## Approximating Certain and Possible Answers beyond Sets Logic and Algebra for Query Evaluation, Logic and Algorithms in Database Theory and AI

## Boris Glavic ${ }^{1}$

## DBGroup

Illinois Institute of Technology


2023-11-14

## Outline

(1) Incomplete Databases, Certain \& Possible Answers
(2) Incomplete K-relations
(3) Approximating Incomplete K-relations
(4) Attribute-level Uncertainty
(5) Conclusions \& Future Work
(6) Appendix and References

## Motivation

## Incomplete \& Probabilistic Databases

- Principled techniques for managing uncertain data
- High computational complexity for query answering
- Many tractable cases based on query / data structure are known [DS12, DS07, FO16, FHO12, RS09, ABS15]
- intractable even for relatively simple queries


## Incomplete Databases

## Definition (Incomplete Database)

An incomplete database is a set $\mathcal{D}=\left\{D_{1}, \ldots D_{n}\right\}$ where each $D_{i}$, called a possible world, is a deterministic database.

## Example

| $D_{1}$ |  | $D_{2}$ |  | $D_{3}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| name | country |  |  | name | country |
| Boris | USA | name | country | Boris | Germany |
| Peter | USA | Boris | Switzerl. | Peter | Netherl. |
| Alexandra | USA | Alexandra |  | Alexandra | USA |

## Possible World Semantics

## Definition (Possible World Semantics)

- Incomplete Databases

$$
Q(\mathcal{D})=\{Q(D) \mid D \in \mathcal{D}\}
$$

## Possible \& Certain Tuples

## Definition (Possible Tuples)

$$
\text { Possible( } \mathcal{D})=\{t \mid \exists D \in \mathcal{D}: t \in D\}
$$

## Definition (Certain Tuples)

$$
\operatorname{Certain}(\mathcal{D})=\{t \mid \forall D \in \mathcal{D}: t \in D\}
$$

## Definition (Possible \& Certain Answers)

$$
\begin{aligned}
\operatorname{Certain}(Q, \mathcal{D}) & =\operatorname{Certain}(Q(\mathcal{D})) \\
\text { Possible }(Q, \mathcal{D}) & =\operatorname{Possible}(Q(\mathcal{D}))
\end{aligned}
$$

## Desiderata

- A practical data model for uncertain data management
(1) Supports bags (in addition to sets)
(2) PTIME data complexity (through approximation)
(3) Support for complex queries (full relational algebra with aggregation, sort-based operations, recursion)
(4) Modular: closed under queries
(5) Compact: be able to trade size for approximation accuracy
(- Compatible: approximate existing uncertain data models and data cleaning paradigms (consistent query answering)


## Outline

(1) Incomplete Databases, Certain \& Possible Answers
(2) Incomplete K-relations
(3) Approximating Incomplete K-relations
(4) Attribute-level Uncertainty
(5) Conclusions \& Future Work
(6) Appendix and References

## Motivation

- Incomplete Databases beyond sets
- Incomplete bag databases
- Incomplete provenance
- Incomplete temporal databases
- Incomplete access control
- ...


## $K$-relations

## Semirings

- semiring $\left(K,+_{K}, \cdot{ }_{K}, 0_{K}, 1_{K}\right)$
- a set $K$
- binary operations $+_{K}$ and $\cdot k$ which are associative and commutative
- $0_{K}\left(1_{K}\right)$ are the neutral elements of $+_{K}$ and $\cdot K$
- $k \cdot K 0_{K}=0_{K}$ for all $k \in K$
- $k_{1} \cdot \kappa\left(k_{2}+\kappa k_{3}\right)=\left(k_{1} \cdot k k_{2}\right)+\left(k_{1} \cdot k k_{3}\right)$


## Semiring-annotated relations

- An $n$-ary $K$-relation $R$ is a function $\mathbb{U}^{n} \rightarrow K$ with finite support: $\left\{t \mid R(t) \neq 0_{K}\right\}([G K T 07, G T 17])$
- Tuples that are annotated with $0_{K}$ do not exist (omit them)


## Natural Order

## Natural order

- $k_{1} \leq_{K} k_{2}$ if exists $k_{3}$ such that $k_{1}+k_{3}=k_{2}$
- Semirings where the natural order has a lattice structure are called I-semirings [KB12]
- Each pair of elements $k_{1}$ and $k_{2}$ has ...
- a greatest lower bound $\Pi_{K}\left(k_{1}, k_{2}\right)$
- a least upper bound $\sqcup_{K}\left(k_{1}, k_{2}\right)$


## Natural order of $\mathbb{N}$

- $k_{1} \leq_{\mathbb{N}} k_{2}$ is the standard order of natural numbers (e.g., $2<3$ )
- $\Pi_{\mathbb{N}}\left(k_{1}, k_{2}\right)=\min \left(k_{1}, k_{2}\right)$
- $\sqcup_{\mathbb{N}}\left(k_{1}, k_{2}\right)=\max \left(k_{1}, k_{2}\right)$


## Incomplete K-relations

## Definition (Incomplete K-database)

An incomplete $K$-database is a set $\mathcal{D}=\left\{D_{1}, \ldots, D_{n}\right\}$ such that each $D_{i}$ is a $K$-database.

## Possible world semantics

- Possible world semantics uses $K$-relational query semantics


## Certain and possible answers

- Consider only I-semirings
- The certain annotation of a tuple

$$
\text { - } \operatorname{Cert}_{K}(\mathcal{D}, t)=\sqcap_{K}\left\{D_{i}(t) \mid D_{i} \in \mathcal{D}\right\}
$$

- The possible annotation of a tuple
- $\operatorname{Poss}_{K}(\mathcal{D}, t)=\sqcup_{K}\left\{D_{i}(t) \mid D_{i} \in \mathcal{D}\right\}$
- Coincides with certain / possible tuples for sets / bags


## Incomplete Bag Relations

## Bag Semantics Example ( $\mathbb{N}$ annotations)

- Annotations encode the multiplicity of tuples under bag semantics
- Possible annotation is max, certain annotation is min
- The certain (possible) annotation of (Peter, USA) is 2 (3)
- The certain (possible) annotation of (Boris, USA) is 0 (2)

$$
D_{1}
$$

| name |  | country |
| :---: | :---: | :---: |
| Boris | $\mathbb{N}$ |  |
| Peter | USA | 2 |
| USA | 3 |  |


| name | country | $\mathbb{N}$ |
| :---: | :---: | :---: |
| Peter | USA | 2 |

## Outline

(1) Incomplete Databases, Certain \& Possible Answers
(2) Incomplete K-relations
(3) Approximating Incomplete K-relations
(4) Attribute-level Uncertainty
(5) Conclusions \& Future Work
(6) Appendix and References

