Experimentation on Networks

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Motivation

Discovery and diffusion of innovations

- Farmers learn about new crop from neighbors (Griliches)
- Doctors learn about new drug from colleagues (Coleman et al)

Discovery and diffusion are tightly intertwined

- Both necessary for sustained progress (Mokyr)
- But social info crowds out acquisition (Grossman-Stiglitz)

Research questions

- How does social info crowd out private experimentation?
- How does network density impact aggregate learning?
- How does network architecture impact social welfare?

Contribution

Canonical model of strategic experimentation

- Agents learn from own and neighbors' successes.
- Widely used class of networks (clique, trees, core-periphery).
- Agents forward-looking and fully Bayesian.

Takeaways

- Asymptotic learning decreasing in network density.
- Welfare single-peaked in network density.
- Compare learning dynamics and welfare across trees.

Some literature

- Bala, Goyal (98), Rosenberg, Solan, Vieille (09), Sahlish (15)
- Keller, Rady, Cripps (05), Bonatti, Horner (11, 17)

Model

Network

- Agents $i = 1, ..., I$ in (possibly) random network G.
- Realized network g; neighbors $N_i(g)$.

Learning Game

- State $\theta \in \{L, H\}$ with prior $\Pr(H) = p_0$.
- Private effort $A_{i,t} \in [0, 1]$ at $t \in [0, \infty)$ at flow cost c.
- Successes at $\{T_i^\iota\}$ with rate $A_{i,t}\mathbb{I}_{\{\theta=H\}}$; payoff $x > c$ to i .
- Observe own and neighbors' successes, and G but not g.

Agent i's problem

$$V_i = \max_{\{A_{i,t}\}_{t\geq 0}} E\left[x \sum_{\iota=1}^{\infty} e^{-rT_i^\iota} - c \int_0^\infty e^{-rt} A_{i,t} dt\right]$$

Best Responses

The Experimentation Problem

- First success times: T_i for agent i, and S_i for her neighbors.
- Continuation value after success: $y = (x - c)/r$.
- Before T_i, S_i, *experimentation* $a_t^{\emptyset}$ and *social learning*

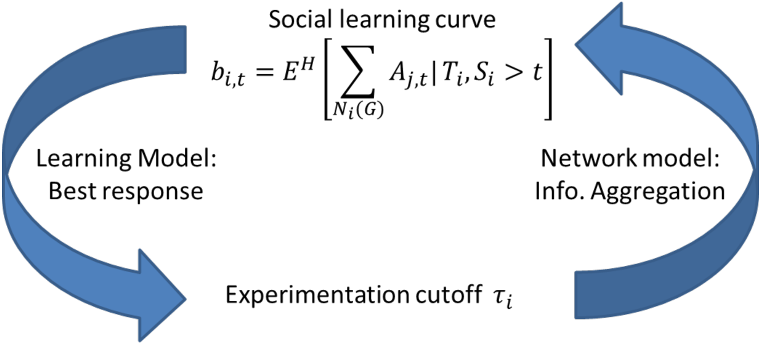
$$b_{i,t} := E^H \left[\sum_{j \in N_i(G)} A_{j,t} \middle| T_i, S_i > t \right]$$

Cutoff Strategies

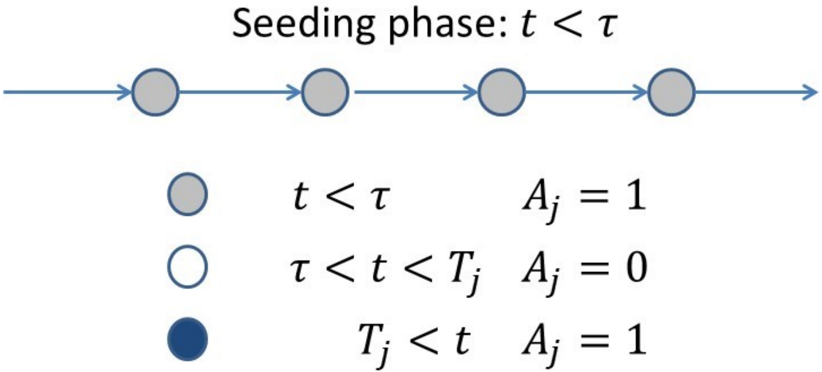
- A cutoff strategy $a_{i,t}^{\emptyset} = \mathbb{I}_{\{t \leq \tau_i\}}$ is uniquely optimal.
- At optimal cutoff τ_i, experimentation incentives vanish:

$$\psi_{i,\tau_i} := p_{i,\tau_i} \left(x + y - \frac{\beta_{i,\tau_i}}{\beta_{i,\tau_i} + r} y \right) - c = 0$$

