Aggregative Efficiency of Bayesian Learning in Networks

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Golub and Sadler (2016): "A significant gap in our knowledge concerns short-run dynamics and rates of learning in these models."

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- express learning rate as simple function of network parameters
- extent of info loss: under a symmetry condition, learning aggregates **no more than 2 signals per gen** asymptotically

Related Social-Learning Literature

Sequential learning: Banerjee (1992), Bikhchandani, Hirshleifer, Welch (1992)

Network structure and Bayesian social learning

- Network does not matter (within "reasonable" class) for long-run learning: Acemoglu, Dahleh, Lobel, and Ozdaglar (2011), Lobel and Sadler (2015), Rosenberg and Vieille (2019)
- Examples and simulations: Sgroi (2002), Lobel, Acemoglu, Dahleh, and Ozdaglar (2009), Arieli and Mueller-Frank (2019)
- Adoption Dynamics: Board and Meyer-ter-Vehn (2021)
- This paper: **analytic ranking of networks on rate of learning**

Other obstructions to the efficient learning rate

- Coarse action space: Harel, Mossel, Strack, Tamuz (2020), Rosenberg and Vieille (2019), Hann-Caruthers, Martynov, Tamuz (2018)
- Endogenous info: Burguet and Vives (2000), Mueller-Frank and Pai (2016), Ali (2018), Lomys (2020), Liang and Mu (2020)
- This paper: **network-based** obstructions to efficient learning

Speed of learning under heuristics: Ellison and Fudenberg (1993), Golub and Jackson (2012), Molavi, Tahbaz-Salehi, Jadbabaie (2018). This paper: **rational learning**

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- Neighborhoods $N(i)$ are common knowledge
- Agents are Bayesian and choose optimal actions (given observations and predecessors' play)

Log-Linearity of Actions

WLOG apply log-transformations and work with log-likelihoods

- **Log-signal** $\lambda_i := \ln\left(\frac{\mathbb{P}[\omega=1|s_i]}{\mathbb{P}[\omega=0|s_i]}\right)$, **log-actions** $\ell_i := \ln\left(\frac{a_i}{1-a_i}\right)$
- These changes are 1-to-1, so there is a (unique) map from i's log-signal and neighbors' log-actions to i's optimal log-action
- Our first result says this map is linear

Proposition 1

For each agent i with $N(i) = \{j(1), ..., j(n_i)\}$, there exist constants $(\beta_{i,j(k)})_{k=1}^{n_i}$ s.t.

$$\ell_i^* = \lambda_i + \sum_{k=1}^{n_i} \beta_{i,j(k)} \ell_{j(k)}^*.$$

- Proof gives explicit formula for coefficients $\beta_{i,j(k)}$

Signal-Counting Interpretation of Accuracy

If i's only info is $n \in \mathbb{N}_+$ indep signals, $\ell_i \sim \mathcal{N}\left(\pm n \cdot \frac{2}{\sigma^2}, n \cdot \frac{4}{\sigma^2}\right)$

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Proposition 2

There exist $(r_i)_{i \geq 1}$ *so that social learning aggregates* r_i *signals by agent* i*. These* $(r_i)_{i \geq 1}$ *depend on the network, but not on* σ^2*.*

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- Can measure each i's accuracy in units of private signals

An Example of Informational Confound

Agent 1

$s_1 = 0.50$

$a_1 = 0.73$

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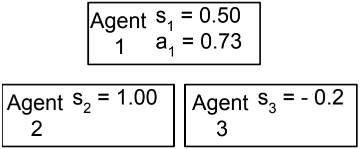

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Agent 1: $s_1 = 0.50$, $a_1 = 0.73$

Agent 2: $s_2 = 1.00$, $a_2 = 0.88$

Agent 3: $s_3 = -0.2$, $a_3 = 0.40$

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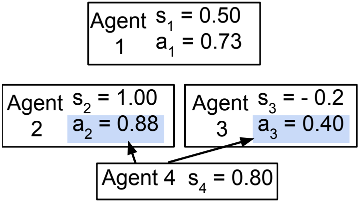

