What is your logical qubit?

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Wanted: Logical Qubits

- Long Q computation needs good qubits and good operations.
- Physical qubits do not meet the bar,
  - though there are interesting states and dynamics
    - Interesting sampling problem
    - Time crystal
    - Topologically ordered states (phase?)

- With logical qubits
  - Factorize integers
  - Estimate ground state energies
  - Probe Q dynamics
  - Simulate materials
  - Try models of AI

- Do you believe you have a Turing machine?
- Why?
- Probably because you’ve seen your laptop
  - Doing calculations with increasing complexity
  - Results are crosschecked,
  - Building up to overwhelming certainty about correctness
    (you only worry of programming bugs and occasional CPU burn-out)

In logical qubit demos…

- logical error rate < physical error rate
- instances of a scalable family of codes.
- error-correction, not detect-and-postselect.
- a universal set of logical components
- Algorithms on small instances

Apples to Oranges

? RISC 1GHz vs CISC 800MHz ?

- With physical qubits
  - maybe never: initialize a qubit in T state, measure two-qubit Paulis.
  - maybe always: unitary CZ, single-qubit arbitrary rotation. Z measurements.

- In surface code architecture in 2d, with code patches
  - Unlikely: CNOT (but usually do with atoms)
  - Always: measure XX, ZZ, and perhaps measure XZ or do H. Embed T into a code patch.

- With a code of large number of encoded qubits (high rate LDPC)
  - maybe never: any Clifford unitary
  - maybe always: measure P1*P2, and inject T.

- If I needed two code blocks to realize single-qubit Clifford unitaries, and made two blocks, did I make one logical qubit or two?

- Despite the differences, a ton of oranges are more useful than a few apples.

Small code demos: does QEC ever work?

- [Duke & Maryland, 2009.11482]
  - Logical Pauli eigenstates on [[9,1,3]]
    → syndrome extraction, deferred to the end
    → logical eigenvalue measurement.
  - Some initialized states are more fiducial when encoded than unencoded. (~1%)
  - Single logical qubit rotation about one axis. S gate.

- [Abobeih et al. 2108.01646]
  - Nitrogen-vacancy in diamond
  - [[5,1,3]] with 2 ancillas, full Clifford on one logical qubit.
  - Two versions of state prep, FT / nFT
  - FT version is better: 95% vs 81%

- [Postler et al. 2111.12654]
  - Ion trap
  - 2x [[7,1,3]], T injection,
  - Encoded T prepared by Chamberland-Cross's flagged H-measurement.
  - Not comparison. Demo of a universal gate set.

Small code demos : break-even?

- [Quantinuum, 2107.07505]
  - Logical Pauli eigenstates on [[7,1,3]]
    → syndrome extraction cycles
    → logical eigenvalue measurement.
  - Logical SPAM error rate 1.7e-3 < Physical SPAM 2.4e-3.
  - “We note that a complete understanding of the overhead associated QEC will have to wait until logical qubit entangling operations are characterized.”
- [Quantinuum, 2208.01863]
  - 2 x [[7,1,3]] with logical CNOT, creating a Bell pair
  - Physical fidelity [0.985,0.990] < Logical fidelity [0.9957,0.9963]
- Lessons (for me)
  - The noise physics does not deviate much from a local model.
  - Ion manipulation technology is pretty good.

Bosonic codes

Gottesmand et al. quant-ph/0008040
Michael et al. 1602.00008
Grimsmo, Puri, 2106.12989

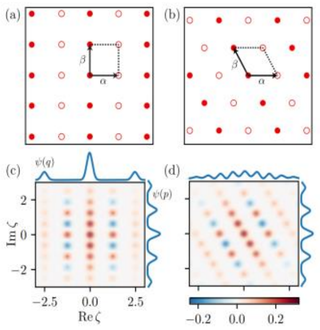
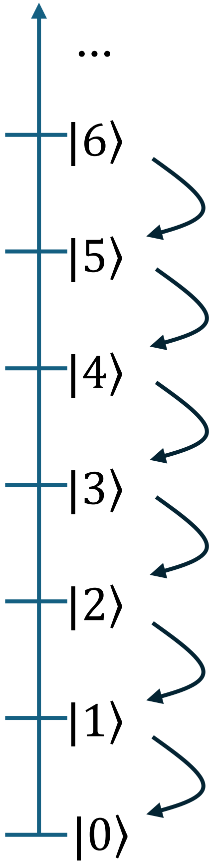


