Matching and Disclosure

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Motivation

- SAT Optional in College Admissions:
  - How does this affect the matching between colleges and students?
  - Do students benefit compared to mandatory SAT taking and disclosure?
  - How about case with voluntary SAT taking but mandatory disclosure?

- Features:
  - Costly pre-match investment
  - Disclosure opportunity
  - Matching

- Another application: Entrepreneurs and VC matching
This Paper

- **Matching model with pre-match investment and disclosure:**
  - Two sides with heterogeneous agents on each side and no transfers
  - One side can costly find out an attribute (payoff-relevant to the other side)
  - Agents choose to disclose the observed attribute
  - Matching takes place between the two sides of the market

- **Key forces at play:**
  - Matching affects incentives to invest and disclose
  - Investment and disclosure affects matching

- **Main results:**
  - Comparison of equilibrium under voluntary/mandatory disclosure
  - Analysis of who benefits from voluntary disclosure
  - Illustration: optional SAT
Related Literature

- **Disclosure:**

- **Matching with pre-match investments:**

- **Optional SAT:**
  - Borghesan (2022), Osaki (2022), Dessein, Frankel, and Kartik (2023)
Model

- Continuum of colleges (measure one) with $s \sim G, g > 0$ on $[0, 1]$

- Continuum of students (measure one) with $t \sim H, h > 0$ on $[0, 1]$
  
  - Type $t$ is only observed by students

  - **Pre-match investment:** student can draw $x \sim F(\cdot \mid t), f(\cdot \mid t) > 0,$ at cost $c > 0$
  
  - $\{f(\cdot \mid t)\}$ common support, strict monotone likelihood ratio property (MLRP)

  - **Disclosure:** If a student observes $x,$ she can then decide whether to disclose it

  - Students who do not invest or invest but do not disclose look identical

- **Payoffs:** utility of students is $s,$ and utility of colleges is $x$

- After pre-match investment and disclosure, matching takes place

- Equilibrium concept: PBE such that matching is stable
Equilibrium

- Wlog, equilibria in threshold strategies:
  - Invest iff $t \geq t_v \in [0, 1]$; then disclose iff $x \geq x_v \in [0, 1]$

- Matching:
  - All $x \geq x_v$ are positively assortatively matched according to $\mu_v(\cdot, t_v)$
  - $s = \mu_v(x, t_v)$ is the college with $s$ who matches with student who disclosed $x$
  - Rest of students (noninvestor/nondisclosers) and colleges randomly matched
  - $x_v$ expectation of $x$ conditional on students who do not invest/disclose and $t_v$

- Pre-match investment and disclosure:
  - Given $t_v$ and $x_v$, a student with $t$ who invested willing to disclose iff $x \geq x_v$
  - Marginal benefit of investment for $t$ given $t_v$ and $x_v$ ($MB_v(t, t_v)$):
    $$\mathbb{P}[x \geq x_v(t_v)|t]E[\mu_v(x, t_v)|x \geq x_v(t_v), t] + (1 - \mathbb{P}[x \geq x_v(t_v)|t])\hat{s}(t_v, x_v(t_v)) - \hat{s}(t_v, x_v(t_v))$$
    where $\hat{s}(t_v, x_v(t_v))$ is the expected payoff from random matching
  - Invest iff $MB_v(t, t_v) \geq c$; unique cutoff $\tilde{t}_v$ by MLRP
In equilibrium, $\tilde{t}_v = t_v$, hence equilibrium condition is

$$MB_v(t_v, t_v) = c$$

If $MB_v(0, 0) \geq c$ then $t_v = 0$; if $MB_v(1, 1) \leq c$ then $t_v = 1$

For any $c$ equilibrium exists; there can be multiple equilibria
Benchmark I: Fully Mandatory Case

- As a benchmark, consider case where students must invest and disclose.

- Matching $\mu_{fm}$ matches marginal distribution $F$ of $x$ and $G$.

- Payoff for each $t$ is $\int_{0}^{1} \mu_{fm}(x)dF(x|t) - c$.

- If student can leave the market, then only $t \geq t_{fm}$ stay, where $t_{fm}$ solves

$$\int_{0}^{1} \mu_{fm}(x, t_{fm})dF(x|t_{fm}) = c$$
Benchmark II: Mandatory Disclosure