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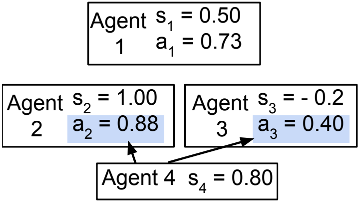

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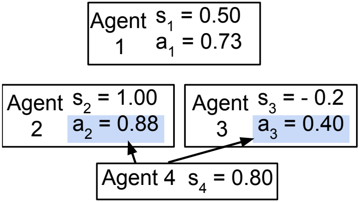

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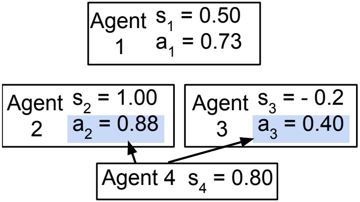

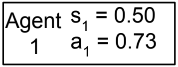

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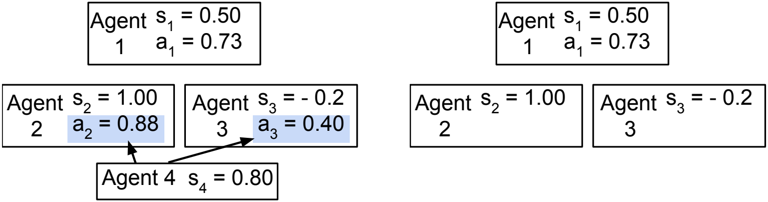

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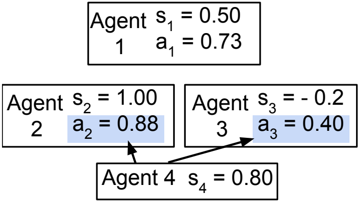

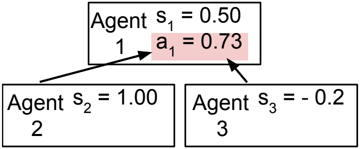

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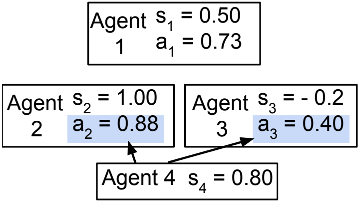

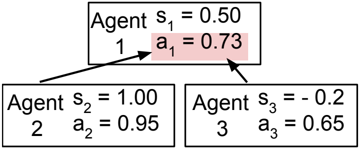

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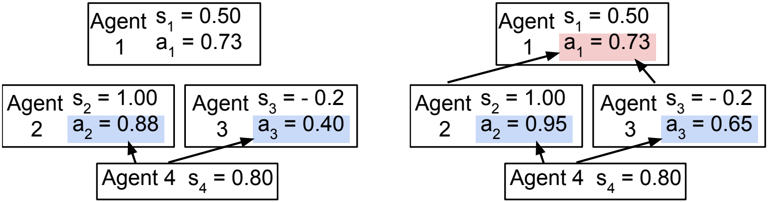

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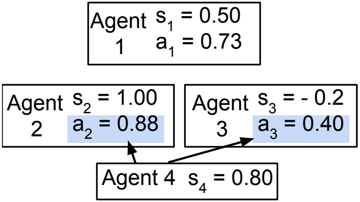

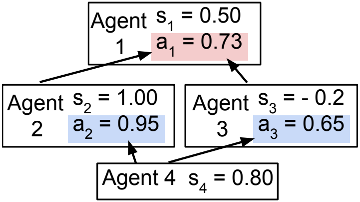

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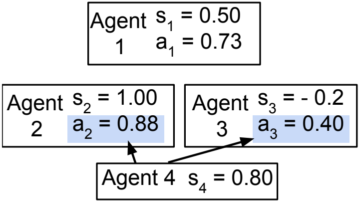

