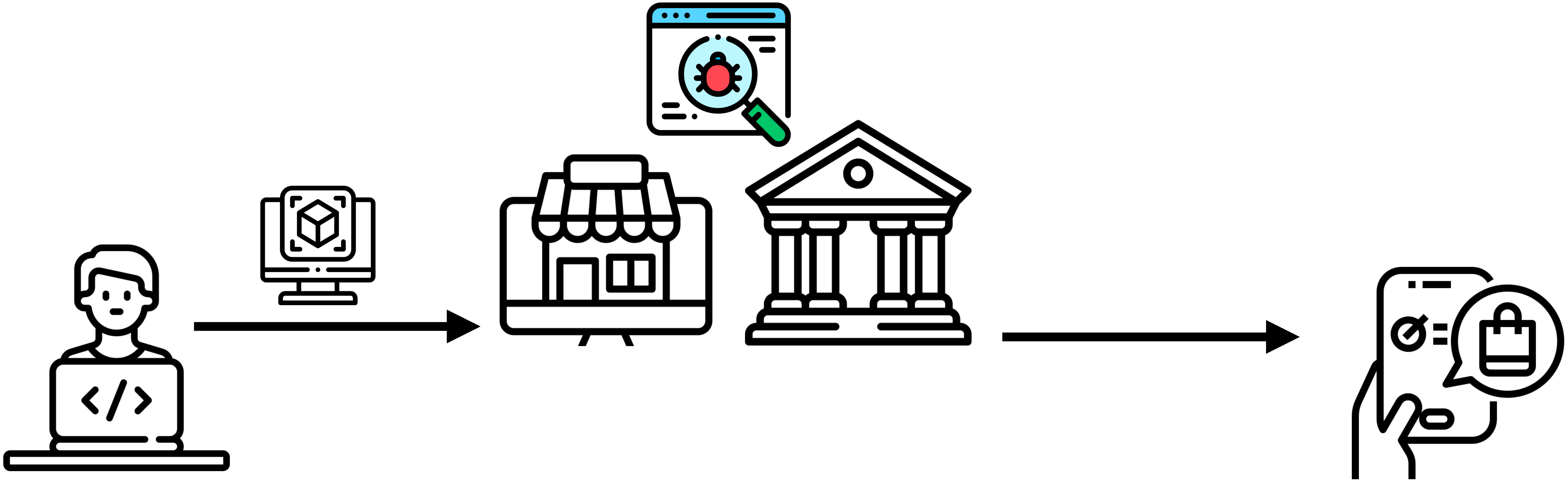
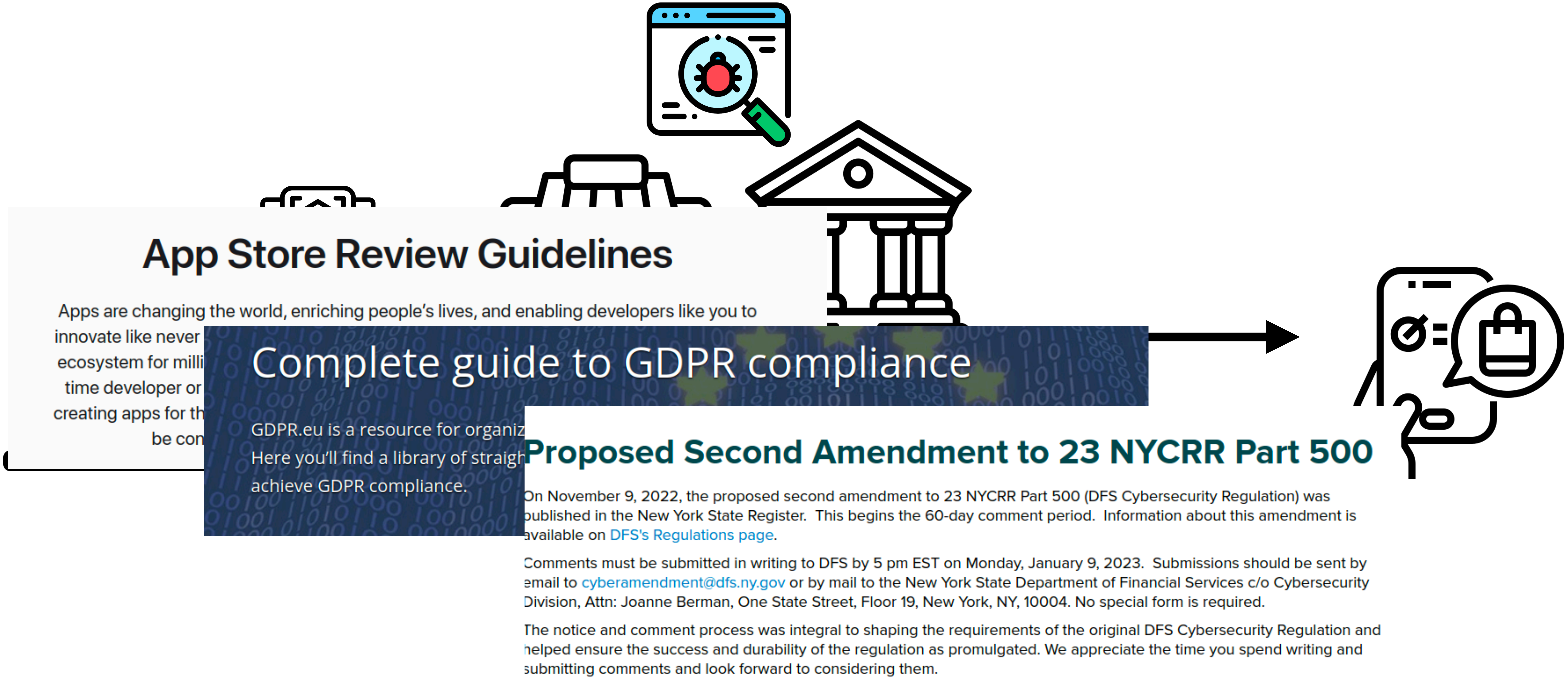
ZKUNSAT
Proving UNSAT in Zero Knowledge

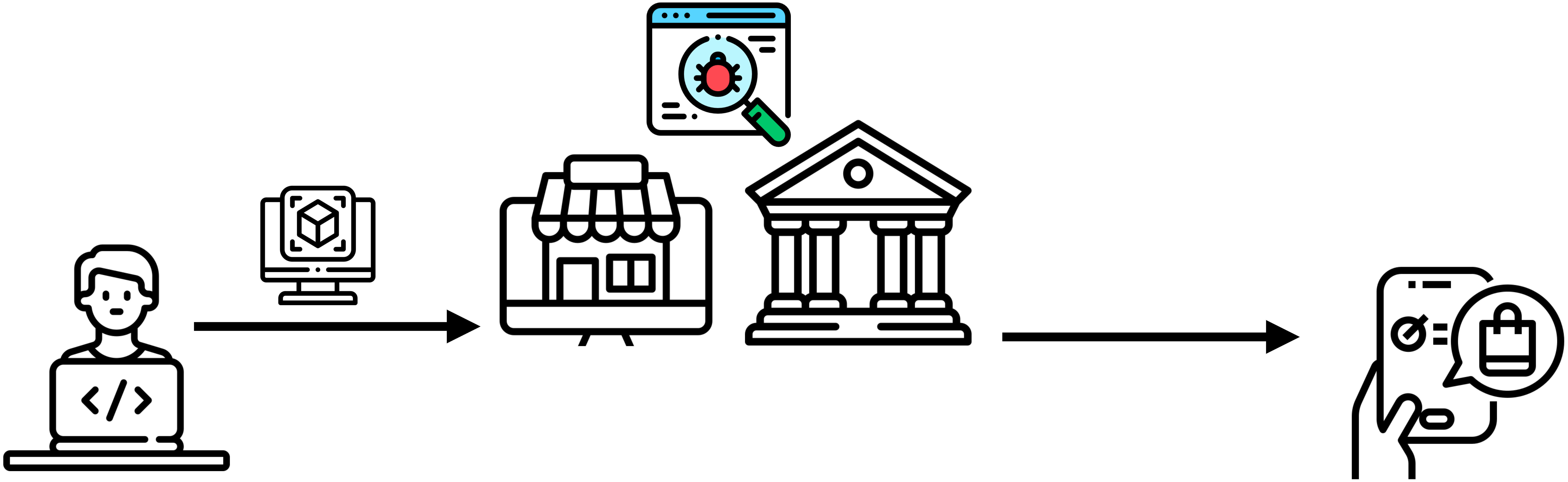
SAT Reunion Workshop, Simons Institute, UC Berkeley

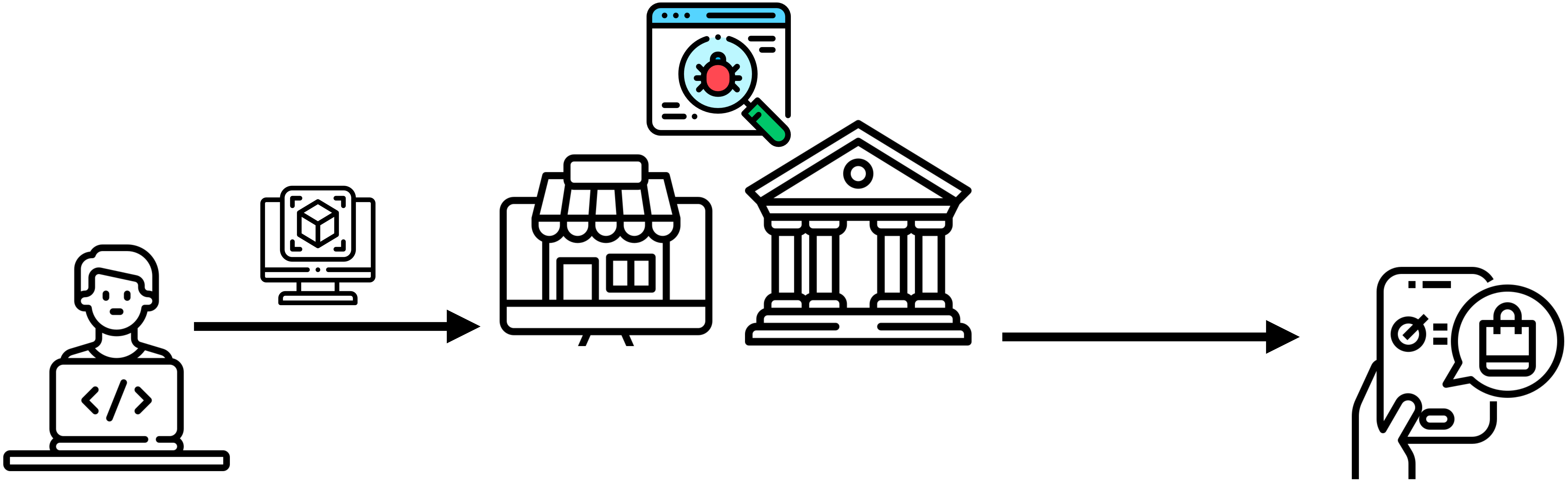
Ning Luo, Timos Antonopoulos, Bill Harris, Ruzica Piskac, Eran Tromer, Xiao Wang

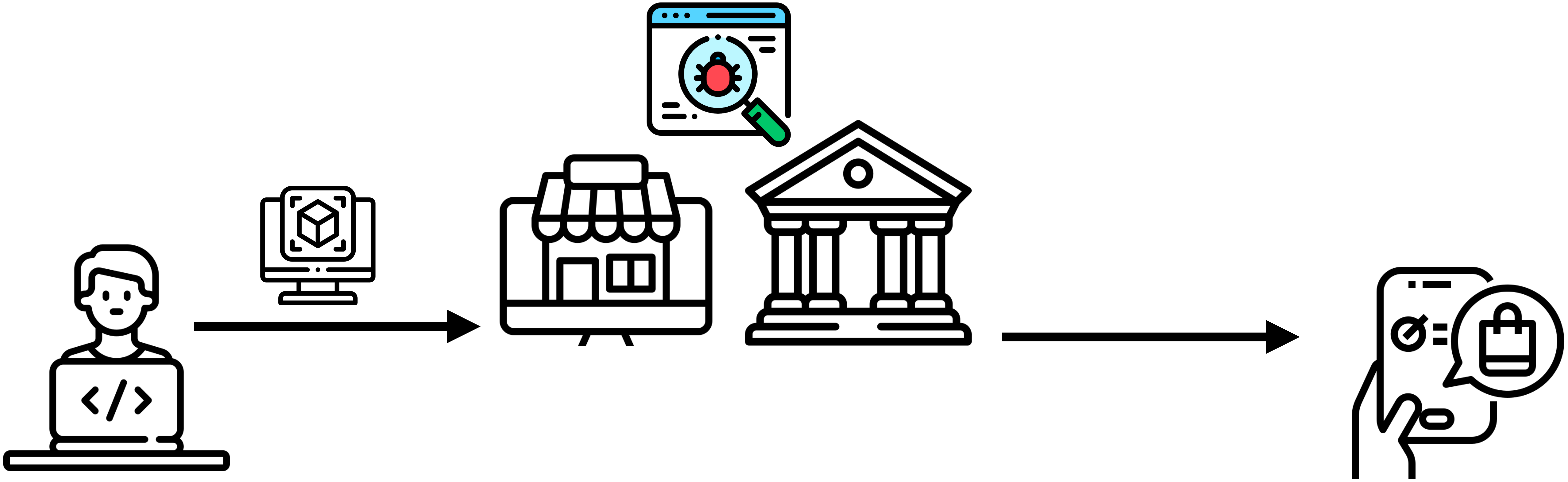




Verifications can be required by outside entities

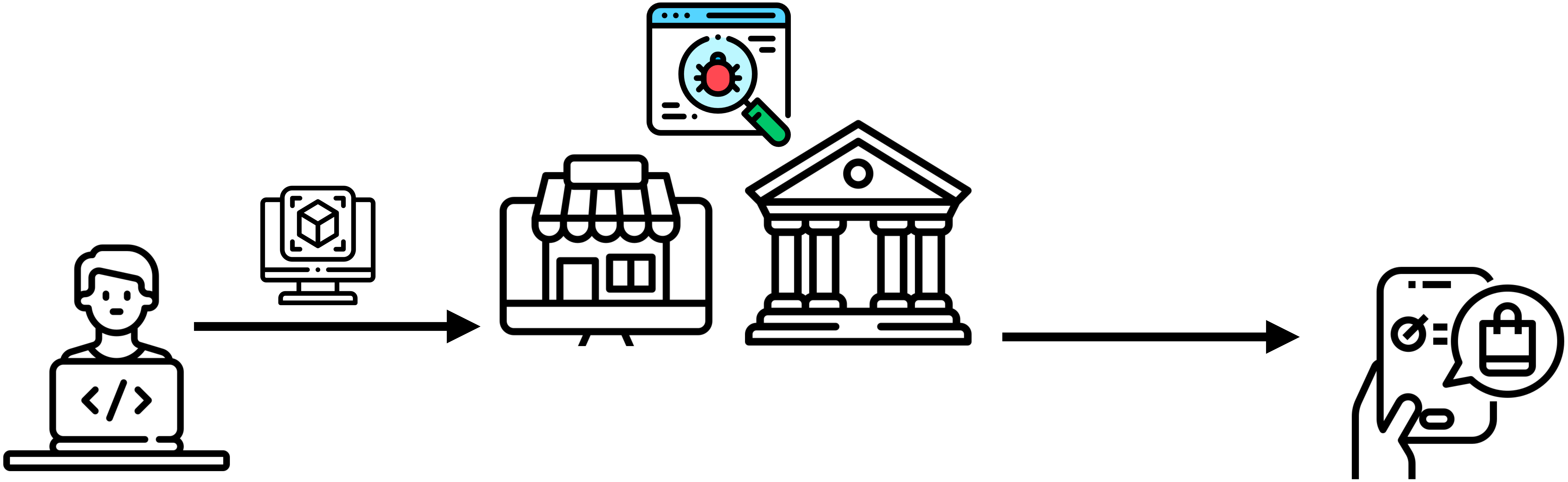


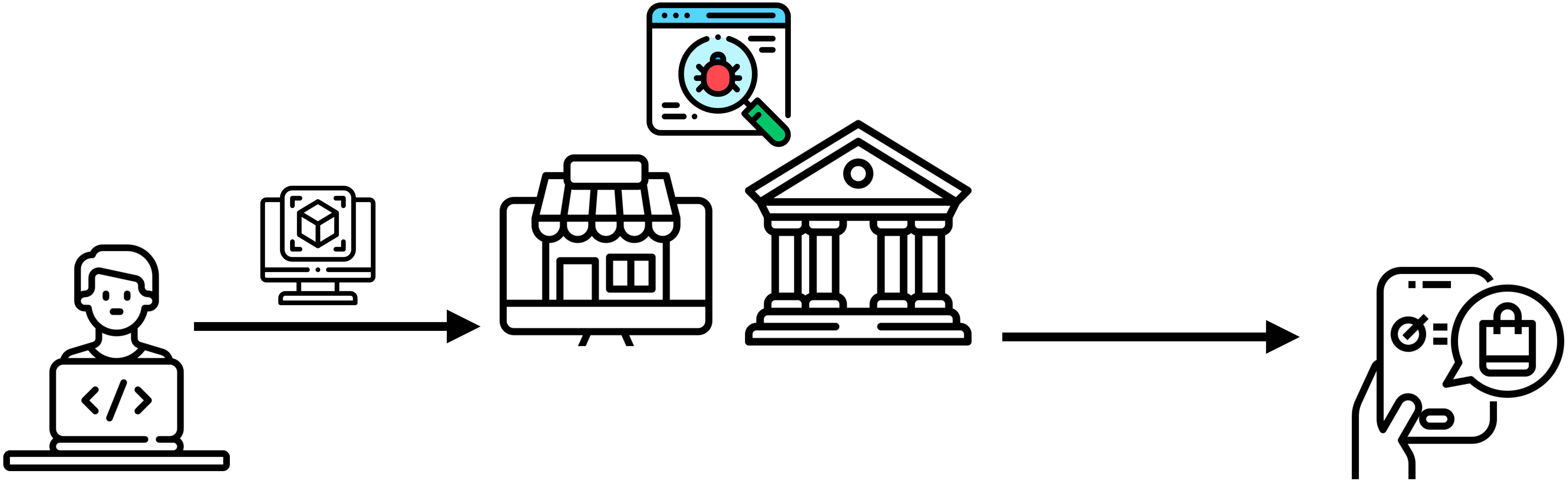




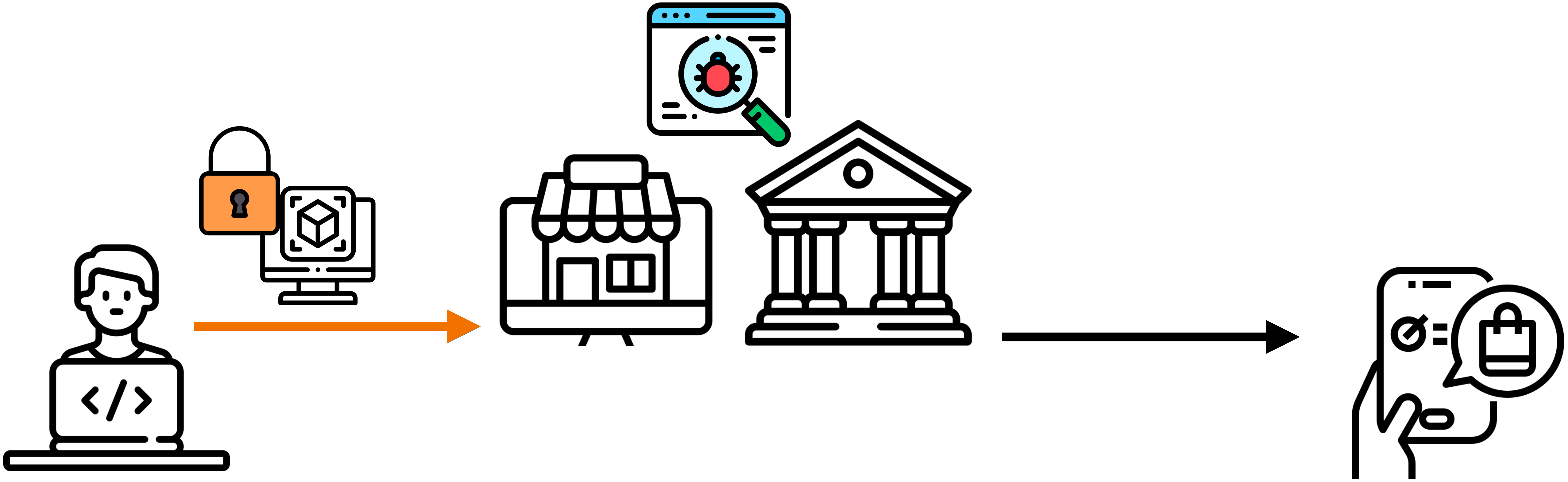
Centralized softwares verification and distribution：

Solution to a lot of problems, eg.supply-chain attacks





Both the developers and users should trust the centralized marketplace



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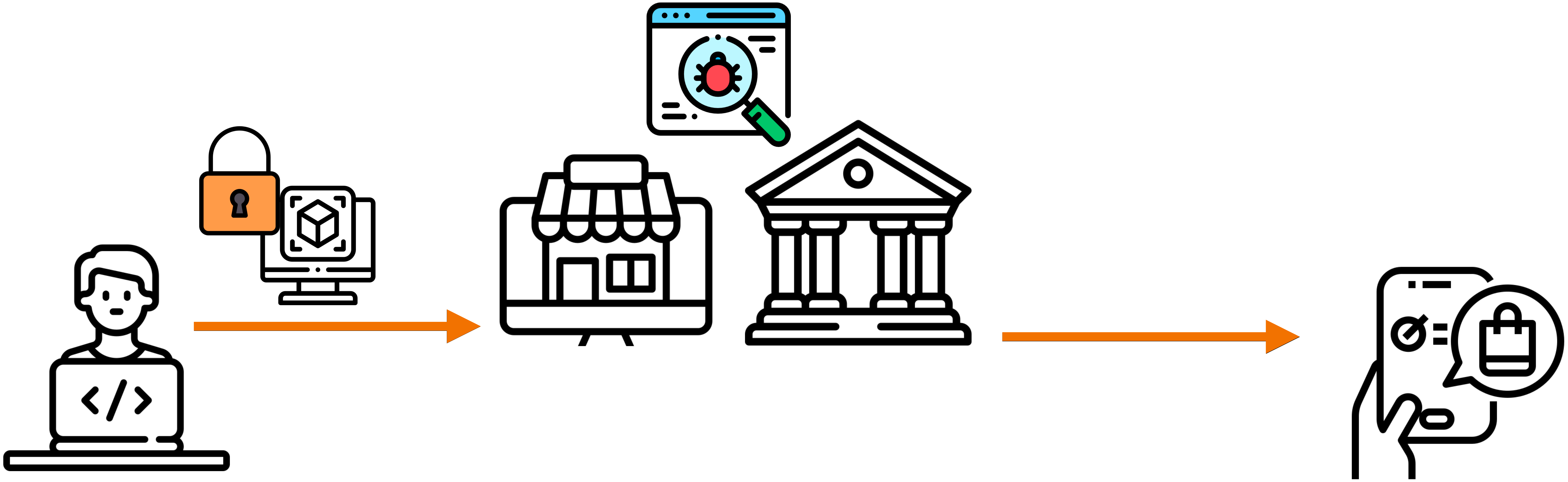

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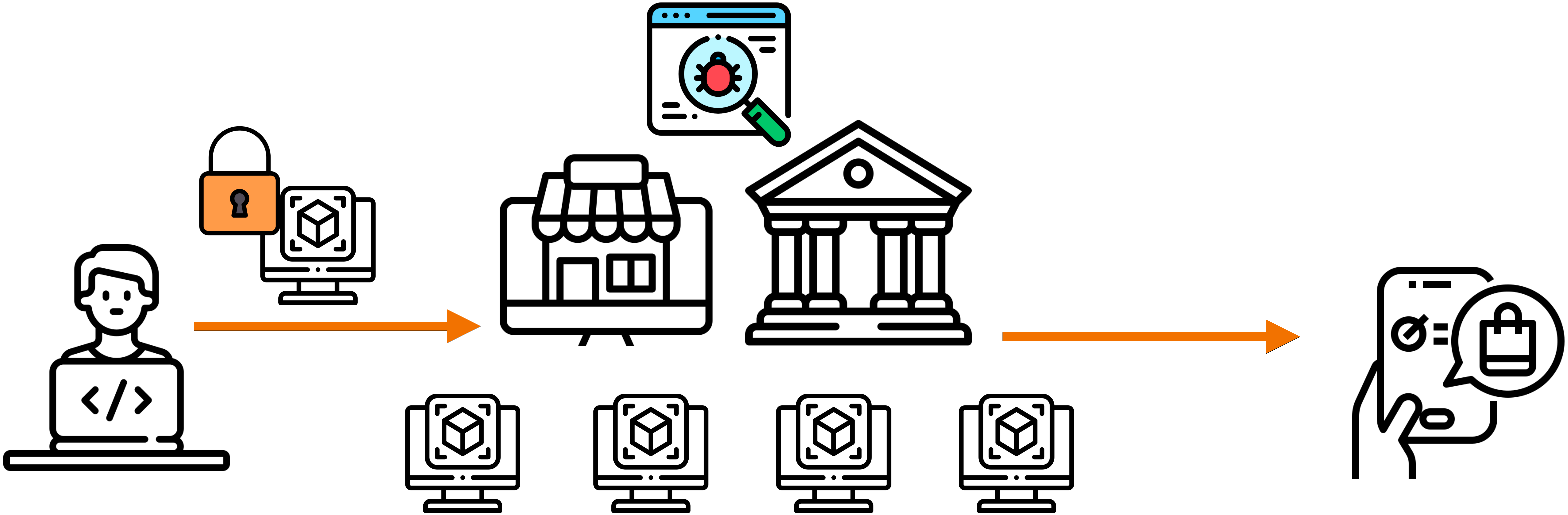

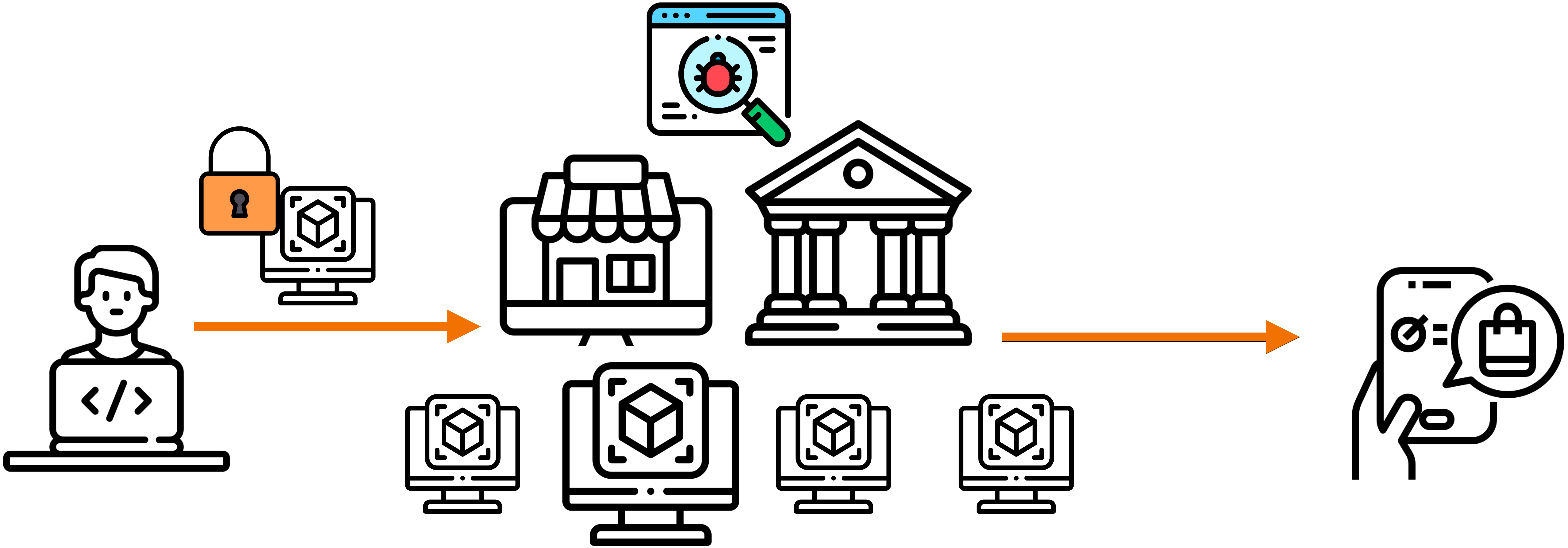

08.11.2021

Blumenthal, Blackburn & Klobuchar Introduce Bipartisan Antitrust Legislation to Promote App Store Competition

Apple Gives Ground in a Strategic Retreat From Strict App Store Rules

The company, under pressure from app developers and regulators, is making concessions while protecting lucrative parts of its App Store.

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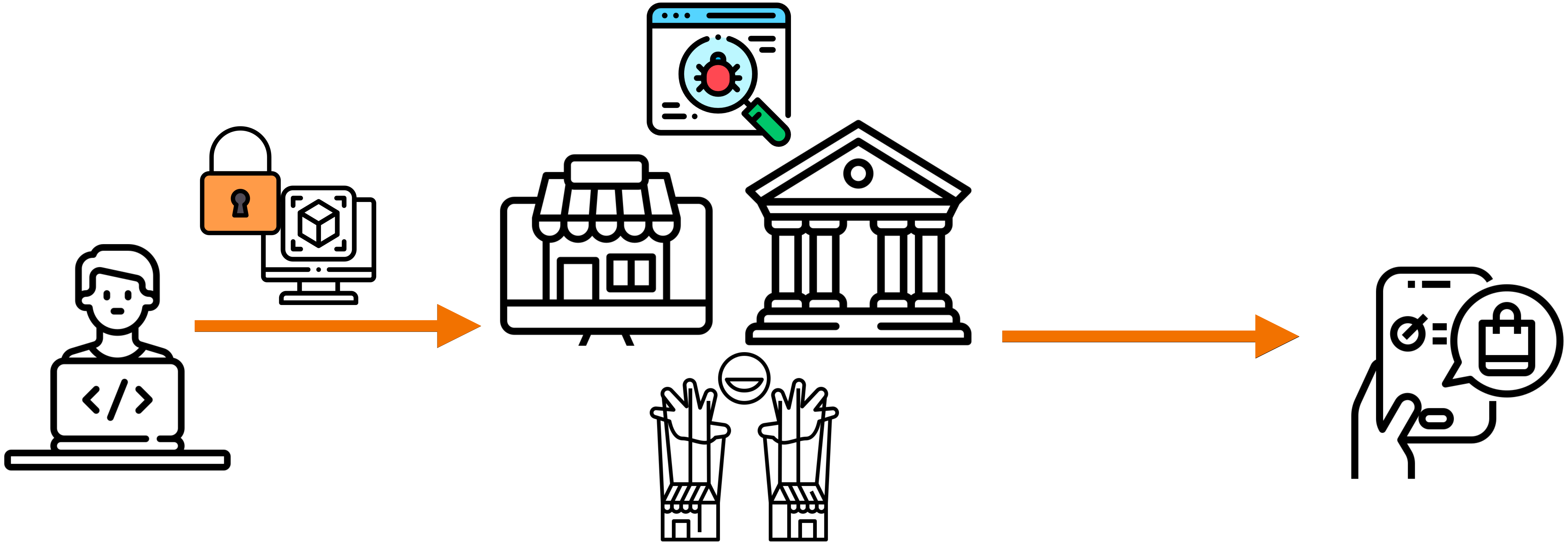
Centralized curation naturally produces a monopoly



THE SEPARATION OF PLATFORMS AND COMMERCE

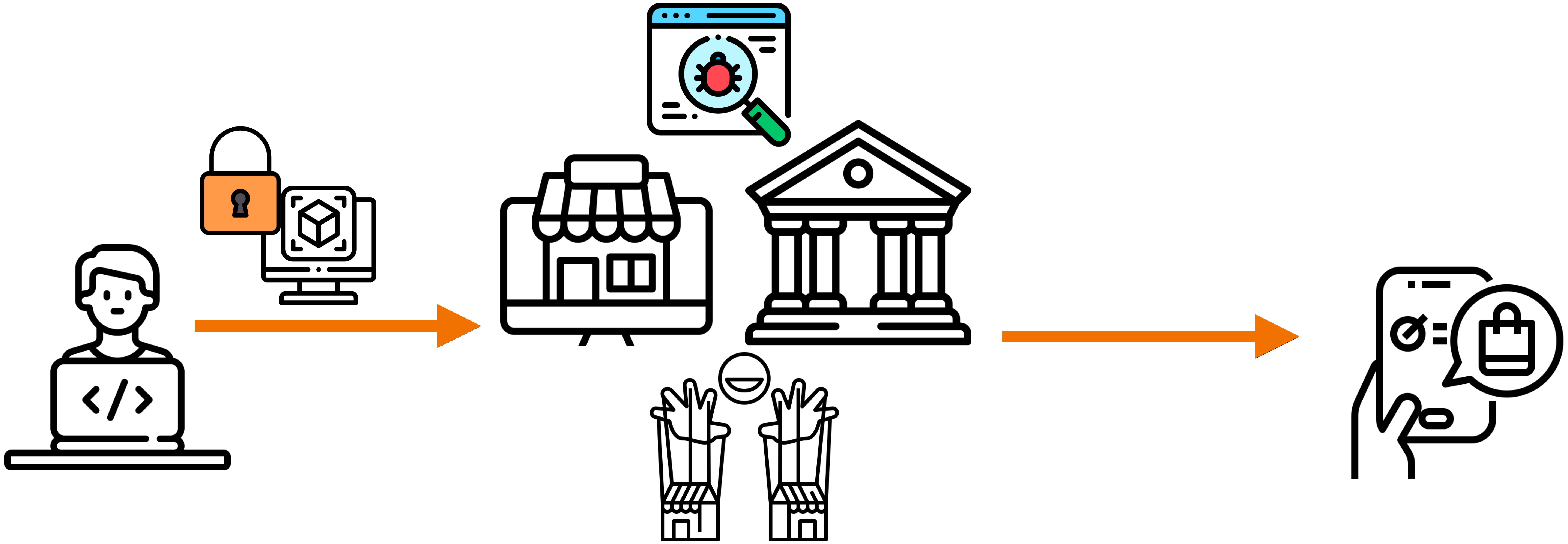
*Lina M. Khan**

A handful of digital platforms mediate a growing share of online commerce and communications. By structuring access to markets, these firms function as gatekeepers for billions of dollars in economic activity. One feature dominant digital platforms share is that they have integrated across business lines such that they both operate a platform and market their own goods and services on it. This structure places dominant platforms in direct competition with some of the businesses that depend on them, creating a conflict of interest that platforms can exploit to further entrench their dominance, thwart competition, and stifle innovation.

Read the Antitrust Lawsuit Against Google

Dozens of States Sue Google Over App Store Fees

Software developers have accused the company of harsh policies and taking a large cut of financial transactions in their apps.



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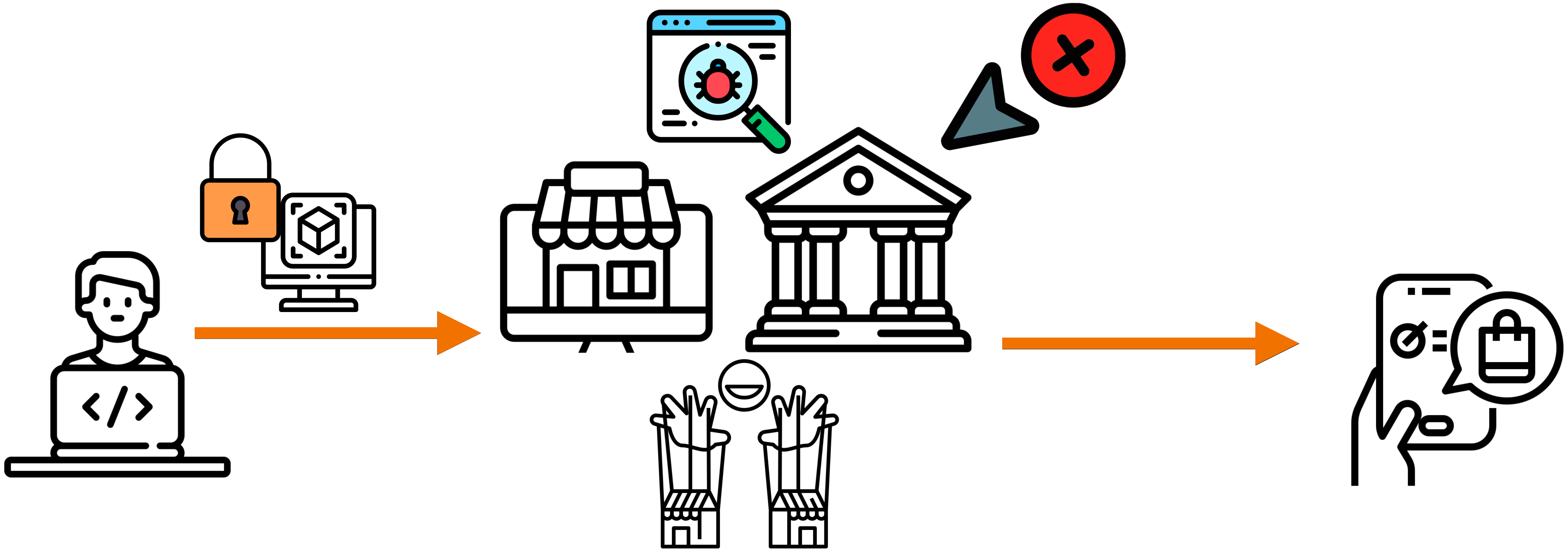
Remove centralized verification and distribution




PPFM
Privacy-Preserving
Formal Methods

ZKUNSAT: The First Step towards PPFM

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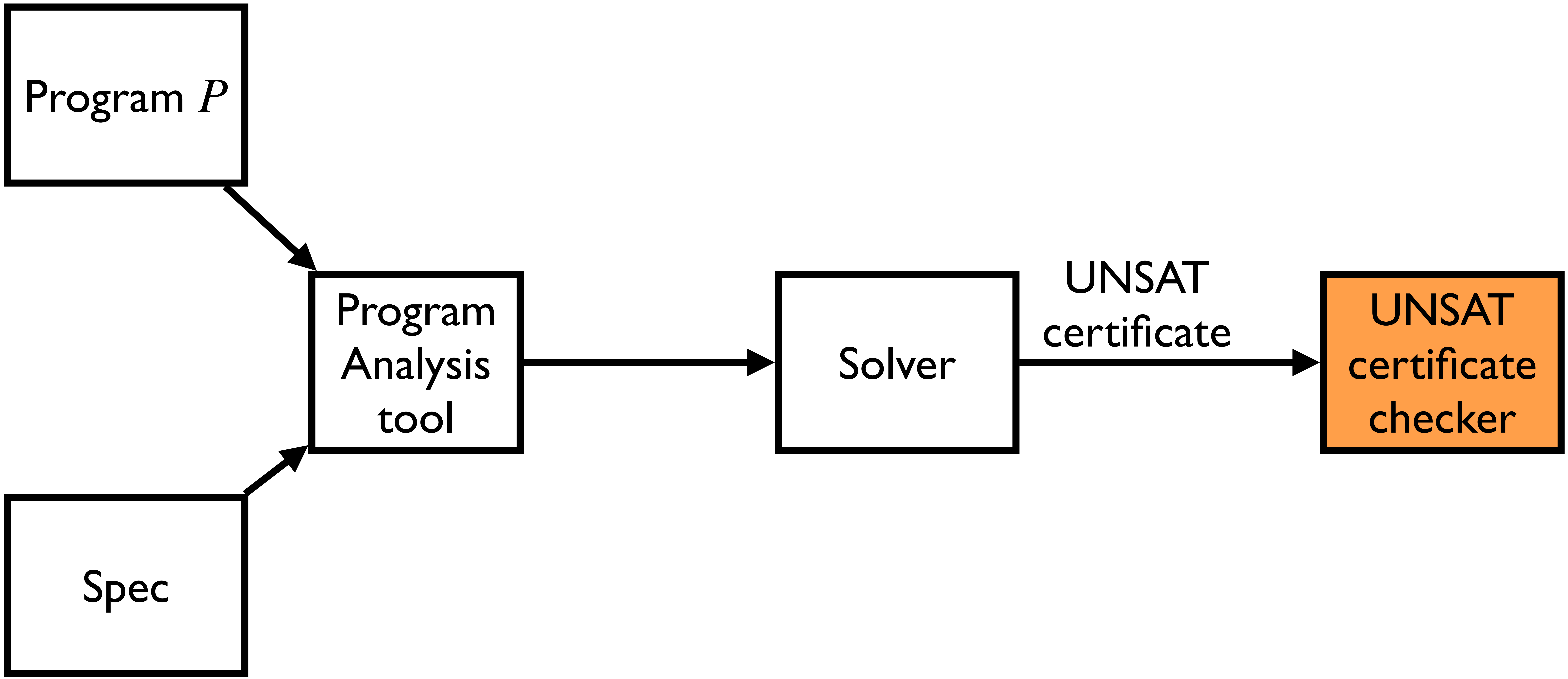
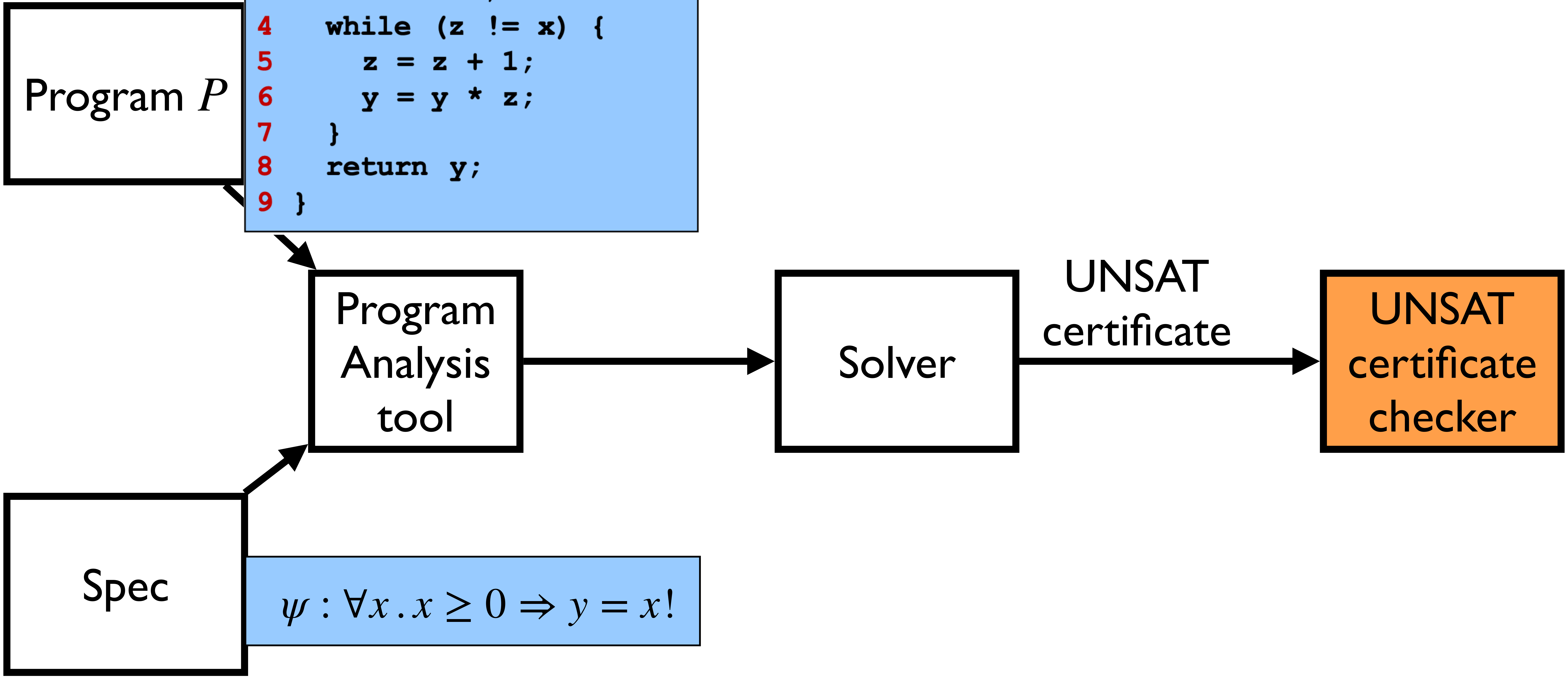