Fig. from Grimsmo, Puri

(Photon) number



$\hat{a}$: annihilation operator

- Studied codes take superposition of either number eigenstates or coherent states
- Cat codes, GKP codes, Binomial codes

Coherent state: $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$

- $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$
- an eigenstate of $\hat{a}$.
- Nonhertmitian operator.
- Eigenvalue = any complex number.
- So, a coherent state = a point in the complex plane

- Pauli stabilizer qubit codes: Many qubits provide a large Hilbert space

- Bosons live in an infinite Hilbert space. Select two-dimensional subspace.

One qubit from many photons in cavity

- Ofek & Petrenko et al. Nature 536, 441 (2016)
  - Cat code: coherent state superpositions $|\alpha\rangle + |-\alpha\rangle$, and $|i\alpha\rangle + |-i\alpha\rangle$
  - Errors push code states to vacuum, the origin of the complex plane
  - Code basis states are not exactly orthogonal.
  - Coherence time gain: 1.1 over best physical qubit (two lowest photon-number eigenstates)

- Sivak et al. Nature 616, 50; Ni et al. ibid, 56 (2023)
  - GKP code: lattice-superposition of coherent states (Yale)
  - Binomial code: superpositions of photon number eigenstates (SUST)
  - Many QEC cycles with sophisticated feedback mechanism.
  - Coherence time gain: 2.27 in Sivak et al., and 1.16 in Ni et al.

- Clifford operations are possible by Gaussian operations
  - Performance not reported on those encoded qubits.

- How do logical operations improve?
- Is the longevity of encoded qubits necessary or sufficient?

Relativity

"A drink that makes you live 300 years."

- Fine print:
  All humans must take it.
- All your activity will last/take longer by a factor of 10.
- Your laughing.
- Your blinking.
- Your walking.
- Your writing.
- Your thinking.

A question of a system admin

- This server burns out every year.
  It is too expensive.
  - Turn down the CPU clock rate by 2x.
    This will make the machine last 18 mos.
- Excuse me? Our revenue is the number of floating-point arithmetic operations.
  - Then, turn up the CPU clock rate by 2x.
    And replace the machine every 9 mos.
- That's great!

Computational tasks

- [Wang et al. 2309.09893] using Quantinuum H1-1
  - One-bit adder: 1.1e-3 < 9.5e-3
  - Used [[8,3,2]]. Error detection.

- [Menendez et al. 2309.08663] on IBM and IonQ
  - CCZ gate performance improves if encoded on [[8,3,2]].

- [Bluvstein et al. 2312.03982]
  - 2^4 x [[8,3,2]] (1 of 4 different experiments. Will discuss later)
  - Fast scrambling circuit. Sampling task. (cf. 2402.03211 faster classical) ({CCZ,CZ,Z} within a block, and CNOT between blocks.) x (many rounds)
  - Linear XEB scored better in postselected encoded qubits than in unencoded counterpart.

“Larger codes work better”

Happens to be all about surface codes

Threshold (& Pseudo…)

- There exists a local noise model, parametrized by ϵ, and a positive number ϵ_0 such that if $\epsilon < \epsilon_0$, then any quantum circuit of size M can be mapped efficiently to a noisy circuit of size $M\ polylog M$, controlled by a perfect classical computer with runtime $M\ polylog(M)$, such that the output distributions have TV-distance ≤ 0.1.
- "Pseudothreshold" is the error rate p of the physical qubits/operations such that the logical error rate is also p.
  - Prerequisite:
    - All noise needs to be parametrized by one real number p.
    - The logical error rate must be defined!
    - Logical Error Rate = $f(p)$

Thresholds for Id, CNOT…

- There exists a local noise model, parametrized by ϵ, and a positive number ϵ_0 such that if $\epsilon < \epsilon_0$, then any quantum circuit of size M can be mapped efficiently to a noisy circuit of size $M\ polylog M$, controlled by a perfect classical computer with runtime $M\ polylog(M)$, such that the output distributions have TV-distance ≤ 0.1.

