semiring-based soft constraint solving and argumentation

Prof. Stefano Bistarelli

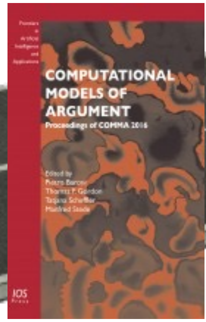
Simons Institute for the Theory of Computing

Logic and Algebra for Query Evaluation

Università degli Studi di Perugia

Semirings since 1995 …

S. Bistarelli, U. Montanari, F. Rossi: **Constraint** Solving over Semirings. IJCAI 1995 (JACM 1997)



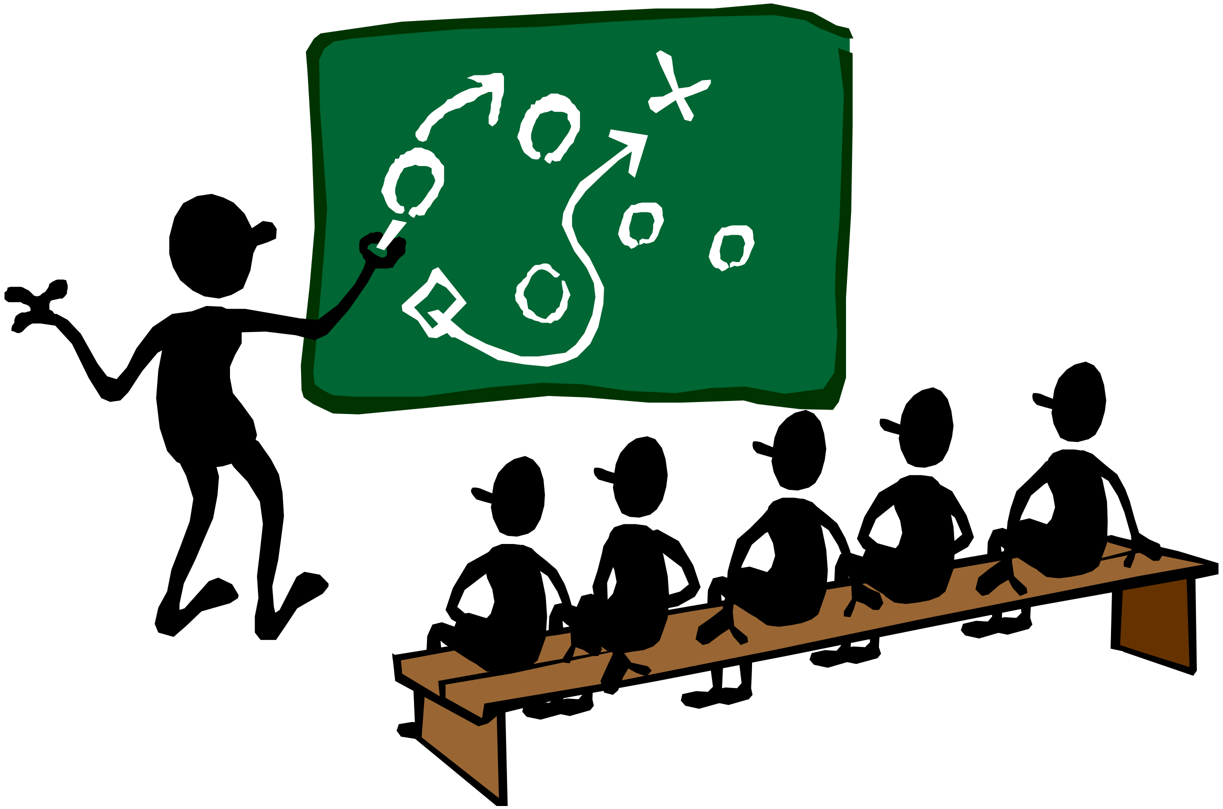
S. Bistarelli, F. Rossi, F. Santini: ConArg: A Tool for Classical and Weighted **Argumentation**. COMMA 2016 (J. Approx. reason 2018)

S. Bistarelli, F. Santini: On merging two **trust-networks** in one with bipolar preferences. Mathematical Structures in Computer Science 2017

Summary

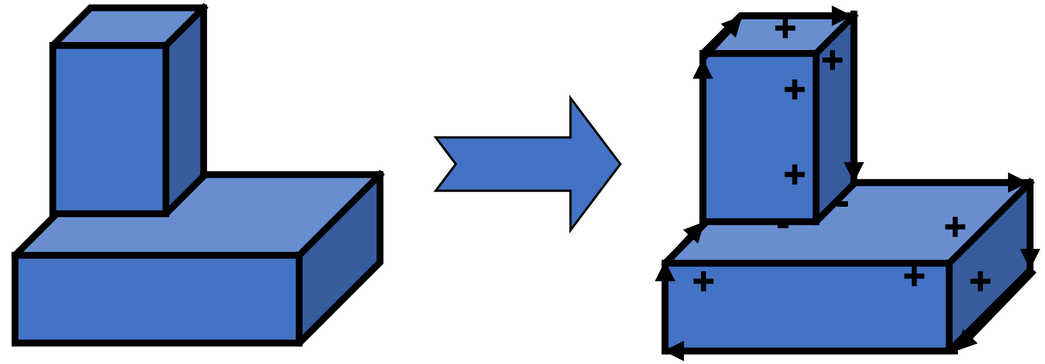
- Constrain Satisfation problems (CSPs)
- Semiring-based CSPs
- Argumentation
- Semiring-based Argumentation
- Some ideas/open problems

Constraint Programming … some background and introduction



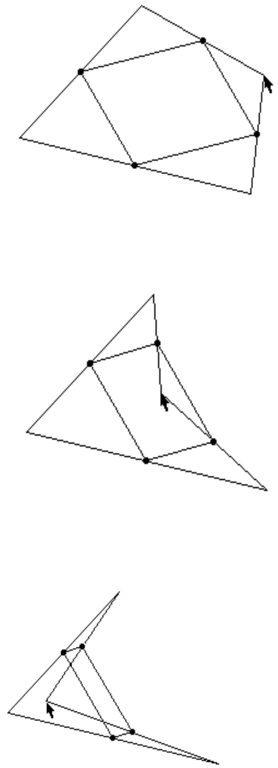
Scene Labelling

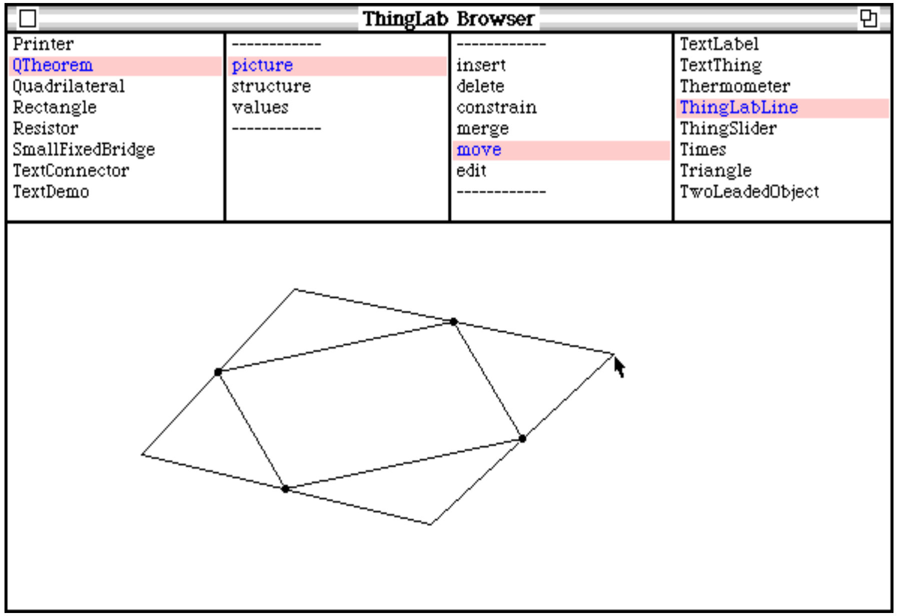
- first constraint satisfaction problem
- *Task:*
  recognise objects in 3D scene by interpreting lines in 2D drawings
- Waltz labelling algorithm (1972)
  - legal labels for junctions only
  - the edge has the same label at both ends

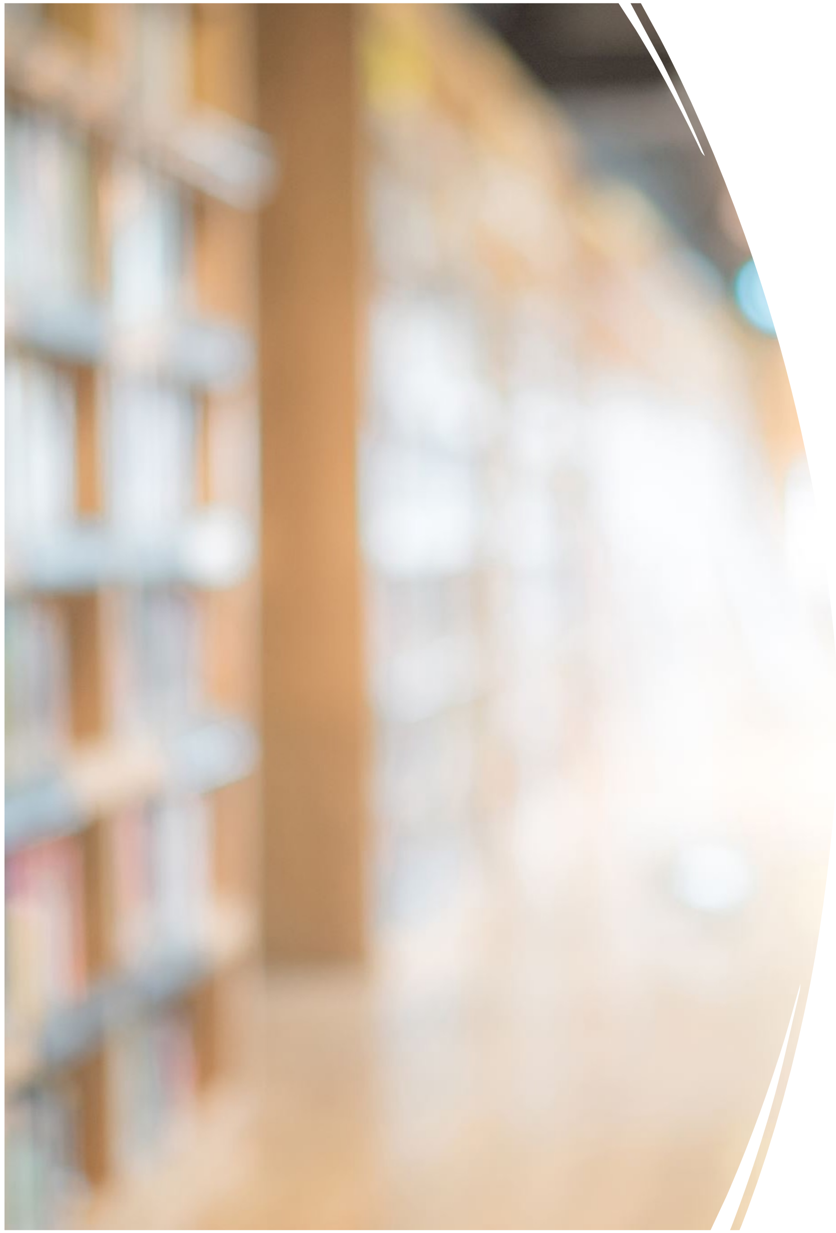


Interactive Graphics

- Sketchpad (Sutherland 1963) geometric constraints
- ThingLab (Borning)
  - allow to draw and manipulate constrained geometric figures in the computer display



