Decoders for Good Quantum LDPC Codes

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Advances in Quantum Coding Theory
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arXiv:2206.06557, **SG**, C.A. Pattison, E. Tang
arXiv:2306.12470, **SG**, E. Tang, L. Caha, S.H. Choe, Z. He, A. Kubica

Quantum error correction

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  - Implementing logical gates
  - Decoding algorithm
- How can we achieve fault-tolerant quantum computation with the lowest overhead?

Outline

1. Space overhead of error correction: quantum LDPC codes
2. Time overhead of error correction: decoders for LDPC codes
    - Decoding good quantum LDPC codes (arXiv:2206.06557)
    - Single-shot decoding (arXiv:2306.12470)
3. Conclusions and open problems

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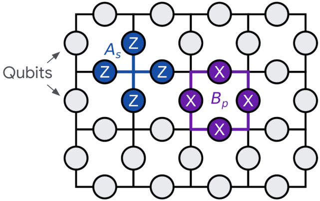
The surface code

Pros

- Geometrically local stabilizers
- Know how to implement, decode, perform logic, etc.



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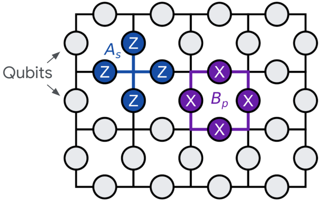

Cons

- Poor code parameters: $[[n, k = 1, d = \Theta(\sqrt{n})]] = [[L^2, 1, L]]$
- High overhead for fault-tolerance

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LDPC codes

- Bravyi-Poulin-Terhal bound [BPT10]: $kd^2 = O(n)$ in 2D

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- Relax condition on geometric locality
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  - Low weight checks are easier to measure
  - Important for fault-tolerance
- Best possible parameters for LDPC codes?

History of quantum LDPC codes

Code	k (Dim)	d (Dist)
Surface code [Kit03] • Geometrically local checks	1	$\Theta(\sqrt{n})$
Hypergraph product codes [TZ14, BH14] • Uses tools from algebraic topology	$\Theta(n)$	$\Theta(\sqrt{n})$
Fibre bundle codes [HHO21]	$\tilde{\Theta}(n^{3/5})$	$\tilde{\Omega}(n^{3/5})$
Lifted product codes [PK22b]	$\tilde{\Theta}(n^{\alpha})$	$\tilde{\Omega}(n^{1-\alpha/2})$
Balanced product codes [BE21]	$\Theta(n^{4/5})$	$\Omega(n^{3/5})$
Expander lifted product codes [PK22a] • First good quantum LDPC code	$\Theta(n)$	$\Theta(n)$
Quantum Tanner codes [LZ22]	$\Theta(n)$	$\Theta(n)$
DHLV codes [DHLV22]	$\Theta(n)$	$\Theta(n)$

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Constant space overhead

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https://arxiv.org/abs/2303.04798

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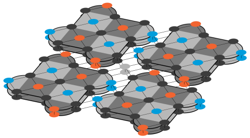
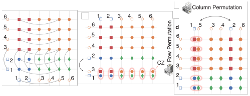
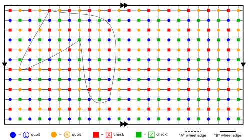
  - Limited number of long-range connections [BCG$^+$23]







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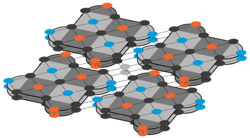
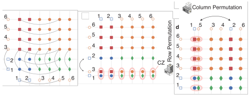
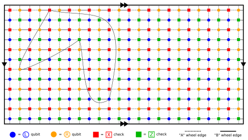
- Need efficient decoder







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- Unknown error e applied to a code state
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- Unknown error e applied to a code state
- Extract syndrome by measuring stabilizers
- Input: syndrome σ of an error e
- Output: a correction $\hat{f}$
- Succeed if $e + \hat{f}$ is a stabilizer