- Another benchmark: voluntary investment but mandatory disclosure
- Wlog, equilibrium in threshold strategies: invest iff \( t \geq t_m \)
- Matching:
  - \( x \geq x_n \) matched positively assortatively with high \( s \)'s according to \( \mu^+_m(\cdot, t_m) \)
  - Randomly match students who do not invest with colleges of intermediate \( s \)'s
  - \( x < x_n \) matched positively assortatively with low \( s \)'s with \( \mu^-_m(\cdot, t_m) \)
  - \( x_n \) is expectation of \( x \) conditional on set of students who do not invest and \( t_m \)
- Pre-match investment:
  - Invest iff \( MB_m(t, t_m) \geq c \) where marginal benefit is
    \[
    \mathbb{P}[x \geq x_n | t] \mathbb{E}[\mu^+_m(x, t_m) | x \geq x_n, t] + (1 - \mathbb{P}[x \geq x_n | t]) \mathbb{E}[\mu^-_m(x, t_m) | x < x_n, t] - \bar{s}(t_m)
    \]
    where \( \bar{s} \) is the expected payoff from random matching
- Equilibrium: \( MB_m(t_m, t_m) = c \) (plus boundary cases); existence, multiplicity
Voluntary Disclosure vs Mandatory Disclosure

- There is more investment under voluntary than under mandatory disclosure:
  - If equilibrium is unique, $t_v \leq t_m$ and $x_v(t_v) \leq x_n(t_m)$ (both strict if interior)
  - If multiple, set of equilibrium thresholds under mandatory “higher set” than under voluntary disclosure
  - If multiple, interval of values of $c$ that sustain interior equilibria under voluntary “higher” than under mandatory cost
  - Intuition is that $MB_v$ is “higher” than $MB_m$

- Amount of disclosure (mass of students disclosing) comparison ambiguous:
  - $1 - H(t_v) - \int_{t_v}^{1} F(x_v(t_v)|t)dH(t)$ versus $1 - H(t_m)$
  - More investment under voluntary but, conditional on investing, less disclosure
  - For low and high $c$’s, amount of disclosure higher under voluntary disclosure
Who Benefits from Voluntary Disclosure?

- Voluntary disclosure versus fully mandatory case:

Proposition (VD versus FM)

(i) Interval of low types \( t \) starting at \( t = 0 \) strictly prefers VD to FM;

(ii) If \( \{f(x|t)\}_{t \in [0,1]} \) is TP3, then either (a) all students strictly prefer VD; or (b) students with \( t \) below a threshold strictly prefer VD, while rest FM; or (c) an interval of intermediate types strictly prefers FM, while rest strictly prefers VD.

- (i) since \( t = 0 \) strictly benefits from not investing in VD comparing to FM

- (ii) from Karlin’s Variation Diminishing Property

- Easy to pin down comparison for \( t < t_v \)

- For \( t \geq t_v \), write difference in payoffs VD–MD as \( \eta(t) = \int_0^1 r(x, t_v) f(x|t) dx \)

- \( r(\cdot, t_v) \) pcw continuous, changes signs at most twice; if twice, then +/−/+−/+−+

- By Karlin’s result, same holds for \( \eta \), and result follows
Who Benefits from Voluntary Disclosure?

- Voluntary disclosure versus mandatory disclosure:

**Proposition (VD versus MD)**

(i) Interval of low types $t$ starting at $t = 0$ strictly prefers MD to VD;
(ii) If $\{f(x|t)\}_{t \in [0,1]}$ is $TP_4$, then either (a) all students strictly prefer MD; or (b) students with $t$ below a threshold strictly prefer MD, while rest VD; or (c) there is one or two intervals of intermediate types that strictly prefer VD, while rest strictly prefers MD.

- (i) since $\bar{s}(t_m) > \hat{s}(t_v, x_v(t_v))$ so all $t \leq t_v$ strictly better off under MD
- (ii) from Karlin’s Variation Diminishing Property
  - All $t < t_v$ better off under MD
  - For $t \geq t_v$, write difference MD–VD as $\delta(t) = \int_0^1 q(x, t_v, t_m) f(x|t)dx$
  - $q(\cdot, t_v, t_m)$ pcw cts, changes signs at most thrice; if thrice, $-/+/-/+$
  - By Karlin’s result, same holds for $\delta$, and result follows
Who Benefits from Voluntary Disclosure?

- Sharper results if we assume binary college characteristics:
  - A measure $\kappa \in (0, 1)$ has characteristic $s_1$; $1 - \kappa$ has $s_2 > s_1$
  - $s_2$ colleges are “top schools” while $s_1$ colleges are “non-top schools”
  - $1 - \kappa$ and $\kappa$ aggregate capacities of top and non-top schools

- Focus on interior equilibria $0 < t_v < t_m < 1$, such that:
  - Measure of students who disclose $x \geq x_v$ strictly less than $1 - \kappa$
  - In mandatory case, measure of students with $x \geq x_n$ strictly less than $1 - \kappa$
  - Easy to ensure from primitives; this is the most interesting case
Who Benefits from Voluntary Disclosure?