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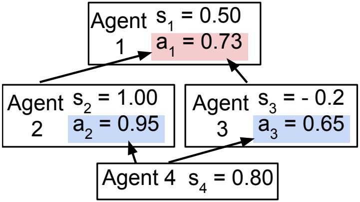

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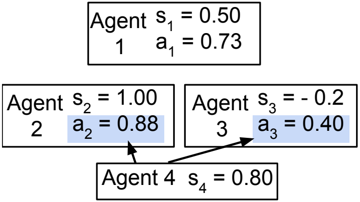

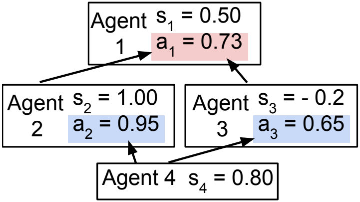

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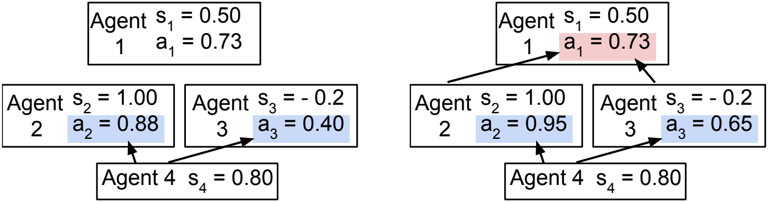

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- With optimal signal extraction, $r_4 =$ "$3.\overline{66}$ signals" (to be formalized soon)

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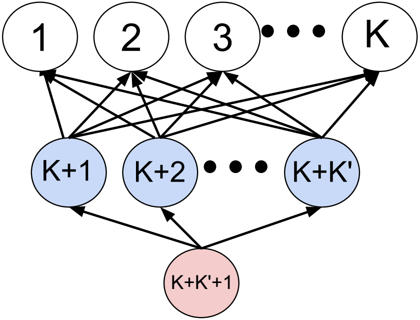

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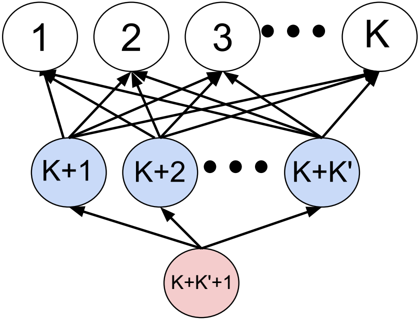

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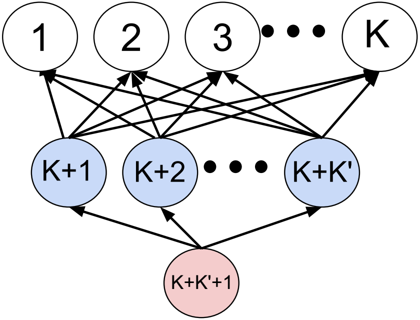

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- Little change in accuracy–even if K small, so little confounding information!
  - Even if K' is also large–many new signals in generation 2, but almost all information lost

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Proposition 3

Society learns completely in the long run if and only if

$$\lim_{i \to \infty} \left[\max_{j \in N(i)} j \right] = \infty.$$

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- Mild and clearly necessary: else for some $C < \infty$, infinitely many i cannot access the signal of any $j > C$ except their own
- Satisfied in all "reasonable" networks, not useful for ranking

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- Rest of the talk: compare networks for social learning by comparing their aggregative efficiency

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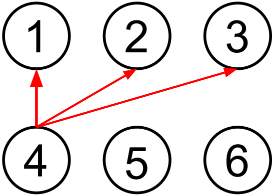

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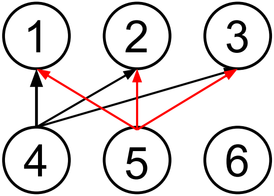

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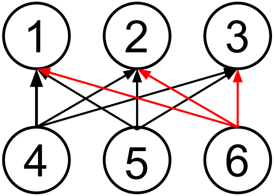