- Any Id/CNOT circuit of size M can be mapped efficiently to another circuit of size $M\ polylog M$, controlled by a perfect classical computer with runtime $M\ polylog(M)$, such that the output distributions have TV-distance ≤ 0.1.

- Therefore, the threshold for id and CNOT is universally ∞.

Nonetheless, we say

- "The threshold of the toric/surface code as a quantum memory is around 1%"
    - Detailed examination of the implementation makes clear what would happen during logical operations.
    - Any logical operation is a minor modification in the limit of large code distances.
- "A quantum code's idling performance is a good proxy for the truth."
    - Assumes: any logical operation will have similar error rate.
    - A priori, this is highly nontrivial.
    - What has happened:
        - (1) Some new code is invented.
        - (2) A logical operation scheme is invented in such a way that error rate per spacetime unit is comparable to what is imagined in the new code.

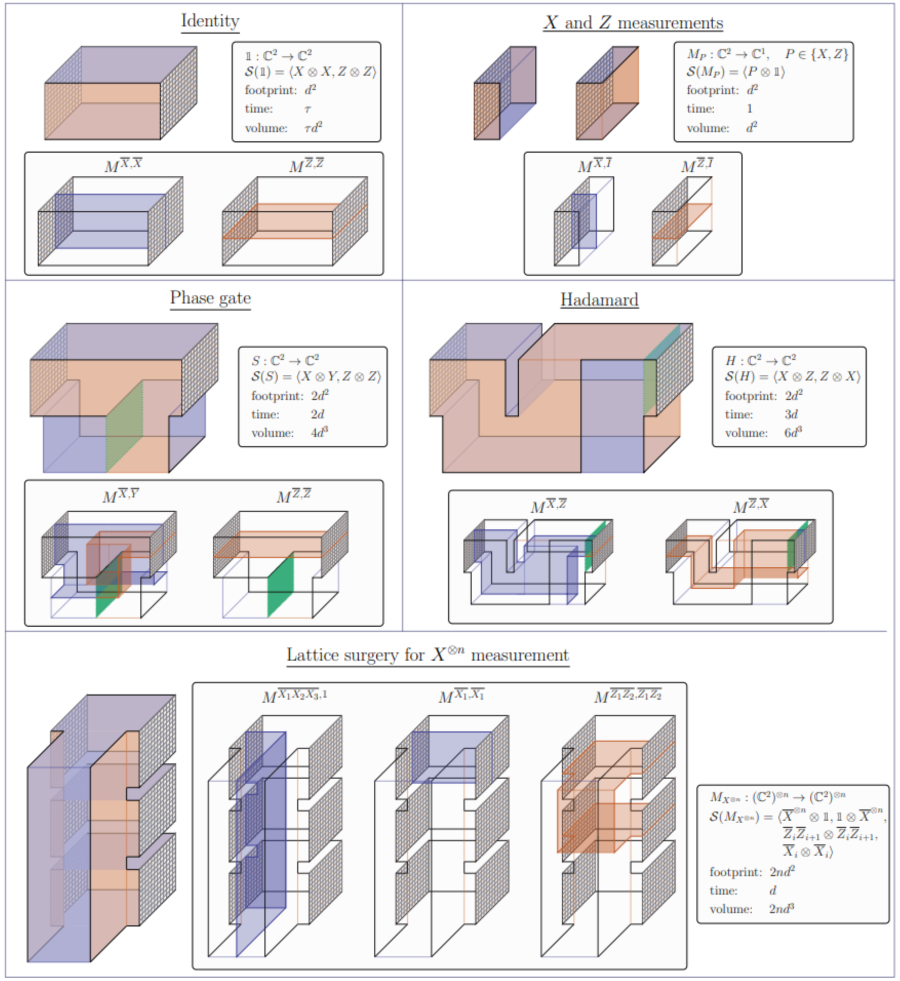


Fig. from PsiQuantum, 2112.12160

Superconducting qubits in a 2d grid

- Surface code. Tens of syndrome extraction cycles
- Error rates per syndrome extraction cycle T
  - 3.03% with $d = 3$
  - 2.91% with $d = 5$
- Acknowledged that it is likely over-threshold.