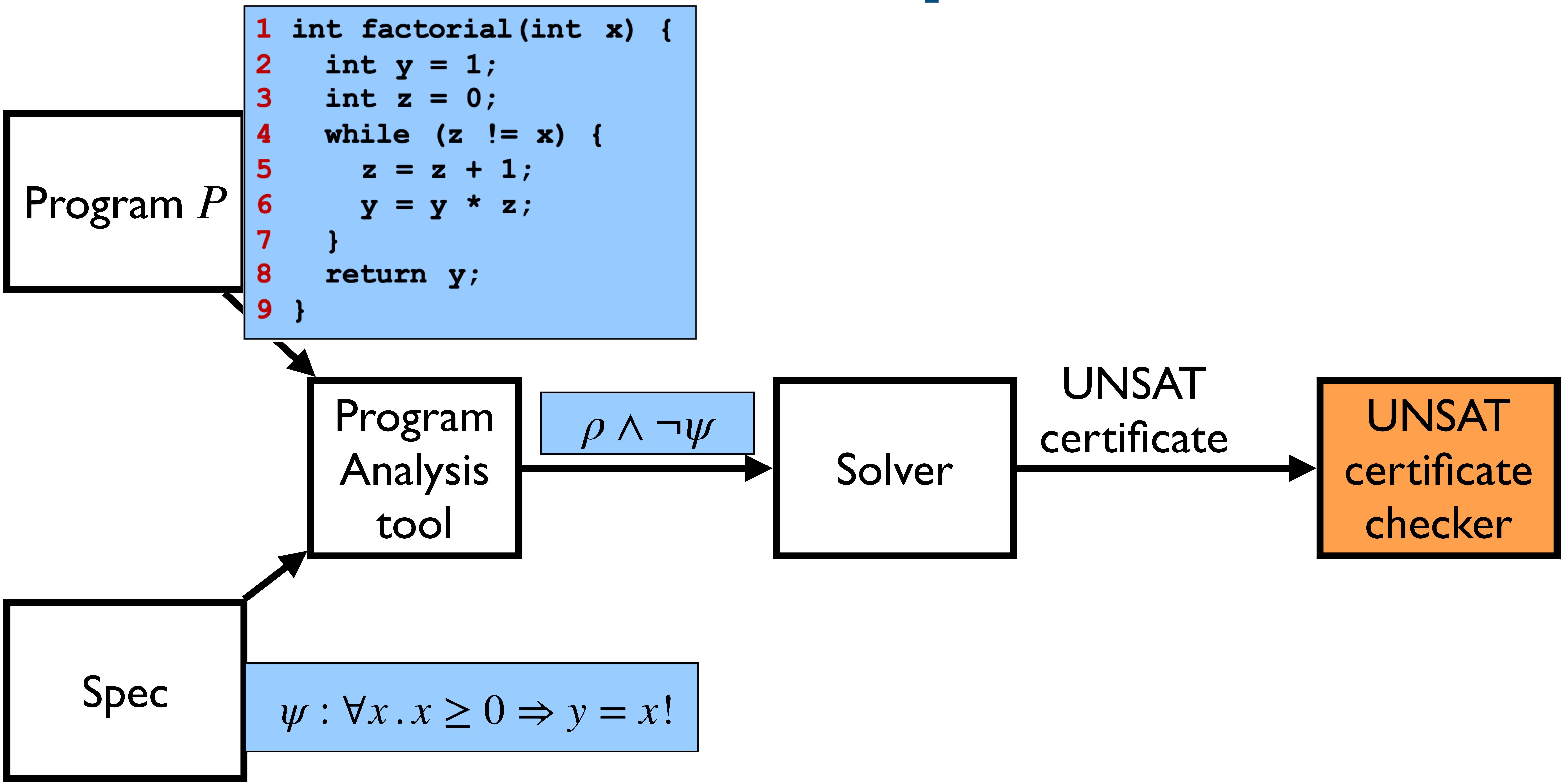
ZKUNSAT: The First Step towards PPFM



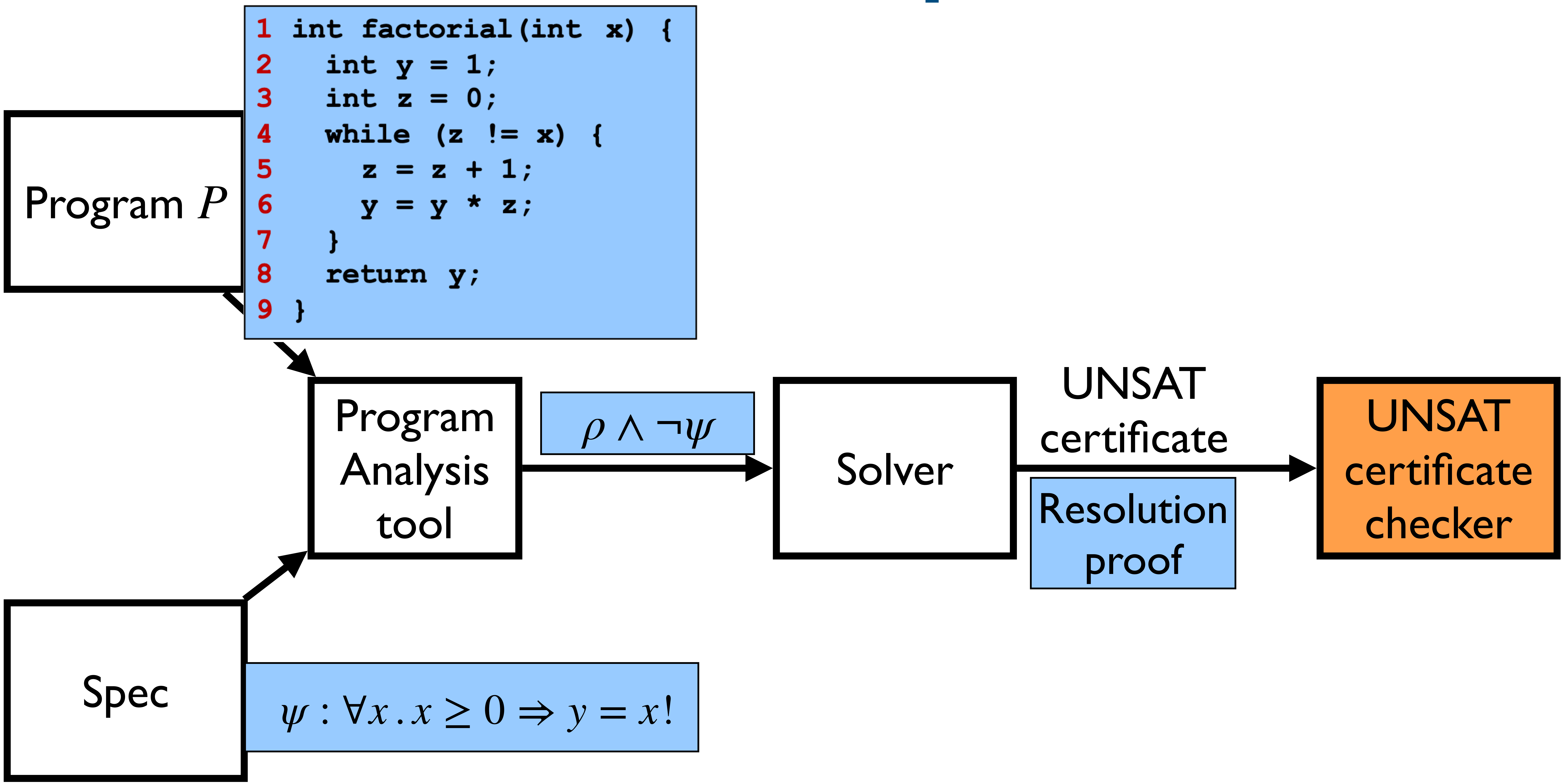

```
int factorial(int x) {
  int y = 1;
  int z = 0;
  while (z != x) {
    z = z + 1;
    y = y * z;
  }
  return y;
}
```

$$\psi : \forall x \,.\, x \geq 0 \Rightarrow y = x!$$

ZKUNSAT: The First Step towards PPFM



ZKUNSAT: The First Step towards PPFM



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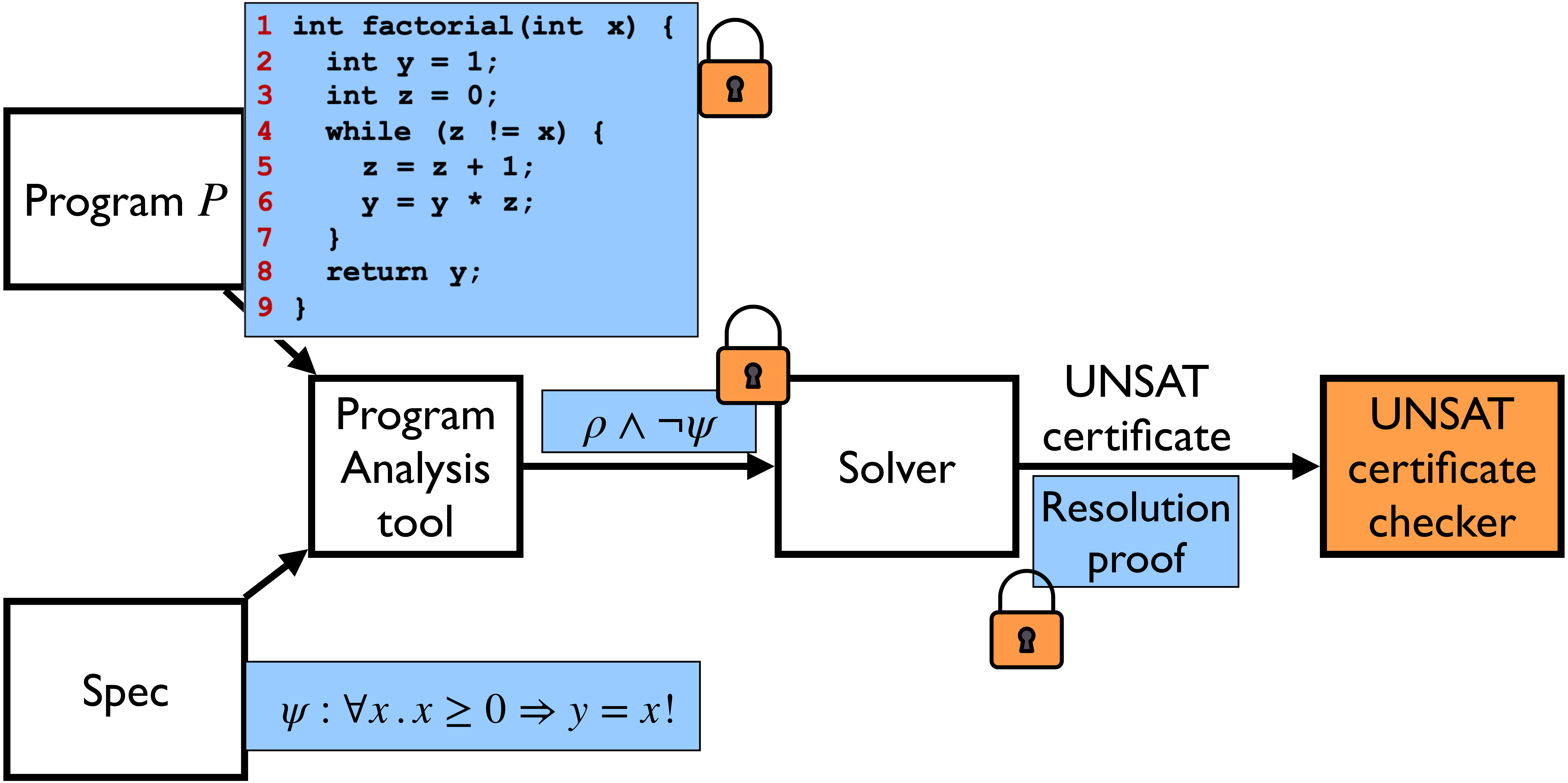

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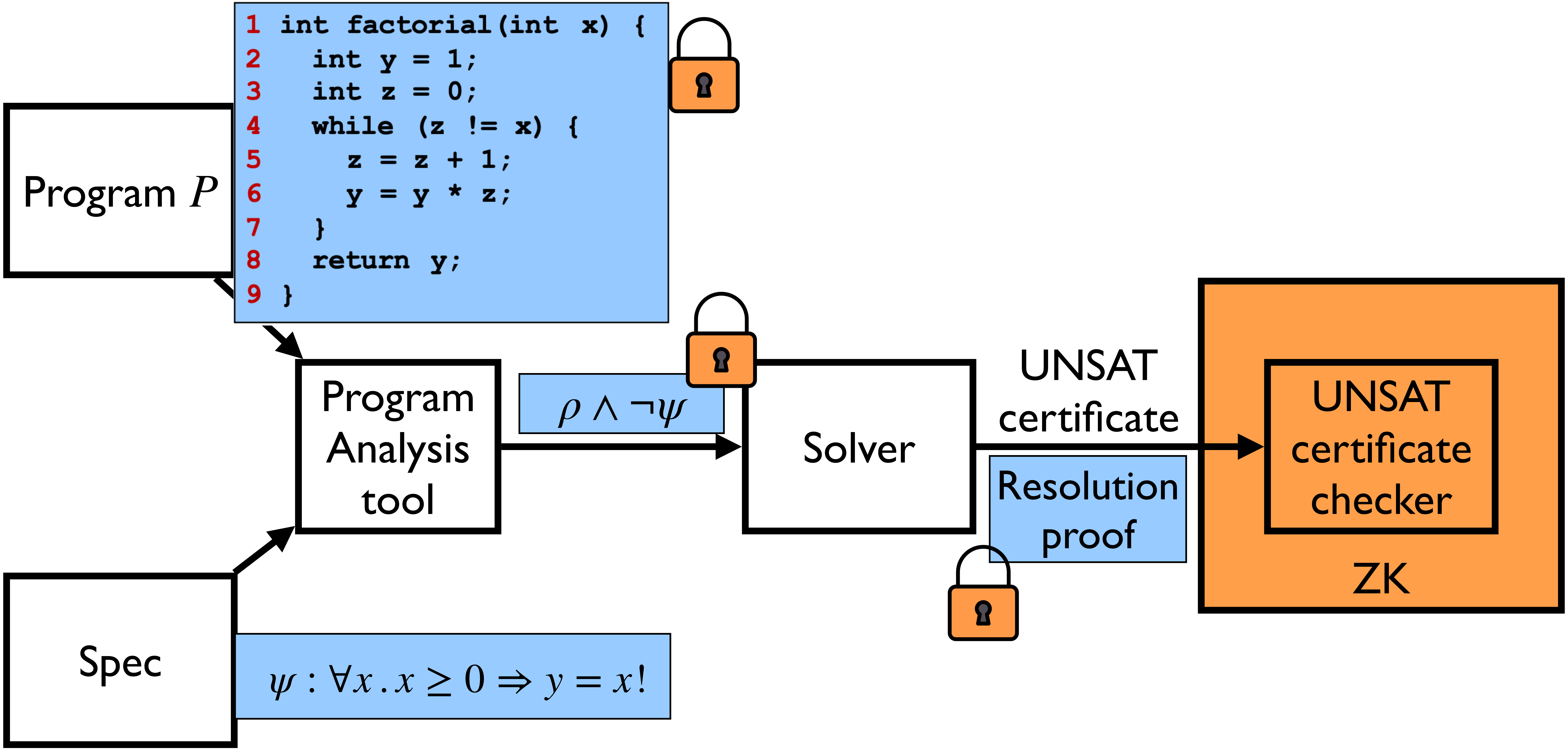
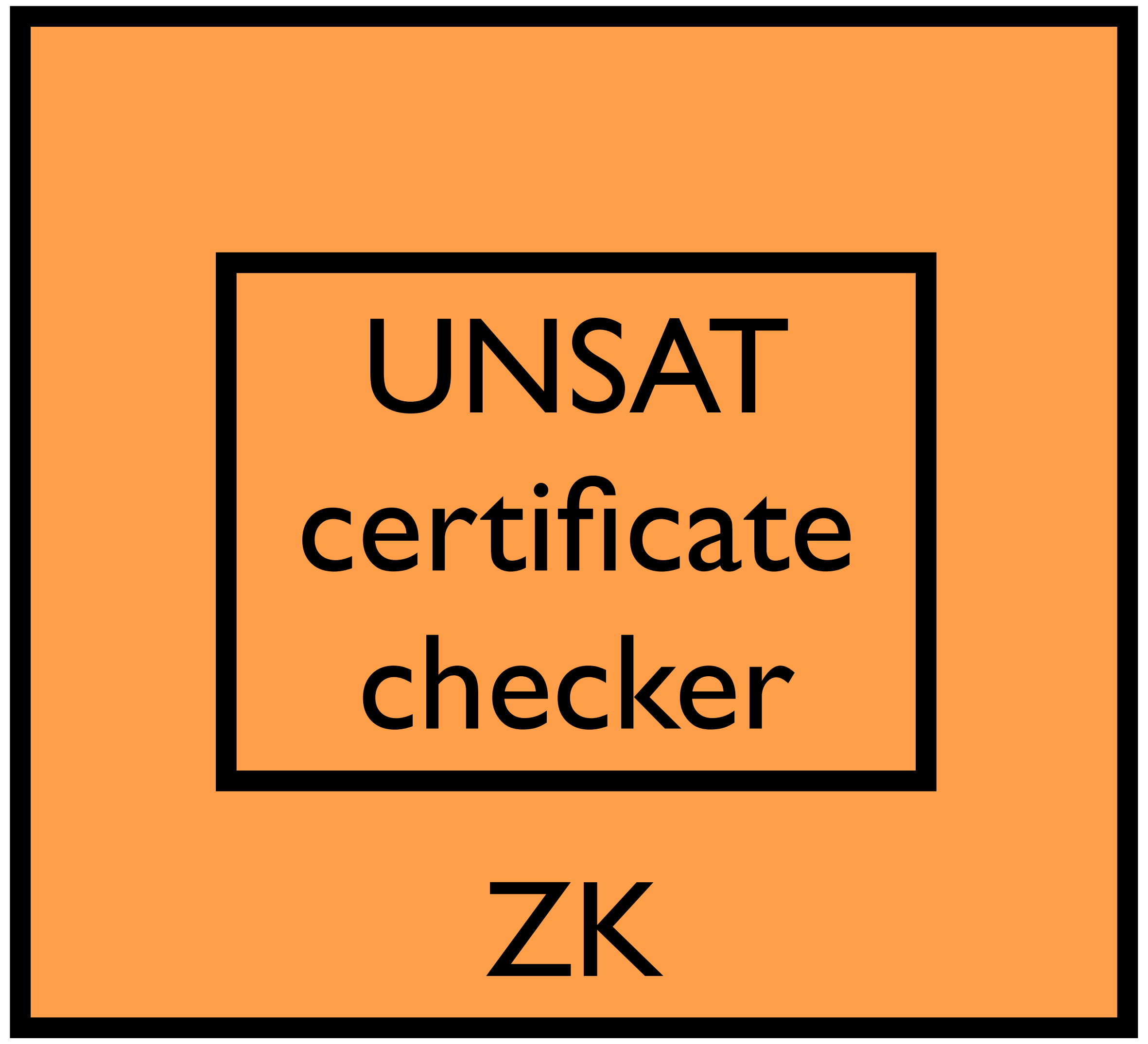
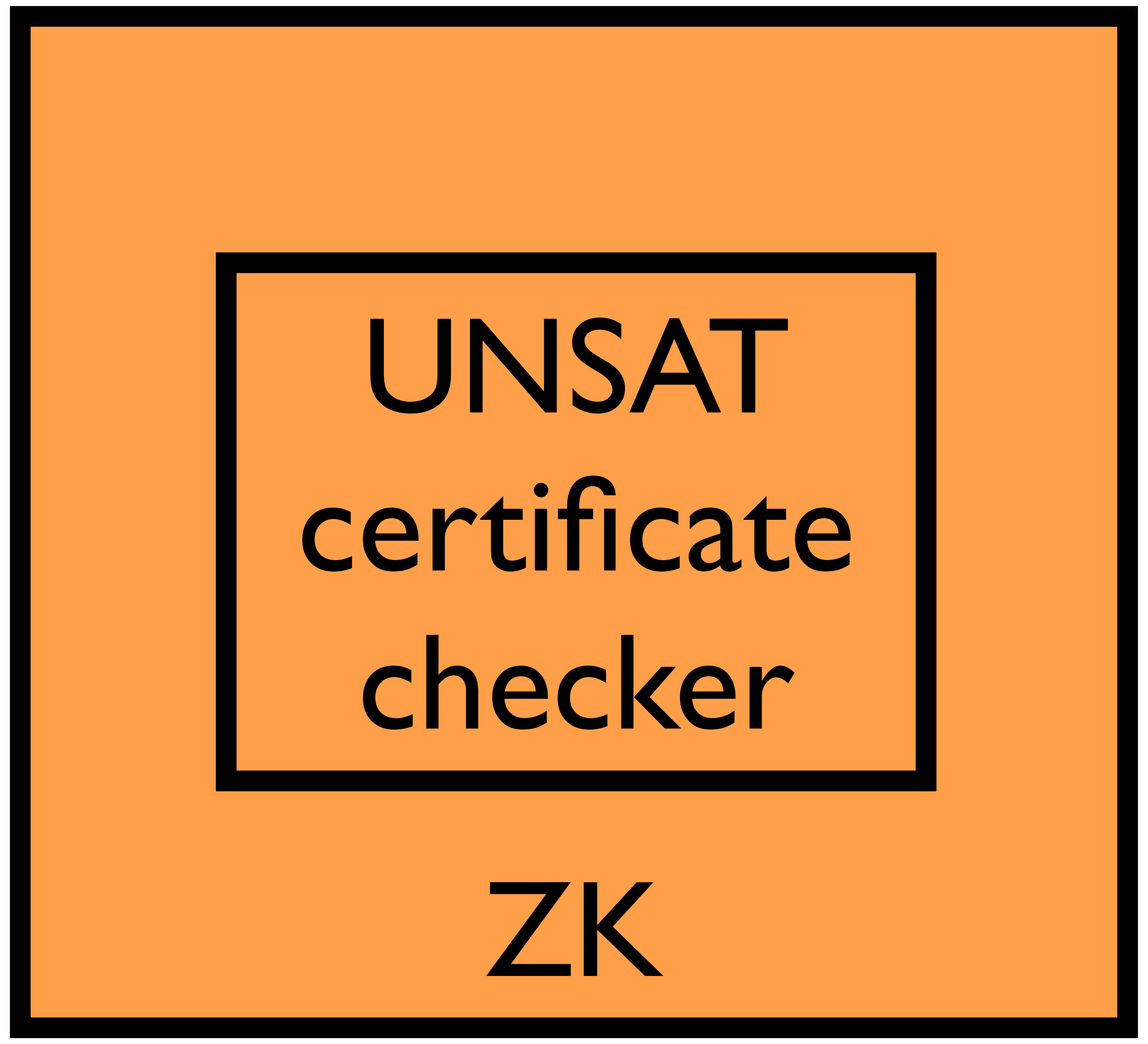


The First Step towards Decentralized Verification



~~**generic implementation of the certificate checker in ZK**~~

The First Step towards Decentralized Verification



The First Step towards Decentralized Verification

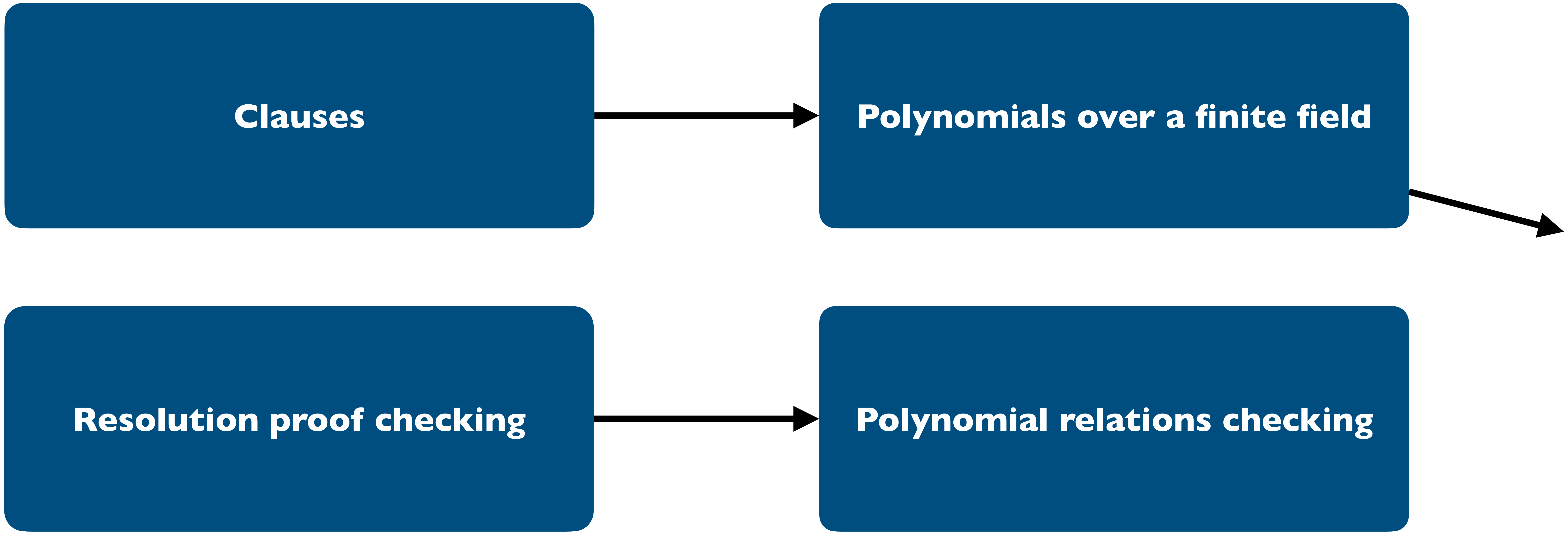
Scales to real-world verification tasks including these for Linux+Windows drivers

The First Step towards Decentralized Verification

Clauses → Polynomials over a finite field

Resolution proof checking → Polynomial relations checking

The First Step towards Decentralized Verification



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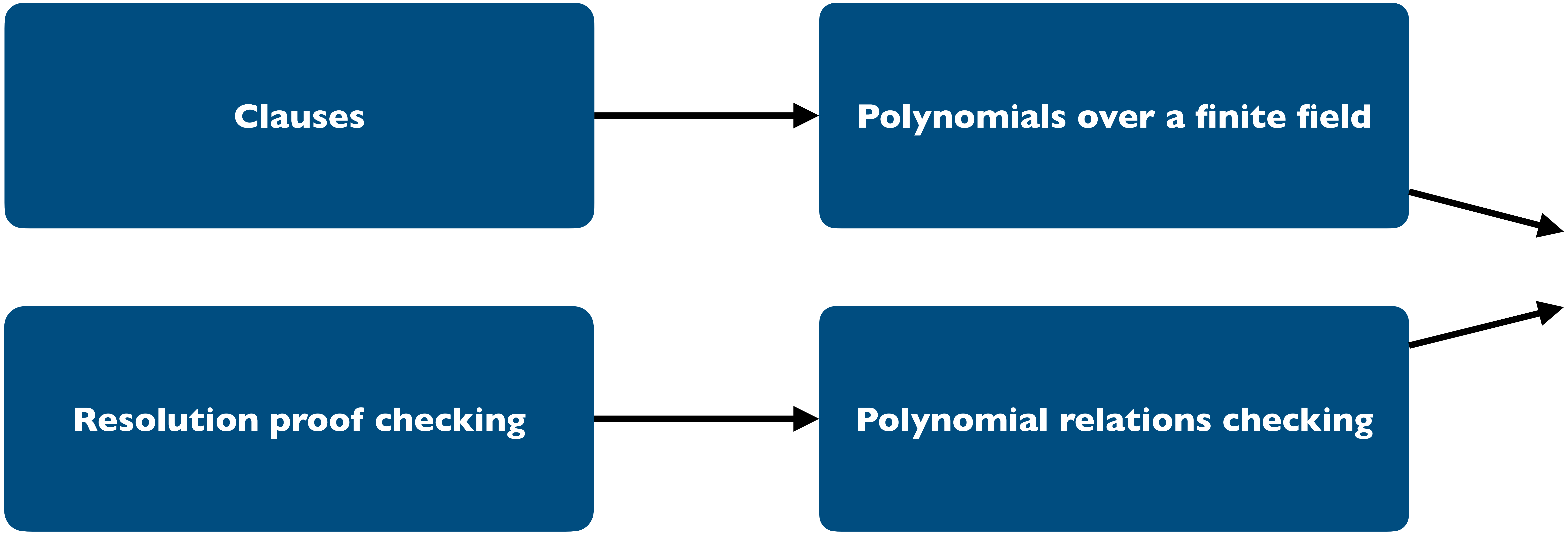

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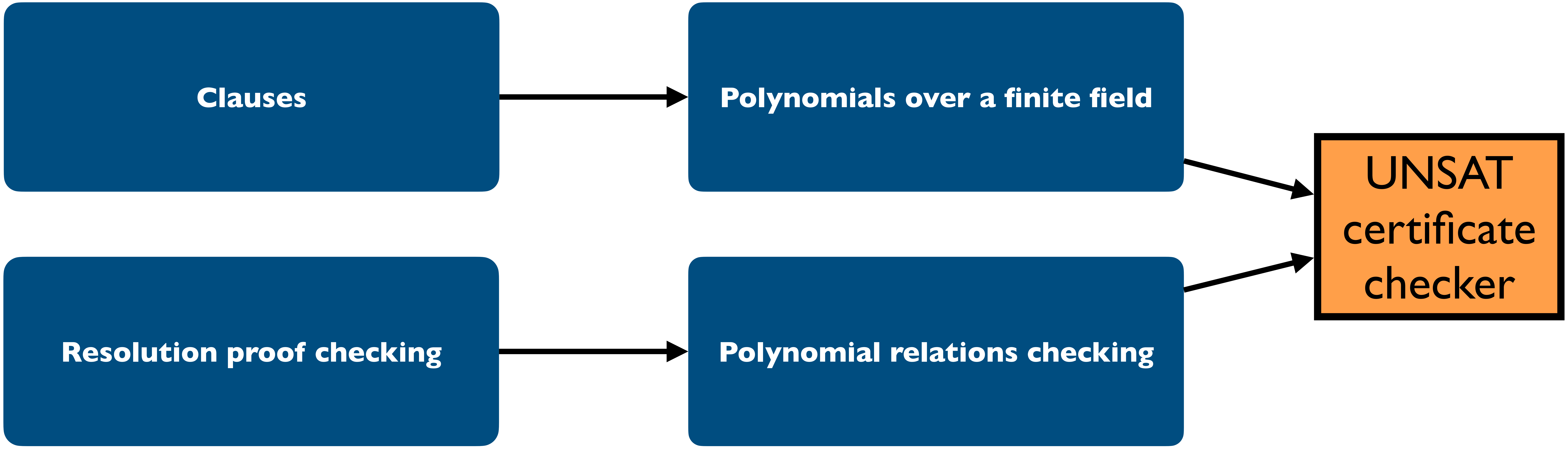

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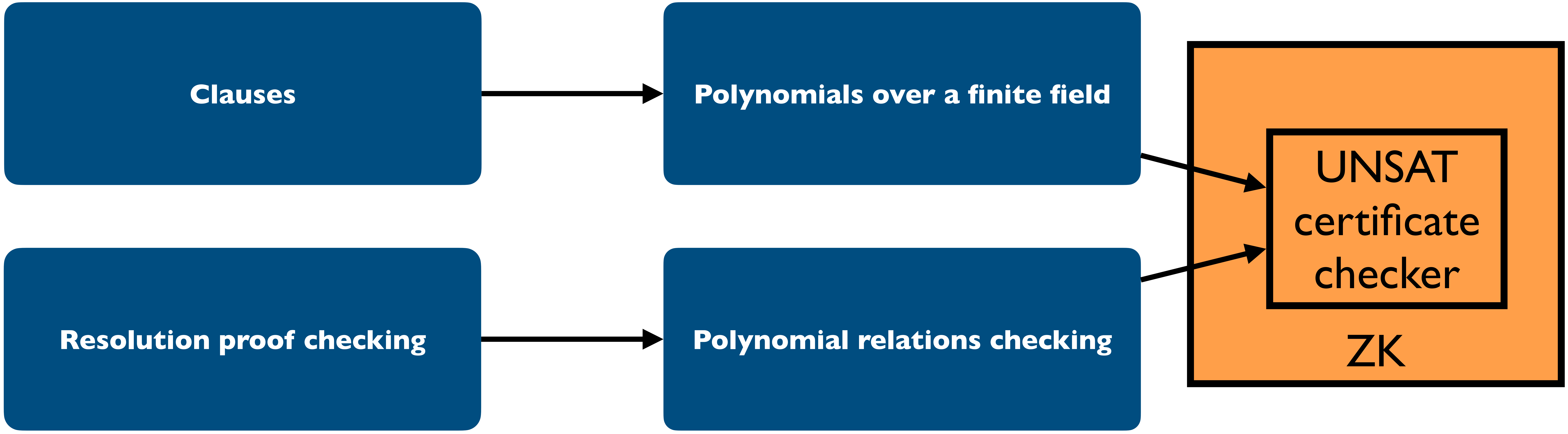
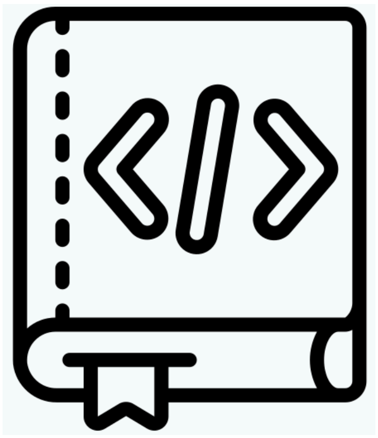


Efficient library for checking relations between polynomials in zero knowledge



Outline

- **Background**
  - Resolution Proof
  - Zero knowledge proof
- **ZKUNSAT**
  - Clause encoding and validating resolvents
  - Clause access and checking consistency
  - Evaluation
- **Future Work**

Refutation Proof

Resolution Rule [Robinson, 65]

$$\frac{c_a = x_1 \vee x_2, \quad c_b = x_3 \vee \neg x_2}{c_r = x_1 \vee x_3}$$

Refutation Proof

Resolution Rule [Robinson, 65]

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Refutation Proof

Resolution Rule [Robinson, 65]

$$\frac{c_a = x_1 \vee \cancel{x_2}, \quad c_b = x_3 \vee \cancel{\neg x_2}}{c_r = x_1 \vee x_3}$$

Refutation Proof

Resolution Proof in Propositional Logic

$$f = c_0 \wedge c_1 \wedge c_2 \wedge c_3$$

$c_0 : (x_1 \vee x_2)$		$-,-$
$c_1 : (\neg x_1 \vee x_2)$		$-,-$
$c_2 : (\neg x_1 \vee \neg x_2)$		$-,-$
$c_3 : (x_1 \vee \neg x_2)$		$-,-$

Refutation Proof

Resolution Proof in Propositional Logic

$$f = c_0 \land c_1 \land c_2 \land c_3$$

$c_0 : (x_1 \lor x_2)$	$-,-$
$c_1 : (\neg x_1 \lor x_2)$	$-,-$
$c_2 : (\neg x_1 \lor \neg x_2)$	$-,-$
$c_3 : (x_1 \lor \neg x_2)$	$-,-$

Theorem: first-order logic is refutationally complete.

- The proof of the theorem relies on the resolution proof of unsatisfiability.
- If a formula is UNSAT, we can always derive $\perp$ using resolution

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-, -$
$c_1 : (\neg x_1 \vee x_2)$	$-, -$
$c_2 : (\neg x_1 \vee \neg x_2)$	$-, -$
$c_3 : (x_1 \vee \neg x_2)$	$-, -$
$c_4 : x_2$	$0, 1$
$c_5 : \neg x_2$	$2, 3$
$c_6 : \perp$	$4, 5$

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-,-$
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Refutation Proof

Resolution Proof in Propositional Logic

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Theorem: first-order logic is refutationally complete.

- A resolution proof consists of a list of

Refutation Proof

Resolution Proof in Propositional Logic

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Theorem: first-order logic is refutationally complete.

- A resolution proof consists of a list of
  - Resolvents

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-, -$
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Theorem: first-order logic is refutationally complete.

- A resolution proof consists of a list of
  - Resolvents
  - How these resolvents could be obtained.