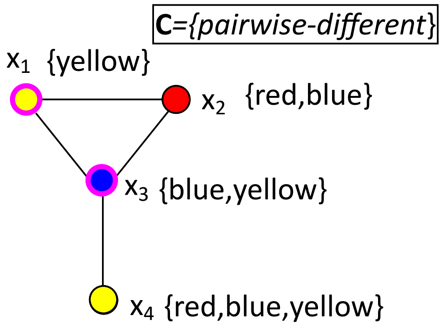




First (and most important) scientific papers

- Networks of Constraints: Fundamental Properties and Applications to Picture Processing, by Ugo Montanari (1974)
- Consistency in Networks of Relations, by Alan K. Mackworth (1980)

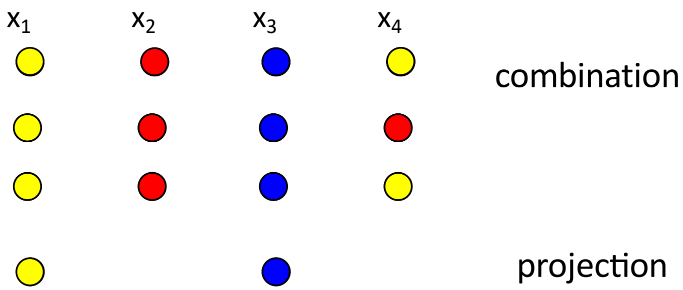
Introduction and Background : Constraint problems



P={ V, D, C, PC, con, def, a}

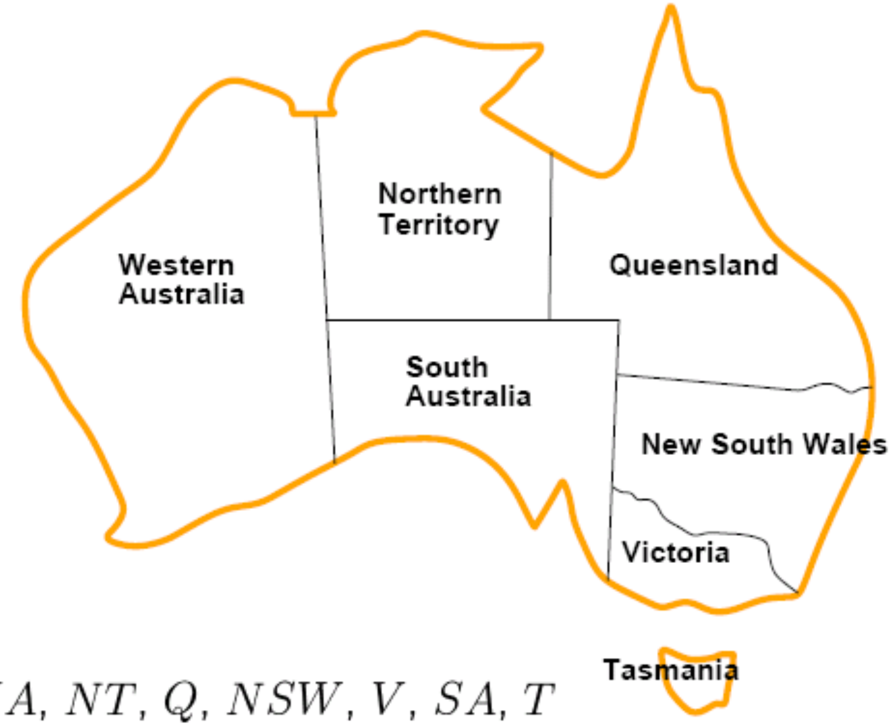
CSP(FD)

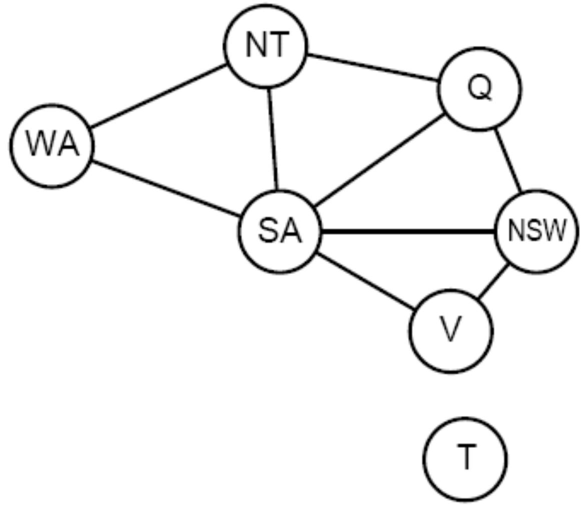
Tuples of domain values



Networks of Constraints: Fundamental Properties and Applications to Picture Processing, by Ugo Montanari (1974)

Example: Map-Coloring





Variables WA, NT, Q, NSW, V, SA, T
Domains $D_i = \{red, green, blue\}$
Constraints: adjacent regions must have different colors
e.g., $WA \neq NT$ (if the language allows this), or
$(WA, NT) \in \{(red, green), (red, blue), (green, red), (green, blue), \ldots\}$

Real-world CSPs

Assignment problems
    e.g., who teaches what class

Timetabling problems
    e.g., which class is offered when and where?

Hardware configuration

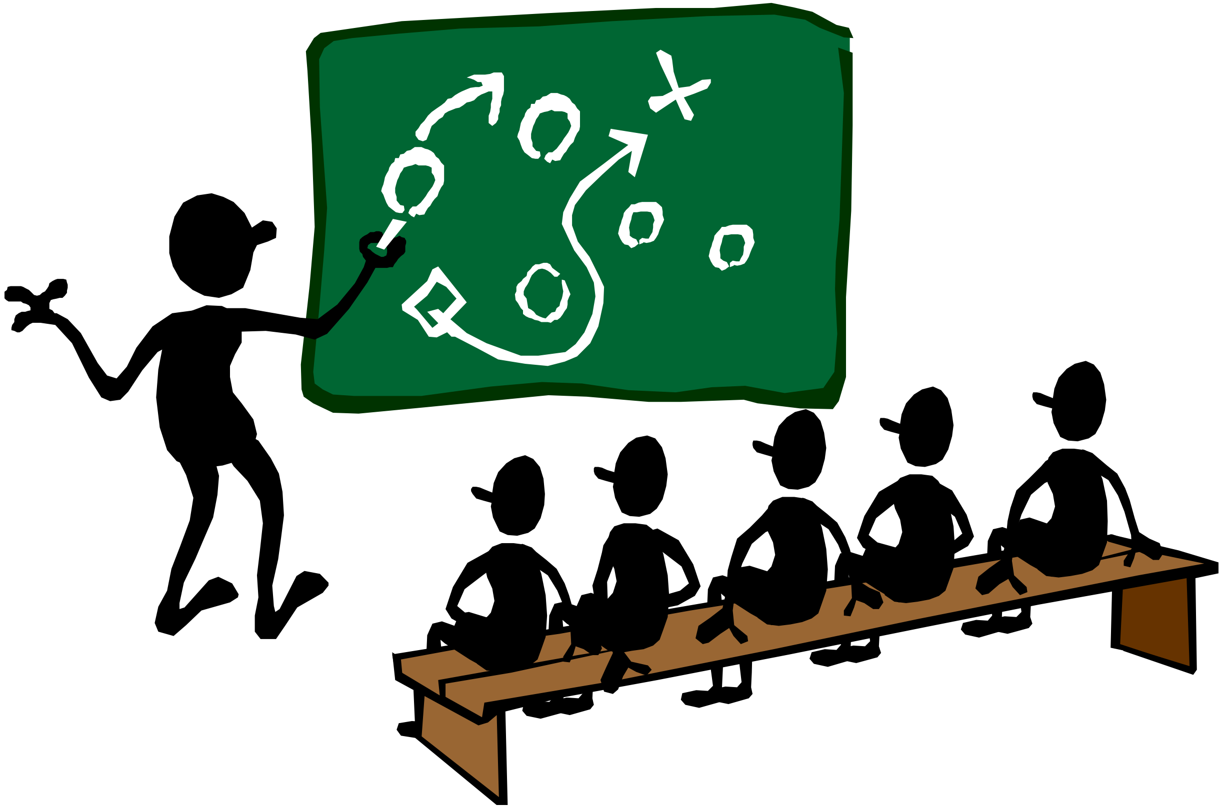
Spreadsheets

Transportation scheduling

Factory scheduling

Floorplanning

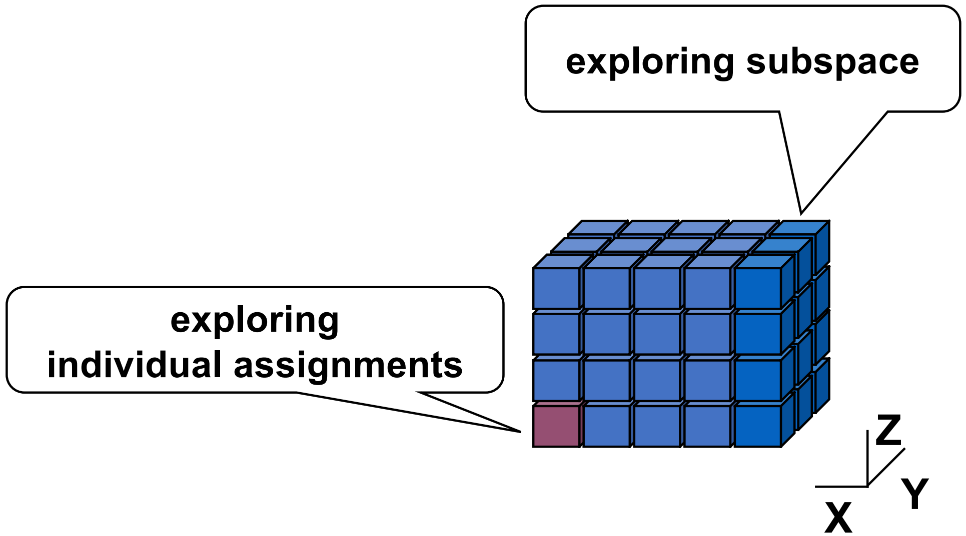
Solve CSPs .. The main idea is local consistency and propagation



Search for a solution

- exploring the solution space
- complete and sound (efficiency issues)

