Matching and Disclosure

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Motivation

- SAT Optional in College Admissions:
  - How does this affect the matching between colleges and students?
  - Do students benefit compared to mandatory SAT taking and disclosure?
  - How about case with voluntary SAT taking but mandatory disclosure?
- Features:
  - Costly pre-match investment
  - Disclosure opportunity
  - Matching
- Another application: Entrepreneurs and VC matching

This Paper

- Matching model with pre-match investment and disclosure:
  - Two sides with heterogeneous agents on each side and no transfers
  - One side can costly find out an attribute (payoff-relevant to the other side)
  - Agents choose to disclose the observed attribute
  - Matching takes place between the two sides of the market
- Key forces at play:
  - Matching affects incentives to invest and disclose
  - Investment and disclosure affects matching
- Main results:
  - Comparison of equilibrium under voluntary/mandatory disclosure
  - Analysis of who benefits from voluntary disclosure
  - Illustration: optional SAT

Related Literature

- Disclosure:
  - Grossman (1981), Milgrom (1981), Verecchia (1983), Dye (1985), Ben-Porath, Dekel, and Lipman (2018)
- Matching with pre-match investments:
  - Cole, Mailath, and Postlewaite (2001), Nöldeke and Samuelson (2015), Bhaskar and Hopkins (2016), Chade and Lindenlaub (2022), Bilancini and Boncinelli (2013)
- Optional SAT:
  - Borghesan (2022), Osaki (2022), Dessein, Frankel, and Kartik (2023)

Model

- Continuum of colleges (measure one) with $s \sim G, g > 0$ on $[0,1]$
- Continuum of students (measure one) with $t \sim H, h > 0$ on $[0,1]$
  - Type t is only observed by students
  - Pre-match investment: student can draw $x \sim F(\cdot|t)$, $f(\cdot|t) > 0$, at cost $c > 0$
  - $\{f(\cdot|t)\}$ common support, strict monotone likelihood ratio property (MLRP)
  - Disclosure: If a student observes x, she can then decide whether to disclose it
  - Students who do not invest or invest but do not disclose look identical
- Payoffs: utility of students is s, and utility of colleges is x
- After pre-match investment and disclosure, matching takes place
- Equilibrium concept: PBE such that matching is stable

Equilibrium

- Wlog, equilibria in threshold strategies:
  - Invest iff $t \geq t_v \in [0,1]$; then disclose iff $x \geq x_v \in [0,1]$
- Matching:
  - All $x \geq x_v$ are positively assortatively matched according to $\mu_v(\cdot, t_v)$ ▸ μ_v
  - $s = \mu_v(x, t_v)$ is the college with s who matches with student who disclosed x
  - Rest of students (noninvestor/nondisclosers) and colleges randomly matched
  - x_v expectation of x conditional on students who do not invest/disclose and t_v
- Pre-match investment and disclosure:
  - Given t_v and x_v, a student with t who invested willing to disclose iff $x \geq x_v$
  - Marginal benefit of investment for t given t_v and x_v ($MB_v(t, t_v)$):

$$\mathbb{P}[x \geq x_v(t_v)|t]\mathbb{E}[\mu_v(x, t_v)|x \geq x_v(t_v), t] + (1 - \mathbb{P}[x \geq x_v(t_v)|t])\hat{s}(t_v, x_v(t_v)) - \hat{s}(t_v, x_v(t_v))$$

    where $\hat{s}(t_v, x_v(t_v))$ is the expected payoff from random matching

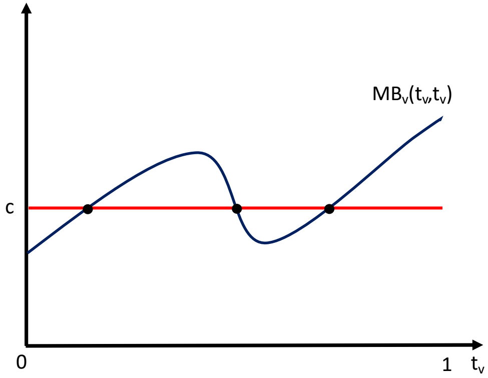
  - Invest iff $MB_v(t, t_v) \geq c$; unique cutoff $\tilde{t}_v$ by MLRP

Equilibrium

- In equilibrium, $\tilde{t}_v = t_v$, hence equilibrium condition is

$$MB_v(t_v, t_v) = c$$

- If $MB_v(0,0) \geq c$ then $t_v = 0$; if $MB_v(1,1) \leq c$ then $t_v = 1$
- For any c equilibrium exists; there can be multiple equilibria