## Approximating Certain \& Possible

## Definition (Containment)

For two $K$-database $D_{1}$ and $D_{2}$ over the same schema, $D_{1}$ is contained in $D_{2}\left(D_{1} \sqsubseteq_{K} D_{2}\right)$ iff

$$
\forall t: D_{1}(t) \leq_{K} D_{2}(t)
$$

## Definition (Under-approximating certain annotations)

A $K$-database $D$ is an under-approximation of an incomplete $K$-database $\mathcal{D}$ iff:

$$
D \sqsubseteq_{K} \operatorname{Certain}(\mathcal{D})
$$

## Definition (Over-approximating possible annotations)

A $K$-database $D$ is an over-approximation of an incomplete $K$-database $\mathcal{D}$ iff:

$$
D \sqsupseteq_{K} \operatorname{Possible}(\mathcal{D})
$$

## Example TIDB

## Definition (Tuple-independent database (TIDB))

A tuple-independent database $\boldsymbol{D}=D_{\text {certain }} \cup D_{\text {possible }}$ encodes an incomplete $\mathbb{B}$-database with the possible worlds $\operatorname{Mod}(\boldsymbol{D})$ :

$$
\operatorname{Mod}(\boldsymbol{D})=\left\{D^{\prime} \mid D_{\text {certain }} \sqsubseteq_{\mathbb{B}} D^{\prime} \wedge D^{\prime} \sqsubseteq_{\mathbb{B}} \boldsymbol{D}\right\}
$$

## Approximating TIDBs

- Under-approximation: $D=D_{\text {certain }}$
- Over-approximation: $D=\boldsymbol{D}$


## Example Key-repair

## Key repair

Consider a $\mathbb{B}$-relation $R\left(A_{1}, \ldots, A_{n}\right)$ with a key $K \subseteq\left\{A_{1}, \ldots, A_{n}\right\}$ and the incomplete database of all S-repairs:

$$
\begin{gathered}
\mathcal{R}=\left\{R^{\prime} \mid R^{\prime} \subseteq R \wedge \forall k \in \pi_{K}(R): \exists t \in R^{\prime}: t . K=k\right. \\
\left.\wedge \nexists t_{1} \neq t_{2} \in R^{\prime}: t_{1} \cdot K=t_{2} . K\right\}
\end{gathered}
$$

## Approximating key repairs

- under-approximation:

$$
R^{\prime}=\left\{t \mid R(t)=T \wedge \nexists t^{\prime} \neq t: t . K=t^{\prime} . K \wedge R\left(t^{\prime}\right)=\top\right\}
$$

- over-approximation:

$$
R^{\prime}=R
$$

## UA-DBs

## Definition (Uncertainty-annotated databases)

- A UA-DB $\boldsymbol{D}$ is a $K_{A U}$-relation where $K_{A U}$ is the product semiring $K \times K$ restricted to $\left\{\left(k_{l}, k_{u}\right) \mid k_{l} \leq_{K} k_{2}\right\}$
- Addition and multiplication in $K^{2}$ are defined point-wise
- Define $(c, p)^{\downarrow}=c$ and $(c, p)^{\uparrow}=p$ and lift this operation pointwise to databases
- $\boldsymbol{D}^{\downarrow}(t)=\boldsymbol{D}(t)^{\downarrow}$ and $\boldsymbol{D}^{\uparrow}(t)=\boldsymbol{D}(t)^{\uparrow}$


## Proposition (Structure $K_{A U}$ is a semiring)

For any naturally-ordered semiring $K, K_{A U}$ is a semiring

## Approximating Certain \& Possible Answers ILINOIS INSTITUTE

## Bounding incomplete $K$-databases

An K-UA-DB $\boldsymbol{D}$ bounds an incomplete $K$-database $\mathcal{D}(\boldsymbol{D} \lessgtr K \mathcal{D})$ iff:

$$
\begin{aligned}
& \boldsymbol{D}^{\downarrow} \sqsubseteq_{\kappa} \operatorname{Certain}(\mathcal{D}) \\
& \boldsymbol{D}^{\uparrow} \sqsupseteq_{K} \operatorname{Possible}(\mathcal{D})
\end{aligned}
$$

## Bound-preserving query semantics

A query semantics for $K_{A U}$ for is called bound-preserving iff for any query $Q$ :

$$
\boldsymbol{D} \lessgtr k \mathcal{D} \Rightarrow Q(\boldsymbol{D}) \lessgtr{ }_{K} Q(\mathcal{D})
$$

## Bound-preserving Queries

## Theorem (Positive relational algebra is bound-preserving)

For any positive relational algebra query $Q$, standard K-relational query semantics is bound preserving.

## Proof Sketch.

Addition and multiplication are monotone wrt. the natural order. If $k_{1} \leq_{K} k_{3}$ and $k_{2} \leq_{K} k_{4}$ then:

- $k_{1}+K k_{2} \leq_{K} k_{3}+K k_{4}$
- $k_{1} \cdot{ }_{K} k_{2} \leq_{K} k_{3} \cdot K k_{4}$


## Bound-preserving Queries

## Theorem (Full relational algebra is bound-preserving)

Full relational algebra is bound-preserving assuming the monus-semiring definition from [GP10] and by defining:

$$
(\boldsymbol{R}-\boldsymbol{S})(t)=\left(\boldsymbol{R}(t)^{\downarrow}{ }_{-}{ }_{\kappa} \boldsymbol{S}(t)^{\uparrow}, \boldsymbol{R}(t)^{\uparrow}{ }_{K} \boldsymbol{S}(t)^{\downarrow}\right)
$$

## Recursive Positive Datalog

## Theorem (Bound preservation)

Recursive Datalog queries over UA-DBs are bound preserving.