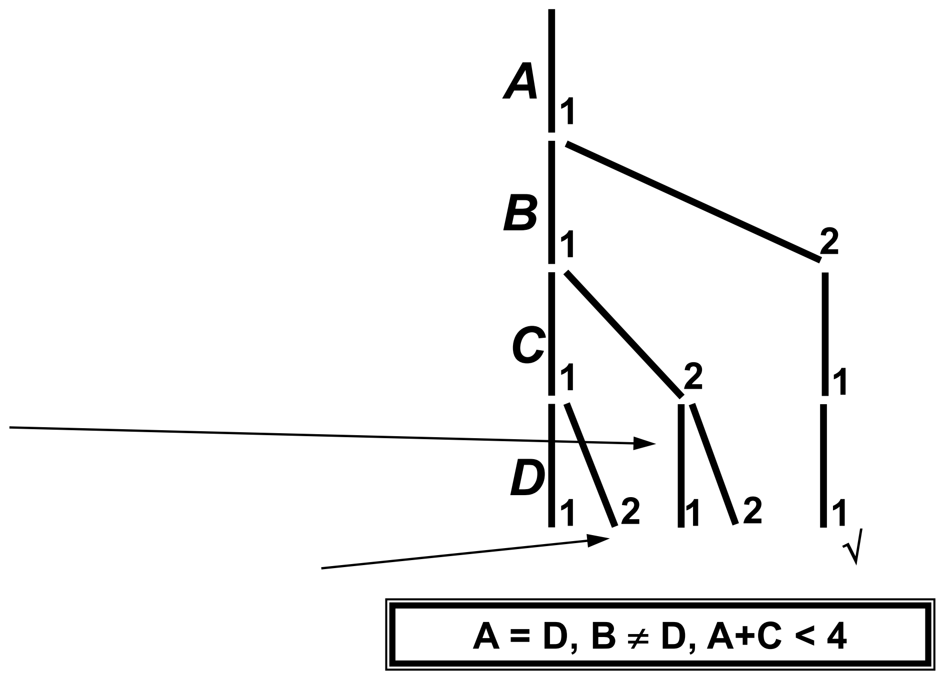
- Generate & Test (GT)
- Backtracking (BT)



Backtracking (BT)

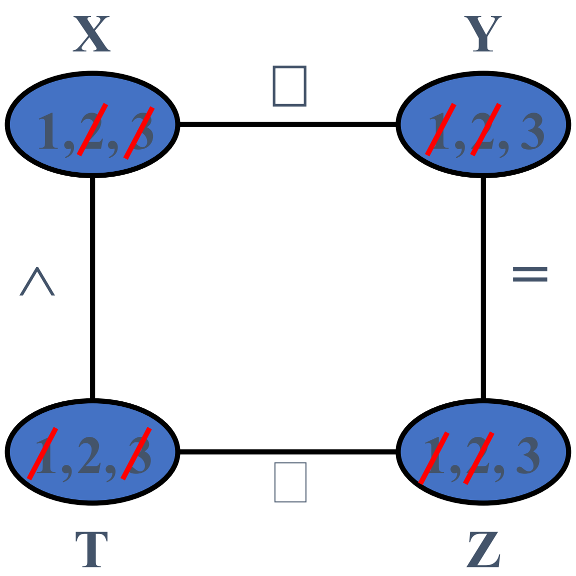
- incrementally extends a partial solution towards a complete solution
- Algorithm:

  assign value to variable
  check consistency
  until all variables labelled
- Drawbacks:
  - thrashing
  - redundant work
  - late detection of conflict



Arc-consistency

1 ≤ X, Y, Z, T ≤ 3
X □ Y
Y = Z
T □ Z
X ≤ T



Consistency in Networks of Relations, by Alan K. Mackworth (1980)

Arc-consistency

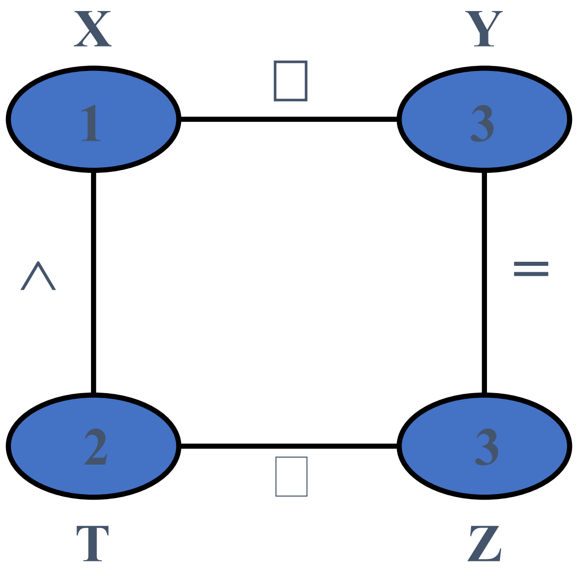
$1 \leq X, Y, Z, T \leq 3$

X □ Y

$Y = Z$

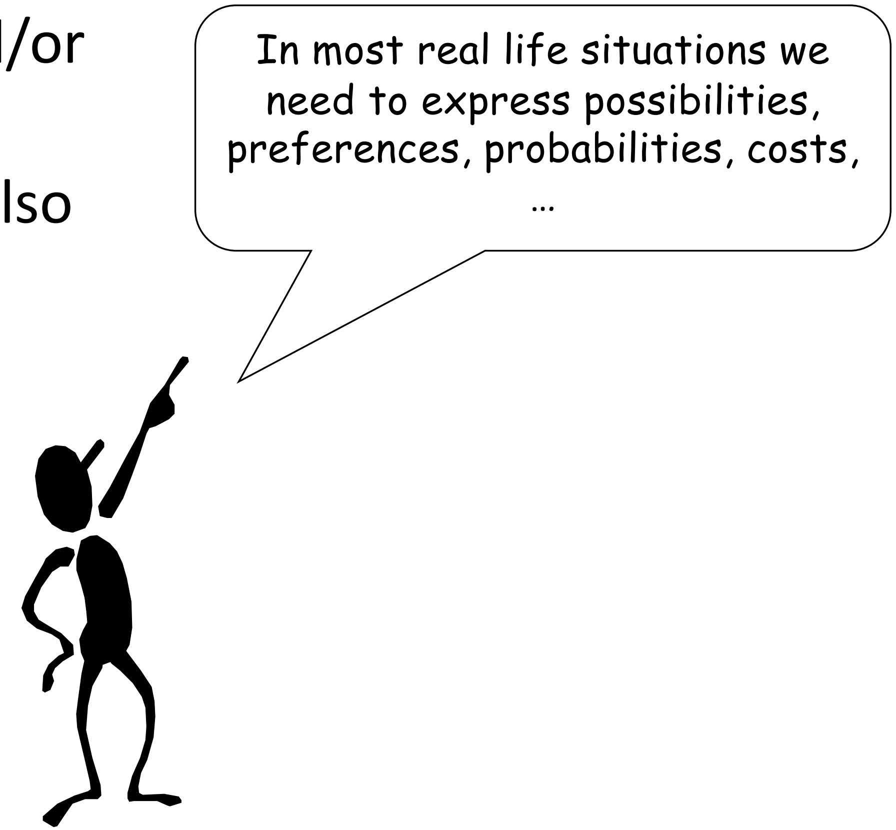
T □ Z

$X \leq T$



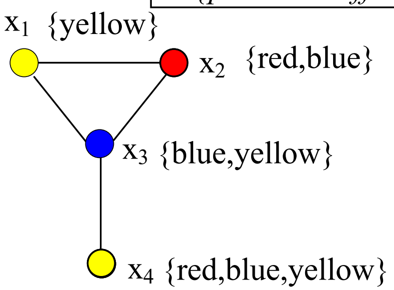
Soft Constraint Satisfaction Problems

- Over-constrained problems
- Problems with both preferences and hard statements, and/or uncertanties
- Optimization problems (also multicriteria)

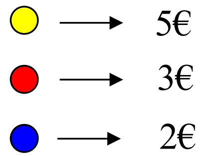


From Crisp CSPs to Soft CSPs

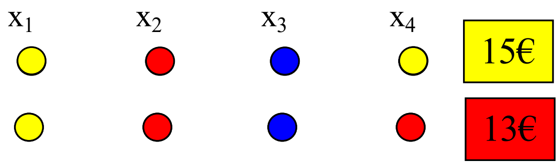
C={*pairwise-different*}



P={V, D,C, supp, def}



combination



using plus to combine costs

<$\Re^+$, +>	<[0,1], min>	<[0,1], $\times$>	<{false,true}, $\wedge$>
Weighted	Fuzzy	Probabilistic	Classical

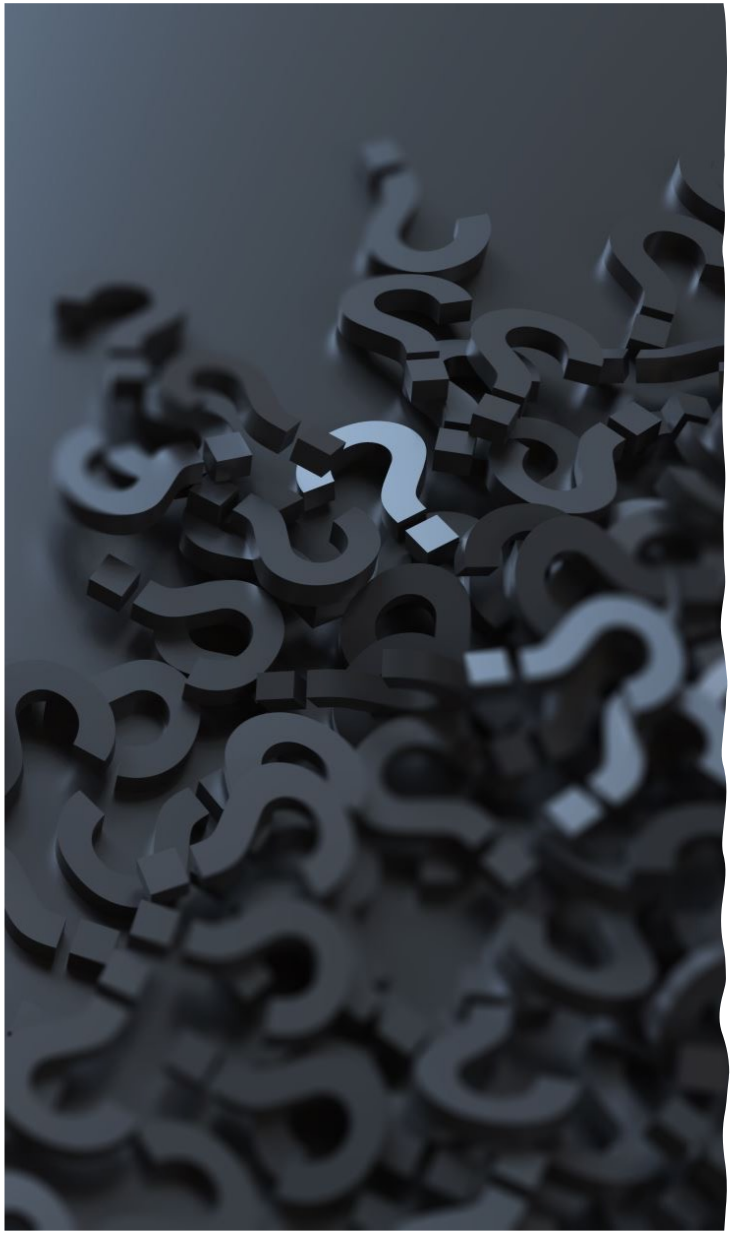
!! A semiring!! <A,+,$\times$,0,1>

$a \leq b \Leftrightarrow a+b=b$

Which properties should be (at least) required for CSPs? 1/2

1. Associativity and commutativity of $\times$ (that is, $(A,\times)$ is a commutative semigroup)
   - Because we consider “sets” of constraints and the order of combination has to be irrelevant

2. Absortiveness of $\times$ that is $(a\times b)+a=a \equiv (a\times b\leq a)$
   - Adding constraints has to decrease the number and the quality of the solutions



Which properties should be (at least) required for CSPs? 2/2

3. $\exists\, 0 \in A$ s.t. $a \times 0 = 0$
   - 0 represents the total dislike of a solution that involves a specific assignment