Illustration of Equilibrium Analysis



Example: The Infinite Directed Line



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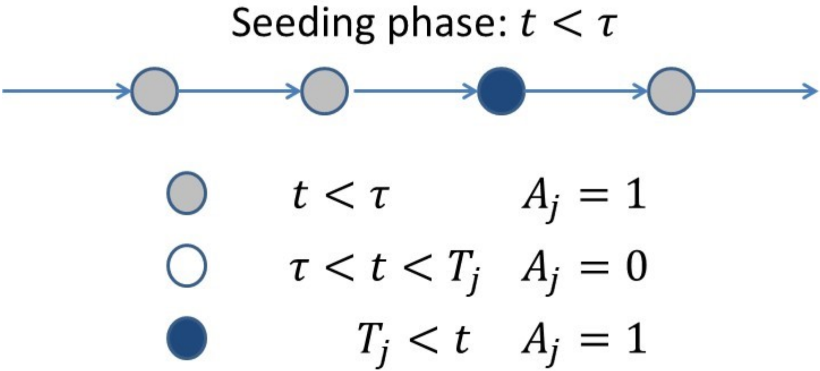

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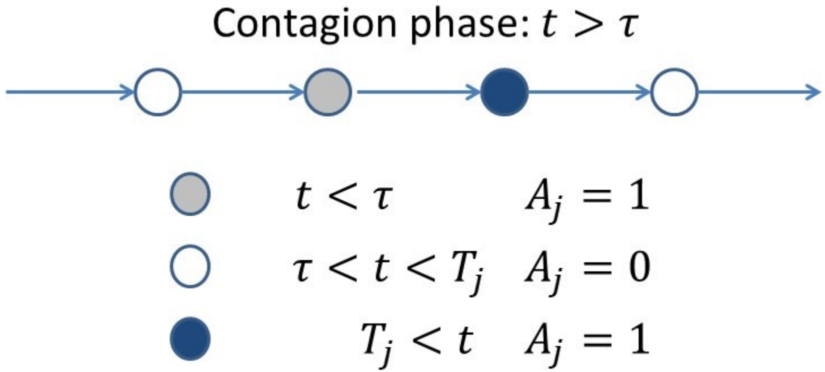

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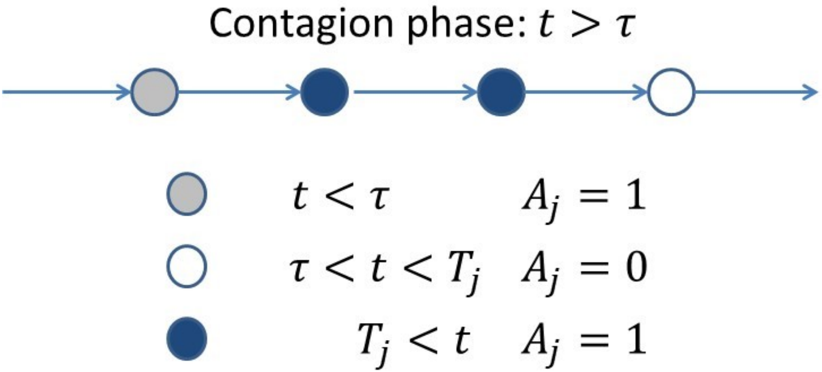

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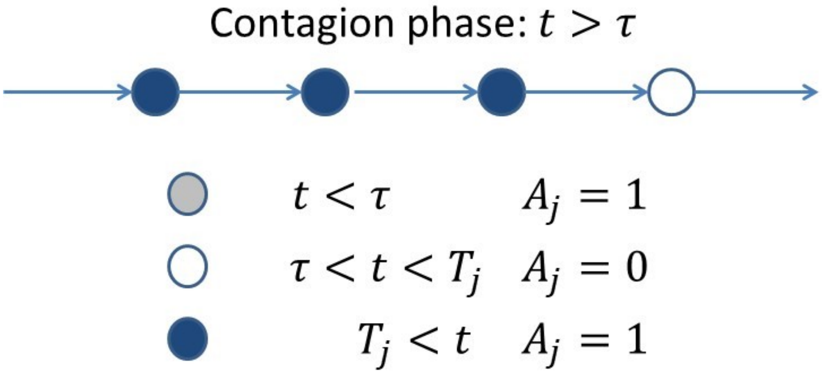