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-, -$
$c_1 : (\neg x_1 \vee x_2)$	$-, -$
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$c_4 : x_2$	$0, 1$
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$$\frac{x_1 \vee x_2, \quad \neg x_1 \vee x_2}{x_2}$$

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-,-$
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Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \lor x_2)$	$-, -$
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$c_6 : \bot$	$4, 5$

$$\frac{x_1 \lor x_2, \quad \neg x_1 \lor x_2}{x_2}$$

Refutation Proof

Resolution Proof in Propositional Logic

$\boxed{c_0 : (x_1 \vee x_2)}$	$\boxed{-,-}$
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$c_2 : (\neg x_1 \vee \neg x_2)$	$-,-$
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Refutation Proof

Resolution Proof in Propositional Logic

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$$\frac{x_1 \lor x_2, \quad \neg x_1 \lor x_2}{x_2}$$

- Fetch two clauses used to derived the resolvent

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-, -$
$c_1 : (\neg x_1 \vee x_2)$	$-, -$
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Refutation Proof

Resolution Proof in Propositional Logic

c_0 :	$(x_1 \lor x_2)$	$-,-$
c_1 :	$(\neg x_1 \lor x_2)$	$-,-$
c_2 :	$(\neg x_1 \lor \neg x_2)$	$-,-$
c_3 :	$(x_1 \lor \neg x_2)$	$-,-$
c_4 :	x_2	$0, 1$ ✓
c_5 :	$\neg x_2$	$2, 3$
c_6 :	$\perp$	$4, 5$

$$\frac{x_1 \lor x_2, \quad \neg x_1 \lor x_2}{x_2}$$

- Fetch two clauses used to derived the resolvent

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-, -$
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$c_6 : \bot$	$4, 5$

✓

$$\frac{x_1 \vee x_2, \quad \neg x_1 \vee x_2}{x_2}$$

- Fetch two clauses used to derived the resolvent
- Check the application of resolution rule is correctly executed

Refutation Proof

Resolution Proof in Propositional Logic

$c_0 : (x_1 \vee x_2)$	$-,-$
$c_1 : (\neg x_1 \vee x_2)$	$-,-$
$c_2 : (\neg x_1 \vee \neg x_2)$	$-,-$
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Repeat for each resolvent until meet a contradiction

Refutation Proof

Resolution Proof in Propositional Logic

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$c_4 : x_2$	$0, 1$ ✓
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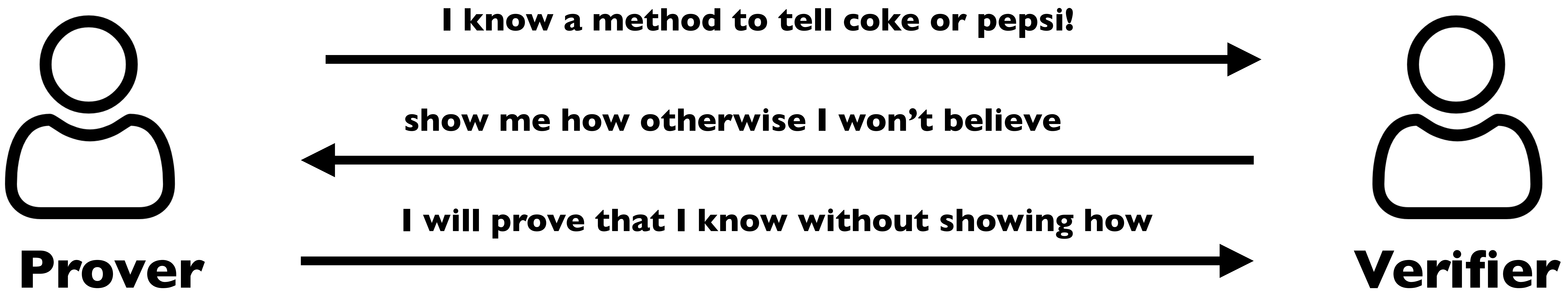
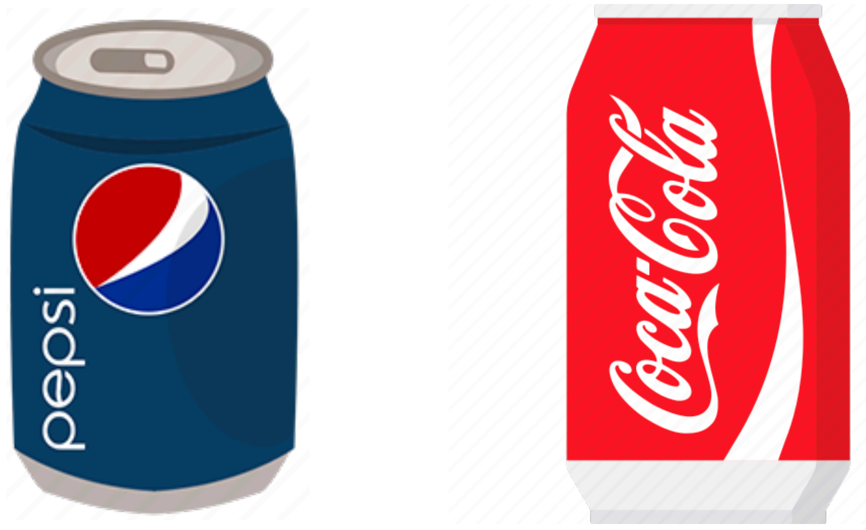
$c_6 : \perp$	$4, 5$ ✓

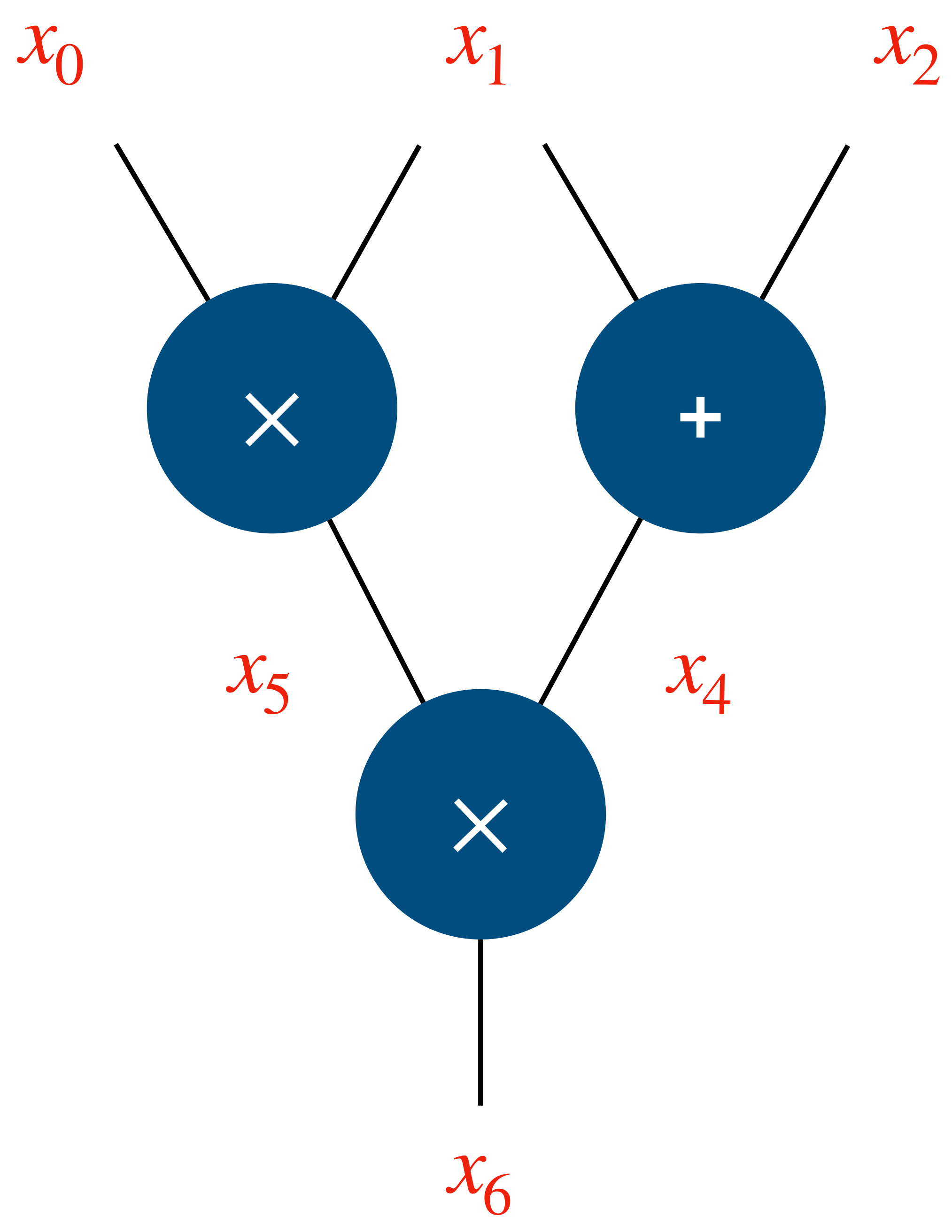
Repeat for each resolvent until meet a contradiction

Zero Knowledge Proof : Coke or Pepsi





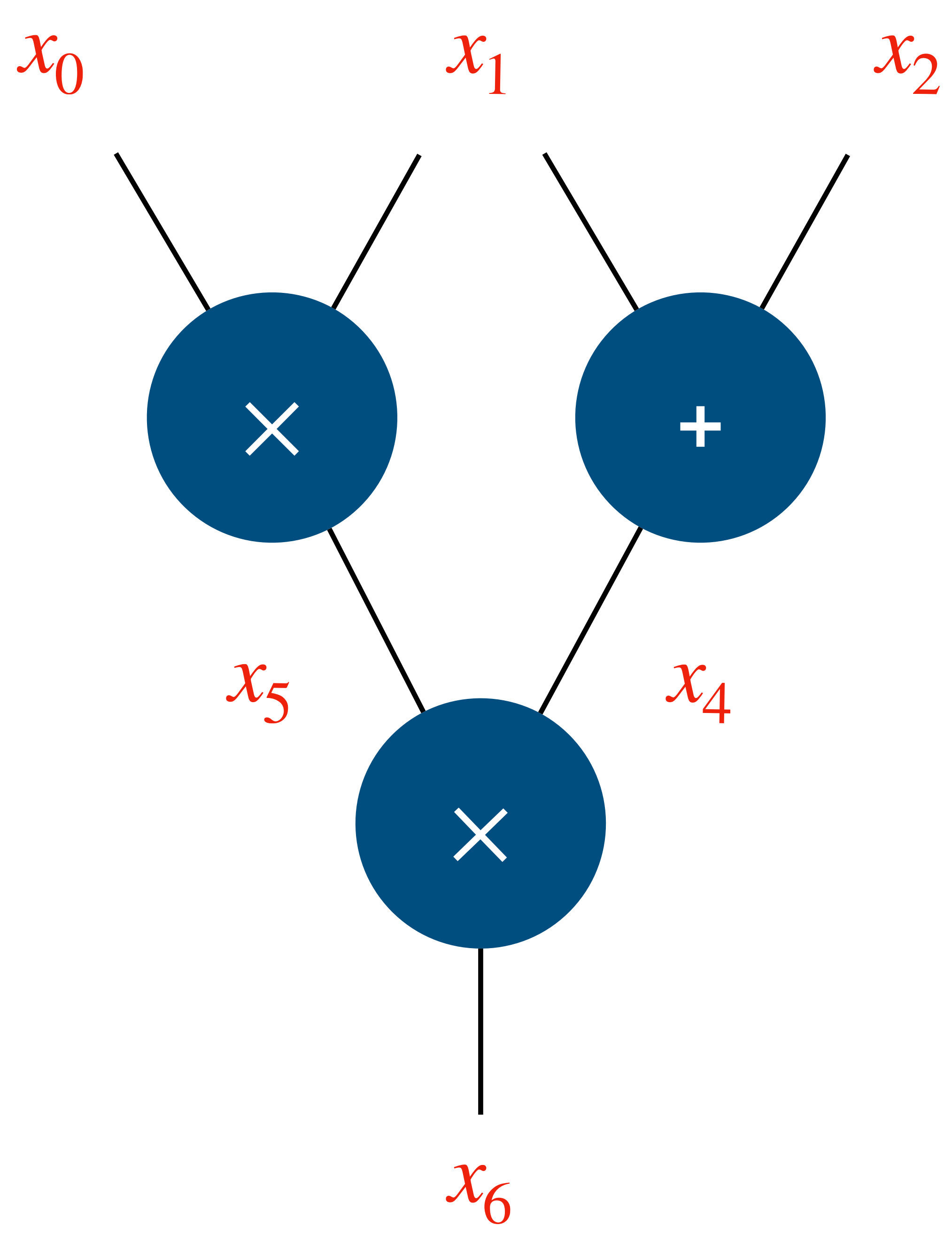
Zero Knowledge Proof



A public circuit C

Zero Knowledge Proof

- **Prover** knows an input w such that $C(w) = 1$, and tries to
  - Convince the verifier that $C(w) = 1$
  - Keep information of w private



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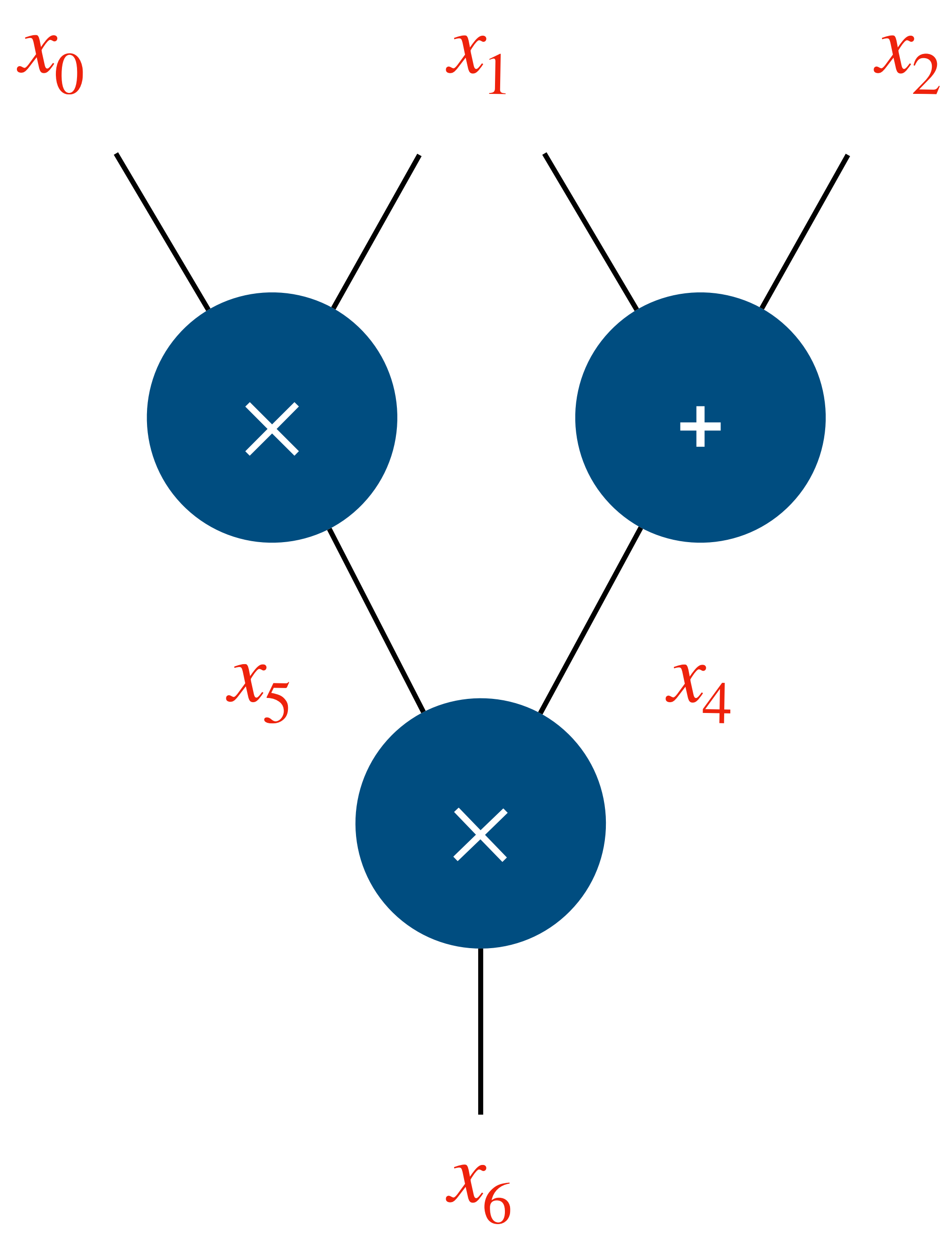

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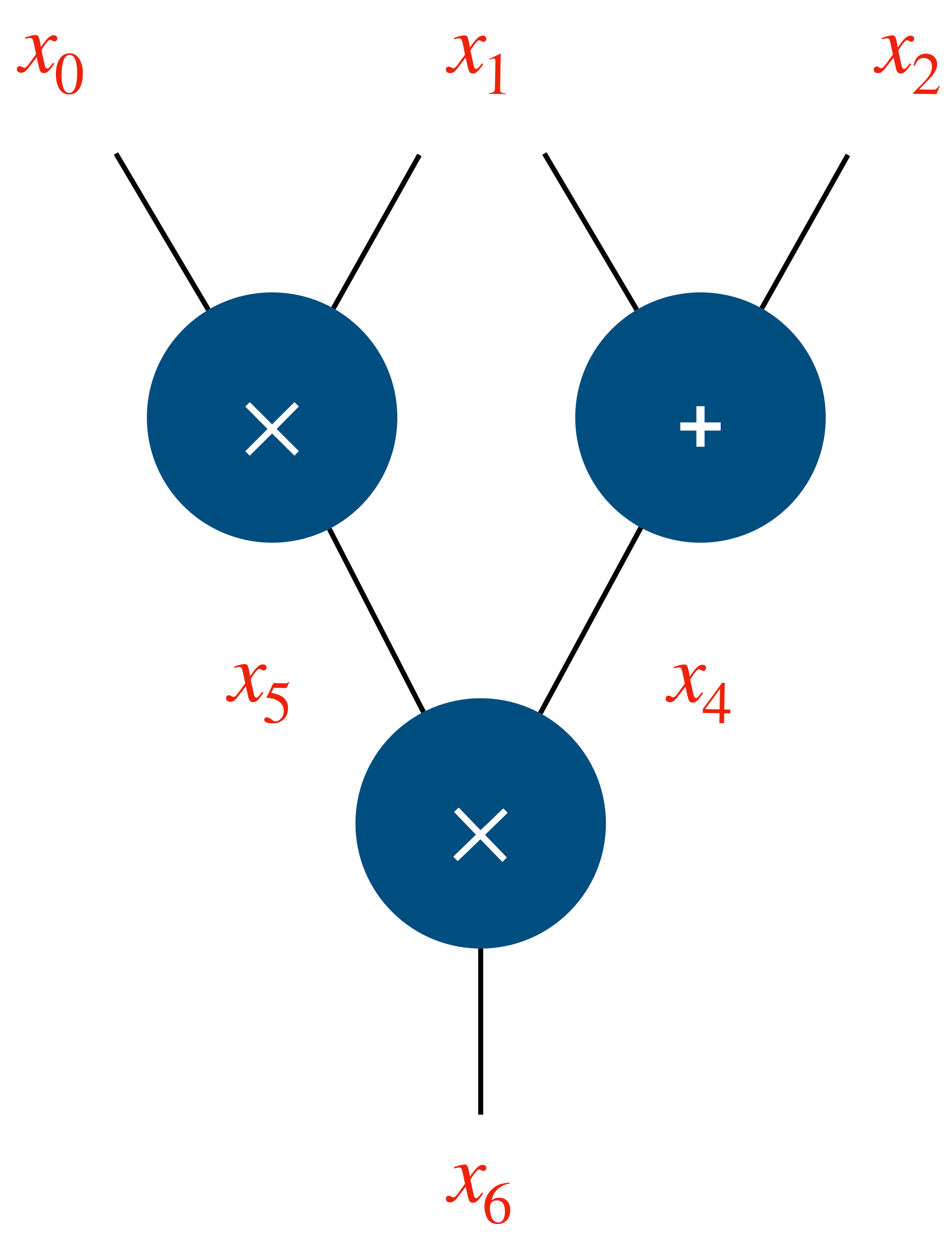

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Both of prover and verifier can be malicious.

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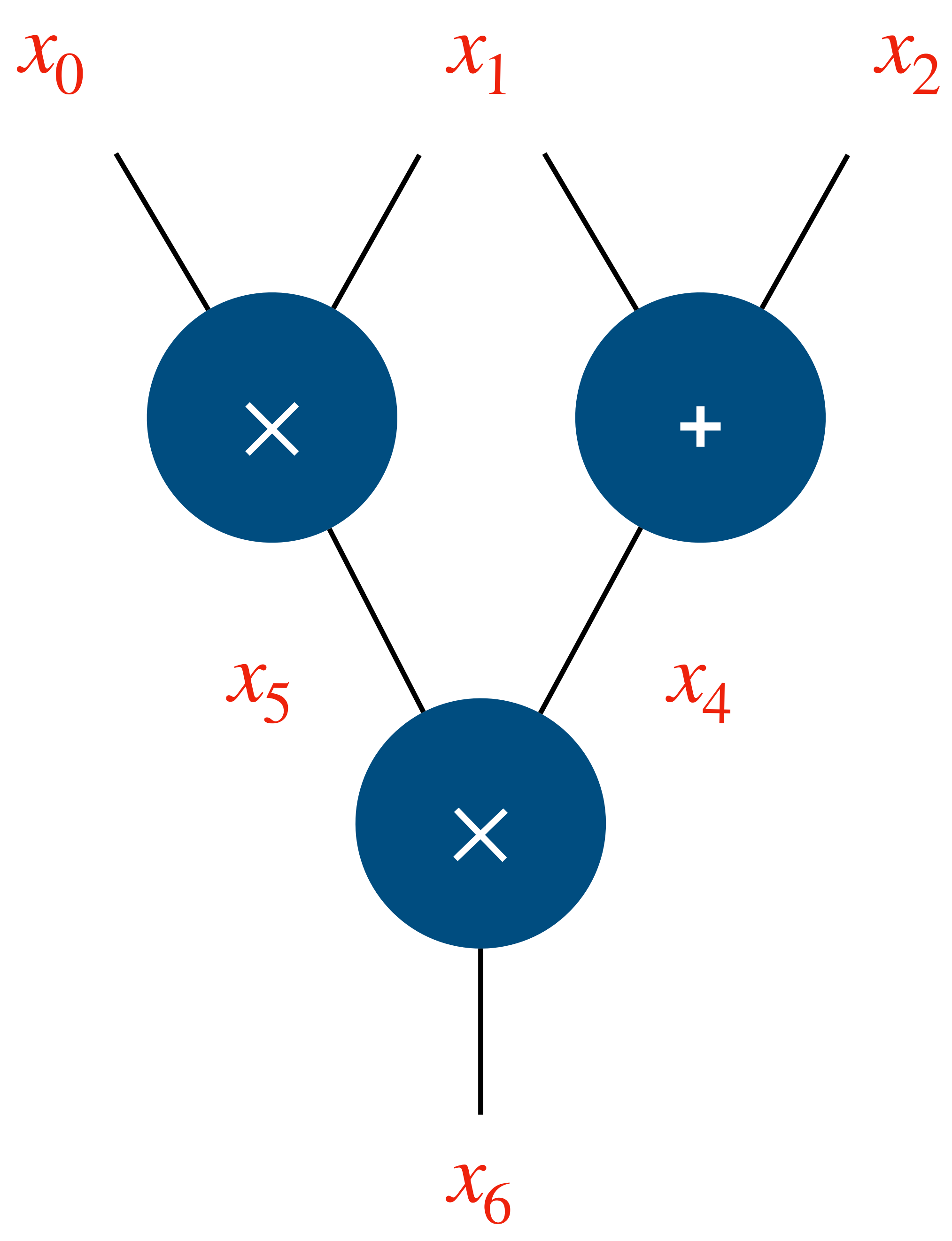

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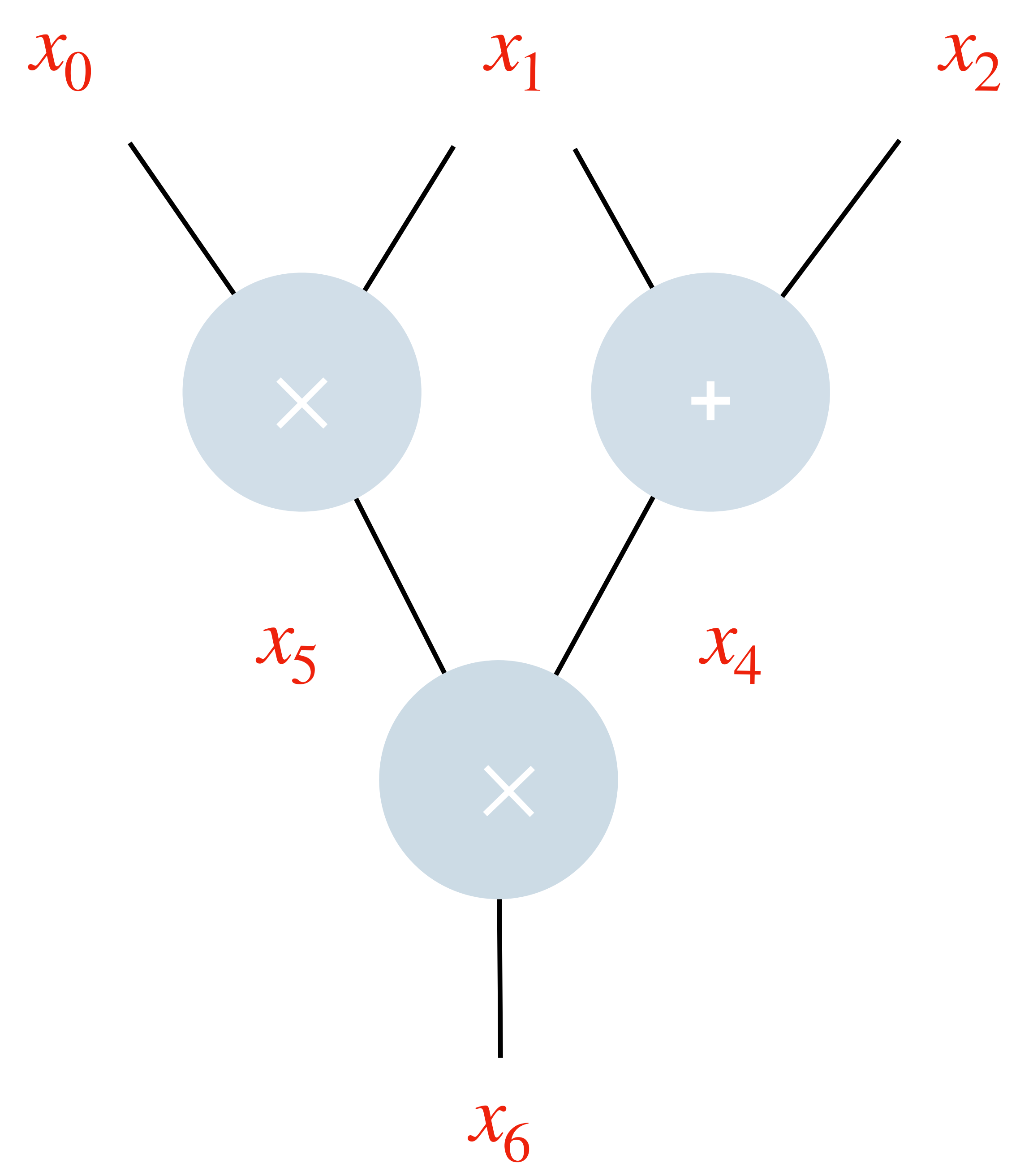

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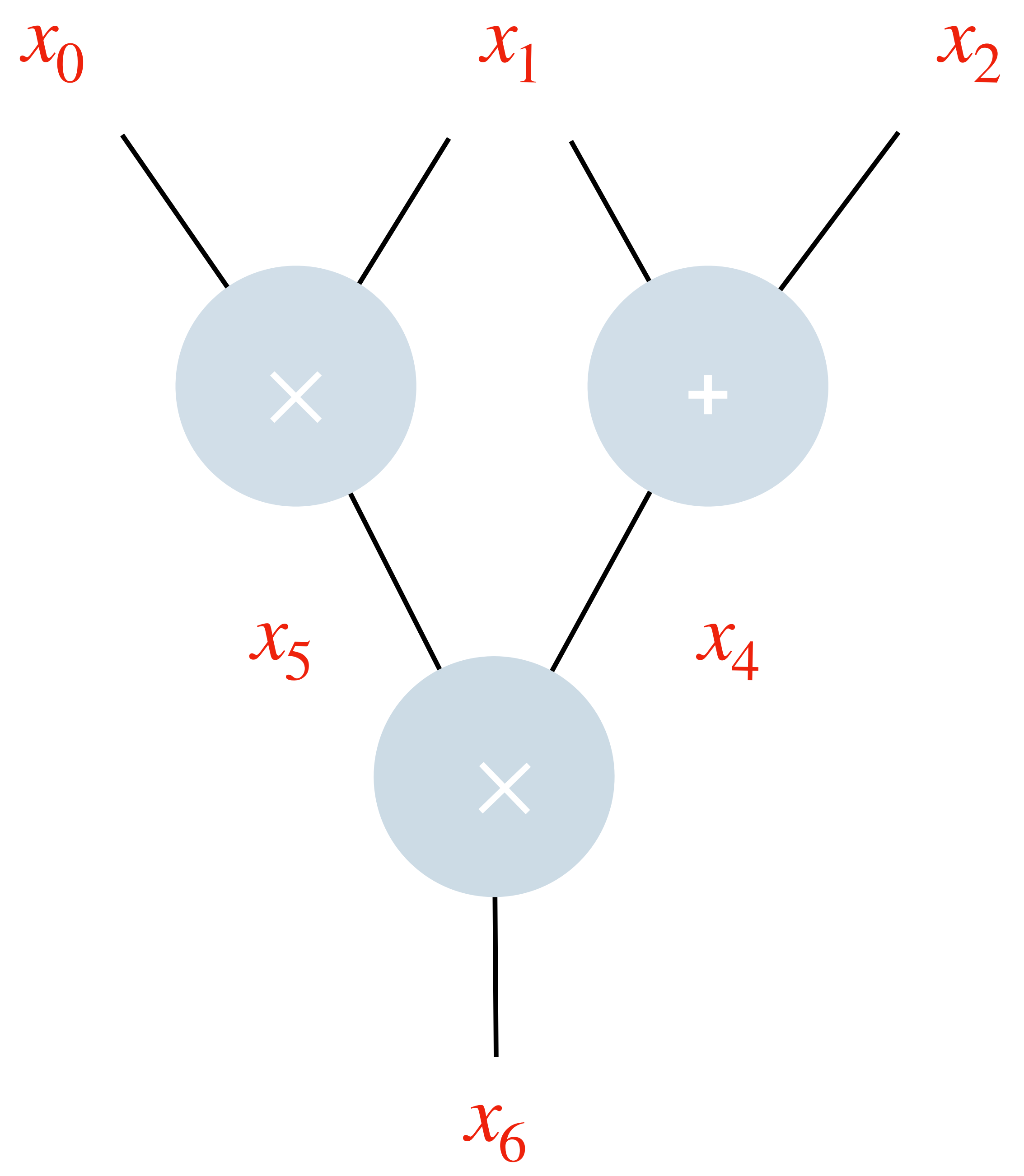

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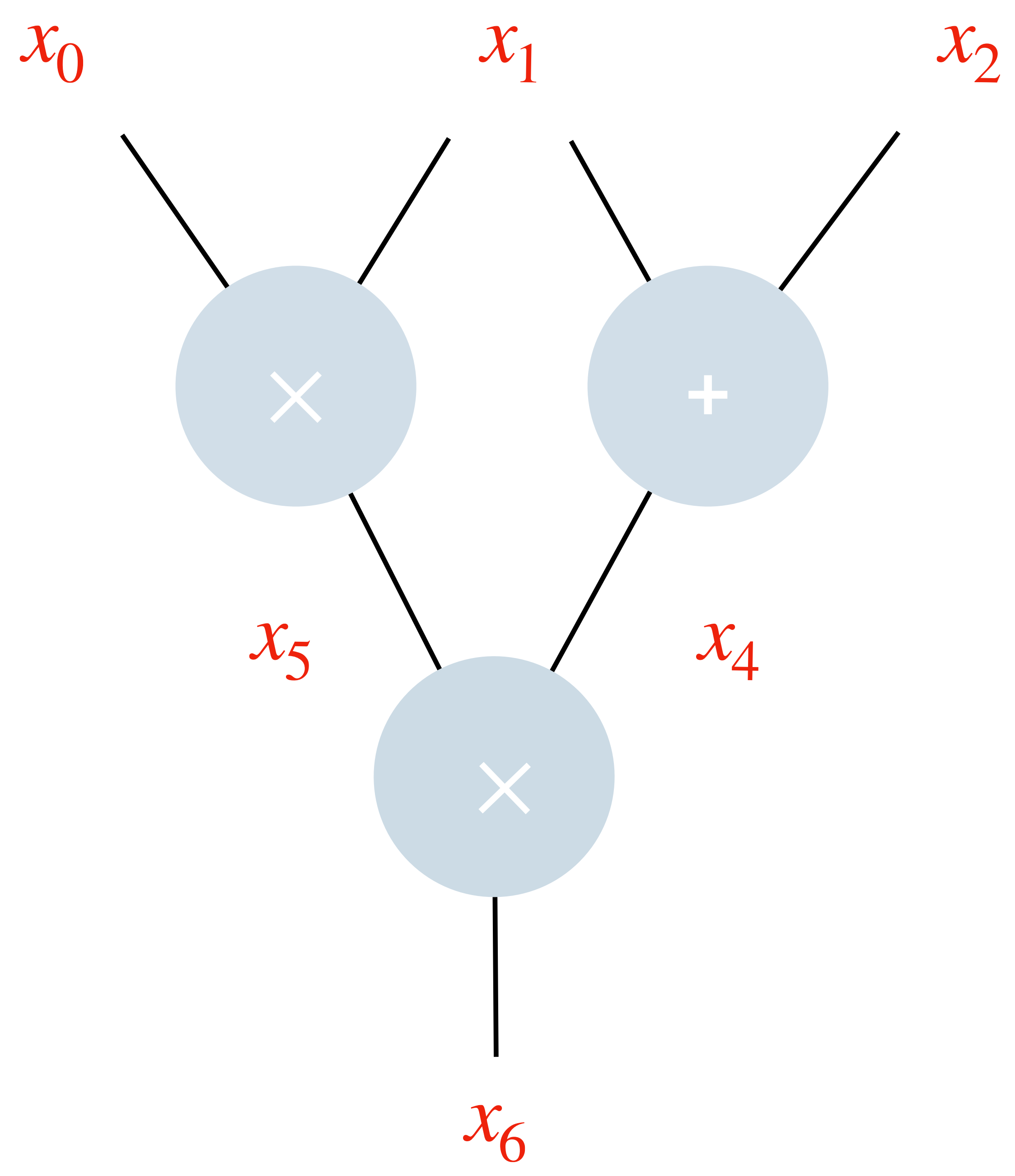

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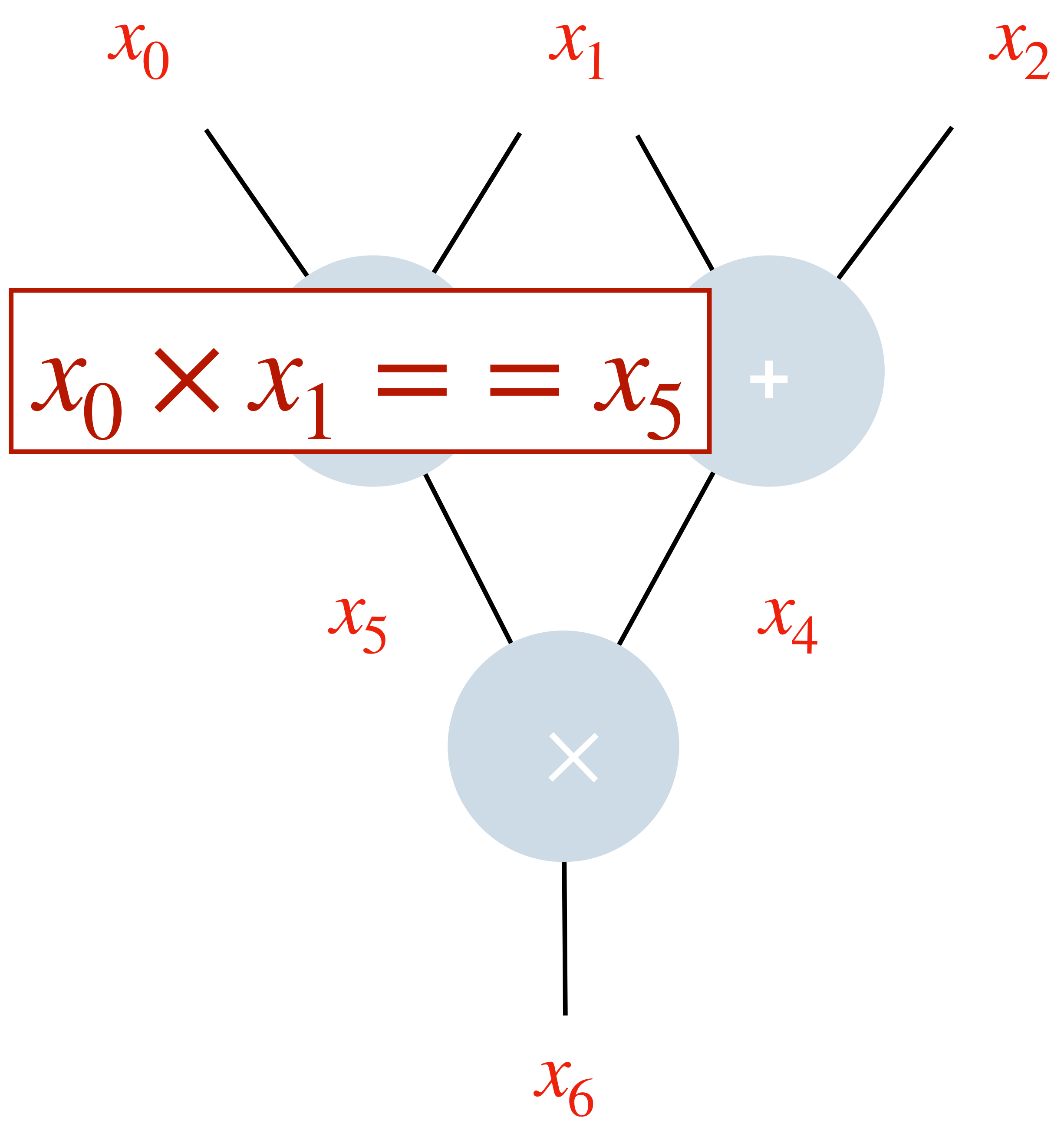

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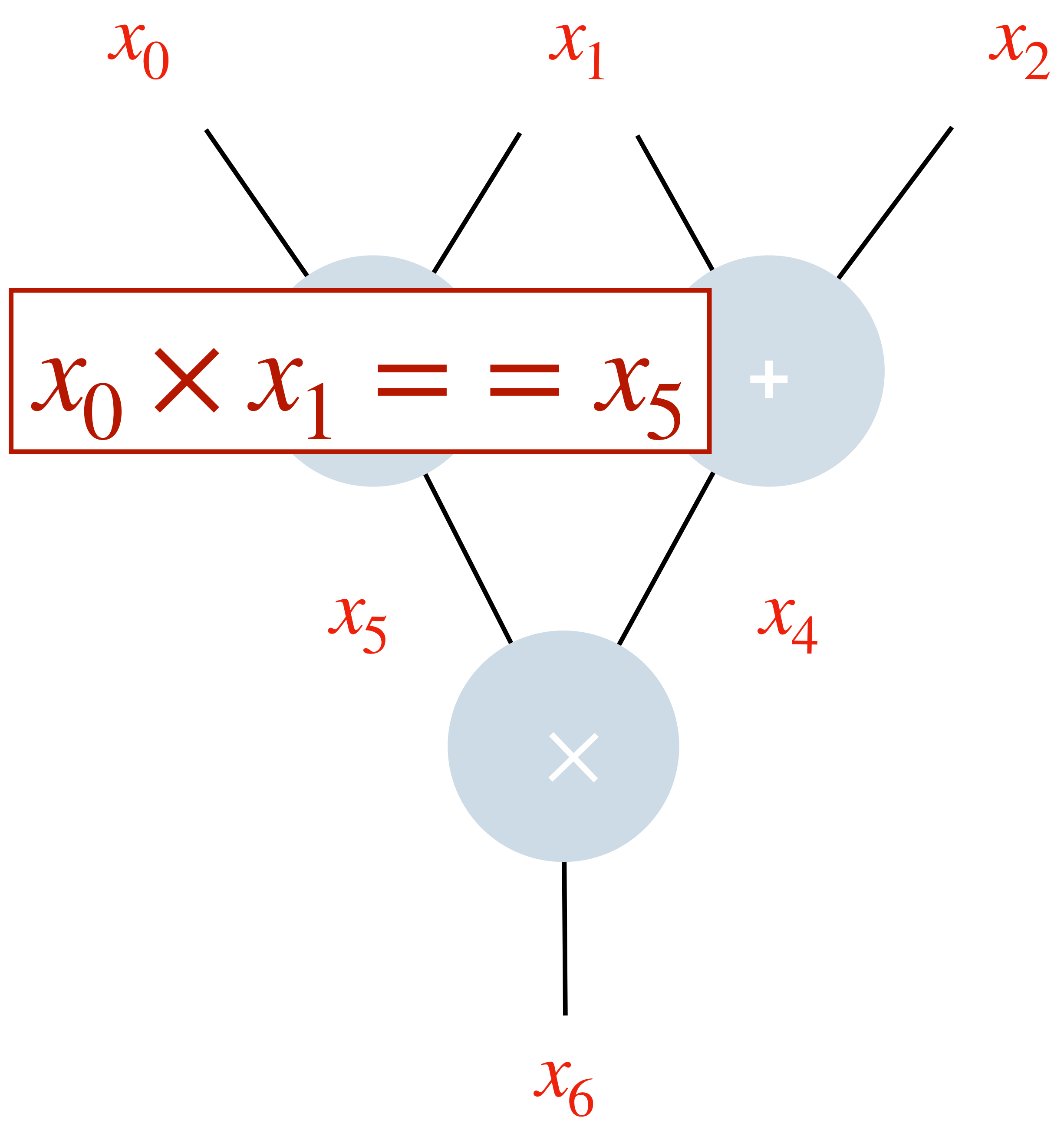

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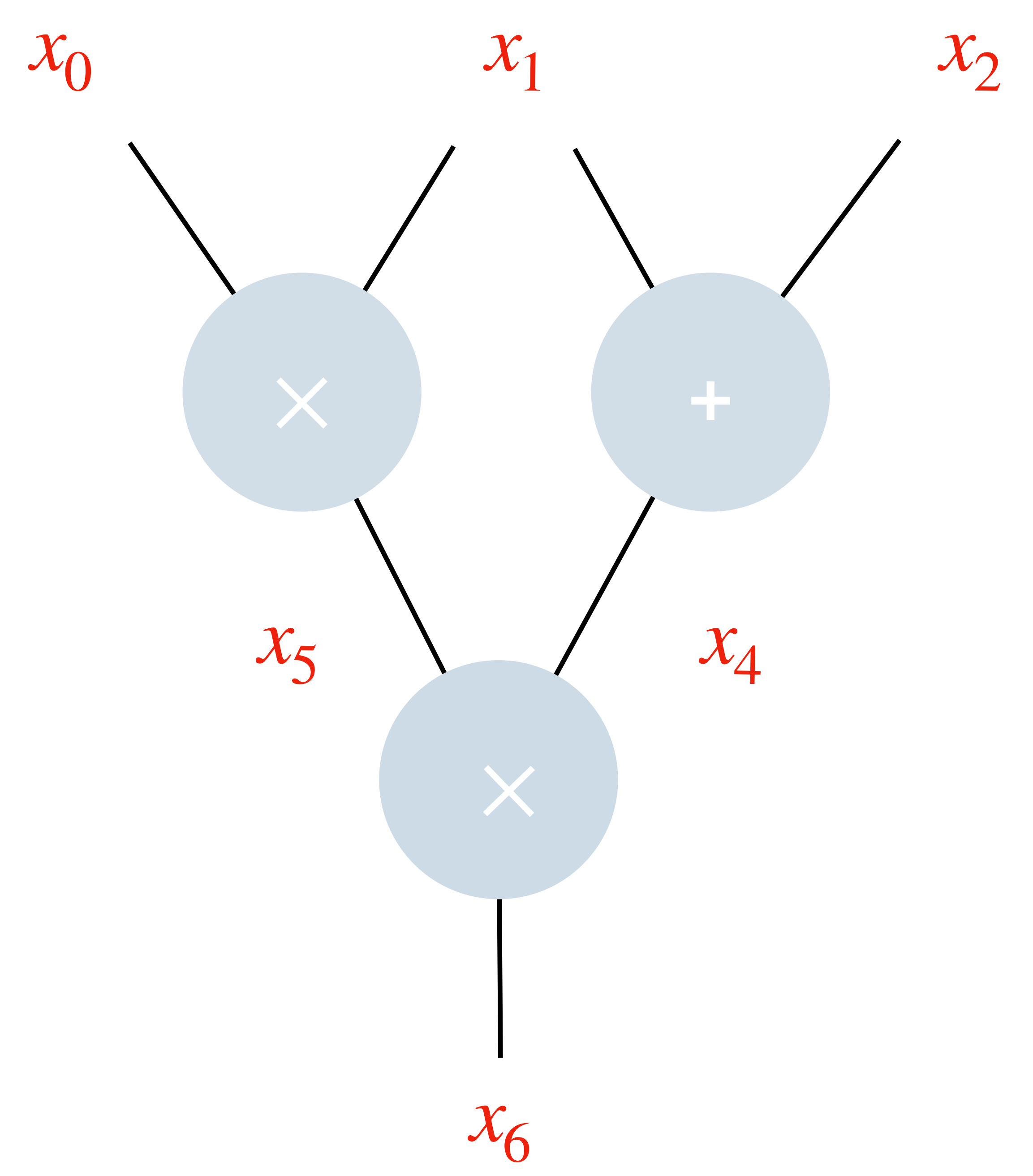

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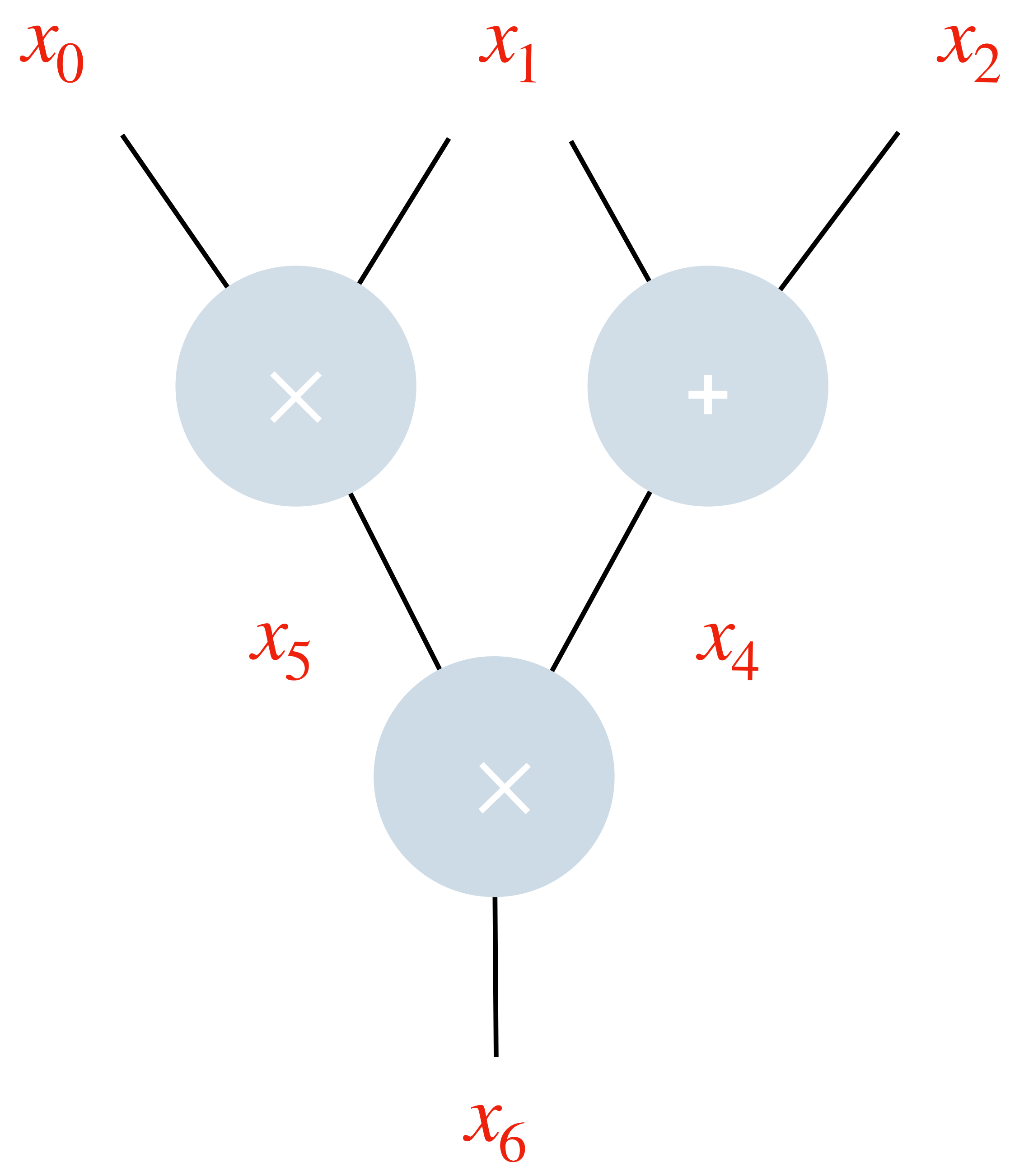