4. $\exists\, 1 \in A$ s.t. $a \times 1 = a$ (that is, $(A, \times)$ is a commutative monoid)
   - 1 represents the lower dislike (indifference) of a solution that involves a specific assignment

Absortive Semirings

- Commutative semirings $(A,+,\times,0,1)$
  - $(A,+,0)$ and $(A,\times,1)$ are commutative monoids
  - $\times$ distributes over $+$
  - $a\times 0=0$ (0 is annihilator)
- Absorptive semirings $(A,+,\times,0,1)$
  - Absortiveness of $\times$ $(a\times b)+a=a$

The semiring based CSPs

Semiring: S = <A,+,×,0,1>   a≤b (b is better than a) iff a+b=b

Constraint System: CS = <C,D,V,S>   Constraint Problem: <C,a>

Constraint: <con,def>, con⊆V (type), def:$D^k \rightarrow A$ (value),

- Constraint $\eta : V \rightarrow D \quad \mathcal{C} : \eta \rightarrow A$

combination: $c = c_1 \otimes c_2$=<def,con=$con_1 \cup con_2$>,

$$def(\mathrm{t}) \square def_1(\mathrm{t} \downarrow^{con}_{con_1}) \times def_2(\mathrm{t} \downarrow^{con}_{con_2})$$

- Combination $(c_1 \otimes c_2)\eta \; = \; c_1\eta \times_S c_2\eta$

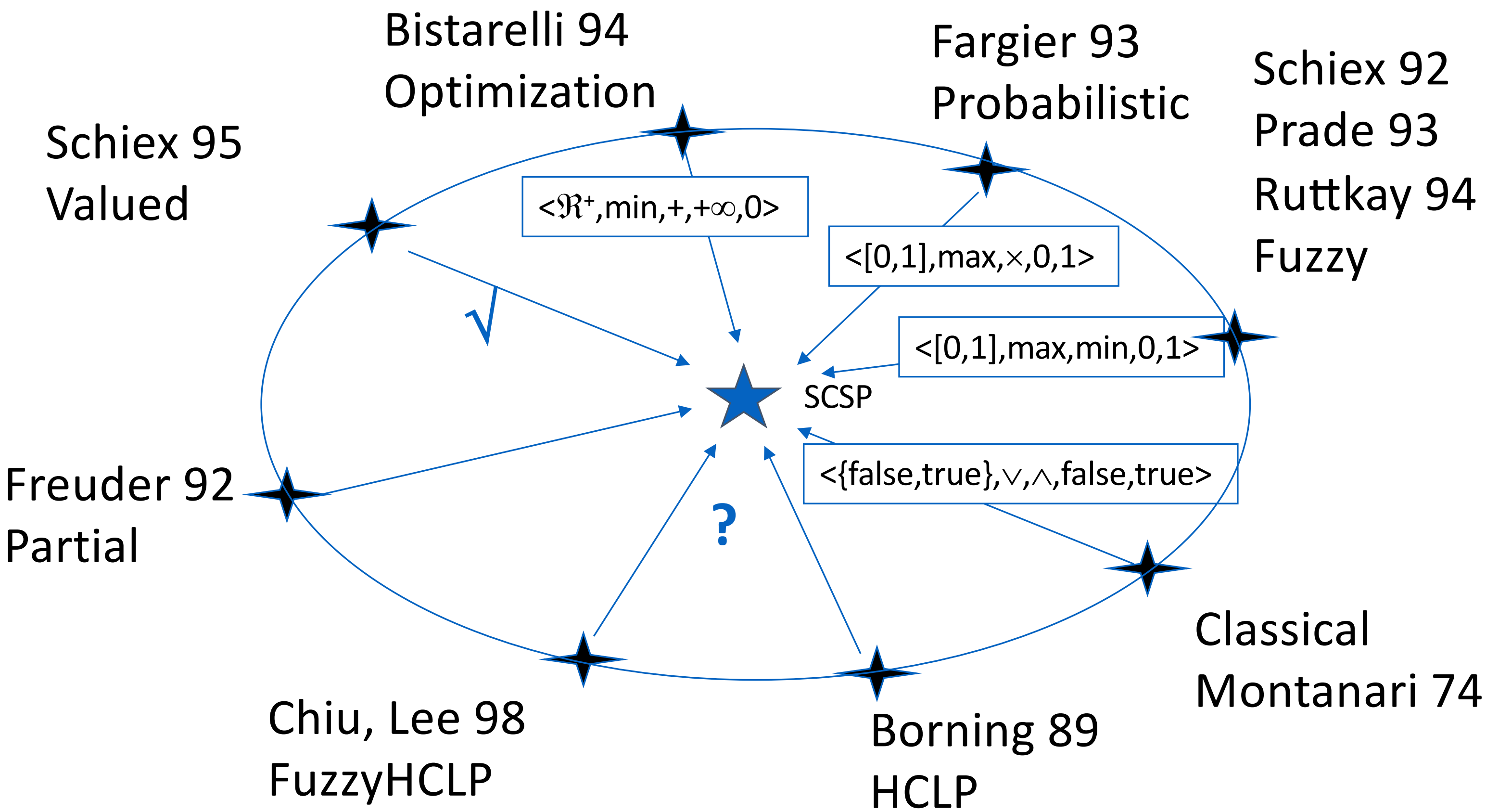
projection: $c \Downarrow_I$=<def,I∩con>,

$$def(\mathrm{t'}) \square \sum_{\{\mathrm{t} | \mathrm{t} \downarrow^{con}_{I \cap con} \square \mathrm{t'}\}} def(\mathrm{t})$$

- Projection $\quad c \Downarrow_{(V - \{v\})} \eta = \sum_{d \in D} c\eta[v := d]$

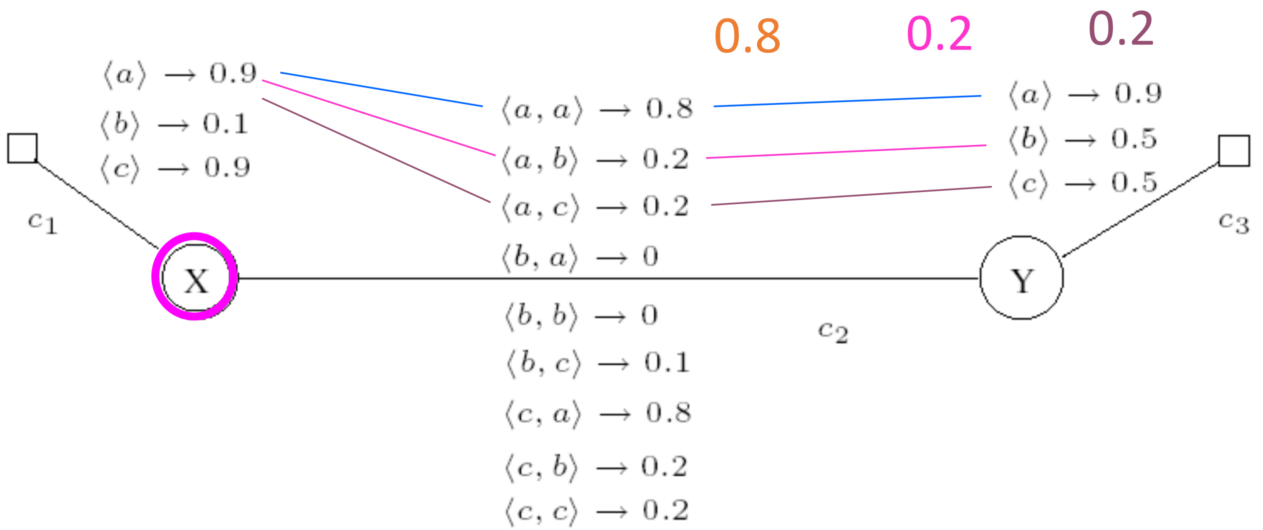
Sol(<C,a>)=(⊗C) $\Downarrow_a$

… One (semi)ring to rule them all …



Some Examples: fuzzy

S_{Fuzzy}<[0,1],max,min,0,1>

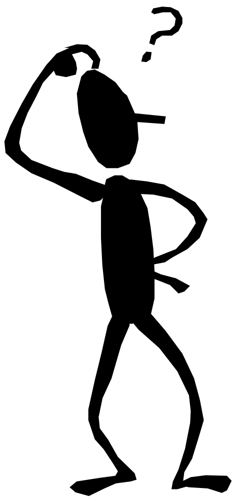


The Solution?

[X:=a]→ 0.8 [X:=b]→ [X:=c]→

(soft) AC … AC, AC again

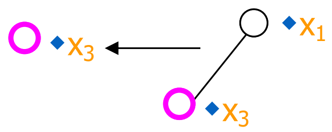
- How to extend classical local consistencies to SCSPs?

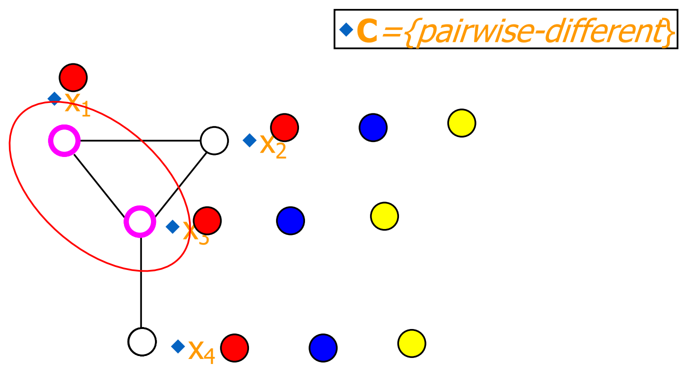


"The consultants are here."

Local consistency …

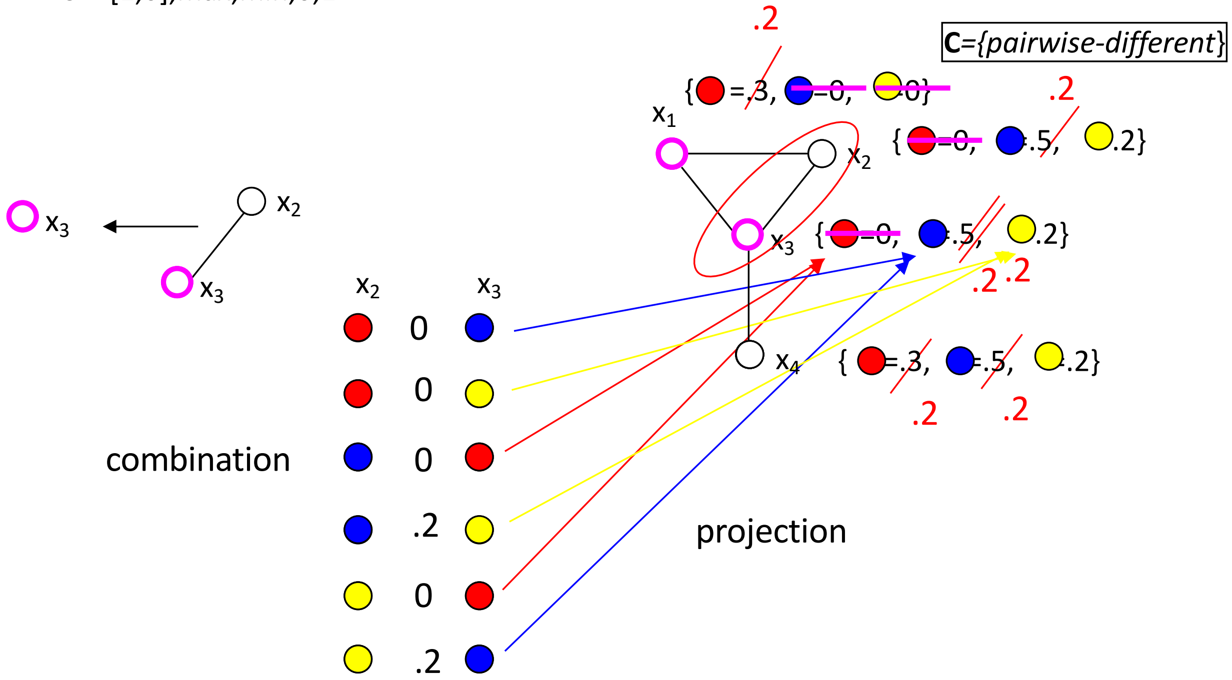
- Classical/crisp CSPs
  - Reduce the domain of the variables