- There is a \( \hat{t} \in (0, 1] \) s.t. \( t \leq \hat{t} \) strictly prefer VD and \( t > \hat{t} \) FM
  - Result holds with \( \{f(\cdot|t)\} \) MLRP; no need for \( TP_3 \)
  - \textbf{Intuition:} Higher probability of \( s_2 \) in fully mandatory; \( \hat{s} > s_1 \)

- There is a \( \tilde{t} \in (t_v, 1] \) s.t. \( t \leq \tilde{t} \) strictly prefer MD and \( t > \tilde{t} \) VD
  - Result holds with \( \{f(\cdot|t)\} \) MLRP; no need for \( TP_4 \)
  - \textbf{Intuition:} for high enough \( t \) disclosure choice provides extra benefit; for low enough \( t \) random matching payoff dominates
Optional SAT

- Interpret incurring $c > 0$ as taking the SAT

- Assume that SAT perfectly reveals student caliber
  - Low caliber students strictly prefer VD to FM
  - But, they benefit even more from MD
  - Ranking reversed for high caliber students in top and non-top schools case
  - More applications under VD than under FM
Other Results

- Comparative statics: so far wrt FOSD shift in $G$

- $t$ observable: $x_v = 0$

- $t$ payoff-relevant: either lots of equilibria, or $x_v = 0$

- Colleges choosing to commit to mandatory disclosure
Conclusion

- Many economic applications combine matching and disclosure
  - This paper analyzes their interaction

- Motivation comes from voluntary SAT taking and reporting
  - All students can be better off than in mandatory SAT taking and reporting
    - **Low-caliber students**: MD $\succ$ VD $\succ$ FM
    - **High-caliber students** FM $\succ$ VD $\succ$ MD in binary case

- Next steps:
  - Welfare college side, efficiency, noise, transferable utility
Matching Function $\mu_v$

- Fix investment threshold $t_v$ and disclosure threshold $x_v$
- Let $\tilde{F}_i(x, t_v) = \int_{t_v}^1 F(x|t)dH(t)$ be the distribution of $x$ given $t_v$, $x \geq x_v$
- $\tilde{F}_i(1, t_v) = 1 - H(t_v)$, mass of students who invest
- $1 - H(t_v) - \tilde{F}_i(x_v, t_v)$ mass of students who invest and disclose, $\tilde{F}_i(x_v, t_v)$ invest but do not disclose, $H(t_v)$ do not invest
- Matching $\mu_v$ given by $1 - G(\mu_v(x, t_v)) = 1 - H(t_v) - \tilde{F}_i(x, t_v)$ for $x \geq x_v$
- Hence $\mu_v(x, t_v) = G^{-1}(H(t_v) + \tilde{F}_i(x, t_v))$ for $x \geq x_v$
Matching Functions $\mu_-$ and $\mu_+$

- Fix and investment threshold $t_m$, which yields $x_n$

- $F_i(x, t_m) = \int_{t_m}^{1} F(x | t) dH(t)$ distribution of $x$ given $t_m$ (investors)

- $F_i(1, t_m) = 1 - H(t_m)$, mass of students who invest

- Hence $\mu_+(x, t_m) = G^{-1}(H(t_m) + F_i(x, t_m))$ for $x \geq x_n$

- Similarly, $\mu_-(x, t_m) = G^{-1}(F_i(x, t_m))$ for $x < x_n$
Equilibrium Comparison

- Let $A$ and $B$ be subsets of $[0, 1]$

- $A$ is higher than $B$ in the weak set order, $A \geq_{ws} B$, if for each $t \in B$ there is a $t' \in A$ such that $t' \geq t$, and for each $t' \in A$, there is a $t \in B$ such that $t \leq t'$ (Che, Kim, and Kojima (2021))

**Proposition**

Assume that $\int_0^1 \mu^{-1}(x) dF(x|0) \leq 1 - F(\mathbb{E}[x]|1) - \mathbb{E}[s]$. Then given any $c > 0$, the set of equilibrium investment thresholds under mandatory disclosure is higher than under voluntary disclosure in the weak set order.

- At any $t_v$ s.t. $MB_v(t_v, t_v) = c$, $MB_m(t_v, t_v) < MB_v(t_v, t_v)$
Equilibrium Comparison

- \([a, b]\) lower than \([c, d]\) if \(a \leq c\) and \(b \leq d\) (similar for other intervals)

Proposition

As a function of \(c\), the following properties hold:

(i) The interval of values of \(c\) that support an equilibrium in which every student invests under mandatory disclosure is lower than the corresponding interval under voluntary disclosure;

(ii) The interval of values of \(c\) that support an equilibrium in which no student invests under mandatory disclosure is lower than the corresponding interval under voluntary disclosure;

(iii) The interval of values of \(c\) that support only interior equilibrium investment thresholds under mandatory disclosure is lower than the corresponding interval under voluntary disclosure.

Follows from shapes of \(MB_v(t_v, t_v)\) and \(MB_m(t_m, t_m)\) functions.