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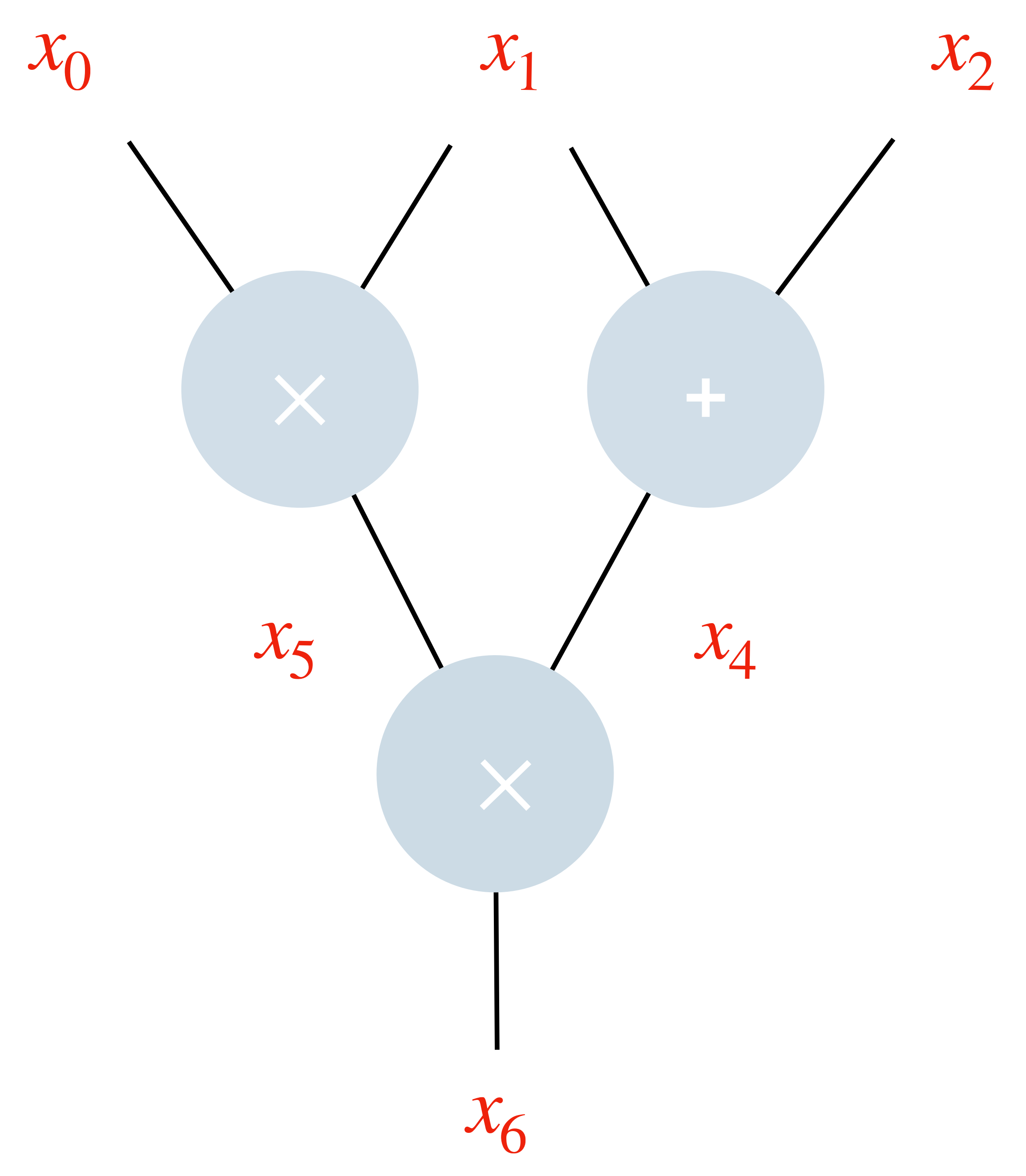

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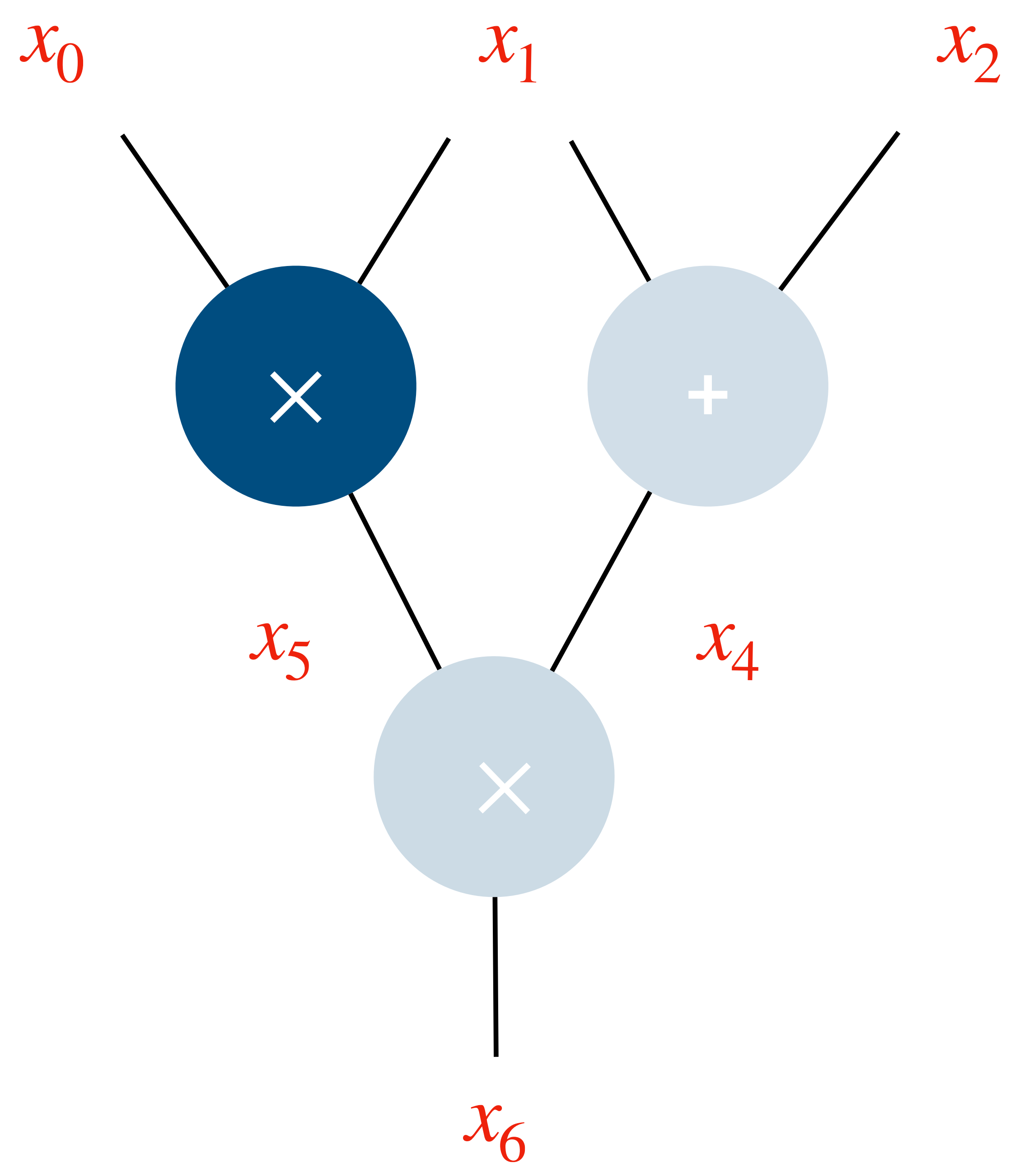

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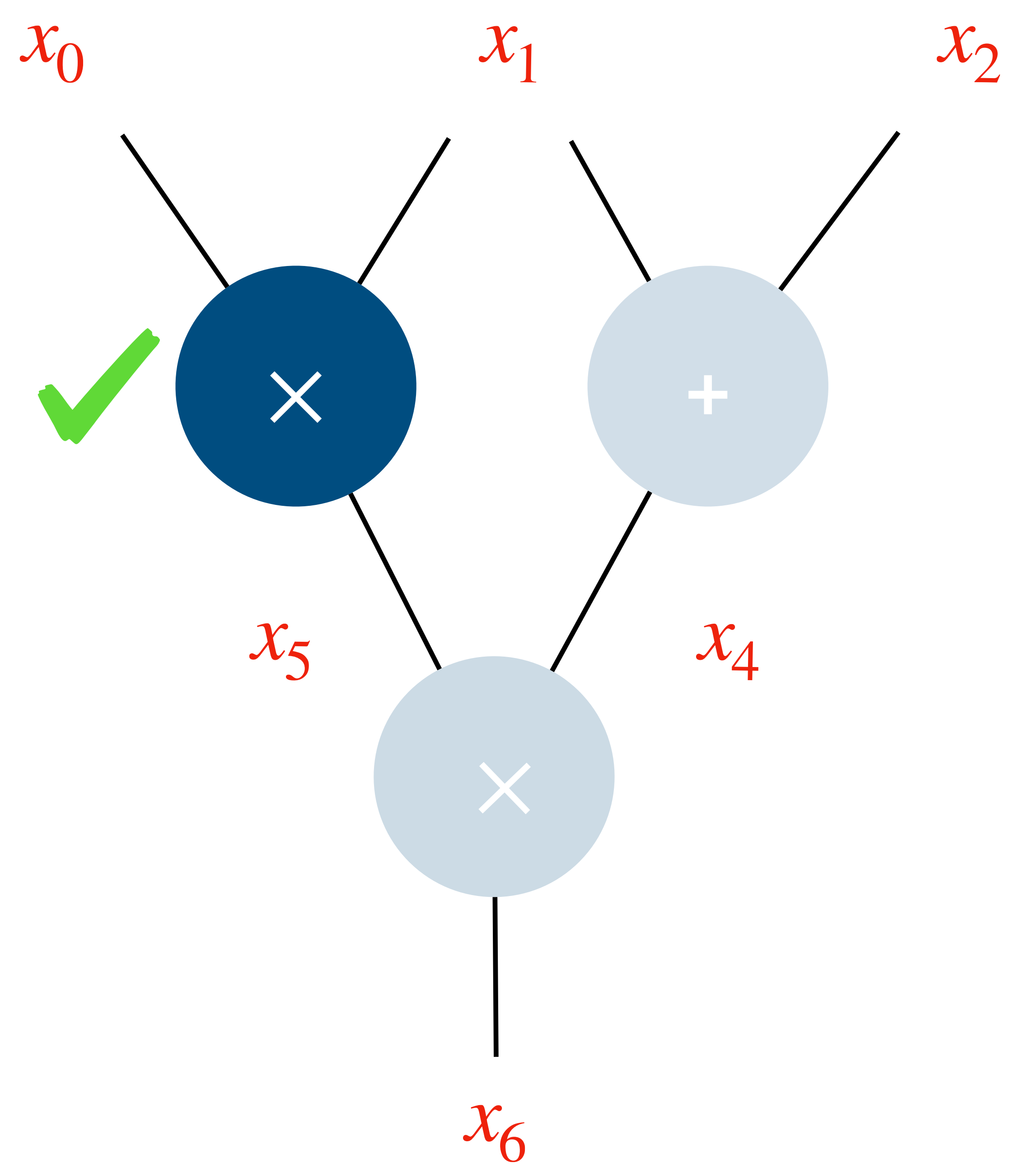

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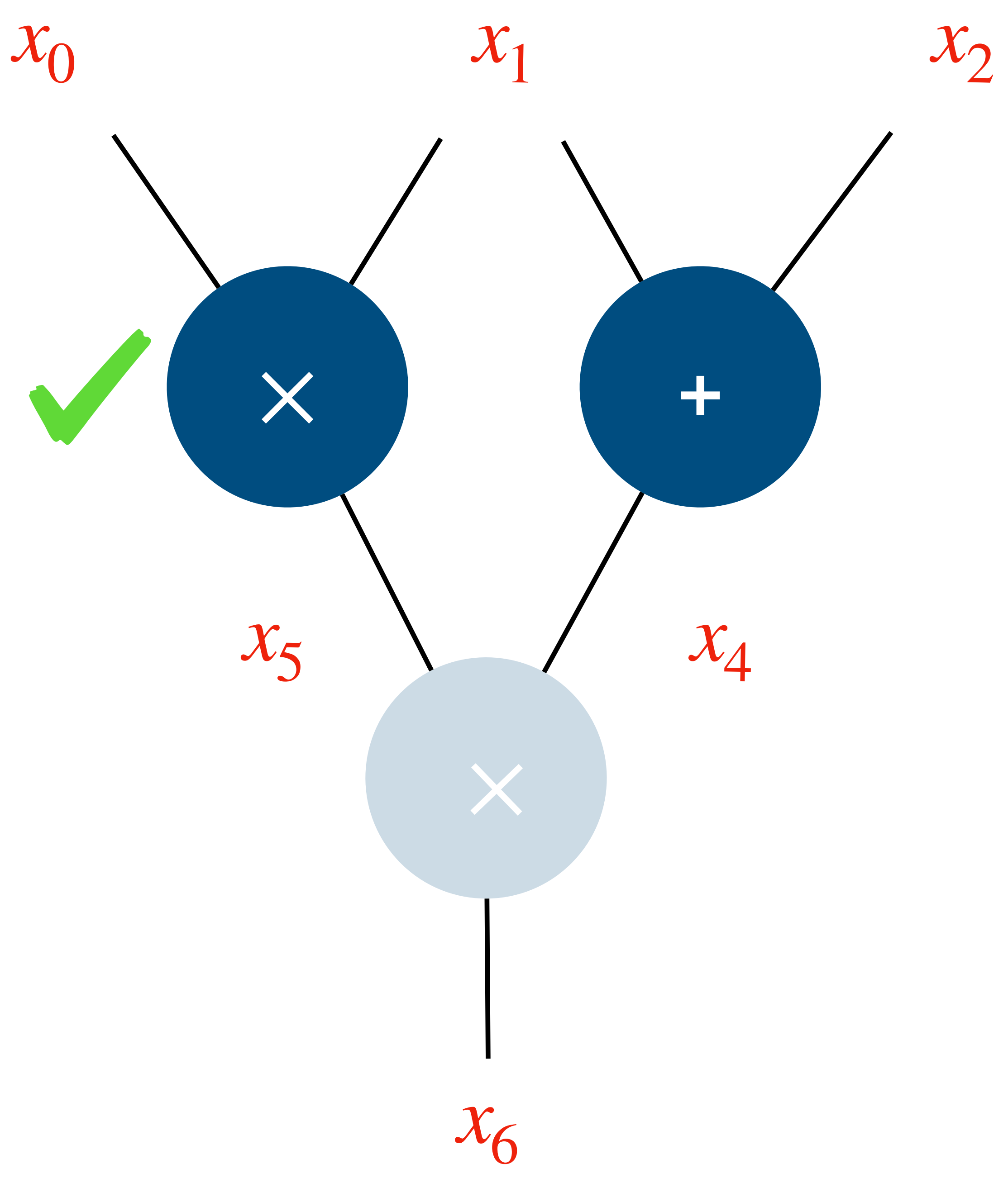

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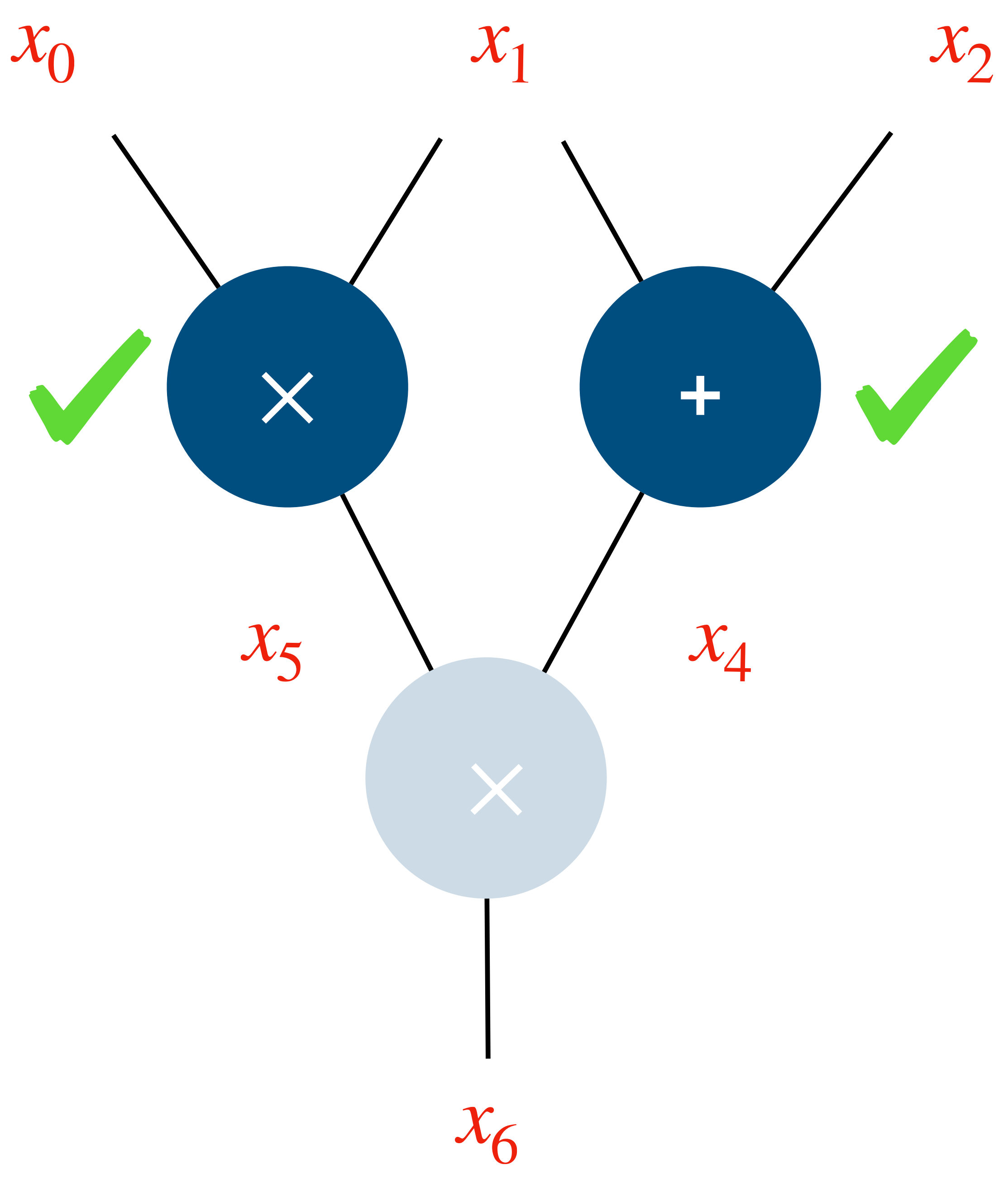

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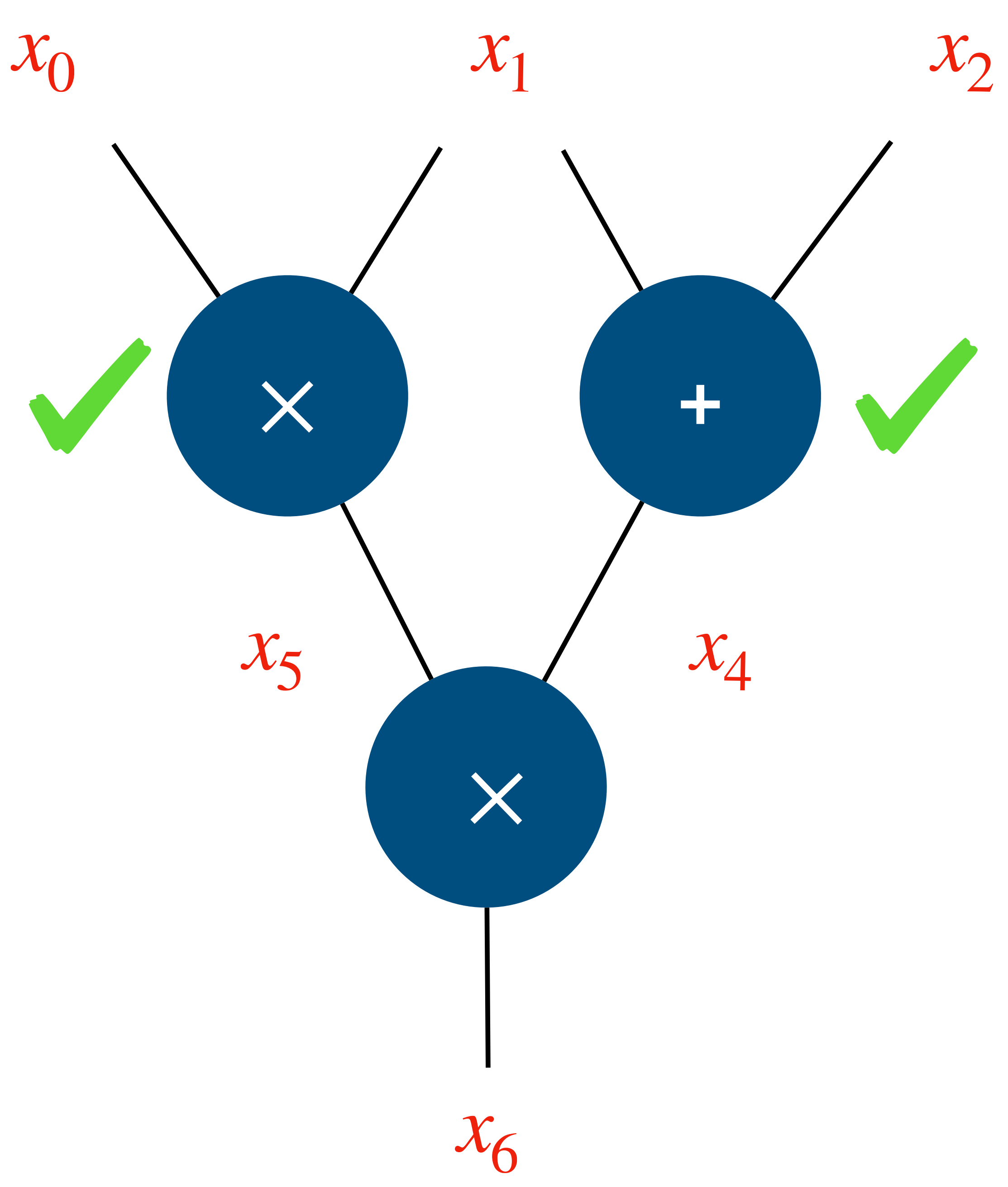

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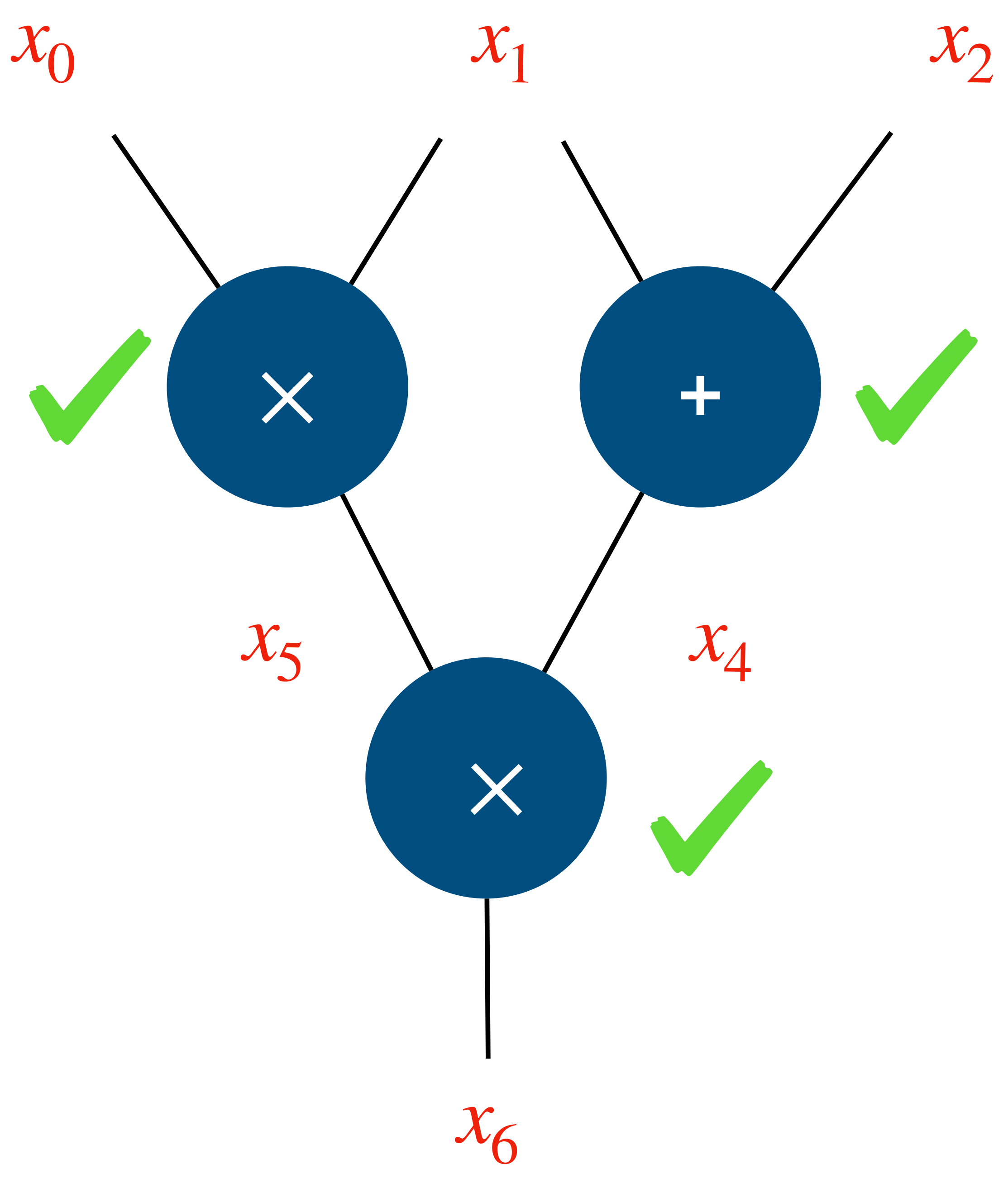

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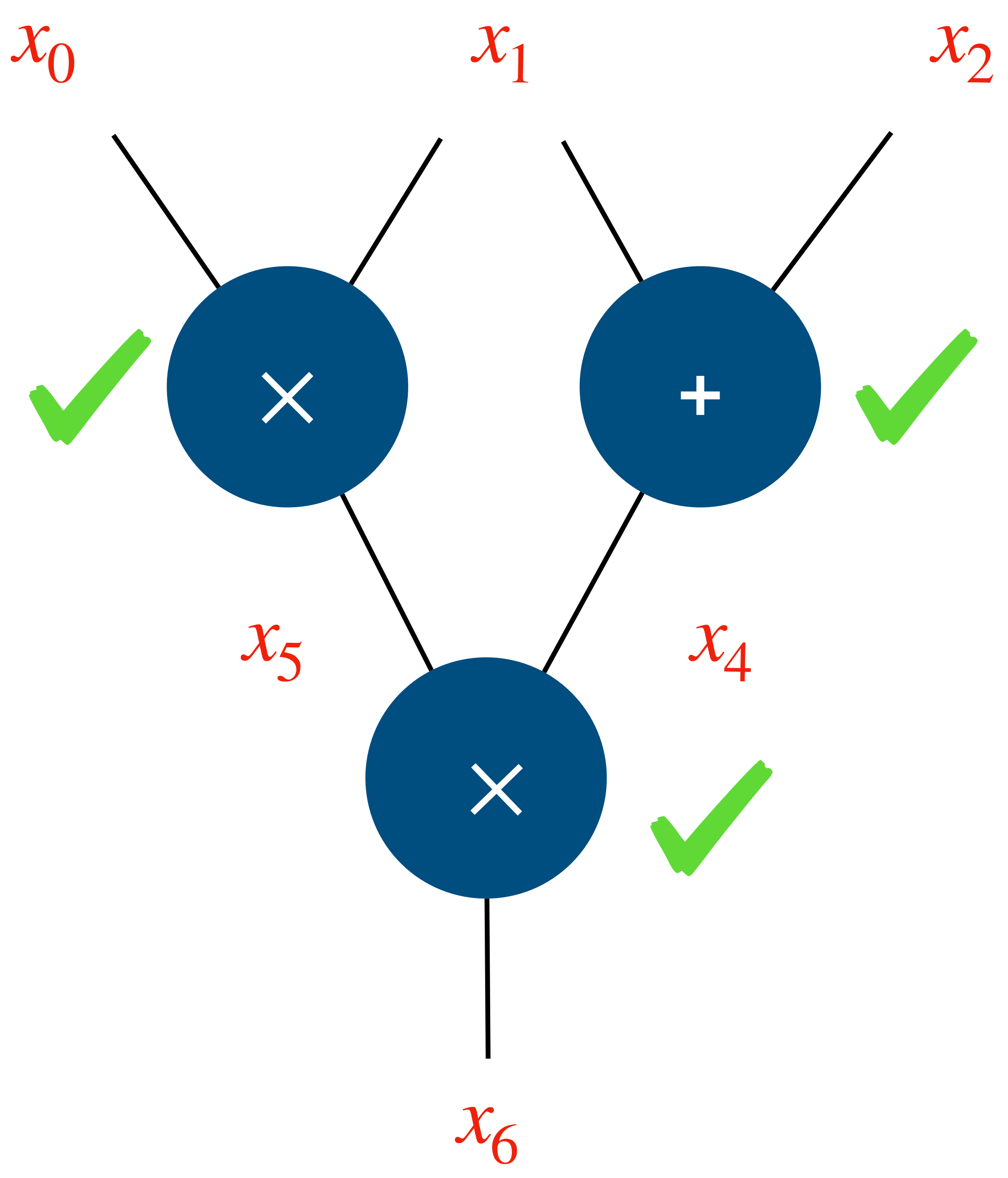

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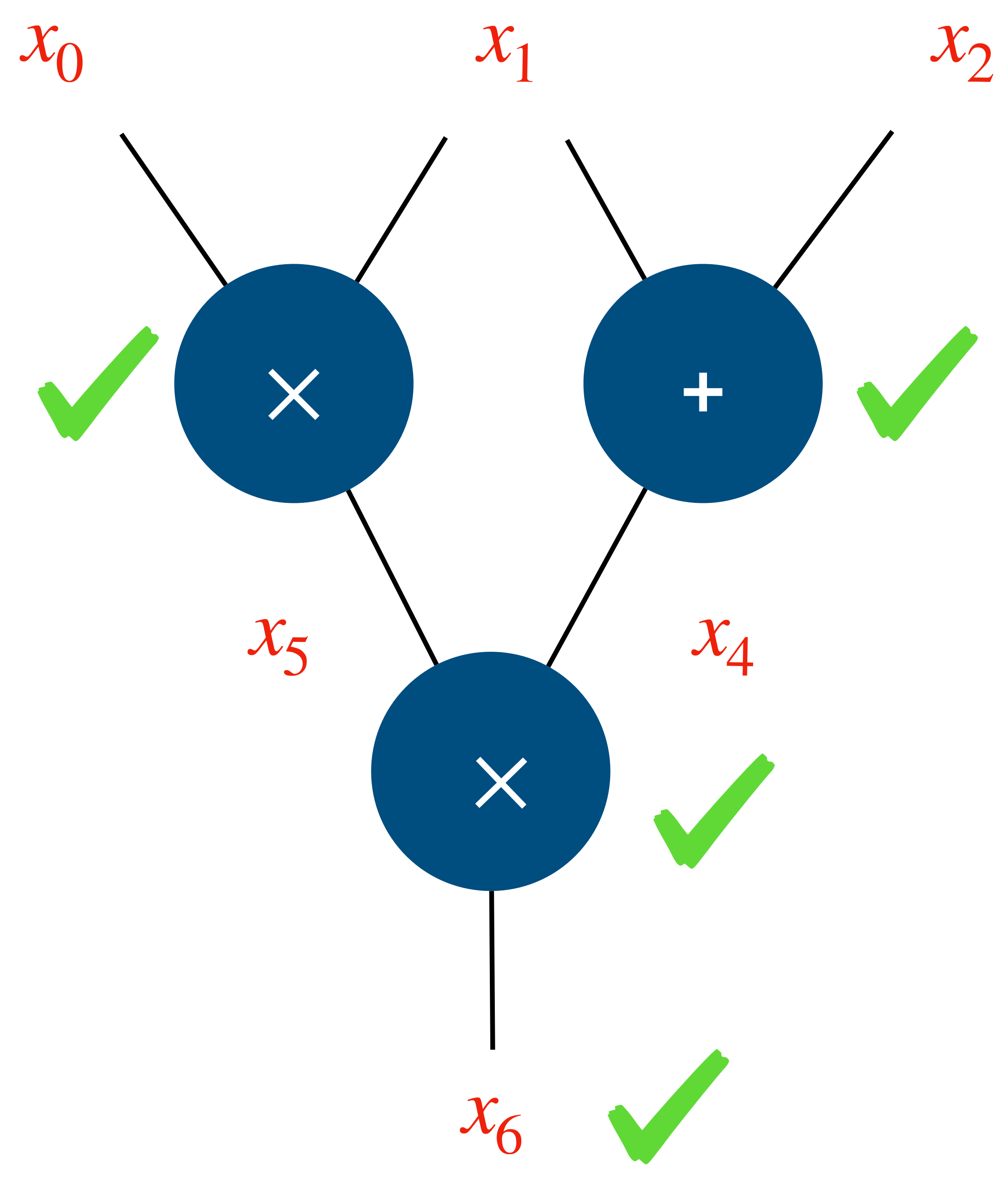

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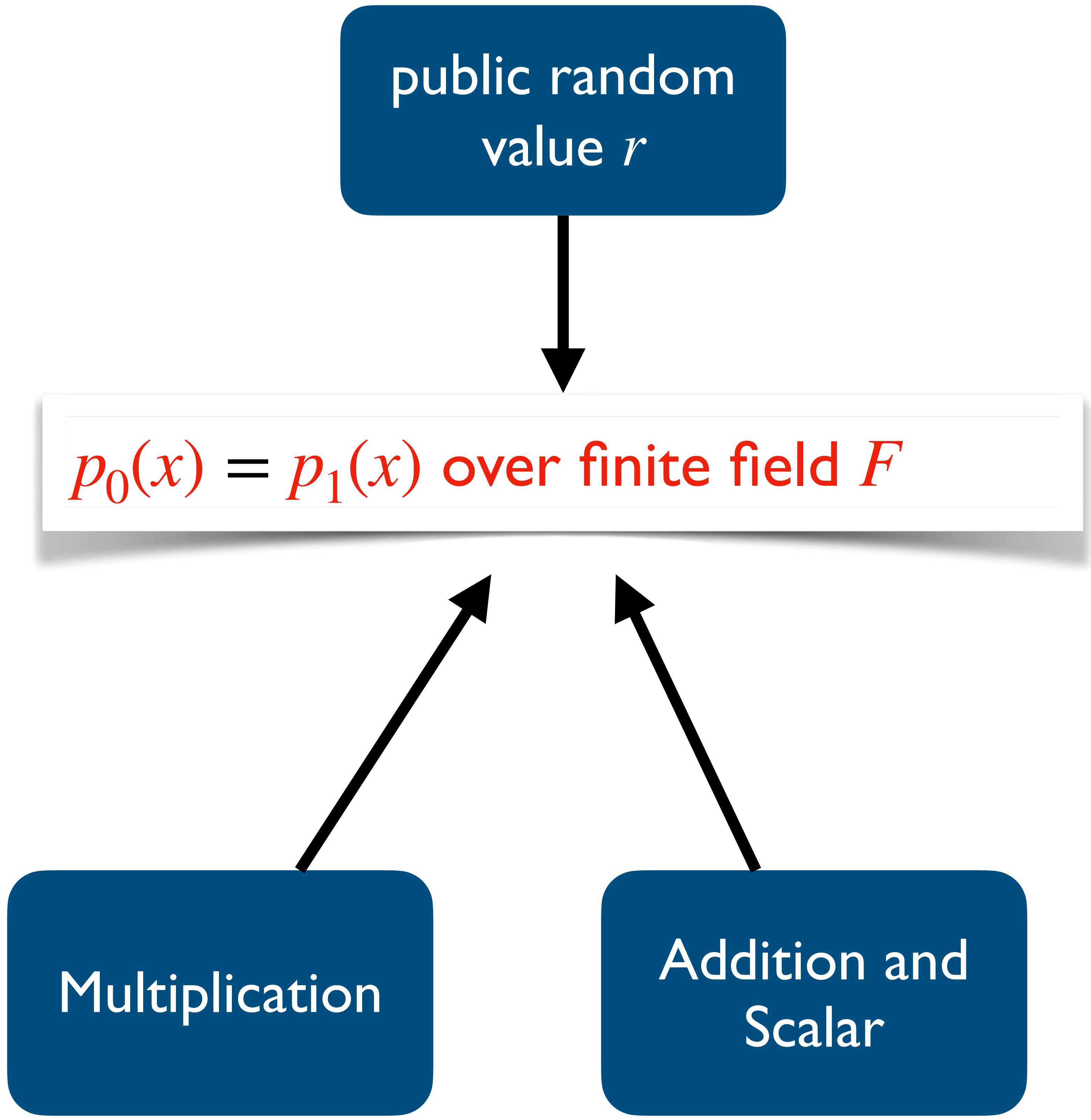
Zero Knowledge Proof

Knowledge for Polynomials[Yang et al. 21]



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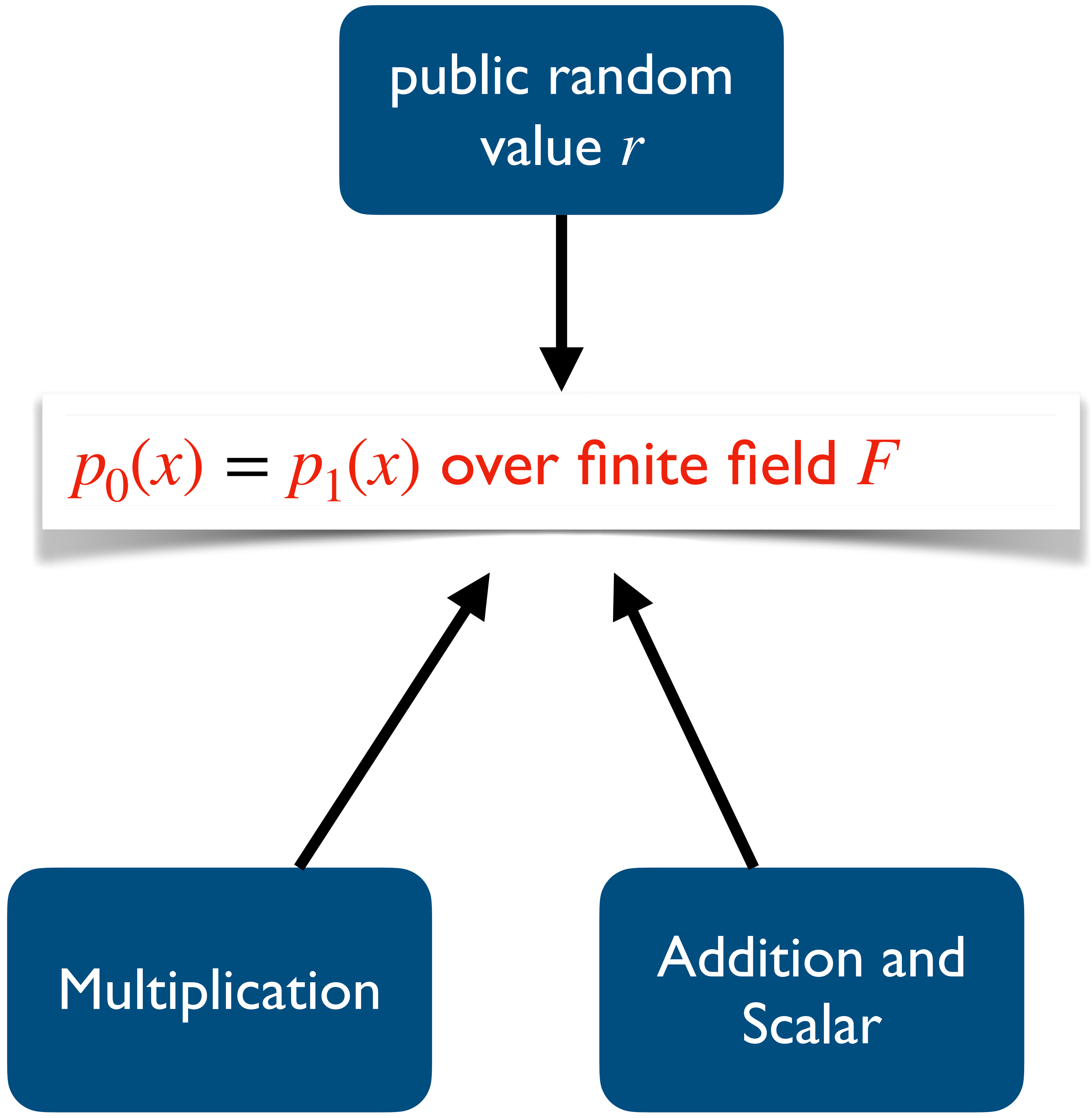