Benchmark I: Fully Mandatory Case

- As a benchmark, consider case where students must invest and disclose
- Matching μ_{fm} matches marginal distribution F of x and G
- Payoff for each t is $\int_0^1 \mu_{fm}(x) dF(x|t) - c$
- If student can leave the market, then only $t \geq t_{fm}$ stay, where t_{fm} solves

$$\int_0^1 \mu_{fm}(x, t_{fm}) dF(x|t_{fm}) = c$$

Benchmark II: Mandatory Disclosure

- Another benchmark: voluntary investment but mandatory disclosure
- Wlog, equilibrium in threshold strategies: invest iff $t \geq t_m$
- Matching:
  - $x \geq x_n$ matched positively assortatively with high s's according to $\mu_m^+(\cdot, t_m)$
  - Randomly match students who do not invest with colleges of intermediate s's
  - $x < x_n$ matched positively assortatively with low s's with $\mu_m^-(\cdot, t_m)$ ▸ μ_m
  - x_n is expectation of x conditional on set of students who do not invest and t_m
- Pre-match investment:
  - Invest iff $MB_m(t, t_m) \geq c$ where marginal benefit is

$$\mathbb{P}[x \geq x_n | t]\mathbb{E}[\mu_m^+(x, t_m) | x \geq x_n, t] + (1 - \mathbb{P}[x \geq x_n | t])\mathbb{E}[\mu_m^-(x, t_m) | x < x_n, t] - \bar{s}(t_m)$$

  where $\bar{s}$ is the expected payoff from random matching
- Equilibrium: $MB_m(t_m, t_m) = c$ (plus boundary cases); existence, multiplicity

Voluntary Disclosure vs Mandatory Disclosure

- There is more investment under voluntary than under mandatory disclosure:
  - If equilibrium is unique, $t_v \leq t_m$ and $x_v(t_v) \leq x_n(t_m)$ (both strict if interior)
  - If multiple, set of equilibrium thresholds under mandatory "higher set" than under voluntary disclosure ▸ thresholds
  - If multiple, interval of values of c that sustain interior equilibria under voluntary "higher" than under mandatory ▸ cost
  - Intuition is that MB_v is "higher" than MB_m
- Amount of disclosure (mass of students disclosing) comparison ambiguous:
  - $1 - H(t_v) - \int_{t_v}^{1} F(x_v(t_v)|t)dH(t)$ versus $1 - H(t_m)$
  - More investment under voluntary but, conditional on investing, less disclosure
  - For low and high c's, amount of disclosure higher under voluntary disclosure

Who Benefits from Voluntary Disclosure?

- Voluntary disclosure versus fully mandatory case:

Proposition (VD versus FM)

(i) Interval of low types t starting at $t = 0$ strictly prefers VD to FM;
(ii) If $\{f(x|t)\}_{t \in [0,1]}$ is TP_3, then either (a) all students strictly prefer VD; or (b) students with t below a threshold strictly prefer VD, while rest FM; or (c) an interval of intermediate types strictly prefers FM, while rest strictly prefers VD.

- (i) since $t = 0$ strictly benefits from not investing in VD comparing to FM
- (ii) from Karlin's Variation Diminishing Property
  - Easy to pin down comparison for $t < t_v$
  - For $t \geq t_v$, write difference in payoffs VD$-$MD as $\eta(t) = \int_0^1 r(x, t_v) f(x|t) dx$
  - $r(\cdot, t_v)$ pcw continuous, changes signs at most twice; if twice, then $+/-/+$
  - By Karlin's result, same holds for η, and result follows

Who Benefits from Voluntary Disclosure?

- Voluntary disclosure versus mandatory disclosure:

Proposition (VD versus MD)

(i) Interval of low types t starting at $t = 0$ strictly prefers MD to VD;
(ii) If $\{f(x|t)\}_{t \in [0,1]}$ is TP_4, then either (a) all students strictly prefer MD; or (b) students with t below a threshold strictly prefer MD, while rest VD; or (c) there is one or two intervals of intermediate types that strictly prefer VD, while rest strictly prefers MD.

- (i) since $\bar{s}(t_m) > \hat{s}(t_v, x_v(t_v))$ so all $t \leq t_v$ strictly better off under MD
- (ii) from Karlin's Variation Diminishing Property
  - All $t < t_v$ better off under MD
  - For $t \geq t_v$, write difference MD$-$VD as $\delta(t) = \int_0^1 q(x, t_v, t_m) f(x|t) dx$
  - $q(\cdot, t_v, t_m)$ pcw cts, changes signs at most thrice; if thrice, $-/+/-/+$
  - By Karlin's result, same holds for δ, and result follows

Who Benefits from Voluntary Disclosure?

- Sharper results if we assume binary college characteristics:
  - A measure $\kappa \in (0,1)$ has characteristic s_1; $1-\kappa$ has $s_2 > s_1$
  - s_2 colleges are "top schools" while s_1 colleges are "non-top schools"
  - $1-\kappa$ and κ aggregate capacities of top and non-top schools
- Focus on interior equilibria $0 < t_v < t_m < 1$, such that:
  - Measure of students who disclose $x \geq x_v$ strictly less than $1-\kappa$
  - In mandatory case, measure of students with $x \geq x_n$ strictly less than $1-\kappa$
  - Easy to ensure from primitives; this is the most interesting case

Who Benefits from Voluntary Disclosure?