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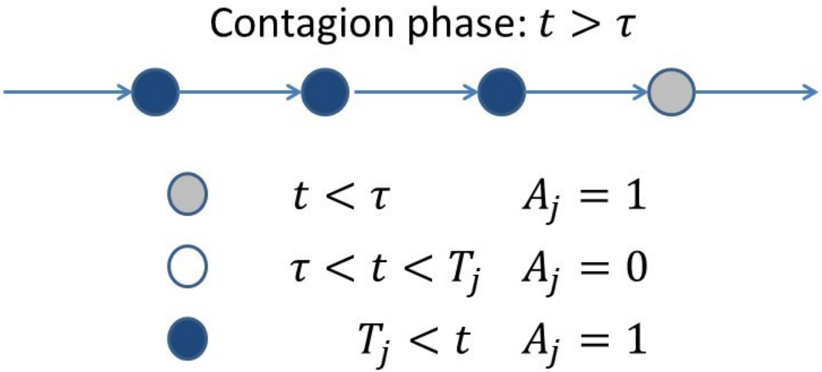

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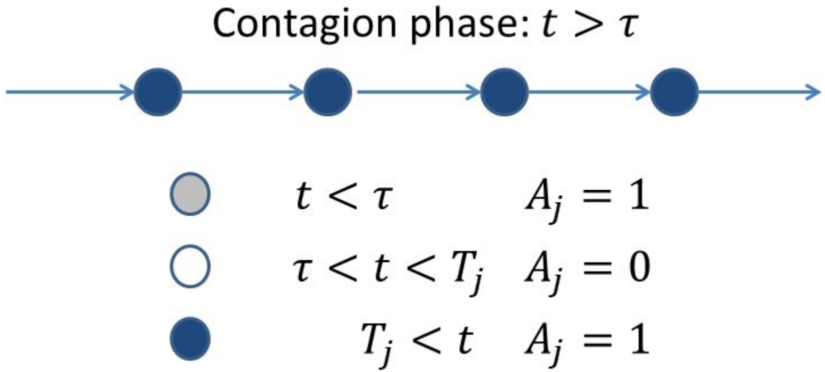


Equilibrium in the Infinite Directed Line

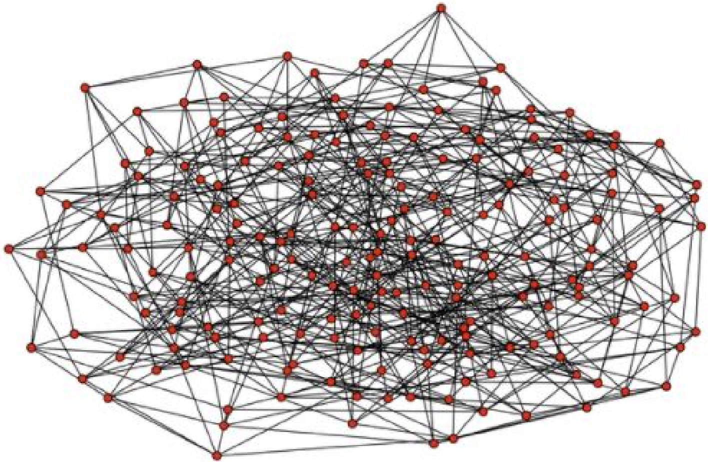
- Assume symmetric cutoffs τ, and focus on $t > \tau$
- Expected effort of i's neighbor j

$$a_t = E^H[A_{j,t}|t < T_i, S_i] = 1 - \Pr^H(t < T_k|t < T_j) = 1 - e^{-\tau}$$

Equilibrium

$$P^{\emptyset}(2\tau)\left(x + \frac{r}{r + (1 - e^{-\tau})}y\right) = c$$

Illustration: Regular Random Network $I = 200, n = 6$



Large Regular Random Networks

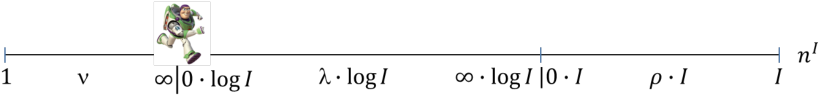
The Configuration Model

- I agents with n^I link stubs.
- Randomly connect pairs of stubs into undirected links.
- Unique equilibrium is symmetric; cutoff τ^I and value V^I.