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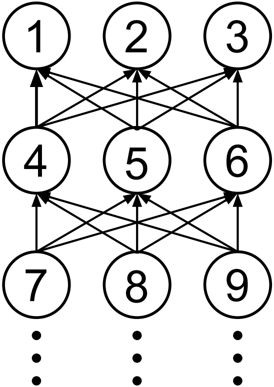

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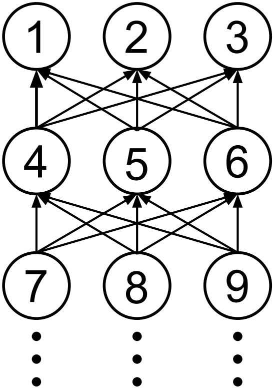

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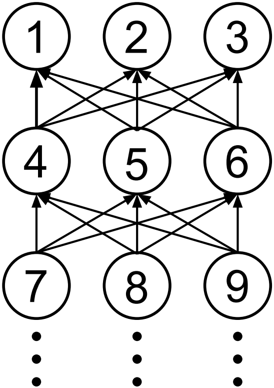

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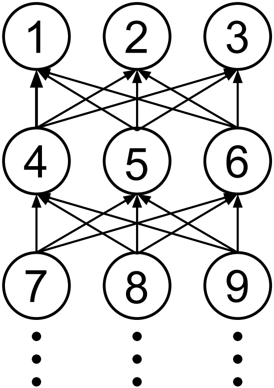

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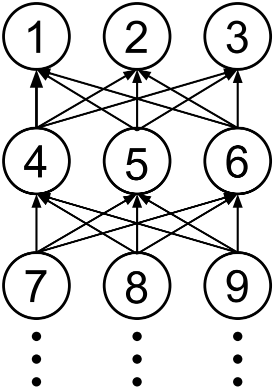

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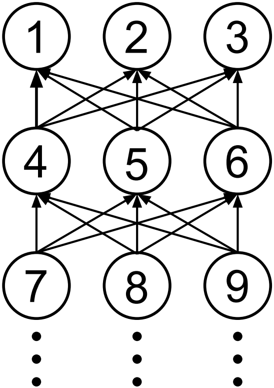

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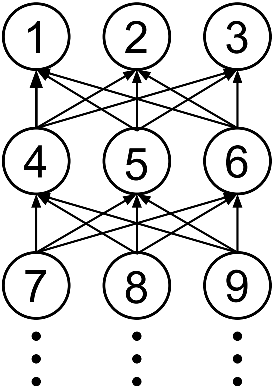

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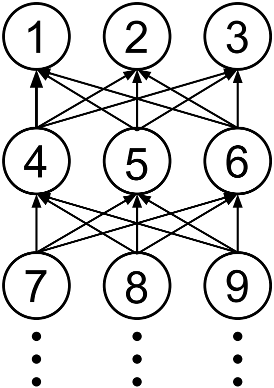

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- *After generation 2, social learning aggregates fewer than 3 signals per generation with any K*

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- For K large, aggregate only an unboundedly small fraction of the private signals

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- When generations are large, optimal action severely underweights private signals from generation t
  - Will see later that total weight on private signals from the generation t is close to 1 for t large
  - Without confounding, would place weight 1 on **each** private signal from generation t

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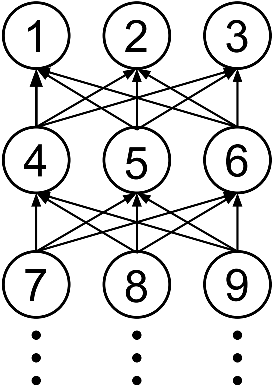
2. Per-generation rate of learning is faster with larger generations:
   - On the other hand larger $K \Rightarrow$ more learning per generation
   - But differences are small, and per-generation rate of learning is bounded above by 2 signals

Which Network Leads to Faster Learning?