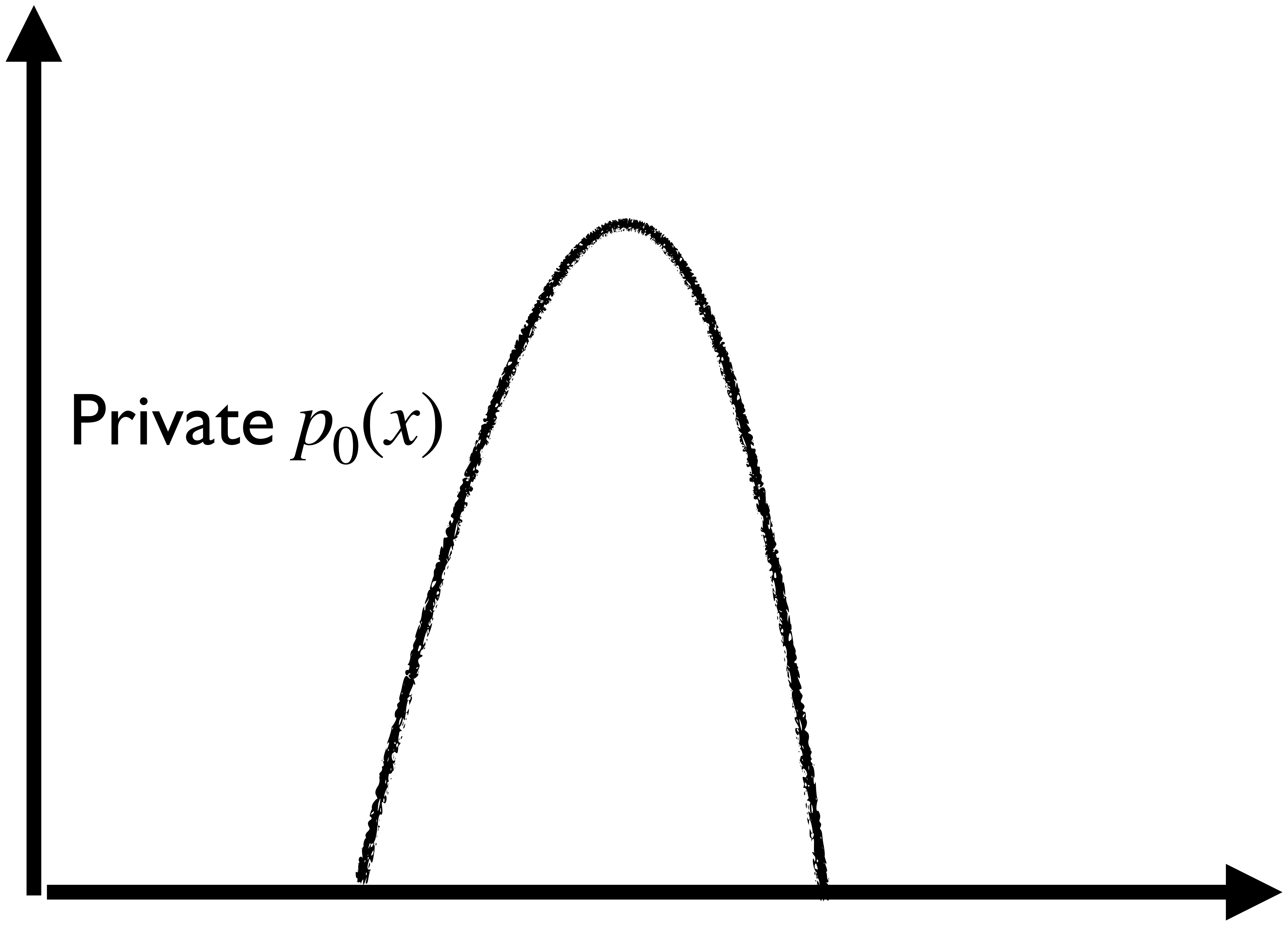

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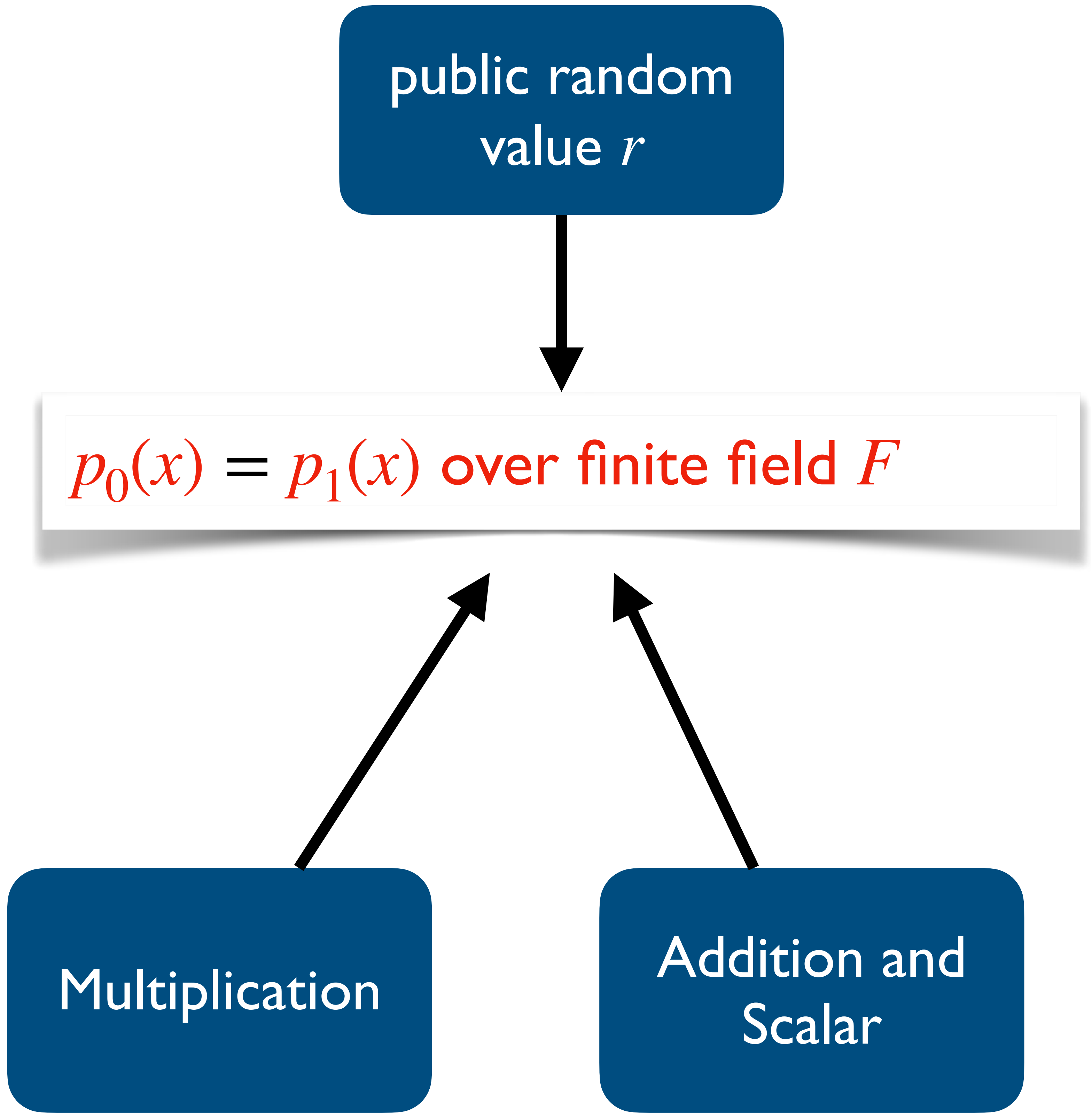

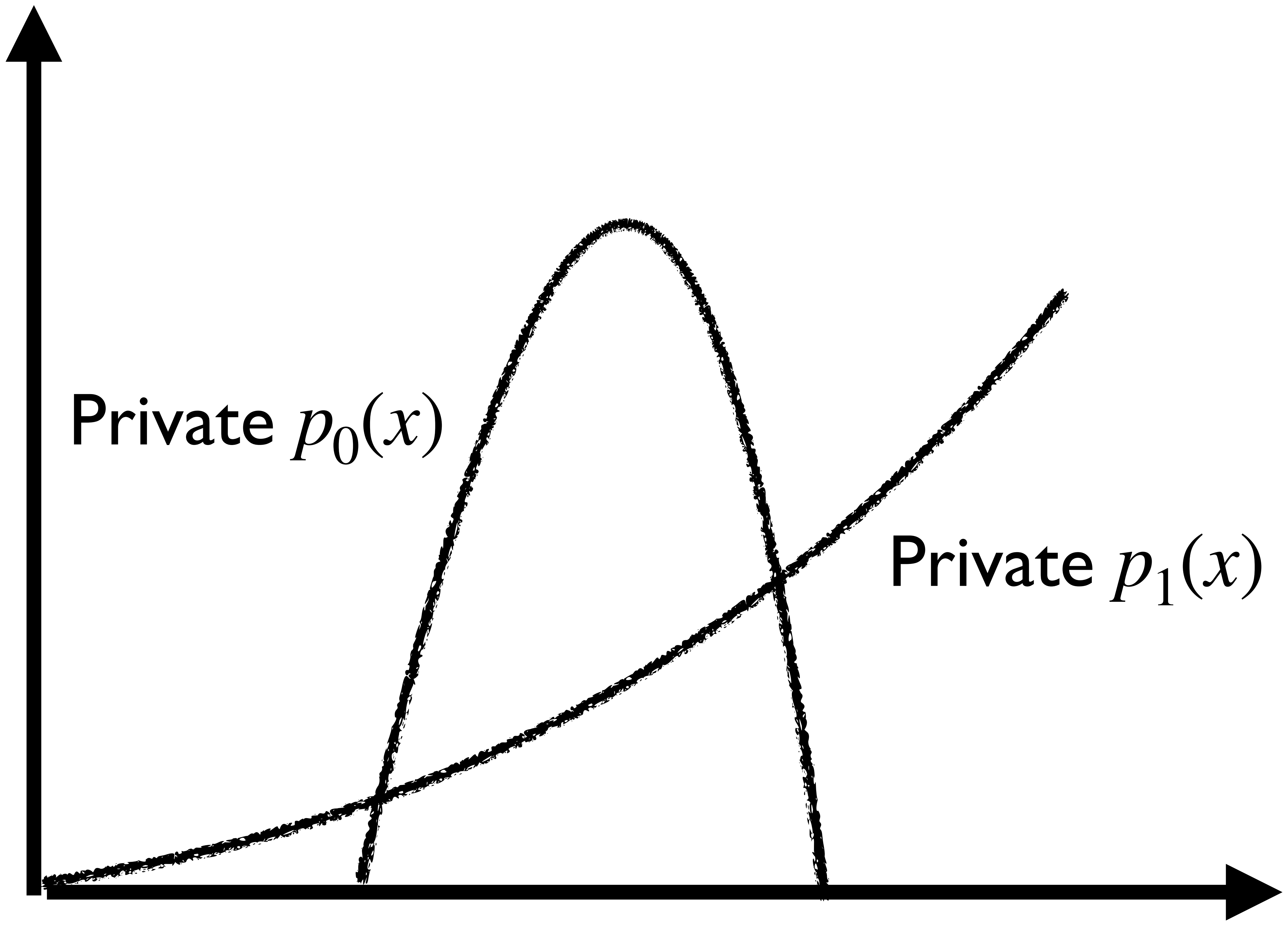

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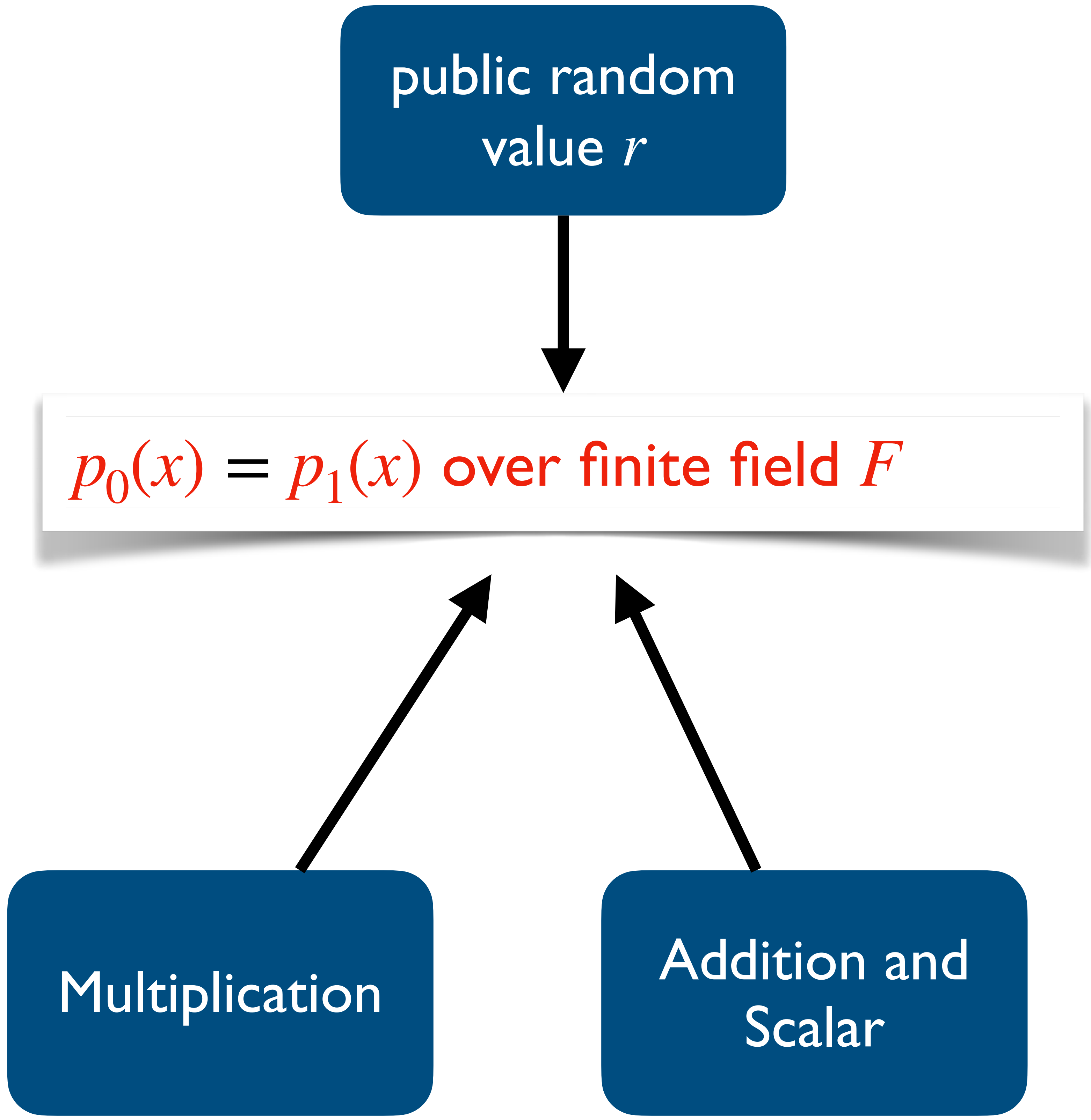

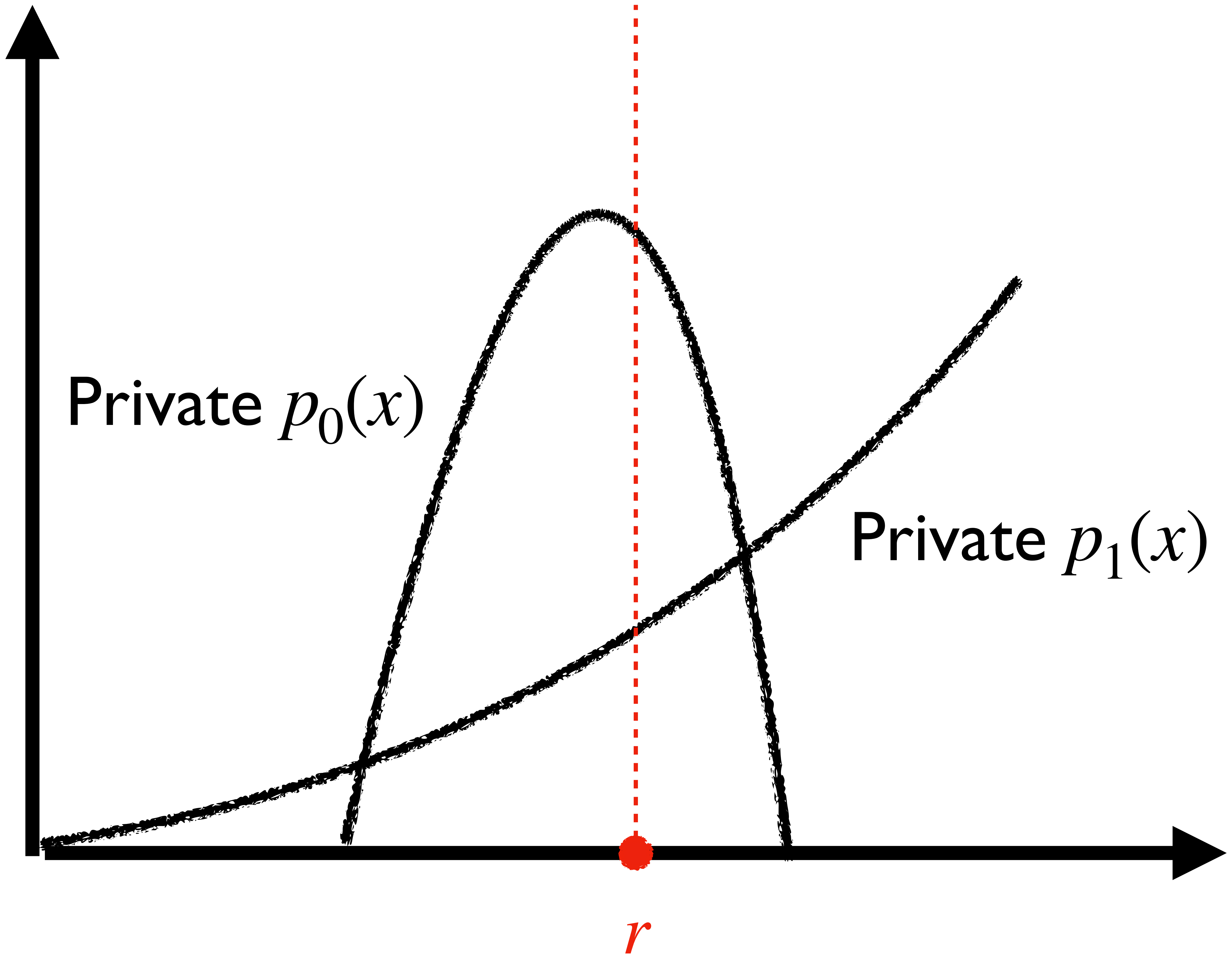

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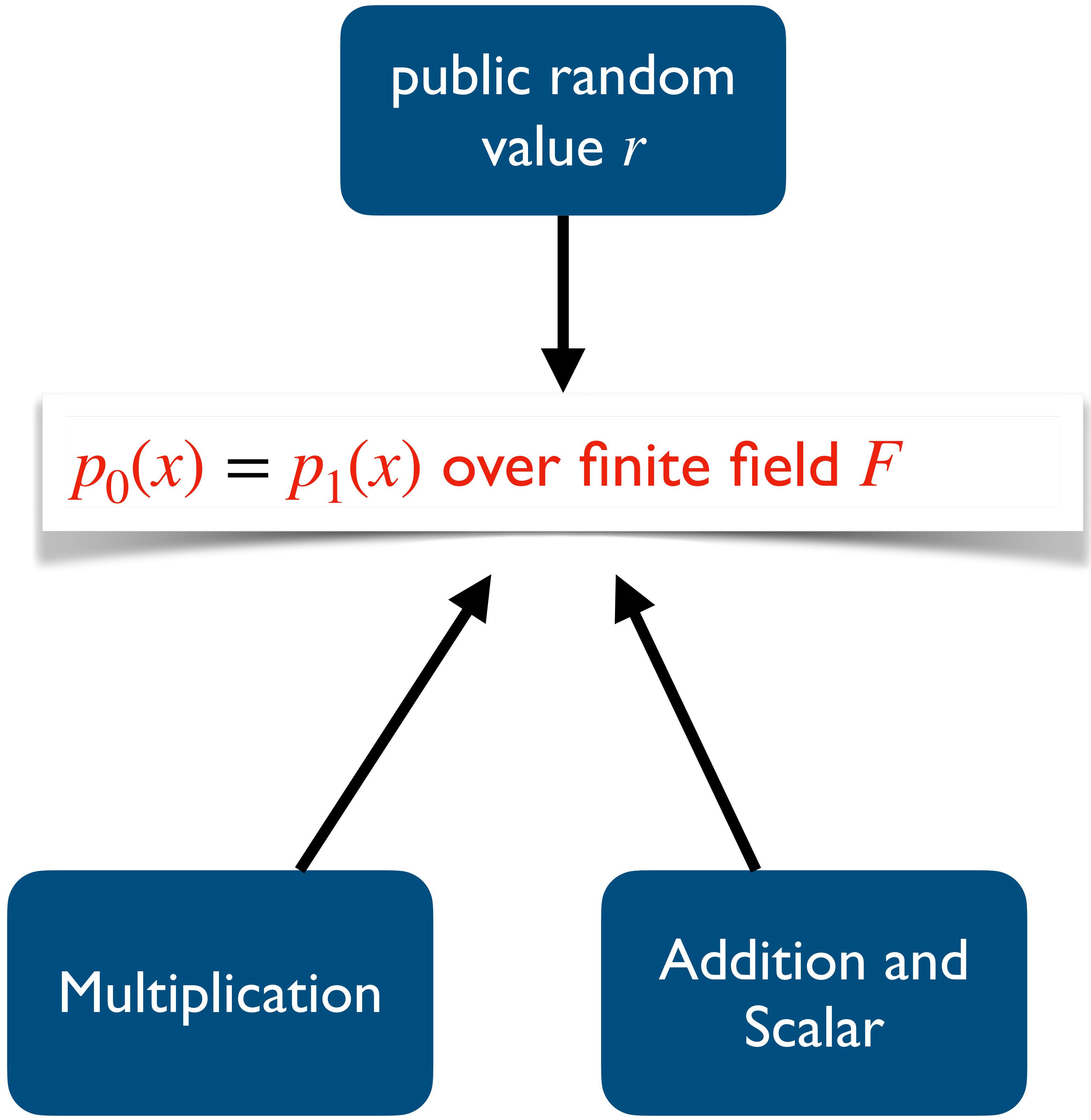
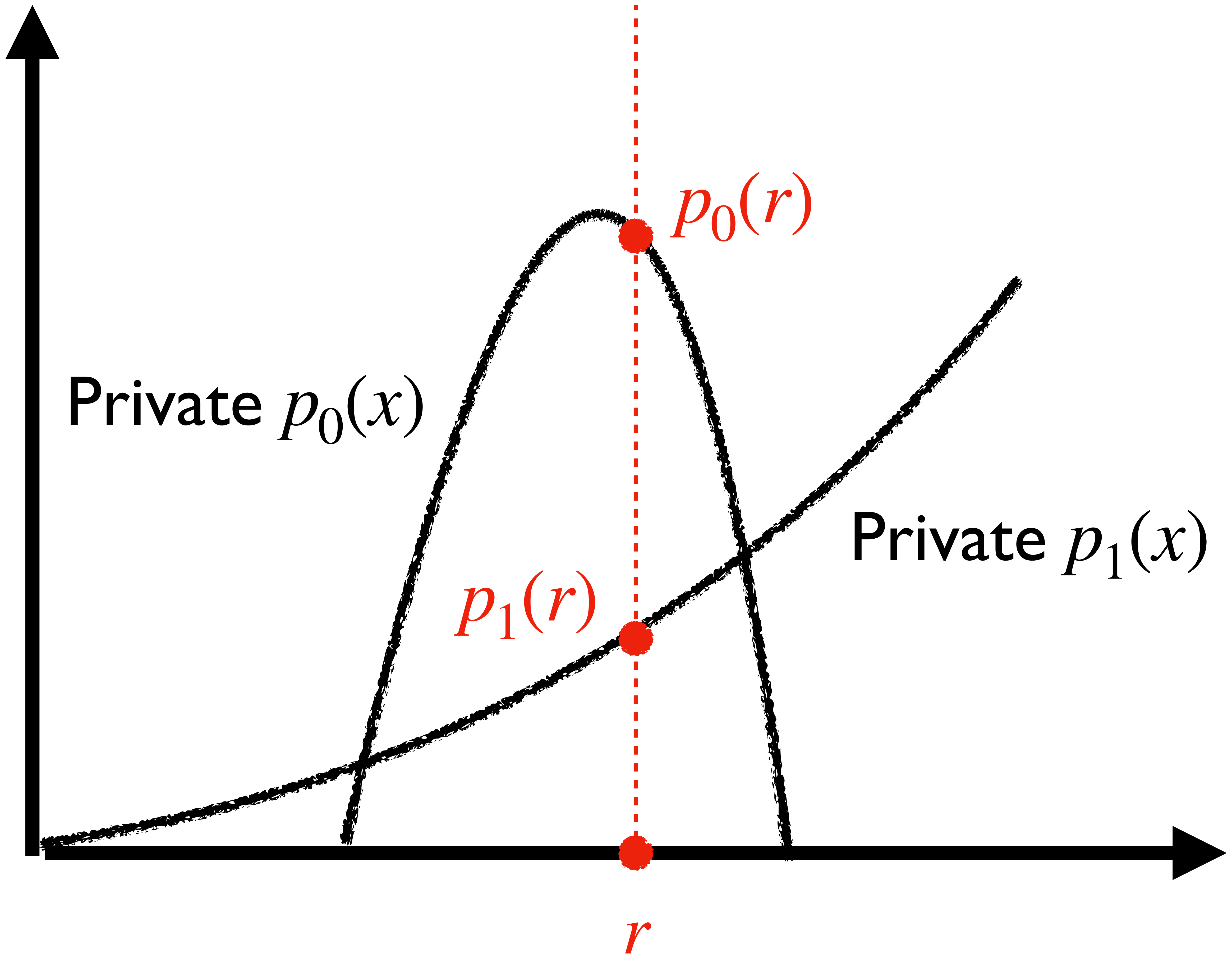




Zero Knowledge Proof

Knowledge for Polynomials[Yang et al. 21]

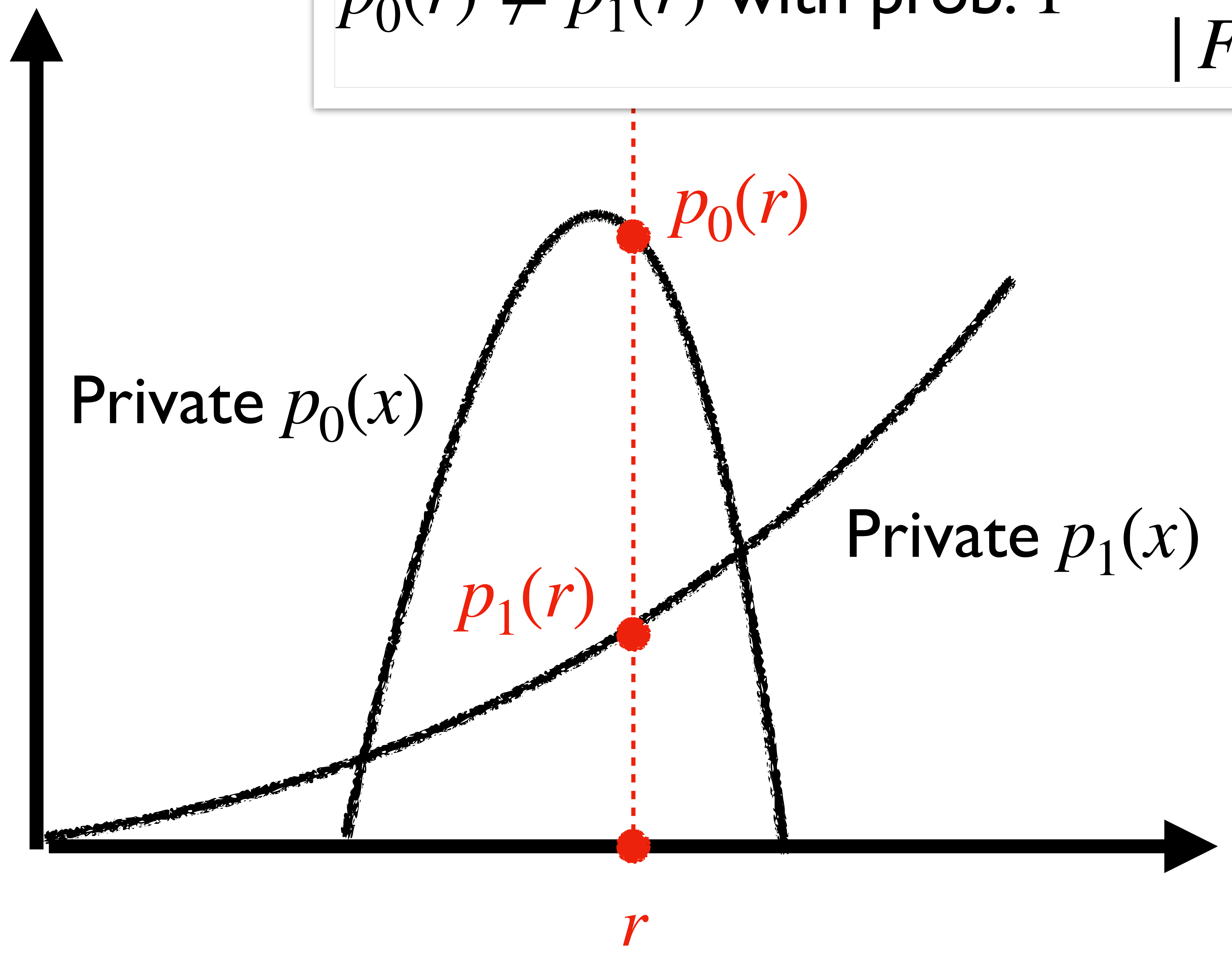
$$p_0(r) \neq p_1(r) \text{ with prob. } 1 - \frac{d}{|F|}$$

public random value r

$p_0(x) = p_1(x)$ over finite field F

Multiplication

Addition and Scalar



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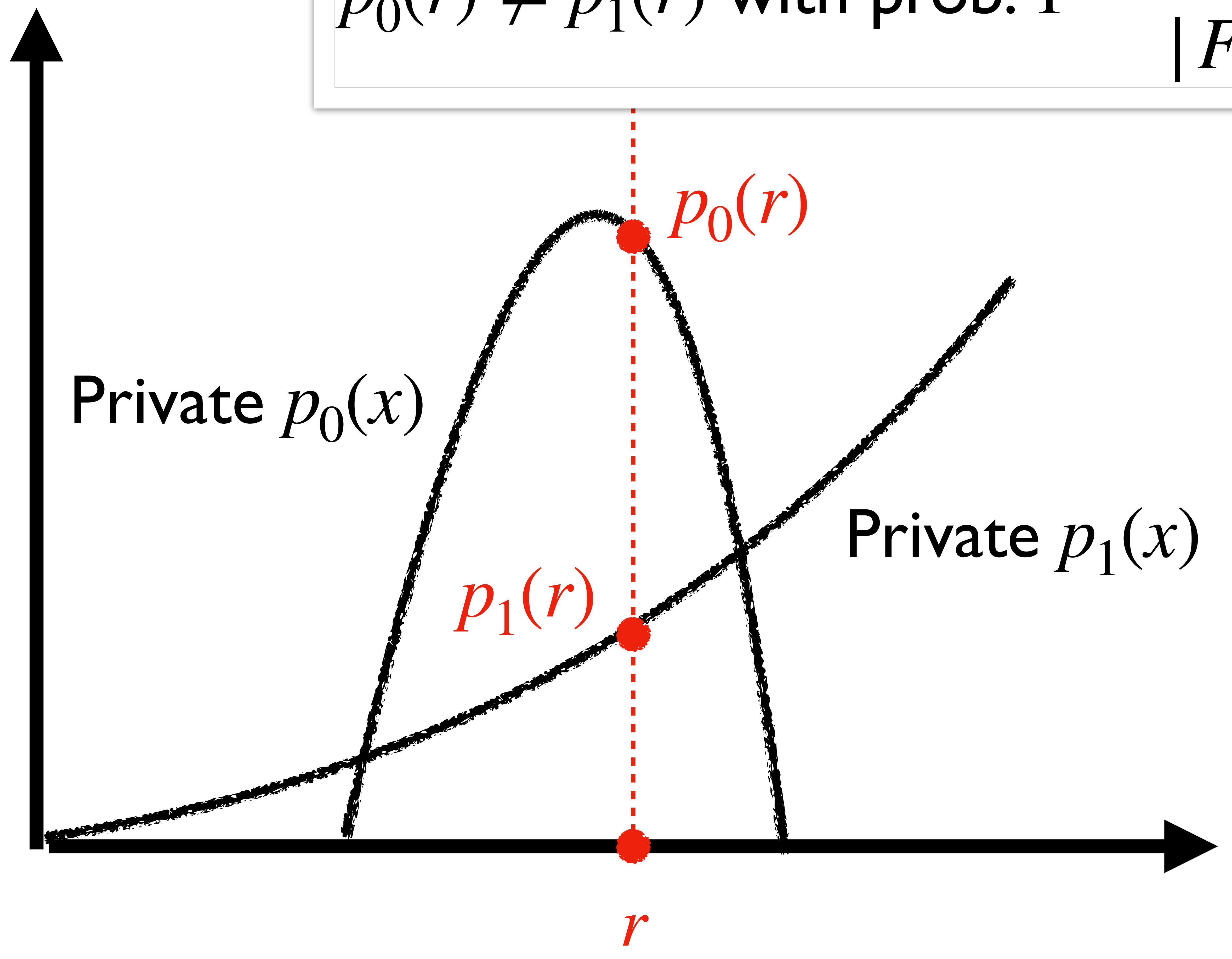

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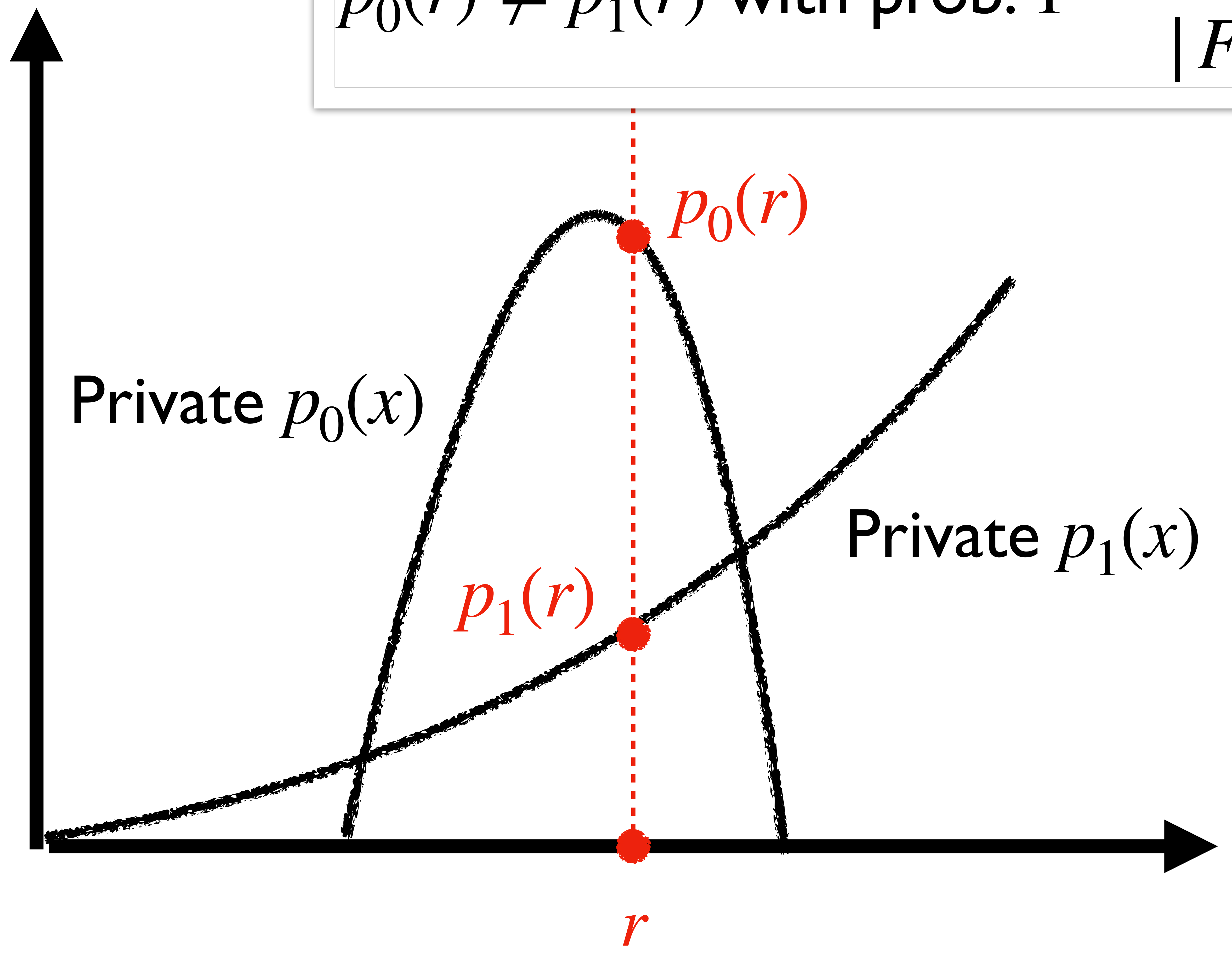

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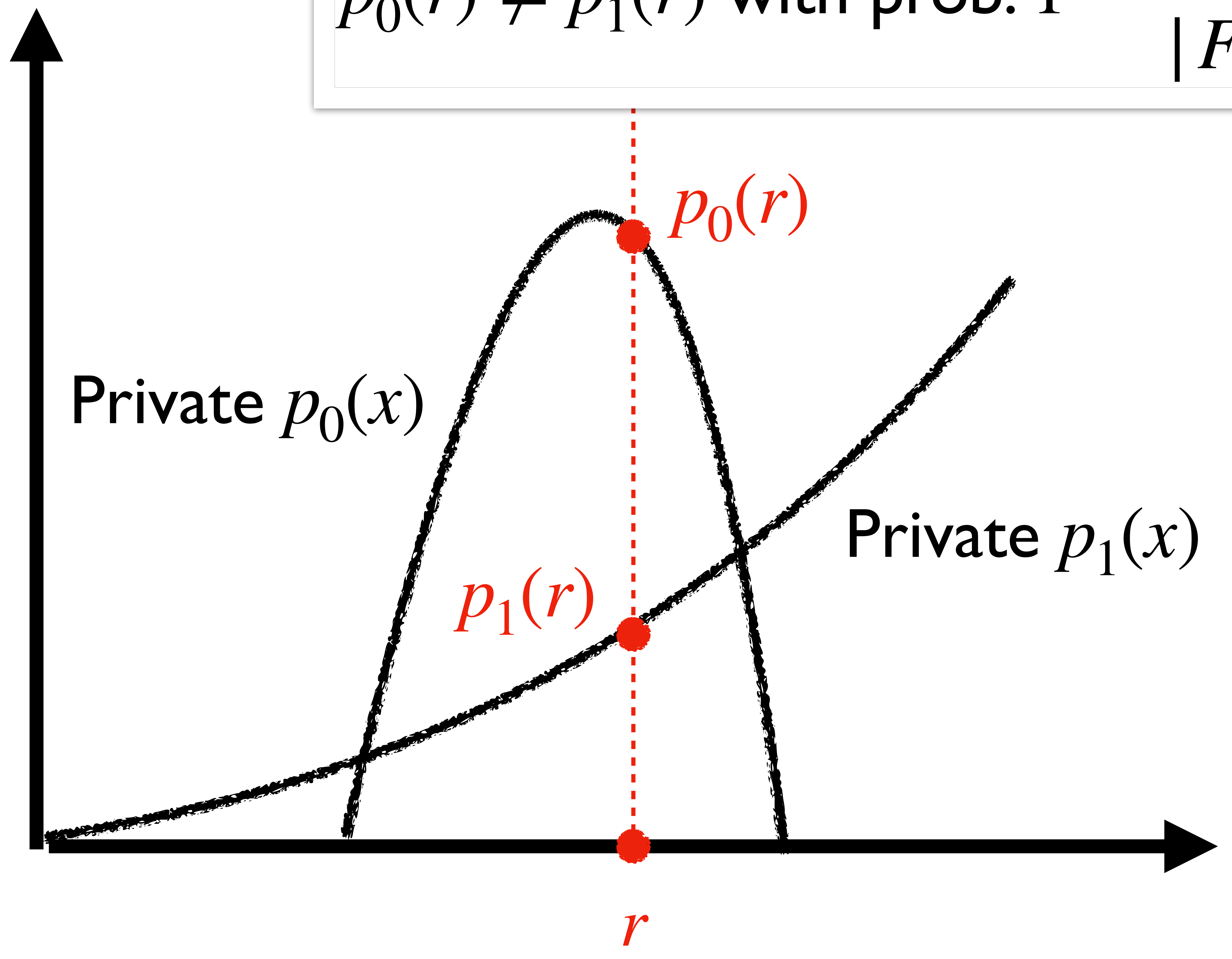
Multiplication