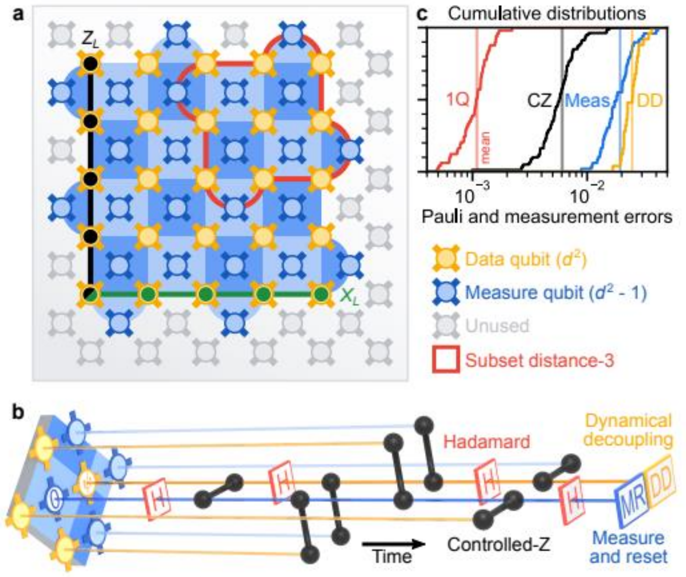


Fig. from 2207.06431

- Surface code-based architecture has logical unit time $\propto d\,T$.
- Hence, the reported numbers imply that one can run only shorter quantum algorithms with a larger code.

[Bluvstein et al. Harvard/Quera/Maryland/MIT 2312.03982]

Reconfigurable neutral atoms

- $> 10^2$ physical qubits
- Virtually all-to-all connectivity
- $> 10^1$ encoded qubits!
- Demos with encoded qubits:
  - CNOT Improving with code distance
  - GHZ state on 4 x [[7,1,3]]
  - {CCZ,CZ,Z}x{CNOT} → Sampling, XEB
  - Entanglement entropy measurement after a scrambling dynamics
- Surface code is error-correcting; others are error-detecting to varying degrees.