## Theorem (Convergence)

As computations in $\boldsymbol{D}^{\downarrow}$ and $\boldsymbol{D}^{\uparrow}$ are standard K-relational query evaluation, existing results for Datalog over K-relations are applicable. Specifically, we get the same convergence guarantees as proven for K-relations in [KNP ${ }^{+}$22]

## Compatibility with Existing Work

## Consistent query answering \& certain answers

- The following approach is bound preserving:
- Answer a supported subquery of a query $Q$ using an existing technique for certain / possible answers
- Interpret the result as a UA-DB
- Evaluate the remainder of the query using UA-DBs


## Outline

(1) Incomplete Databases, Certain \& Possible Answers
(2) Incomplete K-relations
(3) Approximating Incomplete K-relations
(4) Attribute-level Uncertainty
(5) Conclusions \& Future Work
(6) Appendix and References

- Certain answers are often empty
- e.g., operations like aggregation that return different aggregation function results in each world
- Some types of uncertainty cannot be efficiently over-approximated with "concrete" tuples
- This person certainly exists, but their salary is in [100000,200000]
- Operations like sum aggregation may produce exponentially many possible results
- Representation systems that use (labelled) nulls
- C-tables [ILJ84], m-tables [SKL ${ }^{+} 17$ ], Codd-tables, V-tables, ...
- "Factorized" representations
- OR-tables [IVV95], x-tables (block-independent databases) [BSHW06, VdBS17]
- Returning the lowest and highest possible aggregation function result
- $\left[\mathrm{ABC}^{+} 03\right.$, Fux07, $\mathrm{ACK}^{+} 10$, LSV02, AK08]
- Abstract interpretation
- [CC77, CC04]


## Interval domains

## Interval domains

- Domain $\left(\mathbb{D}, \leq_{\mathbb{D}}\right)$ where $\leq_{\mathbb{D}}$ is a partial order
- Interval domain $\mathbb{D} \leq=\{[I, u] \mid I, u \in \mathbb{D} \wedge I \leq \mathbb{D} u\}$
- Representation system $\operatorname{Mod}([I, u])=\{c \mid c \in[I, u]\}$


## Over-approximating operations

- $\left[I_{1}, u_{1}\right]+\left[I_{2}, u_{2}\right]:=\left[I_{1}+I_{2}, u_{1}+u_{2}\right]$
- $\left[I_{1}, u_{1}\right]=\left[I_{2}, u_{2}\right]:=\left[I_{1}=u_{1}=I_{2}=u_{2}, I_{1} \leq u_{2} \wedge I_{2} \leq u_{1}\right]$


## The AU-DB Model

## AU-DB relations

- Special type of K-relations
- Value domains are intervals
- Annotation domain is a UA-semiring $K_{A U}$


## The possible worlds encoded by a UA-DB

- A UA-DB $\boldsymbol{D}$ encodes as possible worlds $\operatorname{Mod}(\boldsymbol{D})$ every K-database $D$ such that we can "redistribute" the annotations of tuples from $D$ to bounding tuples from $\boldsymbol{D}$ such that
- for any tuple $\boldsymbol{t}$ from $\boldsymbol{D}$, the total annotation mass distributed to $\boldsymbol{t}$ is withing its annotation bounds


## UA-DB Example

## UA-DB

| name | salary | $\mathbb{N}^{3}$ |
| :---: | :---: | :---: |
| Boris | $[120 \mathrm{k}, 12 \mathrm{kk}]$ | $[0,2]$ |
| Peter | $[140 \mathrm{k}, 400 \mathrm{k}]$ | $[2,3]$ |

## Two possible worlds

| name | salary | $\mathbb{N}$ |
| :---: | :---: | :---: |
| Boris | 120 k | 1 |
| Peter | 150 k | 1 |
| Peter | 380 k | 2 |


| name | salary | $\mathbb{N}$ |
| :---: | :---: | :---: |
| Peter | 180 k | 2 |

## Bounding Uncertain Databases

## Bounding incomplete databases

- Given an incomplete $\mathbb{N}$ database $\mathcal{D}$ and selected world $D_{s g} \in \mathcal{D}$
- Create UA-DB $\boldsymbol{D}$ with selected-guess $D_{\text {sg }}$ such that $\operatorname{Mod}(\boldsymbol{D}) \supseteq \operatorname{Mod}(\mathcal{D})$


## Incomplete bag database

$D_{1}$

| name | salary | $\mathbb{N}$ |
| :---: | :---: | :---: |
| Boris | 120 k | 2 |
| Peter | 400 k | 3 |

A Bounding UA-DB for SG $D_{1}$

| name | salary | $\mathbb{N}^{3}$ |
| :---: | :---: | :---: |
| Boris | $[120 \mathrm{k}, 120 \mathrm{k}]$ | $[0,2]$ |
| Peter | $[140 \mathrm{k}, 400 \mathrm{k}]$ | $[2,3]$ |

## Alternative Interpretation of Bounding

## Bounding

- Given a UA-DB $\boldsymbol{D}$ that bounds an incomplete $K$-database $\mathcal{D}$

Under-approximation of certain tuples with attribute uncertainty

- The lower bound tuple annotations of $\boldsymbol{D}$
- Under-approximation of certain tuples
- Generalizes certain answers by allowing attribute-level uncertainty

Over-approximation of possible tuples with attribute uncertainty

- The upper bound tuple annotations of $\boldsymbol{D}$
- Over-approximation of possible tuples
- Generalizes possible answers by allowing attribute-level uncertainty


## Query Semantics for $\mathcal{R} \mathcal{A}^{+}$

## Queries over K-relations

- Defined using the semiring addition / multiplication operations
- Disjunctive use of tuples (union, projection) is addition
- Conjunctive use of tuples (join) is multiplication


## $\mathcal{R} \mathcal{A}^{+}$Semantics

$$
\text { Union: }\left(R_{1} \cup R_{2}\right)(t)=R_{1}(t)+\mathcal{K} R_{2}(t)
$$

$$
\text { Join: }\left(R_{1} \bowtie R_{2}\right)(t)=R_{1}\left(t\left[\operatorname{Sch}(R)_{1}\right]\right) \cdot \mathcal{K} R_{2}\left(t\left[\operatorname{Sch}(R)_{2}\right]\right)
$$

Projection: $\left(\pi_{U}(R)\right)(t)=\sum_{t=t^{\prime}[U]} R\left(t^{\prime}\right)$
Selection: $\left(\sigma_{\theta}(R)\right)(t)=R(t) \cdot \mathcal{K} \theta(t)$

## Selections over AU-DBs

## Conditional expressions over range-annotated values

- Expressions over range-annotated values evaluate to a range of Boolean values
- e.g., $[1,3]=[3,3]$ evaluates to $[F, T]$


## Evaluating selections

- in K-relations selection on $\theta$
- false maps to $0_{K}$ and true to $1_{K}$
- tuples get their input annotation if they do fulfill $\theta$
- tuples get a $0_{K}$ annotation in the result if they do not fulfill $\theta$
- in $K_{A U}$ relations: apply this to every dimension
- e.g., $[F, T]$ maps to $[0,1]$


## Semi-module Aggregation?

## Semi-modules

- Polynomial representation system for aggregation results [ADT11]