Addition and Scalar



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Multiplication

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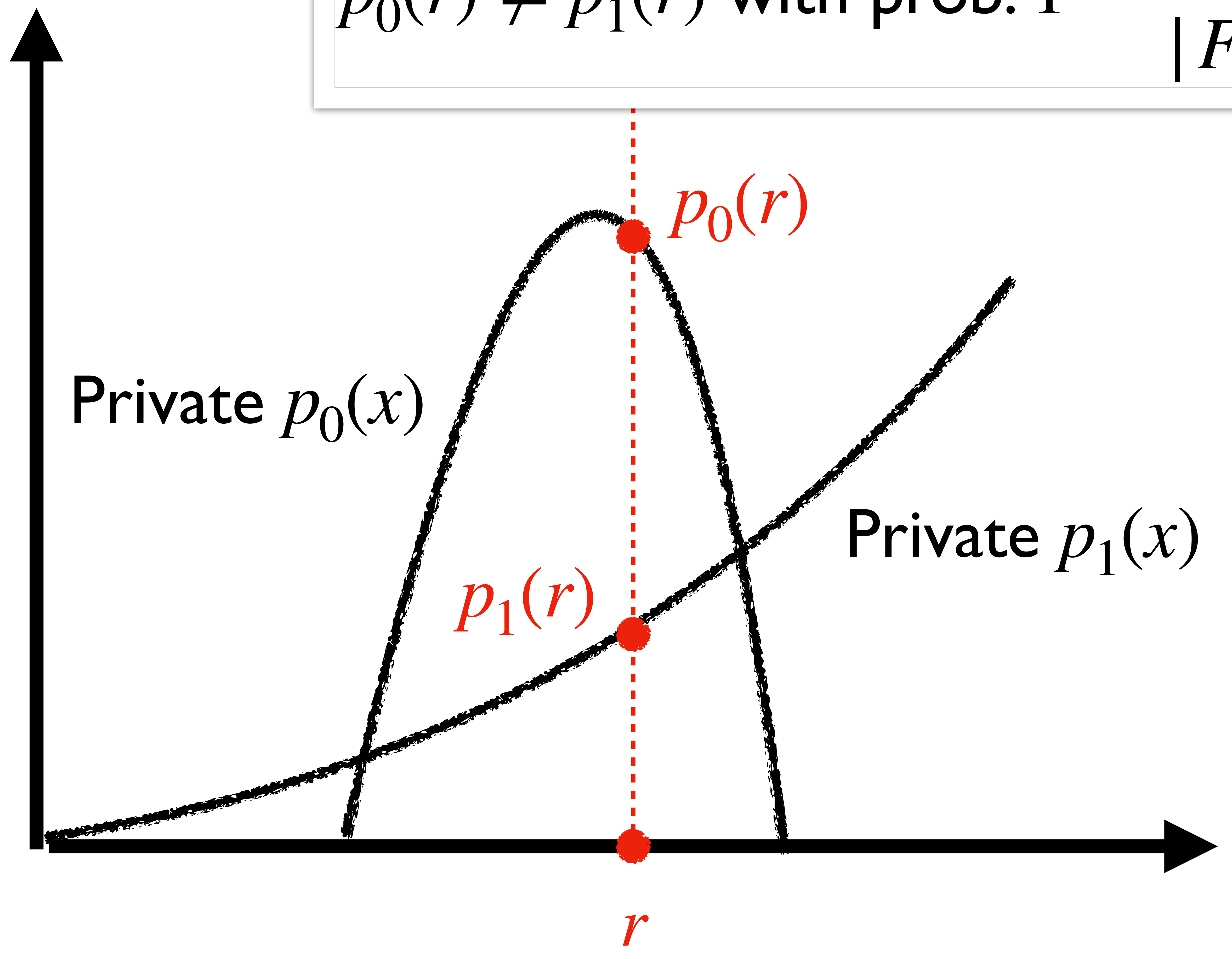

Zero Knowledge Proof: Take Away

Verifier can **verify**
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private values or polynomials
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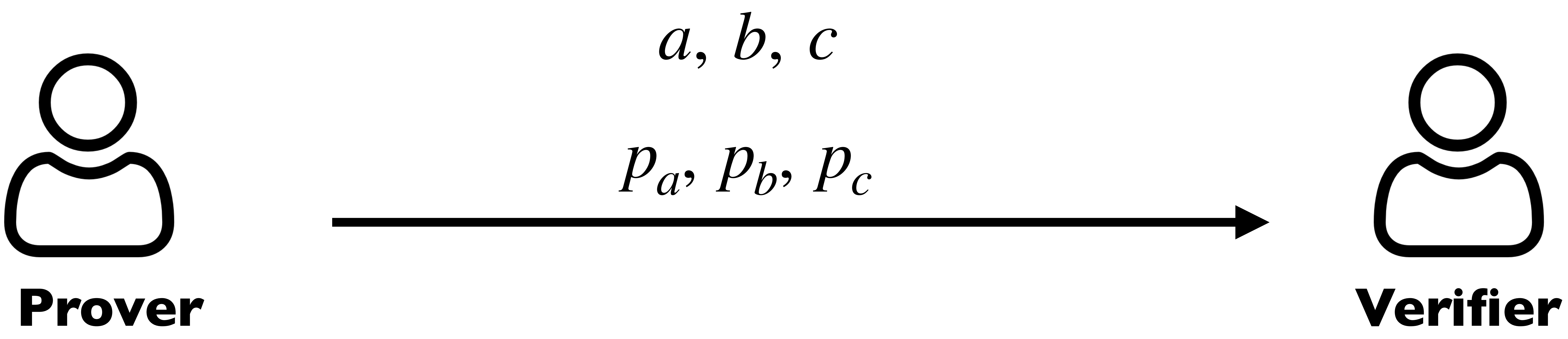

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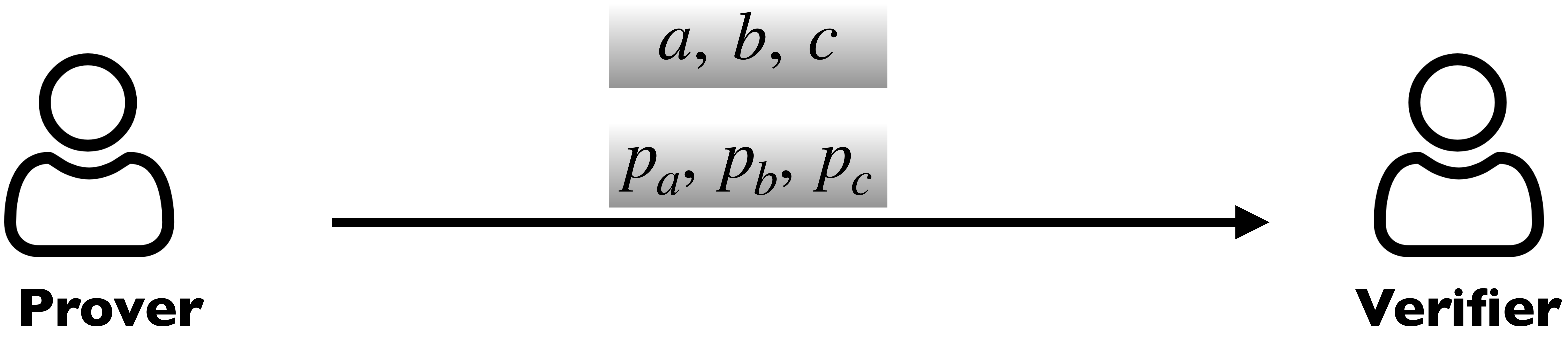

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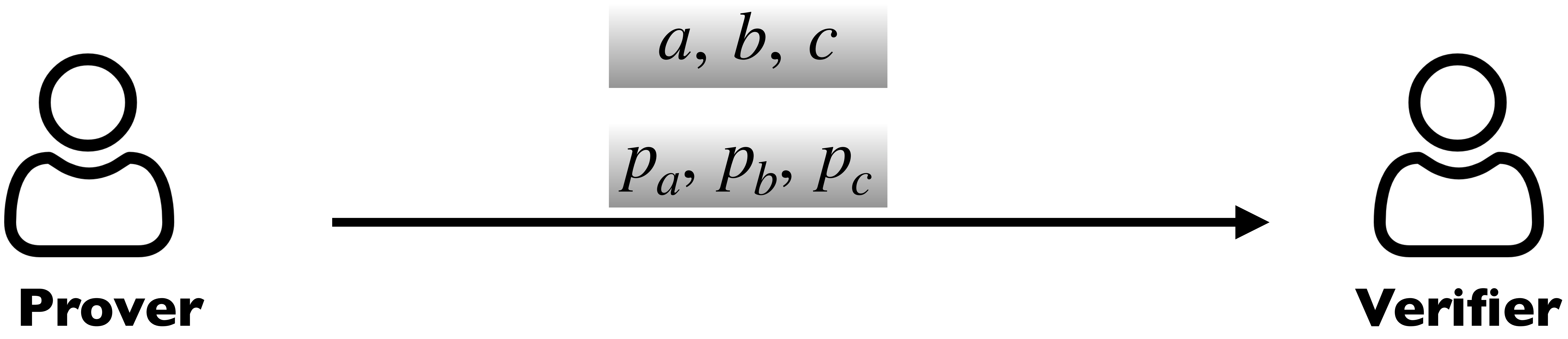

Unsatisfiability in Zero Knowledge Proof

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- Prover knows a resolution proof Prf for a formula ϕ

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$\phi = c_0 \land c_1 \land c_2 \land c_3$

Resolution Proof

Unsatisfiability in Zero Knowledge Proof

- Prover knows a resolution proof Prf for a formula ϕ
  - Convince the verifier that ϕ is **unsatisfiable**

Unsatisfiability in Zero Knowledge Proof

- Prover knows a resolution proof Prf for a formula ϕ
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Unsatisfiability in Zero Knowledge Proof

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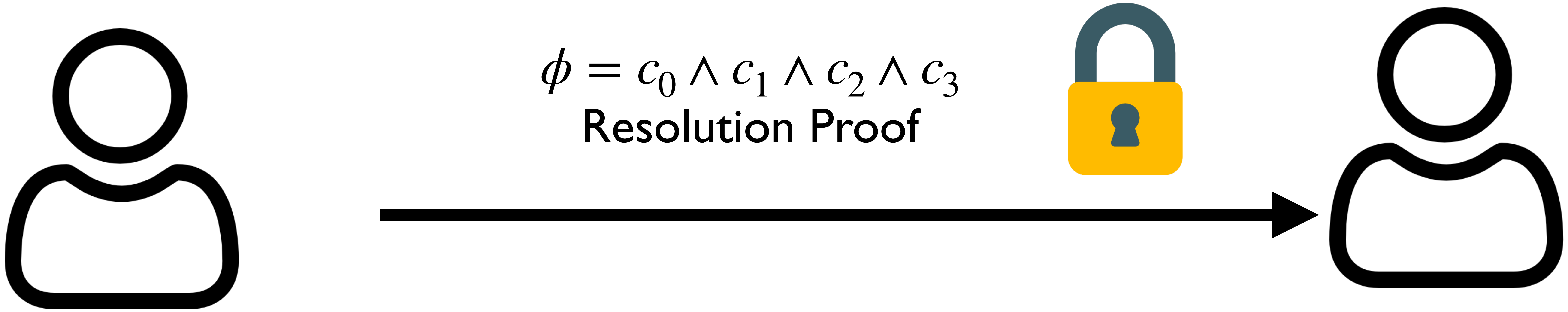
Unsatisfiability in Zero Knowledge Proof



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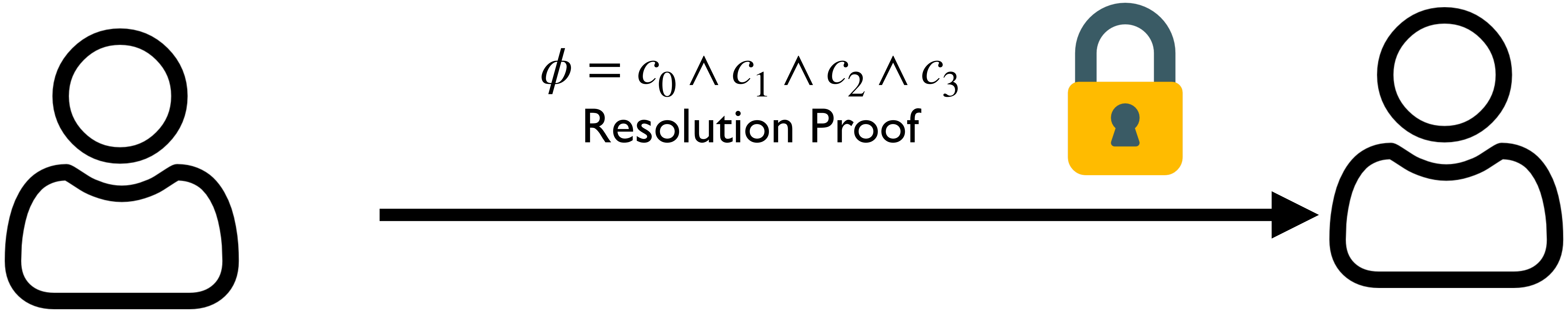

Both Prover and Verifier can be **malicious**.

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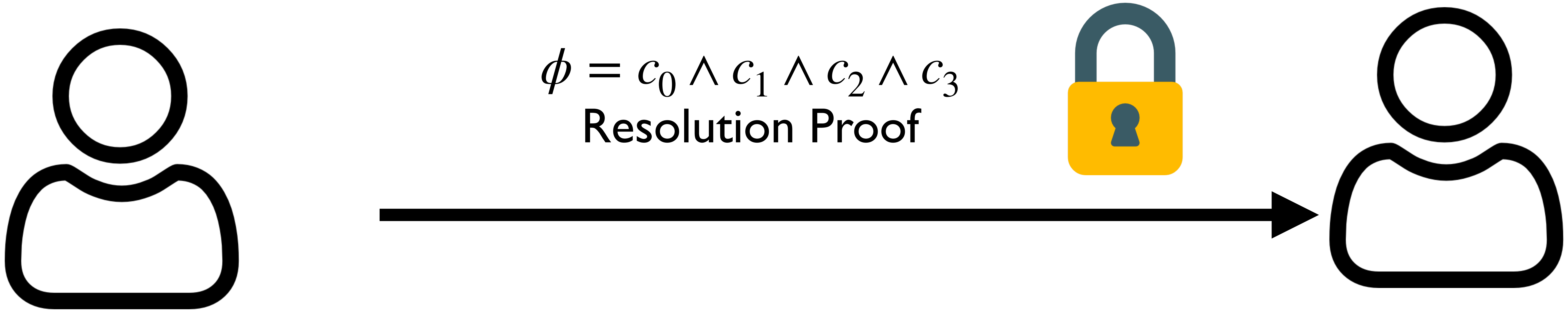

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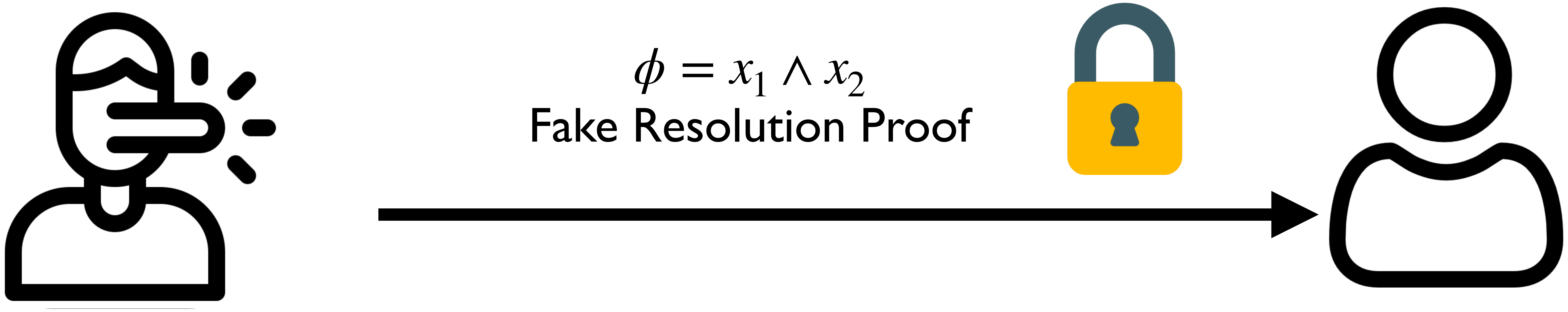

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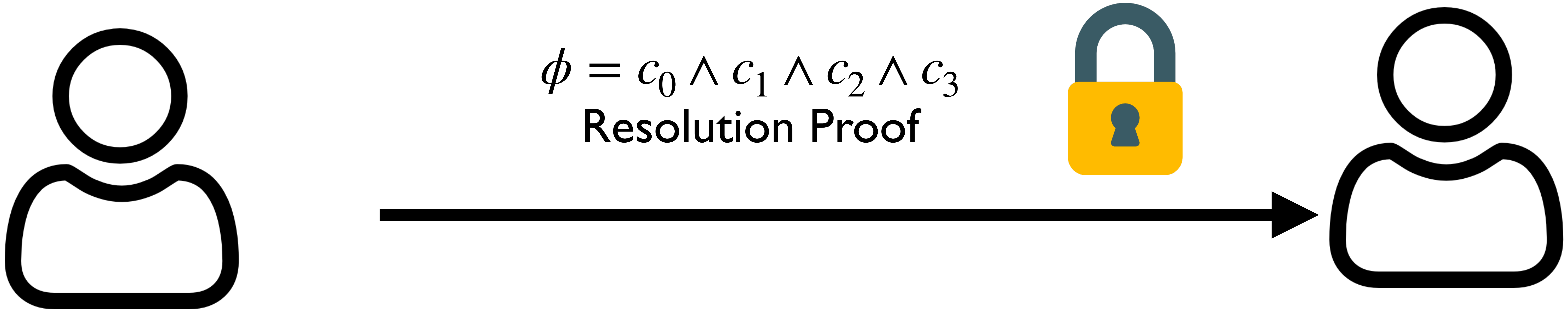

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Unsatisfiability in Zero Knowledge Proof

Both Prover and Verifier can be **malicious**.

- Prover might cheat about unsatisfiability of ϕ
- Verifier tries to learn information about Prf and ϕ

Unsatisfiability in Zero Knowledge Proof

Technique challenges and design overview

ZKUNSAT

$c_0 : (x_1 \vee x_2)$	$-, -$
$c_1 : (\neg x_1 \vee x_2)$	$-, -$
$c_2 : (\neg x_1 \vee \neg x_2)$	$-, -$
$c_3 : (x_1 \vee \neg x_2)$	$-, -$
$c_4 : x_2$	$0, 1$
$c_5 : \neg x_2$	$2, 3$
$c_6 : \perp$	$4, 5$

Unsatisfiability in Zero Knowledge Proof

Technique challenges and design overview

ZKUNSAT

Fetch clauses

$c_0 : (x_1 \vee x_2)$	$-, -$
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Unsatisfiability in Zero Knowledge Proof

Technique challenges and design overview

ZKUNSAT

Fetch clauses

Check application of resolution rule

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$c_1 : (\neg x_1 \vee x_2)$	$-,-$
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Unsatisfiability in Zero Knowledge Proof

Technique challenges and design overview

ZKUNSAT

Fetch clauses

Check application of resolution rule

ZK for integer comparison

$c_0 : (x_1 \vee x_2)$	$-,-$
$c_1 : (\neg x_1 \vee x_2)$	$-,-$
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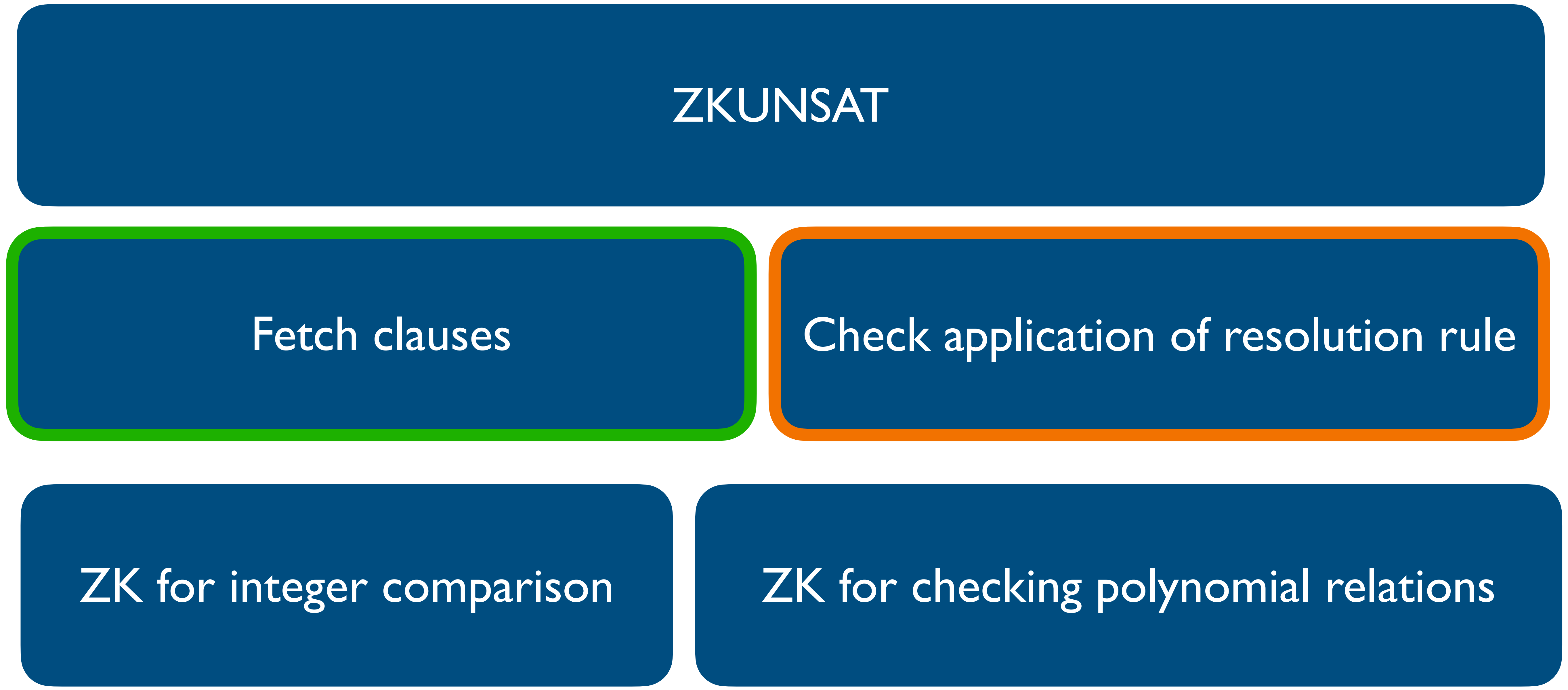
Unsatisfiability in Zero Knowledge Proof

Technique challenges and design overview



$c_0 : (x_1 \lor x_2)$	$-, -$
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Check application of resolution rule

Polynomial-based approach

Clause

$\ell_1 \vee \cdots \vee \ell_k$

Resolution

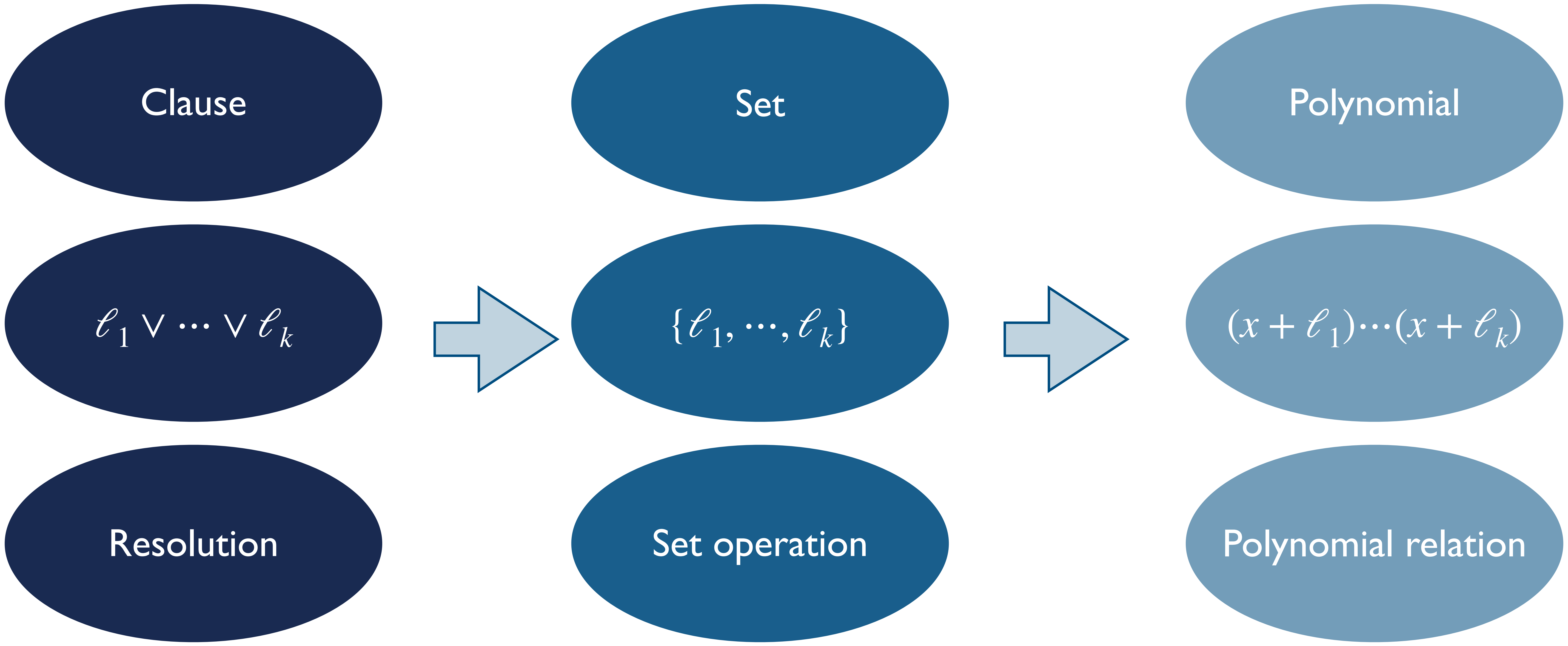
Check application of resolution rule

Polynomial-based approach

Clause		Set
$\ell_1 \vee \cdots \vee \ell_k$	$\Rightarrow$	$\{\ell_1, \cdots, \ell_k\}$
Resolution		Set operation

Check application of resolution rule

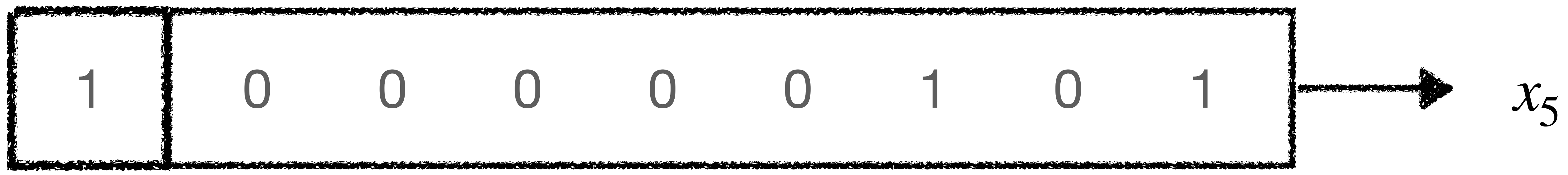
Polynomial-based approach



Check application of resolution rule

Polynomial-based approach

- An Encoding scheme ϵ from literals $\mathscr{L}$ to finite field $\mathbf{F}_{2^k} \backslash \mathbf{0}$



- Any Encoding scheme that satisfies
  - Injective
  - $\epsilon(\ell) + \epsilon(\neg \ell) = c, \ c$ is a public constant

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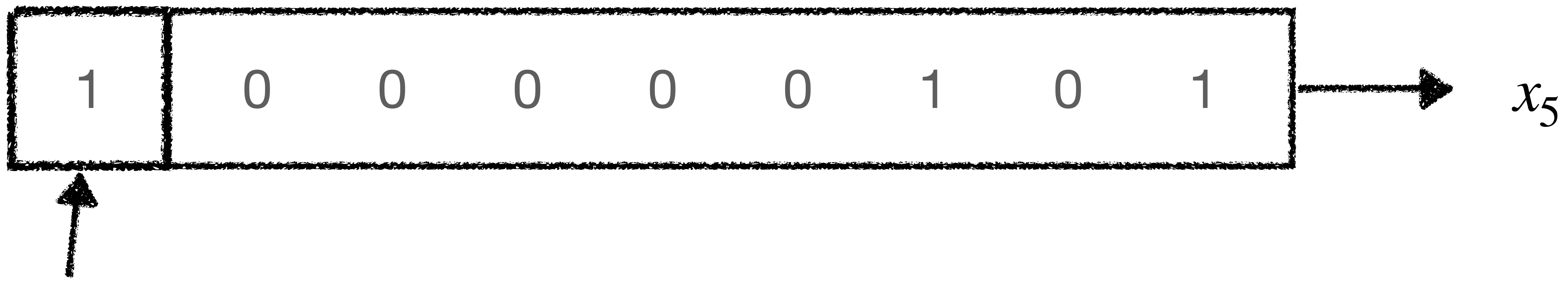

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Check application of resolution rule

Polynomial-based approach

- Clauses are encoded as polynomial over $\mathbf{F}_{2^k}$
  - $c = (\ell_0 \vee \ell_1 \vee \cdots \vee \ell_d) : p_c(x) = (x + \epsilon(\ell_0))\cdots(x + \epsilon(\ell_d))$
- Example

$$c_a = (x_1 \vee x_2) : p_{c_a}(x) = (x+2^{k-1}+1)(x+2^{k-1}+2)$$

$$c_b = (x_1 \vee \neg x_2) : p_{c_b}(x) = (x+2^{k-1}+1)(x+2)$$

Check application of resolution rule

Polynomial-Based Approach

Pivot literal

$$\frac{c_a = x_1 \vee x_2, \quad c_b = x_1 \vee \neg x_2}{c_r = x_1}$$

Check application of resolution rule

Polynomial-Based Approach

- Checking one resolution proof step:

Pivot literal

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Check application of resolution rule

Polynomial-Based Approach

- Checking one resolution proof step:
  - Prover prepares and inputs a pair of pivot polynomials: $(x + \epsilon(\ell_p)), (x + \epsilon(\neg \ell_p))$

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Polynomial-Based Approach

- Checking one resolution proof step:
  - Prover prepares and inputs a pair of pivot polynomials: $(x + \epsilon(\ell_p)), (x + \epsilon(\neg \ell_p))$
    - Pivot polynomials: $(x + 2^{k-1} + 2)$ and $(x + 2)$

Pivot literal

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    - $p_{c_r} = (x + 2^{k-1} + 1)$

Pivot literal

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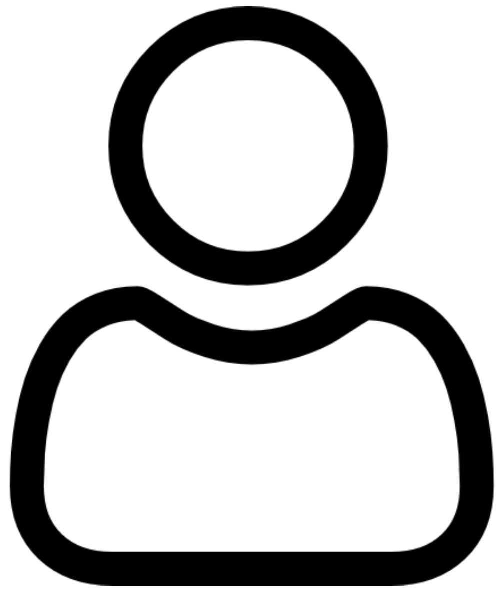
Check application of resolution rule

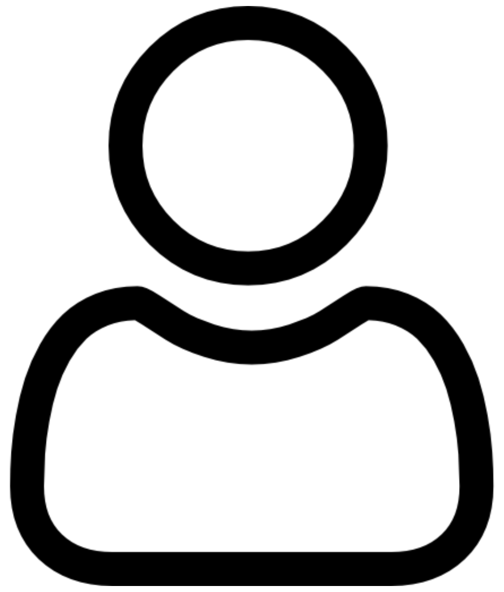
Polynomial-Based Approach

$$\frac{c_a = x_1 \vee x_2, \quad c_b = x_1 \vee \neg x_2}{c_r = x_1}$$

- $C_a \subseteq C_r \cup \{\ell_p\}$, $C_b \subseteq C_r \cup \{\neg \ell_p\}$
  - Proving $p_{c_a} | p_{c_r} \cdot (x + \epsilon(\ell_p))$ and $p_{c_b} | p_{c_r} \cdot (x + \epsilon(\neg \ell_p))$
  - Prover prepares and inputs w_a and w_b
  - Verifier checks
    - $p_{c_a} | p_{c_r} \cdot (x + \epsilon(\ell_p))$ via $p_{c_a} \cdot w_a = p_{c_r} \cdot (x + \epsilon(\ell_p))$
    - $p_{c_b} | p_{c_r} \cdot (x + \epsilon(\neg \ell_p))$ via $p_{c_b} \cdot w_b = p_{c_r} \cdot (x + \epsilon(\neg \ell_p))$

Check application of resolution rule





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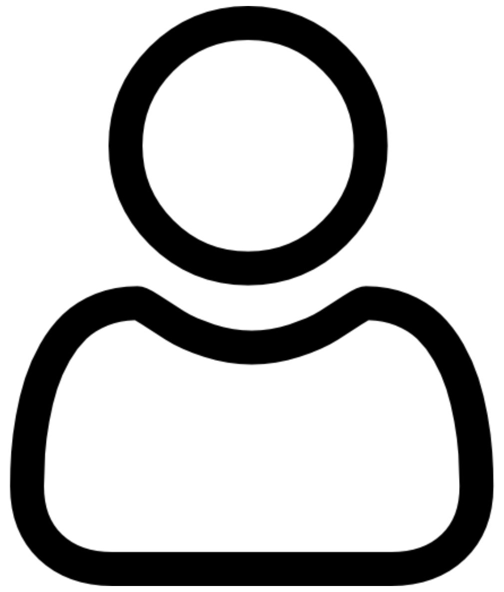

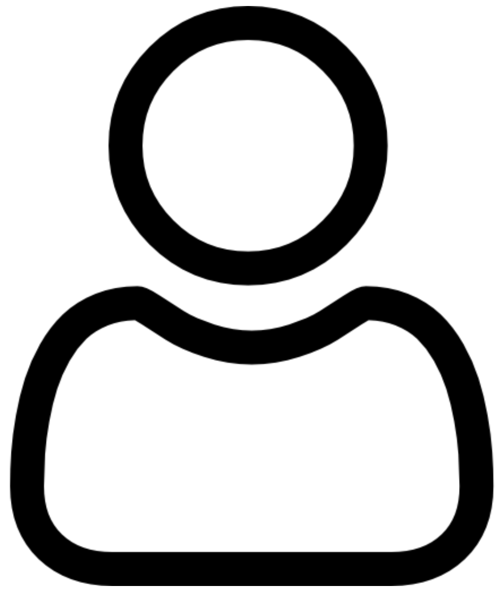

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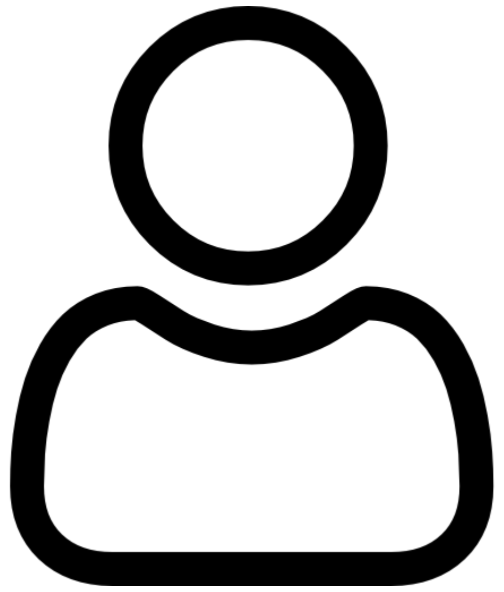

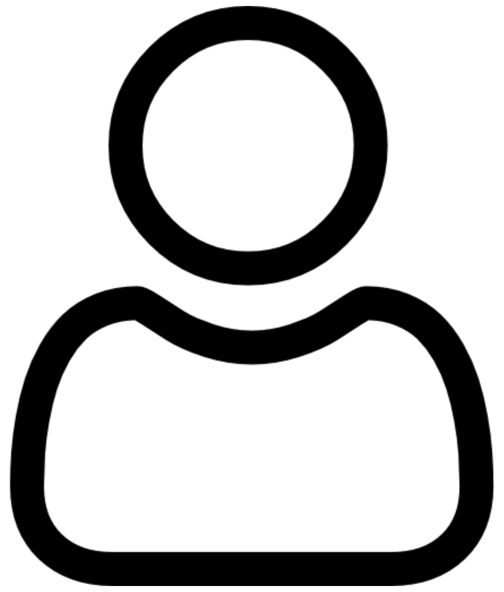

$$p_{c_a} = (x + 2^{k-1} + 1)(x + 2^{k-1} + 2)$$

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$$p_{c_r} = (x + 2^{k-1} + 1)$$

pivot literals: $p_{x_2} = (x + 2^{k-1} + 2),\ p_{\neg x_2} = (x + 2)$

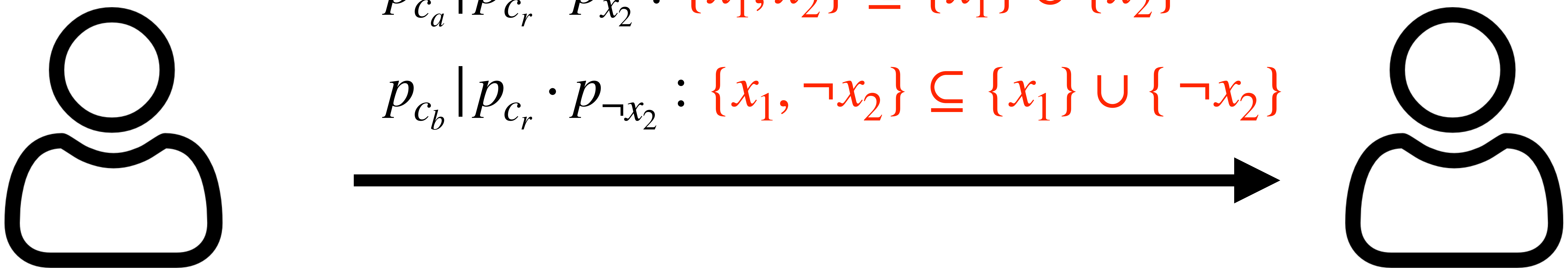
$$\frac{c_a = x_1 \vee x_2, \quad c_b = x_1 \vee \neg x_2}{c_r = x_1}$$

Check application of resolution rule

c_a c_r pivot

$p_{c_a} | p_{c_r} \cdot p_{x_2} : \{x_1, x_2\} \subseteq \{x_1\} \cup \{x_2\}$

$p_{c_b} | p_{c_r} \cdot p_{\neg x_2} : \{x_1, \neg x_2\} \subseteq \{x_1\} \cup \{\neg x_2\}$



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Unsatisfiability in Zero Knowledge Proof

ZKUNSAT

Fetch Clauses

Check application of resolution rule

ZK for integer comparison

ZK for polynomial relations

$$
\begin{array}{ll}
c_0 : (x_1 \vee x_2) & -, - \\
c_1 : (\neg x_1 \vee x_2) & -, - \\
c_2 : (\neg x_1 \vee \neg x_2) & -, - \\
c_3 : (x_1 \vee \neg x_2) & -, - \\
c_4 : x_2 & 0, 1 \\
c_5 : \neg x_2 & 2, 3 \\
c_6 : \perp & 4, 5
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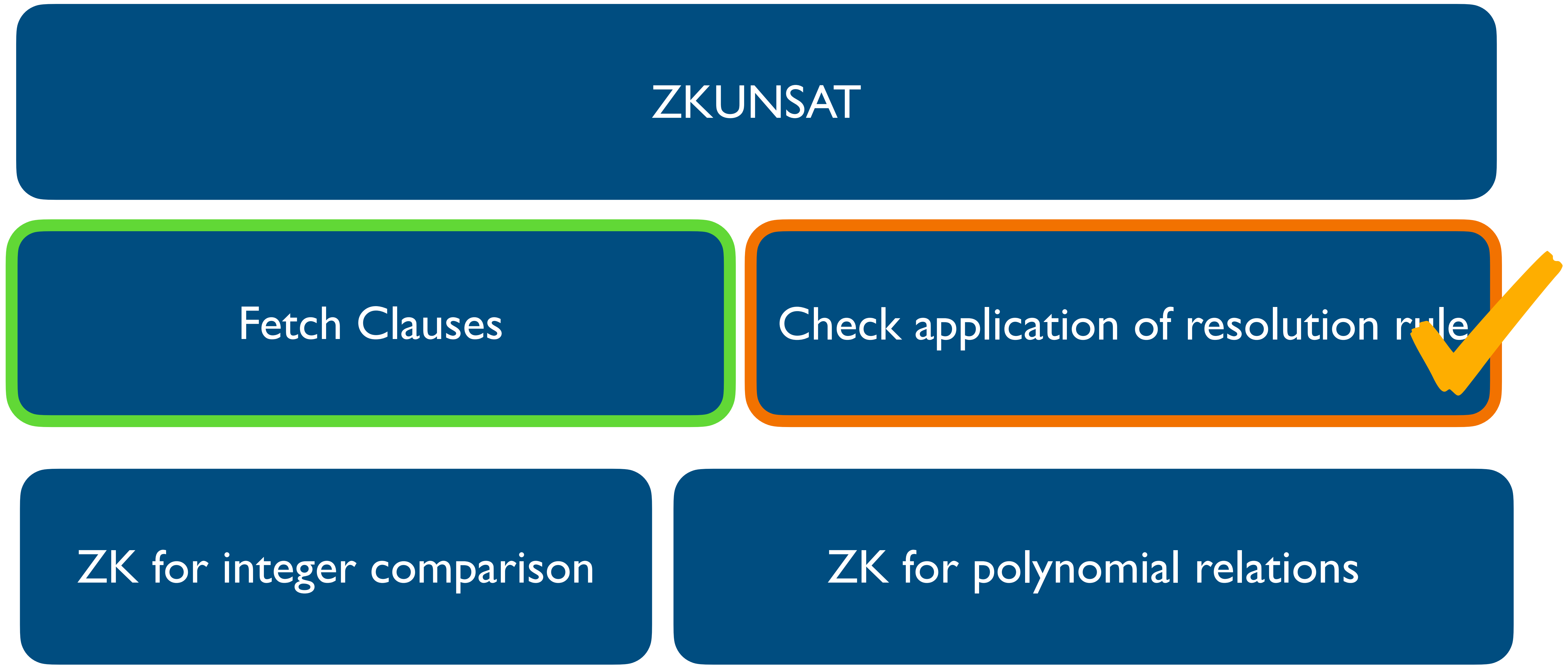
Unsatisfiability in Zero Knowledge Proof

Given clauses c_a, c_b, c_r , we now can check $c_a, c_b \vdash_{res} c_r$

But how we fetch c_a and c_b without revealing the access?



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Zero Knowledge Proof

Read-Only Array in ZK [Franzese, 21]

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Read-Only Array in ZK [Franzese, 21]

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  - Prover sends i, c_i to the verifier when needed.