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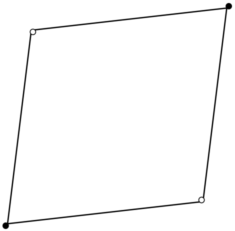
- Intractable problem in general (NP/#P complete)
- Two settings for decoding
  - Adversarial noise: decode any error of weight up to a constant fraction of the distance
  - Stochastic noise: decode random noise with high probability

Lightning overview of quantum Tanner codes [LZ22]

- Left-right Cayley complex
  (two-dimensional expanding object)
  - Vertices $V = V_X \sqcup V_Z$
  - Edges E
  - Squares Q



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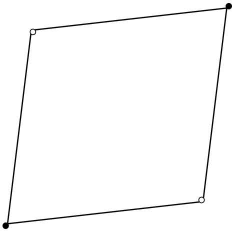

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- Qubits placed on squares Q
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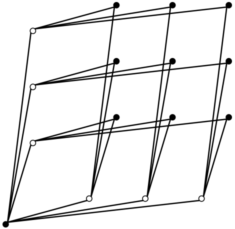

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- S_X generated by checks on faces incident to vertices in V_X
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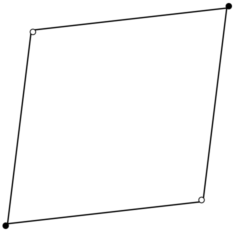
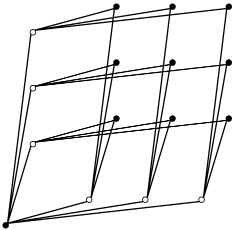
- Gives a code with parameters $[[n, k = \Theta(n), d = \Theta(n)]]$





Decoding quantum Tanner codes

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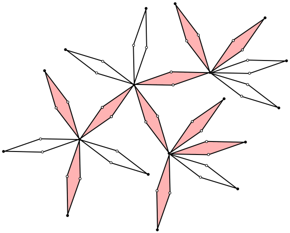

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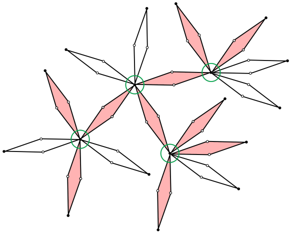

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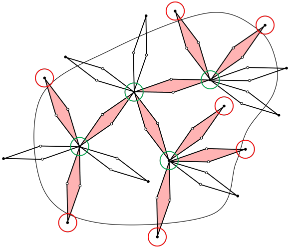

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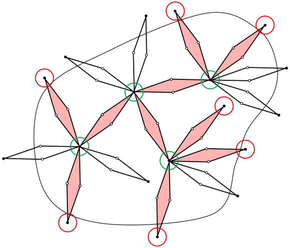
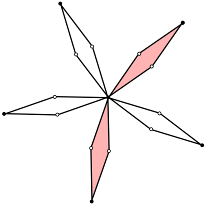
- Flipping qubits at the boundary of e will result in more satisfied checks



Potential-based decoder [GPT23]

1. For each $v \in V_Z$, determine a candidate correction ε_v on the qubits in its neighbourhood
   - Choose ε_v to have minimal weight while satisfying all local checks at v



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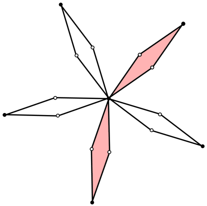

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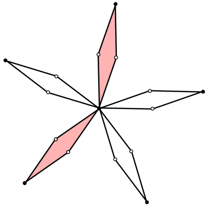

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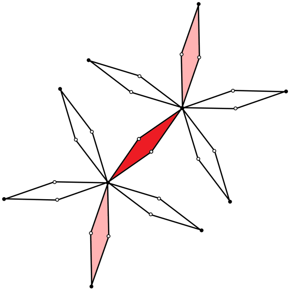

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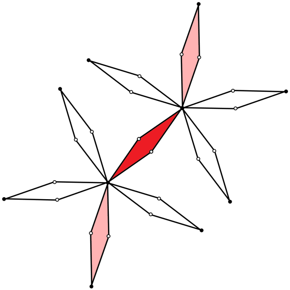