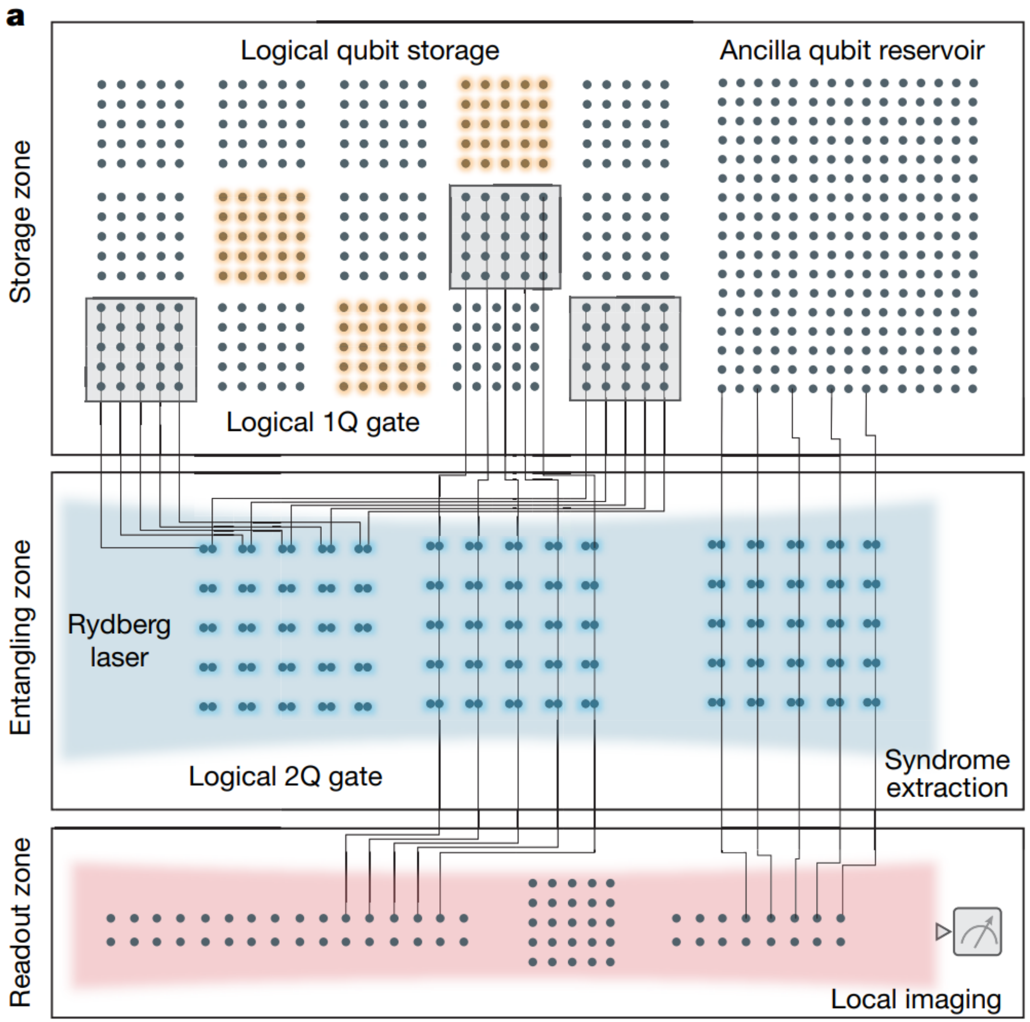
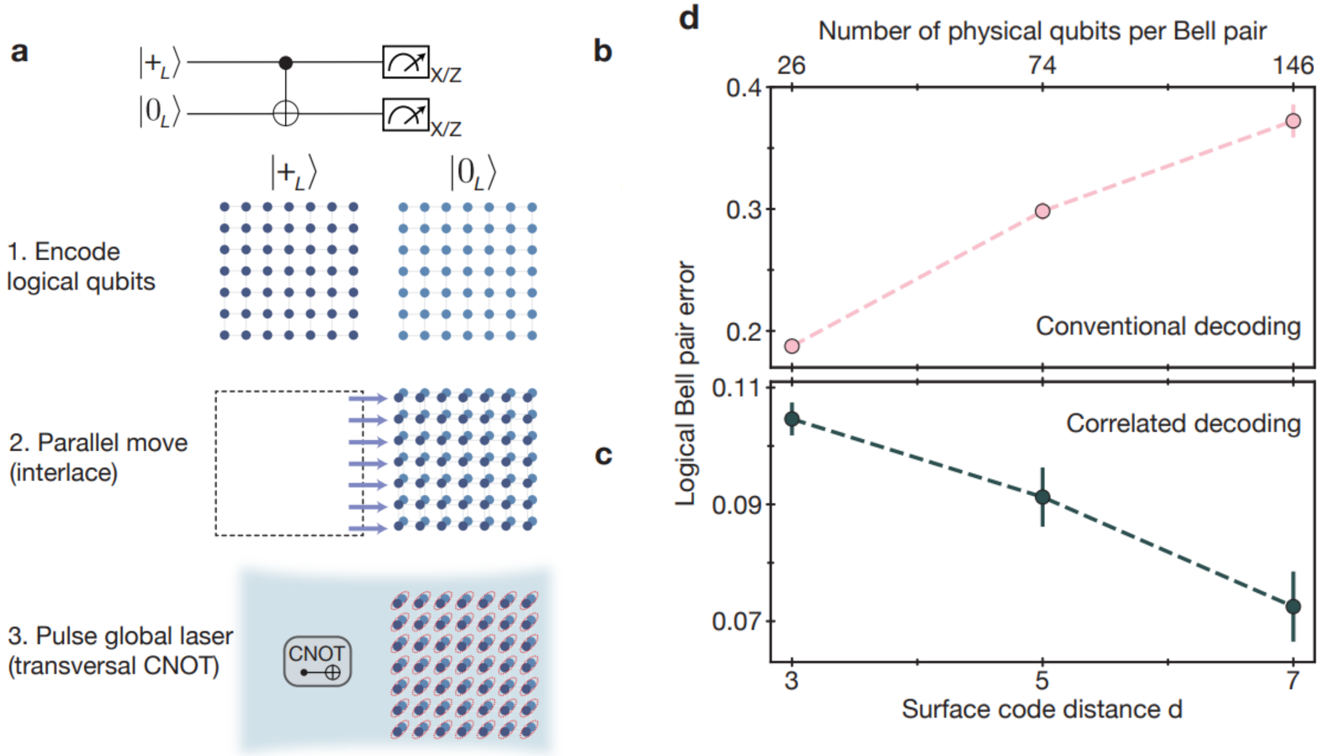