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  - Record $(0,c_0), (1,c_1), (2,c_2), (4,c_4), (5,c_5), (3,c_3), (4,c_4), (6,c_6)$

Zero Knowledge Proof

Read-Only Array in ZK (example)

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- Check if all records are consistent periodically

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        - $L = L'$ in the sense of set (using polynomial equivalence)

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    - In L', either $i'_{t+1} > i'_t$ or $c_{i'_t} = c_{i'_{t+1}}$ when $i'_t = i'_{t+1}$ holds

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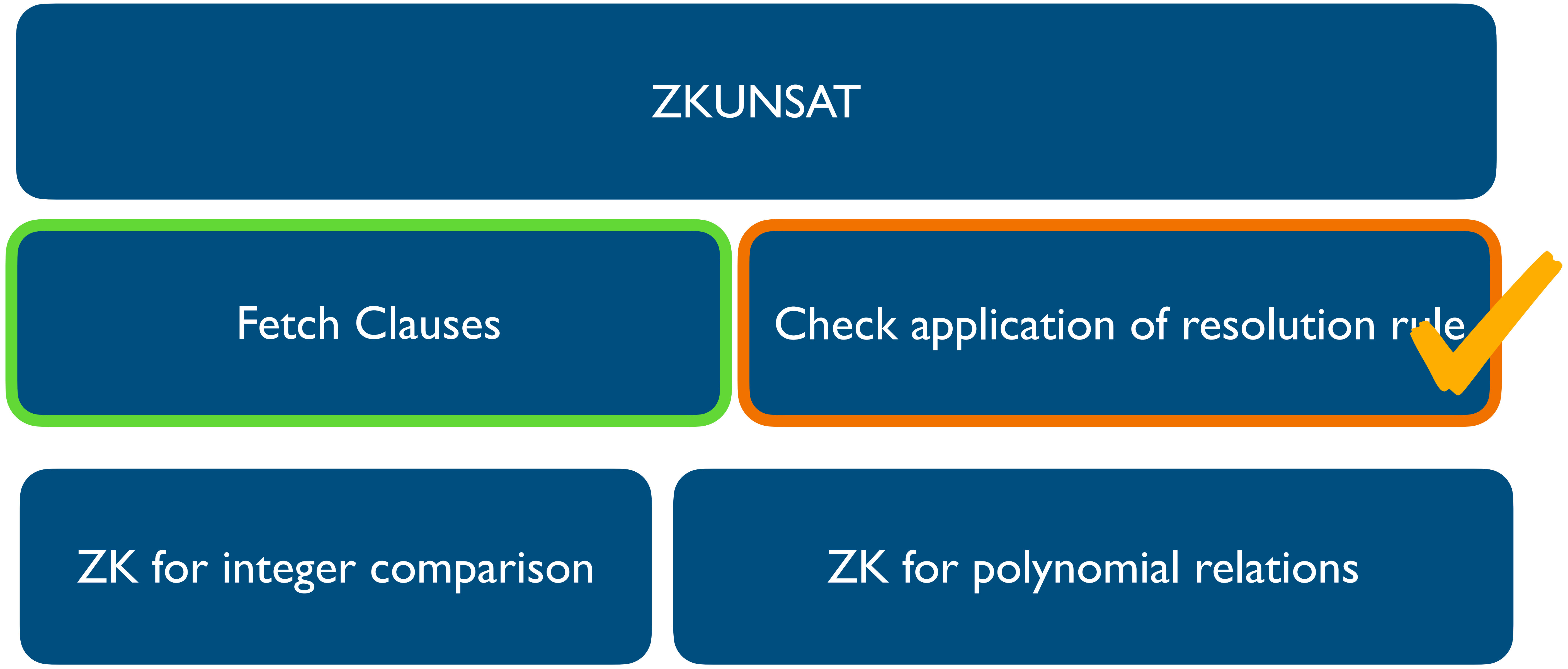

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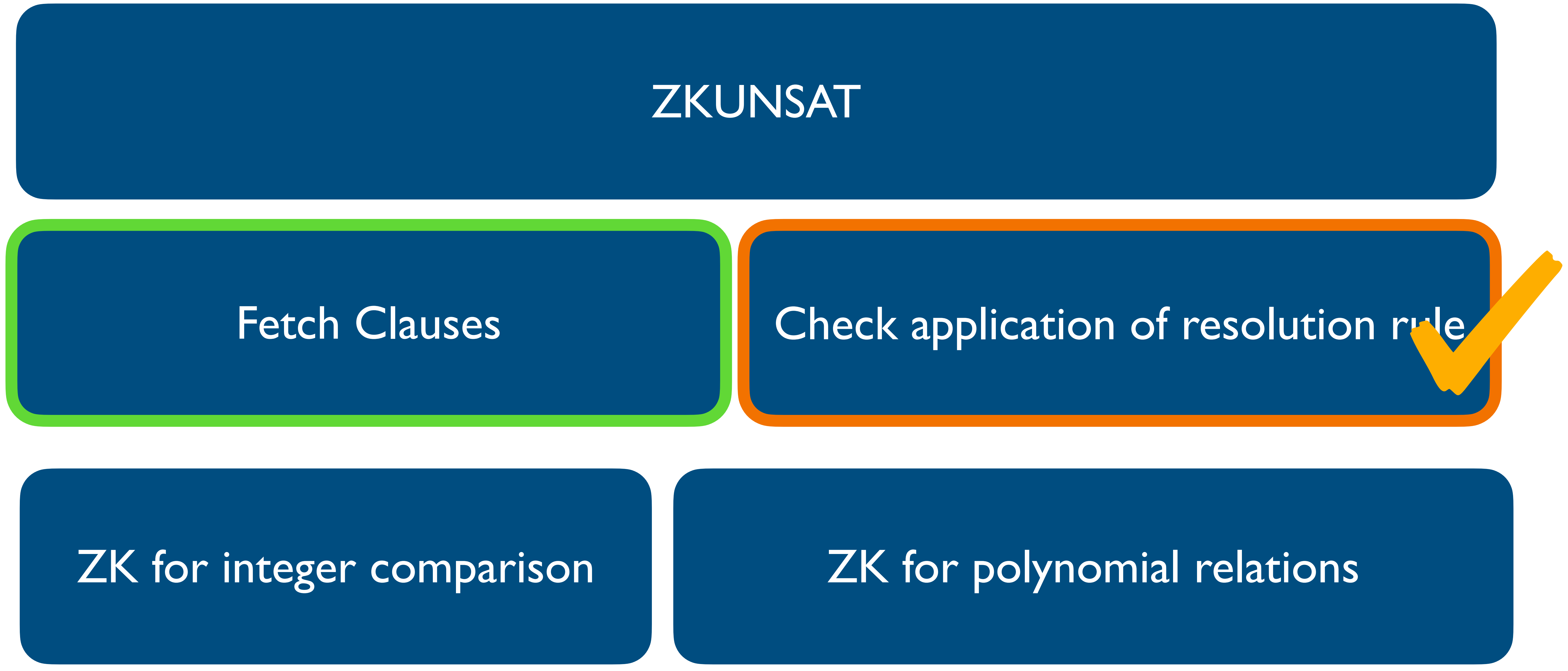
Unsatisfiability in Zero Knowledge Proof

Put them together:

- Fetch input clauses for each resolution using ROARRAY in ZK
- Check application of the resolution rule using polynomial relations in ZK



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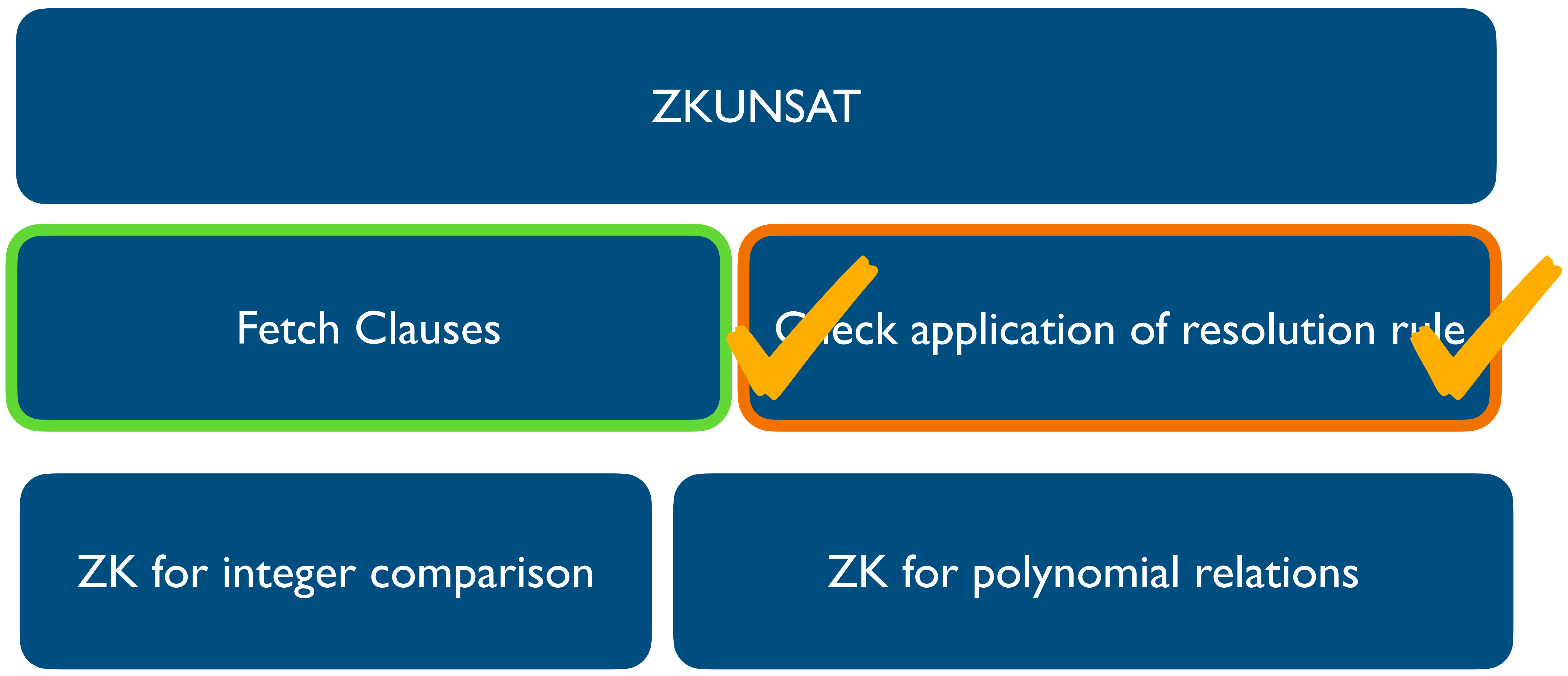

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Evaluation

Benchmark setting

- AWS instances of type r5b.2xlarge
- 64 GB of memory, 16 vCPUs
- 10 Gbps network connection between the prover and the verifier

Evaluation

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Width: the maximum number of literals a clause in the proof can have.

Width = 2 in this example

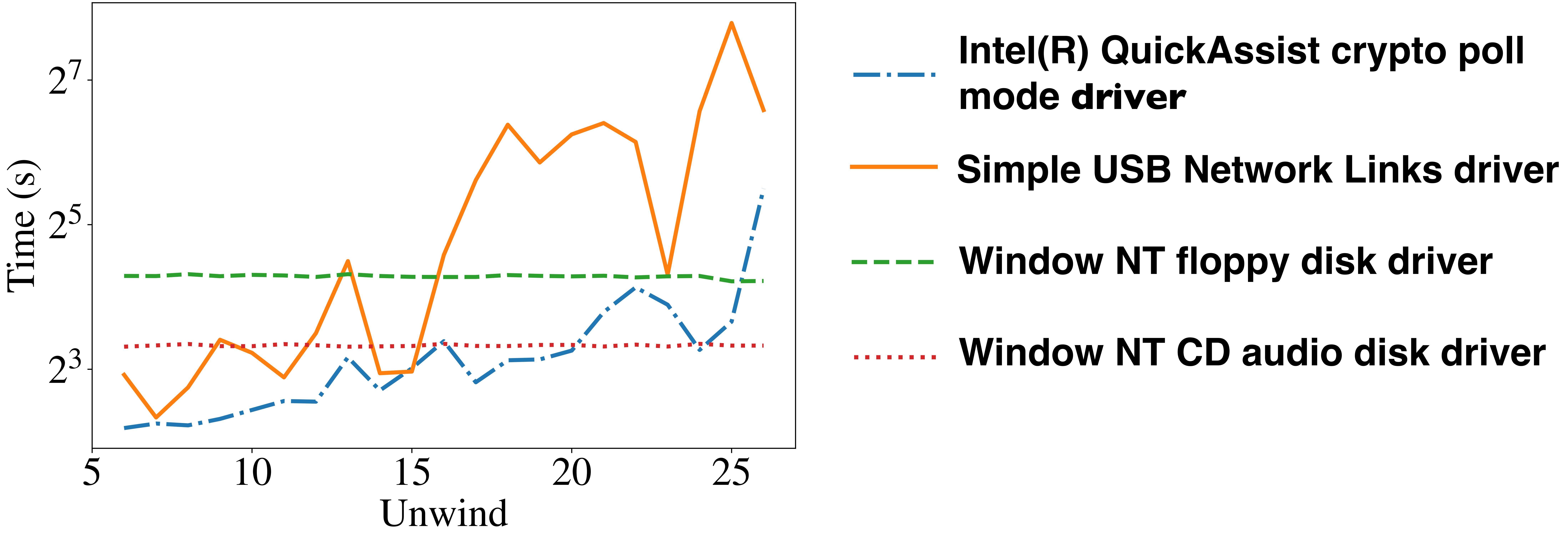
Length: the number of applications of resolution rule.

Length = 3 in this example

- Time and memory requirements **depend on the length and clausal width** of the proof
- ZKUNSAT takes **less than 1 min** to verify proofs of large width (400) and length (8000)

Evaluation

Verification tasks for system drivers



- Unwind is a parameter for translating the verification tasks to Boolean formulae
- Width ≤ 256 and Length $\leq 65K$
- ZKUNSAT can verify UNSAT of formulae from **system drivers verification tasks within 5min**

Evaluation

Other large instances

Program	Len. (K)	Width	Time (s)
inv-square-int	194	414	172.5
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inv-sqrt-quake	182	749	5764.1
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Other large instances

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Further improvement via computing clusters: work to appear in CCS 2023

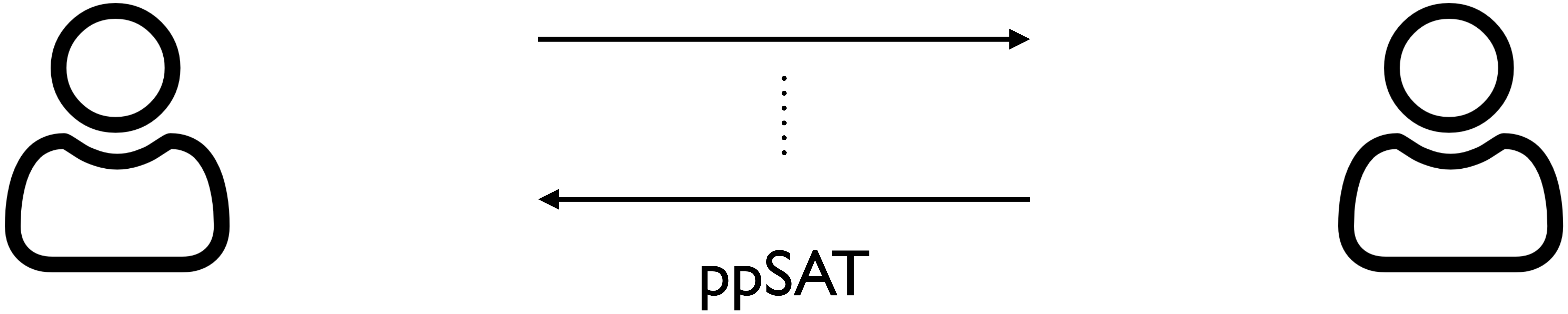
Contribution

- Privacy preserving program verification is in demand
- Encoding resolution proof by polynomials
- UNSAT in ZK is practical

Future Work

Future Work

- **SAT : Privacy-preserving SAT solving (ppSAT)**



$\phi_0 = (x_0 \vee x_1) \wedge (x_0 \vee \neg x_1)$

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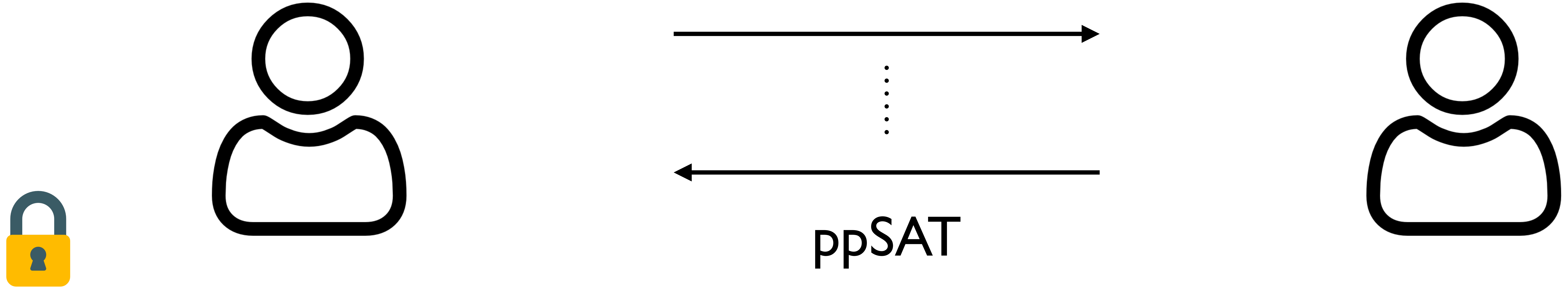

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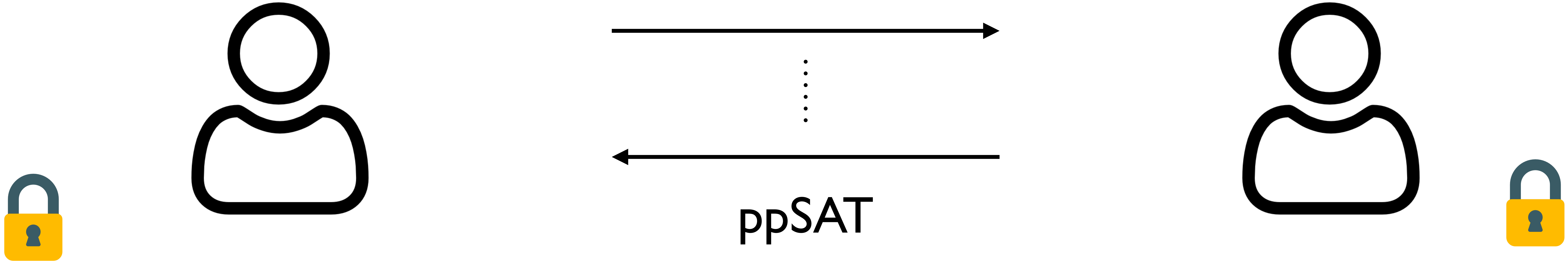

$\phi_0 = (x_0 \lor x_1) \land (x_0 \lor \neg x_1)$

$\phi_1 = (\neg x_0 \lor x_1) \land (\neg x_0 \lor \neg x_1)$

Future Work

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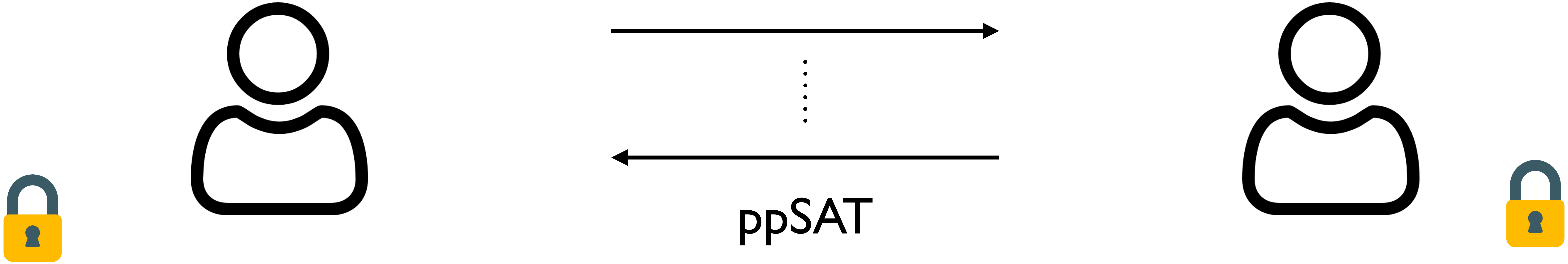

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ppSAT: Towards Two-Party Private SAT Solving, USENIX Security 2022

Boolean SAT Solving: A DPLL Example

$$\phi(x_0, x_1, x_2, x_3) = (x_3 \vee x_0 \vee x_1) \wedge (\neg x_0 \vee x_2) \wedge (x_1 \vee x_2) \wedge \neg x_1 \wedge (\neg x_3 \vee \neg x_0) \wedge (\neg x_0 \vee x_3)$$

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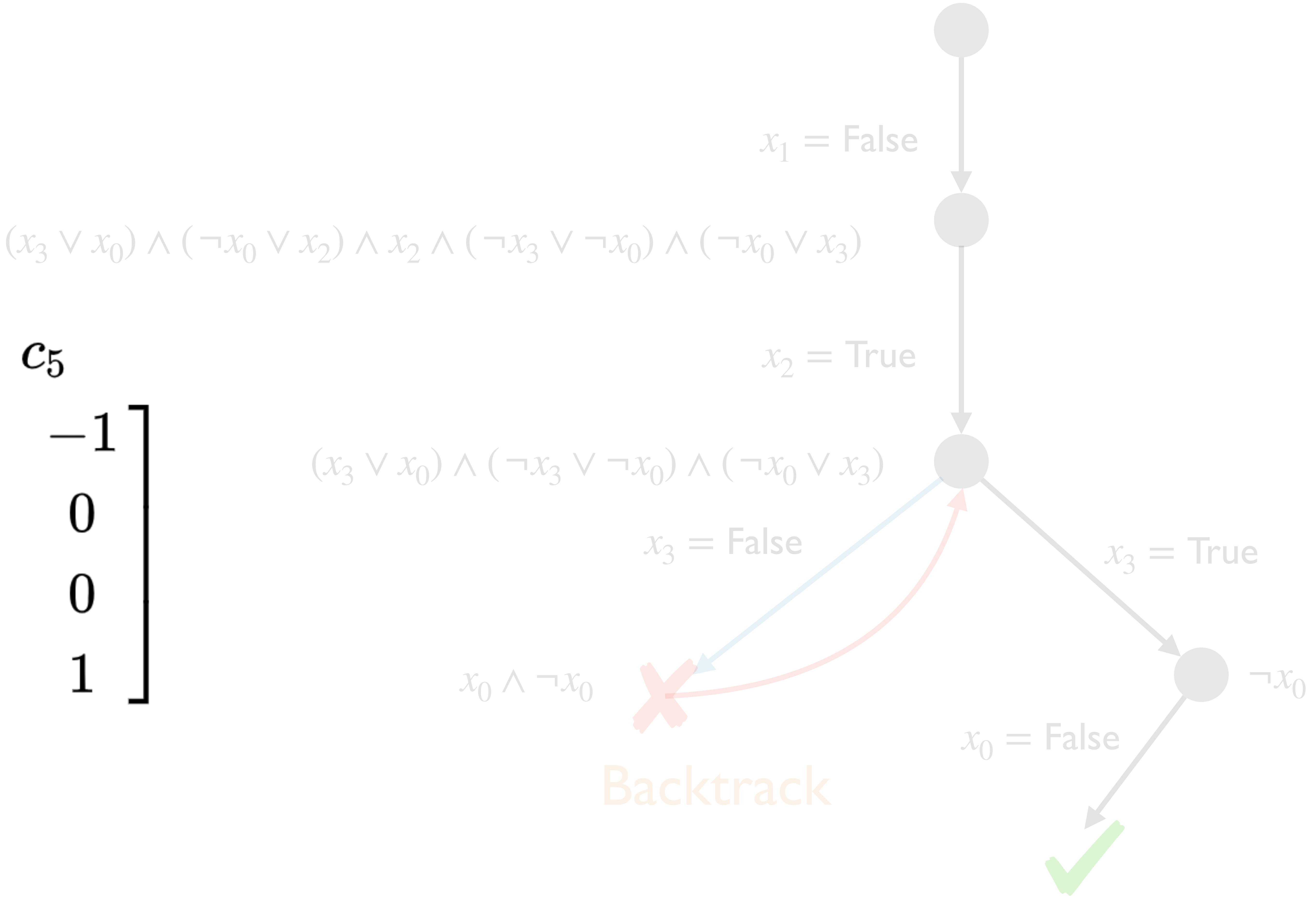
$$\begin{array}{cc} & \begin{array}{cccccc} c_0, & c_1, & c_2, & c_3, & c_4, & c_5 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & -1 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$



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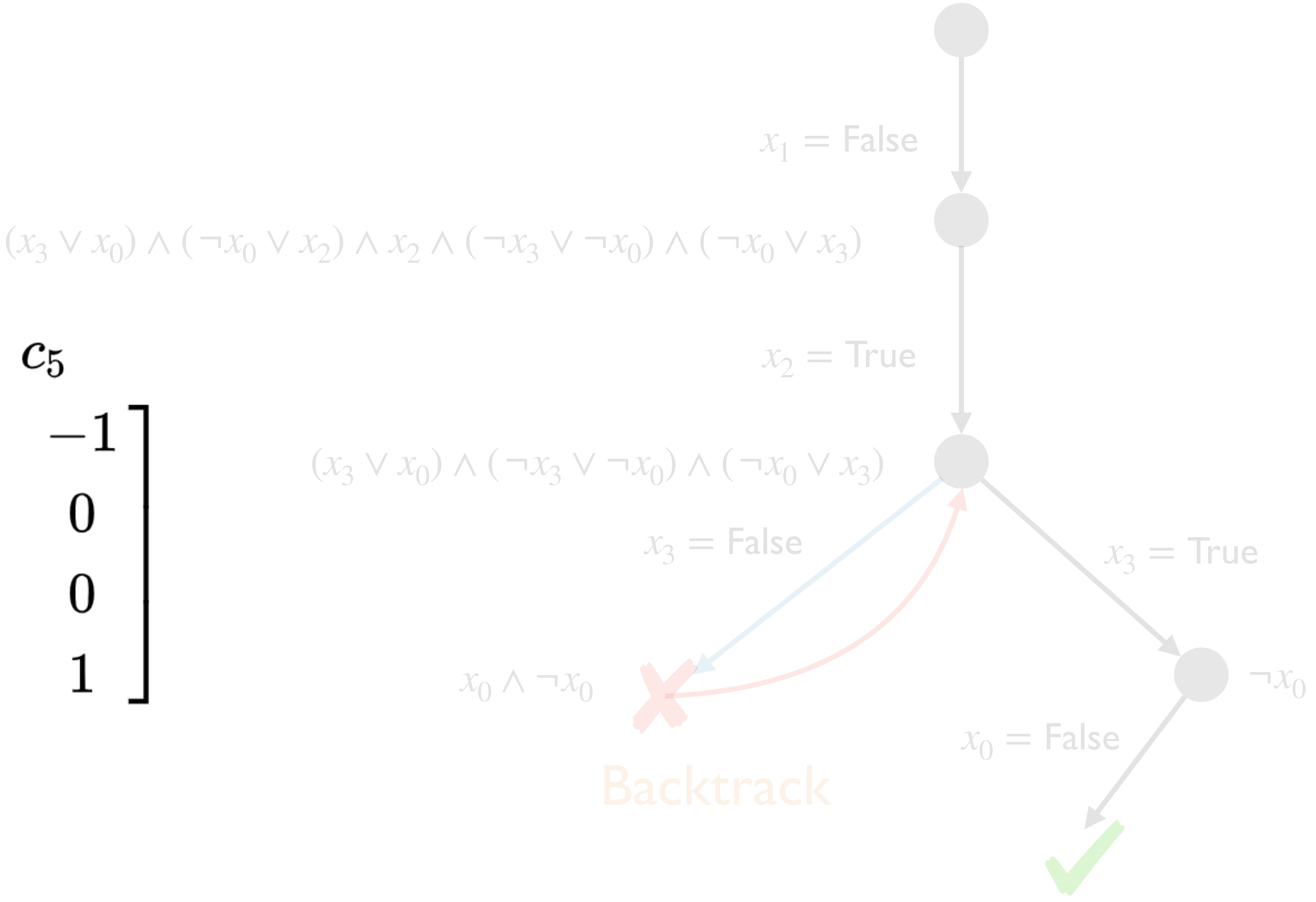

Boolean SAT Solving: A DPLL Example

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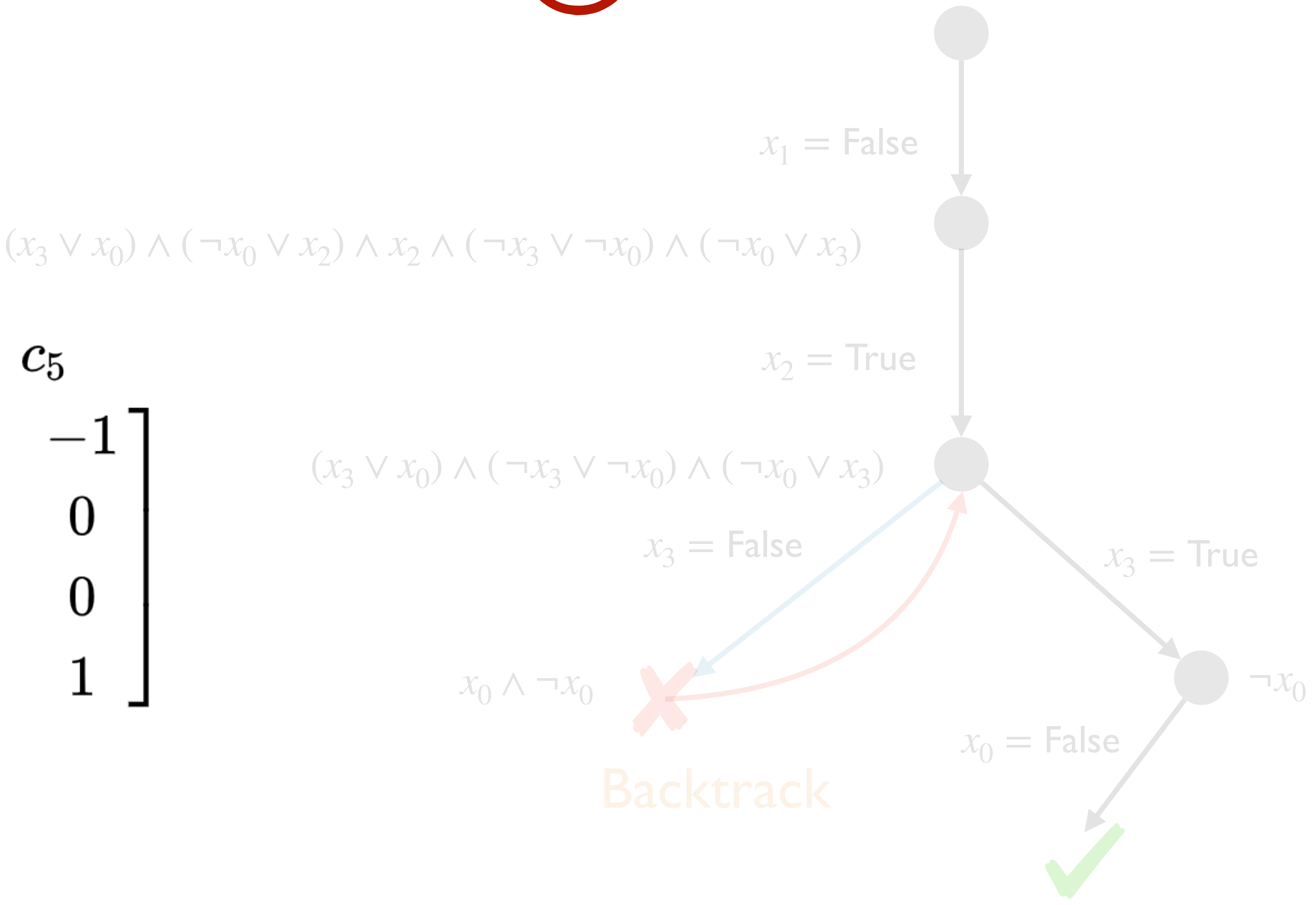

Boolean SAT Solving: A DPLL Example

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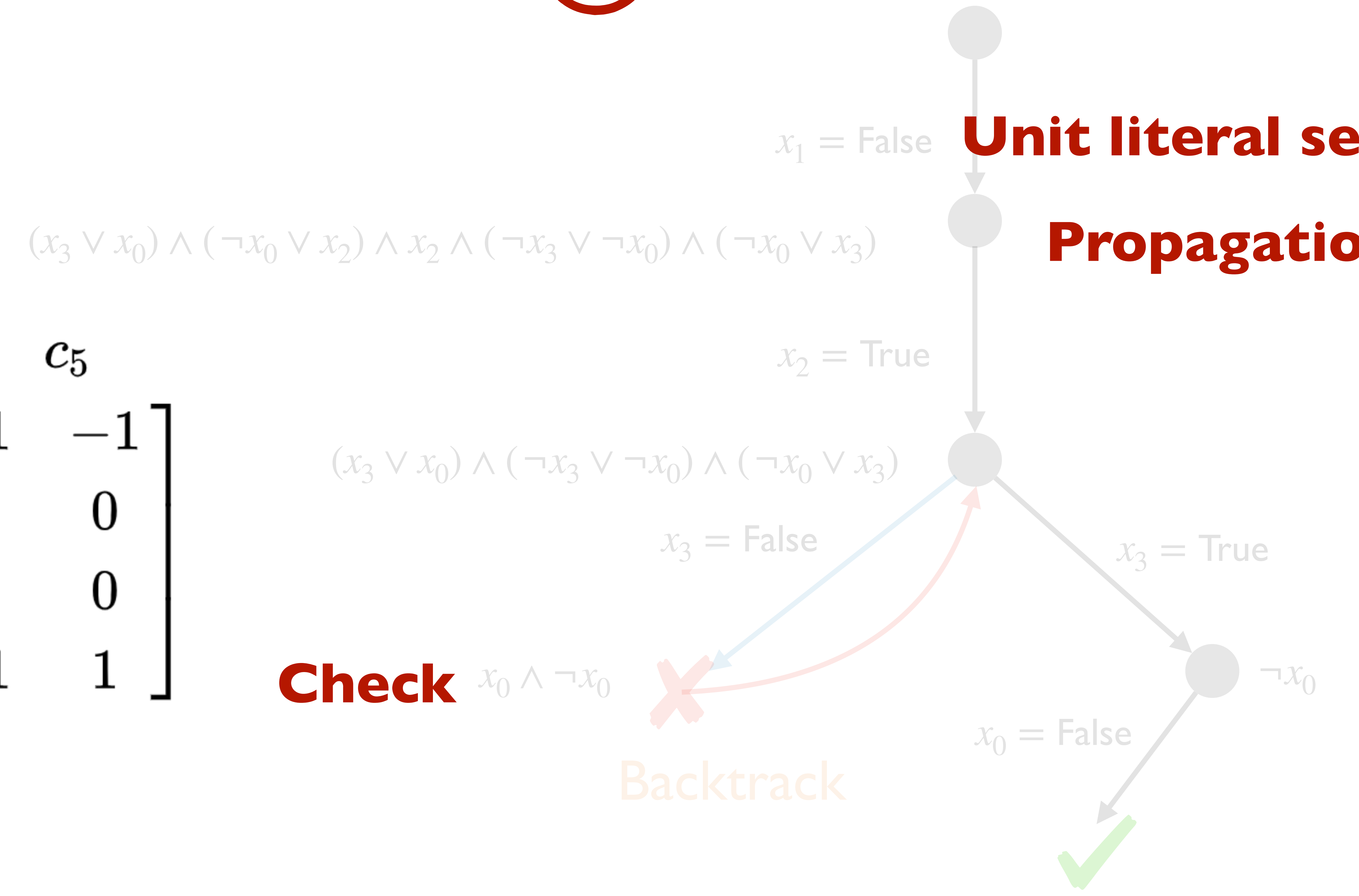
Unit literal search

Propagation



$$\begin{array}{c c} & \begin{matrix} c_0, & c_1, & c_2, & c_3, & c_4, & c_5 \end{matrix} \\ \begin{matrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & -1 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

Check

Boolean SAT Solving: A DPLL Example

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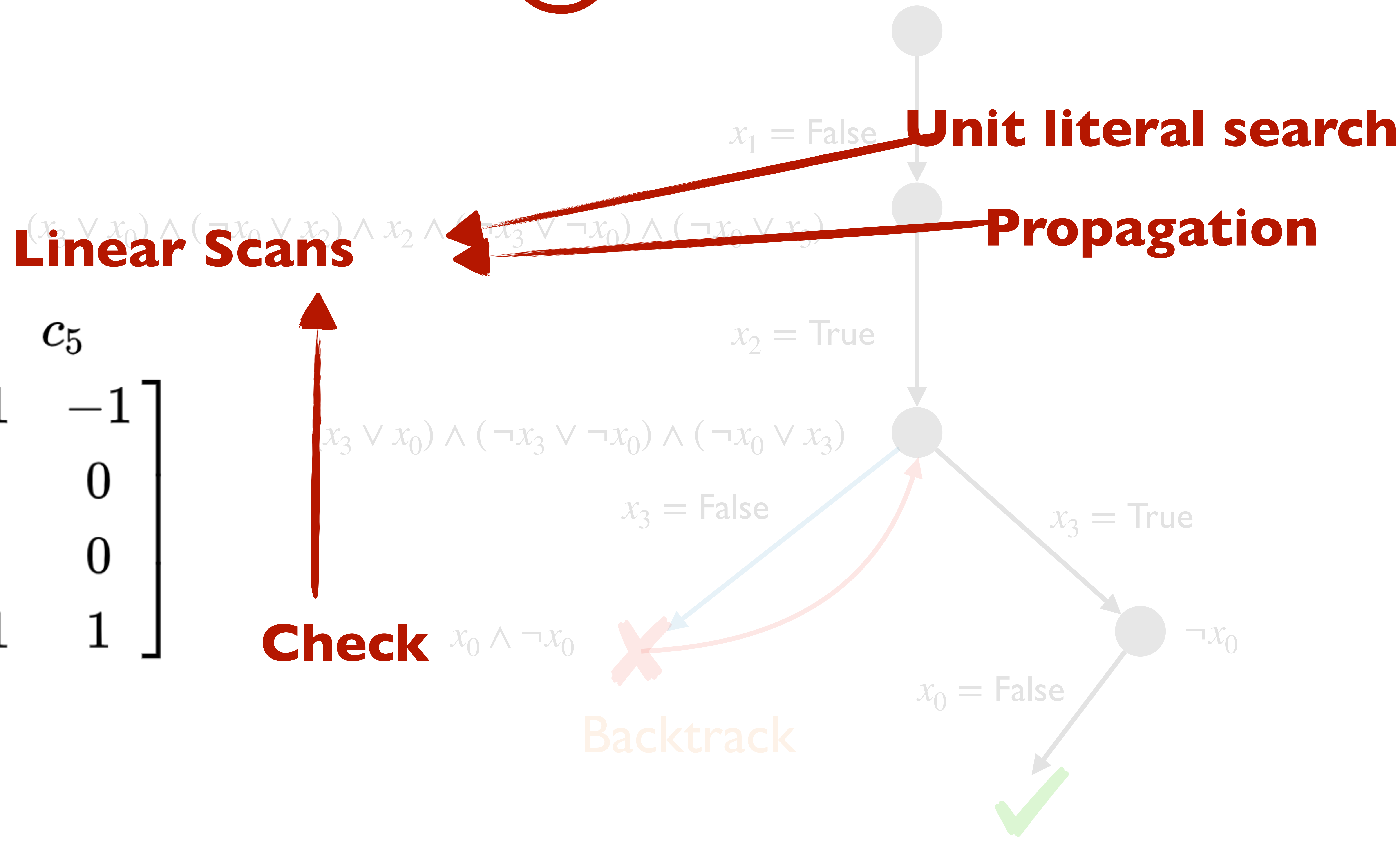

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\end{array}$$

Future Work

- **DLIS**
  - Select the most commonly appearing literal and the smallest index
  - Return the assignment that makes it true
  - Deterministic
  - Example: DLIS will guess $\neg x_0$ for $(x_3 \vee x_0) \wedge (\neg x_3 \vee \neg x_0) \wedge (\neg x_0 \vee x_3)$

Heuristics

- **Rand**
  - Uniformly select a random undecided literal
  - Randomized
  - Example: $(x_3 \vee x_0) \wedge (\neg x_3 \vee \neg x_0) \wedge (\neg x_0 \vee x_3)$

Randomly guess one of $\{x_0, \neg x_0, x_3, \neg x_3\}$ each with $\dfrac{1}{4}$ probability

Future Work

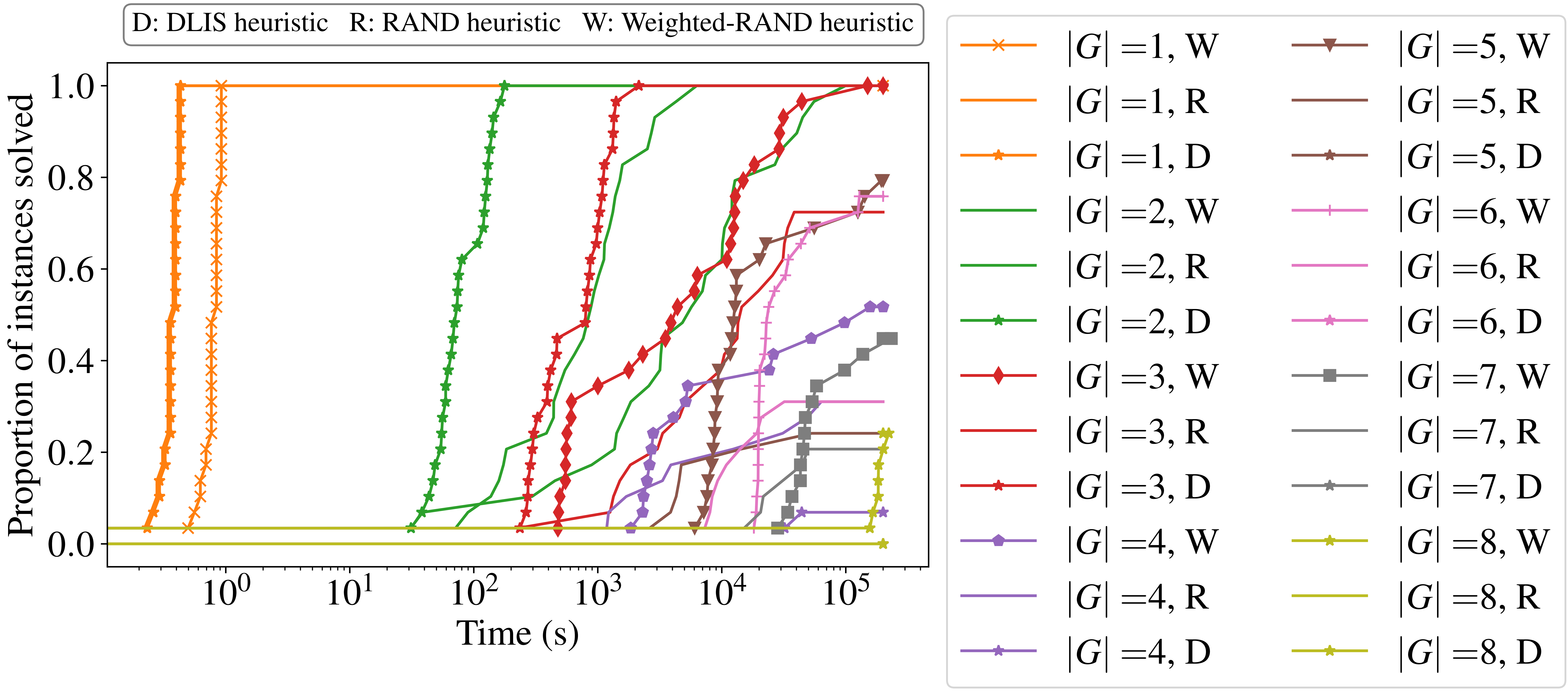
- **Weighted-Rand**
  - Select a random undecided literal according to its frequency
  - Randomized
  - Example: $(x_3 \vee x_0) \wedge (\neg x_3 \vee \neg x_0) \wedge (\neg x_0 \vee x_3)$

guess x_0 with $\frac{1}{6}$ chance, guess $\neg x_0$ with $\frac{2}{6}$ chance, etc.