- There is a $\hat{t} \in (0,1]$ s.t. $t \leq \hat{t}$ strictly prefer VD and $t > \hat{t}$ FM
  - Result holds with $\{f(\cdot|t)\}$ MLRP; no need for TP_3
  - Intuition: Higher probability of s_2 in fully mandatory; $\hat{s} > s_1$
- There is a $\tilde{t} \in (t_v, 1]$ s.t. $t \leq \tilde{t}$ strictly prefer MD and $t > \hat{t}$ VD
  - Result holds with $\{f(\cdot|t)\}$ MLRP; no need for TP_4
  - Intuition: for high enough t disclosure choice provides extra benefit; for low enough t random matching payoff dominates

Optional SAT

- Interpret incurring $c > 0$ as taking the SAT
- Assume that SAT perfectly reveals student caliber
  - Low caliber students strictly prefer VD to FM
  - But, they benefit even more from MD
  - Ranking reversed for high caliber students in top and non-top schools case
  - More applications under VD than under FM

Other Results

- Comparative statics: so far wrt FOSD shift in G
- t observable: $x_v = 0$
- t payoff-relevant: either lots of equilibria, or $x_v = 0$
- Colleges choosing to commit to mandatory disclosure

Conclusion

- Many economic applications combine matching and disclosure
  - This paper analyzes their interaction
- Motivation comes from voluntary SAT taking and reporting
  - All students can be better off than in mandatory SAT taking and reporting
  - Low-caliber students: MD $\succ$ VD $\succ$ FM
  - High-caliber students FM $\succ$ VD $\succ$ MD in binary case
- Next steps:
  - Welfare college side, efficiency, noise, transferable utility

Matching Function μ_v

- Fix investment threshold t_v and disclosure threshold x_v
- Let $\tilde{F}_i(x, t_v) = \int_{t_v}^{1} F(x|t)dH(t)$ be the distribution of x given t_v, $x \geq x_v$
- $\tilde{F}_i(1, t_v) = 1 - H(t_v)$, mass of students who invest
- $1 - H(t_v) - \tilde{F}_i(x_v, t_v)$ mass of students who invest and disclose, $\tilde{F}_i(x_v, t_v)$ invest but do not disclose, $H(t_v)$ do not invesnt
- Matching μ_v given by $1 - G(\mu_v(x, t_v)) = 1 - H(t_v) - \tilde{F}_i(x, t_v)$ for $x \geq x_v$
- Hence $\mu_v(x, t_v) = G^{-1}(H(t_v) + \tilde{F}_i(x, t_v))$ for $x \geq x_v$ back

Matching Functions μ_- and μ_+

- Fix and investment threshold t_m, which yields x_n
- $F_i(x, t_m) = \int_{t_m}^{1} F(x|t)dH(t)$ distribution of x given t_m (investors)
- $F_i(1, t_m) = 1 - H(t_m)$, mass of students who invest
- Hence $\mu_+(x, t_m) = G^{-1}(H(t_m) + F_i(x, t_m))$ for $x \geq x_n$
- Similarly, $\mu_-(x, t_m) = G^{-1}(F_i(x, t_m))$ for $x < x_n$ back

Equilibrium Comparison

- Let A and B be subsets of $[0,1]$
- A is higher than B in the weak set order, $A \geq_{ws} B$, if for each $t \in B$ there is a $t' \in A$ such that $t' \geq t$, and for each $t' \in A$, there is a $t \in B$ such that $t \leq t'$ (Che, Kim, and Kojima (2021))

Proposition

Assume that $\int_0^1 \mu^{-1}(x) dF(x|0) \leq 1 - F(\mathbb{E}[x]|1) - \mathbb{E}[s]$. *Then given any* $c > 0$, *the set of equilibrium investment thresholds under mandatory disclosure is higher than under voluntary disclosure in the weak set order.*

- At any t_v s.t. $MB_v(t_v, t_v) = c$, $MB_m(t_v, t_v) < MB_v(t_v, t_v)$ back

Equilibrium Comparison

- $[a, b]$ lower than $[c, d]$ if $a \leq c$ and $b \leq d$ (similar for other intervals)

Proposition

As a function of c, the following properties hold:

(i) The interval of values of c that support an equilibrium in which every student invests under mandatory disclosure is lower than the corresponding interval under voluntary disclosure;

(ii) The interval of values of c that support an equilibrium in which no student invests under mandatory disclosure is lower than the corresponding interval under voluntary disclosure;

(iii) The interval of values of c that support only interior equilibrium investment thresholds under mandatory disclosure is lower than the corresponding interval under voluntary disclosure.

- Follows from shapes of $MB_v(t_v, t_v)$ and $MB_m(t_m, t_m)$ functions back