Network A



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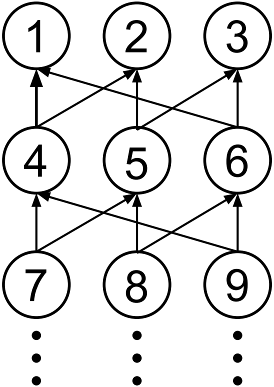

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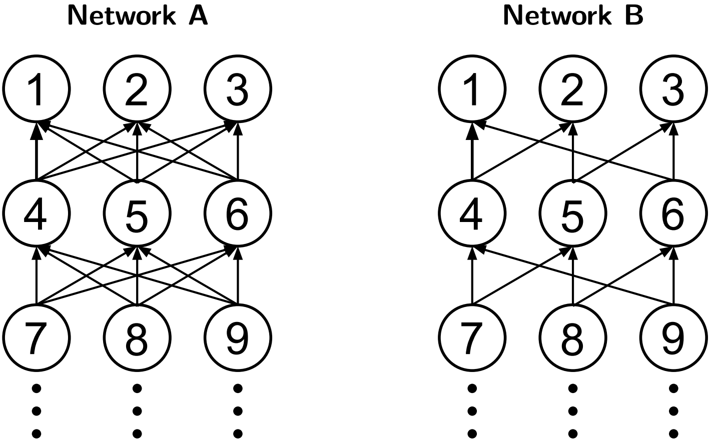

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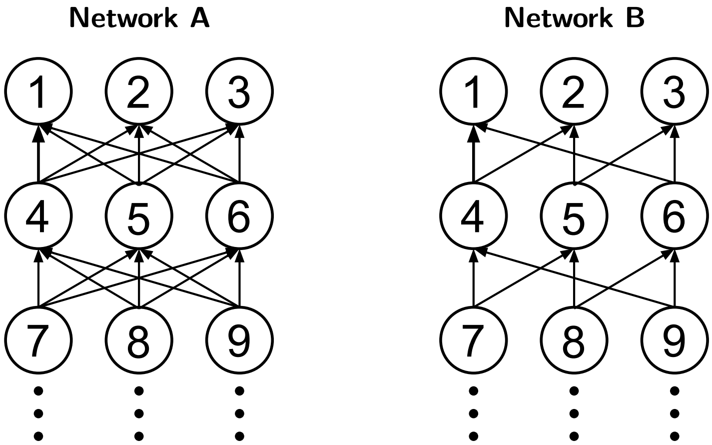

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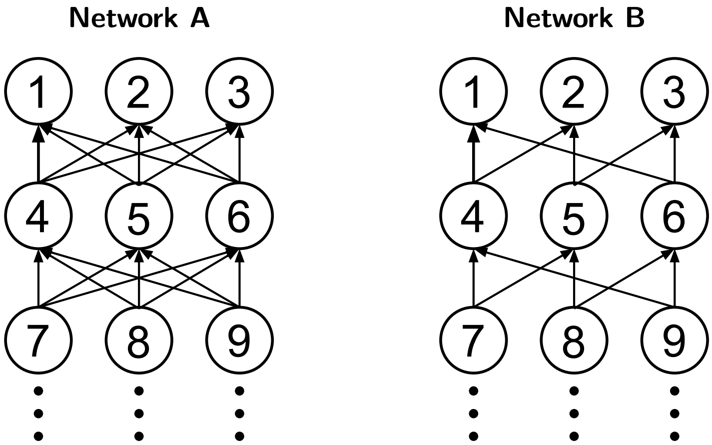