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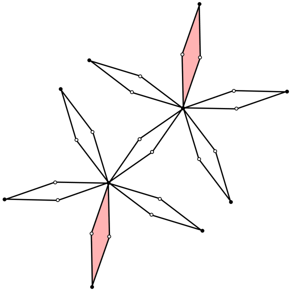

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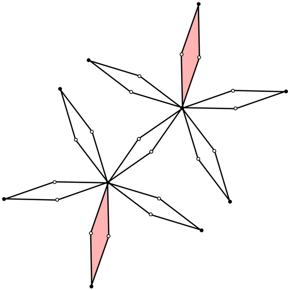

All stabilizer checks are satisfied when $U = 0$.

Main theorem

Theorem (Potential-based decoder [GPT23])

There is a family of quantum Tanner codes with parameters $[[n, \Theta(n), \Theta(n)]]$ such that the potential-based decoder can correct all errors of weight $|e| \leq p^* n$, where p^* is a constant. The time complexity is $O(n)$.

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- First decoder to correct adversarial errors of weight $O(n)$
- Previous best [EKZ22]: $O(\sqrt{n \log n})$

Stochastic errors: existence of threshold

Corollary

For i.i.d. errors with probability $p < p^*$*, the decoder succeeds with probability* $1 - O(e^{-an})$ *with* $a > 0$*.*

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For i.i.d. errors with probability $p < p^*$, the decoder succeeds with probability $1 - O(e^{-an})$ with $a > 0$.

Proof.

- By our main theorem, the decoder succeeds if $|e| \leq p^* n$
- Hoeffding's inequality: $\Pr(|e| > p^* n) < e^{-2n(p^* - p)^2}$

$\square$

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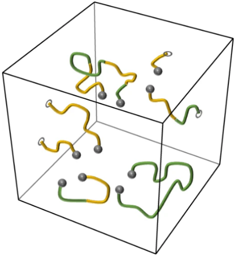
- Alternative approach: single-shot quantum error correction [Bom15]
  - Make progress in decoding with noisy syndrome data
  - Can also consider adversarial or stochastic noise

Existing single-shot decoders

- Topological codes
  - 4D toric code [BDMT17], 3D subsystem toric code [KV22], 3D gauge colour code [Bom15]
  - Use redundancy of checks



https://www.nature.com/articles/s41467-022-33923-4/figures/7

https://arxiv.org/abs/2208.01002

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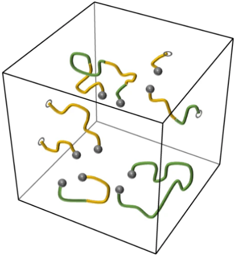
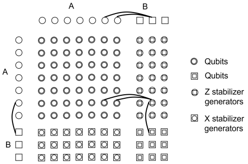
- Expansion based LDPC code
  - Quantum expander codes [FGL18]
  - Expansion provides single-shot property





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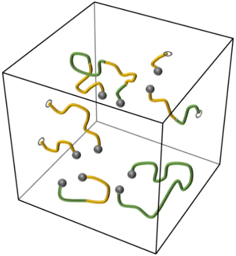
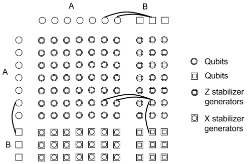
- Arbitrary stabilizer codes can be made single-shot [Cam19]
  - May not keep LDPC property





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Setup

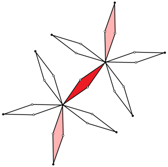
- Input: noisy syndrome $\tilde{\sigma}$
    - Data error e
    - Syndrome error D
- Output: a correction $\hat{f}$

Definition

A decoder is (α, β)-single-shot if for sufficiently low-weight errors,

$$|e + \hat{f}|_R \leq \alpha |e|_R + \beta |D| \,.$$

Mismatch decomposition decoder [LZ23]



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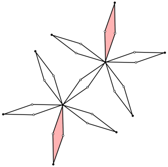
$Z = 2$