## Proposition (SUM,$M A X_{l}$, and $M I N_{l}$ are monoids)

- using our definitions of addition, min, max over the interval domain


## Proposition (Semi-modules cannot be bound preserving)

- Use $\mathbb{N}_{A U}$ and SUM , set $k=(1,2)$ and $m=[0,0]$ and $m_{1}=[-1,-1]$ and $m_{2}=[1,1]$
- Observe that $m=m_{1}+m_{2}$
- Based on semi-module laws we have to have $k \otimes m=k \otimes\left(m_{1}+m_{2}\right)=k \otimes m_{1}+k \otimes m_{2}$
- However, $k \otimes m_{2}=\left(l_{2}, u_{2}\right)$ with $l_{2} \leq 1<2 \leq u_{2}$


## Escape into uncertainty

- Use bound-preserving multiplication of intervals instead of a semimodule
- $k \circledast m=k \cdot m$


## Approach

- Group-by attribute bounds in the result determined based on assigning tuples to groups based on their selected-guess values
- Aggregation function result bounds by reasoning about
- Which tuples could / have to belong to a group
- Taking the lowest / highest possible multiplicities of tuples into account
- Using lowest / highest possible values of the attribute we are aggregating over
- Utilize expression semantics for range-annotated values


## Sorting, Top-k, Windowed Aggregation

## Sort Order as Data

- there is only one physical sort order
- encode sort position as UA-DB range-values!


## Windowed Aggregation

- exploit fixed window sizes



## Outline

(1) Incomplete Databases, Certain \& Possible Answers
(2) Incomplete K-relations
(3) Approximating Incomplete K-relations

4 Attribute-level Uncertainty
(5) Conclusions \& Future Work
(6) Appendix and References

## Conclusions

## UA-DB Data Model

- Lightweight, but powerful uncertain data model
- Under-approximation of certain answers
- Over-approximation of possible answers


## Query semantics

- Bound preservation: the bounding of certain and possible answers is preserved under queries
- Ranges (attribute-level uncertainty) as first-class citizens: generalizes
- certain answers with nulls [LJ84]
- "bounding range semantics" for aggregation over incomplete data


## Future Work

## Even more queries

- Recursive and iterative computations (e.g., training ML models)


## Even better performance

- New specialized physical operators
- Optimizing representations and queries


## Better bounds

- Using more expressive set representations instead of boxes


## Beyond ranges

- Partial orders / Ontologies instead of ranges


## Questions?

## Implementations

- GProM https://github.com/IITDBGroup/gprom
- Vizier https://vizierdb.info/


Su Feng


Oliver Kenndy


Aaron Huber

## Outline

(1) Incomplete Databases, Certain \& Possible Answers
(2) Incomplete K-relations
(3) Approximating Incomplete K-relations

4 Attribute-level Uncertainty
(5) Conclusions \& Future Work

6 Appendix and References

## Experimental Results - TPC-H Queries



Figure: TPC-H (SF=1)

## Experimental Setup - Competitors

## Competitors

- UA-DB
- UA-DB: no over-approximation of possible and no attribute-uncertainty
- AU-DB: the model discussed so far
- Under-approximation of certain for DBs with nulls (Libkin, [Lib16])
- Sampling-based (MCDB w/o probabilities, [JXW $\left.{ }^{+} 08\right]$ )
- All possible answers
- MayBMS w/o probabilities ${ }^{\text {a }}$, [AKO07, OHK09])
- Trio w/o probabilities ${ }^{b}$, [BSHW06]
- Semi-modules w/o probabilities [FHO12]
${ }^{\text {a Does not support aggregation }}$
${ }^{b}$ Does not support uncertain group-by and querying aggregation results


## Experimental Setup - Datasets \& Machines ${ }^{\text {LLINOII INsTITUTE }}$

## Datasets

- PD-Bench (TPC-H data with errors), [AJKO08]
- Real-world open datasets with missing values


## Machines

- $2 \times 6$ core AMD Opteron 4238 CPUs
- 128GB RAM
- $4 \times 1$ TB 7.2 K HDDs (RAID 5)
- Postgres 11


## Experimental Results - SPJ



Figure: PD-Bench Queries (1GB) - Varying input uncertainty

## Experimental Results - Aggregation



Figure: Simple aggregation queries over TPC-H (1GB) - Varying number of aggregation operations

## Experimental Results - Real World Data



Figure: Experiments with Real Datasets (Netflix, Crimes, Healthcare)