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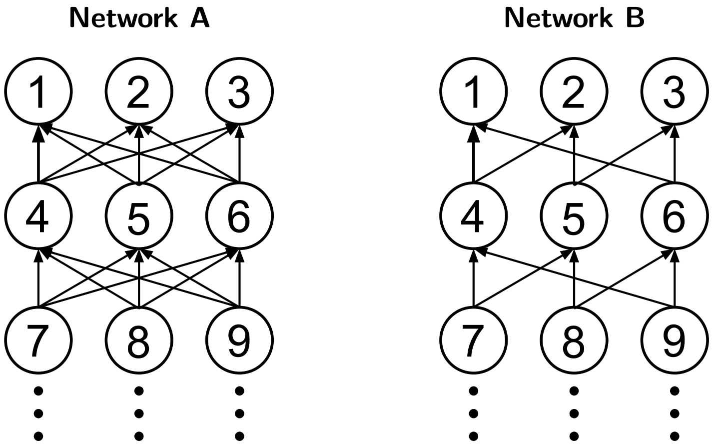

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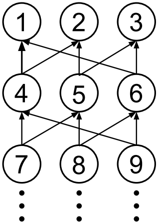

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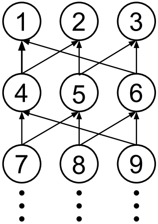

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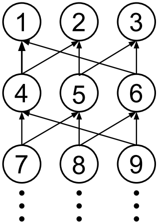

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For example, "**Network B**" is symmetric with $d = 2$, $c = 1$

Speed of Learning with Partial Observations

- Each agent observes d neighbors and pairs of agents in the same generation have c common neighbors

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In symmetric generations networks,

$$\lim_{i \to \infty} (r_i/i) = \left(1 + \frac{d^2 - d}{d^2 - d + c}\right) \frac{1}{K}.$$

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- Theorem 1 shows asymptotic bound of 2 signals per gen applies to **all** such networks, strengthening Proposition 4
- Term in parenthesis increases in d and decreases in c — more obs speeds up rate of learning per gen but more confounding slows it down, all else equal

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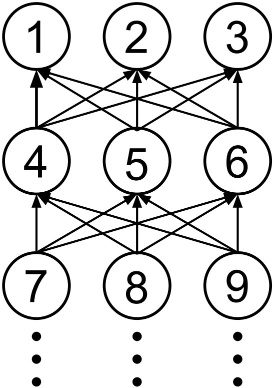

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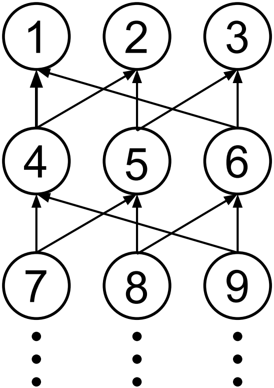

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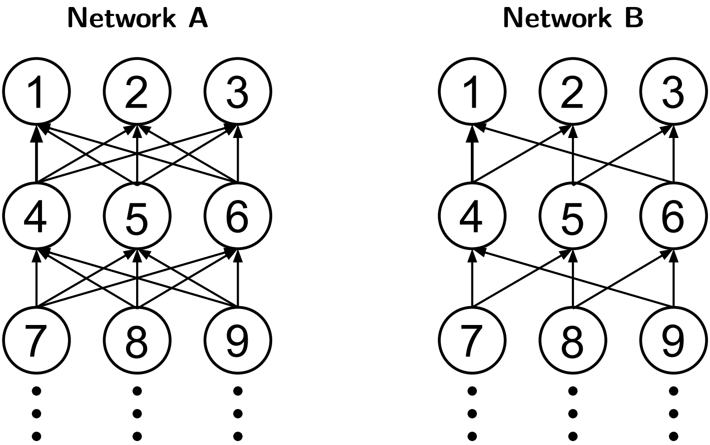

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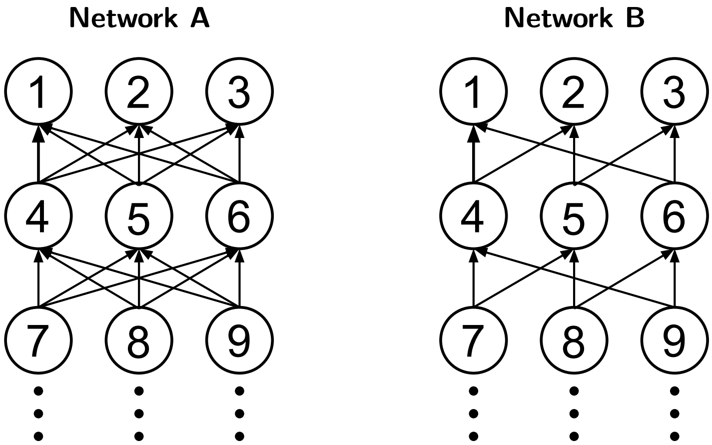