Future Work

Future Work

ppSAT: Towards Two-Party Private SAT Solving, USENIX Security 2022



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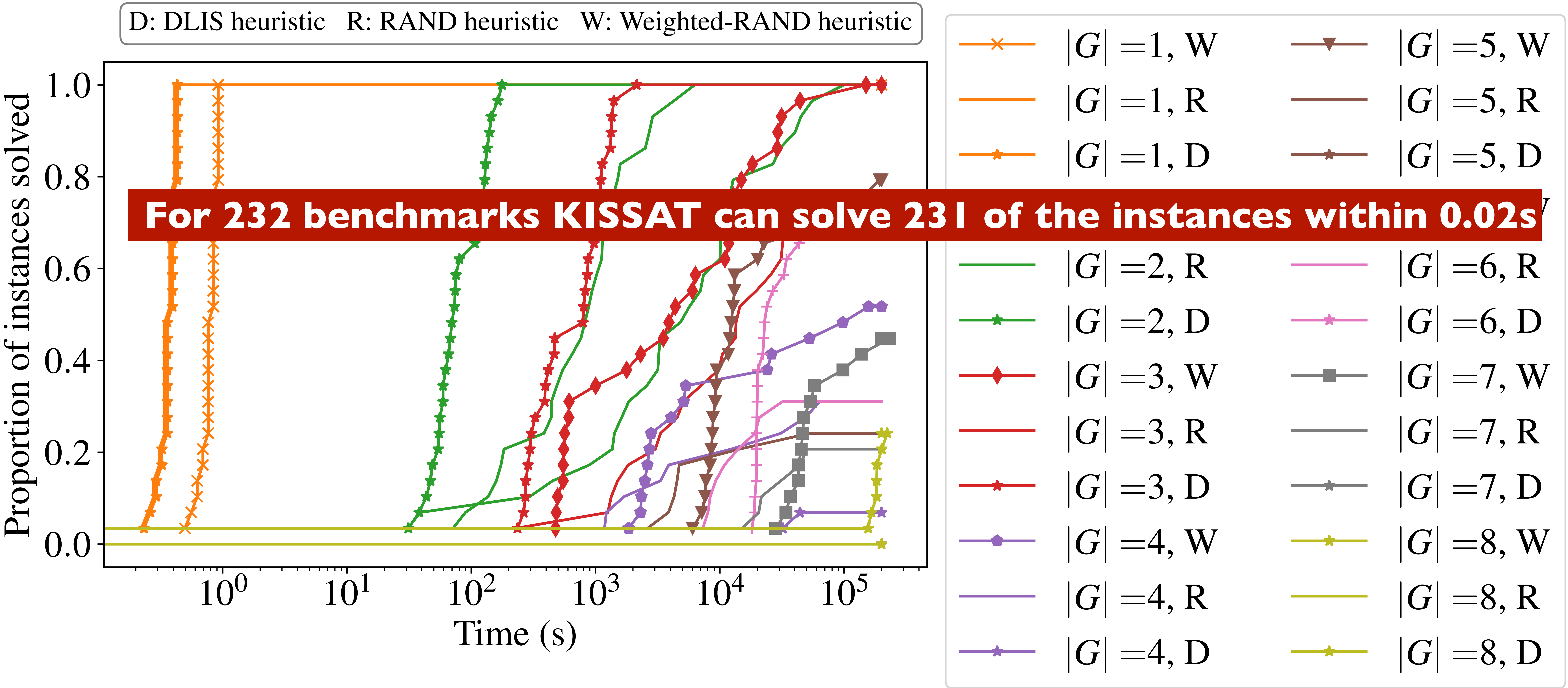

For 232 benchmarks KISSAT can solve 231 of the instances within 0.02s

Future Work

Future Work

Better heuristics when it comes to privacy preserving setting?

Thank you!

Ning Luo: ning.luo@northwestern.edu

https://github.com/PP-FM

I am on job market

Reference

Donald Beaver, Silvio Micali, and Phillip Rogaway. The round complexity of secure protocols (extended abstract). In 22nd ACM STOC, pages 503–513, Baltimore, MD, USA, May 14–16, 1990. ACM Press.

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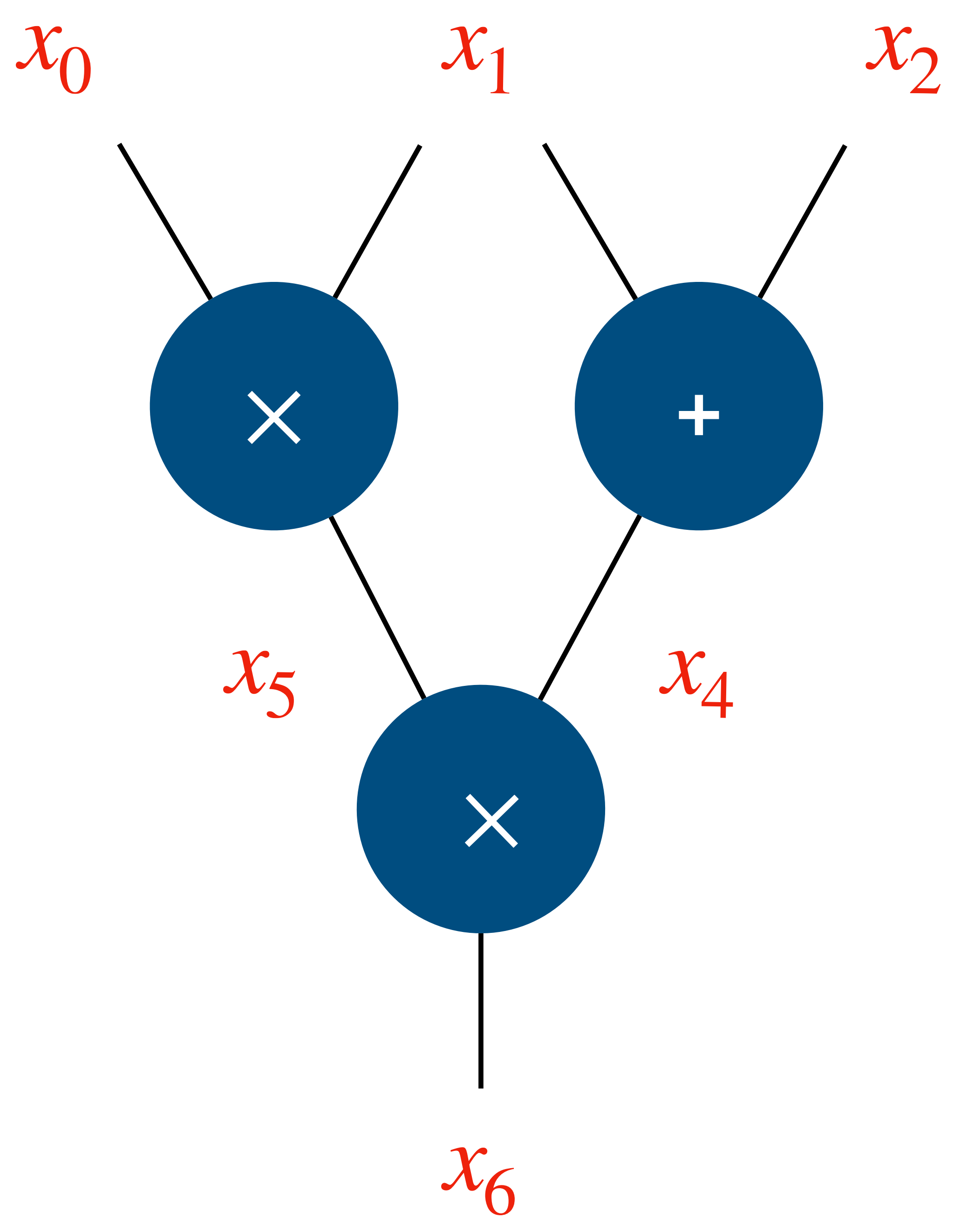
Nicholas Franzese, Jonathan Katz, Steve Lu, Rafail Ostrovsky, Xiao Wang, and Chenkai Weng. 2021. Constant-Overhead Zero-Knowledge for RAM Programs. In <i>Proceedings of the 2021 ACM SIGSAC Conference on Computer and Communications Security</i> (<i>CCS '21</i>). Association for Computing Machinery, New York, NY, USA, 178–191. DOI:https://doi.org/10.1145/3460120.3484800

Zero Knowledge Proof

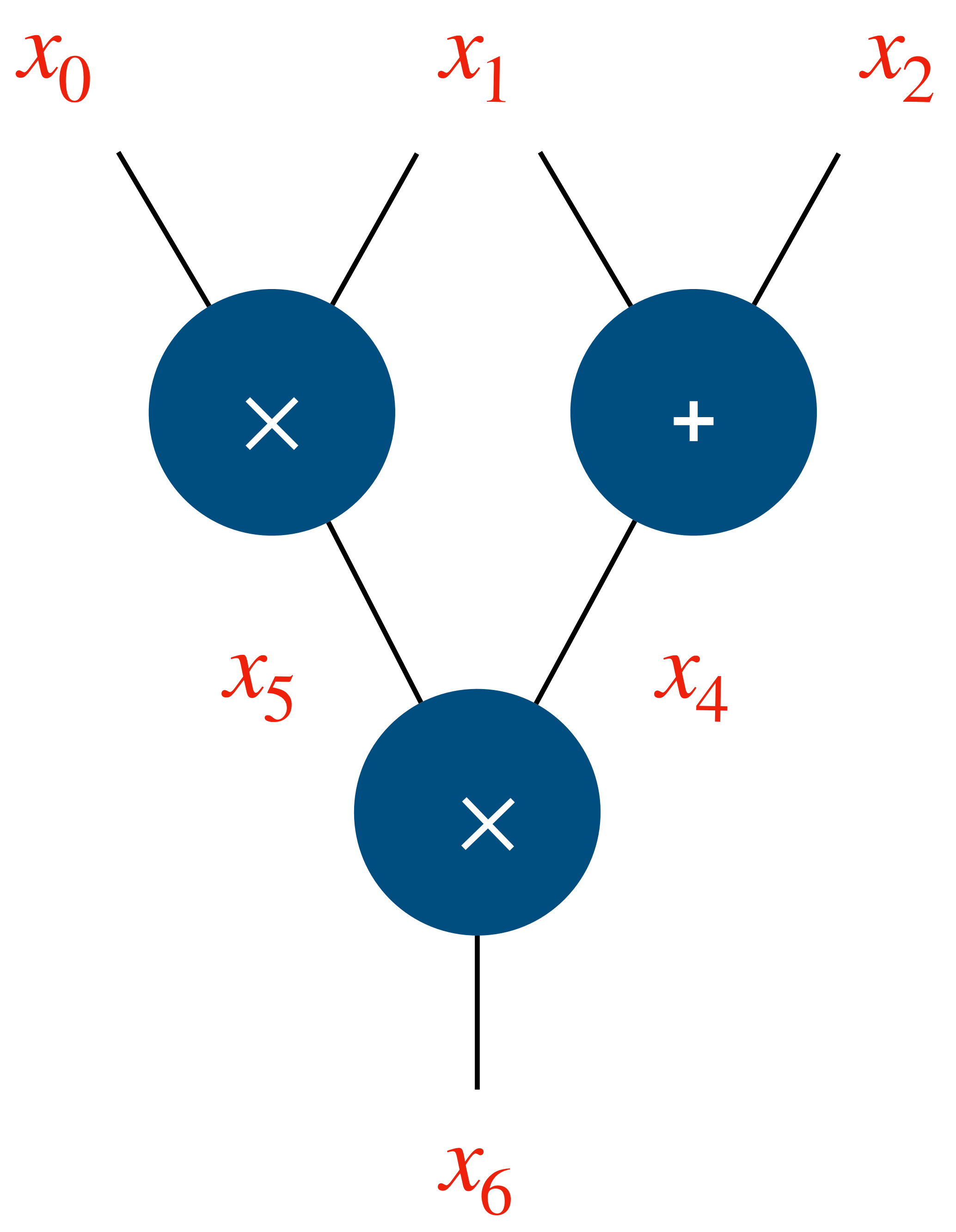
ZKP based on information theoretic MAC



Zero Knowledge Proof

ZKP based on information theoretic MAC

- Information-Theoretic MAC : $k_x = M_x + x \cdot \Delta$



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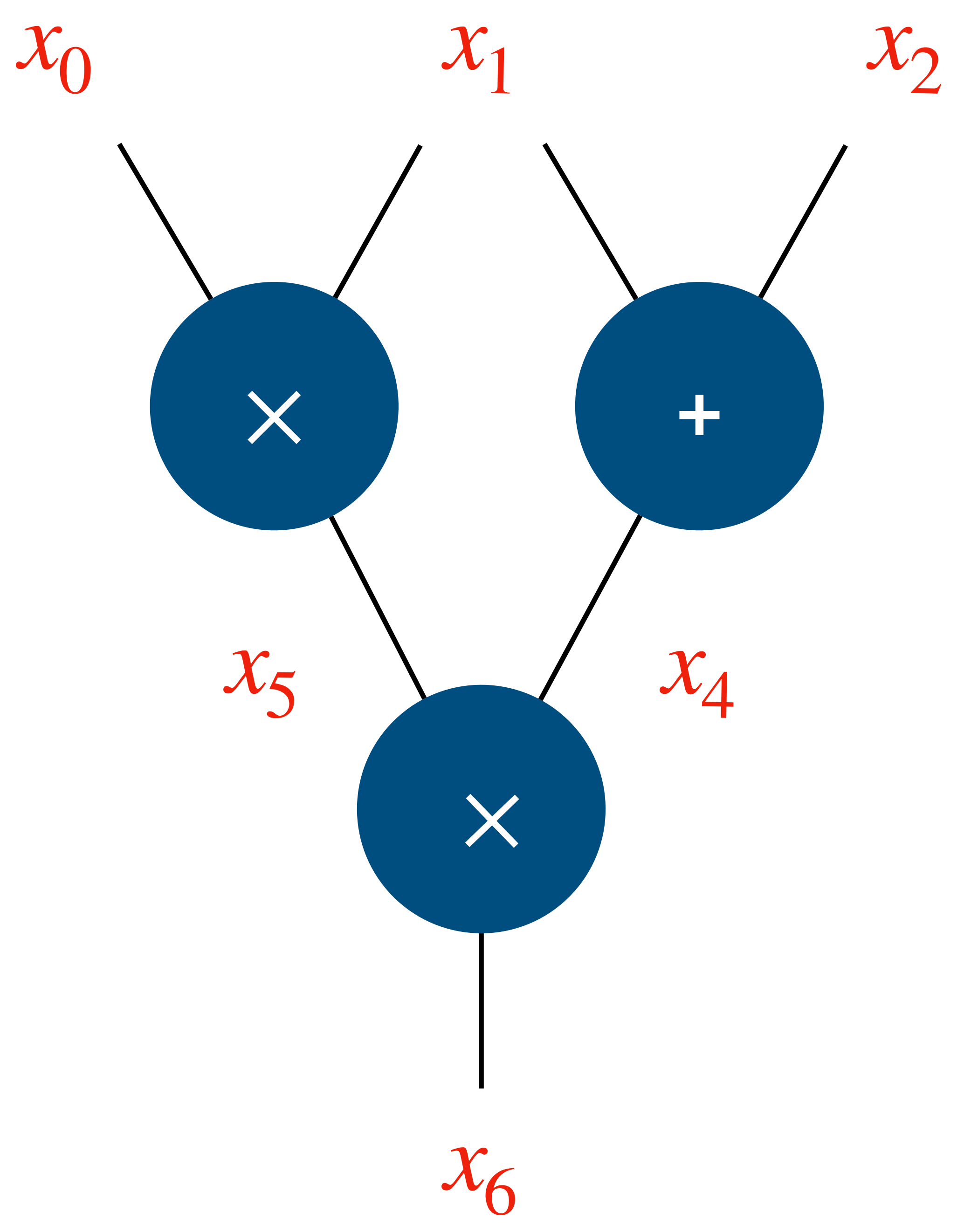

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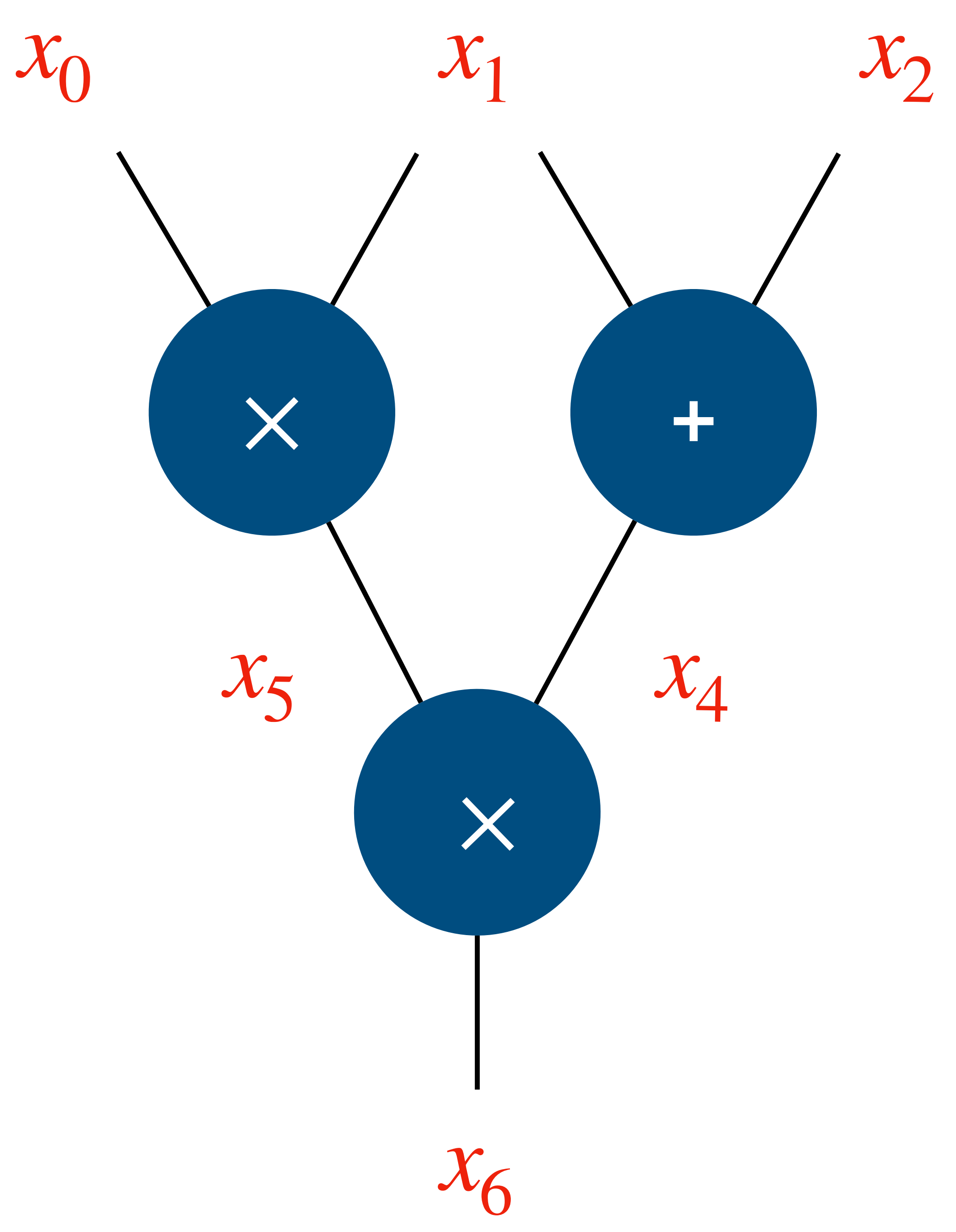
- Information-Theoretic MAC : $k_x = M_x + x \cdot \Delta$
    - x if x is shared in such a way



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- Information-Theoretic MAC : $k_x = M_x + x \cdot \Delta$
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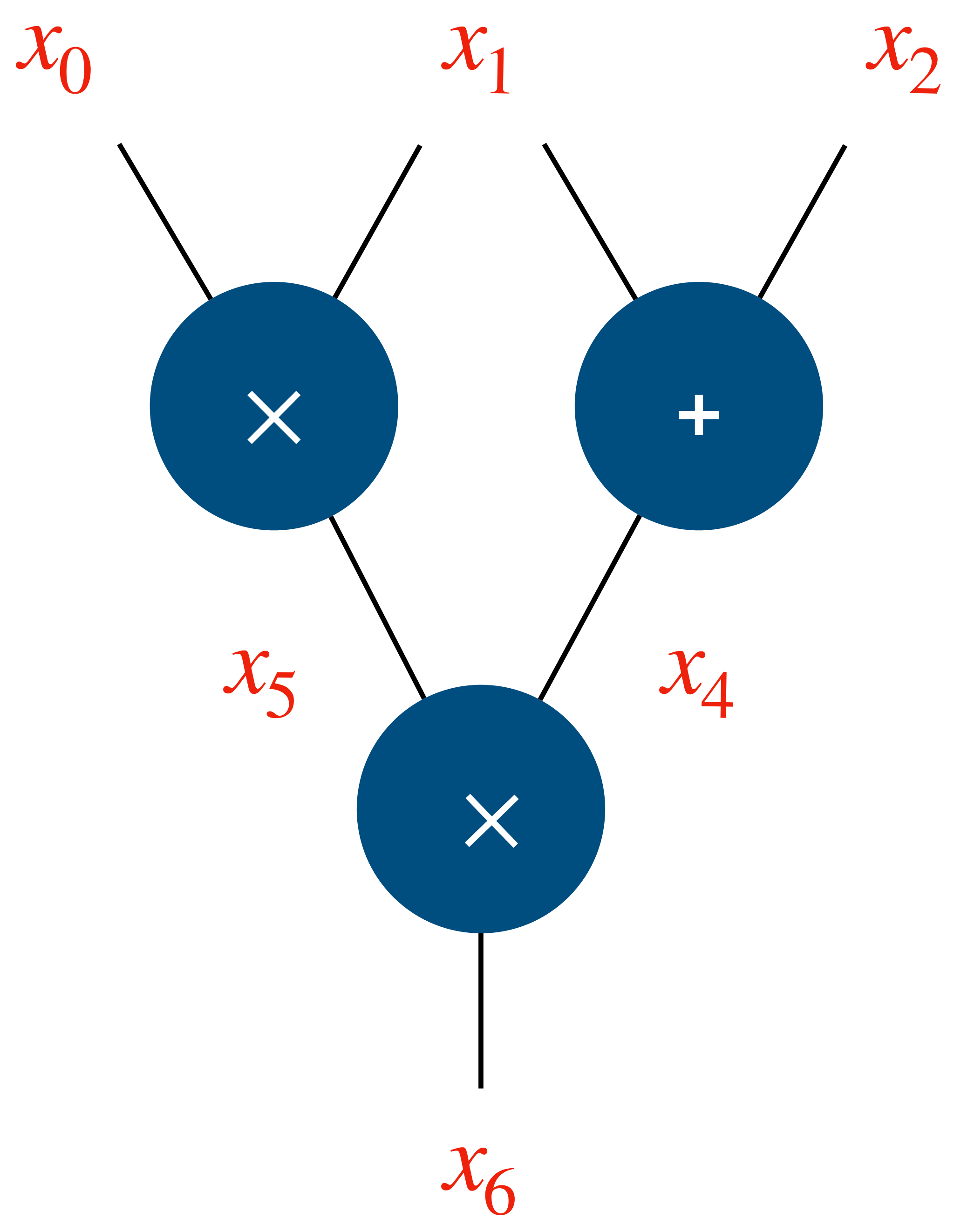
- Addition gate: MAC for $p(x, y) = a \cdot x + b \cdot y$



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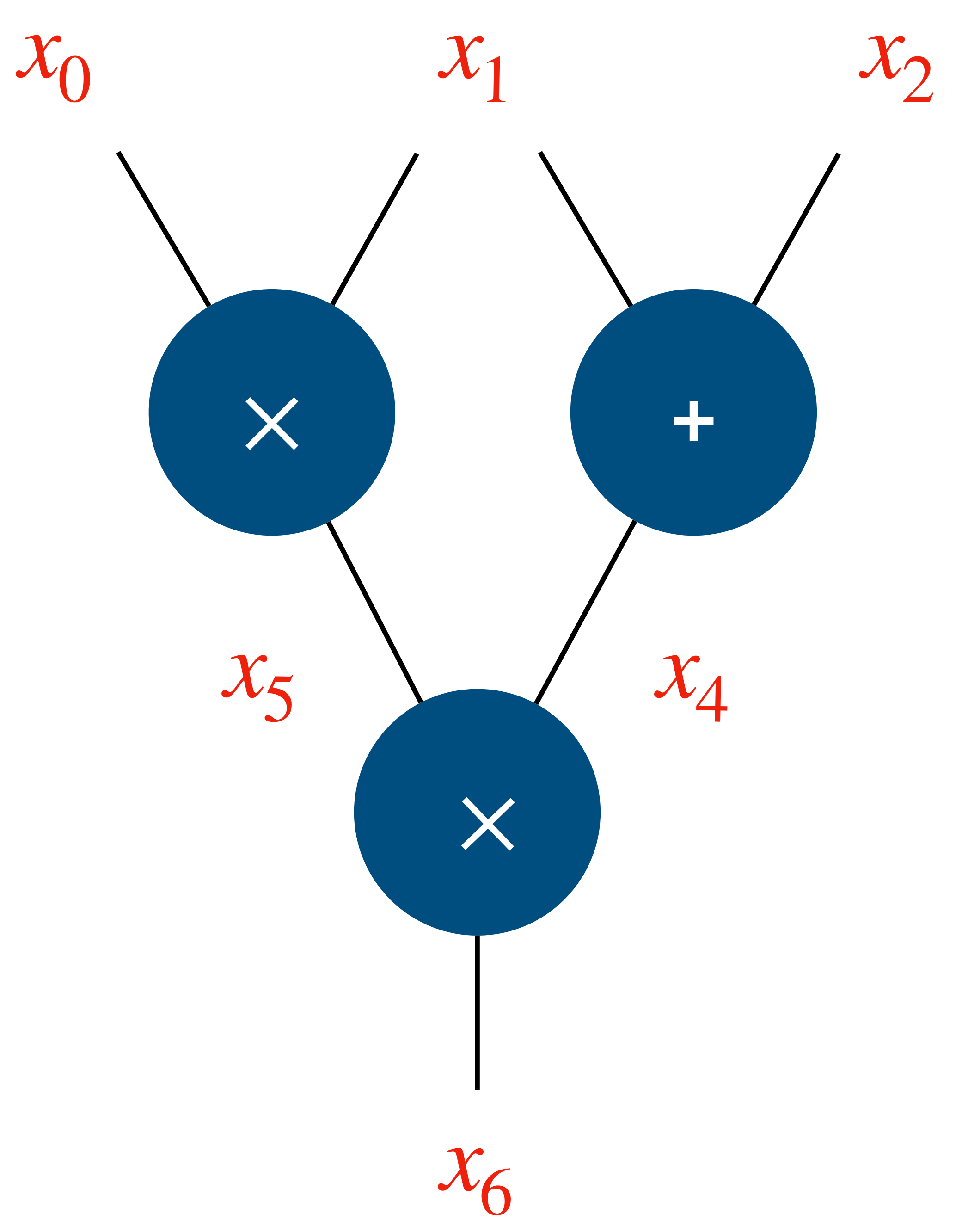

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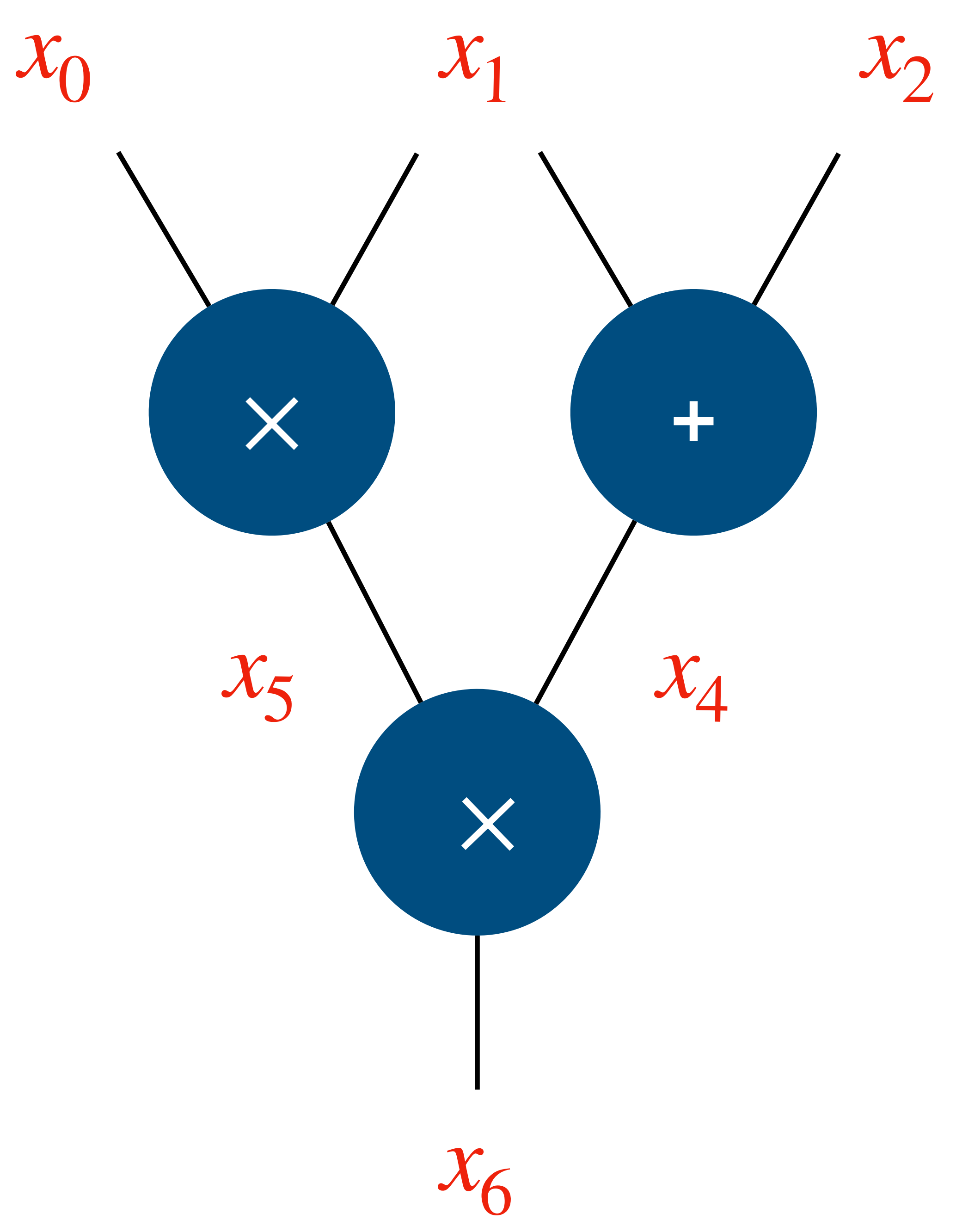

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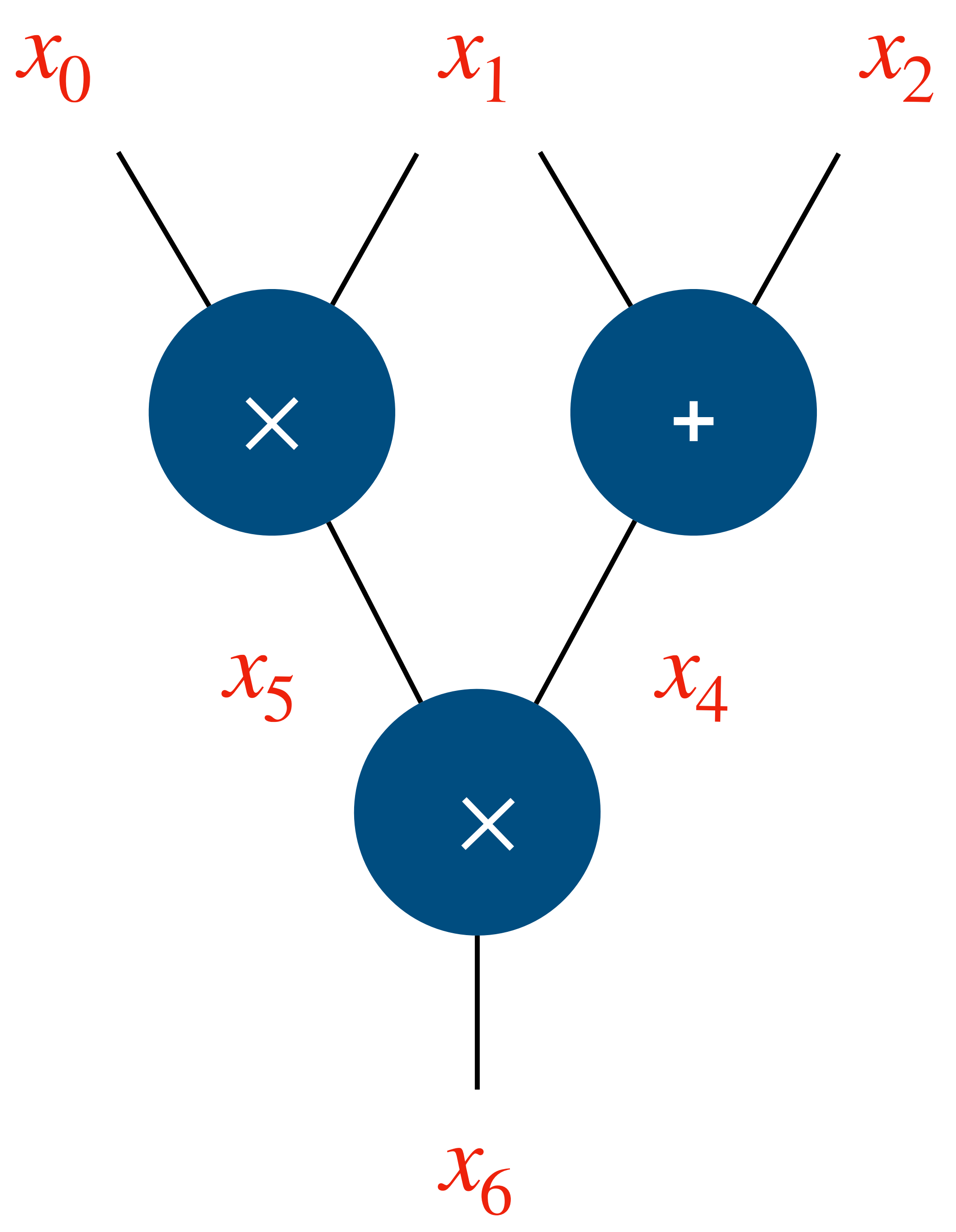
- Multiplication gate: x, y, z verify if $xy = z$ holds



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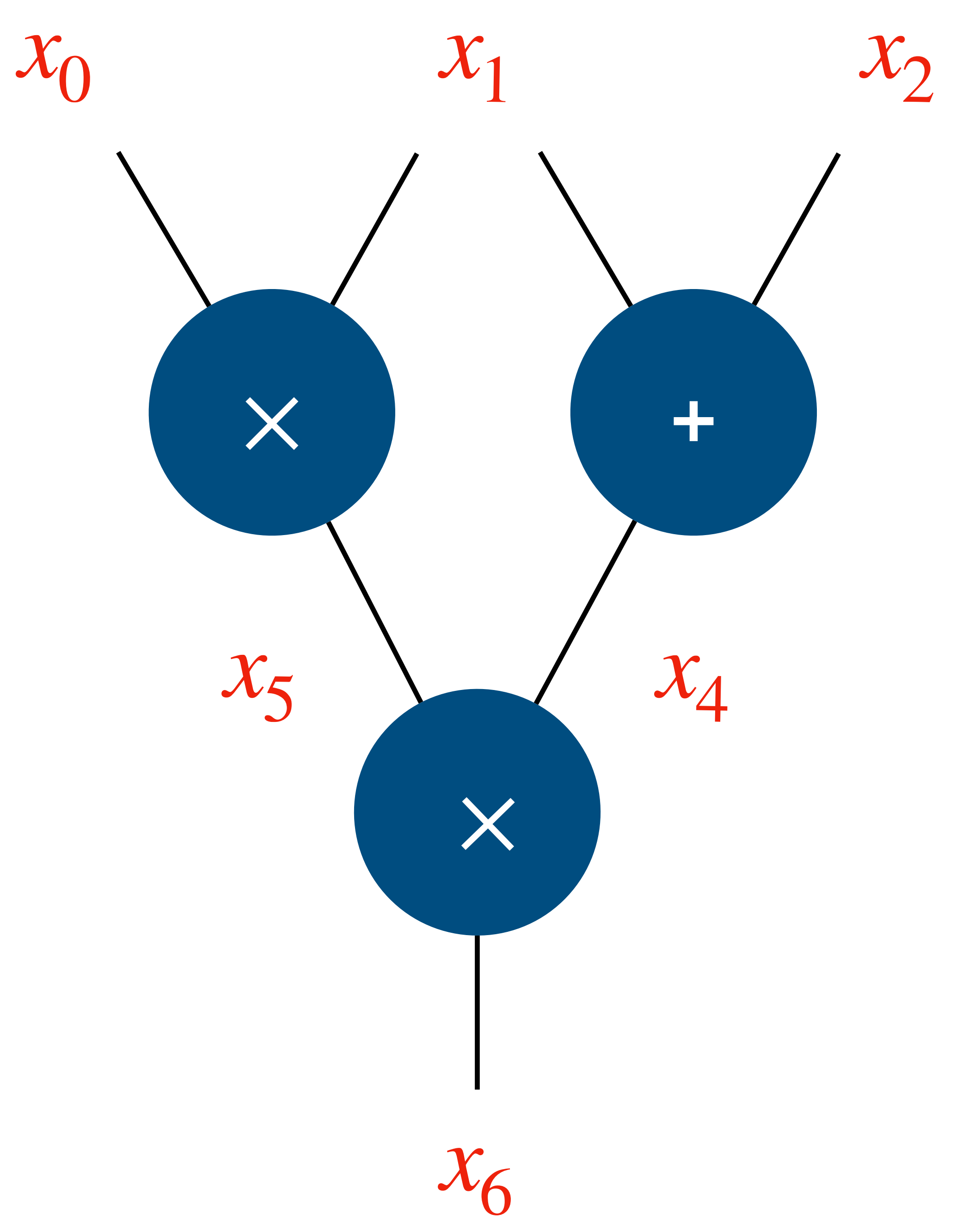

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- Multiplication gate: x, y, z verify if $xy = z$ holds
    - With small probability $k_x \cdot k_y - \Delta \cdot k_z = M_x \cdot M_y + (M_x \cdot y + M_y \cdot x - M_z) \cdot \Delta$ holds

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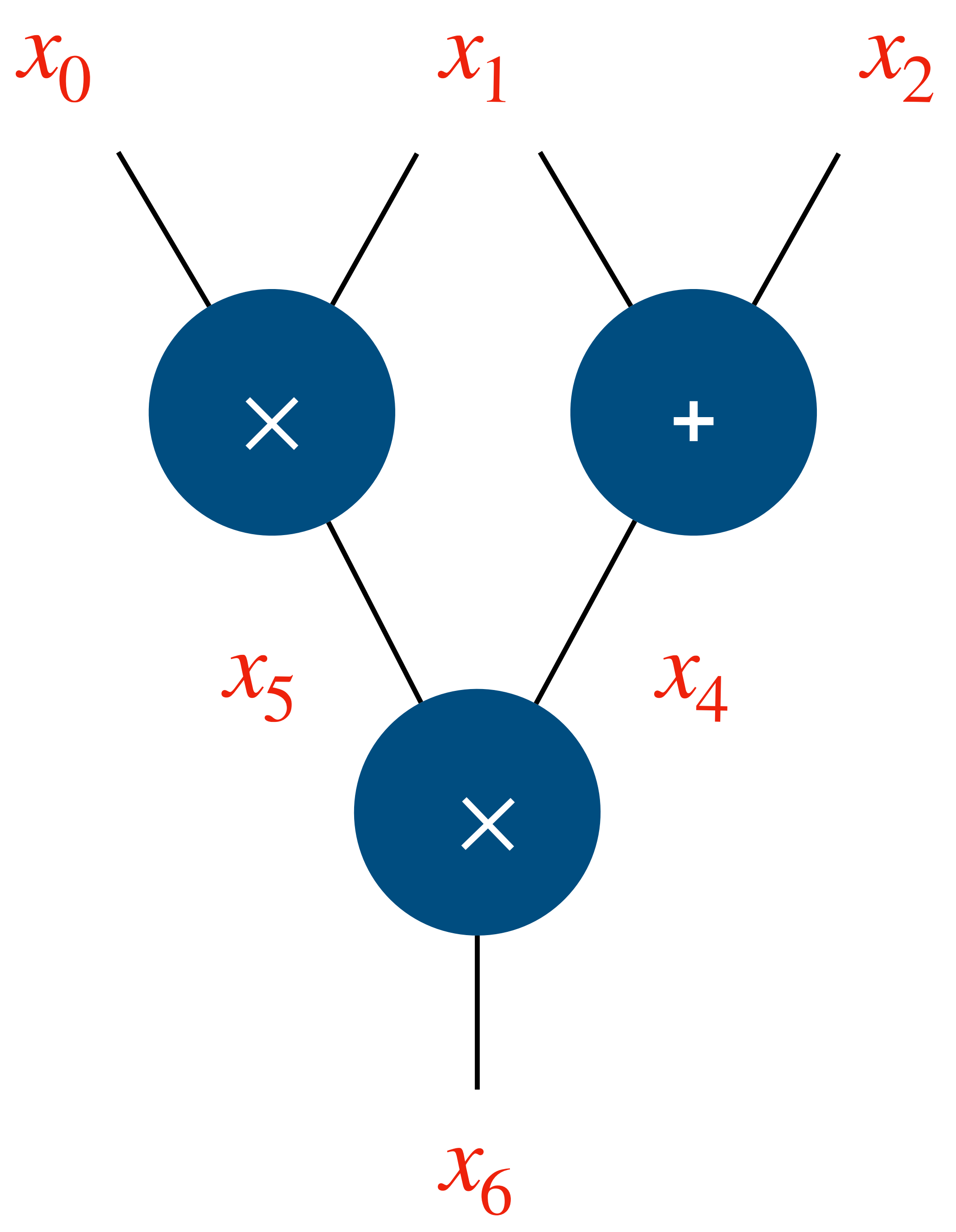

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  - Batching for multiple multiplication gates