Fig. from 2312.03982

[Bluvstein et al. 2312.03982]

Logical Bell pair



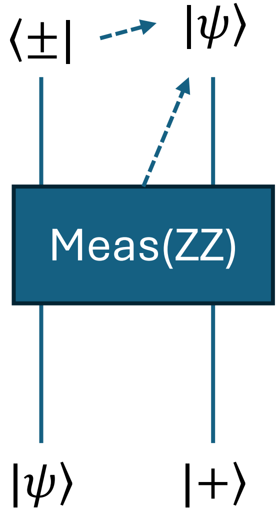
1. Product state prep with physical qubits
2. One round of surface code syndrome measurement.
3. Transverse CNOT
4. Individual physical qubit measurement
5. Report the error-corrected values of logical XX and ZZ

Subthreshold for CNOT ! ...?

Performance of entangling operation for encoded qubits! ...?

Why $\propto d$ rounds with surface code?

- All because of unreliable measurements
- repetition boosts confidence
- To the basic: teleportation



- There is a Pauli correction depending on the measurement outcome.
- Measurement fidelity is the state's fidelity.

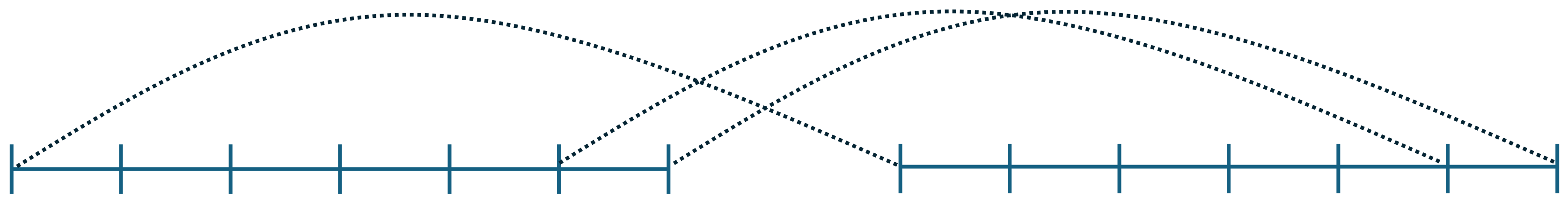
Thought experiment: making a cat state

- Protocol:
  1. Prep $|+\rangle^{\otimes n}$.
  2. Measure $Z_i Z_{i+1}$ once (the only source of error, p)
  3. Apply $X_1 \cdots X_i$ for any outcome -1 at bond i.
     Which correction doesn't matter

- *For any $\ell > 0$, there is an incoherent mixture σ of tensor product of $O(\ell)$-qubit states such that $||\rho - \sigma|| \lesssim n e^{-p\ell}$.*
  - Proof) An error breaks coherence. Bell $\mapsto \frac{1}{2}(++) + \frac{1}{2}(--)$
    For every interval of length ℓ, there is usually an error.
- Take $n \to \infty$, $\ell \approx p^{-1} \log n$. The preparation gives a product state.
- If measurements are each repeated k times, $p \to p^{k/2}$.

Operational aspect

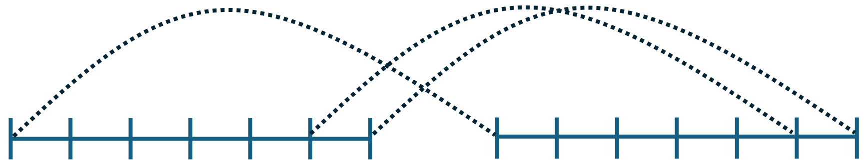
- Suppose error occur only in the measurements. Use repetition code.
- Logical teleportation
    1. Left (L): Bring unknown encoded state.
    2. Right (R): Initialize the other block by measuring $Z_i Z_{i+1}$ once on $|+\rangle^{\otimes n}$ and Pauli correct.
    3. Transversal CNOT with control on (R)
    4. Measure out (L) in Z.



- Because (R) has long X errors w.pr. $O(p)$, the Z measurement outcome on (L) is wrong w.pr. $O(p)$ even with the repetition code's capability.
- Therefore, the teleportation fails w.pr. $O(p)$.