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- Extra social obs exactly cancel out reduced info content of each obs Conclusion

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- Suggests approach for policy to correct inefficiencies
- With much more weight on private signals, can attain aggregative efficiency of 1

Summary

- A tractable model of rational sequential learning that focuses on how the social network affects aggregative efficiency
- Generally, network confounds info content of neighbors' behavior and leads to info loss
- Exact aggregative efficiency in all generations networks with symmetric observation sets
- Significant info loss due to confounding: in any such network, each generation eventually aggregates no more than 2 signals
- Tractability of framework extends beyond generations networks, e.g., canonical random networks

Thank you!

Proof Sketch of Theorem 1

Using symmetry, can show there exist numbers Var_t, Cov_t, β_t s.t.

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Will sketch proof of key lemma:

Lemma 1

$$\lim_{t \to \infty} \beta_t = 1/d.$$

$\beta_t \to 1/d$ Using a Mixing Argument

- Showing $\beta_t \to 1/d$ amounts to showing $\text{Corr}(\ell_i, \ell_{i'} \mid \omega) \to 1$ for $i \neq i'$ in generation t, as $t \to \infty$
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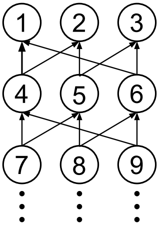
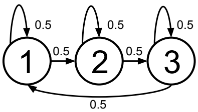


induces the stochastic process



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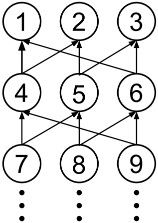

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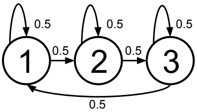

- Markov chain mixing theorem implies steady-state distribution that does not depend on starting state $\Rightarrow$ number of paths to distant j almost independent of i Back

Calculating Aggregative Efficiency

- Expressions for Var_{t+1} and Cov_{t+1} give

$$\mathsf{Var}_{t+1} - \mathsf{Cov}_{t+1} = \frac{4}{\sigma^2} + \beta_{t+1}^2 (d - c)(\mathsf{Var}_t - \mathsf{Cov}_t)$$

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- From this fixed point, can compute the growth rate of Var_t and therefore the aggregative efficiency Back

Finite Populations

Proposition 7

Let $\epsilon > 0$. There exists a constant $C > 0$ such that for any symmetric generations network and any generation $t \geq CK\log(K)$, at most $K\lim_i(r_i/i) + \epsilon$ signals are aggregated between generations t and $t+1$.

- Gives an upper bound on how long it takes for Theorem 1 to apply

Simulation: Observing Multiple Past Generations

Each agent observes all predecessors from past $\tau \geq 1$ generations

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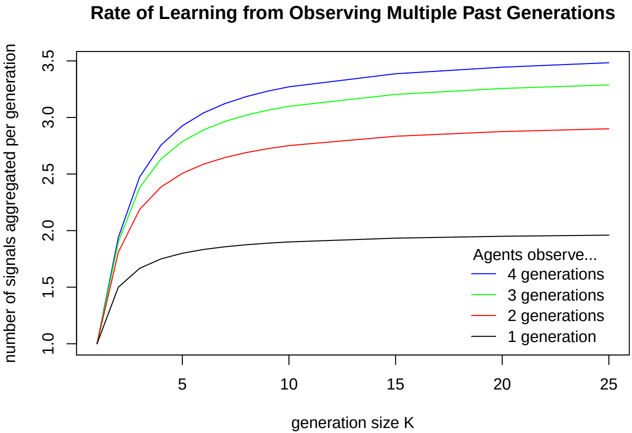