## Experimental Results - Real World Data

| Datasets \& Queries |  |  | Time (sec) | cert. tup. | attr. <br> min | $\begin{aligned} & \text { ounds } \\ & \text { max } \end{aligned}$ | pos.tup. by id | pos.tup by val |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & Q_{n, 1} \\ & \text { SPJ } \end{aligned}$ | AU-DB | $\begin{array}{r} (\mathrm{sec}) \\ \hline 0.011 \end{array}$ | 100\% | 1 | 1 | 100\% | 100\% |
|  |  | Trio | 0.900 | 100\% | 1 | 1 | 100\% | 100\% |
|  |  | MCDB | 0.049 | N/A | 1 | 1 | 99.6\% | 98.5\% |
|  |  | UA-DB | 0.006 | 100\% | N/A | N/A | 99.1\% | 97.3\% |
|  | $\begin{aligned} & Q_{n, 2} \\ & G B \end{aligned}$ | AU-DB | 0.0821.700 | 100\% | 11 | 4 | 100\% | 100\% |
|  |  | Trio |  | 100\% |  | 1 | 98.8\% | 98.0\% |
|  |  | MCDB | 0.118 | $\begin{gathered} \text { N/A } \\ 0 \% \end{gathered}$ | $\begin{gathered} 1 \\ \mathrm{~N} / \mathrm{A} \end{gathered}$ |  | 99.9\% | 97.9\% |
|  |  | UA-DB | 0.009 |  |  | N/A | 99.3\% | 95.7\% |
|  |  | AU-DB | 1.5859.0 | $100 \%$$100 \%$ | 1 | 1 | 100\% | 100\% |
|  |  | Trio |  |  |  | 1N/A | 100\% | 100\% |
|  |  | MCDB |  | $\begin{gathered} \text { N/A } \\ 100 \% \end{gathered}$ | $\begin{gathered} 0.6 \\ \mathrm{~N} / \mathrm{A} \end{gathered}$ |  | $\begin{aligned} & 99.9 \% \\ & 99.9 \% \end{aligned}$ | $\begin{aligned} & 92.1 \% \\ & 87.5 \% \end{aligned}$ |
|  |  | UA-DB |  |  |  |  |  |  |
|  | $\begin{aligned} & Q_{c, 2} \\ & G B \end{aligned}$ | AU-DB | 2.09 | 100\% | / | 1.01 | 100\% |  |
|  |  | Trio | 103.1 | 100\% | 1 | N/A | $\begin{aligned} & 100 \% \\ & 100 \% \\ & 100 \% \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 100 \% \\ & 100 \% \end{aligned}$ |
|  |  | MCDB | 5.24 | $\begin{gathered} \text { N/A } \\ 0 \% \end{gathered}$ | $\begin{aligned} & 0.99 \\ & \mathrm{~N} / \mathrm{A} \end{aligned}$ |  |  | $\begin{aligned} & \sim 0 \% \\ & \sim 0 \% \end{aligned}$ |
|  |  | UA-DB | 0.47 |  |  |  |  |  |
|  | $\begin{aligned} & Q_{h, 1} \\ & \text { SPJ } \end{aligned}$ | AU-DB | $\begin{array}{r} \hline \mathbf{0 . 1 7 9} \\ 20.6 \\ 0.501 \\ 0.042 \end{array}$ | $\begin{gathered} 99.5 \% \\ 100 \% \\ \text { N/A } \\ 98.2 \% \end{gathered}$ | $\begin{gathered} 1 \\ 1 \\ 0.4 \\ \mathrm{~N} / \mathrm{A} \end{gathered}$ | $\begin{gathered} 1 \\ 1 \\ 1 \\ \mathrm{~N} / \mathrm{A} \\ \hline \end{gathered}$ | $\begin{aligned} & 100 \% \\ & 100 \% \\ & 99.9 \% \\ & 99.3 \% \end{aligned}$ | $\begin{aligned} & 100 \% \\ & 100 \% \\ & 87.6 \% \\ & 65.4 \% \end{aligned}$ |
|  |  | Trio |  |  |  |  |  |  |
|  |  | MCDB |  |  |  |  |  |  |
|  |  | UA-DB |  |  |  |  |  |  |
|  | $\begin{aligned} & Q_{h, 2} \\ & G B \end{aligned}$ | AU-DB | $\begin{array}{r} 0.859 \\ 29.2 \\ 2.31 \\ 0.235 \end{array}$ | $\begin{aligned} & \hline 100 \% \\ & 100 \% \\ & \text { N/A } \\ & 0 \% \end{aligned}$ | 1 | 45 | 100\% | 100\% |
|  |  | Trio |  |  | $\begin{array}{\|c\|} 1 \\ 0.78 \\ \mathrm{~N} / \mathrm{A} \end{array}$ | 1 | 100\% | $\begin{gathered} 100 \% \\ \sim 0 \% \end{gathered}$ |
|  |  | MCDB |  |  |  |  | $100 \%$$100 \%$ |  |
|  |  | UA-DB |  |  |  | $\begin{gathered} 1 \\ \mathrm{~N} / \mathrm{A} \end{gathered}$ |  | $\begin{gathered} \sim 0 \% \\ \sim 0 \% \end{gathered}$ |

## Publications \& Links

## Lenses

- Uncertainty-aware Data Cleaning Operations [YMF ${ }^{+}$15]


## UA-DBs

- The basic UA-DB model [FHGK19]
- Attribute-level uncertainty, range values, and aggregation over UA-DBs (the AU-DB model) [FHGK21]
- Sorting, top-k, and windowed aggregation [FGK23]


## Vizer

- http://www.vizierdb.info
- Publications:
http://www.cs.iit.edu/~ dbgroup/publications.html (click on project Vizier to see relevant publications)


## Relational Encoding

## Encode UA-DBs as relational databases

- attribute-level ranges
- for each attribute $A$ add $A_{-} l b$ and $A_{-} u b$
- annotation ranges
- add attributes row_lb, row_sg, row_ub


## UA-DB

| name | salary | $\mathbb{N}^{3}$ |
| :---: | :---: | :---: |
| Boris | $[120 \mathrm{k}, 120 \mathrm{k}, 120 \mathrm{k}]$ | $[0,2,2]$ |
| Peter | $[140 \mathrm{k}, 400 \mathrm{k}, 400 \mathrm{k}]$ | $[2,3,3]$ |

## Relational encoding

| name | salary $_{\text {lb }}$ | salary | salary $_{\text {ub }}$ | row $_{\mathrm{lb}}$ | row $_{\text {sg }}$ | row $_{\text {ub }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Boris | 120 k | 120 k | 120 k | 0 | 2 | 2 |
| Peter | 140 k | 400 k | 400 k | 2 | 3 | 3 |

## Implementing UA-DB Query Semantics

## Rewrite-based approach



Specialized Operator Implementations

- one pass algorithms for sorting and windowed aggregation


## Existing implementations

- GProM (https://github.com/IITDBGroup/gprom)
- Vizier (https://vizierdb.info/)


## Optimizations for Aggregation \& Join

## Performance of Joins and Aggregation

- Aggregation and joins require interval overlap joins
- $O\left(n^{2}\right)$ in many DBMS


## Optimizations

- Split possible from selected-guess and process them separately
- Selected-guess does not require interval overlap joins
- Compress possible, interval overlap joins over smaller inputs
- Trade performance for accuracy of the over-approximation
- This works, because approximate compression is bound preserving



## Sorting and Windowed Aggregation

## Rewrite-based approach

- can be expensive: range-joins and multiple sorting steps


## Native Algorithms

- one pass algorithms
- implemented inside Postgres
- requires reasoning about certain and possible windows and maintain rows in multiple sort orders


## Connected Heaps



Result of h1.pop()


## Running Example

## COVID infection rate data

- COVID infection rate per location from two sources

Query - maximum infection rate per location size SELECT size, max(rate) as mrate FROM R GROUP BY size

## Source 1

| city | size | rate |
| :---: | :---: | :---: |
| Los Angeles | Metro | $2 \%$ |
| Washington | Metro | $6 \%$ |
| Chicago | City | $4 \%$ |
| Springfield | Town | $2 \%$ |

## Source 2

| city | size | rate |
| :---: | :---: | :---: |
| Los Angeles | Metro | $7 \%$ |
| Washington | Village | $8 \%$ |
| Chicago | City | $3 \%$ |

## Running Example - Conflicting Information ILINOIS INSTTTUTE

## Conflicting Information

- Washington may belong to group Metro or Village
- Conflicting rates for Los Angeles
- Does Springfield exist?


