxy for the truth.”
  • Assumes: any logical operation will have similar error rate.
  • A priori, this is highly nontrivial.
  • What has happened:
    • (1) Some new code is invented.
    • (2) A logical operation scheme is invented in such a way that error rate per spacetime unit is comparable to what is imagined in the new code.
Superconducting qubits in a 2d grid

- Surface code. Tens of syndrome extraction cycles
- Error rates per syndrome extraction cycle $T$
  - 3.03% with $d = 3$
  - 2.91% with $d = 5$
- Acknowledged that it is likely over-threshold.

- Surface code-based architecture has logical unit time $\propto d T$.
- Hence, the reported numbers imply that one can run only shorter quantum algorithms with a larger code.
Reconfigurable neutral atoms

- $> 10^2$ physical qubits
- Virtually all-to-all connectivity
- $> 10^1$ encoded qubits!
- Demos with encoded qubits:
  - CNOT Improving with code distance
  - GHZ state on $4 \times [7,1,3]$
  - $\{CCZ,CZ,Z\}x\{CNOT\} \rightarrow$ Sampling, XEB
  - Entanglement entropy measurement after a scrambling dynamics

- Surface code is error-correcting; others are error-detecting to varying degrees.

Fig. from 2312.03982
Logical Bell pair

1. Product state prep with physical qubits
2. One round of surface code syndrome measurement.
3. Transverse CNOT
4. Individual physical qubit measurement
5. Report the error-corrected values of logical XX and ZZ

Subthreshold for CNOT! ...
Performance of entangling operation for encoded qubits! ...?
Why $\propto d$ rounds with surface code?

- All because of unreliable measurements
- repetition boosts confidence
- To the basic: teleportation

- There is a Pauli correction depending on the measurement outcome.
- Measurement fidelity is the state’s fidelity.
Thought experiment: making a cat state

• Protocol:
  1. Prep $|+\rangle^{\otimes n}$.
  2. Measure $Z_i Z_{i+1}$ once (the only source of error, $p$)
  3. Apply $X_1 \cdots X_i$ for any outcome $-1$ at bond $i$.
     Which correction doesn’t matter

• For any $\ell > 0$, there is an incoherent mixture $\sigma$ of tensor product of $O(\ell)$-qubit states such that $||\rho - \sigma|| \lesssim ne^{-p\ell}$.
  • Proof) An error breaks coherence. Bell $\mapsto \frac{1}{2} (+ +) + \frac{1}{2} (− −)$
    For every interval of length $\ell$, there is usually an error.

• Take $n \to \infty$, $\ell \approx p^{-1} \log n$. The preparation gives a product state.
• If measurements are each repeated $k$ times, $p \to p^{k/2}$.
Operational aspect

• Suppose error occur only in the measurements. Use repetition code.

• Logical teleportation
  1. Left (L): Bring unknown encoded state.
  2. Right (R): Initialize the other block by measuring $Z_i Z_{i+1}$ once on $|+\rangle^\otimes n$ and Pauli correct.
  3. Transversal CNOT with control on (R)
  4. Measure out (L) in $Z$.

• Because (R) has long X errors w.pr. $O(p)$, the $Z$ measurement outcome on (L) is wrong w.pr. $O(p)$ even with the repetition code’s capability.

• Therefore, the teleportation fails w.pr. $O(p)$. 
but, Bell correlation measurement is fine.

- Suppose errors occur only in the measurements. Use repetition code.
- Logical Bell creation
  1. Left (L): Bring unknown encoded state. Prepare $|0\rangle^\otimes n$.
  2. Right (R): Initialize the other block by measuring $Z_i Z_{i+1}$ once on $|+\rangle^\otimes n$ and Pauli correct.
  3. Transversal CNOT with control on (R)
  4. Measure everything in $Z$ or everything in $X$.
  5. Think and report logical ZZ or logical XX.
- Long X-errors by meas. error from (R) propagates to (L).
- Logical XX
  - we’re measuring $X$, so $X$ errors don’t affect anything.
- Logical ZZ
  - There are identical errors on identical codes.
  - Both are flipped, or none is flipped. The product is invariant.
Recap

- Toy model - Noise on measurements only.
- Measuring stabilizers once on repetition code.
- Fails to generate a fiducial cat state.
- Hence, it fails to teleport unknown states reliably; it does send some Pauli state.
- Parallel math applies with qubit noise and with 2d toric code.
  - The 2d toric/surface code state at nonzero temperature is basically a mixture of product state. [Hastings 1106.6026]
  - $O(1)$-measurement-prepared surface code is a thermal state.
- [Bluvstein et al. 2312.03982] claims “improving entangling gates with code distance.”
- Is it a scaling demo of a gadget that can be used for a general quantum circuit?
  - If yes, I’d like to learn how to use it!
  - If no, what is it a demo of?
In logical qubit demos

• logical error rate $\ll$ physical error rate
  • The error channels are different. Is it apples-to-apples? No, but order-of-magnitude is meaningful.

• instances of a scalable family of codes.
  • An infinite family is a mathematical ideal.
  • Suffices to show a convincing trend.
  • Completely unnecessary if your demo reaches error rate, say, $10^{-20}$.

• error-correct, not detect-and-postselect.
  • Because the latter is believed to have too small success probability.
  • But, if postselection solves your problem, however rare it may be, please do!

• a universal set of logical components
  • The first and last point of logical qubits.
  • Very demanding, especially because components may have ill-defined time boundaries.
  • Don’t optimize a component so that it won’t fit with others anymore.
Conclusion

• Exciting time for quantum information science
  • Multiple platforms, new designs, new reports
  • QEC demos show consistency with simulation.
    • Locality in noise physics is being justified.

• Request on operational meaning
  • An error rate $\epsilon$, be it physical or logical, must mean that one can do $O(1/\epsilon)$ operations. (error rate per syndrome cycle?)
  • A claim on break-even must be accompanied by a definite task.
  • Test components with scalability in mind.
  • An operation must be reliable, irrespective of other qubits.

• “By QEC, we realized this many logical qubits.”
• “By QEC, we can do this operations more and/or better.”