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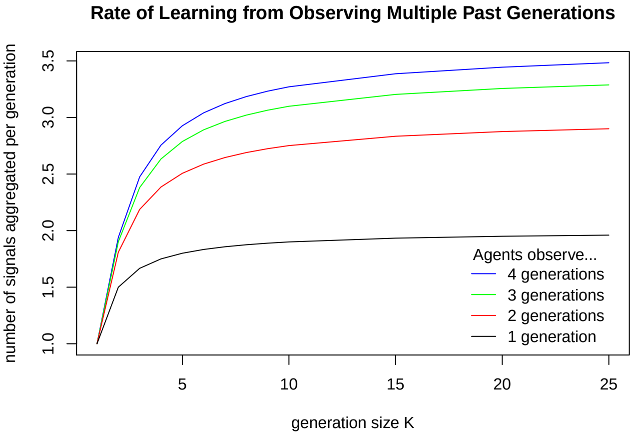
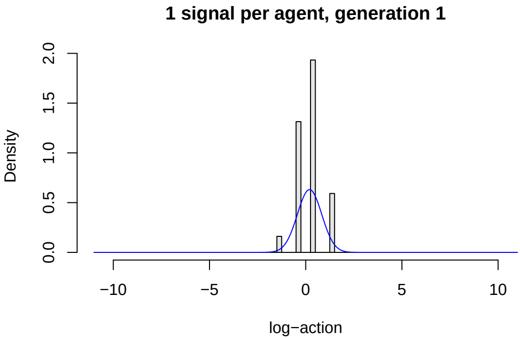


- Limited improvement in aggregative efficiency: removes some confounds but creates new ones

Simulation: General Signal Structures

- Each signal is finitely supported



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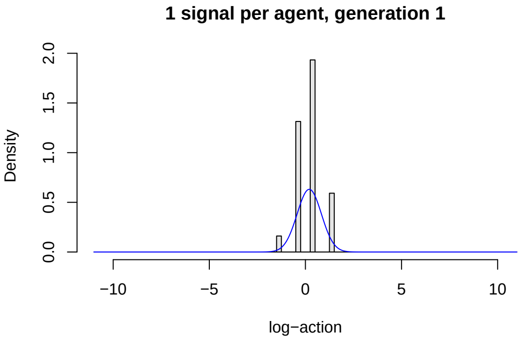

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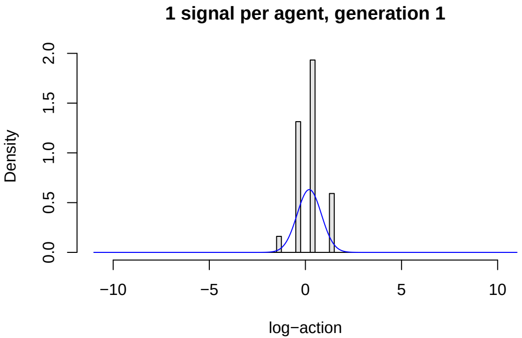

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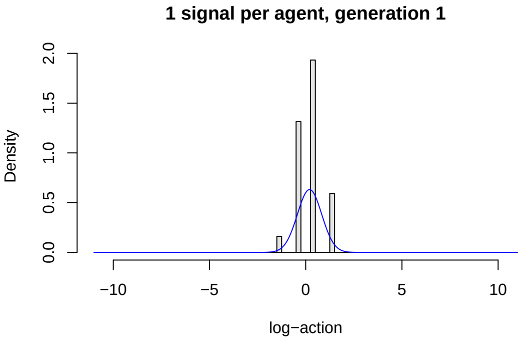

- Each agent has not 1, but n conditionally i.i.d. signals
- Think of agents who gather info over a period of time
- Increase n and scale down informativeness of each signal, fixing mean and SD of private log-belief (based on all n private signals) to match the Gaussian case