## Source 1

| city | size | rate |
| :---: | :---: | :---: |
| Los Angeles | Metro | $2 \%$ |
| Washington | Metro | $6 \%$ |
| Chicago | City | $4 \%$ |
| Springfield | Town | $2 \%$ |

## Source 2

| city | size | rate |
| :---: | :---: | :---: |
| Los Angeles | Metro | $7 \%$ |
| Washington | Village | $8 \%$ |
| Chicago | City | $4 \%$ |

## The Cost of Ignoring Uncertainty

Ignoring uncertainty in data leads to hard to trace errors with severe real world consequences.

Query - maximum infection rate per location size
SELECT size, max(rate) as mrate
FROM R GROUP BY size

## "Cleaned" Sources

| city | size | rate |
| :---: | :---: | :---: |
| Los Angeles | Metro | $2 \%$ |
| Washington | Village | $8 \%$ |
| Chicago | City | $4 \%$ |

## Query Result

| city | rate |
| :---: | :---: |
| Village | $8 \%$ |
| Metro | $2 \%$ |
| City | $4 \%$ |

## Data Cleaning, Curation and Integration

## Why Not Clean Your Data Upfront?

- Heuristic algorithms: pick one repair heuristically
- Highest probability repair: pick the most likely repair


## Cleaned Data $=$ Ground Truth?

- The ground truth is typically hard / impossible to determine

Take-away
Effectively, most data cleaning \& integration techniques select one possible repair

- All information about uncertainty is lost!
- How can we trust any analysis result based on the cleaned data?


## Requirements for Managing Uncertainty

- (R1) Efficiency
- Efficiently generate compact representations of uncertainty
- Running queries over uncertain data should be efficient
- Otherwise, users will just ignore uncertainty


## Requirements for Managing Uncertainty

- (R1) Efficiency
- Efficiently generate compact representations of uncertainty
- Running queries over uncertain data should be efficient
- Otherwise, users will just ignore uncertainty
- (R2) Expressiveness
- Model complex types of uncertainty
- Complex SQL queries / computations
- Unlikely to be adopted otherwise


## Requirements for Managing Uncertainty

- (R1) Efficiency
- Efficiently generate compact representations of uncertainty
- Running queries over uncertain data should be efficient
- Otherwise, users will just ignore uncertainty
- (R2) Expressiveness
- Model complex types of uncertainty
- Complex SQL queries / computations
- Unlikely to be adopted otherwise
- (R3) Guarantees
- Provide as much information about uncertainty as possible
- Queries should take uncertainty into account


## Requirements for Managing Uncertainty

- (R1) Efficiency
- Efficiently generate compact representations of uncertainty
- Running queries over uncertain data should be efficient
- Otherwise, users will just ignore uncertainty
- (R2) Expressiveness
- Model complex types of uncertainty
- Complex SQL queries / computations
- Unlikely to be adopted otherwise
- (R3) Guarantees
- Provide as much information about uncertainty as possible
- Queries should take uncertainty into account
- (R4) Usability
- Interpretable by mere mortals
- Backwards-compatible with existing cleaning \& integration solutions


## Sorting, Top-k, Windowed Aggregation

## Sort Order as Data

- there is only one physical sort order
- encode sort position as UA-DB range-values!


## Windowed Aggregation

- exploit fixed window sizes



## Experimental Results - TPC-H Queries

| Queries |  | 2\%/SF0.1 | 2\%/SF1 | 5\%/SF1 | 10\%/SF1 | 30\%/SF1 |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| Q1 | AU-DB | 1.607 | 15.636 | 15.746 | 15.811 | 16.021 |
|  | Det | 0.560 | 1.833 | 1.884 | 1.882 | 1.883 |
|  | MCDB | 5.152 | 19.107 | 18.938 | 19.063 | 19.279 |
| Q3 | AU-DB | 0.713 | 7.830 | 8.170 | 8.530 | 7.972 |
|  | Det | 0.394 | 1.017 | 1.058 | 1.092 | 1.175 |
|  | MCDB | 4.112 | 11.138 | 11.222 | 10.936 | 11.454 |
| Q5 | AU-DB | 0.846 | 8.877 | 8.803 | 8.839 | 8.925 |
|  | Det | 0.247 | 0.999 | 1.012 | 1.123 | 1.117 |
|  | MCDB | 2.599 | 10.152 | 10.981 | 11.527 | 11.909 |
| Q7 | AU-DB | 0.791 | 7.484 | 7.537 | 7.303 | 7.259 |
|  | Det | 0.145 | 0.977 | 0.985 | 0.989 | 1.044 |
|  | MCDB | 1.472 | 10.123 | 10.277 | 10.749 | 10.900 |
| Q10 | AU-DB | 0.745 | 7.377 | 7.283 | 7.715 | 8.012 |
|  | Det | 0.263 | 1.024 | 0.993 | 1.004 | 1.015 |
|  | MCDB | 2.691 | 10.743 | 10.937 | 11.826 | 11.697 |

## Experimental Results - Ranking + WindowsLINOIS NSSTITUTE <br> OF TECHNOLOGY

- MCDB [JXW $\left.{ }^{+} 08\right]$ - sample-based approach (10 or 20 samples)
- PT-k [HPZL08] - Full certain / possible answers for top-k queries
- Symb - Computing certain (possible) answers with constraint solver

| Datasets \& Queries |  | $\begin{gathered} \operatorname{lmp} \\ \text { (time) } \end{gathered}$ | Det (time) | $\begin{gathered} \text { MCDB20 } \\ \text { (time) } \\ \hline \end{gathered}$ | Rewr <br> (time) | Symb <br> (time) | PT-k <br> (time) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iceberg [ice07] | Rank | 0.816 msms | 0.123 ms | 2.337 ms | 1.269 ms | 278ms | 1s |
| (1.1\%, 167K) | Window | 2.964 ms | 0.363 ms | 7.582 ms | 1.046 ms | 589 ms | N.A. |
| Crimes [crigo] | Rank | 1043.505 ms | 94.306 ms | 2001.12 ms | 14787.723 ms | $>10 \mathrm{~min}$ | 10 min |
| (0.1\%, 1.45M) | Window | 3.050 ms | 0.416 ms | 8.337 ms | 2.226 ms | $>10 \mathrm{~min}$ | N.A. |
| Healthcare [heare] | Rank | 287.515 ms | 72.289 ms | 1451.232 ms | 4226.260 ms | 15s | 8s |
| (1.0\%, 171K) | Window | 130.496 ms | 15.212 ms | 323.911 ms | 13713.218 ms | $>10 \mathrm{~min}$ | N.A. |


| Datasets \& Measures |  | Imp/Rewr | MCDB20 | PT-k/Symb |
| :---: | :---: | :---: | :---: | :---: |
| Iceberg [ice07] | bound accuracy | 0.891 | 1 | 1 |
|  | bound recall | 1 | 0.765 | 1 |
| Crimes [crigo] | bound accuracy | 0.996 | 1 | 1 |
|  | bound recall | 1 | 0.919 | 1 |
| Healthcare [heare] | bound accuracy | 0.990 | 1 | 1 |
|  | bound recall | 1 | 0.767 | 1 |