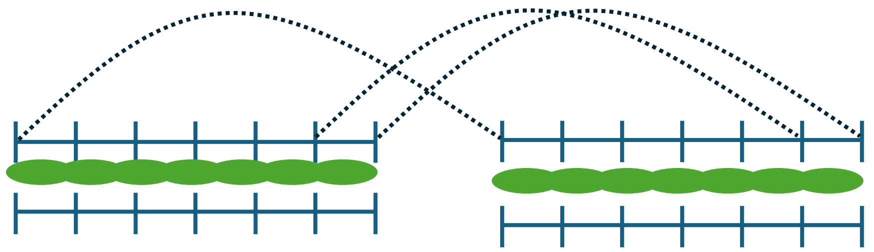
but, Bell correlation measurement is fine.

- Suppose errors occur only in the measurements. Use repetition code.
- Logical Bell creation
  1. Left (L): ~~Bring unknown encoded state.~~ Prepare $|0\rangle^{\otimes n}$.
  2. Right (R): Initialize the other block by measuring $Z_i Z_{i+1}$ once on $|+\rangle^{\otimes n}$ and Pauli correct.
  3. Transversal CNOT with control on (R)
  4. Measure everything in Z or everything in X.
  5. Think and report logical ZZ or logical XX.

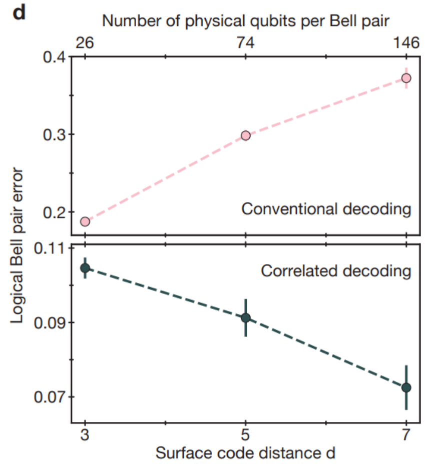


- Long X-errors by meas. error from (R) propagates to (L).
- Logical XX
  - we're measuring X, so X errors don't affect anything.
- Logical ZZ
  - There are identical errors on identical codes.
  - Both are flipped, or none is flipped. The product is invariant.

Recap



- Toy model - Noise on measurements only.
- Measuring stabilizers once on repetition code.
- Fails to generate a fiducial cat state.
- Hence, it fails to teleport unknown states reliably;
  it does send some Pauli state.
- Parallel math applies with qubit noise and with 2d toric code.
  - The 2d toric/surface code state at nonzero temperature is basically a mixture of product state. [Hastings 1106.6026]
  - O(1)-measurement-prepared surface code is a thermal state.
- [Bluvstein et al. 2312.03982] claims "improving entangling gates with code distance."
- Is it a scaling demo of a gadget that can be used for a general quantum circuit?
  - If yes, I'd like to learn how to use it!
  - If no, what is it a demo of?



[Bluvstein et al. 2312.03982]

In logical qubit demos

- logical error rate $\ll$ physical error rate
  - The error channels are different. Is it apples-to-apples? No, but order-of-magnitude is meaningful.
- instances of a scalable family of codes.
  - An infinite family is a mathematical ideal.
  - Suffices to show a convincing trend.
  - Completely unnecessary if your demo reaches error rate, say, 10^{-20}.
- error-correct, not detect-and-postselect.
  - Because the latter is believed to have too small success probability.
  - But, if postselection solves your problem, however rare it may be, please do!
- a universal set of logical components
  - The first and last point of logical qubits.
  - Very demanding, especially because components may have ill-defined time boundaries.
  - Don’t optimize a component so that it won’t fit with others anymore.

Conclusion

- Exciting time for quantum information science
  - Multiple platforms, new designs, new reports
  - QEC demos show consistency with simulation.
    - Locality in noise physics is being justified.
- Request on operational meaning
  - An error rate ϵ, be it physical or logical, must mean that one can do $O(1/\epsilon)$ operations. (error rate per syndrome cycle?)
  - A claim on break-even must be accompanied by a definite task.
  - Test components with scalability in mind.
  - An operation must be reliable, irrespective of other qubits.

- ~~“By QEC, we realized this many logical qubits.”~~
- “By QEC, we can do this operations more and/or better.”