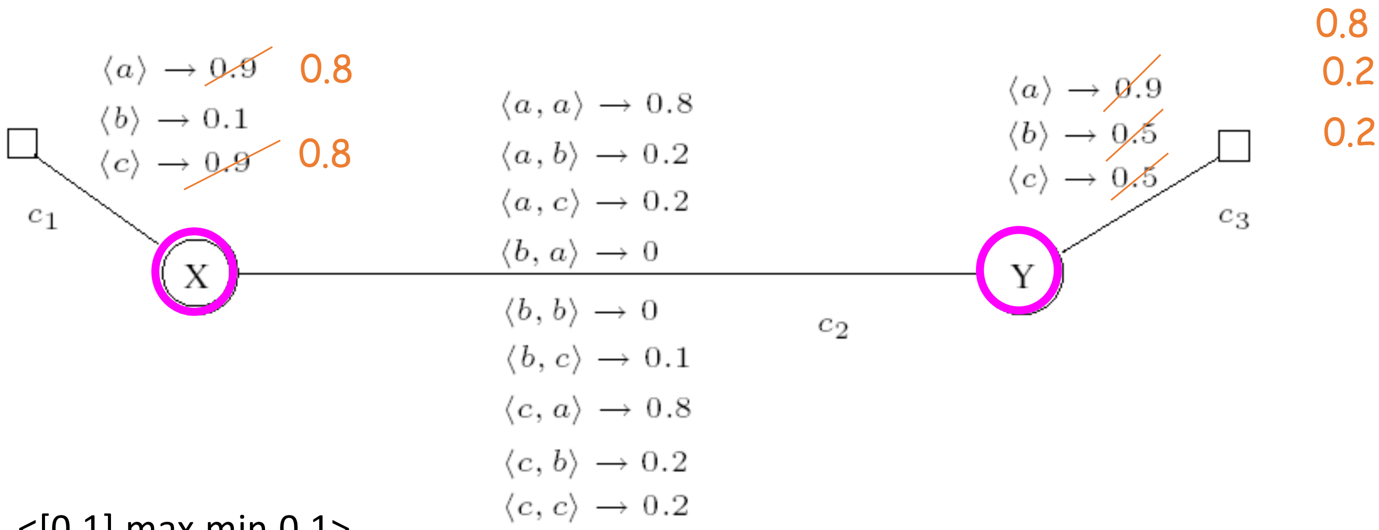
Local consistency

S=<[1,0],max,min,0,1>



$$c'_x = c_x \otimes (c \Downarrow_x)$$

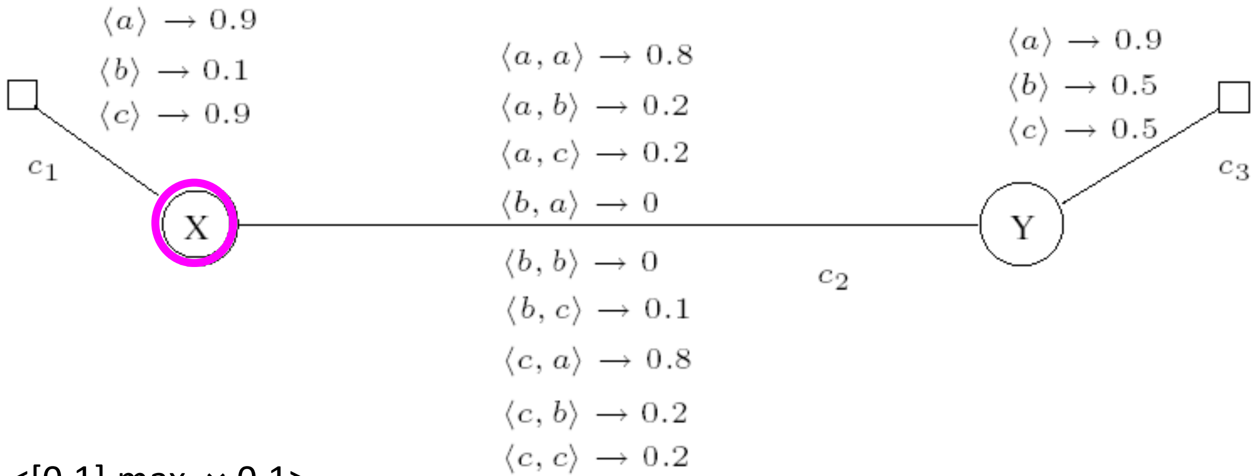
Some Examples: fuzzy



S_{Fuzzy}<[0,1],max,min,0,1>

The new values for X,Y

And for the Probabilistic?



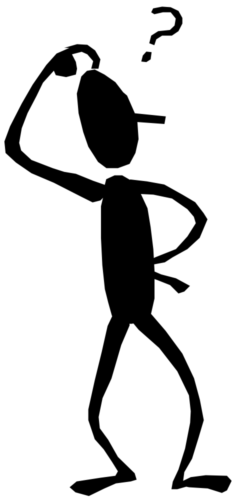
S_{Prob}<[0,1],max, ×,0,1>

$$c'_x = c_x \otimes (c \Downarrow_x)$$

times is not idempotent!!!

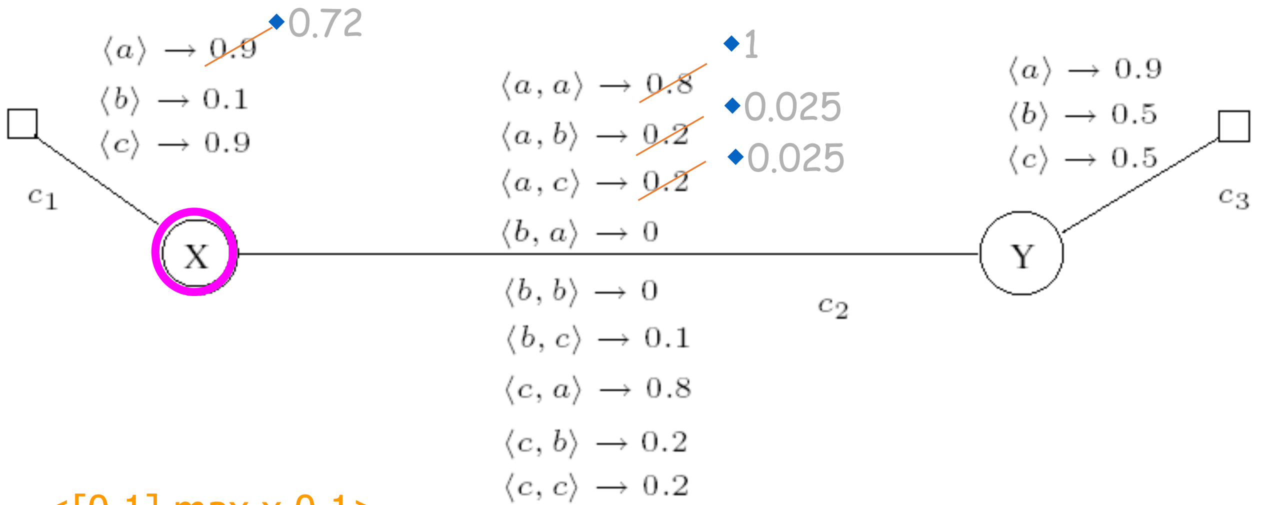
A journey on (soft) AC …

- What to do when times not idempotent?



“The consultants are here.”

And for the Probabilistic?



- S_{Prob}<[0,1],max,x,0,1>

- **times is not idempotent.. But with division!!!**

0.9 x max(0.8, 0.2, 0.2) = 0.9 x 0.8 = 0.72

0.8 : 0.8 = 1
0.2 : 0.8 = 0.025
0.2 : 0.8 = 0.025

Extending local consistency

- Local consistency rules for soft constraints decrease the preference value (instead of removing tuple)
  - A local consistency rule involving a constraint c and a unary constraint c_x with supp(c_x)=x $\subset$ supp(c) consists of the following phases:
    - substituting the original constraint c_x with c'_x

$$c'_x = c_x \otimes (c \Downarrow_x)$$

    - modifying the constraint c in a new constraint c'

$$c' = c \oplus\!\!\!\!\div (c \Downarrow_x)$$

that takes into account the changes performed on c_x: Since constraint c_x is combined with $c\Downarrow_x$, then c' is divided by the same value and the constraint division function is defined as

$$(c_1 \oplus\!\!\!\!\div c_2)\eta = c_1\eta \div c_2\eta.$$

Adding division

✓ Using residuation theory

- Approximating the solution of the equation $b \times x = a$ via the maximal of its subsolutions $\max\{x \mid b \times x \leq a\}$
- Notice that
  - if $\{x \in A \mid b \times x = a\}$ is not empty then $\max \{x \in A \mid b \times x \leq a\} = \max \{x \in A \mid b \times x = a\}$ (if among the subsolutions there is a solution, the maximal element of the set of subsolutions is also the maximal solution)
- $a \div b = \max\{x \in A \mid b \times x \leq a\}$
- $\mathcal{K}$ is residuated if such a maximum exists (all instances of soft CSPs are complete $\Rightarrow$ residuated!)