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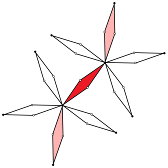

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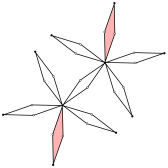

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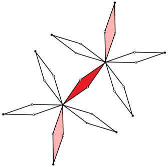
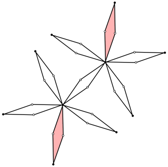
- At every step, flip qubits in some local region to decrease Z
    - Stop when no more flips possible

Mismatch decomposition decoder [LZ23]



$U = 4$

$Z = 2$



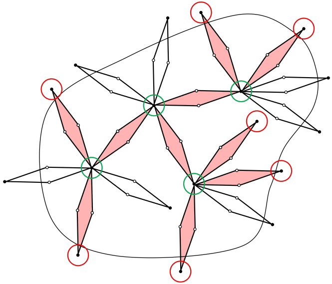
$U = 2$

$Z = 2$

- Also a local greedy decoder, but uses the mismatch $Z = \left|\sum_{v \in V_Z} \varepsilon_v\right|$ instead of the potential $U = \sum_{v \in V_Z} |\varepsilon_v|$
- At every step, flip qubits in some local region to decrease Z
  - Stop when no more flips possible
- The algorithm can be run sequentially or in parallel
  - Sequential decoder: $O(n)$ runtime
  - Parallel decoder: $O(\log n)$ runtime

Why is this decoder single-shot?

- Recall: intuition that valid corrections are near the "boundary" of the error region
- Expansion $\implies$ large boundary $\implies$ many candidate corrections
- Syndrome noise can affect a limited number of these corrections



Main results for single-shot decoding

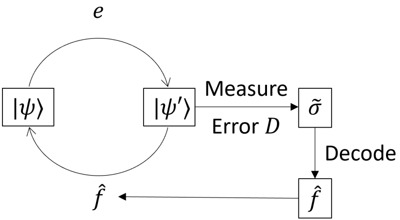
Theorem (Single-shot property [GTC$^+$23])

There exists a constants β such that we have the following:

1. *The sequential decoder is $(\alpha = 0, \beta)$-single-shot.*
2. *The parallel decoder with k-iterations is $(\alpha = 2^{-\Omega(k)}, \beta)$-single-shot.*

(Recall: (α, β)-single-shot means $|e + \hat{f}|_R \leq \alpha |e|_R + \beta |D|$.)

Multiple rounds of errors (stochastic setting)



- For i.i.d. errors (e_i, D_i) with probability $p < p^*$, quantum information is maintained for $\Omega(e^{an})$ rounds with probability $1 - O(e^{-bn})$ with $a, b > 0$

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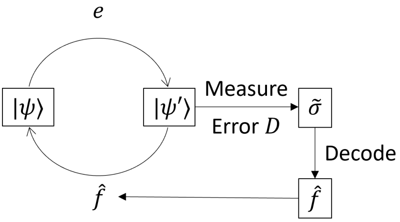

- For i.i.d. errors (e_i, D_i) with probability $p < p^*$, quantum information is maintained for $\Omega(e^{an})$ rounds with probability $1 - O(e^{-bn})$ with $a, b > 0$
- Generalizes to space/time correlated errors
    - E.g. circuit noise

Constant-time decoding of quantum Tanner codes

- The k-iteration parallel decoder is $(\alpha = 2^{-\Omega(k)}, \beta)$-single-shot
    - Choose k a sufficiently large constant
- During the computation: residual errors are small (nonzero)

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- Constant time overhead using quantum Tanner codes

Conclusions

Summary

- Provably correct and efficient decoders for quantum Tanner codes
- Single-shot property of the sequential and parallel decoders
- Quantum error correction with constant space and time overhead

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- Provably correct and efficient decoders for quantum Tanner codes
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Next steps

- Logical gates for LDPC codes
- How to choose the right LDPC code to use?
  - Decrease constants involved in the good code constructions
- General framework for analyzing local greedy decoders

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