## What is Vizier

- Open-source: https://vizierdb.info/
- A Data-centric Workflow Engine Instead of a REPL
- no hidden state!
- incremental workflow execution
- A Hybrid Notebook + Spreadsheet UI
- build workflows incrementally through a notebook interface
- edit datasets as spreadsheets
- Caveats: Data Concerns that Propagate
- document data errors / concerns / uncertainty
- caveats propagate alongside the data
- supports quantification of data uncertainty using UA-DBs
- summaries as overview of problems with a workflow
- Workflow Provenance and Versioning
- Versioning (workflow evolution provenance)


## Data-centric Workflows

## Datasets

- Currently relational tables
- Spreadsheet interpretation (co-ordinate system)


## Workflow Steps (cells)

- Cell executions are isolated from each other
- Dataflow through Vizier's dataset API
- Cells consume and produce datasets


## Vizier Architecture



Figure: Vizier's architecture

## The Notebook UI

## Projects consists of a notebook \& datasets

## A notebook is a sequence of cells

- Cells are steps in a workflow
- User-facing interface like Jupyter or similar
- Cells run in isolation and communicate through datasets


## Datasets

- Datasets are created / read / updated by cells
- Dataset creation
- Loading (special cell type)
- Result of a cell (e.g., SQL cell)
- Dataset update
- Spreadsheet operations (e.g., update value)
- Curation cells (e.g., impute missing values)


## The Spreadsheet UI \& Vizual

## Spreadsheet UI

- Updates are translated into workflow steps


## Spreadsheets as Datasets

- Relation + coordinate system (cell locations)


## Vizual language

- Scripting language for common spreadsheet operations


Figure: Column data distributions shown in the spreadsheet view

## Workflow Evolution and Versioning

## Workflow versioning

- Vizier captures provenance for the evolution of a user's workflow
- Like Git, but automates versioning
- Every edit creates a new version
- Branching is supported
- Versions of the workflow and of datasets have unique URLs


## Incremental Execution model

## Immutable objects

- Cells \& their outputs are immutable objects (at least conceptually)


## Dependency tracking

- Cell ordering is sequential (position in the notebook)
- Dependencies are tracked automatically
- Monitoring Vizier dataset API calls


## Automatic refresh

- On cell update, we determine automatically which cells are dependent and need to be re-executed


## Caveats: Uncertainty and Error Handling

## Caveats

- Caveats are annotations on data (attribute value)
- Record concerns / document assumptions about data
- May list possible alternative values or may mark value as unknown (uncertainty)
- UA-DBs without boolean attribute markers instead of ranges


## Caveat Generation

- Users can manually create caveats
- Data curation operations can annotated their outputs with caveats


## Caveat Example - Key Repair

## Repairing a violated PK constraint

- Primary key: Name
- Key repair: avg(Age)
- Repaired value is caveated (highlighted in red)
- Equivalent to annotating with a range that covers the whole domain

| Name | Age |
| :---: | :---: |
| Peter | 30 |
| Peter | 35 |
| Bob | 25 |


| Name | Age |
| :---: | :---: |
| Peter | 32.5 |
| Bob | 25 |

## UI \& API Support for Caveats

## Dataset error summary

- Per dataset
- Shows counts of caveats grouped by type


## Spreadsheet view

- Caveated values are shown in red
- Users can inspect caveat


## Plots

- Bars / points based on caveated values are show with a red dot


## Dataset API / SQL / Lenses

- Functions for accessing / creating / manipulating caveats
- SQL functions to create / check for caveats


## Cell types - curation \& cleaning

## Data curation (lenses)

- Missing value imputation
- Pivot
- Geocoding
- Type detection
- Schema Matching


## Uncertainty-aware through UA-DBs

- Results stored as UA-DBs


## References I

[ $\left.\mathrm{ABC}^{+} 03\right]$ Marcelo Arenas, Leopoldo Bertossi, Jan Chomicki, Xin He, Vijay Raghavan, and Jeremy Spinrad. Scalar aggregation in inconsistent databases.
Theoretical Computer Science, 296(3):405-434, 2003.
[ABS15] Antoine Amarilli, Pierre Bourhis, and Pierre Senellart.
Probabilities and provenance via tree decompositions.
Preprint: http://a3nm. net/publications/amarilli2015probabilities. pdf. Submitted to PODS, 2015.
[ $\mathrm{ACK}^{+}{ }^{10}$ ] Serge Abiteboul, T.-H. Hubert Chan, Evgeny Kharlamov, Werner Nutt, and Pierre Senellart. Aggregate queries for discrete and continuous probabilistic XML.
In Luc Segoufin, editor, Database Theory - ICDT 2010, 13th International Conference, Lausanne, Switzerland, March 23-25, 2010, Proceedings, ACM International Conference Proceeding Series, pages 50-61. ACM, 2010.
[ADT11] Yael Amsterdamer, Daniel Deutch, and Val Tannen.
Provenance for aggregate queries.
In Proceedings of the thirtieth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems, pages 153-164. ACM, 2011.
[AJKO08] Lyublena Antova, Thomas Jansen, Christoph Koch, and Dan Olteanu.
Fast and simple relational processing of uncertain data.
In Data Engineering, 2008. ICDE 2008. IEEE 24th International Conference on, pages 983-992. IEEE, 2008.
[AK08] Foto N. Afrati and Phokion G. Kolaitis.
Answering aggregate queries in data exchange.
In Proceedings of the Twenty-Seventh ACM SIGMOD-SIGACT-SIGART Symposium on Princtraita Database Systems, PODS 2008, June 9-11, 2008, Vancouver, BC, Canada, pages 129-138,