Residuation Theory - T.S. Blyth and M.F. Janowitz (1972)
Volume 102 in International Series of Monographs on Pure and Applied Mathematics

Division in the soft CSPs istances

- Classical CSPs

$$a \div b = max\{x \mid b \wedge x \leq a\} = (b \implies a)$$

- Fuzzy CSPs

$$a \div b = max\{x \mid min\{b, x\} \leq a\} = \begin{cases} 1 & \text{if } b \leq a \\ a & \text{if } a < b \end{cases}$$

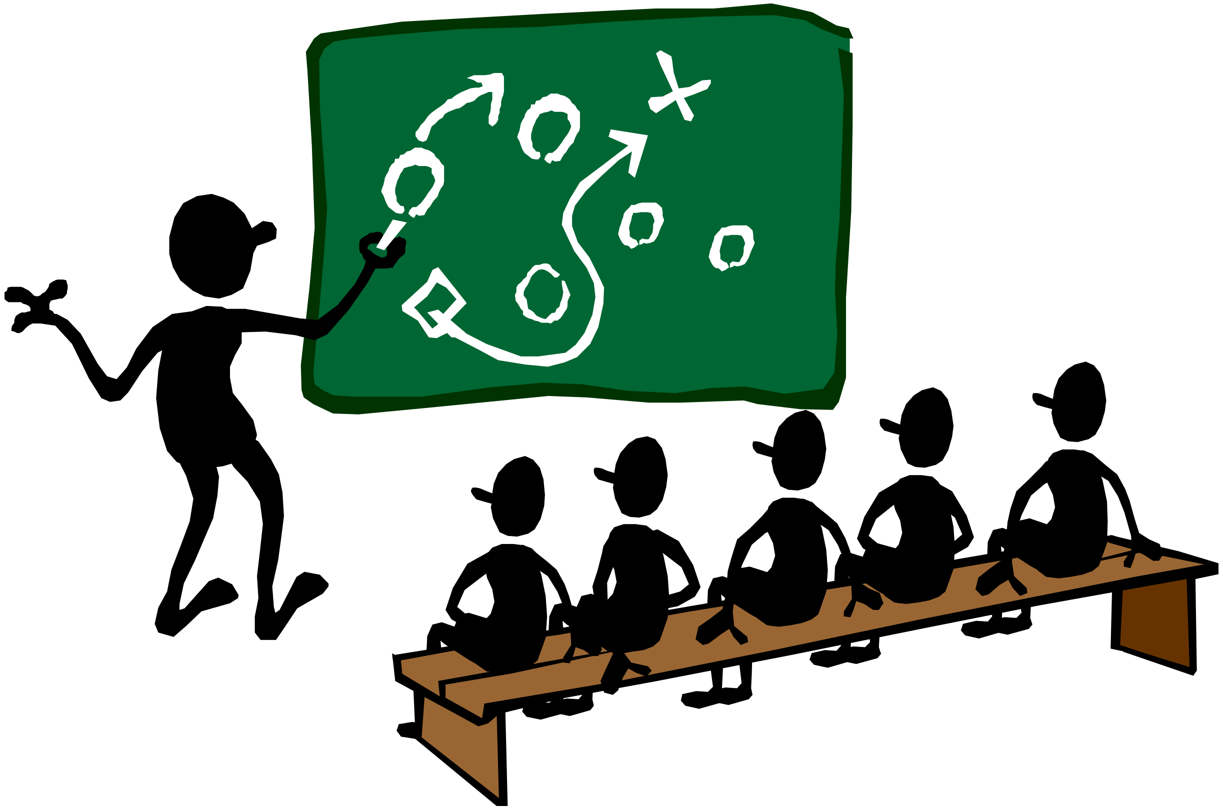
- Weighted CSPs

$$a \div b = min\{x \mid b \hat{+} x \geq a\} = \begin{cases} 0 & \text{if } b \geq a \\ a \hat{-} b & \text{if } a > b \end{cases}$$

- But also Probabilistic CSPs, Set-based CSPs, …

Stefano Bistarelli, Fabio Gadducci, Javier Larrosa, Emma Rollon, Francesco Santini:
Local arc consistency for non-invertible semirings, with an application to multi-objective optimization. Expert Syst. Appl.39(2): 1708-1717 (2012)

Argumentation… some background and introduction



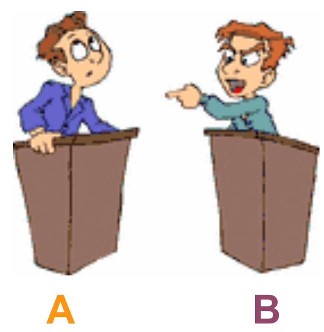
The scenario

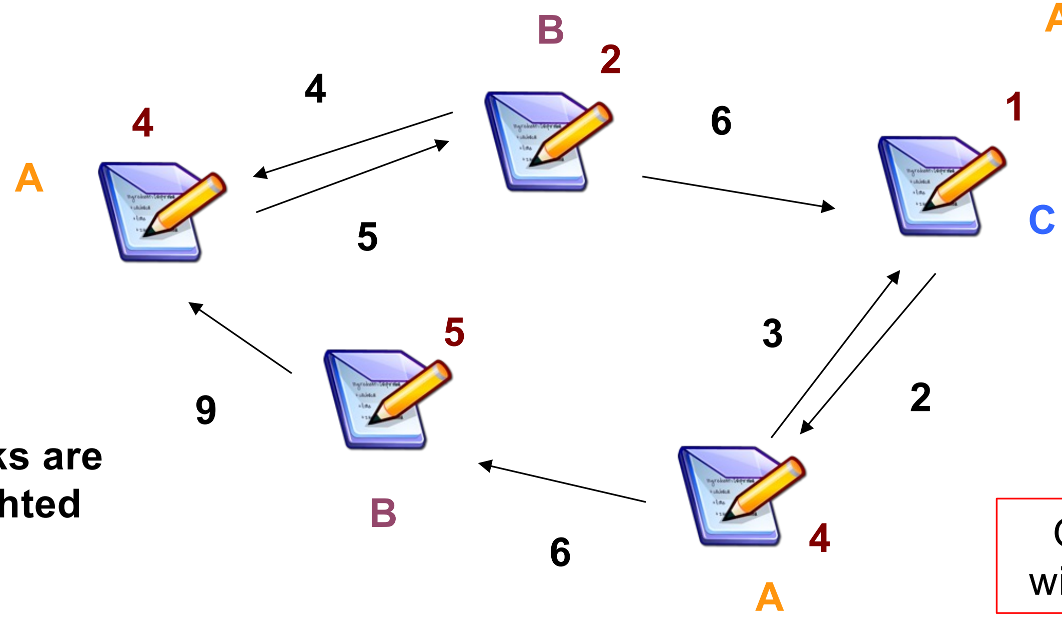
Your country does not want to cooperate

Your country does not want either

Your country is a rogue state

Rogue state is a controversial term





Attacks are weighted

Or Arguments with preferences

Abstract argumentation frameworks (Phan Minh Dung '95, Journal of AI)

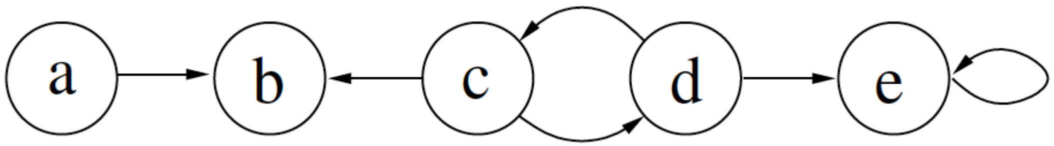
Definition

An argumentation framework (AF) is a pair (A, R) where

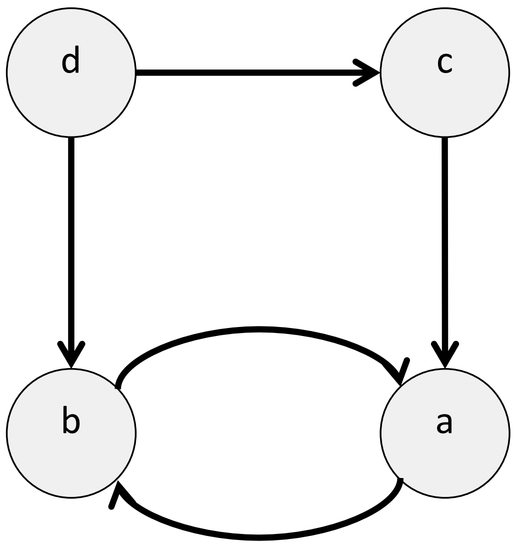
- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

Example

F=({a,b,c,d,e} , {(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)})



Example



a: *Governments should ban smoking*

b: *Governments shant interfere with the right to smoke*

c: *Smoking is a matter of freedom of choice and governments banning smoking would be a violation of rights ought to protect*

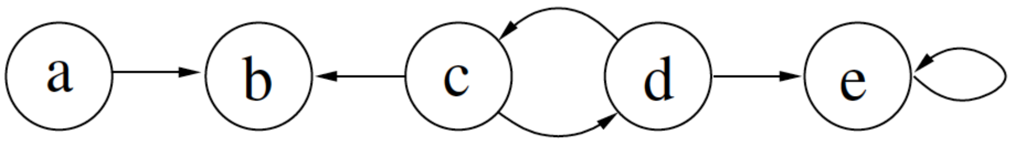
d: *Time after time, clinical research has proven that smoking is highly addictive. Thus, the issue may not be considered as a matter of freedom of choice, and governments are supposed to ban these practices*

Conflict-free Set

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

Example



$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$

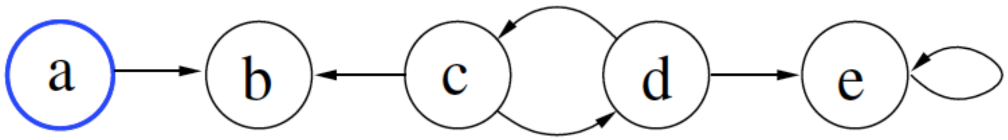
Admissible extensions (no undefended)

Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$adm(F) = \{\{a,c\}, \{a,d\}, \sout{\{b,d\}}, \{a\}, \sout{\{b\}}, \{c\}, \{d\}, \emptyset\}$

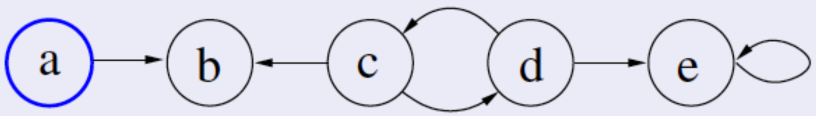
Complete (all defended)

Complete Set

Given an AF $F = (A, R)$. A set $S \subseteq A$ is complete in F, if

- S is admissible in F
- each $a \in A$ defended by S in F is contained in S
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$comp(F) = \{\{a, c\}, \{a, d\}, \{a\},$ ~~$\{c\}, \{d\}, \emptyset$~~$\}$

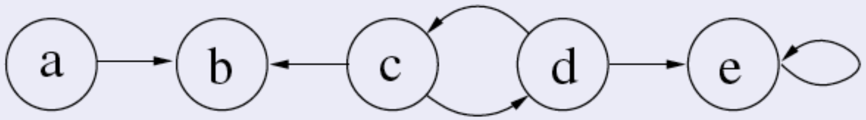
Stable

Stable Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is stable in F, if

- S is conflict-free in F
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$.