Sequences of networks $\{n^I\}$ as $I \to \infty$

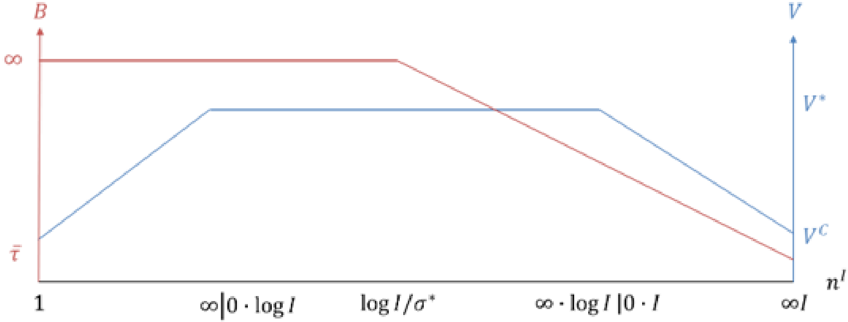
- Limit information $B := \lim \int_0^\infty b_t^I$ and welfare $V := \lim V^I$.
- Network density measures

$$\nu := \lim n^I \qquad \lambda := \lim(n^I / \log I) \qquad \rho := 1 - e^{-\lim(n^I/I)}$$



Learning and Welfare as function of Density

Theorem 1.
Information B falls in density; attains ∞ iff $\lambda \leq 1/\sigma^*$ and $\rho = 0$.
Welfare V is hump-shaped; attains V^* iff $\nu = \infty$ and $\rho = 0$.



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Indifference when learning state at time-σ^*

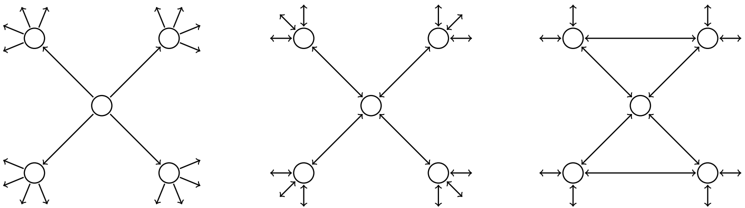
$$p_0 \left(x + (1 - e^{-r\sigma^*}) y \right) = c \qquad \text{(or } \sigma^* = 0 \text{ if } p_0 \geq \bar{p}\text{)}$$

Tension between Learning and Welfare: Network must be...

- ... sparse to sustain information generation.
- ... dense for fast diffusion and welfare benchmark.

(Infinite Regular) Trees $\mathcal{T}^{(n)}$

Directed, $\vec{\mathcal{T}}^{(n)}$ **Undirected, $\bar{\mathcal{T}}^{(n)}$** **Triangle, $\hat{\mathcal{T}}^{(n)}$**



Motivation

- Directed (Twitter), undirected (LinkedIn), cluster (Facebook)
- Approximate large random network (Sadler '20, BMtV '21)
- Highly tractable because of independence across neighbors

How does social learning depend on type of tree?

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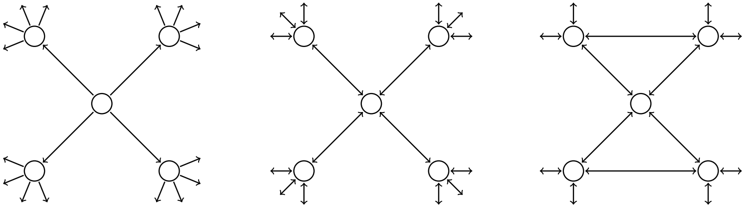

Theorem 3.

$$\begin{aligned} \vec{V}^{(n)} &> \bar{V}^{(n)} > \hat{V}^{(n)} \\ \vec{V}^{(n)} &< \bar{V}^{(n+1)} < \hat{V}^{(n+2)} \end{aligned}$$

Conclusion

An equilibrium model of experimentation on networks

- Forward-looking, fully Bayesian agents.
- Perfect good news learning lead to cutoff strategies.

Findings

- Asymptotic information decreases in network density.
- Welfare single-peaked in network density.
- Compare learning dynamics and welfare across trees.

Check out the paper for...

- Core-periphery networks
- Learning dynamics