## References II

| [AKO07] | L. Antova, C. Koch, and D. Olteanu. <br> MayBMS: Managing incomplete information with probabilistic world-set decompositions. <br> In Data Engineering, 2007. ICDE 2007. IEEE 23rd International Conference on, pages 1479-1480. IEEE, 2007. |
| :---: | :---: |
| [BSHW06] | Omar Benjelloun, Anish Das Sarma, Alon Y. Halevy, and Jennifer Widom. <br> ULDBs: Databases with Uncertainty and Lineage. <br> In Proceedings of the 32th International Conference on Very Large Data Bases (VLDB), pages 953-964, 2006. |
| [CC77] | Patrick Cousot and Radhia Cousot. <br> Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. <br> In Proceedings of the 4 th ACM SIGACT-SIGPLAN symposium on Principles of programming languages, pages 238-252, 1977. |
| [CC04] | P. COUSOT and R. COUSOT. <br> Abstract Interpretation Frameworks. <br> Journal of Logic and Computation, 2(4):511-547, 2004. |
| [crigo] | Chicago crimes dataset. <br> https://www.kaggle.com/currie32/crimes-in-chicago. |
| [DS07] | Nilesh Dalvi and Dan Suciu. <br> The dichotomy of conjunctive queries on probabilistic structures. <br> In Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems, pages 293-302. ACM, 2007. |

## References III

[DS12] Nilesh Dalvi and Dan Suciu.
The dichotomy of probabilistic inference for unions of conjunctive queries.
Journal of the ACM (JACM), 59(6):30, 2012.
[FGK23] Su Feng, Boris Glavic, and Oliver Kennedy.
Efficient approximation of certain and possible answers for ranking and window queries over uncertain data.
Proceedings of the VLDB Endowment, 16(6):1346-1358, 2023.
[FHGK19] Su Feng, Aaron Huber, Boris Glavic, and Oliver Kennedy.
Uncertainty annotated databases - a lightweight approach for approximating certain answers.
In Proceedings of the 44th International Conference on Management of Data, pages 1313-1330, 2019.
[FHGK21] Su Feng, Aaron Huber, Boris Glavic, and Oliver Kennedy.
Efficient uncertainty tracking for complex queries with attribute-level bounds.
In Proceedings of the 46th International Conference on Management of Data, page 528 - 540, 2021.
[FHO12] Robert Fink, Larisa Han, and Dan Olteanu.
Aggregation in probabilistic databases via knowledge compilation.
Proceedings of the VLDB Endowment, 5(5):490-501, 2012.
[FO16] Robert Fink and Dan Olteanu.
Dichotomies for queries with negation in probabilistic databases.
ACM Trans. Database Syst., 41(1):4:1-4:47, 2016.
[Fux07] A.D. Fuxman.
Efficient query processing over inconsistent databases.
PhD thesis, University of Toronto, 2007.


| [GKT07] | Todd J. Green, Gregory Karvounarakis, and Val Tannen. <br> Provenance Semirings. <br> In PODS '07: Proceedings of the 26th Symposium on Principles of Database Systems, pages 31-40, 2007. |
| :---: | :---: |
| [GP10] | F. Geerts and A. Poggi. <br> On database query languages for K-relations. Journal of Applied Logic, 8(2):173-185, 2010. |
| [GT17] | Todd J Green and Val Tannen. <br> The semiring framework for database provenance. <br> In Proceedings of the 36th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems, pages 93-99. ACM, 2017. |
| [heare] | Medicare hospital dataset. <br> https://data.medicare.gov/data/hospital-compare. |
| [HPZL08] | Ming Hua, Jian Pei, Wenjie Zhang, and Xuemin Lin. <br> Ranking queries on uncertain data: a probabilistic threshold approach. <br> In Proceedings of the ACM SIGMOD International Conference on Management of Data, SIGMOD 2008, Vancouver, BC, Canada, June 10-12, 2008, pages 673-686, 2008. |
| [ice07] | Iceberg dataset. <br> https://nsidc.org/data/g00807. |
| [ILJ84] | Tomasz Imieliński and Witold Lipski Jr. <br> Incomplete Information in Relational Databases. <br> Journal of the ACM (JACM), 31(4):761-791, 1984. |

## References V

[IVV95] Tomasz Imielinski, Ron Vandermeyden, and Kumar V Vadaparty.
Complexity tailored design: A new design methodology for databases with incomplete information.
Journal of Computer and System Sciences, 51(3):405-432, 1995.
[JXW ${ }^{+}$08] R. Jampani, F. Xu, M. Wu, L.L. Perez, C. Jermaine, and P.J. Haas.
MCDB: a monte carlo approach to managing uncertain data.
In Proceedings of the 2008 ACM SIGMOD international conference on Management of data, pages 687-700. ACM, 2008.
[KB12] Egor V Kostylev and Peter Buneman.
Combining dependent annotations for relational algebra.
In Proceedings of the 15th International Conference on Database Theory, pages 196-207. ACM, 2012.
$\left[\mathrm{KNP}^{+} 22\right]$ Mahmoud Abo Khamis, Hung Q. Ngo, Reinhard Pichler, Dan Suciu, and Yisu Remy Wang.
Convergence of datalog over (pre-) semirings.
In Leonid Libkin and Pablo Barceló, editors, PODS '22: International Conference on Management of Data, Philadelphia, PA, USA, June 12-17, 2022, pages 105-117. ACM, 2022.
[Lib16] Leonid Libkin.
Sql's three-valued logic and certain answers.
ACM Transactions on Database Systems (TODS), 41(1):1, 2016.
[LJ84] Witold Lipski Jr.
On relational algebra with marked nulls.
In Proceedings of the 3rd ACM SIGACT-SIGMOD symposium on Principles of database systems, pages 201-203. ACM, 1984.
[LSV02] Jens Lechtenbörger, Hua Shu, and Gottfried Vossen.
Aggregate Queries over Conditional Tables.
Journal of Intelligent Information Systems, 19(3):343-362, 2002.

## References VI

```
[OHK09] D. Olteanu, J. Huang, and C. Koch.
    Sprout: Lazy vs. eager query plans for tuple-independent probabilistic databases.
    In Data Engineering, 2009. ICDE'09. IEEE 25th International Conference on, pages 640-651. IEEE,
    2009.
```

[RS09] Christopher Ré and Dan Suciu.
The trichotomy of HAVING queries on a probabilistic database.
VLDB J., 18(5):1091-1116, 2009.
[SKL $\left.{ }^{+} 17\right]$ Bruhathi Sundarmurthy, Paraschos Koutris, Willis Lang, Jeffrey Naughton, and Val Tannen. m-tables: Representing missing data.
In LIPIcs-Leibniz International Proceedings in Informatics, volume 68. Schloss
Dagstuhl-Leibniz-Zentrum fuer Informatik, 2017.
[VdBS17] Guy Van den Broeck and Dan Suciu.
Query processing on probabilistic data: A survey. 2017.
[YMF ${ }^{+}$15] Ying Yang, Niccolo Meneghetti, Ronny Fehling, Zhen Hua Liu, and Oliver Kennedy. Lenses: an on-demand approach to etl.
Proceedings of the VLDB Endowment, 8(12):1578-1589, 2015.