Example



$stable(F) = \{\sout{\{a, c\}}, \{a, d\}, \sout{\{b, d\}}, \sout{\{a\}}, \sout{\{b\}}, \sout{\{c\}}, \sout{\{d\}}, \sout{\emptyset}\}$

Grounded (sceptically accepted)

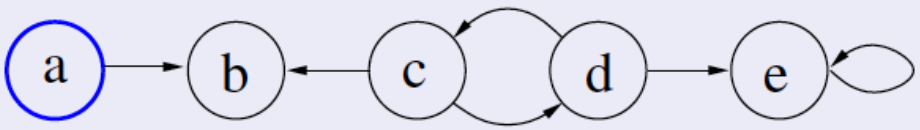
Grounded Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is grounded in F, if

- S is complete in F
- for each $T \subseteq A$ complete in F, $T \not\subset S$

Proposition [Dung 95]: The grounded extension of an AF $F = (A, R)$ is given by the least fix-point of the operator $\Gamma_F : 2^A \to 2^A$, defined as $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Example



$ground(F) = \{$~~$\{a, c\}, \{a, d\},$~~$\{a\}\}$

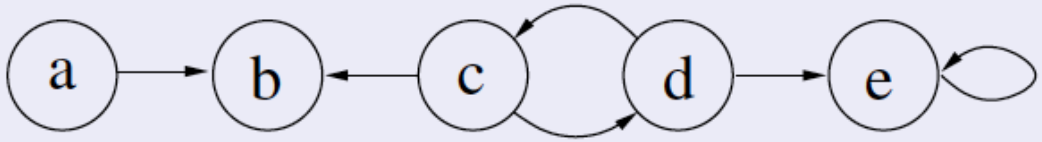
Preferred (guarantee existence)

Preferred Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is preferred in F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in T, $S \not\subset T$

Example



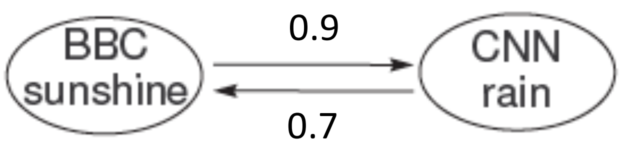
$pref(F) = \{\{a, c\}, \{a, d\},$ ~~$\{a\}, \{c\}, \{d\}, \emptyset$~~ $\}$

Complete, ground, preferred …

Semiring-based argumentation

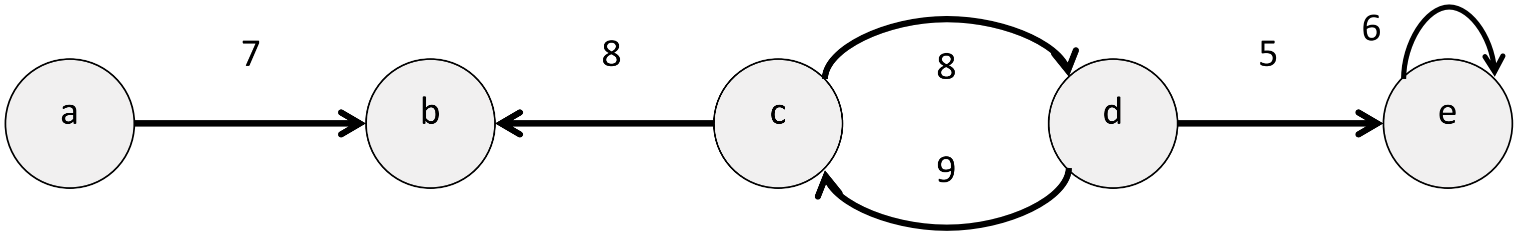
A forecast example

- *A and B claim contradictory conclusions and so attack each other*
  - two different admissible extensions: the sets *{A} and {B}..*
- However, one might reason that *A is preferred to B because the BBC are deemed more trustworthy than CNN.*



Definition and Example

A semiring-based Argumentation Framework ($WAAF_{\mathbb{S}}$) is a quadruple $\langle \mathcal{A}_{rgs}, R, W, \mathbb{S} \rangle$, where $\mathbb{S}$ is a semiring $\langle S, +, \times, \bot, \top \rangle$, $\mathcal{A}_{rgs}$ is a set of arguments, R the attack binary-relation on $\mathcal{A}_{rgs}$, and
$W : \mathcal{A}_{rgs} \times \mathcal{A}_{rgs} \longrightarrow S$ is a binary function.
Given $a, b \in \mathcal{A}_{rgs}$, $\forall (a, b) \in R$, $W(a, b) = s$ means that a attacks b with a weight $s \in S$. Moreover, we require that $R(a, b)$ iff $W(a, b) <_{\mathbb{S}} \top$.



$\mathcal{A}_{rgs} = \{a, b, c, d, e\}$, $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$,
with $W(a, b) = 7, W(c, b) = 8, W(c, d) = 9, W(d, c) = 8,$
$W(d, e) = 5, W(e, e) = 6$, and $\mathbb{S} = \langle \mathbb{R}^+ \cup \{\infty\}, \min, \hat{+}, \infty, 0 \rangle$

$\langle \mathbb{R}^+ \cup \{+\infty\}, min, \hat{+}, \infty, 0 \rangle$

Classical instantiations

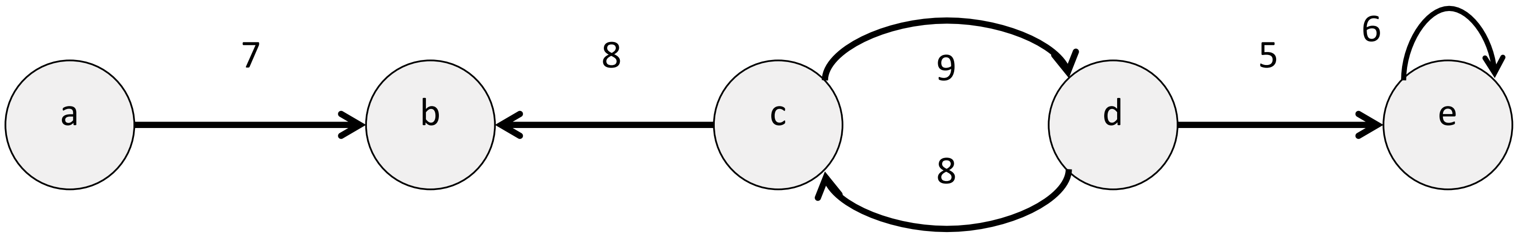
- Weighted $\langle \mathbb{R}^{+} \cup \{+\infty\}, min, \hat{+}, \infty, 0 \rangle$
- Fuzzy $\langle [0..1], \max, \min, 0, 1 \rangle$
- Probabilistic $\langle [0..1], \max, \hat{\times}, 0, 1 \rangle$
- Boolean $\langle \{false, true\}, \vee, \wedge, false, true \rangle$

- Boolean semirings can be used to represent classical defence in Argumentation
- The Cartesian product is still a semiring

$$\langle [0..1], \mathbb{R}^{+} \cup \{+\infty\} \rangle, \langle \max, \min \rangle, \langle \min, \hat{\times} \rangle, \langle 0, +\infty \rangle, \langle 1, 0 \rangle \rangle$$

w-defence (D_w)

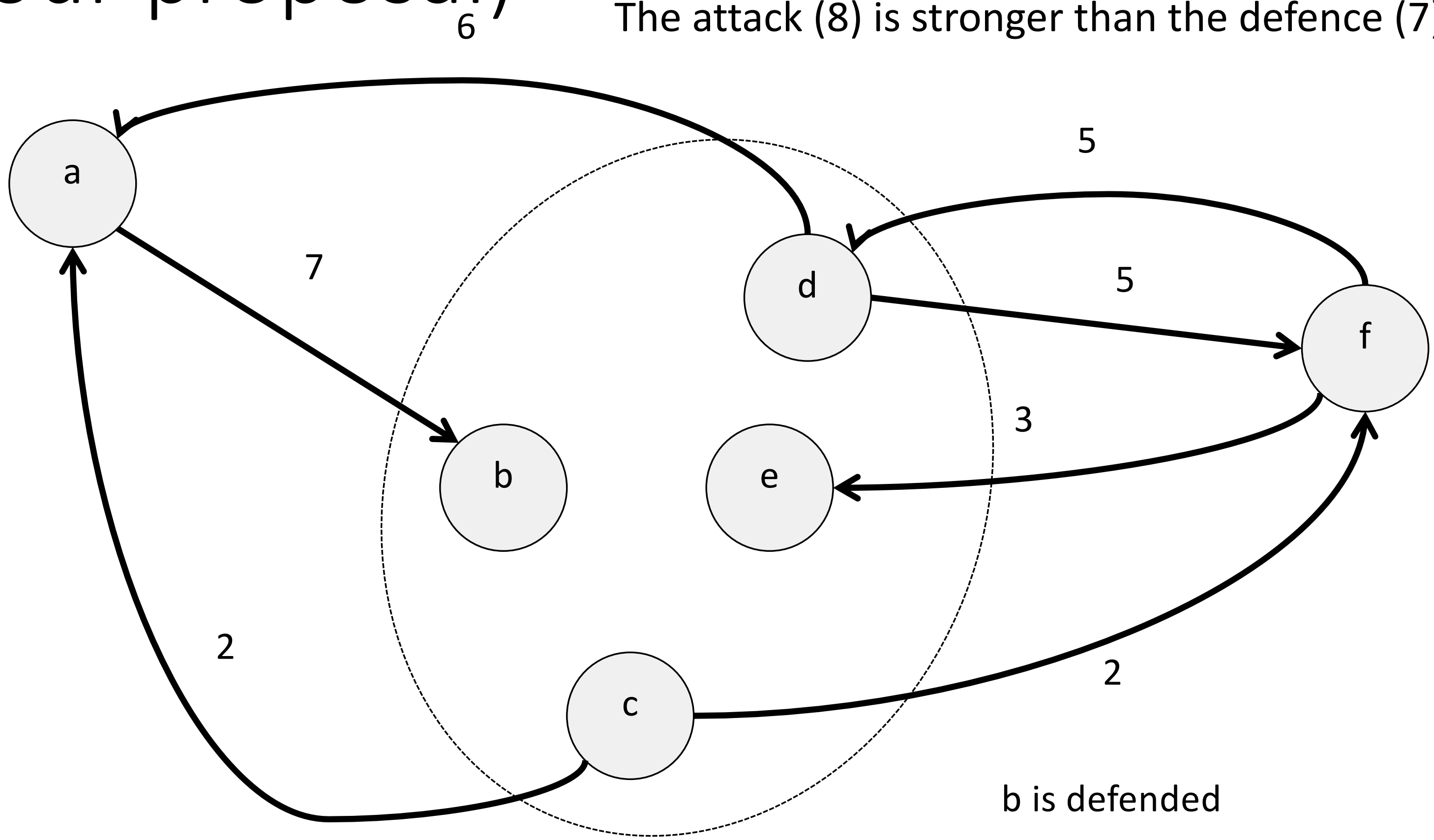
Given a $WAAF_{\$}$, $WF = \langle \mathcal{A}_{rgs}, R, W, S \rangle$, $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ w-defends $b \in \mathcal{A}_{rgs}$ iff, given $a \in \mathcal{A}_{rgs}$ s.t. $R(a, b)$, then $W(a, \mathcal{B} \cup \{b\}) \geq_{\$} W(\mathcal{B}, a)$;
$\mathcal{B}$ w-defends b iff it defends b from any a s.t. $R(a, b)$.



