

# Learning in Repeated Interactions on Networks

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# Motivation and Question

- People learn by interacting with others.
- E.g., what is the most healthy diet?
- You observe your friends' past choices and pick your diet today.
- This influences by your friends' future choices.
- Repeated interactions are hard to model tractably.
- This paper:
  - constant stream of information
  - social network
  - repeated interactions
  - rational framework
- Question: How efficient is the aggregation of information?

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# Model

- A fixed binary state of the world:  $\Theta \in \{\mathbf{g}, \mathbf{b}\}$  with a uniform prior.
- A finite set of agents:  $i \in N = \{1, 2, \dots, n\}$ .
- Discrete time:  $t \in \{1, 2, \dots\}$ .
- Social network:  $i$ 's network neighbors are  $N_i \subseteq N$ .
- Strongly connected network: observation path from each  $i$  to each  $j$ .
- Private signal conditional distributions  $\mu_{\mathbf{g}}, \mu_{\mathbf{b}} \in \Delta(\Omega)$ , finite  $\Omega$ .
- In every period  $t$ , agent  $i$ 
  - observes the past actions of her neighbors  $H_t^i = (a_s^j)_{s < t, j \in N_i}$ ;
  - receives a conditionally independent private signal;
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  - gets payoff function  $u(a_t^i, \Theta) = \mathbb{1}(a_t^i = \Theta)$ .
- Agent  $i$  maximizes expected discounted utility:

$$(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}[\mathbb{1}(a_t^i = \Theta)]$$

- Solution concept: Nash equilibrium.

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# Model

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- Agents do not observe payoffs, only signals and neighbors' actions.
- But can think of private signals as payoff + noise.
- Each agent learns the state in the long run.
- How fast do they learn?
- Do they even learn at the same speed?

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# Learning Speed

- What is the probability of mistake  $\mathbb{P}[a_t^i \neq \Theta]$ ?
- Explicit calculation seems hopeless, even in special cases.
- Classical result in statistics and probability:

- Single agent:

$$\mathbb{P}[a_t \neq \Theta] \approx \exp(-r_1 \cdot t),$$

where  $r_1$  can be explicitly calculated given  $\mu_\Theta$ .

- $n$  agents + **public signals** / **optimal aggregation**:

$$\mathbb{P}[a_t^i \neq \Theta] \approx \exp(-nr_1 \cdot t).$$

- Define the **speed of learning** of agent  $i$  as the exponential rate at which she converges to the correct action:

$$r = \liminf_{t \rightarrow \infty} -\frac{1}{t} \log \mathbb{P}[a_t^i \neq \Theta].$$

- If this limit exists and is equal to  $r$ , then  $\mathbb{P}[a_t^i \neq \Theta] \approx \exp(-r \cdot t)$ .
- $r_1 \leq r \leq nr_1$ .
- Tractable. Asymptotic, no welfare implications.

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# Main Result

- For every pair of conditional signal distributions  $(\mu_g, \mu_b)$  define

$$M = 2 \sup_{\omega \in \Omega} \left| \log \frac{\mu_g(\omega)}{\mu_b(\omega)} \right| \quad (1)$$

This is twice the maximal log-likelihood ratio induced by any signal realization.

## Theorem 1

*Fix  $(\mu_g, \mu_b)$ . In any equilibrium, on any social network of any size, and for any discount factor  $\delta \in [0, 1)$ , the speed of learning is at most  $M$ .*

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# Example

- Symmetric binary signals:  $\mu_\theta(\theta) = 0.9$  for  $\theta \in \{\mathbf{g}, \mathbf{b}\}$ .
- a single agent's learning speed is  $r_1 \approx 0.5$ .
- $M = 2 \log \frac{0.9}{0.1} \approx 4.5$
- So 4.5 is an upper bound to the learning speed in networks of **any size**.
- 10 agents with public signals achieve a speed of learning  $> 4.5$ .
- 1,000,000 agents who observe their neighbors' past actions never learns faster than 10 agents who share their private signals.
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# Myopic and Strategic Behavior

- The posterior log-likelihood ratio is  $L_t^i := \log \frac{\mathbb{P}[\Theta = \mathbf{g} | s_1^i, \dots, s_t^i, H_t^i]}{\mathbb{P}[\Theta = \mathbf{b} | s_1^i, \dots, s_t^i, H_t^i]}$ .
- Myopically optimal action is

$$m_t^i := \arg \max_{a \in \mathcal{A}} \mathbb{P}[\Theta = a | s_1^i, \dots, s_t^i, H_t^i] = \begin{cases} \mathbf{g} & \text{if } L_t^i \geq 0, \\ \mathbf{b} & \text{if } L_t^i < 0. \end{cases}$$

- If  $\delta \in (0, 1)$ , there may be **strategic** incentive to **not** choose  $m_t^i$ .

## Lemma 1 (Eventually Myopic)

- (i) *In equilibrium, if for some time  $t$  it holds that  $|L_t^i| \geq -\log(1 - \delta)$  then  $a_t^i = m_t^i$ .*
- (ii) *There exists a random time  $T < \infty$  such that in equilibrium, **all** agents behave myopically after  $T$ , i.e.  $t \geq T \Rightarrow a_t^i = m_t^i$  for all  $i$  almost surely.*

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## Lemma 1 (Eventually Myopic)

- (i) *In equilibrium, if for some time  $t$  it holds that  $|L_t^i| \geq -\log(1 - \delta)$  then  $a_t^i = m_t^i$ .*
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# Common Learning Rate

## Lemma 2 (All Agents Learn at the Same Rate)

- (i) *If agent  $i$  can observe agent  $j$ , i.e.,  $i \in N_i$ , then in equilibrium  $i$  learns at a (weakly) higher rate, i.e.,  $r_i \geq r_j$ .*
  - (ii) *All agents learn at the same rate, i.e.,  $r_i = r_j$  for all  $i, j$ , in any strongly connected network.*
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# The Imitation Principle

- (i) **Imitation Principle for Myopic Agents** if  $i$  observes  $j$ , then  $i$ 's actions are not worse than  $j$ 's:

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# Proof sketch

## Theorem

*Fix  $(\mu_{\mathfrak{g}}, \mu_{\mathfrak{b}})$ . In any equilibrium, on any social network of any size, and for any discount factor  $\delta \in [0, 1)$ , the speed of learning is at most  $M$ .*

- Suppose speed of learning is  $r > M$ .
- Condition on  $\Theta = \mathfrak{g}$ .
- $a_t^j = \mathfrak{g}$  eventually.
- $\mathbb{P}[a_t^j \neq \mathfrak{g}] \approx e^{-rt}$ .
- Observing  $j$  provides a log-likelihood ratio

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  - who interact **repeatedly**.
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  - Measure of efficiency of information aggregation.
  - Asymptotic.
  - Tractable.
- Main result: The speed of learning is **bounded above** by a constant, which depends only on the private signal structure.
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  - Asymptotic.
  - Tractable.
- Main result: The speed of learning is **bounded above** by a constant, which depends only on the private signal structure.
- Aggregation is highly **inefficient**.

Thank You!

# Conclusion

- Social learning in a
  - **rational** setting,
  - on **general** networks,
  - with **forward-looking**/strategic agents
  - who interact **repeatedly**.
- **speed of learning**:
  - Measure of efficiency of information aggregation.
  - Asymptotic.
  - Tractable.
- Main result: The speed of learning is **bounded above** by a constant, which depends only on the private signal structure.
- Aggregation is highly **inefficient**.

Thank You!