$\{c\}$ defends c from d because $W(d, \{c\}) \geq_{\$} W(\{c\}, d)$, i.e., $(8 \leq 9)$.
On the other hand, $\{d\}$ does not defend d because $W(c, \{d\}) \not\geq_{\$} W(\{d\}, c)$

D_W (our proposal)

The attack (8) is stronger than the defence (7)



b is defended

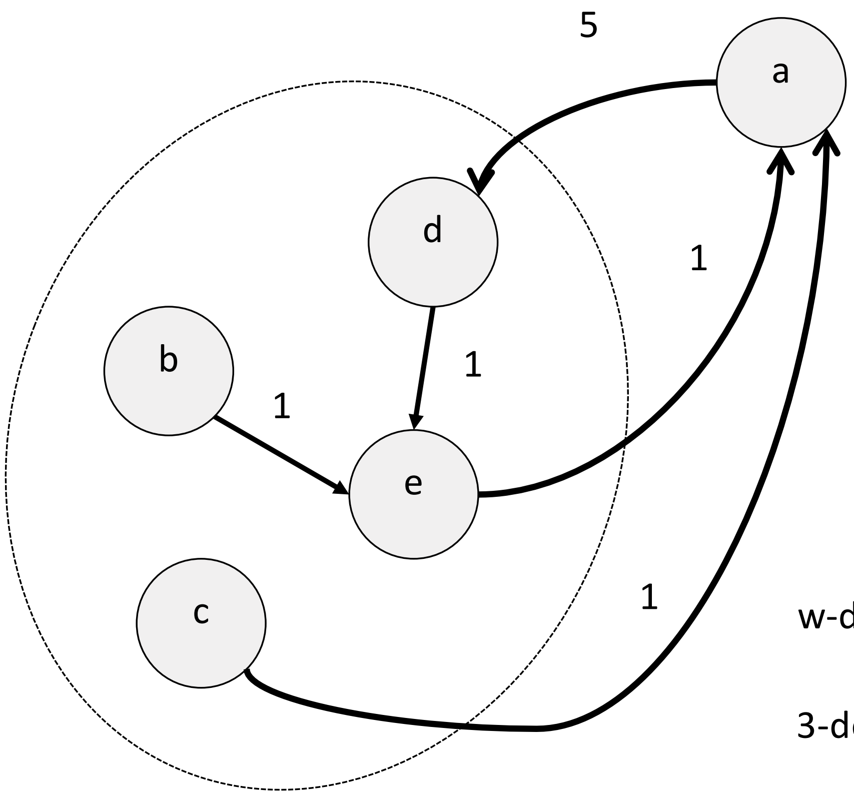
d and e are not defended

$$\langle \mathbb{R}^+ \cup \{+\infty\}, min, \hat{+}, \infty, 0 \rangle$$

Relaxing defence

α^γ-semantics

$$(W(a, \mathcal{B} \cup \{b\}) \div W(\mathcal{B}, a)) \geq_{\mathbb{S}} \gamma$$



$\alpha = 2$

$\gamma = 3$

{b,c,d,e} is a 2^3-admissible

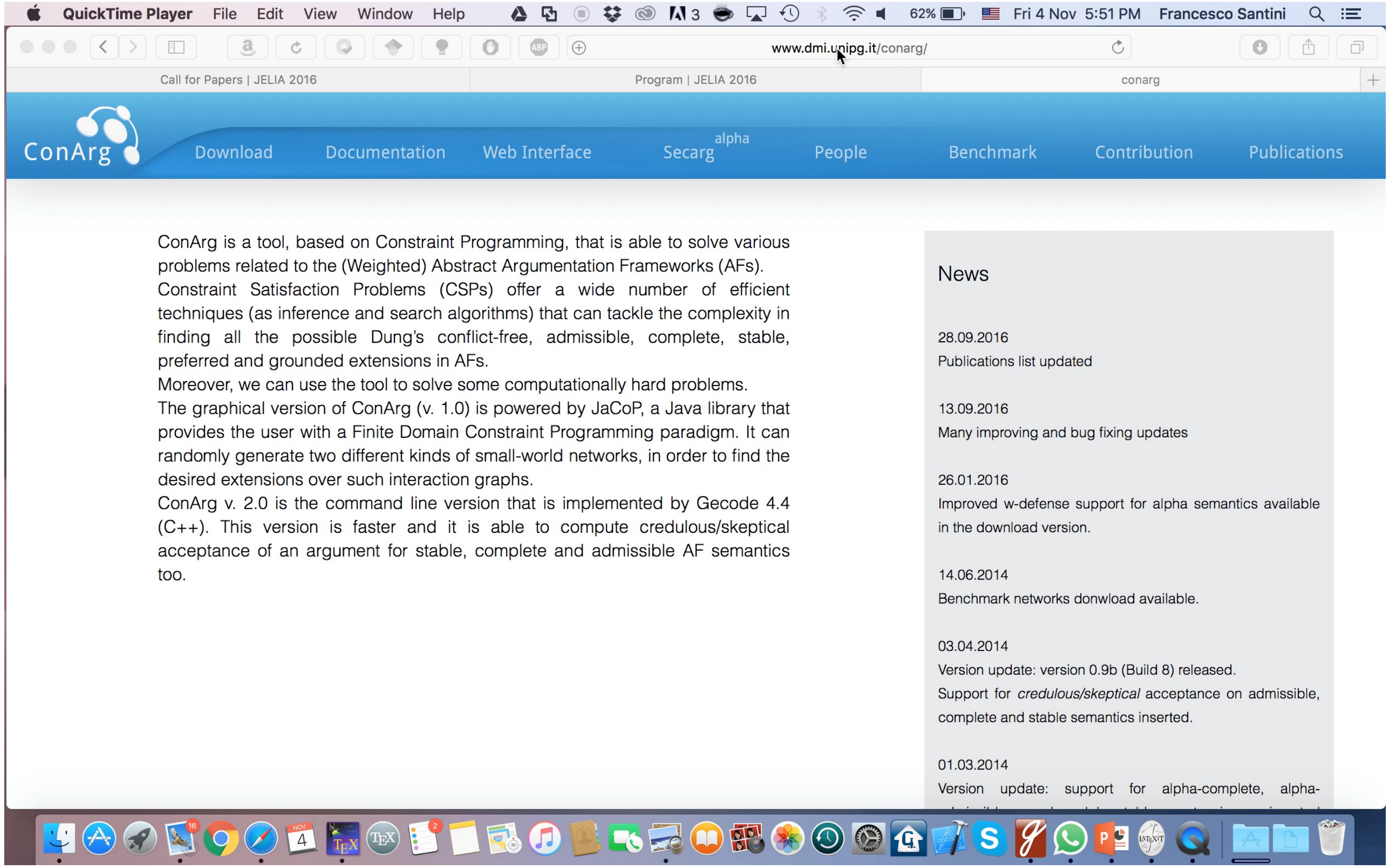
w-defended (0-defended)? NO!

3-defended? YES!

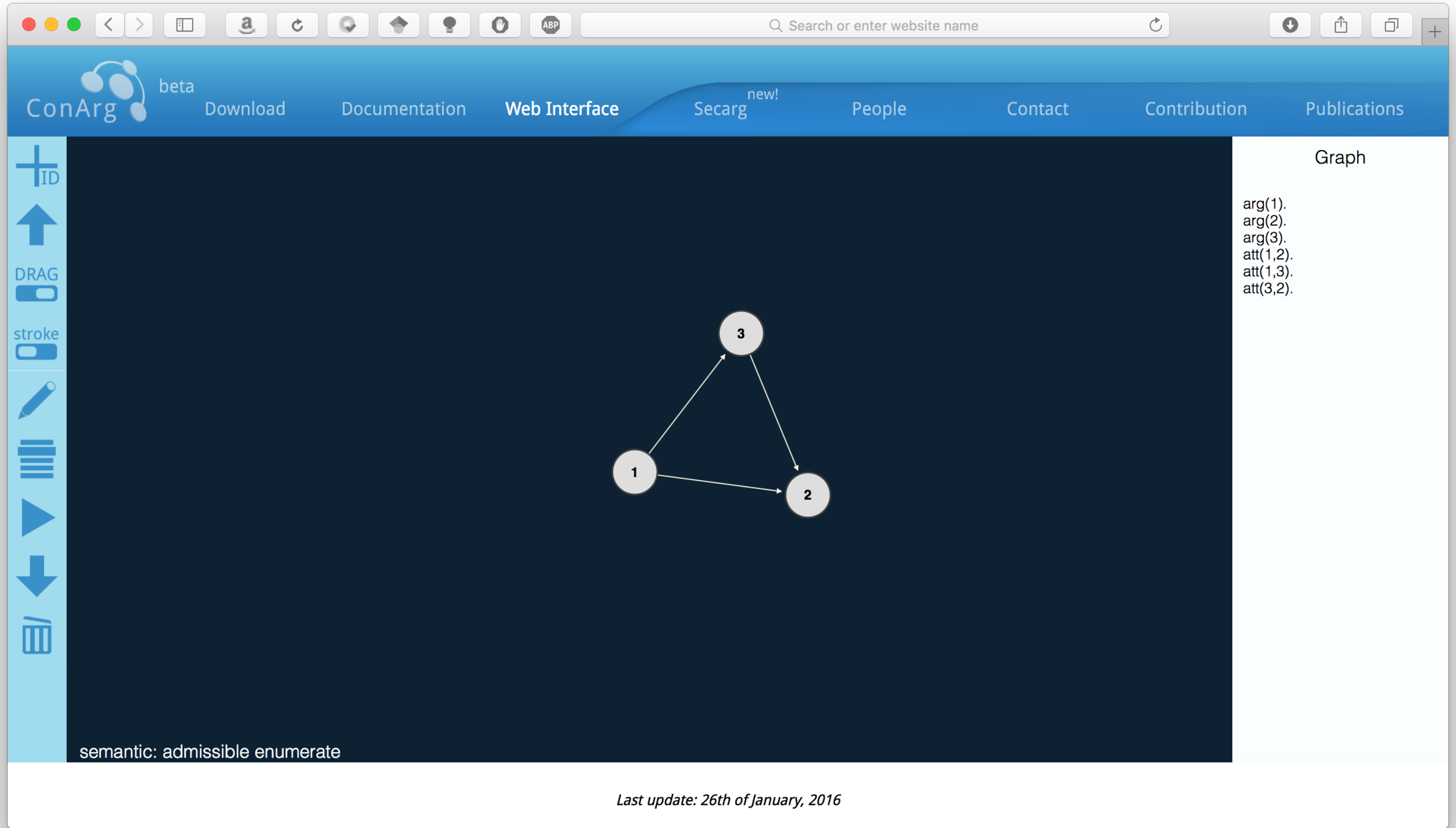
{b,c,d,e} is a 4^5-admissible as well

Tool

www.dmi.unipg.it/conarg/



https://conarg.dmi.unipg.it



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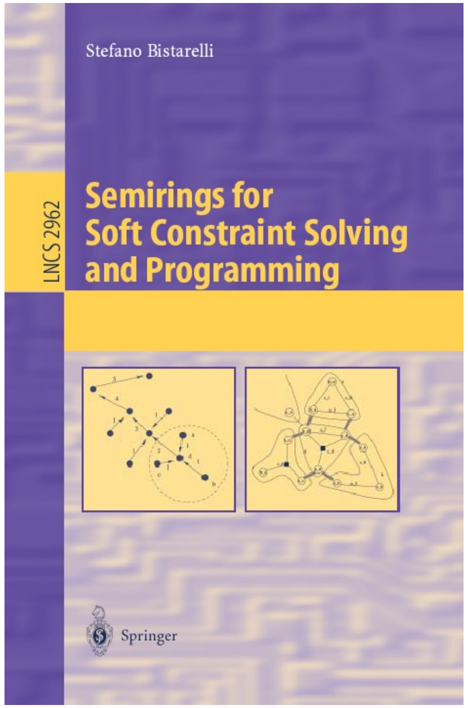

open problems / questions / ideas

- map partial CSPs HCLP, and fuzzy HCLP
- Study complexity when passing from classical (boolean) instances to the semiring-based ones
  - Increase of complexity can be quantified and generalized (wrt the semiring used)?
- Trying to use semiring in security, blockchain and explainable AI
- Extend semiring –based quantitative approach to other fields
  - Quantify security and trust
  - «measure» quality of solutions obtained with Machine learning output (using some probability computation)?
  - Match from (operations of) neural network to algebraic/semiring operators?

Questions?

- Thank you for your attention